JEE Main 2023 April 13 Shift 1 Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathmatics

Section-A

1.

$$\int_0^\infty \frac{6}{e^{3x}+6e^{2x}+11e^x+6}dx$$

(1) $\log_e\left(\frac{32}{27}\right)$

(2) $\log_e\left(\frac{256}{81}\right)$

(3) $\log_e\left(\frac{512}{81}\right)$

(4) $\log_e\left(\frac{64}{27}\right)$

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2. Among

(S1) $\lim_{n\to\infty}\frac{1}{n}(2+4+6+\cdots+2n)=1$

(S2) $\lim_{n\to\infty}\frac{1}{16}\left(1^{15}+2^{15}+3^{15}+\cdots+n^{15}\right)=\frac{1}{16}$

(1) Only (S1) is true

(2) Both (S1) and (S2) are true

(3) Both (S1) and (S2) are false

(4) Only (S2) is true

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3. The number of symmetric matrices of order 3, with all the entries from the set $\{0,1,2,3,4,5,6,7,8,9\}$, is:

(1) 10^9

(2) 10^6

(3) 9^{10}

(4) 6^{10}

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4. Let $\mathrm{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\mathbf{b}=3\hat{i}-2\hat{j}+7\hat{k}$, $\mathbf{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector d satisfies $\hat{d}\times\hat{b}=\hat{c}\times\hat{b}$ and $\hat{d}\cdot\hat{a}=24$, then $|\hat{d}|^2$ is equal to:

(1) 323

(2) 423

(3) 413

(4) 313

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5. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the

mean of X is:

(1) $\frac{21}{16}$

(2) $\frac{15}{16}$

(3) $\frac{81}{64}$

(4) $\frac{37}{16}$

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6. Find the maximum value of the function

$$\max_{0\le x\le\pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$$

(1) 0

(2) π

(3) $\frac{5\pi+2+3\sqrt{3}}{6}$

(4) $\frac{\pi+2-3\sqrt{3}}{6}$

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7. The set of all $a\in\mathbb{R}$ for which the equation

$$x-|x-1|+|x+2|+a=0$$

has exactly one real root is:

(1) $(-\infty,-3)$

(2) $(-\infty,\infty)$

(3) $(-6,\infty)$

(4) $(-6,-3)$

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8. Let PQ be a focal chord of the parabola $y^2=36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that $PM:MQ=3:1$. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

(1) $(3, 33)$

(2) $(6, 29)$

(3) $(-6, 45)$

(4) $(-3, 43)$

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9. For the system of linear equations

2x + 4y + 2az = b,

x + 2y + 3z = 4,

2x - 5y + 2z = 8,

which of the following is NOT correct?

(1) It has infinitely many solutions if $a = 3, b = 8$

(2) It has unique solution if $a = b = 8$

(3) It has unique solution if $a = b = 6$

(4) It has infinitely many solutions if $a = 3, b = 6$

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10. Let $s_1, s_2, s_3, \ldots, s_{10}$ **respectively be the sum to 12 terms of 10 A.P.s whose first terms are** 1, 2, 3, . . . , 10 **and the common differences are** 1, 3, 5, . . . , 19 **respectively. Then**

$$\sum_{i=1}^{10} s_i \text{ is equal to:}$$

(1) 7260

(2) 7380

(3) 7220

(4) 7360

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11. For the differentiable function $f : \mathbb{R} - \{0\} \to \mathbb{R}$, **let** $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, **then** $\left|f(3) + f\left(\frac{1}{4}\right)\right|$ **is equal to:**

(1) 13

(2) $\frac{29}{5}$

(3) $\frac{33}{5}$

(4) 7

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12. The negation of the statement $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ **is:**

(1) equivalent to $B \vee \sim C$

(2) a fallacy

(3) equivalent to $\sim C$

(4) equivalent to $\sim A$

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13. Let the tangent and normal at the point $\left(3\sqrt{3}, 1\right)$ on the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$

meet the y-axis at the points A and B respectively.

Let the circle C be drawn taking AB as a diameter and the line x = $2\sqrt{5}$

intersect C at the points P and Q.

If the tangents at the points P and Q on the circle intersect at the point (α, β),

then $\alpha^2 - \beta^2$

(1) $\frac{304}{5}$

(2) 60

(3) $\frac{314}{5}$

(4) 61

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14. The distance of the point (-1, 2, 3) **from the plane** $\mathbf{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ **parallel to the line of the shortest distance between the lines** r = (-) +

$\lambda(2\hat{i} + \hat{k})$ and $\mathbf{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$**is:**

(1) $2\sqrt{5}$

(2) $3\sqrt{5}$

(3) $3\sqrt{6}$

(4) $2\sqrt{6}$

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15. Let

$$\text{Let } B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix},$$

$\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$.

Then

$$[\alpha \quad -2\alpha \quad \alpha] B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix} \text{ is equal to}$$

(1) 16

(2) 32

(3) 0

(4) -16

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16. For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that,

$$g(x) = \sqrt{x} + 1 \quad \text{and} \quad f \circ g(x) = x + 3 - \sqrt{x}.$$

Then $f(0)$ is equal to:

(1) 5

(2) 0

(3) -3

(4) 1

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17. Let the equation of the plane passing through the line of intersection of the planes x + 2y + az = 2 and x - y + bz = 6a - 1 **be** x + y + tz = 5x. **For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then** a + b $\overline{c\textbf{is equal to:}}$

(1) -4

(2) 2

(3) -2

(4) 4

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18. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to:

(1) $\frac{4}{15}$

(2) $\frac{8}{15}$

(3) $\frac{1}{15}$

(4) $\frac{14}{15}$

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19. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation

$$\frac{dy}{dx} = y + 7$$

with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at:

(1) no point

(2) infinite number of points

(3) one point

(4) two points

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20. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}$, where $-\pi \leq x \leq \pi$ and the x-axis is:

(1) $2\sqrt{2}(\sqrt{2}+1)$

(2) $4(\sqrt{2})$

(3) 4

(4) $2(\sqrt{2}+1)$

Section-B

21. The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - \ldots$ is equal to $--$.

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22. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5. If m and

σ^2 are respectively the mean deviation about the mean and the variance of the data, then

$$\frac{3\alpha}{m+\sigma^2} \text{ is equal to———-}$$

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23. Let α be the constant term in the binomial expansion of

$$\left(\sqrt{x} - \frac{6}{3x^2}\right)^n, \; n \leq 15.$$

If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to——

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24. Let $\omega = zz + k_1 z + k_2 iz + \lambda(1+i), k_1, k_2 \in \mathbb{R}$. **Let** $\text{Re}(\omega) = 0$

be the circle C of radius 1 in the first quadrant touching the line $y = 1$

and the y-axis. If the curve $\text{Im}(\omega) = 0$ intersects C at A and B, then $30(AB)^2$

is equal to———-

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25. Let a $= 3\hat{i} + \hat{j} - \hat{k}$ and $\hat{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If

$\hat{b}$ is a vector such that $\hat{a} = \hat{b} \times \hat{c}$ and $||\hat{b}||^2 = 50$, then $\left|72 - ||\hat{b} - \hat{c}||^2\right|$ is equal to—-

26. Let m_1 and m_2 be the slopes of the tangents drawn from the point P(4,1) to the hyperbola

$$\frac{y^2}{25} - \frac{x^2}{16} = 1.$$

If Q is the point from which the tangents drawn to H have slopes —m$_1$| and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to——

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27. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, 8\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point Q(6, -2, -2), $\alpha > 0,$ from P is 13, then α is equal to——

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28. Let for $x \in \mathbb{R},\ S_0(x) = x,\ S_k(x) = C_k x + k\int_0^x S_{k-1}(t)dt$ where $C_0 = 1,\ C_k = 1 - \int_0^1 S_{k-1}(x)dx,\ k = 1, 2, 3, \ldots$. Then S$_2(3) + 6C_3$ is equal to——

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29. If $S = \left\{x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}\right\}$, then S is equal to——

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30. The number of seven digit positive integers formed using the digits 1, 2, 3, and 4 only and the sum of the digits equal to 12 is:
