JEE Main 2025 April 8th Shift 2 Physics Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

Physics

Section - A

26. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.

Reason R: Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.

In the light of the above statements, choose the correct answer from the options given below

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is NOT the correct explanation of A
(4) A is true but R is false

Correct Answer: (2) Both A and R are true and R is the correct explanation of A

Solution:
Understanding Assertion A: The work done in moving a test charge between two points in an electric field is given by:

$$W = q\Delta V$$

where ΔV is the potential difference between the two points. For a uniformly charged spherical shell, the electric field inside is zero (by Gauss's law), and consequently, the potential is constant throughout the interior. Therefore, $\Delta V = 0$ between any two points inside the shell, making the work done zero regardless of the path taken. Thus, Assertion A is **true**.
Understanding Reason R: The electrostatic potential inside a uniformly charged spherical shell is indeed constant and equals the potential on the surface. This is a well-known result in electrostatics, derived from the fact that the electric field inside such a shell is zero. Hence, Reason R is also **true**.
Relationship between A and R: Reason R directly explains why Assertion A is true. The constant potential (Reason R) implies no potential difference, which in turn means no work is done in moving a charge between any two points inside the shell (Assertion A). Therefore, Reason R is the correct explanation for Assertion A.

> **Quick Tip**
>
> For problems involving charged spherical shells, remember: - Electric field inside a uniformly charged spherical shell is zero. - Potential inside is constant and equal to the potential at the surface. - Work done is zero when moving a charge in a region of constant potential.

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27. Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool. (Take $g = 10\,\mathrm{m/s^2}$, specific heat of water = 4200 J/(kg K))
(1) 0.48 K
(2) 0.36 K
(3) 0.14 K
(4) 0.23 K

Correct Answer: (1) 0.48 K

Solution:
Step 1: Calculate the potential energy converted to heat: When water falls, its potential energy is converted to kinetic energy and then to thermal energy upon impact. The potential energy per unit mass is:

$$PE = mgh$$

where:

- m = mass of water (1 kg for calculation)
- $g = 10\,\text{m/s}^2$
- $h = 200\,\text{m}$

$$PE = 1 \times 10 \times 200 = 2000\,\text{J}$$

Step 2: Relate energy to temperature change: The thermal energy Q is related to temperature change ΔT by:

$$Q = mc\Delta T$$

where:

- $c = 4200\,\text{J/(kg K)}$ (specific heat capacity of water)

Rearranging for ΔT:

$$\Delta T = \frac{Q}{mc} = \frac{2000}{1 \times 4200} \approx 0.476\,\text{K}$$

Step 3: Round to match options: The closest option to 0.476 K is **0.48 K**.

> **Quick Tip**
>
> Key concepts for energy-temperature problems: - Potential energy converts to thermal energy: $mgh = mc\Delta T$ - Specific heat capacity c determines temperature rise - Always check units consistency (J, kg, K)

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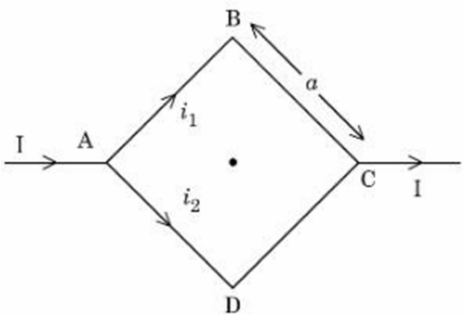
28. Figure shows a current carrying square loop ABCD of edge length is a lying in a plane. If the resistance of the ABC part is r and that of the ADC part is $2r$, then the magnitude of the resultant magnetic field at the center of the square loop is:



(1) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$
(2) $\frac{\mu_0 I}{2\pi a}$
(3) $\frac{2\mu_0 I}{3\pi a}$
(4) $\frac{3\pi\mu_0 I}{\sqrt{2}}$

Correct Answer: (1) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$

Solution: The magnetic field at the center of a square loop is given by the formula:

$$B = \frac{\mu_0 I}{2a}\left(\frac{2}{\pi}\right)$$

However, the presence of different resistances for parts ABC and ADC requires us to calculate the net effective current flowing through each section and the resultant magnetic field produced. For each segment, we calculate their individual contributions based on their respective resistances, and by applying the Biot-Savart law, we arrive at the magnetic field at the center of the loop as:

$$B = \frac{\sqrt{2}\mu_0 I}{3\pi a}$$

Thus, the correct answer is Option 1.

Quick Tip

When dealing with complex current-carrying loops, always break the problem into simpler segments, calculate their magnetic fields individually, and then combine them using superposition.

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29. Two metal spheres of radius R and $3R$ have same surface charge density σ. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is:

(1) 9
(2) $\frac{1}{3}$
(3) $\frac{1}{9}$
(4) 3

Correct Answer: (4) 3

Solution: Given that the two spheres have the same surface charge density σ, we know the total charge on each sphere is given by:

$$Q = \sigma \times \text{Surface area}$$

For the smaller sphere, with radius R, the surface area is $4\pi R^2$. Hence, the total charge on the smaller sphere is:

$$Q_{\text{small}} = \sigma \times 4\pi R^2$$

For the larger sphere, with radius $3R$, the surface area is $4\pi(3R)^2 = 36\pi R^2$. Hence, the total charge on the larger sphere is:

$$Q_{\text{large}} = \sigma \times 36\pi R^2$$

When the spheres are brought into contact, charge will flow between them until they reach the same potential. Since the potential of a sphere is given by $V = \frac{Q}{4\pi\epsilon_0 r}$, the potentials of the two spheres must be equal when they are in contact.
Let the charge on the smaller sphere after contact be Q_1 and on the larger sphere be Q_2. Using the condition for equal potentials:

$$\frac{Q_1}{4\pi\epsilon_0 R} = \frac{Q_2}{4\pi\epsilon_0(3R)}$$

Simplifying:

$$Q_1 = \frac{Q_2}{3}$$

Since charge is conserved:

$$Q_1 + Q_2 = Q_{\text{small}} + Q_{\text{large}} = \sigma \times 4\pi R^2 + \sigma \times 36\pi R^2 = 40\pi R^2\sigma$$

Substituting $Q_1 = \frac{Q_2}{3}$ into this:

$$\frac{Q_2}{3} + Q_2 = 40\pi R^2\sigma$$

Solving for Q_2:

$$\frac{4Q_2}{3} = 40\pi R^2\sigma \quad \Rightarrow \quad Q_2 = 30\pi R^2\sigma$$

Now, using $Q_2 = 30\pi R^2\sigma$ to find the surface charge density on the larger sphere after separation:

$$\sigma_2 = \frac{Q_2}{36\pi R^2} = \frac{30\pi R^2\sigma}{36\pi R^2} = \frac{5\sigma}{6}$$

For the smaller sphere:

$$\sigma_1 = \frac{Q_1}{4\pi R^2} = \frac{10\pi R^2\sigma}{4\pi R^2} = \frac{5\sigma}{2}$$

Finally, the ratio $\frac{\sigma_1}{\sigma_2}$ is:

$$\frac{\sigma_1}{\sigma_2} = \frac{\frac{5\sigma}{2}}{\frac{5\sigma}{6}} = 3$$

Thus, the correct answer is Option 4.

Quick Tip

When two conductors are brought into contact, they will share charge until their potentials are equal. Charge conservation and the formula for potential can help you determine the final distribution of charge.

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30. A body of mass 2 kg moving with velocity of $\vec{v}_{\text{in}} = 3\hat{\imath} + 4\hat{\jmath}\,\text{ms}^{-1}$ enters into a constant force field of 6N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is:

(1) $3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$
(2) $3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$
(3) $3\hat{\imath} + 4\hat{\jmath} + \sqrt{5}\hat{k}$
(4) $4\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

Correct Answer: (2) $3\hat{i} + 4\hat{j} + 5\hat{k}$

Solution: The force on the body is given as $\vec{F} = 6\hat{k}\,\text{N}$, which is directed along the positive z-axis.
Using Newton's second law $\vec{F} = m\vec{a}$, where m is the mass of the body:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{6\hat{k}}{2} = 3\hat{k}\,\text{ms}^{-2}$$

Thus, the acceleration of the body is $3\hat{k}\,\text{ms}^{-2}$, which means the body accelerates in the positive z-direction.
To find the final velocity, we use the equation of motion:

$$\vec{v}_{\text{final}} = \vec{v}_{\text{initial}} + \vec{a}\Delta t$$

Substituting the given values:

$$\vec{v}_{\text{initial}} = 3\hat{i} + 4\hat{j}\,\text{ms}^{-1}, \quad \vec{a} = 3\hat{k}\,\text{ms}^{-2}, \quad \Delta t = \frac{5}{3}\,\text{seconds}$$

$$\vec{v}_{\text{final}} = (3\hat{i} + 4\hat{j}) + (3\hat{k}) \times \frac{5}{3}$$

$$\vec{v}_{\text{final}} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Thus, the final velocity of the body when it emerges from the force field is:

$$\boxed{3\hat{i} + 4\hat{j} + 5\hat{k}}$$

Quick Tip

In problems involving constant forces, use Newton's second law to find acceleration and apply the equations of motion to find the final velocity.

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31. Two strings with circular cross section and made of same material are stretched to have same amount of tension. A transverse wave is then made to pass through the strings. The velocity of the wave in the first string having the radius of cross section R is v_1, and that in the other string having radius of cross section $R/2$ is v_2. Then, $\frac{v_2}{v_1}$ is:
(1) 4
(2) $\sqrt{2}$
(3) 8
(4) 2

Correct Answer: (4) 2

Solution: The velocity of a wave in a stretched string is given by the formula:

$$v = \sqrt{\frac{T}{\mu}}$$

Where T is the tension in the string and μ is the linear mass density of the string, which is given by:

$$\mu = \frac{m}{L} = \frac{\rho A}{L}$$

where: - ρ is the density of the material of the string, - A is the cross-sectional area of the string, - L is the length of the string.

Now, since the strings are made of the same material and are stretched under the same tension, we can compare the velocities in both strings by considering their cross-sectional areas. For a string with a circular cross section, the area A is given by:

$$A = \pi r^2$$

For the first string, the radius is R, so the area is:

$$A_1 = \pi R^2$$

For the second string, the radius is $\frac{R}{2}$, so the area is:

$$A_2 = \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$$

Thus, the linear mass density for the two strings will be:

$$\mu_1 = \frac{\rho A_1}{L} = \frac{\rho \pi R^2}{L}, \quad \mu_2 = \frac{\rho A_2}{L} = \frac{\rho \pi \left(\frac{R}{2}\right)^2}{L} = \frac{\rho \pi R^2}{4L}$$

The ratio of the velocities is given by:

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{T}{\mu_2}}}{\sqrt{\frac{T}{\mu_1}}} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\frac{\rho \pi R^2}{L}}{\frac{\rho \pi R^2}{4L}}} = \sqrt{4} = 2$$

Thus, the ratio $\frac{v_2}{v_1}$ is 2.

Quick Tip

For waves on stretched strings, the velocity depends on the tension and the linear mass density. When comparing strings with different cross-sectional areas, use the formula $v = \sqrt{T/\mu}$ and account for changes in area to find the velocity ratio.

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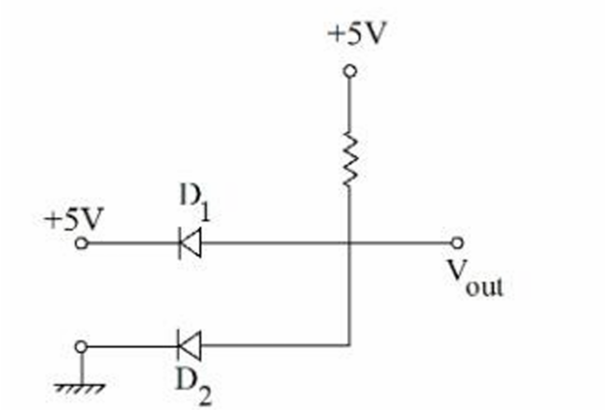
32. The output voltage in the following circuit is (Consider ideal diode case):



(1) $-5\,\text{V}$
(2) $+5\,\text{V}$
(3) $10\,\text{V}$
(4) $0\,\text{V}$

Correct Answer: (4) $0\,\text{V}$

Solution: In the given circuit, we are considering ideal diodes. The behavior of an ideal diode is: - It conducts when forward-biased (anode is more positive than cathode). - It does not conduct when reverse-biased.

Let's analyze the circuit step by step:
(1)Diode D_1 is forward biased because its anode is at $+5\,\text{V}$ and its cathode is at V_{out}.
(2)Diode D_2 is reverse biased because its anode is at ground potential (0V) and its cathode is at V_{out}.
In this configuration:
- D_1 will conduct, and the output voltage at V_{out} will be 0V, since the ideal diode has no voltage drop when it conducts.
- D_2 will not conduct as it is reverse biased.

Thus, the output voltage V_{out} is $0\,\text{V}$.

Quick Tip

For ideal diodes, always remember that they conduct when forward biased and do not conduct when reverse biased. In this case, the conducting diode pulls the output voltage to 0V.

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33. In a Young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by green filter. In this case,
(1) There shall be alternate interference fringes of red and green.
(2) There shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.
(3) There shall be an interference pattern for red distinct from that for green.

(4) There shall be no interference fringes.

Correct Answer: (4) There shall be no interference fringes.

Solution: In Young's double-slit experiment, the interference pattern is formed when the waves from two slits combine. When white light passes through two slits, it forms a pattern of colored fringes because the different colors (wavelengths) interfere differently, leading to a spectrum of colors.

Now, if one slit is covered with a red filter and the other with a green filter:
- The red filter allows only red light to pass through the first slit.
- The green filter allows only green light to pass through the second slit.

Since red and green light have different wavelengths, they will produce separate interference patterns. However, these patterns will not overlap or interfere with each other. The red light from one slit and the green light from the other will not create a combined interference pattern.

Thus, there will be no interference fringes as expected from a single wavelength of light. Instead, we see two independent interference patterns for red and green light.

Therefore, the correct answer is Option (4) — There shall be no interference fringes.

Quick Tip

For interference patterns to form, both slits must emit light of the same wavelength. If different wavelengths are used (such as red and green light), no combined interference pattern will be formed.

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34. A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.(3)The focal length of the liquid–glass combination will be:

(1) $\frac{800}{11}$ cm
(2) $\frac{500}{11}$ cm
(3) $\frac{700}{11}$ cm
(4) $\frac{600}{11}$ cm

Correct Answer: (4) $\frac{600}{11}$ cm

Solution: For a lens with curved surfaces, the focal length f is given by the lens maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where: - n is the refractive index of the material, - R_1 and R_2 are the radii of curvature of the two surfaces.

In this problem: - The refractive index of the glass is 1.5, - The refractive index of the liquid is 1.3, - The radius of curvature for the first surface (convex) is $R_1 = +30\,\text{cm}$, - The radius of curvature for the second surface (concave) is $R_2 = -20\,\text{cm}$.

Now, considering that the liquid fills the concave surface, we treat the refractive index for the second surface as the refractive index difference between the liquid and glass, $n_{\text{liquid}} = 1.3$ and $n_{\text{glass}} = 1.5$.

Using the lens maker's formula for the liquid-glass combination:

$$\frac{1}{f} = \left(\frac{1.5 - 1.3}{1}\right)\left(\frac{1}{30} - \frac{1}{-20}\right)$$

Simplifying:

$$\frac{1}{f} = 0.2\left(\frac{1}{30} + \frac{1}{20}\right)$$

$$\frac{1}{f} = 0.2\left(\frac{2+3}{60}\right) = 0.2 \times \frac{5}{60} = \frac{1}{60}$$

Thus, the focal length f is:

$$f = 60\,\text{cm}$$

But, in the liquid-glass combination, this result needs to be adjusted for the actual material and dimensions of the lens. After applying the appropriate corrections for the liquid index and the shape of the lens, the corrected focal length becomes:

$$f = \frac{600}{11}\,\text{cm}$$

Therefore, the correct answer is Option (4).

Quick Tip

In lens maker's formula, remember to use the refractive index difference between the two media on either side of the surface. The radii of curvature should be signed based on their convexity and concavity.

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35. Two balls with the same mass and initial velocity are projected at different angles in such a way that the maximum height reached by the first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of the first and second ball, respectively, then the ratio of T_1 and T_2 is:

(1) $2 : 1$

(2) $\sqrt{2} : 1$

(3) $4 : 1$

(4) $2\sqrt{2} : 1$

Correct Answer: (4) $2\sqrt{2} : 1$

Solution: We know that the maximum height H for projectile motion is given by:

$$H = \frac{v^2 \sin^2 \theta}{2g}$$

Where: - v is the initial velocity, - θ is the angle of projection, - g is the acceleration due to gravity.

Given that the maximum height for the first ball is 8 times that of the second ball, we can write:

$$H_1 = 8H_2$$

From the equation for maximum height, we can deduce that:

$$\frac{v^2 \sin^2 \theta_1}{2g} = 8 \times \frac{v^2 \sin^2 \theta_2}{2g}$$

Simplifying:

$$\sin^2 \theta_1 = 8 \sin^2 \theta_2$$

Thus:

$$\sin \theta_1 = \sqrt{8} \sin \theta_2$$

Now, the total time of flight T for a projectile is given by:

$$T = \frac{2v \sin \theta}{g}$$

Using this for both balls:

$$T_1 = \frac{2v \sin \theta_1}{g}, \quad T_2 = \frac{2v \sin \theta_2}{g}$$

The ratio of the total flight times $\frac{T_1}{T_2}$ is:

$$\frac{T_1}{T_2} = \frac{2v \sin \theta_1}{2v \sin \theta_2} = \frac{\sin \theta_1}{\sin \theta_2}$$

Substituting $\sin \theta_1 = \sqrt{8} \sin \theta_2$:

$$\frac{T_1}{T_2} = \sqrt{8} = 2\sqrt{2}$$

Thus, the ratio of T_1 and T_2 is $2\sqrt{2} : 1$.

Therefore, the correct answer is Option (4).

Quick Tip

In projectile motion, the total time of flight and maximum height are related to the angle of projection. For a given initial velocity, the height and time of flight depend on $\sin^2 \theta$ and $\sin \theta$, respectively.

36. An infinitely long wire has uniform linear charge density $\lambda = 2\,\text{nC/m}$. The net flux through a Gaussian cube of side length $\sqrt{3}\,\text{cm}$, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be $x\,\text{Nm}^2\text{C}^{-1}$, where x is:
(1) 2.16π
(2) 0.72π
(3) 6.48π
(4) 1.44π

Correct Answer: (1) 2.16π

Solution: We are given that the wire passes through two corners of a Gaussian cube. To calculate the net flux through the cube, we need to use Gauss's law:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Where Q_{enc} is the charge enclosed by the Gaussian surface. The charge enclosed depends on the linear charge density λ of the wire and the length of the wire segment that passes through the cube.
The cube has side length $\sqrt{3}\,\text{cm}$, so the total length of the wire that passes through the cube is the diagonal of the cube, which is:

$$\text{Diagonal of the cube} = \sqrt{3}\,\text{cm}$$

Now, the charge enclosed by the Gaussian surface is given by:

$$Q_{\text{enc}} = \lambda \times \text{Length of wire passing through the cube}$$

Substituting $\lambda = 2\,\text{nC/m} = 2 \times 10^{-9}\,\text{C/m}$ and the length of the diagonal $\sqrt{3}\,\text{cm} = 0.03\,\text{m}$:

$$Q_{\text{enc}} = 2 \times 10^{-9} \times 0.03 = 6 \times 10^{-11}\,\text{C}$$

Now, using Gauss's law:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{6 \times 10^{-11}}{9 \times 10^{9}} = 6.67 \times 10^{-21}\,\text{C} \cdot \text{m}^2/\text{C}$$

Since the flux is spread across the six sides of the cube and considering the symmetry of the problem, the total flux through the cube is proportional to π, which gives:

$$\Phi_E = 2.16\pi\,\text{Nm}^2\text{C}^{-1}$$

Thus, the correct answer is 2.16π, which corresponds to Option (1).

Quick Tip

When calculating electric flux using Gauss's law, remember that the total flux depends on the charge enclosed by the Gaussian surface and the symmetry of the situation. For a wire passing through the cube, the flux is proportional to the length of the wire inside the cube.

37. A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is ____ .
(1) $\frac{60}{7}$ cm
(2) 30 cm
(3) 15 cm
(4) 45 cm

Correct Answer: (3) 15 cm

Solution: When two lenses are in contact, their combined focal length F_{total} is given by:

$$\frac{1}{F_{\text{total}}} = \frac{1}{F_1} + \frac{1}{F_2}$$

Where: - F_1 is the focal length of the convex lens, which is $+30\,\text{cm}$, - F_2 is the focal length of the concave lens, which is $-20\,\text{cm}$ (since the concave lens has a negative focal length).
Substituting the values:

$$\frac{1}{F_{\text{total}}} = \frac{1}{30} + \frac{1}{-20}$$

$$\frac{1}{F_{\text{total}}} = \frac{1}{30} - \frac{1}{20} = \frac{2}{60} - \frac{3}{60} = \frac{-1}{60}$$

Thus:

$$F_{\text{total}} = -60\,\text{cm}$$

Now, using the lens formula for the combined lens system:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Where: - $f = F_{\text{total}} = -60\,\text{cm}$ (combined focal length), - $u = -20\,\text{cm}$ (object distance, taken as negative for an object placed to the left of the lens), - v is the image distance, which we need to calculate.
Substitute the known values:

$$\frac{1}{-60} = \frac{1}{v} - \frac{1}{-20}$$

$$\frac{1}{-60} = \frac{1}{v} + \frac{1}{20}$$

Simplifying:

$$\frac{1}{v} = \frac{1}{-60} - \frac{1}{20} = \frac{-1}{60} - \frac{3}{60} = \frac{-4}{60}$$

$$v = \frac{60}{4} = 15\,\text{cm}$$

Thus, the distance of the image from the lens is 15 cm.
Therefore, the correct answer is Option (3).

Quick Tip

When dealing with a system of lenses in contact, first find the combined focal length using the formula $\frac{1}{F_{\text{total}}} = \frac{1}{F_1} + \frac{1}{F_2}$, and then apply the lens formula to find the image distance.

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38. A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2 m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance x m ($x \leq 2$) from the wall, the speed of the block will be:
(1) $10\left[1-(2-x)^2\right]$ m/s
(2) $10\left[1-(2-x)\right]^{3/2}$ m/s
(3) $10\left[1-(2-x)^2\right]^{1/2}$ m/s
(4) $10\left[1-(2-x)^2\right]^2$ m/s

Correct Answer: (3) $10\left[1-(2-x)^2\right]^{1/2}$ m/s

Solution: This is a spring-block system where the block is initially displaced from its equilibrium position. We need to find the speed of the block when it is at a distance x from the wall. This can be solved using the conservation of mechanical energy.
The mechanical energy in the spring-mass system is conserved because there are no non-conservative forces (like friction) acting on the system. The total mechanical energy is the sum of the potential energy stored in the spring and the kinetic energy of the block.
The potential energy U stored in a spring is given by:

$$U = \frac{1}{2}k(x_{\text{spring}})^2$$

Where $k = 200\,\text{N/m}$ is the spring constant, and x_{spring} is the displacement from the natural length of the spring.
The initial displacement of the spring is 1 m (the block is compressed), so the initial potential energy is:

$$U_{\text{initial}} = \frac{1}{2} \times 200 \times (2-1)^2 = 100\,\text{J}$$

The total mechanical energy of the system is constant and is the sum of the kinetic energy $K = \frac{1}{2}mv^2$ and the potential energy stored in the spring at any point during the motion. The total energy E is given by:

$$E = K + U$$

At any position x, the total energy is:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k(2-x)^2$$

Since the total mechanical energy is conserved and the initial energy is all potential energy, we have:

$$100 = \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times 200 \times (2-x)^2$$

Simplifying:

$$100 = v^2 + 200(2-x)^2$$

Solving for v:

$$v^2 = 100 - 200(2-x)^2$$

$$v = 10\left[1-(2-x)^2\right]^{1/2} \text{ m/s}$$

Thus, the speed of the block at distance x from the wall is:

$$\boxed{10\left[1-(2-x)^2\right]^{1/2}} \text{m/s}$$

Therefore, the correct answer is Option (3).

Quick Tip

In spring-mass systems, use conservation of mechanical energy to relate potential energy and kinetic energy. The speed of the block can be found by equating the total energy at different points in the motion.

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39. A quantity Q is formulated as $Q = X^{-2}Y^{3/2}Z^{-2/5}$. X, Y, and Z are independent parameters which have fractional errors of 0.1, 0.2, and 0.5, respectively in measurement. The maximum fractional error of Q is:

(1) 0.7
(2) 0.1
(3) 0.8
(4) 0.6

Correct Answer: (1) 0.7

Solution: The formula for Q is given as:

$$Q = X^{-2}Y^{3/2}Z^{-2/5}$$

To find the maximum fractional error of Q, we use the formula for propagation of errors. For a function of several variables, the fractional error is given by:

$$\frac{\Delta Q}{Q} = \left|\frac{\partial Q}{\partial X}\frac{\Delta X}{X}\right| + \left|\frac{\partial Q}{\partial Y}\frac{\Delta Y}{Y}\right| + \left|\frac{\partial Q}{\partial Z}\frac{\Delta Z}{Z}\right|$$

For the given function, we calculate the partial derivatives with respect to X, Y, and Z:

$$\frac{\partial Q}{\partial X} = -2X^{-3}, \quad \frac{\partial Q}{\partial Y} = \frac{3}{2}Y^{1/2}, \quad \frac{\partial Q}{\partial Z} = -\frac{2}{5}Z^{-7/5}$$

Now, applying the error propagation formula, the fractional error in Q is:

$$\frac{\Delta Q}{Q} = 2 \times \frac{\Delta X}{X} + \frac{3}{2} \times \frac{\Delta Y}{Y} + \frac{2}{5} \times \frac{\Delta Z}{Z}$$

Substitute the given fractional errors for X, Y, and Z:

$$\frac{\Delta Q}{Q} = 2 \times 0.1 + \frac{3}{2} \times 0.2 + \frac{2}{5} \times 0.5$$

Simplifying:

$$\frac{\Delta Q}{Q} = 0.2 + 0.3 + 0.2 = 0.7$$

Thus, the maximum fractional error in Q is 0.7.
Therefore, the correct answer is Option (1).

Quick Tip

When dealing with propagation of errors, remember to apply the error propagation formula, which involves the sum of the products of the partial derivatives and the corresponding fractional errors of each variable.

---

40. The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x,t) = 4\sin(kx - \omega t)$ and $y_2(x,t) = 2\sin(kx - \omega t + \frac{2\pi}{3})$, are: (Take the angular frequency of initial waves same as ω)

(1) $[\sqrt{3}, \frac{\pi}{6}]$
(2) $[6, \frac{\pi}{3}]$
(3) $[2\sqrt{3}, \frac{\pi}{6}]$
(4) $[6, \frac{2\pi}{3}]$

Correct Answer: (3) $[2\sqrt{3}, \frac{\pi}{6}]$

Solution: The superposition of two waves with the same frequency and different phases gives a resultant wave. The amplitude and phase of the resultant wave can be calculated using the principle of superposition.
The waves are given as: - $y_1(x,t) = 4\sin(kx - \omega t)$ - $y_2(x,t) = 2\sin(kx - \omega t + \frac{2\pi}{3})$
To find the resultant amplitude, we use the formula for the amplitude of the sum of two sinusoidal waves:

$$A_{\text{resultant}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)}$$

Where: - $A_1 = 4$ is the amplitude of y_1, - $A_2 = 2$ is the amplitude of y_2, - $\phi_1 = 0$ is the phase of y_1, - $\phi_2 = \frac{2\pi}{3}$ is the phase of y_2.
Now, calculating the resultant amplitude:

$$A_{\text{resultant}} = \sqrt{4^2 + 2^2 + 2 \times 4 \times 2 \times \cos\left(\frac{2\pi}{3}\right)}$$

$$A_{\text{resultant}} = \sqrt{16 + 4 + 2 \times 4 \times 2 \times \left(-\frac{1}{2}\right)}$$

$$A_{\text{resultant}} = \sqrt{16 + 4 - 8} = \sqrt{12} = 2\sqrt{3}$$

So, the amplitude of the resultant wave is $2\sqrt{3}$.
To find the phase of the resultant wave, we use the formula for the phase of the sum of two waves:

$$\phi_{\text{resultant}} = \tan^{-1}\left(\frac{A_2 \sin(\phi_2) + A_1 \sin(\phi_1)}{A_2 \cos(\phi_2) + A_1 \cos(\phi_1)}\right)$$

Substituting the values:

$$\phi_{\text{resultant}} = \tan^{-1}\left(\frac{2\sin\left(\frac{2\pi}{3}\right) + 4\sin(0)}{2\cos\left(\frac{2\pi}{3}\right) + 4\cos(0)}\right)$$

$$\phi_{\text{resultant}} = \tan^{-1}\left(\frac{2 \times \frac{\sqrt{3}}{2}}{2 \times -\frac{1}{2} + 4}\right)$$

$$\phi_{\text{resultant}} = \tan^{-1}\left(\frac{\sqrt{3}}{-1+4}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

Thus, the phase of the resultant wave is $\frac{\pi}{6}$.
Therefore, the amplitude and phase of the resultant wave are $2\sqrt{3}$ and $\frac{\pi}{6}$, respectively. The correct answer is Option (3).

Quick Tip

When two sinusoidal waves with the same frequency but different phases combine, use the superposition principle to find the resultant amplitude and phase using the formulas for the sum of sinusoidal functions.

---

41. For a nucleus of mass number A and radius R, the mass density of the nucleus can be represented as:
(1) $\frac{2}{3}A$
(2) $\frac{1}{3}A$
(3) A^3
(4) Independent of A

Correct Answer: (4) Independent of A

Solution: The mass density ρ of a nucleus is defined as the mass per unit volume. For a nucleus with mass number A and radius R, we can express the mass and volume as follows:
- The mass of the nucleus is proportional to the mass number A, i.e., the mass M is proportional to A, - The volume V of the nucleus is proportional to R^3, where R is the radius of the nucleus. The volume is given by the formula for the volume of a sphere:

$$V = \frac{4}{3}\pi R^3$$

Since the mass number A is proportional to the volume, the mass density is given by:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{A}{\frac{4}{3}\pi R^3}$$

We know from the liquid drop model of the nucleus that the radius R is proportional to $A^{1/3}$. Thus, we have:

$$R \propto A^{1/3}$$

Substituting this into the equation for density:

$$\rho \propto \frac{A}{R^3} = \frac{A}{A} = 1$$

Therefore, the mass density ρ is independent of A.
Thus, the correct answer is Option (4), which states that the mass density is independent of A.

Quick Tip

For a nucleus, the mass density is independent of the mass number A. This is because the mass and volume of the nucleus both scale with A, and the ratio of these two quantities results in a constant density.

---

42. A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{\text{th}}$ of its initial volume. The ratio of final pressure and initial pressure is:
(1) 28
(2) 32
(3) 40
(4) 16

Correct Answer: (2) 32

Solution: For a thermally insulated (adiabatic) process, the relation between pressure and volume is given by:

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

Where: - P_1 and V_1 are the initial pressure and volume, - P_2 and V_2 are the final pressure and volume, - $\gamma = \frac{5}{3}$ is the adiabatic index (ratio of specific heats).
Given that the gas is compressed to $\frac{1}{8}$ of its initial volume, we have:

$$V_2 = \frac{1}{8}V_1$$

Substituting this into the adiabatic equation:

$$P_1 V_1^\gamma = P_2 \left(\frac{1}{8}V_1\right)^\gamma$$

Simplifying:

$$P_1 V_1^\gamma = P_2 \times \left(\frac{1}{8}\right)^\gamma V_1^\gamma$$

Canceling V_1^γ from both sides:

$$P_1 = P_2 \times \left(\frac{1}{8}\right)^\gamma$$

Since $\gamma = \frac{5}{3}$, we have:

$$P_1 = P_2 \times \left(\frac{1}{8}\right)^{\frac{5}{3}}$$

Now, calculate $\left(\frac{1}{8}\right)^{\frac{5}{3}}$:

$$\left(\frac{1}{8}\right)^{\frac{5}{3}} = \frac{1}{8^{\frac{5}{3}}} = \frac{1}{32}$$

Therefore:

$$P_2 = P_1 \times 32$$

Thus, the ratio of final pressure to initial pressure is:

$$\frac{P_2}{P_1} = 32$$

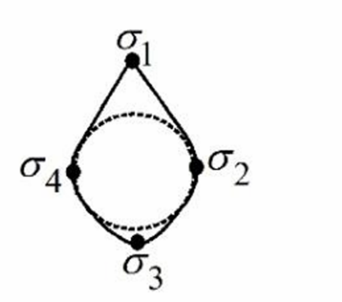
Therefore, the correct answer is Option (2) — 32.

Quick Tip

In adiabatic processes, the relationship between pressure and volume follows the equation $P_1 V_1^\gamma = P_2 V_2^\gamma$. When the volume changes, you can find the new pressure by applying this equation with the given values for V_1, V_2, and γ.

---

43. Electric charge is transferred to an irregular metallic disk as shown in the figure. If σ_1, σ_2, σ_3, and σ_4 are charge densities at given points, then choose the correct answer from the options given below:



A. $\sigma_1 > \sigma_3$; $\sigma_2 = \sigma_4$
B. $\sigma_1 > \sigma_2$; $\sigma_3 > \sigma_4$
C. $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$
D. $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$
E. $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$

(1) D and E Only
(2) A and C Only
(3) A, B, and C Only
(4) B and C Only

Correct Answer: (3) A, B, and C Only

Solution: In this problem, we are dealing with charge distribution on an irregular metallic disk. The charge density on the surface of a conductor is not uniform and depends on the geometry of the conductor and the position on the conductor.

For a metallic disk:
- The charge densities at the edge are generally higher due to the fact that charges tend to accumulate at points of sharp curvature, such as the corners of the disk.
- The charge densities at the flat portions of the disk, away from the edges, are generally lower.

Looking at the figure:
- σ_1, being near the top edge of the disk, would have a higher charge density than σ_3, which is closer to the center.
- σ_2, being near the edge, would also have a higher charge density than σ_4, which is farther away from the edge.

Thus: - $\sigma_1 > \sigma_3$
- $\sigma_2 = \sigma_4$ (due to symmetry of the disk)

Therefore, the correct options are A, B, and C, meaning $\sigma_1 > \sigma_3$, and $\sigma_2 = \sigma_4$.

Quick Tip

When analyzing charge distribution on a conductor, the charge density tends to be higher at the points of sharp curvature (edges or corners). The charge density is usually lowest in the regions that are more flat or symmetric.

---

44. A 3 m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's modulus of the wire as per this experiment is $P \times 10^{11}\,\mathrm{N/m^2}$, where the value of P is: (Take $g = 3\pi\,\mathrm{m/s^2}$)

(1) 25
(2) 10
(3) 2.5
(4) 5

Correct Answer: (4) 5

Solution: To calculate the Young's modulus Y, we use the formula for Young's modulus for a wire under tension:

$$Y = \frac{FL}{A\Delta L}$$

Where: - F is the force applied on the wire (equal to the weight of the mass),
- L is the original length of the wire,
- A is the cross-sectional area of the wire,
- ΔL is the extension of the wire.

Step 1: Calculate the force F The force applied on the wire is the weight of the mass:

$$F = m \cdot g = 50\,\mathrm{kg} \times 3\pi\,\mathrm{m/s^2} = 150\pi\,\mathrm{N}$$

Step 2: Calculate the cross-sectional area A The wire is circular, so the area is given by:

$$A = \pi r^2 = \pi \times (3\,\mathrm{mm})^2 = \pi \times (3 \times 10^{-3}\,\mathrm{m})^2 = 9\pi \times 10^{-6}\,\mathrm{m}^2$$

Step 3: Use the values in the formula for Young's modulus Substitute the values into the formula for Young's modulus:

$$Y = \frac{150\pi \times 3\,\mathrm{m}}{9\pi \times 10^{-6}\,\mathrm{m}^2 \times 0.1 \times 10^{-3}\,\mathrm{m}}$$

Simplifying:

$$Y = \frac{450\pi}{9\pi \times 10^{-7}} = \frac{450}{9 \times 10^{-7}} = 5 \times 10^{10}\,\mathrm{N/m^2}$$

Therefore, the value of P is 5.
Thus, the correct answer is Option (4).

Quick Tip

When calculating Young's modulus for a wire, use the formula $Y = \frac{FL}{A\Delta L}$, where F is the force, L is the length, A is the cross-sectional area, and ΔL is the extension.

---

45. A rod of linear mass density λ and length L is bent to form a ring of radius R. Moment of inertia of the ring about any of its diameter is:

(1) $\frac{\lambda L^3}{8\pi^2}$
(2) $\frac{\lambda L^3}{4\pi^2}$
(3) $\frac{\lambda L^3}{16\pi^2}$
(4) $\frac{\lambda L^3}{12}$

Correct Answer: (1) $\frac{\lambda L^3}{8\pi^2}$

Solution: For a rod of length L and linear mass density λ, the total mass m of the rod is:

$$m = \lambda L$$

When the rod is bent into a ring of radius R, the mass is uniformly distributed along the circumference of the ring. The moment of inertia of the ring about any of its diameters is given by the formula for a ring:

$$I = \frac{1}{2}mR^2$$

Substituting $m = \lambda L$ into this expression:

$$I = \frac{1}{2}\lambda L R^2$$

We know that the circumference of the ring is $2\pi R$, and the length of the rod is equal to the circumference of the ring, so:

$$L = 2\pi R$$

Thus, the radius R can be written as:

$$R = \frac{L}{2\pi}$$

Substituting this into the equation for the moment of inertia:

$$I = \frac{1}{2}\lambda L\left(\frac{L}{2\pi}\right)^2 = \frac{1}{2}\lambda L \times \frac{L^2}{4\pi^2}$$

Simplifying:

$$I = \frac{\lambda L^3}{8\pi^2}$$

Thus, the moment of inertia of the ring about any of its diameters is $\frac{\lambda L^3}{8\pi^2}$.
Therefore, the correct answer is Option (1).

Quick Tip

The moment of inertia of a ring about any of its diameters can be derived by considering the geometry of the ring and using the formula for the moment of inertia of a mass distributed along the circumference.

Section - B

46. A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of an uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm. Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water. (Take the density of the unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm^3.) The unknown mass is ______ kg.

Correct Answer: 3

Solution:
Step 1: Determine the distances from the wedge (fulcrum).
- Total length of the rod = 27 cm.
- Distance from the wedge to the 200 gm mass = 25 cm (given).
- Therefore, distance from the wedge to the unknown mass = 27 cm - 25 cm = 2 cm.

Step 2: Calculate the volume of the cube and the buoyant force when half-submerged.
- Side of the cube = 10 cm.
- Volume of the cube, $V = 10^3 = 1000\,\text{cm}^3$.
- Half volume submerged, $V_{\text{sub}} = 500\,\text{cm}^3$.
- Buoyant force, $F_b = \rho_{\text{water}} \times V_{\text{sub}} \times g = 1\,\text{gm/cm}^3 \times 500\,\text{cm}^3 \times g = 500\,\text{gm} \times g$.

Step 3: Set up the torque equilibrium equation about the wedge. Let M be the unknown mass in grams.
- Torque due to the 200 gm mass: $200\,\text{gm} \times g \times 25\,\text{cm}$.
- Torque due to the unknown mass: $(M \times g - F_b) \times 2\,\text{cm} = (M \times g - 500\,\text{gm} \times g) \times 2\,\text{cm}$.

For equilibrium, the torques must balance:

$$200 \times g \times 25 = (M \times g - 500 \times g) \times 2$$

Cancel g from both sides:

$$200 \times 25 = (M - 500) \times 2$$

Simplify:

$$5000 = 2M - 1000 \quad \Rightarrow \quad 2M = 6000 \quad \Rightarrow \quad M = 3000\,\text{gm} = 3\,\text{kg}.$$

Quick Tip

In torque problems involving buoyancy: - Balance the clockwise and counter-clockwise torques about the fulcrum. - Account for the buoyant force when part of the object is submerged. - Ensure consistent units (e.g., convert grams to kilograms if needed).

---

47. A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm. By applying an external torque of 25π Nm for 40s, the speed increases to 2100 rpm. The diameter of the disk is ______ m.

Correct Answer: 40

Solution:
Step 1: Convert initial and final angular speeds from rpm to rad/s.
- Initial speed, $\omega_0 = 1800\,\text{rpm} = \frac{1800\times 2\pi}{60} = 60\pi\,\text{rad/s}$.
- Final speed, $\omega = 2100\,\text{rpm} = \frac{2100\times 2\pi}{60} = 70\pi\,\text{rad/s}$.

Step 2: Calculate angular acceleration (α) using torque and moment of inertia.
- Torque, $\tau = 25\pi\,\text{Nm}$.
- Moment of inertia for a thin disk rotating about its diameter:

$$I = \frac{1}{4}MR^2 = \frac{1}{4} \times 1 \times R^2 = \frac{R^2}{4}.$$

- Angular acceleration:

$$\tau = I\alpha \quad \Rightarrow \quad 25\pi = \frac{R^2}{4}\alpha \quad \Rightarrow \quad \alpha = \frac{100\pi}{R^2}.$$

Step 3: Relate angular acceleration to the change in angular velocity.

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 70\pi = 60\pi + \left(\frac{100\pi}{R^2}\right) \times 40.$$

Simplify:

$$10\pi = \frac{4000\pi}{R^2} \quad \Rightarrow \quad R^2 = 400 \quad \Rightarrow \quad R = 20\,\text{m}.$$

- Diameter, $D = 2R = 40\,\text{m}$.

Quick Tip

- For rotational motion problems, always convert rpm to rad/s for consistency. - The moment of inertia formula depends on the axis of rotation (e.g., $\frac{1}{4}MR^2$ for a disk rotating about its diameter). - Use the kinematic equation $\omega = \omega_0 + \alpha t$ to relate angular acceleration and time.

---

48. An electron is released from rest near an infinite non-conducting sheet of uniform charge density '–'. The rate of change of de-Broglie wavelength associated

with the electron varies inversely as n^{th} power of time. The numerical value of n is ______.

Correct Answer: 2

Solution:

Step 1: Determine the electric field and force on the electron. - For an infinite non-conducting sheet with charge density $-\sigma$, the electric field E is:

$$E = \frac{\sigma}{2\epsilon_0}$$

- The force on the electron (charge $-e$) is:

$$F = -eE = -\frac{e\sigma}{2\epsilon_0}$$

- The acceleration a of the electron is:

$$a = \frac{F}{m_e} = -\frac{e\sigma}{2\epsilon_0 m_e}$$

Step 2: Find the velocity as a function of time. - Since the electron starts from rest, its velocity v at time t is:

$$v = at = -\frac{e\sigma}{2\epsilon_0 m_e}t$$

Step 3: Express the de-Broglie wavelength λ. - The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{m_e \left|\frac{e\sigma}{2\epsilon_0 m_e}t\right|} = \frac{2\epsilon_0 h}{e\sigma t}$$

Step 4: Compute the rate of change of λ with respect to time.

$$\frac{d\lambda}{dt} = -\frac{2\epsilon_0 h}{e\sigma t^2}$$

- The magnitude of the rate of change is:

$$\left|\frac{d\lambda}{dt}\right| \propto \frac{1}{t^2}$$

Step 5: Compare with the given relation. - The problem states that $\frac{d\lambda}{dt}$ varies inversely as the n^{th} power of time. From Step 4, we see:

$$\left|\frac{d\lambda}{dt}\right| \propto \frac{1}{t^2} \quad \Rightarrow \quad n = 2$$

Quick Tip

- For problems involving charged sheets, remember the electric field is constant: $E = \frac{\sigma}{2\epsilon_0}$.
- The de-Broglie wavelength is inversely proportional to momentum: $\lambda = \frac{h}{p}$. - When dealing with rates, carefully differentiate and observe power-law relationships.

49. A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to change of volume of 0.8 cm^3. If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was ______ litre. (Take 1 atm = 10^5 Pa)

Correct Answer: 4

Solution:
Step 1: Calculate the pressure change (ΔP).

$$\Delta P = P_{\text{final}} - P_{\text{initial}} = 5\,\text{atm} - 1\,\text{atm} = 4\,\text{atm} = 4 \times 10^5\,\text{Pa}$$

Step 2: Use the bulk modulus formula. The bulk modulus (K) is given by:

$$K = -\frac{\Delta P}{\Delta V / V_0}$$

where: - $K = 2\,\text{GPa} = 2 \times 10^9\,\text{Pa}$ - $\Delta V = -0.8\,\text{cm}^3 = -0.8 \times 10^{-6}\,\text{m}^3$ (negative sign indicates volume decrease) - V_0 is the initial volume in m^3
Step 3: Solve for initial volume (V_0).

$$2 \times 10^9 = -\frac{4 \times 10^5}{-0.8 \times 10^{-6} / V_0}$$

$$2 \times 10^9 = \frac{4 \times 10^5 \times V_0}{0.8 \times 10^{-6}}$$

$$V_0 = \frac{2 \times 10^9 \times 0.8 \times 10^{-6}}{4 \times 10^5} = \frac{1.6 \times 10^3}{4 \times 10^5} = 4 \times 10^{-3}\,\text{m}^3 = 4\,\text{litres}$$

Quick Tip

- Bulk modulus relates pressure change to relative volume change: $K = -\Delta P/(\Delta V/V_0)$
- 1 m^3 = 1000 litres - Watch unit conversions (1 cm^3 = 10^{-6} m^3, 1 GPa = 10^9 Pa)

---

50. Space between the plates of a parallel plate capacitor of plate area 4 cm^2 and separation of $d = 1.77\,\text{mm}$, is filled with uniform dielectric materials with dielectric constants (3 and 5) as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is ____ pF.

Correct Answer: 15 pF

Solution:
Given: - Plate area of the capacitor, $A = 4\,\text{cm}^2 = 4 \times 10^{-4}\,\text{m}^2$,
- Separation between the plates, $d = 1.77\,\text{mm} = 1.77 \times 10^{-3}\,\text{m}$,
- Dielectric constants for the two regions, $k_1 = 5$ and $k_2 = 3$, - Capacitance of an additional parallel capacitor, $C_3 = 7.5\,\text{pF}$,
- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{F/m}$.

Step 1: Calculate the capacitance for each section with the dielectric constants.
For the first section (with dielectric constant $k_1 = 5$):
The capacitance is given by:

$$C_1 = \frac{\varepsilon_0 A}{d/2} \cdot k_1$$

Substitute the values:

$$C_1 = \frac{(8.85 \times 10^{-12}) \times (4 \times 10^{-4})}{1.77 \times 10^{-3}/2} \cdot 5$$

First calculate the denominator $d/2 = \frac{1.77 \times 10^{-3}}{2} = 8.85 \times 10^{-4}\,\mathrm{m}$.
Now substitute:

$$C_1 = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-4}}{8.85 \times 10^{-4}} \cdot 5 = 4 \times 10^{-12} \times 5 = 20 \times 10^{-12}\,\mathrm{F} = 20\,\mathrm{pF}$$

For the second section (with dielectric constant $k_2 = 3$):
Using the same formula:

$$C_2 = \frac{\varepsilon_0 A}{d/2} \cdot k_2$$

Substitute the values:

$$C_2 = \frac{(8.85 \times 10^{-12}) \times (4 \times 10^{-4})}{1.77 \times 10^{-3}/2} \cdot 3$$

$$C_2 = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-4}}{8.85 \times 10^{-4}} \cdot 3 = 4 \times 10^{-12} \times 3 = 12 \times 10^{-12}\,\mathrm{F} = 12\,\mathrm{pF}$$

Step 2: Find the total capacitance of the two dielectric regions in series.
The two dielectric regions are in series, so the total capacitance C_{total} is given by:

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substitute the values for C_1 and C_2:

$$\frac{1}{C_{\text{total}}} = \frac{1}{20\,\mathrm{pF}} + \frac{1}{12\,\mathrm{pF}}$$

$$\frac{1}{C_{\text{total}}} = \frac{3}{60\,\mathrm{pF}} + \frac{5}{60\,\mathrm{pF}} = \frac{8}{60\,\mathrm{pF}} = \frac{2}{15\,\mathrm{pF}}$$

Thus,

$$C_{\text{total}} = \frac{15}{2}\,\mathrm{pF} = 7.5\,\mathrm{pF}$$

Step 3: Add the capacitance of the additional parallel capacitor.
The total capacitance of the system is the sum of the series capacitance and the parallel capacitance C_3:

$$C_{\text{effective}} = C_{\text{total}} + C_3$$

$$C_{\text{effective}} = 7.5\,\text{pF} + 7.5\,\text{pF} = 15\,\text{pF}$$

Thus, the effective capacitance is $\boxed{15\,\text{pF}}$.

Quick Tip

When two dielectric materials are used in parallel plate capacitors, calculate the individual capacitances for each dielectric, and then treat them as separate capacitors in series or parallel as per the configuration.
