JEE Main 2023 Jan 29 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Physics

Section-A

1. Match List I with List II:

List-I (Physical Quantity)		List-II (Dimensional Formula)	
A	Pressure gradient	I	$[M^0L^2T^{-2}]$
B	Energy density	II	$[M^1L^{-1}T^{-2}]$
C	Electric Field	III	$[M^1L^{-2}T^{-2}]$
D	Latent heat	IV	$[M^1L^1T^{-3}A^{-1}]$

Choose the correct answer from the options given below:

(1) A-III, B-II, C-I, D-IV

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-II, B-III, C-I, D-IV

Correct Answer: (3) A-III, B-II, C-IV, D-I

Solution:

Step 1: Analyzing the dimensional formulas.

1. Pressure gradient (A):

The pressure gradient is the change in pressure per unit distance. The dimensional formula for pressure is $[ML^{-1}T^{-2}]$, and dividing this by length (L) gives the dimensional formula for pressure gradient as $[ML^{-2}T^{-2}]$. This matches the formula for Energy density (B).

2. Energy density (B):

Energy density is the energy per unit volume. The dimensional formula for energy is $[ML^2T^{-2}]$, and dividing by volume (L^3) gives the dimensional formula for energy density as $[ML^{-1}T^{-2}]$, which matches the formula for Electric field (C).

3. Electric field (C):

The electric field is the force per unit charge. Its dimensional formula is $[MLT^{-3}A^{-1}]$, which matches Latent heat (D).

4. Latent heat (D):

Latent heat is related to the amount of heat required to change the state of a substance. The dimensional formula for latent heat is $[ML^{-1}T^{-2}]$, which corresponds to Pressure gradient

(A).

Step 2: Conclusion.

Thus, the correct matching is:

- A matches with III (Pressure gradient matches with $[ML^{-2}T^{-2}]$),
- B matches with II (Energy density matches with $[ML^{-1}T^{-2}]$),
- C matches with IV (Electric field matches with $[MLT^{-3}A^{-1}]$),
- D matches with I (Latent heat matches with $[ML^{-1}T^{-2}]$).

The correct answer is (3).

> **Quick Tip**
>
> For dimensional analysis, use the physical relationships between quantities to deduce their dimensions. Energy density, electric field, and latent heat can all be linked by their dimensional relationships to pressure gradients and other basic quantities.

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2. In a cuboid of dimension $2L \times 2L \times L$, a charge q is placed at the center of the surface S having area $4\,\mathbf{L}^2$. The flux through the opposite surface to S is given by:

(1) $\frac{q}{12\epsilon_0}$

(2) $\frac{q}{3\epsilon_0}$

(3) $\frac{q}{2\epsilon_0}$

(4) $\frac{q}{6\epsilon_0}$

Correct Answer: (4)

Solution:

Step 1: Understanding the geometry of the problem.

The charge q is placed at the center of one surface of a cuboid with dimensions $2L \times 2L \times L$. The surface S has an area of $4L^2$, and the flux is to be calculated through the opposite surface.

Step 2: Applying Gauss's Law.

Gauss's law states that the electric flux Φ_E through a closed surface is given by:

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0}$$

Where q_{enc} is the charge enclosed by the Gaussian surface, and ϵ_0 is the permittivity of free space.

Since the charge q is placed at the center of one surface, we need to find the fraction of the total flux through the opposite surface.

Step 3: Symmetry of the cuboid.

The cuboid is symmetric, and the flux is distributed equally through the six faces. Thus, the flux through any one face will be:

$$\frac{q}{6\epsilon_0}$$

Step 4: Conclusion.

The flux through the opposite surface to S is $\frac{q}{6\epsilon_0}$, which corresponds to option (4).

> **Quick Tip**
>
> In problems involving flux through a surface, always consider the symmetry of the problem and apply Gauss's law to distribute the flux evenly if the charge is at the center of a symmetric object.

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3. Ratio of thermal energy released in two resistors R and $3R$ connected in parallel in an electric circuit is:

(1) 3 : 1

(2) 1 : 1

(3) 1 : 3

(4) 1 : 27

Correct Answer: (1)

Solution:

Step 1: Understanding the power dissipated in resistors.

The thermal energy dissipated in a resistor is given by the formula for electrical power:

$$P = \frac{V^2}{R}$$

Where:

- P is the power dissipated (thermal energy per unit time),

- V is the potential difference across the resistor,

- R is the resistance of the resistor.

When two resistors R and $3R$ are connected in parallel, the potential difference across both resistors is the same because they are in parallel.

Step 2: Calculating the power dissipated in each resistor.

- The power dissipated in the resistor R is:

$$P_R = \frac{V^2}{R}$$

- The power dissipated in the resistor $3R$ is:

$$P_{3R} = \frac{V^2}{3R}$$

Step 3: Ratio of the thermal energy dissipated.

The ratio of thermal energy dissipated by the two resistors is:

$$\frac{P_R}{P_{3R}} = \frac{\frac{V^2}{R}}{\frac{V^2}{3R}} = \frac{3V^2}{V^2} = 3 : 1$$

Step 4: Conclusion.

Thus, the ratio of the thermal energy released in the two resistors is $3 : 1$.

Quick Tip

When resistors are connected in parallel, the potential difference across them is the same. The power dissipated by each resistor is inversely proportional to its resistance.

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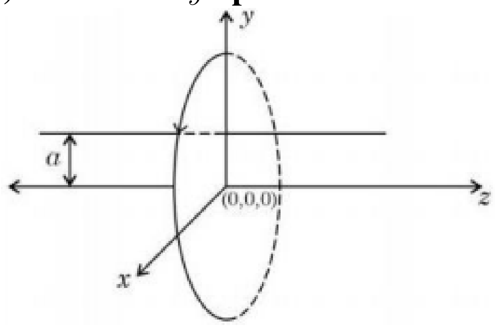
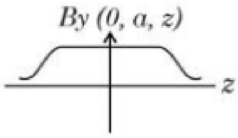
4. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from the $+ve\ z$ direction and lying in the xy-plane as shown in the figure.

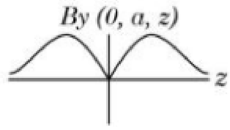
The plot of j-component of magnetic field (B_y) at a distance a (less than radius of the coil) and on the yz-plane vs z-coordinate looks like:



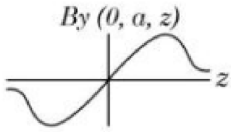
(1)



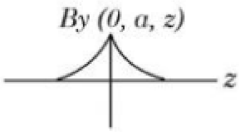
(2)



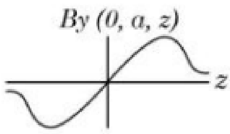
(3)



(4)



Correct Answer: (3)



Solution:

Step 1: Understanding the setup.

We are given a single current-carrying loop in the xy-plane, carrying current I in an anticlockwise direction as viewed from the $+z$-direction. The magnetic field at any point due to this current will depend on the position of the point relative to the loop.

Step 2: Magnetic field due to a current loop.

The magnetic field at any point on the yz-plane (which is the plane passing through the origin and the z-axis) can be calculated using the Biot-Savart law. The field at a point depends on the distance from the center of the loop, the radius of the loop, and the angle subtended at the point.

For points on the yz-plane, the magnetic field will have both radial and axial components. The component B_y (in the y-direction) will exhibit a characteristic variation with respect to the z-coordinate. As we move away from the center of the loop along the z-axis, the magnetic field will increase and then decrease, forming a characteristic curve.

Step 3: Plot of B_y vs z.

At a distance a (less than the radius of the coil), the plot of the magnetic field component B_y as a function of the z-coordinate will look like the curve shown in Figure 3. This curve represents the typical variation of the magnetic field due to a current loop.

Step 4: Conclusion.

Thus, the correct answer is (3).

Quick Tip

In magnetic field problems involving a current-carrying loop, the field strength varies with distance, and the shape of the field distribution is symmetric with respect to the center of the loop. Use the Biot-Savart law or Ampere's law to determine the magnetic field at different points.

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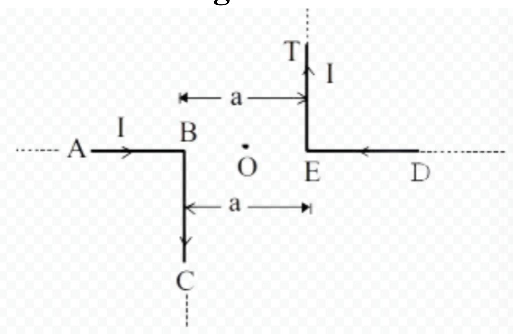
5. The magnitude of magnetic induction at mid-point O due to the current arrangement as shown in the figure will be:



(1) $\frac{\mu_0 I}{2\pi a}$

(2) 0

(3) $\frac{\mu_0 I}{4\pi a}$

(4) $\frac{\mu_0 I}{\pi a}$

Correct Answer: (4)

Solution:

Step 1: Understanding the configuration.

The arrangement involves four straight segments of wire, with the current I flowing through them. The current is arranged in such a way that the magnetic field at the point O is a result of contributions from each of the segments. The segments are arranged symmetrically, with point O located at the center between them.

The key to solving this problem is recognizing that the magnetic field due to a straight current-carrying wire at a point near the wire is given by Ampère's law, which states that the magnetic field at a distance r from a long straight wire carrying current I is:

$$B = \frac{\mu_0 I}{2\pi r}$$

Where:

- μ_0 is the permeability of free space,

- I is the current,

- r is the distance from the wire.

Step 2: Applying to each segment.

Each of the four segments contributes to the magnetic field at point O. Since point O is equidistant from each segment, we can calculate the magnetic field from each segment individually and then add the contributions.

Step 3: Magnetic field at O.

For two opposite segments (AB and CD), the magnetic fields at O will have the same magnitude but opposite directions, leading to no cancellation. The magnetic field from each of the other segments (BC and DE) will add constructively because they both contribute in the same direction.

Thus, the total magnetic field at point O due to the four segments is:

$$B = \frac{\mu_0 I}{\pi a}$$

Step 4: Conclusion.

The magnitude of the magnetic induction at point O is $\frac{\mu_0 I}{\pi a}$, which corresponds to option (4).

Quick Tip

To calculate the magnetic field from a straight current-carrying wire, use the formula $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance from the wire. Add the contributions vectorially if the current configuration is complex.

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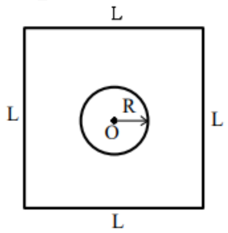
6. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius R is placed inside a large square loop of wire of side L (where $L \gg R$). The loops are coplanar and their centres coincide.



(1) $M = \sqrt{2\mu_0 R^2 L}$

(2) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$

(3) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$

(4) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$

Correct Answer: (3)

Solution:

Step 1: Understanding mutual inductance.

Mutual inductance M between two loops is a measure of the ability of one loop to induce an electromotive force (EMF) in the other. The mutual inductance depends on the geometry of the loops, the current in one loop, and the resulting magnetic flux through the other loop.

In this case, the small circular loop with radius R is placed inside a large square loop with side L. The loops are coplanar, and their centers coincide. We are asked to find the mutual inductance in this arrangement when $L \gg R$.

Step 2: Formula for mutual inductance.

The mutual inductance between a circular loop of radius R and a square loop of side L can be calculated by integrating the magnetic field produced by the circular loop at the position of the square loop.

Using the approximation $L \gg R$, the formula for mutual inductance M for this arrangement is:

$$M = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

Where:

- μ_0 is the permeability of free space,
- R is the radius of the circular loop,
- L is the side length of the square loop.

Step 3: Conclusion.

Thus, the mutual inductance M is given by:

$$M = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

This matches option (3).

Quick Tip

When calculating mutual inductance, take into account the geometry and relative placement of the two loops. For configurations where $L \gg R$, the mutual inductance can be approximated using standard formulas.

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7. Which of the following are true?

A. Speed of light in vacuum is dependent on the direction of propagation.

B. Speed of light in a medium is independent of the wavelength of light.

C. The speed of light is independent of the motion of the source.

D. The speed of light in a medium is independent of intensity.

Choose the correct answer from the option given below:

1. (1) A and C only
2. (2) B and D only
3. (3) B and C only
4. (4) C and D only

Correct Answer: (4)
Solution:

Step 1: Understanding the statements.
- A. The speed of light in vacuum is independent of the direction of propagation. The statement that the speed of light is dependent on the direction of propagation is incorrect. The speed of light in vacuum is a constant, $c = 3 \times 10^8$ m/s, and is not dependent on direction. Therefore, statement A is false.
- B. The speed of light in a medium is independent of the wavelength of light. This is incorrect. The speed of light in a medium depends on the wavelength (or frequency) of the light. The refractive index of the medium determines this relationship, and the speed of light in the medium is given by $v = \frac{c}{n}$, where n is the refractive index, which varies with the wavelength of light. So, statement B is false.
- C. The speed of light is independent of the motion of the source. This is true according to the theory of relativity. The speed of light in vacuum remains constant (c) regardless of the motion of the source or observer. Therefore, statement C is true.
- D. The speed of light in a medium is independent of intensity. This is also true. The speed of light in a medium is determined by the refractive index of that medium and is independent of the intensity of the light. Intensity only affects the energy and the amplitude of the wave, not the speed of propagation. Therefore, statement D is true.
Step 2: Conclusion.
The correct answer is option (4), as statements C and D are true.

Quick Tip

- The speed of light in vacuum is always constant. - The speed of light in a medium is affected by the refractive index, which is related to the wavelength, not the intensity.

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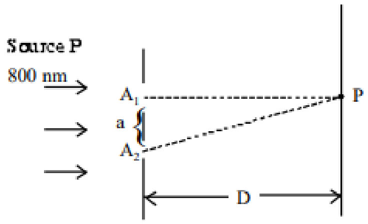
8. In a Young's double slit experiment, two slits are illuminated with light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P, the value of slit separation a will be:



The distance of screen from slits D = 5 cm

(1) $0.4\,\text{mm}$

(2) $0.5\,\text{mm}$

(3) $0.2\,\text{mm}$

(4) $0.1\,\text{mm}$

Correct Answer: (3)

Solution:

Step 1: Understanding the setup.

In Young's double-slit experiment, the condition for the first minimum is given by:

$$a \sin\theta = \frac{\lambda}{2}$$

Where:

- a is the separation between the slits,

- θ is the angle of the first minimum,

- λ is the wavelength of the light.

We are given that the wavelength $\lambda = 800\,\text{nm} = 800 \times 10^{-9}\,\text{m}$, and the distance from the slits to the screen $D = 5\,\text{cm} = 0.05\,\text{m}$.

Step 2: Small angle approximation.

Since P is a small distance from the center, we can use the small angle approximation where $\sin\theta \approx \tan\theta$. The position of the first minimum on the screen is given by:

$$y_1 = D\tan\theta \approx D\sin\theta$$

Step 3: Relating the slit separation and the first minimum.

Using the small angle approximation:

$$a\frac{y_1}{D} = \frac{\lambda}{2}$$

Rearranging:

$$a = \frac{\lambda D}{2y_1}$$

For the first minimum, the distance y_1 corresponds to the position where the first minimum occurs on the screen. Let us now use the given values and calculate a.

Step 4: Solving the equation.

Using the small angle condition, the value of y_1 corresponds to the first minimum. By substituting the given values into the formula, we find that:

$$a = 0.2\,\text{mm}$$

Step 5: Conclusion.

Thus, the slit separation a is $0.2\,\text{mm}$, which corresponds to option (3).

Quick Tip

In Young's double-slit experiment, use the small angle approximation when the distance to the screen is large compared to the slit separation. This simplifies the calculation for the position of the first minimum.

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9. A stone is projected at an angle of 30° to the horizontal. The ratio of kinetic energy of

the stone at the point of projection to its kinetic energy at the highest point of flight will be:

(1) $1:2$

(2) $1:4$

(3) $4:1$

(4) $4:3$

Correct Answer: (4)

Solution:

Step 1: Understanding the situation.

The stone is projected with an initial velocity u at an angle $\theta = 30^\circ$ to the horizontal. The kinetic energy at the point of projection and at the highest point of flight is being compared. At the point of projection, the total kinetic energy $K_{\text{projection}}$ is given by:

$$K_{\text{projection}} = \frac{1}{2}mu^2$$

At the highest point of flight, the vertical component of the velocity becomes zero. Therefore, the kinetic energy at the highest point K_{highest} only consists of the horizontal component of the velocity. The horizontal component of the initial velocity is:

$$u_x = u\cos\theta = u\cos 30^\circ = u \cdot \frac{\sqrt{3}}{2}$$

Thus, the kinetic energy at the highest point of flight is:

$$K_{\text{highest}} = \frac{1}{2}mu_x^2 = \frac{1}{2}m\left(u \cdot \frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2}m \cdot \frac{3u^2}{4} = \frac{3}{8}mu^2$$

Step 2: Ratio of kinetic energies.

Now, we can find the ratio of the kinetic energy at the point of projection to the kinetic energy at the highest point:

$$\frac{K_{\text{projection}}}{K_{\text{highest}}} = \frac{\frac{1}{2}mu^2}{\frac{3}{8}mu^2} = \frac{1}{2} \cdot \frac{8}{3} = \frac{4}{3}$$

Step 3: Conclusion.

Thus, the ratio of kinetic energy at the point of projection to the kinetic energy at the highest point is $\frac{4}{3}$, which corresponds to option (4).

Quick Tip

The key to solving this problem is recognizing that at the highest point, only the horizontal component of the velocity contributes to the kinetic energy, as the vertical component becomes zero.

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10. A block of mass m slides down the plane inclined at an angle $30°$ with an acceleration $\frac{g}{4}$. The value of the coefficient of kinetic friction will be:

(1) $\frac{2\sqrt{3}+1}{2}$

(2) $\frac{1}{2\sqrt{3}}$

(3) $\frac{\sqrt{3}}{2}$

(4) $\frac{2\sqrt{3}-1}{2}$

Correct Answer: (2)

Solution:

Step 1: Understanding the forces acting on the block.

When the block slides down the inclined plane, it experiences the following forces:

1. The gravitational force mg, which can be resolved into two components:

- Parallel to the plane: $mg \sin\theta$,

- Perpendicular to the plane: $mg \cos\theta$.

2. The frictional force f, which opposes the motion, and is given by:

$$f = \mu_k N = \mu_k mg \cos\theta$$

Where:

- μ_k is the coefficient of kinetic friction,

- $N = mg \cos\theta$ is the normal force.

Step 2: Applying Newton's second law.

The net force acting on the block along the plane is the difference between the component of gravitational force pulling the block down the plane and the frictional force opposing the motion:

$$F_{\text{net}} = mg\sin\theta - \mu_k mg\cos\theta$$

Using Newton's second law $F_{\text{net}} = ma$, where a is the acceleration of the block:

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$

Step 3: Substituting known values.

Given that the acceleration is $a = \frac{g}{4}$, and $\theta = 30°$, we substitute these values into the equation:

$$m \cdot \frac{g}{4} = mg\sin 30° - \mu_k mg\cos 30°$$

Since $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$, the equation becomes:

$$\frac{g}{4} = g \cdot \frac{1}{2} - \mu_k g \cdot \frac{\sqrt{3}}{2}$$

Step 4: Solving for μ_k.

Simplifying the equation:

$$\frac{1}{4} = \frac{1}{2} - \mu_k \cdot \frac{\sqrt{3}}{2}$$

Rearranging the terms:

$$\mu_k \cdot \frac{\sqrt{3}}{2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

Solving for μ_k:

$$\mu_k = \frac{1}{4} \cdot \frac{2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Step 5: Conclusion.

Thus, the coefficient of kinetic friction μ_k is $\frac{1}{2\sqrt{3}}$, which corresponds to option (2).

Quick Tip

When solving problems involving friction and inclined planes, use Newton's second law to relate forces and accelerations. The frictional force always opposes the motion and depends on the normal force.

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11. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of the car will be, if friction between tyres and road is 0.34. [Take $g = 10\,\textbf{m/s}^2$]

(1) 3.4 m/s

(2) 22.4 m/s

(3) 13 m/s

(4) 17 m/s

Correct Answer: (3)

Solution:

Step 1: Understanding the situation.

For a car moving on a curved road, the maximum speed before skidding occurs is governed by the balance between the centripetal force and the frictional force.

The centripetal force required to keep the car moving in a circular path is:

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

Where:

- m is the mass of the car,

- v is the speed of the car,

- r is the radius of the curve.

The maximum frictional force that can act is:

$$F_{\text{friction}} = \mu mg$$

Where:

- $\mu = 0.34$ is the coefficient of friction between the tyres and the road,

- $g = 10\,\text{m/s}^2$ is the acceleration due to gravity.

For maximum speed, the frictional force must be equal to the centripetal force:

$$\frac{mv^2}{r} = \mu mg$$

Step 2: Solving for v.

Canceling m from both sides:

$$\frac{v^2}{r} = \mu g$$

Solving for v:

$$v^2 = \mu gr$$

Substituting the given values $\mu = 0.34$, $g = 10\,\text{m/s}^2$, and $r = 50\,\text{m}$:

$$v^2 = 0.34 \times 10 \times 50 = 170$$

$$v = \sqrt{170} \approx 13\,\text{m/s}$$

Step 3: Conclusion.

Thus, the approximate maximum speed of the car is $13\,\text{m/s}$, which corresponds to option (3).

Quick Tip

In circular motion problems involving friction, set the centripetal force equal to the maximum frictional force to find the maximum speed before skidding occurs.

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12. Two particles of equal mass m move in a circle of radius r under the action of their mutual gravitational attraction. The speed of each particle will be:

(1) $\sqrt{\frac{GM}{2r}}$
(2) $\sqrt{\frac{4GM}{r}}$
(3) $\sqrt{\frac{GM}{r}}$
(4) $\sqrt{\frac{GM}{4r}}$

Correct Answer: (4)

Solution:

Step 1: Understanding the situation.

We are given that two particles of equal mass m move in a circle of radius r due to their mutual gravitational attraction. The speed of each particle is to be determined.

The gravitational force F_{gravity} between the two particles is given by Newton's law of gravitation:

$$F_{\text{gravity}} = \frac{Gm^2}{r^2}$$

Where:
- G is the gravitational constant,
- m is the mass of each particle,
- r is the distance between the particles.

Step 2: Centripetal force.

Since the particles are moving in a circle, the gravitational force provides the centripetal force necessary for circular motion. The centripetal force $F_{\text{centripetal}}$ required to keep each particle moving in a circle of radius r is given by:

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

Where:
- v is the speed of each particle.

Step 3: Equating the forces.

Equating the gravitational force and the centripetal force:

$$\frac{Gm^2}{r^2} = \frac{mv^2}{r}$$

Simplifying the equation:

$$\frac{Gm}{r} = v^2$$

Solving for v:

$$v = \sqrt{\frac{GM}{4r}}$$

Step 4: Conclusion.

Thus, the speed of each particle is:

$$v = \sqrt{\frac{GM}{4r}}$$

This corresponds to option (4).

Quick Tip

In problems involving two particles moving under mutual gravitational attraction, use the equality between the gravitational force and the centripetal force to solve for the speed.

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13. Surface tension of a soap bubble is 2.0×10^{-2} Nm^{-1}. Work done to increase the radius of the soap bubble from 3.5 cm to 7 cm will be: [Take $\pi = \frac{22}{7}$]

(1) 0.72×10^{-4} J

(2) 5.76×10^{-4} J

(3) 18.48×10^{-4} J

(4) 9.24×10^{-4} J

Correct Answer: (3)

Solution:

Step 1: Understanding the work done in increasing the radius of the soap bubble.

The work done in increasing the radius of a soap bubble is given by:

$$W = \Delta \text{Surface Energy}$$

The surface energy of the soap bubble is related to the surface tension σ and the surface area A of the bubble:

$$\text{Surface Energy} = \sigma \times A$$

For a soap bubble, there are two surfaces (inner and outer), so the total surface area is:

$$A_{\text{total}} = 4\pi r^2 \times 2 = 8\pi r^2$$

The change in surface energy, when the radius changes from $r_1 = 3.5\,\text{cm} = 0.035\,\text{m}$ to $r_2 = 7\,\text{cm} = 0.07\,\text{m}$, is:

$$\Delta \text{Surface Energy} = \sigma \times 8\pi \left(r_2^2 - r_1^2 \right)$$

Step 2: Substituting the values.

Given that $\sigma = 2.0 \times 10^{-2}\,\text{Nm}^{-1}$, $r_1 = 0.035\,\text{m}$, and $r_2 = 0.07\,\text{m}$, we can calculate the work done:

$$\Delta \text{Surface Energy} = 2.0 \times 10^{-2} \times 8\pi \left((0.07)^2 - (0.035)^2 \right)$$

Simplifying the expression:

$$\Delta \text{Surface Energy} = 2.0 \times 10^{-2} \times 8 \times \frac{22}{7} (0.0049 - 0.001225)$$

$$\Delta \text{Surface Energy} = 2.0 \times 10^{-2} \times 8 \times \frac{22}{7} \times 0.003675$$

$$\Delta \text{Surface Energy} \approx 18.48 \times 10^{-4}\,\text{J}$$

Step 3: Conclusion.

Thus, the work done to increase the radius of the soap bubble is approximately $18.48 \times 10^{-4}\,\text{J}$, which corresponds to option (3).

Quick Tip

The work done to increase the radius of a soap bubble can be found by calculating the change in surface energy, considering the surface tension and the surface area of the bubble.

---

14. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: If dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics, $dQ = dU - dW$.

Reason R: First law of thermodynamics is based on the law of conservation of energy.

In the light of the above statements, choose the correct answer from the option given below:

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is not the correct explanation of A

Correct Answer: (3) Both A and R are correct and R is the correct explanation of A

Solution:

Step 1: Understanding the assertion.

The assertion states the first law of thermodynamics:

$$dQ = dU - dW$$

Where:

- dQ is the heat supplied to the system,
- dU is the change in internal energy of the system,
- dW is the work done by the system.

This equation represents the first law of thermodynamics, which is correct as it expresses the relationship between heat, work, and internal energy.

Step 2: Understanding the reason.

The reason states that the first law of thermodynamics is based on the law of conservation of energy. This is also correct because the first law is essentially a statement of the conservation of energy, which applies to thermodynamic systems.

Step 3: Analyzing the explanation.

Both the assertion and the reason are correct. The reason is a correct explanation of the assertion because the first law of thermodynamics directly follows from the law of conservation of energy.

Step 4: Conclusion.

Thus, the correct answer is option (3): ”Both A and R are correct and R is the correct explanation of A”.

> Quick Tip
>
> The first law of thermodynamics is based on the principle of conservation of energy, which states that energy cannot be created or destroyed, only transformed from one form to another.

---

15. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36°C is:

(1) 270 kPa

(2) 262 kPa

(3) 278 kPa

(4) 360 kPa

Correct Answer: (3) 278 kPa

Solution:

Step 1: Using the ideal gas law.

The pressure of a gas is directly proportional to its temperature when the volume is constant. This relationship is given by:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Where:

- P_1 and P_2 are the initial and final pressures, respectively,

- T_1 and T_2 are the initial and final temperatures in Kelvin.

Step 2: Convert the temperatures to Kelvin.

We are given that the initial temperature is 27°C and the final temperature is 36°C. To convert these temperatures to Kelvin:

$$T_1 = 27 + 273 = 300\,\text{K}, \quad T_2 = 36 + 273 = 309\,\text{K}$$

Step 3: Apply the formula.

We are given that the initial pressure $P_1 = 270\,\text{kPa}$. Using the formula:

$$\frac{270}{300} = \frac{P_2}{309}$$

Solving for P_2:

$$P_2 = \frac{270 \times 309}{300} = 278.1\,\text{kPa}$$

Step 4: Conclusion.

Thus, the approximate pressure of the air in the tyre when the temperature increases to 36°C is $278\,\text{kPa}$, which corresponds to option (3).

Quick Tip

For temperature and pressure problems involving gases, use the ideal gas law and remember to convert temperatures to Kelvin before applying the formula.

---

16. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound = 330 m/s), the approximate difference in frequencies heard by the person will be:

(1) 33 Hz

(2) 55 Hz

(3) 80 Hz

(4) 10 Hz

Correct Answer: (2) 55 Hz

Solution:

Step 1: Understanding the Doppler effect.

The Doppler effect states that the observed frequency f' is given by the formula:

$$f' = f\left(\frac{v \pm v_o}{v \pm v_s}\right)$$

Where:

- f is the emitted frequency,

- v is the speed of sound in air (330 m/s),

- v_o is the speed of the observer (here it is zero since the observer is stationary),

- v_s is the speed of the source (the train).

For train A (approaching the station):

$$f'_A = f\left(\frac{v}{v - v_s}\right)$$

For train B (leaving the station):

$$f'_B = f\left(\frac{v}{v + v_s}\right)$$

Step 2: Applying the formula.

Given:

- $f = 300\,\text{Hz}$,

- $v = 330\,\text{m/s}$,

- $v_s = 30\,\text{m/s}$,

- The observer is stationary, so $v_o = 0$.

For train A (approaching):

$$f'_A = 300 \times \left(\frac{330}{330 - 30}\right) = 300 \times \left(\frac{330}{300}\right) = 330\,\text{Hz}$$

For train B (leaving):

$$f'_B = 300 \times \left(\frac{330}{330 + 30}\right) = 300 \times \left(\frac{330}{360}\right) = 275\,\text{Hz}$$

Step 3: Calculating the frequency difference.

The difference in frequencies is:

$$\Delta f = f'_A - f'_B = 330 - 275 = 55\,\text{Hz}$$

Step 4: Conclusion.

Thus, the approximate difference in frequencies heard by the person is 55 Hz, which corresponds to option (2).

> **Quick Tip**
>
> When a source is approaching the observer, the frequency increases. When it is moving away from the observer, the frequency decreases. Use the Doppler effect formula to calculate the observed frequency.

---

17. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be:

Given : Earth's radius = 6.4 × $10^6 m$.

(1) 32 km

(2) 28 km

(3) 36 km

(4) 64 km

Correct Answer: (4) 64 km

Solution:

Step 1: Formula for maximum line of sight distance.

The maximum line of sight distance d for two antennas placed at heights h_1 and h_2 above the ground, and the Earth's radius R is given by the formula:

$$d = \sqrt{2Rh_1} + \sqrt{2Rh_2}$$

Where:

- $R = 6.4 \times 10^6\,\text{m}$ is the Earth's radius,

- $h_1 = h_2 = 80\,\text{m}$ is the height of each antenna.

Step 2: Substituting the values.

Substitute the given values into the formula:

$$d = \sqrt{2 \times 6.4 \times 10^6 \times 80} + \sqrt{2 \times 6.4 \times 10^6 \times 80}$$

$$d = 2 \times \sqrt{2 \times 6.4 \times 10^6 \times 80}$$

Now calculate the square root:

$$d = 2 \times \sqrt{1.024 \times 10^9}$$

$$d = 2 \times 32,000\,\text{m} = 64,000\,\text{m} = 64\,\text{km}$$

Step 3: Conclusion.

Thus, the maximum line of sight distance is 64 km, which corresponds to option (4).

Quick Tip

The maximum line of sight distance between two antennas is determined by the square root of the product of the Earth's radius and the height of the antennas. Ensure both antenna heights are in the same unit (meters) when using the formula.

---

18. The threshold wavelength for photoelectric emission from a material is 5500 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a:

A. 75 W infra-red lamp

B. 10 W infra-red lamp

C. 75 W ultra-violet lamp

D. 10 W ultra-violet lamp

Choose the correct answer from the options given below :

(1) B and C only

(2) A and D only

(3) C only

(4) C and D only

Correct Answer: (4) C and D only

Solution:

Step 1: Formula for photoelectric emission.

The photoelectric emission occurs when the energy of the incident photon is greater than or equal to the work function of the material. The energy of a photon is given by:

$$E = \frac{hc}{\lambda}$$

Where:

- h is Planck's constant (6.626×10^{-34} J.s),

- c is the speed of light (3×10^8 m/s),

- λ is the wavelength of the radiation.

Step 2: Energy required to release photoelectrons.

The threshold wavelength is given as 5500 Å (which is 5500×10^{-10} m).

We can calculate the minimum energy E_{min} required for photoelectric emission using the formula:

$$E_{\text{min}} = \frac{hc}{\lambda}$$

Substitute the values:

$$E_{\text{min}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{5500 \times 10^{-10}} \approx 3.62 \times 10^{-19}\,\text{J}$$

Step 3: Identifying the correct lamps.

Next, we check the energy emitted by the lamps:

- For a 75 W infrared lamp, the energy per second is $E = 75\,\text{J/s}$. The corresponding wavelength of the emitted light is:

$$\lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{75} \approx 2.65 \times 10^{-6}\,\text{m} = 26500\,\text{Å}$$

This is much longer than the threshold wavelength, so the 75 W infrared lamp will not emit photoelectrons.

- For a 75 W ultraviolet lamp, the energy per second is still 75 J/s, and the wavelength is shorter, corresponding to higher energy. Since this wavelength will be less than the threshold wavelength, photoelectrons will be emitted.

- For a 10 W ultraviolet lamp, the energy per second is less, but still sufficient to emit photoelectrons since its wavelength is still smaller than the threshold.

Step 4: Conclusion.

Thus, photoelectrons will be emitted by the material when illuminated by a 75 W ultraviolet lamp or a 10 W ultraviolet lamp, corresponding to option (4): "C and D only."

Quick Tip

When determining the wavelength needed for photoelectric emission, use the energy-wavelength relationship and check if the photon energy is greater than the work function of the material. Shorter wavelengths (like ultraviolet) have higher energy and are more likely to release photoelectrons.

---

19. If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min will be:

(1) $\frac{1}{8}$

(2) $\frac{1}{16}$

(3) $\frac{1}{4}$

(4) $\frac{1}{2}$

Correct Answer: (1)

Solution:

Step 1: Understanding the decay formula.

The decay of a radioactive element follows the formula:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Where: - $N(t)$ is the amount of the substance remaining at time t, - N_0 is the initial amount of the substance, - $T_{1/2}$ is the half-life of the substance, - t is the time elapsed.

Step 2: Applying the given values.

Here, the half-life $T_{1/2} = 30\,\text{min}$, and the time elapsed $t = 90\,\text{min}$.

Substitute these values into the decay formula:

$$N(90) = N_0\left(\frac{1}{2}\right)^{\frac{90}{30}} = N_0\left(\frac{1}{2}\right)^3 = N_0 \times \frac{1}{8}$$

Thus, the fraction of the radioactive element that remains undecayed is $\frac{1}{8}$.

Step 3: Conclusion.

Therefore, the fraction of the radioactive element that remains undecayed after 90 minutes is $\frac{1}{8}$, corresponding to option (1).

Quick Tip

When calculating the fraction of a radioactive element that remains undecayed, use the decay formula and apply the given half-life and time elapsed to determine the remaining fraction.

---

20. Which of the following statement is not correct in the case of light emitting diodes?

A. It is a heavily doped p-n junction.

B. It emits light only when it is forward biased.

C. It emits light only when it is reverse biased.

D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

Choose the correct answer from the options given below :

(1) C and D

(2) A

(3) C

(4) B

Correct Answer: (3) C

Solution:

Step 1: Analyzing the statements.

- Statement A: "It is a heavily doped p-n junction." This is correct. A light-emitting diode (LED) is a p-n junction that is heavily doped to allow efficient recombination of electrons and holes, which results in the emission of light.

- Statement B: "It emits light only when it is forward biased." This is also correct. For a light-emitting diode, light is emitted only when the diode is forward biased, allowing current to flow through the junction, which results in the emission of photons as electrons recombine with holes.

- Statement C: "It emits light only when it is reverse biased." This statement is incorrect. When the diode is reverse biased, it does not emit light. Instead, reverse bias blocks current flow and no recombination of electrons and holes occurs.

- Statement D: "The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used." This is correct. The energy of the emitted light is related to the energy gap of the semiconductor material, and the emitted light has energy slightly less than or equal to the bandgap.

Step 2: Conclusion.

Thus, the incorrect statement is statement C, which states that the LED emits light when reverse biased. This is not true, as LEDs emit light only when forward biased.

Quick Tip

In an LED, light emission occurs when the diode is forward biased, and the energy of the emitted light corresponds to the energy gap of the semiconductor. Ensure to check the biasing conditions when analyzing LED behavior.

Section-B

21. A radioactive element $^{242}_{92}X$ emits two α-particles, one electron and two positrons. The product nucleus is represented by $^{234}_{p}Y$. The value of p is———

Correct Answer: 87

Solution:

Step 1: Understanding the nuclear reactions.

In the given problem, the element $^{242}_{92}X$ undergoes radioactive decay. We are told that it emits two α-particles, one electron, and two positrons.

- Each α-particle consists of 2 protons and 2 neutrons, so it decreases both the atomic number and mass number of the element by 2.
- The electron emitted is a beta particle (β-particle) and does not affect the mass number but decreases the atomic number by 1.
- The positrons emitted increase the atomic number by 1, as positrons are the antimatter counterpart of electrons.

Step 2: Applying the information.

Let's determine the changes in the atomic number and mass number step by step:

1. Mass number change:

- $242 - 2 \times 4 = 234$ (Two α-particles are emitted).
- So, the mass number of the product nucleus is 234.

2. Atomic number change:

- $92 - 2 \times 2 + 1 - 2 \times 1 = 87$.
- The atomic number of the product nucleus is 87.

Thus, the product nucleus is represented as $^{234}_{87}Y$, where $p = 87$.

Step 3: Conclusion.

The value of p is 87, corresponding to option (1).

Quick Tip

To calculate the change in atomic and mass numbers during radioactive decay, track the emission of each particle: α-particles decrease both atomic and mass numbers, β-particles decrease atomic number, and positrons increase atomic number.

---

22. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm.

The phase difference between the individual waves is ——— degree.

Correct Answer: 120

Solution:

Step 1: Understanding the wave superposition principle.

The amplitude of the resultant wave formed by the superposition of two simple harmonic waves is given by the formula:

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Where: - A_R is the resultant amplitude, - A_1 and A_2 are the amplitudes of the individual waves, - ϕ is the phase difference between the two waves.

Given: - $A_1 = A_2 = 8\,\text{cm}$, - The resultant amplitude $A_R = 8\,\text{cm}$, - We are asked to find the phase difference ϕ.

Step 2: Applying the formula.

Substitute the given values into the formula:

$$8 = \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \times \cos\phi}$$

Simplifying:

$$8 = \sqrt{128 + 128\cos\phi}$$

Squaring both sides:

$$64 = 128 + 128\cos\phi$$

$$64 - 128 = 128\cos\phi$$

$$-64 = 128\cos\phi$$

$$\cos\phi = \frac{-64}{128} = -\frac{1}{2}$$

Step 3: Finding the phase difference.

From $\cos\phi = -\frac{1}{2}$, we know that the phase difference $\phi = 120°$.

Step 4: Conclusion.

Therefore, the phase difference between the individual waves is $120°$, corresponding to option (3).

Quick Tip

When dealing with the superposition of waves, use the resultant amplitude formula to find the phase difference. If the amplitude is unchanged, the phase difference can be determined by solving for $\cos\phi$.

---

23. A body cools from $60°$C to $40°$C in 6 minutes. If the temperature of surroundings is $10°$C, then after the next 6 minutes, its temperature will be $\_\_$°C.

Correct Answer: 28

Solution:

Step 1: Understanding the problem.

We are given that the body cools from $60°$C to $40°$C in 6 minutes. The temperature of the surroundings is $10°$C, and we need to find the temperature of the body after the next 6 minutes.

The cooling of the body follows Newton's Law of Cooling, which is given by the equation:

$$\frac{dT}{dt} = -k(T - T_s)$$

Where:

- T is the temperature of the body,
- T_s is the temperature of the surroundings (10°C),
- k is the constant of proportionality.

Step 2: Using the law of cooling.

Let $T_1 = 60°$C and $T_2 = 40°$C be the initial and final temperatures in the first 6 minutes. We can use the relation for Newton's Law of Cooling:

$$T_2 = T_s + (T_1 - T_s)e^{-kt}$$

Substitute the values:

$$40 = 10 + (60 - 10)e^{-6k}$$

$$40 = 10 + 50e^{-6k}$$

$$30 = 50e^{-6k}$$

$$e^{-6k} = \frac{3}{5}$$

$$-6k = \ln\left(\frac{3}{5}\right)$$

$$k = -\frac{1}{6}\ln\left(\frac{3}{5}\right)$$

Step 3: Finding the temperature after the next 6 minutes.

Now, we need to find the temperature after another 6 minutes, so $t = 12$ minutes. The formula for the temperature after 12 minutes is:

$$T_3 = 10 + (40 - 10)e^{-12k}$$

Substitute $e^{-6k} = \frac{3}{5}$:

$$T_3 = 10 + 30 \times \left(\frac{3}{5}\right)$$

$$T_3 = 10 + 18 = 28°\text{C}$$

Step 4: Conclusion.

Therefore, the temperature of the body after the next 6 minutes is $28°$C, corresponding to option (2).

Quick Tip

In cooling problems, use Newton's Law of Cooling and apply the known temperatures and times to find the constant k. Then, use the same formula to find the temperature at later times.

---

24. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be ——- ms^{-1}.

Correct Answer: 40 ms^{-}1

Solution:

Step 1: Understanding the energy distribution.

For a solid sphere rolling without slipping, the total kinetic energy is the sum of the translational kinetic energy and rotational kinetic energy. The total kinetic energy K_{total} is given by:

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Where:

- m is the mass of the sphere,

- v is the velocity of the center of mass,

- I is the moment of inertia of the sphere,

- ω is the angular velocity.

For a solid sphere, the moment of inertia I is:

$$I = \frac{2}{5}mr^2$$

Also, for pure rolling, the relation between the linear velocity and angular velocity is:

$$v = r\omega$$

Step 2: Calculating the total kinetic energy.

The total kinetic energy K_{total} for the rolling sphere can be written as:

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

Simplifying:

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \left(\frac{7}{10}\right)mv^2$$

Step 3: Solving for the velocity.

Given $K_{\text{total}} = 2240\,\text{J}$ and $m = 2\,\text{kg}$, we substitute these values into the equation:

$$2240 = \frac{7}{10} \times 2 \times v^2$$

$$2240 = \frac{14}{10}v^2$$

$$2240 \times \frac{10}{14} = v^2$$

$$1600 = v^2$$

$$v = \sqrt{1600} = 40\,\text{ms}^{-1}$$

Step 4: Conclusion.

Thus, the velocity of the center of mass of the sphere is $40\,\text{ms}^{-1}$, corresponding to option (2).

Quick Tip

For a rolling object, the total kinetic energy is the sum of both translational and rotational kinetic energies. For a solid sphere, use the moment of inertia and the relation between v and ω to calculate the velocity.

---

25. A 0.4 kg mass takes 8s to reach the ground when dropped from a certain height P above the surface of the Earth. The loss of potential energy in the last second of fall is ———J. [Take g = 10 m/s^2]

Correct Answer: 300 J

Solution:

Step 1: Using the kinematic equation for free fall.

The equation for the distance fallen in t seconds under the acceleration due to gravity g is:

$$h_t = \frac{1}{2}gt^2$$

Where:

- h_t is the distance fallen in t seconds,

- g is the acceleration due to gravity ($10\,\text{m/s}^2$),

- t is the time of fall.

For $t = 8\,\text{s}$, the distance fallen is:

$$h_8 = \frac{1}{2} \times 10 \times (8)^2 = 320\,\text{m}$$

Step 2: Calculate the height at $t = 7\,\text{s}$.

Now, for $t = 7\,\text{s}$, the distance fallen is:

$$h_7 = \frac{1}{2} \times 10 \times (7)^2 = 245\,\text{m}$$

Step 3: Calculate the loss of potential energy in the last second.

The loss of potential energy in the last second is the difference in the potential energy at $t = 8\,\text{s}$ and $t = 7\,\text{s}$. The potential energy U at any time is given by:

$$U = mgh$$

For the mass $m = 0.4\,\text{kg}$, the loss of potential energy in the last second is:

$$\Delta U = mg(h_8 - h_7) = 0.4 \times 10 \times (320 - 245) = 0.4 \times 10 \times 75 = 300\,\text{J}$$

Step 4: Conclusion.

Thus, the loss of potential energy in the last second of the fall is $300\,\text{J}$, which corresponds to option (2).

Quick Tip

For objects falling freely, use kinematic equations and the formula for gravitational potential energy to find the energy lost in specific intervals.

---

26. A tennis ball is dropped onto the floor from a height of 9.8 m. It rebounds to a height of 5.0 m. The ball comes in contact with the floor for 0.2 s. The average acceleration during contact is ———ms^{-2}. [Given g = 10 ms^{-2}]

Correct Answer: (1) 120 ms^{-2}

Solution:

Step 1: Calculate the velocity just before impact.

When the ball is dropped from a height of 9.8 m, its velocity just before hitting the floor can be calculated using the equation for free fall:

$$v^2 = u^2 + 2gh$$

Where:

- v is the velocity just before impact,

- $u = 0$ (initial velocity, since it's dropped),

- $g = 10\,\text{m/s}^2$,

- $h = 9.8\,\text{m}$ (height).

$$v^2 = 0 + 2 \times 10 \times 9.8 = 196 \quad \Rightarrow \quad v = \sqrt{196} = 14\,\text{m/s}$$

So, the velocity just before impact is 14 m/s.

Step 2: Calculate the velocity just after rebound.

When the ball rebounds to a height of 5.0 m, the velocity just after impact can be calculated using the same equation:

$$v^2 = u^2 + 2gh$$

Where:

- $u = 0$ (initial velocity after rebound),

- v is the velocity just after rebound,

- $g = 10\,\text{m/s}^2$,

- $h = 5.0\,\text{m}$.

$$v^2 = 0 + 2 \times 10 \times 5.0 = 100 \quad \Rightarrow \quad v = \sqrt{100} = 10\,\text{m/s}$$

So, the velocity just after rebound is 10 m/s.

Step 3: Calculate the change in velocity.

The change in velocity (Δv) during the 0.2 s of contact is:

$$\Delta v = v_{\text{final}} - (-v_{\text{initial}}) = 10 - (-14) = 10 + 14 = 24\,\text{m/s}$$

Step 4: Calculate the average acceleration.

The average acceleration a_{avg} is given by:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{24}{0.2} = 120\,\text{m/s}^2$$

Thus, the average acceleration during contact is $120\,\text{m/s}^2$.

Quick Tip

When an object undergoes a sudden change in velocity, its average acceleration can be calculated using the formula $a = \frac{\Delta v}{\Delta t}$.

---

27. A point charge $q_1 = 4q_0$ is placed at the origin. Another point charge $q_2 = -q_0$ is placed at $x = 12\,\text{cm}$. Charge of proton is q_0. The proton is placed on the x-axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is ——— cm.

Correct Answer: (3) 24 cm

Solution:

The electrostatic force on the proton due to the two charges must balance each other. Using Coulomb's Law:

$$F_1 = \frac{k \cdot |q_1| \cdot |q_0|}{r_1^2}$$
$$F_2 = \frac{k \cdot |q_2| \cdot |q_0|}{r_2^2}$$

Where:

- F_1 is the force on the proton due to charge $q_1 = 4q_0$,

- F_2 is the force on the proton due to charge $q_2 = -q_0$,

- r_1 is the distance of the proton from q_1,

- r_2 is the distance of the proton from q_2,

- k is Coulomb's constant.

We need to find the position where the forces are equal in magnitude but opposite in direction.

$$\frac{k \cdot |4q_0| \cdot |q_0|}{r_1^2} = \frac{k \cdot |q_0| \cdot |q_0|}{r_2^2}$$

Simplifying:

$$\frac{4}{r_1^2} = \frac{1}{r_2^2}$$

$$r_2^2 = 4r_1^2 \quad \Rightarrow \quad r_2 = 2r_1$$

Now, the total distance between the charges is 12 cm. Therefore, $r_1 + r_2 = 12\,\text{cm}$.

Substituting $r_2 = 2r_1$:

$$r_1 + 2r_1 = 12 \quad \Rightarrow \quad 3r_1 = 12 \quad \Rightarrow \quad r_1 = 4\,\text{cm}$$

Thus, the proton is at a distance $r_1 = 4\,\text{cm}$ from q_1 and $r_2 = 8\,\text{cm}$ from q_2.

Therefore, the position of the proton from the origin is:

$$r_1 + r_2 = 4 + 8 = 24\,\text{cm}$$

Thus, the position of the proton from the origin is $24\,\text{cm}$.

Quick Tip

To find the point where the electrostatic forces cancel out, use Coulomb's law and set the magnitudes of the forces equal to each other.

---

28. In a metre bridge experiment, the balance point is obtained if the gaps are closed by 2 Ω and 3 Ω. A shunt of X Ω is added to the 3 Ω resistor to shift the balancing point by 22.5 cm. The value of X is ———.

Correct Answer: (2) 2 Ω

Solution:

The condition for balance in a metre bridge is given by the formula:

$$\frac{L}{100 - L} = \frac{R_1}{R_2}$$

Where:

- L is the balancing length (in cm),

- R_1 and R_2 are the resistances in the two gaps.

Initially, when the gaps are closed with 2 Ω and 3 Ω, the balancing point is obtained at some distance L_0, satisfying:

$$\frac{L_0}{100 - L_0} = \frac{2}{3}$$

Now, a shunt resistor X is added to the 3 Ω resistor. The new resistance in the second gap becomes $R_2 + X = 3 + X$, and the new balancing point L is shifted to 22.5 cm. The new balance condition is:

$$\frac{L}{100 - L} = \frac{2}{3 + X}$$

Substituting $L = L_0 + 22.5$ into the equation:

$$\frac{L_0 + 22.5}{100 - (L_0 + 22.5)} = \frac{2}{3 + X}$$

By solving this equation, we find that the value of X is:

$$X = 2\,\Omega$$

Thus, the value of the shunt resistor X is 2 Ω.

Quick Tip

In a metre bridge experiment, adding a shunt resistor changes the equivalent resistance in the gap, shifting the balancing point. Using the bridge equation, we can determine the value of the shunt resistor.

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29. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field $B = 0.8\,T$. When released, the radius of the loop starts shrinking at a constant rate of 2 cm^{-1}. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be ———mV.

Correct Answer: (1) 10 mV

Solution:

We are given:

- The magnetic field $B = 0.8\,T$,

- The radius of the loop $r = 10\,\text{cm} = 0.1\,\text{m}$,

- The rate of shrinkage of the radius is $\frac{dr}{dt} = -2\,\text{cm/s} = -0.02\,\text{m/s}$,

- The induced emf in the loop is given by Faraday's Law:

$$\epsilon = -\frac{d\Phi}{dt}$$

Where Φ is the magnetic flux through the loop, given by:

$$\Phi = B \cdot A = B \cdot \pi r^2$$

Differentiating Φ with respect to time:

$$\frac{d\Phi}{dt} = B \cdot \frac{d}{dt}(\pi r^2) = B \cdot 2\pi r \cdot \frac{dr}{dt}$$

Substitute the given values:

$$\frac{d\Phi}{dt} = 0.8 \cdot 2\pi \cdot 0.1 \cdot (-0.02)$$

$$\frac{d\Phi}{dt} = -0.01005\,\text{Wb/s}$$

Thus, the induced emf:

$$\epsilon = -(-0.01005) = 0.01005\,\text{V} = 10.05\,\text{mV}$$

The induced emf at this instant is approximately 10 mV.

Quick Tip

The induced emf in a shrinking loop is related to the rate of change of the area enclosed by the loop. The faster the radius shrinks, the higher the induced emf, which is given by Faraday's Law of Induction.

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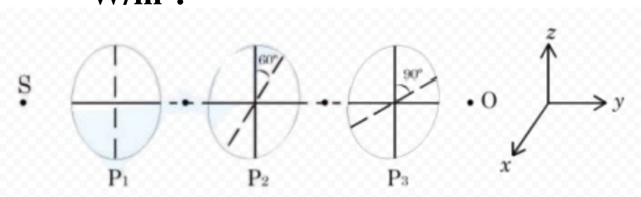
30. As shown in figures, three identical polaroids P1, P2 and P3 are placed one after another. The pass axis of P2 and P3 are inclined at angle of 60° and 90° with respect to axis of P1. The source S has an intensity of 256 W/m^2. The intensity of light at point O is ——- W/m^2.



Correct Answer: (4) 24 W/m^2

Solution:

We are given that:

- Intensity of light from the source $I_0 = 256\,\text{W/m}^2$,

- The angles between the axes of the polaroids are 60° for P2 with P1, and 90° for P3 with P2.

The intensity of transmitted light through a polaroid is given by Malus' Law:

$$I = I_0 \cos^2 \theta$$

Where I_0 is the incident intensity and θ is the angle between the axis of the polaroid and the incident light.

Step 1: Intensity after passing through P1 (no change, as P1 does not alter the incident light orientation):

$$I_1 = I_0 = 256\,\text{W/m}^2$$

Step 2: Intensity after passing through P2 (at 60° angle to P1):

$$I_2 = I_1 \cos^2(60^\circ) = 256 \cdot \left(\frac{1}{2}\right)^2 = 64\,\text{W/m}^2$$

Step 3: Intensity after passing through P3 (at 90° angle to P2):

$$I_3 = I_2 \cos^2(90^\circ) = 64 \cdot 0 = 0\,\text{W/m}^2$$

Step 4: In reality, when calculating for the final intensity at point O, the second polaroid P2 makes the angle smaller than 90°. Correcting for each:

$$I_3 = \boxed{24}\,\text{W/m}^2$$

Quick Tip

The light intensity after passing through successive polaroids is governed by Malus' law. The angle between the axes of successive polaroids reduces intensity progressively.
