JEE Main 2023 April 15 Shift 1 Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

(Mathematics) Section-A

1. Let S be the set of all values of λ, for which the shortest distance between the lines

$$\frac{x-0}{1}=\frac{y-4}{3}=\frac{z+\lambda}{6} \quad \text{and} \quad \frac{x-3}{1}=\frac{y+\lambda}{-4}=\frac{z}{0}$$

is 13. Then, $\sum \lambda \in S$ is equal to:

(1) 302

(2) 306

(3) 304

(4) 308

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2. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then

$$\sum_{(\lambda,\mu)\in S} 80(\lambda^2 + \mu^2)$$

is equal to:

(1) 2130

(2) 2210

(3) 2290

(4) 2370

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3. Let the foot of perpendicular of the point $P(3, -2, -9)$ on the plane passing through the points $(1, -2, -3), (9, 3, 4), (9, -2, 1)$ be $Q(\alpha, \beta, \gamma)$. Then the distance of Q from the origin is:

(1) $\sqrt{29}$

(2) $\sqrt{38}$

(3) $\sqrt{42}$

(4) $\sqrt{35}$

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4. If the set $\left\{\textbf{Re}\left(\frac{z-\bar{z}+z^2}{2-3z+5z^2}\right) : z \in \mathbb{C}, \textbf{Re}(z) = 3\right\}$ is equal to the interval (α, β), then $24(\beta - \alpha)$ is equal to:

(1) 36

(2) 27

(3) 30

(4) 42

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5. Let $x = y$ be the solution of the differential equation

$$2(y + 2)\log(y + 2)\,dx + (x + 4) - 2\log(x + 2)\,dy = 0, \quad \text{with} \quad x(1) = -2.$$

Then, $x'(-2)$ is equal to:

(1) $\frac{4}{9}$

(2) $\frac{32}{9}$

(3) $\frac{10}{3}$

(4) 3

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6. If

$$\int_0^2 \frac{1}{(5+2x-2x^2)\,(1+(e^{2-4x}))}\,dx = \frac{1}{\alpha}\log\left(\frac{\alpha+1}{\beta}\right), \quad \alpha, \beta > 0,$$

then $\alpha^4 - \beta^4$ is equal to:

(1) 19

(2) -21

(3) 21

(4) 0

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7. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is

(1) 4

(2) 1

(3) 3

(4) 2

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8. Let ABCD be a quadrilateral. If E and F are the midpoints of the diagonals AC and BD respectively and

$$(AB - BC) + (AD - DC) = k\,FE \quad \text{then} \quad k \text{ is equal to:}$$

(1) 4

(2) 2

(3) -2

(4) -4

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9. Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} P_i x^i$, where $a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to:

(1) 8

(2) 12

(3) 6

(4) 15

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10. Let $[x]$ denote the greatest integer function and $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}$, where $0 \le x \le 2$. Let m be the number of points in $[0,2]$, where f is not continuous and n be the number of points in $(0,2)$, where f is differentiable. Then $(m+n)^2+2$ is equal to:

(1) 6

(2) 2

(3) 3

(4) 11

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11. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is:

(1) $\frac{1}{4}$

(2) $\frac{9}{50}$

(3) $\frac{11}{50}$

(4) $\frac{1}{5}$

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12. If the domain of the function

$$f(x) = \log_e\left(4x^2+11x+6\right) + \sin^{-1}(4x+3) + \cos^{-1}\left(\frac{10x+6}{3}\right),$$

then $36|\alpha+\beta|$ is equal to:

(1) 72

(2) 63

(3) 45

(4) 54

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13. Let the determinant of a square matrix A of order m be $m-n$, where m and n satisfy $4m+n=22$ and $17m+4n=93$. If det$(n\,$adj$($adj$(mA)))=3^a5^b6^c$, then $a+b+c$ is equal to:

(1) 101

(2) 84

(3) 109

(4) 96

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14. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is:

(1) 14

(2) 11

(3) 12

(4) 13

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15. If (α, β) **is the orthocenter of the triangle ABC with vertices** $A(3, -7), B(-1, 2), C(4, 5)$**, then** $9\alpha - 6\beta + 60$ **is equal to:**

(1) 30

(2) 35

(3) 40

(4) 25

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16. The number of real roots of the equation

$$x|x| - 5|x + 2| + 6 = 0,$$

is:

(1) 5

(2) 6

(3) 4

(4) 3

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17. Let the system of linear equations

$$-x + 2y - 9z = 7$$

-x + 3y + 7z = 9

-2x + y + 5z = 8

-3x + y + 13z = λ **has a unique solution** $x = \alpha, y = \beta, z = \gamma$**. Then the distance of the point** (α, β, γ) **from the plane** $2x - 2y + z = \lambda$ **is:**

(1) 7

(2) 9

(3) 13

(4) 11

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18. Let A_1 **and** A_2 **be two arithmetic means and** G_1, G_2, G_3 **be three geometric means of two distinct positive numbers. Then**

$$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2 \text{ is equal to:}$$

(1) $2(A_1 + A_2)G_1G_3$

(2) $(A_1 + A_2)^2 G_1G_3$

(3) $2(A_1 + A_2)^2 G_1^2 G_3^2$

(4) $(A_1 + A_2)G_1^2 G_2^2 G_3^2$

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19. Negation of $p \wedge (q \wedge \neg(p \wedge q))$ **is:**

(1) $\neg(p \wedge q) \wedge q$

(2) $\neg(p \vee q)$

(3) $p \vee q$

(4) $(\neg(p \wedge q)) \vee p$

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20. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is:

(1) 21

(2) 18

(3) 20

(4) 22

Section-B

21. Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by

$$R = \{(a, b), (c, d) : 2a + 3b = 4c + 5d\}.$$

Then the number of elements in R is:

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22. The number of elements in the set

$\{n \in \mathbb{N} : 10 \leq n \leq 100$ **and** $3n^3 - 3$ **is a multiple of 7**$\}$ **is:**

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23. Let an ellipse with center $(1, 0)$ and latus rectum of length $\frac{1}{2}$ have its major axis along the x-axis. If its minor axis subtends an angle of 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to:

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24. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$, and outside the circle $(x - 3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to:

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25. Consider the triangles with vertices $A(2, 1)$, $B(0, 0)$ and $C(t, 4)$, $t \in [0, 4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to:

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26. Let the plane P contain the line $2x + y - z = 3 = 0$, $5x - 3y + 4z + 9 = 0$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{4} = \frac{z-7}{5}$. Then the distance of the point $A(8, -1, -19)$ from the plane P, measured parallel to the line is equal to:

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27. If the sum of the series

$$\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{2 \cdot 3^2}\right) + \left(\frac{1}{3^2} + \frac{1}{2^2 \cdot 3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{2^2 \cdot 3^3}\right) + \dots$$

is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to:

28. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is:

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29. If the line $x = y = z$ **intersects the line** $x \sin A + y \sin B + z \sin C - 18 = 0$ **and** $x \sin 2A + y \sin 2B + z \sin 2C - 9 = 0$**, where A, B, C are the angles of a triangle ABC, then** $80 \left(\frac{\sin A}{\sin B} \frac{\sin C}{\sin B} \right)$ **is equal to:**

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30. Let $f(x) = \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$**,** $|x| < \frac{2}{\sqrt{3}}$**, and** $f(0) = 0$**. If** $f(0) = 0$ **and** $f(1) = 1$**, then** $\alpha\beta > 0$**, then** $\alpha^2 + \beta^2$ **is equal to:**
