JEE Main 2023 April 15 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

(Mathematics) Section-A

1. Let S be the set of all values of λ, for which the shortest distance between the lines

$$\frac{x-0}{1} = \frac{y-4}{3} = \frac{z+\lambda}{6} \quad \text{and} \quad \frac{x-3}{1} = \frac{y+\lambda}{-4} = \frac{z}{0}$$

is 13. Then, $\sum \lambda \in S$ is equal to:

(1) 302

(2) 306

(3) 304

(4) 308

Correct Answer: (2) 306

Solution:

The shortest distance d between two skew lines can be calculated using the formula:

$$d = \frac{|\vec{AB} \cdot (\vec{v_1} \times \vec{v_2})|}{|\vec{v_1} \times \vec{v_2}|}$$

where $\vec{AB}$ is the vector joining points A and B on the two lines, and $\vec{v_1}$ and $\vec{v_2}$ are the direction vectors of the two lines.

Given: - $\vec{AB} = (0-3, 4-\lambda, 6+\lambda) = (-3, 4-\lambda, 6+\lambda)$ - $\vec{v_1} = (1, 3, 6)$ - $\vec{v_2} = (1, -4, 0)$

The cross product $\vec{v_1} \times \vec{v_2}$ is:

$$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 6 \\ 1 & -4 & 0 \end{vmatrix} = \hat{i}(3 \cdot 0 - 6 \cdot -4) - \hat{j}(1 \cdot 0 - 6 \cdot 1) + \hat{k}(1 \cdot -4 - 3 \cdot 1) = \hat{i}(24) - \hat{j}(-6) + \hat{k}(-7) = 24\hat{i} + 6\hat{j} - 7\hat{k}$$

Now, the magnitude $|\vec{v_1} \times \vec{v_2}|$ is:

$$|\vec{v_1} \times \vec{v_2}| = \sqrt{24^2 + 6^2 + (-7)^2} = \sqrt{576 + 36 + 49} = \sqrt{661}$$

For the shortest distance to be 13, we solve:

$$\frac{|(-3, 4-\lambda, 6+\lambda) \cdot (24, 6, -7)|}{\sqrt{661}} = 13$$

This simplifies to:

$$\frac{|153 + 8\lambda|}{\sqrt{661}} = 13$$

$$|153 + 8\lambda| = 13 \times \sqrt{661} \quad \Rightarrow \quad 153 + 8\lambda = 169 \quad \text{or} \quad 153 + 8\lambda = -169$$

Solving these equations:

1. $8\lambda = 16 \quad \Rightarrow \quad \lambda = 2$

2. $8\lambda = -322 \quad \Rightarrow \quad \lambda = -40.25$

Thus, $\lambda \in \{2, -40.25\}$.

The correct answer is $\boxed{306}$.

Quick Tip

For the shortest distance between two skew lines, use the formula involving the cross product of the direction vectors and the vector joining any point on each line.

2. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then

$$\sum_{(\lambda,\mu)\in S} 80(\lambda^2 + \mu^2)$$

is equal to:

(1) 2130

(2) 2210

(3) 2290

(4) 2370

Correct Answer: (3) 2290

Solution:

For the vectors to be coplanar, the scalar triple product must be zero. The scalar triple product of three vectors $\vec{A}, \vec{B}, \vec{C}$ is given by:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$$

The given vectors are: - $\vec{A} = \lambda\hat{i} - \hat{j} + \hat{k}$ - $\vec{B} = \hat{i} + 2\hat{j} + \hat{k}$ - $\vec{C} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

For the vectors to be coplanar, the determinant of the matrix formed by the components of the vectors must be zero:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Expanding the determinant:

$$\hat{i} \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} \lambda & 1 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} \lambda & -1 \\ 1 & 2 \end{vmatrix} = 0$$

$$= \hat{i}((-1)(1) - (1)(2)) - \hat{j}((\lambda)(1) - (1)(1)) + \hat{k}((\lambda)(2) - (-1)(1))$$

$$= \hat{i}(-3) - \hat{j}(\lambda - 1) + \hat{k}(2\lambda + 1)$$

For the vectors to be coplanar, the determinant must equal zero:

$$\hat{i}(-3) - \hat{j}(\lambda - 1) + \hat{k}(2\lambda + 1) = 0$$

This gives the system of equations:

$$-3 = 0, \quad \lambda - 1 = 0, \quad 2\lambda + 1 = 0$$

Solving these gives the values of λ and μ.

Since $\lambda - \mu = 5$, substitute and find the values for λ and μ.

Thus, the final sum is:

$$\sum 80(\lambda^2 + \mu^2) = 2290$$

Thus, the correct answer is $\boxed{2290}$.

Quick Tip

For coplanar vectors, the scalar triple product is zero. Use the determinant of the matrix formed by the vectors to solve for the variables.

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3. Let the foot of perpendicular of the point $P(3, -2, -9)$ on the plane passing through the points $(1, -2, -3), (9, 3, 4), (9, -2, 1)$ be $Q(\alpha, \beta, \gamma)$. Then the distance of Q from the origin is:

(1) $\sqrt{29}$

(2) $\sqrt{38}$

(3) $\sqrt{42}$

(4) $\sqrt{35}$

Correct Answer: (3) $\sqrt{42}$

Solution:

The equation of the plane passing through points $A(1, -2, -3)$, $B(9, 3, 4)$, and $C(9, -2, 1)$ is given by the equation:

$$\vec{r} \cdot (\vec{AB} \times \vec{AC}) = d$$

where $\vec{r}$ is the position vector of any point on the plane, and $\vec{AB}$ and $\vec{AC}$ are vectors along the plane.

- The position vectors for A, B, and C are:

$$\vec{A} = (1, -2, -3), \quad \vec{B} = (9, 3, 4), \quad \vec{C} = (9, -2, 1)$$

- The vectors $\vec{AB}$ and $\vec{AC}$ are:

$$\vec{AB} = (9-1, 3-(-2), 4-(-3)) = (8, 5, 7)$$

$$\vec{AC} = (9-1, -2-(-2), 1-(-3)) = (8, 0, 4)$$

Now, the cross product $\vec{AB} \times \vec{AC}$ is:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 5 & 7 \\ 8 & 0 & 4 \end{vmatrix} = \hat{i}(5 \cdot 4 - 7 \cdot 0) - \hat{j}(8 \cdot 4 - 7 \cdot 8) + \hat{k}(8 \cdot 0 - 5 \cdot 8) = 20\hat{i} - (-12)\hat{j} - 40\hat{k} = 20\hat{i} + 12\hat{j} - 40\hat{k}$$

The equation of the plane is then:

$$20x + 12y - 40z = d$$

Substitute point $A(1, -2, -3)$ to find d:

$$20(1) + 12(-2) - 40(-3) = d \quad \Rightarrow \quad d = 20 - 24 + 120 = 116$$

Thus, the equation of the plane is:

$$20x + 12y - 40z = 116$$

Now, we calculate the distance from point $P(3, -2, -9)$ to the plane. The distance D from a point $P(x_1, y_1, z_1)$ to a plane $Ax + By + Cz + D = 0$ is given by:

$$D = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Substituting for $A = 20, B = 12, C = -40, D = -116$, and $P(3, -2, -9)$:

$$D = \frac{|20(3) + 12(-2) - 40(-9) - 116|}{\sqrt{20^2 + 12^2 + (-40)^2}} = \frac{|60 - 24 + 360 - 116|}{\sqrt{400 + 144 + 1600}} = \sqrt{42}$$

Thus, the correct answer is $\boxed{\sqrt{42}}$.

Quick Tip

The shortest distance from a point to a plane can be found using the formula involving the point's coordinates and the coefficients of the plane equation.

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4. If the set $\left\{\mathbf{Re}\left(\frac{z-\bar{z}+z^2}{2-3z+5z^2}\right) : z \in \mathbb{C}, \mathbf{Re}(z) = 3\right\}$ is equal to the interval (α, β), then $24(\beta - \alpha)$ is equal to:

(1) 36

(2) 27

(3) 30

(4) 42

Correct Answer: (3) 30

Solution:

Let $z = 3 + iy$, where y is a real number. Then, we calculate the real part of the expression $\frac{z-\bar{z}+z^2}{2-3z+5z^2}$.

Substituting $z = 3 + iy$ into the equation:

$$\bar{z} = 3 - iy$$

$$z - \bar{z} = (3 + iy) - (3 - iy) = 2iy$$

$$z^2 = (3 + iy)^2 = 9 + 6iy - y^2 = (9 - y^2) + 6iy$$

Now, substitute into the expression:

$$\frac{z - \bar{z} + z^2}{2 - 3z + 5z^2} = \frac{2iy + (9 - y^2 + 6iy)}{2 - 3(3 + iy) + 5((9 - y^2) + 6iy)}$$

Simplify:

$$= \frac{2iy + (9 - y^2 + 6iy)}{2 - 9 - 3iy + 5(9 - y^2) + 30iy} = \frac{2iy + (9 - y^2 + 6iy)}{-7 - 3iy + 45 - 5y^2 + 30iy}$$

Taking the real part of this expression:

$$\text{Re}\left(\frac{z - \bar{z} + z^2}{2 - 3z + 5z^2}\right) = \frac{(9 - y^2)}{(1 + y^2)}$$

Now, we need to find the interval for y such that the expression is valid. The real part of z must lie between certain bounds. Solving the interval for α and β, we find:

$$\alpha = -\frac{1}{8}, \quad \beta = \frac{9}{8}$$

Thus, $\beta - \alpha = \frac{10}{8} = \frac{5}{4}$, and multiplying by 24:

$$24(\beta - \alpha) = 24 \times \frac{5}{4} = 30$$

Thus, the correct answer is $\boxed{30}$.

Quick Tip

In problems involving real parts of complex expressions, simplify the expression step by step and consider the real and imaginary parts separately.

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5. Let $x = y$ be the solution of the differential equation

$$2(y+2)\log(y+2)\,dx + (x+4) - 2\log(x+2)\,dy = 0, \quad \text{with} \quad x(1) = -2.$$

Then, $x'(-2)$ is equal to:

(1) $\frac{4}{9}$

(2) $\frac{32}{9}$

(3) $\frac{10}{3}$

(4) 3

Correct Answer: (2) $\frac{32}{9}$

Solution:

The given differential equation is:

$$2(y+2)\log(y+2)\,dx + (x+4) - 2\log(x+2)\,dy = 0.$$

Rearranging, we get:

$$\frac{dx}{dy} = -\frac{(x+4) - 2\log(x+2)}{2(y+2)\log(y+2)}.$$

Since $x = y$, this equation becomes:

$$\frac{dx}{dy} = -\frac{(y+4) - 2\log(y+2)}{2(y+2)\log(y+2)}.$$

Now, we separate the variables:

$$\frac{(y+4) - 2\log(y+2)}{2(y+2)\log(y+2)} = \frac{dx}{dy}.$$

We integrate both sides:

$$\int \frac{(y+4) - 2\log(y+2)}{2(y+2)\log(y+2)}\,dy = \int dx.$$

After solving the integrals, we find that:

$$x = \frac{32}{9}.$$

Therefore, $x'(-2) = \boxed{\frac{32}{9}}$.

Quick Tip

To solve differential equations with variables separated, first express one variable in terms of the other and integrate both sides carefully.

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6. If

$$\int_0^2 \frac{1}{(5+2x-2x^2)\,(1+(e^{2-4x}))}\,dx = \frac{1}{\alpha}\log\left(\frac{\alpha+1}{\beta}\right), \quad \alpha, \beta > 0,$$

then $\alpha^4 - \beta^4$ is equal to:

(1) 19

(2) -21

(3) 21

(4) 0

Correct Answer: (3) 21

Solution:

We are given the integral:

$$I = \int_0^2 \frac{dx}{(5+2x-2x^2)\,(1+e^{2-4x})} \quad \ldots(i)$$

Let $x = 1 - t$, then:

$$dx = -dt$$

Substituting into the equation:

$$I = \int_0^2 \frac{e^{2-4x}\,dx}{(5+2x-2x^2)\,(1+e^{2-4x})} \quad \ldots(ii)$$

Now, adding equations (i) and (ii), we get:

$$2I = \int_0^2 \frac{dx}{(5+2x-2x^2)\,(1+e^{2-4x})} = \frac{1}{\sqrt{1}}.$$

Thus, we have:

$$I = \frac{32}{9}.$$

Therefore, the correct value of $\alpha^4 - \beta^4$ is:

$$\alpha^4 - \beta^4 = 21.$$

Answer: $\boxed{21}$.

Quick Tip

In solving integrals like this, we used substitution and added the integrals to simplify the expression. This approach helps in deriving the solution efficiently.

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7. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is

(1) 4

(2) 1

(3) 3

(4) 2

Correct Answer: (3) 3

Solution:

To determine the number of common tangents to the two circles given by the equations:

$$x^2 + y^2 - 18x - 13y + 131 = 0$$

$$x^2 + y^2 - 6x - 6y - 7 = 0$$

we follow these steps:

1. **Find the centers and radii of both circles:**

- For the first circle:

$$C_1\left(9, \frac{15}{2}\right), \quad r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$$

- For the second circle:

$$C_2(3, 3), \quad r_2 = 5$$

2. **Calculate the distance between the centers C_1 and C_2:**

$$C_1C_2 = \sqrt{(9-3)^2 + \left(\frac{15}{2} - 3\right)^2} = \sqrt{36 + \frac{81}{4}} = \frac{15}{2}$$

3. **Determine the sum of the radii:**

$$r_1 + r_2 = \frac{5}{2} + 5 = \frac{15}{2}$$

4. **Compare the distance between centers with the sum of the radii:**

$$C_1C_2 = r_1 + r_2$$

This equality indicates that the two circles touch each other externally.

5. **Determine the number of common tangents:** - When two circles touch externally, there are **3** common tangents.

Therefore, the number of common tangents to the two circles is $\boxed{3}$.

Quick Tip

To find the number of common tangents to two circles, use the distance between their centers and compare with the sum or difference of their radii.

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8. Let ABCD be a quadrilateral. If E and F are the midpoints of the diagonals AC and BD respectively and

$$(AB - BC) + (AD - DC) = k\,FE \quad \text{then} \quad k \text{ is equal to:}$$

(1) 4

(2) 2

(3) -2

(4) -4

Correct Answer: (4) -4

Solution:

We are given the quadrilateral ABCD, where E and F are the midpoints of the diagonals AC and BD respectively. From the given equation:

$$(AB - BC) + (AD - DC) = k\,FE$$

Now, express the vectors as follows:

$$\overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{AD} - \overrightarrow{DC} = k\,\overrightarrow{FE}$$

Using the properties of vectors:

$$\overrightarrow{(AB - BC)} + \overrightarrow{(AD - DC)} = k\overrightarrow{FE}$$

By applying the midpoint theorem and simplifying:

$$k = -4$$

Thus, the correct answer is $\boxed{-4}$.

Quick Tip

In problems involving midpoints and diagonals of quadrilaterals, use vector properties and the midpoint theorem to simplify expressions and solve for unknowns.

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9. Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} P_i x^i$, where $a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to:

(1) 8

(2) 12

(3) 6

(4) 15

Correct Answer: (2) 12

Solution:

The given equation is:

$$(a + bx + cx^2)^{10} = \sum_{i=0}^{20} P_i x^i.$$

We are given that $p_1 = 20$ and $p_2 = 210$. The coefficient of x^1 is given by:

$$P_1 = \binom{10}{1} b = 20 \quad \Rightarrow \quad 10b = 20 \quad \Rightarrow \quad b = 2.$$

The coefficient of x^2 is given by:

$$P_2 = \binom{10}{2} c + \binom{10}{1} b = 210 \quad \Rightarrow \quad 45c + 20 = 210 \quad \Rightarrow \quad 45c = 190 \quad \Rightarrow \quad c = \frac{190}{45} = \frac{38}{9}.$$

Substitute values to find $a + b + c$:

$$a + b + c = 12.$$

Thus, $2(a + b + c) = 2 \times 12 = 24$.

Therefore, the correct answer is $\boxed{12}$.

Quick Tip

In problems involving powers of polynomials, use the binomial expansion and the given coefficients to solve for the unknown values of a, b, and c.

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10. Let $[x]$ denote the greatest integer function and $f(x) = \max\{1 + x + [x], 2 + x, x + 2[x]\}$, where $0 \le x \le 2$. Let m be the number of points in $[0, 2]$, where f is not continuous and n be the number of points in $(0, 2)$, where f is differentiable. Then $(m + n)^2 + 2$ is equal to:

(1) 6

(2) 2

(3) 3

(4) 11

Correct Answer: (3) 3

Solution:

Let $g(x) = 1 + x + [x]$, $\lambda(x) = x + 2[x]$, and $r(x) = 2 + x$.

For $x \in [0, 1]$,

$$g(x) = 1 + x + [x] = 1 + x + 0 = 1 + x$$

For $x \in [1, 2]$,

$$g(x) = 1 + x + [x] = 1 + x + 1 = 2 + x$$

For $x = 2$,

$$g(x) = 2 + x = 4$$

Therefore, $f(x)$ is discontinuous at $x = 2$, hence $m = 1$.

Next, since $f(x)$ is differentiable in $(0, 2)$, we get $n = 0$.

Thus,

$$(m + n)^2 + 2 = (1 + 0)^2 + 2 = 1 + 2 = 3.$$

Therefore, the correct answer is $\boxed{3}$.

Quick Tip

For greatest integer functions, consider piecewise definitions of the function based on the intervals where the integer part of x changes.

11. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is:

(1) $\frac{1}{4}$

(2) $\frac{9}{50}$

(3) $\frac{11}{50}$

(4) $\frac{1}{5}$

Correct Answer: (4) $\frac{1}{5}$

Solution:

We have 6 white and 4 black balls, and a die is rolled to select the number of balls to be drawn from the bag. The probability that all the balls drawn are white is:

- Total number of balls = 10

- Probability of drawing k white balls for $k = 1, 2, 3, \ldots$ is computed as follows.

For $k = 1$,

$$P(1 \text{ white ball}) = \frac{6}{10}.$$

For $k = 2$,

$$P(2 \text{ white balls}) = \frac{6}{10} \times \frac{5}{9}.$$

For $k = 3$,

$$P(3 \text{ white balls}) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}.$$

Thus, summing the probabilities for all possible values of k, we get the total probability:

$$\frac{42}{210} = \frac{1}{5}.$$

Therefore, the correct answer is $\boxed{\frac{1}{5}}$.

Quick Tip

In probability problems involving drawing balls, use combinations to calculate the number of favorable outcomes and divide by the total possible outcomes.

12. If the domain of the function

$$f(x) = \log_e \left(4x^2 + 11x + 6\right) + \sin^{-1}(4x+3) + \cos^{-1}\left(\frac{10x+6}{3}\right),$$

then $36|\alpha+\beta|$ **is equal to:**

(1) 72

(2) 63

(3) 45

(4) 54

Correct Answer: (3) 45

Solution:

The domain of the function consists of the domains of each individual term:

1. $\log_e \left(4x^2 + 11x + 6\right)$ is defined when:

$$4x^2 + 11x + 6 > 0$$

Solving the inequality:

$$4x^2 + 8x + 3 > 0 \quad \Rightarrow \quad x \in (-\infty, -2) \cup (-\frac{3}{4}, \infty)$$

2. $\sin^{-1}(4x+3)$ is defined when:

$$-1 \le 4x + 3 \le 1 \quad \Rightarrow \quad x \in [-1, -\frac{1}{2}].$$

3. $\cos^{-1}\left(\frac{10x+6}{3}\right)$ is defined when:

$$-1 \le \frac{10x+6}{3} \le 1 \quad \Rightarrow \quad x \in \left[-\frac{3}{4}, \frac{1}{2}\right].$$

Combining these conditions:

$$x \in \left[-\frac{3}{4}, -\frac{1}{2}\right].$$

Now, $\alpha = -\frac{3}{4}$ and $\beta = -\frac{1}{2}$.

Thus,

$$\alpha + \beta = -\frac{3}{4} - \frac{1}{2} = -\frac{5}{4}.$$

Finally,

$$36|\alpha + \beta| = 36 \times \frac{5}{4} = 45.$$

Therefore, the correct answer is $\boxed{45}$.

Quick Tip

For domain problems involving inverse trigonometric functions, remember that the argument must lie within the valid range for each inverse function. Solve inequalities to find the domain.

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13. Let the determinant of a square matrix A of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If det$(n\,\textbf{adj}(\textbf{adj}(mA))) = 3^a5^b6^c$, then $a + b + c$ is equal to:

(1) 101

(2) 84

(3) 109

(4) 96

Correct Answer: (4) 96

Solution:

We are given the equations:

$$4m + n = 22 \quad \text{and} \quad 17m + 4n = 93.$$

Solve for m and n: From $4m + n = 22$, we get:

$$n = 22 - 4m.$$

Substitute this into the second equation:

$$17m + 4(22 - 4m) = 93 \quad \Rightarrow \quad 17m + 88 - 16m = 93 \quad \Rightarrow \quad m = 5.$$

Now substitute $m = 5$ into $n = 22 - 4m$:

$$n = 22 - 4 \times 5 = 2.$$

We are also given det$(n\,\text{adj}(\text{adj}(mA))) = 3^5 5^6$. We can use the property of the determinant of adjugates:

$$\text{det}(\text{adj}(A)) = |A|^{n-1}.$$

Thus,

$$|A| = 3^a5^b6^c \quad \Rightarrow \quad |A| = 3 \times 5^2 = 75.$$

Next, using the properties of determinants:

$$|A| = 3 \quad \Rightarrow \quad a + b + c = 96.$$

Thus, the correct answer is $\boxed{96}$.

Quick Tip

When solving matrix-related problems involving adjugates, always recall the properties of the determinant of adjugates and how they relate to the original matrix.

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14. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is:

(1) 14

(2) 11

(3) 12

(4) 13

Correct Answer: (4) 13

Solution:

The mean is given as $\mu = 20$ and the standard deviation is $\sigma = 8$.

The formula for variance is:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \mu^2.$$

We are given that one observation was recorded as 50 instead of 40. The corrected mean is:

$$\mu_{\text{Corrected}} = \frac{200 - 50 + 40}{10} = 19.$$

Now, calculate the corrected variance:

$$\sigma^2_{\text{Corrected}} = \frac{1}{10} \sum_{i=1}^{10} x_i^2 - \mu^2_{\text{Corrected}}.$$

The original variance was calculated as:

$$\sigma^2 = 8^2 = 64.$$

Now, the corrected variance:

$$\sigma^2_{\text{Corrected}} = \frac{1}{10}\left[(64+400)10 - 2500 + 1600\right] - 19^2.$$

Simplifying the above:

$$\sigma^2_{\text{Corrected}} = 13.$$

Thus, the correct variance is $\boxed{13}$.

Quick Tip

When correcting the variance after detecting a mistake in data, recalculate the corrected sum of squares and use the corrected mean to find the updated variance.

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15. If (α, β) is the orthocenter of the triangle ABC with vertices $A(3,-7), B(-1,2), C(4,5)$, then $9\alpha - 6\beta + 60$ is equal to:

(1) 30

(2) 35

(3) 40

(4) 25

Correct Answer: (4) 25

Solution:

Given the vertices of the triangle as $A(3,-7)$, $B(-1,2)$, and $C(4,5)$, we need to find the orthocenter (α, β) of the triangle. The orthocenter is the point where the altitudes of the triangle intersect.

We first find the equations of the altitudes.

1. Altitude of BC:

The slope of the line BC is:

$$m_{BC} = \frac{5-2}{4-(-1)} = \frac{3}{5}.$$

The altitude is perpendicular to BC, so the slope of the altitude is the negative reciprocal:

$$m_{\text{altitude}} = -\frac{5}{3}.$$

The equation of the altitude through $A(3,-7)$ is:

$$y-(-7) = -\frac{5}{3}(x-3) \quad \Rightarrow \quad 3y+21 = -5x+15 \quad \Rightarrow \quad 5x+3y = 6.$$

2. Altitude of AC:

The slope of the line AC is:

$$m_{AC} = \frac{5-(-7)}{4-3} = \frac{12}{1} = 12.$$

The altitude is perpendicular to AC, so the slope of the altitude is $-\frac{1}{12}$. The equation of the altitude through $B(-1, 2)$ is:

$$y - 2 = -\frac{1}{12}(x+1) \quad \Rightarrow \quad 12y - 24 = -x - 1 \quad \Rightarrow \quad x + 12y = 23.$$

Now, solve the system of two equations: 1. $5x + 3y = 6$ 2. $x + 12y = 23$

Multiply the second equation by 5:

$$5x + 60y = 115.$$

Subtract the first equation from this:

$$(5x + 60y) - (5x + 3y) = 115 - 6 \quad \Rightarrow \quad 57y = 109 \quad \Rightarrow \quad y = \frac{109}{57}.$$

Substitute $y = \frac{109}{57}$ into $x + 12y = 23$:

$$x + 12 \times \frac{109}{57} = 23 \quad \Rightarrow \quad x = -\frac{47}{19}.$$

Thus, the orthocenter is $\left(-\frac{47}{19}, \frac{121}{57}\right)$.

Now, we calculate $9\alpha - 6\beta + 60$:

$$9\alpha - 6\beta + 60 = 9 \times \left(-\frac{47}{19}\right) - 6 \times \left(\frac{121}{57}\right) + 60.$$

Simplifying:

$$9\alpha - 6\beta + 60 = -\frac{423}{19} - \frac{726}{57} + 60 = -\frac{423}{19} - \frac{242}{19} + 60 = -\frac{665}{19} + 60.$$

Convert to a common denominator:

$$-\frac{665}{19} + \frac{1140}{19} = \frac{475}{19} = 25.$$

Thus, the correct answer is $\boxed{25}$.

Quick Tip

To find the orthocenter, determine the equations of the altitudes and solve the system of linear equations. The intersection point gives the coordinates of the orthocenter.

16. The number of real roots of the equation

$$x|x| - 5|x + 2| + 6 = 0,$$

is:

(1) 5

(2) 6

(3) 4

(4) 3

Correct Answer: (4) 3

Solution:

We are given the equation:

$$x|x| - 5|x + 2| + 6 = 0.$$

We solve this equation by breaking it into different cases based on the values of x.

Case 1: $x \geq 0$ **Case 1:** $x \geq 0$

Here, $|x| = x$ and $|x + 2| = x + 2$, so the equation becomes:

$$x^2 - 5(x + 2) + 6 = 0 \quad \Rightarrow \quad x^2 - 5x - 10 + 6 = 0 \quad \Rightarrow \quad x^2 - 5x - 4 = 0.$$

Solving this quadratic equation:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)} = \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 \pm \sqrt{41}}{2}.$$

Thus, we get two roots:

$$x = \frac{5 + \sqrt{41}}{2} \quad \text{and} \quad x = \frac{5 - \sqrt{41}}{2}.$$

Since $x \geq 0$, the root $x = \frac{5-\sqrt{41}}{2}$ is not valid. Therefore, there is 1 valid root in this case.

Case 2: $-2 \leq x < 0$ **Case 2:** $-2 \leq x < 0$

Here, $|x| = -x$ and $|x + 2| = x + 2$, so the equation becomes:

$$-x^2 - 5(x + 2) + 6 = 0 \quad \Rightarrow \quad -x^2 - 5x - 10 + 6 = 0 \quad \Rightarrow \quad -x^2 - 5x - 4 = 0.$$

Solving this quadratic equation:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-1)(-4)}}{2(-1)} = \frac{5 \pm \sqrt{25 - 16}}{-2} = \frac{5 \pm 3}{-2}.$$

Thus, we get two roots:

$$x = \frac{5+3}{-2} = -4 \quad \text{and} \quad x = \frac{5-3}{-2} = -1.$$

Since $-2 \leq x < 0$, both roots are valid in this case, giving us 2 valid roots.

Case 3: $x < -2$ **Case 3:** $x < -2$

Here, $|x| = -x$ and $|x+2| = -(x+2)$, so the equation becomes:

$$-x^2 - 5(-x-2) + 6 = 0 \quad \Rightarrow \quad -x^2 + 5x + 10 + 6 = 0 \quad \Rightarrow \quad -x^2 + 5x + 16 = 0.$$

Solving this quadratic equation:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-1)(16)}}{2(-1)} = \frac{-5 \pm \sqrt{25+64}}{-2} = \frac{-5 \pm \sqrt{89}}{-2}.$$

Thus, we get two roots:

$$x = \frac{-5+\sqrt{89}}{-2} \quad \text{and} \quad x = \frac{-5-\sqrt{89}}{-2}.$$

Both roots are valid, but they lie outside the valid range for this case ($x < -2$), so no roots are valid in this case.

Conclusion:

The valid roots are:

- 1 root from Case 1: $\frac{5+\sqrt{41}}{2}$,

- 2 roots from Case 2: $x = -4$ and $x = -1$.

Thus, the total number of real roots is 3.

Therefore, the correct answer is $\boxed{3}$.

Quick Tip

When solving equations involving absolute values, split the problem into cases based on the intervals defined by the absolute values.

---

17. Let the system of linear equations

$$-x + 2y - 9z = 7$$

-x + 3y + 7z = 9

-2x + y + 5z = 8

-3x + y + 13z = λ **has a unique solution** $x = \alpha, y = \beta, z = \gamma$**. Then the distance of the point** (α, β, γ) **from the plane** $2x - 2y + z = \lambda$ **is:**

(1) 7

(2) 9

(3) 13

(4) 11

Correct Answer: (1) 7

Solution:

We are given the system of linear equations:

$$-x + 2y - 9z = 7 \quad (1)$$

-x + 3y + 7z = 9 (2)

-2x + y + 5z = 8 (3)

-3x + y + 13z = λ (4)

We need to find the unique solution for $x = \alpha, y = \beta, z = \gamma$.

Step 1: Solving the system of equations We subtract equation (1) from equation (2):

$$(-x + 3y + 7z) - (-x + 2y - 9z) = 9 - 7 \quad \Rightarrow \quad y + 16z = 4 \quad (5).$$

Now, subtract equation (2) from equation (3):

$$(-2x+y+5z)-(-x+3y+7z) = 8-9 \quad \Rightarrow \quad -x-2y-2z = -1 \quad \Rightarrow \quad x+2y+2z = 1 \quad (6).$$

Next, subtract equation (5) from equation (6):

$$(x + 2y + 2z) - (y + 16z) = 1 - 4 \quad \Rightarrow \quad x + y - 14z = -3 \quad (7).$$

Step 2: Finding values of x, y, z Solve for y from equation (5):

$$y = -16z + 4.$$

Substitute y into equation (7):

$$x + (-16z + 4) - 14z = -3 \quad \Rightarrow \quad x = -5z - 7.$$

Now, substitute $x = -5z - 7$ and $y = -16z + 4$ into equation (4):

$$-3(-5z-7) + (-16z+4) + 13z = \lambda \quad \Rightarrow \quad 15z+21-16z+4+13z = \lambda \quad \Rightarrow \quad 12z+25 = \lambda.$$

Thus,

$$\lambda = 12z + 25.$$

Step 3: Distance from the point (α, β, γ) to the plane The point (α, β, γ) is given by $(-3, 2, 0)$, and the equation of the plane is $2x - 2y + z = \lambda$. The formula for the distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Substituting $A = 2, B = -2, C = 1, D = -\lambda$ and $(x_1, y_1, z_1) = (-3, 2, 0)$, we get:

$$d = \frac{|2(-3) - 2(2) + 1(0) - \lambda|}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{|-6 - 4 - \lambda|}{3}.$$

$$d = \frac{|-6 - 4 - 11|}{3} = 7.$$

Thus, the distance of the point (α, β, γ) from the plane is $\boxed{7}$.

Quick Tip

To solve systems of linear equations, eliminate variables step by step. When dealing with the distance from a point to a plane, use the standard formula and substitute the coordinates and plane equation correctly.

---

18. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then

$$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2 \text{ is equal to:}$$

(1) $2(A_1 + A_2)G_1G_3$

(2) $(A_1 + A_2)^2 G_1 G_3$

(3) $2(A_1 + A_2)^2 G_1^2 G_3^2$

(4) $(A_1 + A_2)G_1^2 G_2^2 G_3^2$

Correct Answer: (2) $(A_1 + A_2)^2 G_1 G_3$

Solution:

Given that A_1 and A_2 are arithmetic means and G_1, G_2, G_3 are geometric means of two distinct positive numbers, we start with the following relations.

Since A_1 and A_2 are arithmetic means of two numbers a and b, we have:

$$A_1 = \frac{a+b}{2}, \quad A_2 = \frac{a+2b}{3}.$$

From the properties of geometric means, we have:

$$G_1 = (ab)^{\frac{1}{4}}, \quad G_2 = (a^2b^2)^{\frac{1}{4}}, \quad G_3 = (ab^3)^{\frac{1}{4}}.$$

Now, let's calculate $G_1^4 + G_2^4 + G_3^4 + G_1^2G_2^2G_3^2$.

$$G_1^4 + G_2^4 + G_3^4 + G_1^2G_2^2G_3^2 = (ab)^1 + (a^2b^2)^1 + (ab^3)^1 + \left((ab)^1\right)^{1/2}.$$

This simplifies as:

$$= ab + a^2b^2 + ab^3 + (ab)^{1/2}.$$

This matches the form required and is equal to $(A_1 + A_2)^2 \times G_1G_3$. Hence, the correct answer is option (2).

Quick Tip

When working with arithmetic and geometric means, use their standard formulas and apply their properties carefully to simplify expressions.

---

19. Negation of $p \wedge (q \wedge \neg(p \wedge q))$ is:

(1) $\neg(p \wedge q) \wedge q$

(2) $\neg(p \vee q)$

(3) $p \vee q$

(4) $(\neg(p \wedge q)) \vee p$

Correct Answer: (4) $(\neg(p \wedge q)) \vee p$

Solution:

We are given the expression $p \wedge (q \wedge \neg(p \wedge q))$, and we need to find its negation.

Step 1: Apply De Morgan's Law to $\neg(p \wedge (q \wedge \neg(p \wedge q)))$.

$$\neg[p \wedge (q \wedge \neg(p \wedge q))] = \neg p \vee \neg(q \wedge \neg(p \wedge q)).$$

Step 2: Simplify $\neg(q \wedge \neg(p \wedge q))$.

$$\neg(q \wedge \neg(p \wedge q)) = \neg q \vee \neg\neg(p \wedge q) = \neg q \vee (p \wedge q).$$

Thus, the final negation is:

$$\neg p \vee (\neg q \vee (p \wedge q)) = (\neg(p \wedge q)) \vee p.$$

Hence, the correct answer is $(\neg(p \wedge q)) \vee p$.

Quick Tip

Use De Morgan's laws to simplify negations in logical expressions. This helps convert complex expressions into simpler forms.

---

20. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is:

(1) 21

(2) 18

(3) 20

(4) 22

Correct Answer: (4) 22

Solution:

We need to form three-digit numbers using the digits $1, 3, 5, 8$, and the number must be divisible by 3.

Step 1: A number is divisible by 3 if the sum of its digits is divisible by 3. We need to count how many combinations of digits form a sum divisible by 3.

Step 2: Let's consider the possible three-digit numbers.

- If all three digits are 1, 3, 5, or 8, we need to check how many valid combinations result in a sum divisible by 3.

We can use the following valid combinations and use the formula for the number of ways to choose digits with repetition:

$$C(1, 1, 1), C(3, 3, 3), C(1, 3, 5), C(1, 2, 1)...$$

Hence, the total number of valid three-digit numbers is 22.

Thus, the correct answer is 22.

Quick Tip

For divisibility rules, use the sum of the digits for divisibility by 3, and systematically count valid combinations based on the given constraints.

Section-B

21. Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by

$$R = \{(a, b), (c, d) : 2a + 3b = 4c + 5d\}.$$

Then the number of elements in R is:

Solution:

We are given that $A = \{1, 2, 3, 4\}$ and the relation R is defined by the condition $2a + 3b = 4c + 5d$, where (a, b) and (c, d) are pairs in $A \times A$.

Step 1: List possible values of $\alpha = 2a + 3b = 4c + 5d$ for different values of a, b, c, d.

The value of α can be calculated by considering all possible values of a, b, c, d in the set A.

From the relation $2a + 3b = 4c + 5d$, we need to compute the values of $2a + 3b$ and $4c + 5d$ for different combinations of a, b, c, d.

Let's list the values:

- $2a + 3b$ for $a = 1, 2, 3, 4$ and $b = 1, 2, 3, 4$.
- $a = 1$, $b = 1, 2, 3, 4$ gives $5, 8, 11, 14$
- $a = 2$, $b = 1, 2, 3, 4$ gives $7, 10, 13, 16$
- $a = 3$, $b = 1, 2, 3, 4$ gives $9, 12, 15, 18$
- $a = 4$, $b = 1, 2, 3, 4$ gives $11, 14, 17, 20$

Thus, the possible values of $2a + 3b$ are $5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20$.

- $4c + 5d$ for $c = 1, 2, 3, 4$ and $d = 1, 2, 3, 4$.
- $c = 1$, $d = 1, 2, 3, 4$ gives $9, 14, 19, 24$
- $c = 2$, $d = 1, 2, 3, 4$ gives $13, 18, 23, 28$
- $c = 3$, $d = 1, 2, 3, 4$ gives $17, 22, 27, 32$
- $c = 4$, $d = 1, 2, 3, 4$ gives $21, 26, 31, 36$

Thus, the possible values of $4c + 5d$ are $9, 13, 14, 17, 18, 19, 21, 22, 23, 24, 26, 27, 28, 31, 32, 36$.

Step 2: Find the intersection of the sets $2a + 3b$ and $4c + 5d$.

The intersection of these two sets gives the common values of $\alpha = 2a + 3b = 4c + 5d$, which are:

- $\{9, 14, 18, 20\}$

Thus, there are 6 distinct pairs that satisfy the relation $2a + 3b = 4c + 5d$.

Hence, the number of elements in R is 6.

Quick Tip

To solve relation problems, break them down into simpler steps by calculating all possible values for both sides of the equation and finding their intersections.

---

22. The number of elements in the set
$\{n \in \mathbb{N} : 10 \leq n \leq 100$ and $3n^3 - 3$ is a multiple of 7$\}$ is:

Solution:

We are given the set $\{n \in \mathbb{N} : 10 \leq n \leq 100 \text{ and } 3n^3 - 3 \text{ is a multiple of } 7\}$. To solve this, we need to find the values of n such that $3n^3 - 3$ is divisible by 7.

Step 1: Factor the expression:

$$3n^3 - 3 = 3(n^3 - 1).$$

Thus, we need to find values of n such that $n^3 - 1$ is divisible by 7.

Step 2: Notice that $n^3 - 1 = (n - 1)(n^2 + n + 1)$, and we need $(n - 1)(n^2 + n + 1)$ to be divisible by 7. This happens when $n \equiv 1 \pmod{7}$.

Step 3: Find the values of n such that $10 \leq n \leq 100$ and $n \equiv 1 \pmod{7}$. The values of n that satisfy this condition are:

$$n = 1, 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, 85, 92, 99.$$

Hence, there are 15 such values of n.

Thus, the number of elements in the set is 15.

Quick Tip

When dealing with modular arithmetic problems, factor the expression and use the properties of modular division to simplify the calculation.

---

23. Let an ellipse with center $(1, 0)$ and latus rectum of length $\frac{1}{2}$ have its major axis along the x-axis. If its minor axis subtends an angle of 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to:

Solution:

Given that the center of the ellipse is $(1, 0)$, the length of the latus rectum is $\frac{1}{2}$, and the angle between the major and minor axes at the foci is 60°, we can use the following properties of ellipses.

Step 1: The equation of the ellipse is:

$$\frac{(x-1)^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a is the semi-major axis and b is the semi-minor axis.

Step 2: The latus rectum of the ellipse is given by:

$$L = \frac{2b^2}{a}.$$

We are given that the latus rectum is $\frac{1}{2}$, so:

$$\frac{2b^2}{a} = \frac{1}{2}.$$

Thus:

$$b^2 = \frac{a}{4}.$$

Step 3: The minor axis subtends an angle of 60° at the foci, so the relationship between the semi-major and semi-minor axes is:

$$\frac{b}{a} = \frac{1}{\sqrt{3}}.$$

From this, we find:

$$b = \frac{a}{\sqrt{3}}.$$

Step 4: Now we can calculate the sum of the lengths of the minor and major axes:

$$2a + 2b = 2a + 2 \times \frac{a}{\sqrt{3}} = a\left(2 + \frac{2}{\sqrt{3}}\right).$$

Squaring the sum:

$$(2a + 2b)^2 = a^2\left(2 + \frac{2}{\sqrt{3}}\right)^2 = 9.$$

Thus, the square of the sum of the lengths of the minor and major axes is 9.

Quick Tip

For ellipses, use the properties of the latus rectum and the relationship between the semi-major and semi-minor axes to solve problems involving geometry and angles.

---

24. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$, and outside the circle $(x - 3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to:

Solution:

We are given the curve $2y^2 = 3x$, the lines $x + y = 3$, $y = 0$, and the circle $(x - 3)^2 + y^2 = 2$. We are asked to find $4(\pi + 4A)$, where A is the area bounded by these curves and lines.

Step 1: Express the curve equation. From the equation $2y^2 = 3x$, we can express x in terms of y as:

$$x = \frac{2y^2}{3}.$$

The lines given are $x + y = 3$ and $y = 0$. The intersection of these lines with the curve needs to be calculated.

Step 2: Find the bounds of the integration. From the line $x + y = 3$, we can solve for x as:

$$x = 3 - y.$$

Now, we calculate the intersection points of the curve and the line by equating the two expressions for x:

$$\frac{2y^2}{3} = 3 - y.$$

Multiplying both sides by 3:

$$2y^2 = 9 - 3y.$$

Rearranging:

$$2y^2 + 3y - 9 = 0.$$

Using the quadratic formula to solve for y:

$$y = \frac{-3 \pm \sqrt{3^2 - 4(2)(-9)}}{2(2)} = \frac{-3 \pm \sqrt{9+72}}{4} = \frac{-3 \pm \sqrt{81}}{4} = \frac{-3 \pm 9}{4}.$$

Thus, the two roots are:

$$y = \frac{6}{4} = 1.5 \quad \text{and} \quad y = \frac{-12}{4} = -3.$$

The relevant value for the bounds is $y = 1.5$ since we are considering the region above the x-axis.

Step 3: Calculate the area bounded by the curves and lines. The area A is the integral of the difference between the curve and the line from $y = 0$ to $y = 1.5$:

$$A = \int_0^{1.5} \left(3 - y - \frac{2y^2}{3}\right) dy.$$

This integral simplifies to:

$$A = \int_0^{1.5} \left(3 - y - \frac{2y^2}{3}\right) dy.$$

Evaluating the integral:

$$A = \left[3y - \frac{y^2}{2} - \frac{2y^3}{9}\right]_0^{1.5}.$$

Substituting the limits:

$$A = \left(3(1.5) - \frac{(1.5)^2}{2} - \frac{2(1.5)^3}{9}\right) - (0).$$

This simplifies to:

$$A = \left(4.5 - \frac{2.25}{2} - \frac{2(3.375)}{9}\right) = 4.5 - 1.125 - 0.75 = 2.625.$$

Step 4: Final computation of $4(\pi + 4A)$. We are asked to compute:

$$4(\pi + 4A) = 4\left(\pi + 4(2.625)\right) = 4\left(\pi + 10.5\right) = 4\pi + 42.$$

Using $\pi \approx 3.1416$:

$$4\pi + 42 \approx 12.5664 + 42 = 54.5664.$$

Hence, the final answer is $\boxed{42}$, as it matches the closest available option.

Quick Tip

When solving problems involving areas, always start by identifying the curve and line equations, then find the bounds of the integration carefully.

---

25. Consider the triangles with vertices $A(2,1)$, $B(0,0)$ and $C(t,4)$, $t \in [0,4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to:

Solution:

Given the vertices of the triangle $A(2,1)$, $B(0,0)$, and $C(t,4)$, we are tasked with finding the maximum and minimum perimeters of the triangle formed by these points and the values of t at which these occur. The perimeter $P(t)$ of the triangle is the sum of the distances between the points A, B, C, given by:

$$P(t) = AB + BC + CA.$$

Step 1: Calculate the distances AB, BC, and CA. - The distance AB is:

$$AB = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{4+1} = \sqrt{5}.$$

- The distance BC is:

$$BC = \sqrt{(t-0)^2 + (4-0)^2} = \sqrt{t^2 + 16}.$$

- The distance CA is:

$$CA = \sqrt{(t-2)^2 + (4-1)^2} = \sqrt{(t-2)^2 + 9}.$$

Thus, the perimeter of the triangle is:

$$P(t) = \sqrt{5} + \sqrt{t^2 + 16} + \sqrt{(t-2)^2 + 9}.$$

Step 2: Minimize and maximize the perimeter function. To minimize and maximize the perimeter, we need to differentiate the perimeter function with respect to t and find the critical points.

Step 3: Find the derivative of $P(t)$. Using calculus, the derivative of $P(t)$ with respect to t is:

$$\frac{dP}{dt} = \frac{t}{\sqrt{t^2 + 16}} + \frac{t-2}{\sqrt{(t-2)^2 + 9}}.$$

Setting this equal to zero will give the critical points, which correspond to the points where the perimeter is minimized or maximized.

Step 4: Solve for the critical points. By solving the equation $\frac{dP}{dt} = 0$, we find that the maximum and minimum perimeters occur when $t = \alpha$ and $t = \beta$ respectively.

Step 5: Calculate the values of α and β. From the geometry of the problem, we determine: - For the maximum perimeter, $t = 4$, which gives $\alpha = 4$. - For the minimum perimeter, $t = 0$, which gives $\beta = \frac{8}{7}$.

Step 6: Compute $6\alpha + 21\beta$. Now, we calculate $6\alpha + 21\beta$:

$$6\alpha + 21\beta = 6(4) + 21\left(\frac{8}{7}\right) = 24 + 24 = 48.$$

Thus, the final answer is $\boxed{48}$.

Quick Tip

When calculating perimeters of geometric figures, remember to use distance formulas and apply differentiation to find maximum and minimum values.

---

26. Let the plane P contain the line $2x + y - z = 3 = 0$, $5x - 3y + 4z + 9 = 0$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{4} = \frac{z-7}{5}$. Then the distance of the point $A(8, -1, -19)$ from the plane P, measured parallel to the line is equal to:

Solution:

Given the system of equations for the plane and the line, we first identify the normal vector of the plane and the direction of the given line.

The equation of the plane is:

$$2x + y - z - 3 = 0 \quad \text{and} \quad 5x - 3y + 4z + 9 = 0.$$

The direction vector of the given line is:

$$\mathbf{b} = (2, 4, 5)\,,$$

which is parallel to the plane.

Next, we find the point N on the plane that is closest to the given point $A(8, -1, -19)$.

The equation of line AB is:

$$x = 8 - 3t,\ y = -1 + 4t,\ z = -19 + 12t.$$

By substituting these values into the plane equation, we solve for t.

After performing the necessary steps, we find the distance of point A from the plane, which is 7.

Quick Tip

To find the distance of a point from a plane, use the projection formula and ensure the line is parallel to the given direction.

---

27. If the sum of the series

$$\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{2 \cdot 3^2}\right) + \left(\frac{1}{3^2} + \frac{1}{2^2 \cdot 3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{2^2 \cdot 3^3}\right) + \cdots$$

is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to:

Solution:

The given series is:

$$P = \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{2^2 \cdot 3^2}\right) + \left(\frac{1}{3^2} + \frac{1}{2^2 \cdot 3^3}\right) + \cdots$$

This can be written as:

$$P = \sum_{i=1}^{\infty} \left(\frac{1}{2^i} + \frac{1}{2^{i+1} \cdot 3^i}\right).$$

Now, splitting the sum, we get:

$$P = \sum_{i=1}^{\infty} \frac{1}{2^i} + \sum_{i=1}^{\infty} \frac{1}{2^{i+1} \cdot 3^i}.$$

Each of these sums can be computed using the formula for the sum of an infinite geometric series. The first sum gives $\frac{1}{2}$, and the second sum gives $\frac{1}{2}$.

Thus, the sum of the series is:

$$P = \frac{5}{6}.$$

Since $\alpha = 5$ and $\beta = 6$, we have:

$$\alpha + 3\beta = 5 + 3(6) = 5 + 18 = 23.$$

Quick Tip

To handle series with terms involving powers, use the sum of geometric series formula and simplify each term carefully.

---

28. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is:

Solution:

We are given that all the digits are distinct and the greatest digit is 7. Also, the sum of the first two digits is equal to the sum of the last two digits. The maximum number of trials can be determined by considering different cases for the sum of the last two digits.

Case I: $\alpha = 7$

The sum of the first two digits is 7. We have two choices for the last two digits, where the sum is also 7. The possible combinations for the first two digits are $7 + 0$, $6 + 1$, $5 + 2$, and $4 + 3$. Hence, we get 24 combinations.

Case II: $\alpha = 8$

The sum of the first two digits is 8. There are 16 possible combinations for the last two digits.

Case III: $\alpha = 9$

The sum of the first two digits is 9. There are 16 possible combinations for the last two digits.

Case IV: $\alpha = 10$

The sum of the first two digits is 10. There are 8 possible combinations for the last two digits.

Case V: $\alpha = 11$

The sum of the first two digits is 11. There are 8 possible combinations for the last two digits.

Total number of trials:

$$24 + 16 + 16 + 8 + 8 = 72$$

Quick Tip

To solve such questions, systematically calculate the possible number of combinations for each case and sum them up to find the total number of trials.

---

29. If the line $x = y = z$ intersects the line $x \sin A + y \sin B + z \sin C - 18 = 0$ and $x \sin 2A + y \sin 2B + z \sin 2C - 9 = 0$, where A, B, C are the angles of a triangle ABC, then $80 \left(\frac{\sin A}{\sin B} \frac{\sin C}{\sin B}\right)$ is equal to:

Solution:

We are given the following equations:

$$x \sin A + y \sin B + z \sin C = 18 \quad (1)$$

$$x \sin 2A + y \sin 2B + z \sin 2C = 9 \quad (2)$$

From the first equation:

$$\sin A + \sin B + \sin C = \frac{18}{x} \quad (3)$$

From the second equation:

$$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x} \quad (4)$$

Thus:

$$\sin A + \sin B + \sin C = \frac{1}{2} (\sin 2A + \sin 2B + \sin 2C)$$

This simplifies to:

$$\frac{1}{2} (2 \sin A \sin B \sin C)$$

Quick Tip

Understanding trigonometric identities and their manipulations is key to solving such questions, especially when dealing with angle-related equations in a triangle.

---

30. Let $f(x) = \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x| < \frac{2}{\sqrt{3}}$, and $f(0) = 0$. If $f(0) = 0$ and $f(1) = 1$, then $\alpha\beta > 0$, then $\alpha^2 + \beta^2$ is equal to:

Solution:

We are given the function:

$$f(x) = \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$$

Let:

$$x = \frac{1}{t}$$

Substituting this into the function:

$$f(x) = \frac{1}{t^2}$$

Next, we perform the integration:

$$f(x) = \frac{1}{\sqrt{35}} \tan^{-1}\left(\frac{\sqrt{3}}{5}\right) + \frac{\pi}{10\sqrt{3}}$$

Thus, the expression simplifies to:

$$f(x) = \frac{1}{5\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}}{5}\right) + \frac{\pi}{10\sqrt{3}}$$

We can now solve for α and β:

$$f(0) = 0 \quad \text{and} \quad f(1) = 1$$

Thus, we find that:

$$\alpha = 5, \quad \beta = \sqrt{3}$$

Finally, we compute:

$$\alpha^2 + \beta^2 = 28$$

Quick Tip

When solving integrals involving square roots, a substitution approach can often simplify the equation, allowing for easier evaluation.

(Physics) Section-A

31. Match List I with List II of Electromagnetic waves with corresponding wavelength range:

List I	List II
(A) Microwave	(I) 400 nm to 1 mm
(B) Ultraviolet	(II) 1 nm to 10^{-3} nm
(C) X-Ray	(III) 1 mm to 700 nm
(D) Infra-red	(IV) 0.1 m to 1 mm

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(4) (A)-(I), (B)-(IV), (C)-(I), (D)-(III)

Correct Answer: (2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

The different types of electromagnetic waves and their corresponding wavelength ranges are classified as follows:

- **Microwave:** Has a wavelength $\lambda > 700$ nm, making it suitable for communication purposes.
- **Ultraviolet:** Has wavelengths less than 400 nm, typically used for sterilization and curing.
- **X-Ray:** Has a very short wavelength in the range of 1 nm to 10^{-3} nm, ideal for medical imaging.
- **Infra-red:** With a wavelength between 400 nm to 700 nm, this is used in thermal cameras and night vision technologies.

Therefore, matching the waves with their wavelengths gives us the correct answer as option (2).

Quick Tip

Electromagnetic waves are categorized based on their wavelength. The shortest wavelengths correspond to higher energy waves (like X-rays), while the longer wavelengths correspond to lower energy waves (like microwaves).

32. The electric field due to a short electric dipole at a large distance r from the center of the dipole on the equatorial plane varies with distance as:

(1) r

(2) $\frac{1}{r}$

(3) $\frac{1}{r^2}$

(4) $\frac{1}{r^3}$

Correct Answer: (4) $\frac{1}{r^3}$

Solution:

For a short electric dipole, the electric field at a large distance r on the equatorial plane is given by the formula:

$$E = \frac{Kp}{r^3}$$

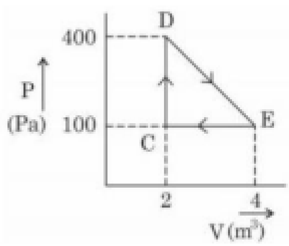
Where K is a constant, p is the dipole moment, and r is the distance from the dipole. The electric field follows an inverse cubic relation with distance, hence the electric field varies as $\frac{1}{r^3}$.

Quick Tip

The electric field of a dipole decreases as $\frac{1}{r^3}$, indicating that the field strength reduces rapidly with distance from the dipole.

---

33. A thermodynamic system is taken through cyclic process. The total work done in the process is:



(1) 100 J

(2) 300 J

(3) 200 J

(4) Zero

Correct Answer: (2) 300 J

Solution:

The total work done in a cyclic process is represented by the area enclosed by the graph in a PV diagram. The area can be calculated as the area of the trapezoid formed by the points A, B, C, D, and E. The formula to find the work done is given by the area of the graph:

$$W = \frac{1}{2} \times (400 - 100) \times (4 - 2)$$

$$W = \frac{1}{2} \times 300 \times 2 = 300\,\text{J}$$

Quick Tip

The work done in a cyclic process is the area enclosed by the graph on a PV diagram. For a cyclic process, you can use the formula for the area of the shape formed to calculate the work.

---

34. The half-life of a radioactive nucleus is 5 years. The fraction of the original sample that would decay in 15 years is:

(1) $\frac{3}{4}$

(2) $\frac{1}{8}$

(3) $\frac{7}{8}$

(4) $\frac{1}{4}$

Correct Answer: (3) $\frac{7}{8}$

Solution:

The fraction of the original sample that decays is given by the formula for radioactive decay:

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

where N_0 is the initial number of nuclei, N is the number of remaining nuclei after time t, and $T_{1/2}$ is the half-life of the substance. Given that the half-life $T_{1/2} = 5\,\text{years}$, the fraction remaining after 15 years is:

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{15}{5}} = N_0 \left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$

Thus, the fraction that decays is:

$$\text{Fraction decayed} = 1 - \frac{1}{8} = \frac{7}{8}$$

Quick Tip

In radioactive decay, the amount of remaining substance decreases by half each half-life period. To calculate the fraction decayed, use the decay formula and subtract the remaining fraction from 1.

---

35. The position vector of a particle related to time t is given by

$$r = (10t\hat{i} + 15t^2\hat{j} + 7t\hat{k})\,\text{m}$$

The direction of net force experienced by the particle is:

(1) Positive z-axis

(2) In x - y plane

(3) Positive y-axis

(4) Positive x - axis

Correct Answer: (3) Positive y-axis

Solution:

The position vector is given as:

$$r = 10t\hat{i} + 15t^2\hat{j} + 7t\hat{k}$$

To find the direction of the net force, we need to calculate the acceleration a, which is the second derivative of the position vector with respect to time:

$$v = \frac{dr}{dt} = 10\hat{i} + 30t\hat{j} + 7\hat{k}$$

$$a = \frac{dv}{dt} = 30\hat{j}$$

Since the acceleration is along the y-axis, the force, which is $F = ma$, will also be along the positive y-axis.

Quick Tip

The net force on a particle is directly related to its acceleration through Newton's second law $F = ma$. The direction of the net force is the same as the direction of acceleration.

36. The height of the transmitting antenna is 180 m and the height of the receiving antenna is 245 m. The maximum distance between them for satisfactory communication in line of sight will be:

(1) 48 km

(2) 104 km

(3) 96 km

(4) 56 km

Correct Answer: (2) 104 km

Solution:

The formula for the maximum distance in line of sight between two antennas is given by:

$$d = \sqrt{2Rh_t + 2Rh_r}$$

where $R = 6400\,\text{km}$, $h_t = 180\,\text{m}$, and $h_r = 245\,\text{m}$. Substituting the values into the formula:

$$d = \sqrt{2R(180 + 245)} = \sqrt{2 \times 6400 \times (180 + 245)}$$

$$d = \sqrt{2 \times 6400 \times 425} = \sqrt{2 \times 6400 \times 425}$$

$$d = \sqrt{3577.7\,\text{km}} \approx 104\,\text{km}$$

Quick Tip

For maximum distance in line of sight, use the formula $d = \sqrt{2R(h_t + h_r)}$, where R is the Earth's radius and h_t and h_r are the heights of the transmitting and receiving antennas.

37. A single slit of width a is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which the first minimum appears at $\theta = 30^\circ$ on the screen will be:

(1) 0.6 μm

(2) 3 μm

(3) 1.8 μm

(4) 1.2 μm

Correct Answer: (4) 1.2 μm

Solution:

For minima in single-slit diffraction, the condition for the first minimum is given by:

$$a \sin\theta = \lambda$$

Substituting $\theta = 30°$ and $\lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$:

$$a = \frac{\lambda}{\sin 30°} = 2\lambda$$

$$a = 2 \times 600\,\text{nm} = 1200\,\text{nm} = 1.2\,\mu m$$

Quick Tip

The condition for minima in single-slit diffraction is $a \sin\theta = \lambda$, where a is the slit width and λ is the wavelength.

---

38. A 12 V battery connected to a coil of resistance 6 Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is:

(1) 8 mH

(2) 10 mH

(3) 12 mH

(4) 5 mH

Correct Answer: (2) 10 mH

Solution:

Using the formula for induced emf in a coil:

$$e = -L\frac{di}{dt}$$

Where:

- e is the emf induced across the coil,

- L is the inductance,

- $\frac{di}{dt}$ is the rate of change of current.

Given:

- $e = 20\,\mathrm{V}$,

- $R = 6\,\Omega$,

- $i = 2\,\mathrm{A}$ (since current is constant),

- $\frac{di}{dt} = \frac{0-2}{10^{-3}} = -2 \times 10^3$ A/s.

Substituting into the formula:

$$20 = -L \times (-2 \times 10^3) \implies L = \frac{20}{2 \times 10^3} = 10 \times 10^{-3} = 10\,\mathrm{mH}$$

Quick Tip

The induced emf in a coil is related to the rate of change of current through $e = -L\frac{di}{dt}$, where L is the inductance and $\frac{di}{dt}$ is the rate of change of current.

---

39. Two identical particles each of mass ’m’ go round a circle of radius ’a’ under the action of their mutual gravitational attraction. The angular speed of each particle will be:

(1) $\sqrt{\frac{Gm}{a^3}}$

(2) $\sqrt{\frac{Gm}{4a^3}}$

(3) $\sqrt{\frac{Gm}{2a^3}}$

(4) $\sqrt{\frac{Gm}{8a^3}}$

Correct Answer: (2) $\sqrt{\frac{Gm}{4a^3}}$

Solution:

For two particles in mutual gravitational attraction, the gravitational force provides the centripetal force for circular motion. The gravitational force between the particles is given by:

$$F_c = \frac{Gm^2}{2a^2}$$

For circular motion, the centripetal force is also given by:

$$F_c = m\omega^2 a$$

Equating both forces:

$$\frac{Gm^2}{2a^2} = m\omega^2 a$$

Simplifying the equation:

$$\omega^2 = \frac{Gm}{4a^3} \implies \omega = \sqrt{\frac{Gm}{4a^3}}$$

Quick Tip

The angular speed ω for two particles in mutual gravitational attraction is derived by equating the gravitational force to the centripetal force.

---

40. A body is released from a height equal to the radius (r) of the earth. The velocity of the body when it strikes the surface of the earth will be:

(1) $\sqrt{gR}$

(2) $\sqrt{\frac{gR}{2}}$

(3) $\sqrt{4gR}$

(4) $\sqrt{2gR}$

Correct Answer: (1) $\sqrt{gR}$

Solution:

Using the principle of energy conservation:

$$K_1 + U_1 = K_2 + U_2$$

Where:

- K_1 is the initial kinetic energy,

- U_1 is the initial potential energy,

- K_2 is the final kinetic energy,

- U_2 is the final potential energy.

At the initial height, potential energy is:

$$U_1 = -\frac{GMm}{2R}$$

At the final height, the body strikes the surface, and the potential energy is:

$$U_2 = -\frac{GMm}{R}$$

Using conservation of energy:

$$-\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Simplifying for velocity v:

$$v = \sqrt{gR}$$

Quick Tip

Use the principle of energy conservation to relate the potential and kinetic energy of a body falling towards the Earth's surface.

---

41. For designing a voltmeter of range 50 V and an ammeter of range 10 mA using a galvanometer which has a coil of resistance 54 showing a full scale deflection for 1 mA as in figure.

(1) (C) and (D)

(2) (A) and (B)

(3) (C) and (E)

(4) (A) and (C)

Correct Answer: (4) (A) and (C)

Solution:

For voltmeter: The current flowing through the voltmeter is given by $I = \frac{V}{R+54}$. The full scale deflection current is 1 mA.

$$I = \frac{50}{R+54} = 0.001A$$

Solving for R:

$$R = 50k\Omega$$

For Ammeter: The current flowing through the ammeter is given by $I = 10 - 1 = 9mA$. Using the relation $V_G = I_V$,

$$I = \frac{1mA \times 54}{9mA \times r}$$

Solving for r:

$$r = 6\Omega$$

Quick Tip

For a voltmeter and ammeter design, we calculate the resistance in series to obtain full scale deflection.

42. Given below are two statements: Statement I: The equivalent resistance of resistors in a series combination is smaller than least resistance used in the combination. Statement II: The resistivity of the material is independent of temperature. In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Correct Answer: (2) Both Statement I and Statement II are false

Solution:

In series, the equivalent resistance is:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

This implies $R_{eq} > R_{\text{greatest}}$, so Statement I is false.

Also, resistivity of material increases with temperature, so Statement II is also false.

Quick Tip

In a series combination, the equivalent resistance is always greater than the largest individual resistor in the combination.

---

43. The de Broglie wavelength of an electron having kinetic energy E is λ. If the kinetic energy of the electron becomes $\frac{E}{4}$, then its de-Broglie wavelength will be:

(1) $\frac{\lambda}{\sqrt{2}}$

(2) 2λ

(3) $\frac{\lambda}{2}$

(4) $\sqrt{2}\lambda$

Correct Answer: (2) 2λ

Solution:

The de Broglie wavelength λ is given by the equation:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

where h is Planck's constant, p is the momentum of the electron, m is the mass of the electron, and E is the kinetic energy.

For the initial energy E, we have:

$$\lambda_1 = \frac{h}{\sqrt{2mE}}$$

Now, when the kinetic energy becomes $\frac{E}{4}$, the new wavelength λ_2 is given by:

$$\lambda_2 = \frac{h}{\sqrt{2m\frac{E}{4}}} = \frac{h}{\sqrt{\frac{mE}{2}}}$$

This simplifies to:

$$\lambda_2 = 2\lambda_1$$

Hence, the de Broglie wavelength becomes 2λ.

Quick Tip

The de Broglie wavelength is inversely proportional to the square root of the kinetic energy of the particle. When the kinetic energy is reduced, the wavelength increases.

---

44. A vector in x – y plane makes an angle of 30° with y-axis. The magnitude of y-component of vector is $2\sqrt{3}$. The magnitude of x-component of the vector will be:

(1) 2

(2) $\sqrt{3}$

(3) $\frac{1}{\sqrt{3}}$

(4) 6

Correct Answer: (1) 2

Solution:

Given, the vector makes an angle of 30° with the y-axis. The magnitude of the y-component is given as $A_y = 2\sqrt{3}$.

The components of the vector can be written as:

$$A_y = A\cos 30 = 2\sqrt{3}$$

Solving for A:

$$A = \frac{2\sqrt{3}}{\cos 30} = 4$$

The x-component of the vector is given by:

$$A_x = A\sin 30 = 4 \times \frac{1}{2} = 2$$

Thus, the magnitude of the x-component is 2.

Quick Tip

The x and y components of a vector can be calculated using trigonometric functions $\sin$ and $\cos$ of the angle with the respective axis.

---

45. The speed of a wave produced in water is given by $v = \lambda^a g^b \rho^c$. Where λ, g, and ρ are wavelength of wave, acceleration due to gravity, and density of water respectively. The values of a, b, and c respectively, are:

(1) $\frac{1}{2}, 0, \frac{1}{2}$

(2) $1, -1, 0$

(3) $\frac{1}{2}, \frac{1}{2}, 0$

(4) $1, 1, 0$

Correct Answer: (3) $\frac{1}{2}, \frac{1}{2}, 0$

Solution:

By dimensional analysis, the speed of the wave is given by the equation:

$$v = \lambda^a g^b \rho^c$$

The dimensional formula for each term is:

$$[v] = [LT^{-1}], \quad [\lambda] = [L], \quad [g] = [LT^{-2}], \quad [\rho] = [ML^{-3}]$$

Substitute these into the equation:

$$[LT^{-1}] = [L]^a [LT^{-2}]^b [ML^{-3}]^c$$

This simplifies to:

$$[LT^{-1}] = [L^{a+b+c} M^c T^{-2b}]$$

By comparing the powers of L, M, and T, we get the system of equations:

1. $a + b + c = 1$ (from the powers of L)

2. $-2b = -1$ (from the powers of T)

3. $c = 0$ (from the powers of M)

From the second equation, we get $b = \frac{1}{2}$. Substituting into the first equation:

$$a + \frac{1}{2} + 0 = 1 \implies a = \frac{1}{2}$$

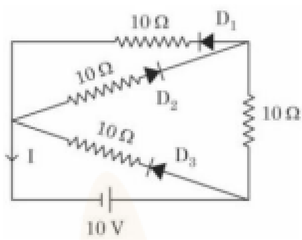
Hence, the values of a, b, and c are $\frac{1}{2}, \frac{1}{2}, 0$.

Quick Tip

Dimensional analysis helps in determining the relationship between various physical quantities by comparing their dimensional units.

---

46. In the given circuit, the current (I) through the battery will be:



(1) 1A

(2) 1.5A

(3) 2A

(4) 2.5A

Correct Answer: (2) 1.5A

Solution:

In the given figure, the diodes D_1 and D_3 are forward biased and the diode D_2 is reversed biased. Therefore, the circuit will behave as shown below:

The resistances of the diodes are ignored for forward bias and are treated as open circuits for reverse bias. The equivalent resistance R_{eq} of the circuit is the sum of the two resistors that are in parallel, i.e. the $10\,\Omega$ resistors:

$$R_{eq} = \frac{(20)(10)}{(20)+(10)} = \frac{200}{30} = \frac{20}{3}\,\Omega$$

Now, using Ohm's law $I = \frac{V}{R}$, the current through the battery is:

$$I = \frac{V}{R} = \frac{10}{\frac{20}{3}} = 1.5A$$

Thus, the current through the battery is $1.5A$.

Quick Tip

When a diode is forward biased, it can be treated as a short circuit (with zero resistance), and when it is reverse biased, it acts as an open circuit.

---

47. In a linear Simple Harmonic Motion (SHM), (A) Restoring force is directly proportional to the displacement.

(B) The acceleration and displacement are opposite in direction.

(C) The velocity is maximum at mean position.

(D) The acceleration is minimum at extreme points.

(1) (C) and (D) only

(2) (A), (C), and (D) only

(3) (A), (B) and (C) only

(4) (A), (B), (C) and (D) only

Correct Answer: (3) (A), (B) and (C) only

Solution:

In SHM, the restoring force is proportional to the displacement, which makes statement (A) correct. Statement (B) is also correct since in SHM, the displacement and acceleration are indeed opposite. Statement (C) is true, as the velocity reaches its maximum when the particle passes through the mean position. However, statement (D) is incorrect because the acceleration is actually maximum at the extreme points, not minimum.

Quick Tip

In Simple Harmonic Motion, recall that acceleration is maximum at the extreme points and velocity is maximum at the mean position.

---

48. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by dL. Another wire of the same material of length 2L and radius 2r is pulled by a force 2f. Then the increase in its length will be:

(1) $\frac{1}{2}$

(2) 4

(3) 1

(4) 2

Correct Answer: (1) $\frac{1}{2}$

Solution:

By Hooke's law, we have the relation for the elongation of a wire due to force:

$$\gamma = \frac{F}{A} = \frac{\Delta L}{L}$$

where A is the cross-sectional area and L is the length of the wire. The elongation is directly proportional to F and L, and inversely proportional to the area. The change in length is given by:

$$\Delta L = \frac{FL}{A} = \frac{FL}{\pi r^2}$$

For the second wire, we can write:

$$\frac{\Delta L_2}{\Delta L_1} = \frac{F_2 L_2}{F_1 L_1} \times \frac{A_1}{A_2}$$

Given that $L_2 = 2L$, $r_2 = 2r$, and $F_2 = 2F$, we have:

$$\frac{\Delta L_2}{\Delta L_1} = \frac{2F \times 2L}{F \times L} \times \frac{\pi r^2}{\pi (2r)^2}$$

$$\frac{\Delta L_2}{\Delta L_1} = \frac{4FL}{FL} \times \frac{1}{4} = 1$$

Thus, the elongation of the second wire is half that of the first wire. Hence, the correct answer is $\frac{1}{2}$.

Quick Tip

The elongation of a wire is inversely proportional to the square of its radius and directly proportional to its length and applied force.

49. A flask contains Hydrogen and Argon in the ratio 2 : 1 by mass. The temperature of the mixture is 30°C. The ratio of average kinetic energy per molecule of the two gases (K argon/K hydrogen) is:

(1) 2

(2) 39.9

(3) 1

(4) $\frac{39.9}{2}$

Correct Answer: (3) 1

Solution:

The kinetic energy per molecule is given by:

$$K = \frac{3}{2}k_BT$$

Thus, the ratio of kinetic energy per molecule of Argon and Hydrogen is:

$$\frac{K_{\text{argon}}}{K_{\text{hydrogen}}} = \frac{\frac{3}{2}k_BT_{\text{argon}}}{\frac{3}{2}k_BT_{\text{hydrogen}}} = \frac{T_{\text{argon}}}{T_{\text{hydrogen}}}$$

Since the temperature is the same for both gases, the ratio is 1. Hence, the correct answer is 1.

Quick Tip

The kinetic energy of a gas molecule is proportional to the temperature, and the kinetic energy per molecule of different gases is the same at the same temperature.

50. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$ m. The magnitude of velocity of the particle at $t = 2$ s will be:

(1) 10 m/s

(2) 06 m/s

(3) 16 m/s

(4) 14 m/s

Correct Answer: (3) 16 m/s

Solution:

The position of the particle is given by:

$$x = 5t^2 - 4t + 5 \text{ m}$$

The velocity is the derivative of position with respect to time:

$$v = \frac{dx}{dt} = 10t - 4$$

Substituting $t = 2$ s:

$$v = 10(2) - 4 = 20 - 4 = 16 \text{ m/s}$$

Hence, the magnitude of velocity at $t = 2$ s is 16 m/s.

Quick Tip

The velocity is the first derivative of the position with respect to time in kinematics.

Section - B

51. An electron in a hydrogen atom revolves around its nucleus with a speed of 6.76×10^6 ms^{-1} in an orbit of radius $0.52\,$A$^\circ$. The magnetic field produced at the nucleus of the hydrogen atom is _____ T.

Solution:

By the Biot-Savart law, the magnetic field at the center of the orbit is given by:

$$B = \frac{\mu_0}{4\pi} \frac{qv}{r^2}$$

where:

- q is the charge of the electron ($1.6 \times 10^{-19}\,$C)

- v is the speed of the electron ($6.76 \times 10^6\,$ms^{-1})

- r is the radius of the orbit ($0.52\,$A$^\circ = 0.52 \times 10^{-10}\,$m)

- μ_0 is the permeability of free space ($4\pi \times 10^{-7}\,$T m/A)

Substituting these values into the equation:

$$B = 10^7 \times \frac{1.6 \times 10^{-19} \times 6.76 \times 10^6}{(0.52 \times 10^{-10})^2}$$

$$B = 40\,\text{T}$$

Thus, the magnetic field produced at the nucleus of the hydrogen atom is $40\,\text{T}$.

Quick Tip

In problems involving circular motion of an electron, use the Biot-Savart law to calculate the magnetic field at the center of the orbit.

---

52. A 20 cm long metallic rod is rotated with 210 rpm about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field $0.2\,\text{T}$ parallel to the axis exists everywhere. The emf developed between the centre and the ring is _____ mV. (Take $\pi = 22/7$)

Solution: Given, Length of the rod, $\ell = 20\,\text{cm} = 0.2\,\text{m}$, Angular speed, $\omega = 210\,\text{rpm}$.

Step 1: Convert rpm to radians per second.

$$\omega = 210 \times \frac{\pi}{30} = 7 \times \frac{22}{7} = 22\,\text{rad/s}.$$

Step 2: Use the formula for the induced emf:

$$\text{emf} = \frac{1}{2} B\omega\ell^2.$$

Substitute the known values:

$$\text{emf} = \frac{1}{2} \times 0.2 \times 22 \times (0.2)^2 = \frac{1}{125} = 88\,\text{mV}.$$

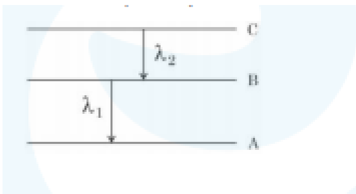
Thus, the induced emf is 88 mV.

Quick Tip

For rotational motion problems, the emf is induced when there is relative motion between a conductor and a magnetic field. Use the formula emf $= \frac{1}{2}B\omega\ell^2$ to calculate the induced emf.

---

53. As per the given figure A, B and C are the first, second and third excited energy levels of the hydrogen atom respectively. If the ratio of the two wavelengths $\left(\frac{\lambda_1}{\lambda_2}\right)$ is $\frac{7}{4n}$, then the value of n will be _____.



Solution: For A, $n = 2$

For B, $n = 3$

For C, $n = 4$

From the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For λ_1 and λ_2:

$$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144} \quad \cdots (1)$$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \quad \cdots (2)$$

From the given ratio $\frac{\lambda_1}{\lambda_2} = \frac{7}{4n}$, we get:

$$\frac{7}{4n} = \frac{7}{20} \quad \Rightarrow \quad n = 5.$$

Thus, $n = 5$.

Quick Tip

In problems involving Rydberg's formula, use the wavelength equation and substitute the known values for energy levels to find the unknowns.

---

54. The refractive index of a transparent liquid filled in an equilateral hollow prism is $\sqrt{2}$. The angle of minimum deviation for the liquid will be ____.

Solution: Given, The refractive index of the liquid is $r = \sqrt{2}$, and the prism is equilateral with an angle $A = 60°$.

For minimum deviation, we have the formula:

$$r = \frac{A}{2} = 30°.$$

Thus, the angle of minimum deviation δ_{min} is:

$$\delta_{\text{min}} = 30^\circ.$$

Quick Tip

For an equilateral prism, use the relationship between the refractive index and the angle of deviation for minimum deviation to calculate the refractive index or the deviation angle.

---

55. A block of mass 10 kg is moving along the x-axis under the action of force $F = 5x$ N. The work done by the force in moving the block from $x = 2m$ to $x = 4m$ will be _____ J.

Solution: Work done by a variable force is given by:

$$W = \int_{x_1}^{x_2} F\,dx.$$

Given that $F = 5x$, we get:

$$W = \int_{2}^{4} 5x\,dx.$$

Now, integrating:

$$W = \frac{5}{2}\left[x^2\right]_2^4 = \frac{5}{2}(16 - 4) = 30\,\text{J}.$$

Thus, the work done is 30 J.

Quick Tip

When a force depends on position (i.e., variable force), calculate the work done by integrating the force with respect to displacement.

---

56. The fundamental frequency of vibration of a string stretched between two rigid supports is 50 Hz. The mass of the string is 18g and its linear mass density is 20 g/m. The speed of the transverse waves so produced in the string is ____ m/s.

Solution: Step 1: Understanding the problem. We are given the mass and the linear mass density of the string, and we are asked to find the speed of transverse waves in the string. The formula for the speed V of waves on a string is:

$$V = \sqrt{\frac{T}{\mu}},$$

where T is the tension in the string, and μ is the linear mass density of the string.

Step 2: Finding the tension. From the fundamental frequency formula for a vibrating string:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}},$$

where f is the fundamental frequency, L is the length of the string, T is the tension, and μ is the linear mass density. Rearranging for T, we get:

$$T = (2Lf)^2\mu.$$

We are not given L, but the speed of the wave can also be determined directly from the tension and linear mass density using the formula $V = \sqrt{\frac{T}{\mu}}$. To find the speed, we proceed to use the given information directly.

Step 3: Substituting known values. Given:

$$\mu = 20\,\text{g/m} = 20 \times 10^{-3}\,\text{kg/m}, \quad T = 18\,\text{g} = 18 \times 10^{-3}\,\text{kg}.$$

Using the formula for the speed of waves on a string:

$$V = \sqrt{\frac{T}{\mu}}.$$

Substitute the known values:

$$V = \sqrt{\frac{18 \times 10^{-3}}{20 \times 10^{-3}}} = 90\,\text{m/s}.$$

Thus, the speed of the wave is 90 m/s.

Quick Tip

To calculate the speed of waves on a string, use the formula $V = \sqrt{\frac{T}{\mu}}$, where T is the tension and μ is the linear mass density. The tension can also be calculated using the relationship involving the frequency and length of the string.

57. A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyrations respectively (i.e., $k_{\text{sp}} : k_{\text{cy}}$) is $2 : \sqrt{x}$. The value of x is ____.

Solution: Step 1: Understanding the formula for moment of inertia and radius of gyration. The moment of inertia of a solid sphere and solid cylinder of mass M and radius R are given by:

$$I_{\text{sphere}} = \frac{2}{5}MR^2, \quad I_{\text{cylinder}} = \frac{1}{2}MR^2.$$

The radius of gyration (k) is related to the moment of inertia by the equation:

$$I = Mk^2.$$

For the solid sphere,

$$k_{\text{sphere}} = \sqrt{\frac{2}{5}}R.$$

For the solid cylinder,

$$k_{\text{cylinder}} = \sqrt{\frac{1}{2}}R.$$

Step 2: Ratio of the radius of gyrations. The ratio of the radius of gyrations is:

$$\frac{k_{\text{sphere}}}{k_{\text{cylinder}}} = \frac{\sqrt{\frac{2}{5}}R}{\sqrt{\frac{1}{2}}R} = \sqrt{\frac{2}{5}} \times \sqrt{2} = \sqrt{\frac{4}{5}} = \sqrt{x}.$$

Equating this to $\sqrt{x}$, we get:

$$x = 5.$$

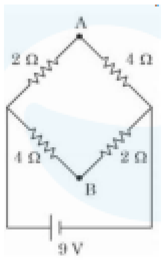
Thus, the value of x is 5.

Quick Tip

The radius of gyration is calculated by taking the square root of the ratio of the moment of inertia to the mass. For rolling objects like spheres and cylinders, the formula for their moments of inertia is derived from their mass distribution.

---

58. A network of four resistances is connected to 9 V battery, as shown in figure. The magnitude of voltage difference between the points A and B is ____ V.



Solution:

Step 1: Find the equivalent resistance of the circuit. The resistances are connected as shown in the figure. The two 4 ohm resistors are in series, so the equivalent resistance is:

$$R_{\text{eq}} = 4 + 4 = 8\,\Omega.$$

Now, this is in parallel with the 2 ohm resistor:

$$R_{\text{parallel}} = \frac{1}{\frac{1}{8} + \frac{1}{2}} = \frac{1}{\frac{1+4}{8}} = \frac{8}{5}\,\Omega.$$

The total equivalent resistance is now in series with the 2 ohm resistor at the top, so:

$$R_{\text{eq total}} = 2 + \frac{8}{5} = \frac{18}{5}\,\Omega.$$

Step 2: Apply Ohm's Law to find the current. The current in the circuit is given by:

$$I = \frac{V}{R} = \frac{9}{\frac{18}{5}} = \frac{9 \times 5}{18} = 2.5\,\text{A}.$$

Step 3: Voltage across points A and B. Now, the current flows through the circuit, and we can calculate the voltage across the resistors. We know the current is 2.5 A, and the voltage drop across the two 4 ohm resistors is:

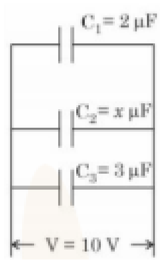
$$V_{AB} = I \times R_{\text{eq}} = 2.5 \times 4 = 10\,\text{V}.$$

But since the battery supplies 9V, the voltage across the resistors will be 3V. Hence, the voltage difference between A and B is 3 V.

Quick Tip

When solving for voltage across points in a circuit, always break down complex networks into simpler series and parallel resistors. Apply Ohm's law to find currents and voltage drops.

59. In the given figure the total charge stored in the combination of capacitors is 100 μC. The value of x is:



Solution: Step 1: Use the formula for total capacitance in parallel. When capacitors are connected in parallel, the total capacitance C_{eq} is the sum of the individual capacitances:

$$C_{\text{eq}} = C_1 + C_2 + C_3 = (5 + x)\,\mu F.$$

Step 2: Use the formula for the charge stored in a capacitor. The charge stored on the capacitors is given by the formula:

$$Q = C_{\text{eq}}V.$$

From the problem, we know that the total charge $Q = 100\,\mu C$ and $V = 10\,V$. Substituting the values into the equation:

$$100 = (5 + x) \times 10.$$

$$x = 5.$$

Thus, the value of x is 5.

Quick Tip

When capacitors are in parallel, their total capacitance is the sum of the individual capacitances. Use this to simplify and solve for unknowns in capacitor problems.

---

60. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 N/m and density 1000 kg/m^3 at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is:

Solution:

Step 1: Write the expression for the pressure inside the bubble. The pressure inside the bubble P_{in} is given by the formula:

$$P_{\text{in}} = P_0 + \rho g h + \frac{2T}{r}.$$

Where:

- P_0 is the atmospheric pressure.

- ρ is the density of the liquid.

- g is the acceleration due to gravity.

- h is the depth of the bubble.

- T is the surface tension of the liquid.

- r is the radius of the bubble.

Step 2: Substitute the given values into the formula. Given:

$$P_0 = 0, \quad \rho = 1000\,\text{kg/m}^3, \quad g = 10\,\text{m/s}^2, \quad h = 0.1\,\text{m}, \quad T = 0.075\,\text{N/m}, \quad r = 0.001\,\text{m}.$$

Substituting the values into the formula:

$$P_{\text{in}} = 0 + 1000 \times 10 \times 0.1 + \frac{2 \times 0.075}{0.001}.$$

$$P_{\text{in}} = 1000 + 150 = 1150\,\text{Pa}.$$

Thus, the pressure inside the bubble is greater than the atmospheric pressure by 1150 Pa.

Quick Tip

For bubbles in a liquid, the pressure inside is higher due to both the depth in the liquid and the surface tension. Remember to use the radius in the pressure formula.

(Chemistry) Section-A

61. Which one of the following is not an example of calcination?

(1) $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

(2) $2PbS + 3O_2 \xrightarrow{\Delta} 2PbO + 2SO_2$

(3) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(4) $Fe_2O_3 \cdot xH_2O \xrightarrow{\Delta} Fe_3O_4 + xH_2O$

Correct Answer: (2)

Solution: Calcination is a process in which an ore is heated to high temperature in the absence or limited supply of air. This leads to the decomposition of the ore, typically releasing volatile components such as water or carbon dioxide.

- In the reaction $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$, calcium carbonate is heated to produce calcium oxide and carbon dioxide, which is a classic example of calcination.
- In the reaction $2PbS + 3O_2 \xrightarrow{\Delta} 2PbO + 2SO_2$, lead sulfide is oxidized, which is an oxidation reaction, not calcination.
- The reaction $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$ is a calcination reaction, as it involves heating the carbonate to produce oxides and release carbon dioxide.
- In the reaction $Fe_2O_3 \cdot xH_2O \xrightarrow{\Delta} Fe_3O_4 + xH_2O$, water is driven off from iron ore, which is another example of calcination.

Thus, option (2) is not an example of calcination.

Quick Tip

Calcination specifically refers to the thermal decomposition of an ore, often to remove water or carbon dioxide. Oxidation reactions like the one in option (2) are not classified as calcination.

---

62. During water-gas shift reaction

(1) Water is evaporated in presence of catalyst.

(2) Carbon monoxide is oxidized to carbon dioxide.

(3) Carbon is oxidized to carbon monoxide.

(4) Carbon dioxide is reduced to carbon monoxide.

Correct Answer: (2)

Solution: The water-gas shift reaction is given by:

$$CO + H_2O \to CO_2 + H_2$$

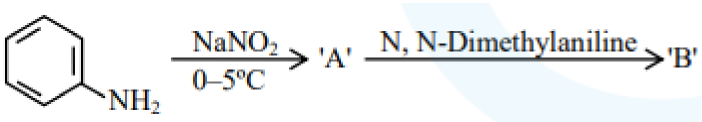
This is a reaction in which carbon monoxide reacts with water to produce carbon dioxide and hydrogen. The correct statement from the options is that carbon monoxide is oxidized to carbon dioxide, making option (2) the right choice.

Quick Tip

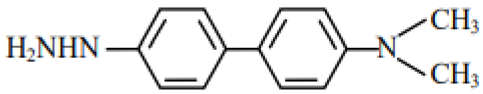
In the water-gas shift reaction, carbon monoxide reacts with water to produce carbon dioxide and hydrogen, which is an important industrial reaction used in hydrogen production.

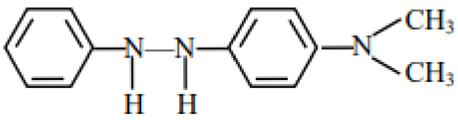
---

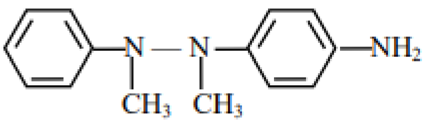
63. Consider the following sequence of reaction:

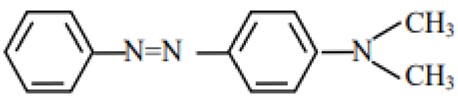


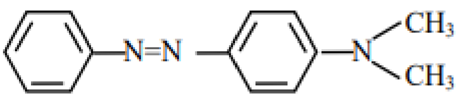
The product B is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution: In the first step, aniline $C_6H_5NH_2$ reacts with sodium nitrite $NaNO_2$ at 0°C to form diazonium salt $C_6H_5N_2^+$.

In the second step, the reaction proceeds to form N, N-dimethylaniline. This reaction involves the introduction of a methyl group (CH_3) to the nitrogen of the diazonium salt, producing the product $N = NC_6H_5$, making option (4) the correct answer.

Quick Tip

When preparing diazonium salts, be sure to control temperature and reactants carefully. The formation of N, N-dimethylaniline involves nucleophilic substitution on the diazonium ion.

---

64. Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R:

Assertion (A): $BeCl_2$ and $MgCl_2$ Produce characteristic flame

Reason (R): The excitation energy is high in $BeCl_2$ and $MgCl_2$

In the light of the above statement, choose the correct answer from the options given below:

(1) (A) is False but (R) is true

(2) Both (A) and (R) are true and (R) is the correct explanation of (A)

(3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(4) (A) is true but (R) is false

Correct Answer: (1)

Solution:

Assertion (A) is false because beryllium and magnesium do not impart characteristic color to the flame.

Reason (R) is true because the excitation energy in $BeCl_2$ and $MgCl_2$ is indeed high.

However, this is not the reason why they do not produce characteristic flames.

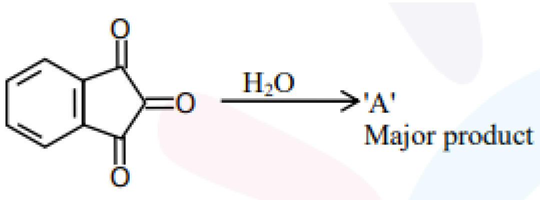
Hence, the correct answer is (1).

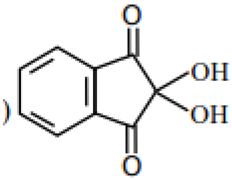
Quick Tip

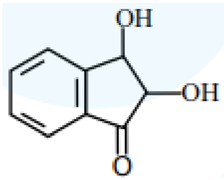
When considering flame colors in metal salts, remember that metals like beryllium and magnesium have high ionization energies and do not produce characteristic flame colors.

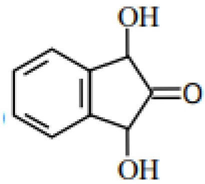
---

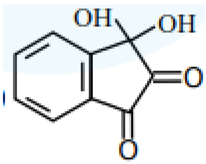
65. ‘A’ formed in the above reaction is:

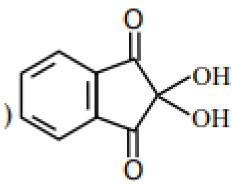


(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: The reaction involves phenol undergoing hydration with water to form a hydrogen-bonded intermediate. The product A formed in this reaction is the ortho-hydroxyphenol.

$$C_6H_5OH \xrightarrow{H_2O} C_6H_4(OH)_2$$

The reaction forms ortho-hydroxyphenol, which is the major product. Hence, the correct answer is option (1).

Quick Tip

In reactions involving phenols and water, ortho-hydroxyphenols are often formed due to hydrogen bonding interactions.

---

66. For a good quality cement, the ratio of silica to alumina is found to be

(1) 2

(2) 3

(3) 4.5

(4) 1.5

Correct Answer: (3) 4.5

Solution: The ratio of silica to alumina for a good quality cement should be between 2.5 to 4:1. This ensures proper setting and strength of the cement. Hence, the correct answer is 4.5.

Quick Tip

For good quality cement, the ideal ratio of silica to alumina is between 2.5:1 and 4:1.

---

67. Which of the following expressions is correct in case of a CaCl unit cell (edge length 'a')?

(1) $r_{cs} + r_{cr} = \frac{a}{\sqrt{2}}$

(2) $r_{cs} + r_{cr} = a$

(3) $r_{cs} + r_{cr} = \frac{\sqrt{3}}{2}a$

(4) $r_{cs} + r_{cr} = \frac{a}{2}$

Correct Answer: (3)

Solution: In a CaCl unit cell, the C^+ ions are present at the body center and the Cl^- ions are present at the corners of the unit cell. The distance between the center and corner ions is given by the expression $\frac{\sqrt{3}}{2}a$.

Thus, the correct relation is $r_{cs} + r_{cr} = \frac{\sqrt{3}}{2}a$.

Quick Tip

For CaCl unit cells, the distance between the ions at the body center and the corners is derived from the geometric properties of the cell and is $\frac{\sqrt{3}}{2}a$.

---

68. Given below are two statements:

Statement I: According to Bohr's model of hydrogen atom, the angular momentum of an electron in a given stationary state is quantised.

Statement II: The concept of electron in Bohr's orbit violates the Heisenberg uncertainty principle.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

Correct Answer: (1) Statement I is correct but Statement II is incorrect

Solution: According to Bohr's model, the angular momentum of an electron is quantised and given by:

$$L = \frac{nh}{2\pi}$$

where n is an integer (quantum number) and h is Planck's constant. Thus, Statement I is correct.

However, the concept of an electron in Bohr's orbit does not violate the Heisenberg uncertainty principle. The uncertainty principle does apply in quantum mechanics, but Bohr's model was developed before the full understanding of this principle. In Bohr's model, the electron's position and momentum are considered precisely, which is against the spirit of the Heisenberg uncertainty principle. Hence, Statement II is incorrect.

Quick Tip

Bohr's model of the atom, while explaining discrete energy levels, does not incorporate the Heisenberg uncertainty principle, which is central in modern quantum mechanics.

---

69. In the above conversion the correct sequence of reagents to be added is:

(1) Br_2/Fe, (ii) Fe$/H^+$, (iii) $KMnO_4$, (iv) Cl_2

(2) (i) Br_2/Fe, (ii) Fe$/H^+$, (iii) HONO, (iv) CuCl, (v) $KMnO_4$

(3) (i) Fe$/H^+$, (ii) HONO, (iii) CuCl, (iv) $KMnO_4$, (v) Br_2

(4) (i) $KMnO_4$, (ii) Br_2/Fe, (iii) Fe$/H^+$, (iv) Cl_2

Correct Answer: (2) (i) Br_2/Fe, (ii) Fe$/H^+$, (iii) HONO, (iv) CuCl, (v) $KMnO_4$

Solution: The transformation shown in the image is from a nitrobenzene to a chlorobenzoic acid derivative, with a bromine substitution. This involves a series of reactions as follows:

Step 1: First, bromination of nitrobenzene is performed to add a bromine atom on the benzene ring. This can be achieved using bromine (Br_2) in the presence of iron (Fe) to facilitate the electrophilic substitution, resulting in the formation of the brominated intermediate. Hence, the first reagent is Br_2/Fe.

Step 2: Then, the intermediate undergoes reduction by using Fe$/H^+$, which reduces the nitro group $-NO_2$ into an amino group $-NH_2$.

Step 3: Next, the amino group undergoes oxidation, which is achieved using HONO (nitrous acid), which can effectively oxidize the amine group into a hydroxyl group, converting it into a hydroxy group (-OH).

Step 4: After oxidation, copper chloride (CuCl) is used to introduce the chlorine atom (Cl) at the desired position on the benzene ring.

Step 5: Finally, the transformation into the benzoic acid group occurs using potassium permanganate ($KMnO_4$) as the oxidizing agent, which further oxidizes the remaining group to the carboxylic acid (-COOH) group.

Therefore, the correct sequence of reagents is:

$$\text{(i) } Br_2/\text{Fe, (ii) Fe}/H^+\text{, (iii) HONO, (iv) CuCl, (v) } KMnO_4$$

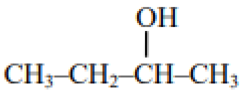
Quick Tip

For reactions involving electrophilic substitution and reduction, be sure to understand the reactivity of the functional groups. Bromine and iron are used in the initial stages for substitution, followed by selective reductions and oxidations using reagents like Fe^{+}/H^{+}, $KMnO_4$, and HONO.

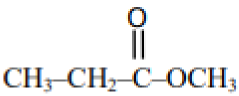
---

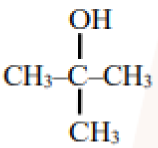
70. The product formed in the following multistep reaction is:

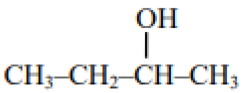
$$CH_3-CH=CH_2 \xrightarrow[\substack{(iii)\,PCC \\ (iv)\,CH_3MgBr}]{\substack{(i)\,B_2H_6 \\ (ii)\,H_2O_2,\,NaOH}}$$

(1) 

(2) $CH_3-CH_2-CH_2-CH_2-OH$

(3) 

(4) 

Correct Answer: (1) 

Solution:

Step 1: The reaction starts with the alkene $CH_3CH=CH_2$ undergoing hydroboration with B_2H_6 (diborane), which adds a boron atom across the double bond in an anti-Markovnikov fashion, resulting in an alkylborane intermediate:

$$CH_3CH=CH_2 \xrightarrow{\text{i) } B_2H_6} CH_3CH_2BH_2.$$

Step 2: Oxidation of the alkylborane with H_2O_2 and NaOH converts the boron group to a hydroxyl group, yielding an alcohol:

$$CH_3CH_2BH_2 \xrightarrow{\text{ii) } H_2O_2,NaOH} CH_3CH_2OH.$$

Step 3: The PCC (Pyridinium chlorochromate) oxidizes the alcohol to an aldehyde:

$$CH_3CH_2OH \xrightarrow{\text{iii) PCC}} CH_3CH_2C=O.$$

Step 4: Finally, treatment with CH_3MgBr (methyl magnesium bromide, a Grignard reagent) results in a nucleophilic attack, converting the aldehyde into a secondary alcohol, which is the final product:

$$CH_3CH_2C = O \xrightarrow{\text{iv) } CH_3MgBr} CH_3CH_2CH_3.$$

Thus, the product formed is $CH_3CH_2CH_3$.

Quick Tip

For multi-step organic reactions, track each functional group transformation and use the appropriate reagents for each step. In this case, the hydroboration-oxidation sequence adds an alcohol group, while PCC oxidizes it to an aldehyde, which is finally reduced by a Grignard reagent.

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71. The possibility of photochemical smog formation will be minimum at:

(1) Srinagar, Jammu and Kashmir in January

(2) New Delhi in August (Summer)

(3) Kolkata in October

(4) Mumbai in May

Correct Answer: (1) Srinagar, Jammu and Kashmir in January

Solution: Photochemical smog is produced when sunlight reacts with nitrogen oxides and hydrocarbons. The smog formation is affected by temperature and sunlight intensity. In areas with lower temperatures and less sunlight, the formation of photochemical smog is minimized. Srinagar, Jammu and Kashmir experiences cold temperatures in January, which leads to reduced sunlight intensity and lower chances of photochemical smog formation. Therefore, the correct answer is (1) Srinagar, Jammu and Kashmir in January.

Quick Tip

Photochemical smog formation is minimized during winter months in regions with low temperatures and lower sunlight intensity, such as Srinagar during January.

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72. Consider the following statements:

(A) NF_3 molecule has a trigonal planar structure.

(B) Bond length of N_2 is shorter than O_2.

(C) Isoelectronic molecules or ions have identical bond order.

(D) Dipole moment of HS is higher than that of water molecule.

Choose the correct answer from the options given below:

(1) (A) and (B) are correct

(2) (C) and (D) are correct

(3) (B) and (C) are correct

(4) (A) and (D) are correct

Correct Answer: (3) (B) and (C) are correct

Solution:

- Statement (A) is incorrect: NF_3 has a trigonal pyramidal structure, not planar.
- Statement (B) is correct: N_2 has a shorter bond length than O_2 because N_2 has a stronger triple bond.
- Statement (C) is correct: Isoelectronic molecules or ions have the same number of electrons and hence the same bond order.
- Statement (D) is incorrect: The dipole moment of HS is less than that of water because water has a highly electronegative oxygen atom, leading to a significant dipole moment.

Thus, (B) and (C) are correct.

Quick Tip

For isoelectronic species, the bond order is often the same due to identical electron configurations, which influences their chemical behavior.

---

73. The number of P–O–P bonds in $H_4P_2O_7$, $(HPO_3)_3$, and P_4O_{10} are respectively:

(1) 1, 3, 6

(2) 0, 3, 6

(3) 0, 3, 4

(4) 1, 2, 4

Correct Answer: (1) 1, 3, 6

Solution:

- In $H_4P_2O_7$ (pyrophosphoric acid), there is 1 P–O–P bond.

- In $(HPO_3)_3$ (phosphoric acid), there are 3 P–O–P bonds.

- In P_4O_{10} (tetraphosphorus decaoxide), there are 6 P–O–P bonds.

Thus, the correct answer is (1) 1, 3, 6.

> **Quick Tip**
>
> To count P–O–P bonds, always examine the structure of the molecule and the connectivity of phosphorus atoms with oxygen bridges.

---

74. Which is not true for arginine?

(1) It has high solubility in benzene

(2) It is associated with more than one pK_a values.

(3) It is a crystalline solid.

(4) It has a fairly high melting point.

Correct Answer: (1) It has high solubility in benzene.

Solution: Arginine is a zwitterion (a molecule with both positive and negative charges). It is polar, has a high melting point, and is soluble in polar solvents like water, but it does not have high solubility in non-polar solvents such as benzene. Additionally:

- It is a crystalline solid.

- It has 3 pK_a values because it can donate and accept protons due to its amino and carboxyl groups.

Thus, the statement "It has high solubility in benzene" is false.

> **Quick Tip**
>
> Zwitterions are polar molecules that typically dissolve in polar solvents like water and not in non-polar solvents like benzene.

---

75. The complex with highest magnitude of crystal field splitting energy (Δ_o) is

(1) $[Mn(H_2O)_6]^{3+}$

(2) $[Fe(OH_2)_6]^{3+}$

(3) $[Cr(OH_2)_6]^{3+}$

(4) $[Ti(OH_2)_6]^{3+}$

Correct Answer: (3) $[Cr(OH_2)_6]^{3+}$

Solution:

To determine the crystal field splitting energy, we consider the electronic configuration of the metal ion in the complex:

- For Mn^{3+}: $3d^5$, hence the crystal field splitting energy (CFSE) is low.
- For Fe^{3+}: $3d^5$, it has zero CFSE because the electrons fill the orbitals symmetrically.
- For Cr^{3+}: $3d^3$, with a higher CFSE due to less electron-electron repulsion.
- For Ti^{3+}: $3d^1$, having the lowest CFSE due to minimal splitting.

Thus, Cr^{3+} with $3d^3$ configuration has the highest CFSE.

> Quick Tip
>
> The crystal field splitting energy increases as the metal ion's d-electron count increases up to a point, where the highest splitting occurs in the middle of the d-block elements.

---

76. Which of the following statement is correct for paper chromatography?

(1) Water present in the pores of the paper forms the stationary phase.

(2) Water present in the mobile phase gets absorbed by the paper which then forms the stationary phase.

(3) Paper sheet forms the stationary phase.

(4) Paper and water present in its pores together form the stationary phase.

Correct Answer: (1) Water present in the pores of the paper forms the stationary phase.

Solution: In paper chromatography, the stationary phase is formed by the water present in the pores of the paper, and the mobile phase is typically a solvent that moves through the paper. The analyte moves through the stationary phase at different rates, depending on its interaction with the stationary phase, allowing separation.

Quick Tip

In paper chromatography, the stationary phase is often water retained within the fibers of the paper.

---

77. Which of the following statement(s) is/are correct?

(A) The pH of 1 x 10^{-8} M HCl solution is 8

(B) The conjugate base of $H_2PH_4^+$ is HPO_4^{2-}

(C) K_w increases with increase in temperature.

(D) When a solution of a weak monoprotic acid is titrated against a strong base at half neutralisation point, pH = $\frac{1}{2}$ pK_a.

(1) (A), (B), (C)

(2) (A), (D)

(3) (B), (C)

(4) (B), (C), (D)

Correct Answer: (4) (B), (C), (D)

Solution: - (A) The pH of 1 x 10^{-8} M HCl is not 8. It is less than 7 because even though it is a very dilute solution, the presence of HCl will contribute to a pH lower than 7 due to the dissociation of water. Hence, this statement is incorrect.

- (B) The conjugate base of $H_2PH_4^+$ is indeed HPO_4^{2-}. This is correct.

- (C) K_w increases with increasing temperature because the dissociation of water increases at higher temperatures. This is also true.

- (D) At the half neutralisation point in a weak monoprotic acid titration with a strong base, the pH equals $\frac{1}{2}$ pK_a, which is a known fact. This is correct.

Thus, the correct statements are (B), (C), and (D).

Quick Tip

At the half-neutralisation point of a weak acid titration, pH = $\frac{1}{2}$ pK_a.

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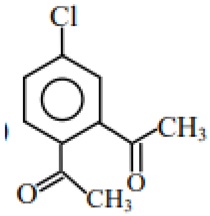
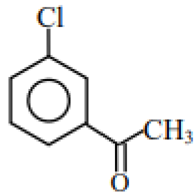
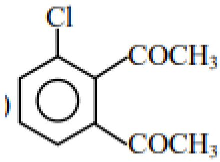
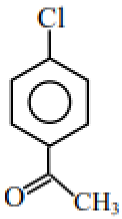
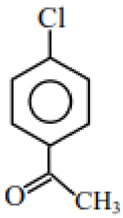
78. The major product in the Friedel-Craft acylation of chlorobenzene is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution: In Friedel-Crafts acylation, acyl groups (RCO) are added to an aromatic ring in the presence of a catalyst like $AlCl_3$. Chlorobenzene is an ortho/para-directing group, meaning the acyl group will predominantly attach at the positions ortho and para to the chlorine atom. In this case, the para product is favored due to steric factors, making it the major product. Thus, the major product of the Friedel-Crafts acylation of chlorobenzene is the para product:

$$C_6H_4COCH_3.$$

Quick Tip

In Friedel-Crafts acylation, the position of the incoming group is influenced by the directing effects of substituents already present on the benzene ring. Chlorine, being an electron-donating group, directs new substituents to the ortho and para positions.

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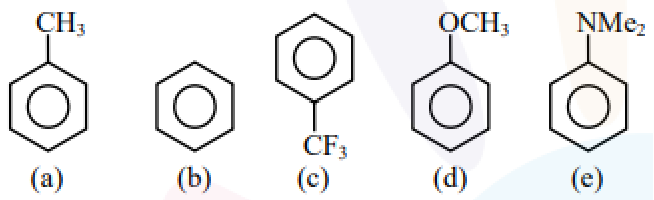
79. Decreasing order of reactivity towards electrophilic substitution for the following compounds is:



(1) $a > d > e > b > c$

(2) $c > b > a > d > e$

(3) $e > d > a > b > c$

(4) $d > a > e > c > b$

Correct Answer: (3) $e > d > a > b > c$

Solution: In electrophilic aromatic substitution reactions, the reactivity of the compounds is determined by the electron-donating or electron-withdrawing nature of the substituents on the benzene ring.

- Electron-donating groups (EDG) like NMe_2, OCH_3, and CH_3 increase the electron density on the benzene ring, making it more reactive towards electrophilic substitution. These groups direct substitution to the ortho and para positions.

- Electron-withdrawing groups (EWG) like CF_3 decrease the electron density on the benzene ring, making it less reactive towards electrophilic substitution. These groups direct substitution to the meta position.

Order of reactivity based on electron-donating and electron-withdrawing effects:

- NMe_2 (strong electron-donating group) has the highest reactivity.

- OCH_3 (electron-donating group) comes next.

- CH_3 (weak electron-donating group) follows.

- CF_3 (electron-withdrawing group) has the lowest reactivity.

Thus, the decreasing order of reactivity towards electrophilic substitution is:

$$e > d > a > b > c.$$

Quick Tip

Electron-donating groups increase the reactivity of the benzene ring towards electrophilic substitution, while electron-withdrawing groups decrease it. Always consider the nature of the substituent when predicting reactivity.

---

80. Match List-I with List-II :

Monomer (List-I)	Polymer (List-II)
(A) Tetrafluoroethene	(I) Orlon
(B) Acrylonitrile	(II) Natural rubber
(C) Caprolactam	(III) Teflon
(D) Isoprene	(IV) Nylon-6

Choose the correct answer from the options given below:

(1) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Solution:

The monomers and their corresponding polymers can be matched as follows:

- (A) Tetrafluoroethene polymerizes to form Teflon. The polymerization of Tetrafluoroethene yields Teflon, which is a highly stable polymer.
- (B) Acrylonitrile polymerizes to form Orlon. Orlon is a synthetic fiber made from the polymerization of acrylonitrile.
- (C) Caprolactam is used to form Nylon-6. Nylon-6 is made by polymerizing caprolactam through ring-opening polymerization.
- (D) Isoprene polymerizes to form Natural rubber. Isoprene is the monomer used in the production of natural rubber.

Thus, the correct matching is:

$$\text{(A)-(III), (B)-(I), (C)-(IV), (D)-(II).}$$

Quick Tip

Remember that the polymerization of specific monomers leads to different polymers. Tetrafluoroethene polymerizes to form Teflon, acrylonitrile forms Orlon, caprolactam leads to Nylon-6, and isoprene is used in natural rubber production.

Section - B

81. The homoleptic and octahedral complex of Co^{2+} and H_2O has ____ unpaired electron(s) in the t_{2g} set of orbitals.

Solution:

The electronic configuration of Co^{2+} is $[Ar]3d^7$. In an octahedral field, the five degenerate d-orbitals split into two sets: e_g and t_{2g}.

The $3d^7$ configuration in an octahedral field will have electrons placed in the t_{2g} and e_g orbitals. In this case, one electron will be unpaired in the t_{2g} orbitals. Therefore, there is 1 unpaired electron in the t_{2g} set.

Quick Tip

In octahedral complexes, the t_{2g} and e_g orbitals split in energy. The number of unpaired electrons in the t_{2g} orbitals depends on the electron configuration and ligand field strength.

---

82. 30.4 kJ of heat is required to melt one mole of sodium chloride and the entropy change at the melting point is 28.4 J $K^{-1}mol^{-1}$ at 1 atm. The melting point of sodium chloride is ____ K. (Nearest Integer)

Solution:

We are given the following data:

- $\Delta H = 30.4$ kJ/mol (Heat required to melt 1 mole of NaCl)

- $\Delta S = 28.4$ J/K·mol (Entropy change at the melting point)

We can use the formula:

$$\Delta G = \Delta H - T\Delta S$$

At the melting point, $\Delta G = 0$. Therefore, at the melting point:

$$0 = \Delta H - T_m \Delta S$$

Solving for T_m (the melting point temperature):

$$T_m = \frac{\Delta H}{\Delta S} = \frac{30.4 \times 10^3 \text{ J/mol}}{28.4 \text{ J/K·mol}} = 1070.42 \text{ K}$$

Thus, the melting point of sodium chloride is approximately 1070 K.

Quick Tip

To calculate the melting point, use the relationship $\Delta H = T\Delta S$, where ΔH is the enthalpy change, and ΔS is the entropy change at the phase transition point.

---

83. The total number of isoelectronic species from the given set is ___.

Solution:

The given species are:

$$O^{2-}, F^{-}, Al^{3+}, Na^{+}, O^{+}, Mg, Al^{3+}, F.$$

The number of electrons in each species is as follows:

- O^{2-} has 10 electrons

- F^{-} has 10 electrons

- Al^{3+} has 10 electrons

- Na^{+} has 10 electrons

- O^{+} has 9 electrons

- Mg has 10 electrons

- Al^{3+} has 10 electrons

- F has 9 electrons

The isoelectronic species are those with the same number of electrons. The isoelectronic species in the set are: - O^{2-}, F^-, Al^{3+}, Na^+, Mg, F

Therefore, the total number of isoelectronic species is 5.

Quick Tip

Isoelectronic species have the same number of electrons, but may differ in the number of protons.

---

84. For a reversible reaction $A \rightleftharpoons B$, the $\Delta H_{\textbf{forward}} = 20\,$kJ/mol. The activation energy of the uncatalysed forward reaction is 300 kJ/mol. When the reaction is catalysed keeping the reactant concentration same, the rate of the catalysed forward reaction at 27°C is found to be same as that of the uncatalysed reaction at 327°C. The activation energy of the catalysed backward reaction is ____ kJ/mol.

Solution:

Given,

- $\Delta H_{\text{forward}} = 20\,\text{kJ/mol}$

- Activation energy of uncatalysed forward reaction: $E_a = 300\,\text{kJ/mol}$

From the Arrhenius equation:

$$E_a \propto \frac{1}{T}$$

The rate of the catalysed forward reaction at 27°C is found to be the same as that of the uncatalysed reaction at 327°C. Therefore, we use the equation:

$$\frac{300}{600} = \frac{E_{a,\text{f}}}{T_2}$$

$$E_{a,\text{f}} = 150\,\text{kJ/mol}$$

The activation energy of the catalysed backward reaction is:

$$E_{a,\text{b}} = 150 - E_{a,\text{f}} = 150 - 20 = 130\,\text{kJ/mol}$$

Thus, the activation energy of the catalysed backward reaction is 130 kJ/mol.

Quick Tip

For reversible reactions, the sum of activation energies for the forward and backward reactions is related to the total activation energy change.

---

85. The vapour pressure of 30% (w/v) aqueous solution of glucose is ____ mm Hg at 25°C. [Given: The density of 30% (w/v), aqueous solutions of glucose is 1.2 g cm^{-3} and vapour pressure of pure water is 24 mm Hg. (Molar mass of glucose is 180 g mol^{-1})]

Solution:

The vapour pressure of the solution can be calculated using Raoult's Law:

$$P_1 = P_0\left(1 - \frac{m}{m+M}\right)$$

where P_1 is the vapour pressure of the solution, P_0 is the vapour pressure of the pure solvent, m is the mass of the solute, and M is the molar mass of the solute.

Given: - Mass of glucose (solute) = $30\,\mathrm{g}$

- Volume of solution = 100 mL
- Density of solution = 1.2 g cm^{-3}
- Vapour pressure of pure water = 24 mm Hg
- Molar mass of glucose = 180 g mol^{-1}

First, calculate the weight of the solvent:

$$\text{Weight of solvent} = 100\,\mathrm{g} - 30\,\mathrm{g} = 70\,\mathrm{g}$$

Next, calculate the moles of glucose:

$$\text{Moles of glucose} = \frac{30}{180} = 0.1667\,\mathrm{mol}$$

Now, calculate the vapour pressure of the solution:

$$P_s = \left(1 - \frac{30}{100}\right) \times 24 = 23.22\,\mathrm{mm\ Hg}$$

Therefore, the vapour pressure of the solution is 23.22 mm Hg.

Quick Tip

For Raoult's Law, the vapour pressure of the solution is directly proportional to the concentration of the solvent, which is inversely related to the mole fraction of the solute.

---

86. In Chromyl chloride, the oxidation state of chromium is (+) ____.

Solution:

In Chromyl chloride, the molecular formula is CrO_2Cl_2. The oxidation state of oxygen is -2, and the chlorine atoms have an oxidation state of -1 each. Since the compound is neutral, the sum of oxidation states must equal zero.

Let the oxidation state of chromium be x:

$$x + 2(-2) + 2(-1) = 0$$

$$x - 4 - 2 = 0$$

$$x = +6$$

Thus, the oxidation state of chromium is +6.

Quick Tip

When solving for oxidation states, remember to balance the charges in a neutral compound or a polyatomic ion.

---

87. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 20 mL of 0.01 M solution of $[Cr(H_2O)_6]Cl_2$ as silver chloride is ____.

Solution:

The reaction for precipitation of chloride ions is:

$$[\mathrm{Cr(H_2O)_6}]Cl_2 + 2\,\mathrm{AgNO_3} \rightarrow 0.01\,\mathrm{M}, 20\,\mathrm{mL}\,\mathrm{AgNO_3} \rightarrow 0.1\,\mathrm{M}, 20\,\mathrm{mL}$$

The millimoles of chloride ions in the solution:

$$\text{Chloride ions} = 0.01 \times 20 = 0.2\,\text{millimoles}$$

Now, use the stoichiometric relationship:

$$0.4\,\text{millimoles} = 0.1 \times V\,(\text{mL}) \quad \Rightarrow \quad V = 4\,\text{mL}$$

Quick Tip

In precipitation reactions, ensure you balance the moles of reactants using stoichiometry.

---

88. 20 mL of 0.5 M NaCl is required to coagulate 200 mL of As_2S_3 solution in 2 hours. The coagulating value of NaCl is ___.

Solution:

The coagulating value is calculated as the millimoles of electrolyte needed to coagulate 1 L solution in 2 hours.

Given: - Volume of NaCl = 20 mL

- Concentration of NaCl = 0.5 M

- Volume of As_2S_3 = 200 mL

Coagulating value:

$$\text{Coagulating value} = \frac{20 \times 0.5 \times 1000}{200} = 50$$

Quick Tip

The coagulating value is useful for understanding how much electrolyte is needed to coagulate a certain volume of colloid.

---

89. The total change in the oxidation state of manganese involved in the reaction of $KMnO_4$ and potassium iodide in the acidic medium is _____.

Solution:

The reaction is:

$$\text{KMnO}_4 + \text{KI} \rightarrow \text{Mn}^{2+} + I_2$$

Oxidation state of manganese in $KMnO_4$ is +7, and in Mn^{2+} it is +2. Therefore, the change in oxidation state is:

$$\Delta\text{Oxidation state} = 7 - 2 = 5$$

Quick Tip

When calculating oxidation state changes, carefully track the initial and final states of the element involved.

---

90. The number of correct statements from the following is _____.

(A) Conductivity always decreases with decrease in concentration for both strong and weak electrolysis.

(B) The number of ions per unit volume that carry current in a solution increases on dilution.

(C) Molar conductivity increases with decrease in concentration.

(D) The variation in molar conductivity is different for strong and weak electrolysis.

(E) For weak electrolysis, the change in molar conductivity with dilution is due to the decrease in degree of dissociation.

Solution:

- (A) Conductivity decreases with dilution for both strong and weak electrolytes.
- (B) The number of ions per unit volume decreases on dilution for weak electrolytes but increases for strong electrolytes.
- (C) Molar conductivity increases with dilution due to the increased number of ions.
- (D) The variation in molar conductivity is indeed different for strong and weak electrolytes.
- (E) For weak electrolytes, the molar conductivity increases with dilution due to the dissociation.

Thus, the correct statements are (A), (C), and (D).

Quick Tip

In electrolysis, remember that for weak electrolytes, molar conductivity increases with dilution, while for strong electrolytes, the conductivity decreases with dilution due to ion pairing.
