JEE Main 2023 24 Jan Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This Question paper consists of 90 Questions.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Question 1. Two long straight wires P and Q carrying equal currents of 10A each were kept parallel to each other at a distance of 5cm. The magnitude of the magnetic force experienced by a 10cm length of wire P is F_1. If the distance between the wires is halved and the currents in them are doubled, the force F_2 on a 10cm length of wire P will be:

(1) $8F_1$

(2) $10F_1$

(3) $\frac{F_1}{8}$

(4) $\frac{F_1}{10}$

Correct Answer: (1) $8F_1$

Solution The magnetic force between two parallel current-carrying wires is given by the formula:

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

where:

- F = magnetic force,
- μ_0 = permeability of free space ($4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$),
- I_1 and I_2 = currents in the wires,
- L = length of the wires,
- r = distance between the wires.

Given:

$$I_1 = I_2 = 10\ \mathrm{A}$$

$$r = 5\ \mathrm{cm} = 0.05\ \mathrm{m}$$

$$L = 10\ \mathrm{cm} = 0.1\ \mathrm{m}$$

$$F_1 = \frac{\mu_0 \times 10 \times 10 \times 0.1}{2\pi \times 0.05}$$

When the distance is halved and the currents are doubled:

$$I_1' = I_2' = 20\ \mathrm{A}$$

$$r' = \frac{5\ \mathrm{cm}}{2} = 2.5\ \mathrm{cm} = 0.025\ \mathrm{m}$$

$$L = 10\ \mathrm{cm} = 0.1\ \mathrm{m}$$

$$F_2 = \frac{\mu_0 \times 20 \times 20 \times 0.1}{2\pi \times 0.025}$$

Calculating the ratio $\frac{F_2}{F_1}$:

$$\frac{F_2}{F_1} = \frac{\frac{\mu_0 \times 20 \times 20 \times 0.1}{2\pi \times 0.025}}{\frac{\mu_0 \times 10 \times 10 \times 0.1}{2\pi \times 0.05}} = \frac{20 \times 20 \times 0.05}{10 \times 10 \times 0.025} = \frac{400 \times 0.05}{100 \times 0.025} = \frac{20}{2.5} = 8$$

$$\Rightarrow F_2 = 8F_1$$

Quick Tip

The magnetic force between parallel wires is directly proportional to the product of the currents and inversely proportional to the distance between them. Adjusting these parameters significantly affects the force experienced.

Question 2. Given below are two statements:

Statement-I : An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement-II : Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Correct Answer: (2) Statement I is true but Statement II is false

Solution : **Statement-I:**An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

When an elevator moves with a uniform speed (constant velocity), it implies that there is no acceleration . According to Newton's First Law of Motion, if there is no acceleration, the net force acting on the elevator must be zero .

$$\text{Net Force} = 0 \Rightarrow \text{Tension (T)} = \text{Weight (W)}$$

Where:

$$W = mg$$

$$T = mg$$

Here, m is the mass of the elevator and g is the acceleration due to gravity. Since the tension in the cable exactly balances the weight of the elevator, the elevator can move upwards or downwards at a uniform speed without any change in tension or weight.

$$\Rightarrow \text{Statement-I is True}$$

Statement-II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.
When the elevator is going down with increasing speed , it is undergoing downward acceleration . According to Newton's Second Law of Motion:

$$\text{Net Force} = ma$$

For a person standing in the elevator:

$$N - W = -ma$$

$$N = W - ma$$

Where:

$$N = \text{Normal force (force exerted by the floor)}$$

$$W = \text{Weight of the person} = mg$$

$$a = \text{Downward acceleration}$$

Since $a > 0$ (increasing speed downwards),

$$N = mg - ma = m(g - a)$$

Here, $g > a$ to ensure that the elevator is not in free fall. Therefore,

$$N < mg$$

This means the normal force exerted by the floor on the person is less than the person's actual weight.

$$\Rightarrow \text{Statement-II is False}$$

Quick Tip

When analyzing forces in accelerating systems, always consider the direction of acceleration. Downward acceleration in an elevator reduces the normal force experienced by a person standing inside.

Question 3. From the photoelectric effect experiment, the following observations are made. Identify which of these are correct:
A. The stopping potential depends only on the work function of the metal.
B. The saturation current increases as the intensity of incident light increases.
C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.
D. Photoelectric effect can be explained using wave theory of light. Choose the correct answer from the options given below:

(1) B, C only
(2) A, C, D only
(3) B only
(4) A, B, D only

Correct Answer: (3) B only

Solution : **Statement A:** *The stopping potential depends only on the work function of the metal.*

The stopping potential in the photoelectric effect is given by:

$$V_s = \frac{K_{\text{max}}}{e}$$

where K_{max} is the maximum kinetic energy of the emitted electrons and e is the elementary charge. According to Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

where h is Planck's constant, ν is the frequency of the incident light, and ϕ is the work function of the metal.

Thus, the stopping potential depends on both the frequency of the incident light and the work function of the metal, not solely on the work function.

Statement A is False.

Statement B: *The saturation current increases as the intensity of incident light increases.*

The saturation current in the photoelectric effect is directly proportional to the number of incident photons, provided that each photon has enough energy to eject an electron. An increase in the intensity of the incident light means more photons are striking the metal surface per unit time, leading to an increase in the number of photoelectrons emitted, and hence, an increase in the saturation current.

Statement B is True.

Statement C: *The maximum kinetic energy of a photo electron depends on the intensity of the incident light.*

From Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

The maximum kinetic energy depends only on the frequency of the incident light and the work function of the metal. It is independent of the intensity of the incident light, which affects the number of photoelectrons but not their kinetic energy.

Statement C is False.

Statement D: *Photoelectric effect can be explained using wave theory of light.*

The wave theory of light fails to explain certain observations of the photoelectric effect, such as the existence of a threshold frequency and the instantaneous emission of electrons regardless of light intensity. These phenomena are successfully explained by the particle theory of light, where light consists of photons with quantized energy.

Statement D is False.

Conclusion: Only Statement B is correct.

Quick Tip

In the photoelectric effect, the intensity of light affects the number of emitted electrons (saturation current) but not their kinetic energy, which is determined by the light's frequency.

Question 4. The weight of a body at the surface of Earth is 18 N. The weight of the body at an altitude of 3200 km above the Earth's surface is (given, radius of Earth $R_e = 6400$ **km):**

(1) 9.8N

(2) 4.9N

(3) 19.6N

(4) 8N

Correct Answer: (4) 8 N

Solution : The weight of a body is given by:

$$W = mg$$

where m is the mass of the body and g is the acceleration due to gravity.

At the surface of the Earth:

$$W_{\text{surface}} = mg = 18 \text{ N}$$

Thus,

$$mg = 18 \text{ N} \quad \Rightarrow \quad g = \frac{18}{m}$$

At an altitude $h = 3200$ km, the distance from the center of the Earth becomes:

$$r = R_e + h = 6400 \text{ km} + 3200 \text{ km} = 9600 \text{ km} = 9.6 \times 10^6 \text{ m}$$

The acceleration due to gravity at a distance r from the center of the Earth is given by:

$$g' = g\left(\frac{R_e}{r}\right)^2$$

Substituting the known values:

$$g' = 9.8 \text{ m/s}^2 \times \left(\frac{6400}{9600}\right)^2 = 9.8 \text{ m/s}^2 \times \left(\frac{2}{3}\right)^2 = 9.8 \text{ m/s}^2 \times \frac{4}{9} \approx 4.355 \text{ m/s}^2$$

Therefore, the weight at altitude h is:

$$W' = mg' = m \times 4.355 \text{ m/s}^2$$

Given that $mg = 18$ N, we can express W' in terms of W:

$$W' = W \times \left(\frac{g'}{g}\right) = 18 \text{ N} \times \frac{4.355}{9.8} \approx 8 \text{ N}$$

$$\Rightarrow W' \approx 8 \text{ N}$$

Quick Tip

The gravitational force decreases with the square of the distance from the center of the Earth. Always convert all distances to the same units before performing calculations.

Question 5. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be :

(1) 6.25×10^{-3} m

(2) 4×10^{-4} m

(3) 6.25×10^{-6} m

(4) 4×10^{-3} m

Correct Answer: (4) 4×10^{-3} m

Solution : Elongation in the wire can be calculated using Young's modulus formula:

$$\Delta L = \frac{FL}{AE}$$

where:

- $F = 250$ N (load applied),
- $L = 100$ m (original length of the wire),
- $A = 6.25 \times 10^{-4}$ m^2 (cross-sectional area),
- $E = 10^{10}$ Nm^{-2} (Young's modulus).

Substituting the given values:

$$\Delta L = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}} = \frac{25000}{6.25 \times 10^{6}} = 4 \times 10^{-3} \text{ m}$$

$$\Rightarrow \Delta L = 4 \times 10^{-3} \text{ m}$$

Quick Tip

Young's modulus relates the stress and strain in a material, allowing the calculation of elongation under a given load.

Question 6. 1 g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be :

(1) 4320 J

(2) 432000 J

(3) 4800 J

(4) 4.32×10^8 J

Correct Answer: (1) 4320J

Solution : The work done during the phase change is given by:

$$\text{Work done} = P\Delta V$$

where:

- $P = 3 \times 10^5$ Pa (pressure),
- $\Delta V = 1600 \text{ cm}^3 = 1600 \times 10^{-6} \text{ m}^3 = 1.6 \times 10^{-3} \text{ m}^3$ (change in volume).

Calculating the work done:

$$\text{Work done} = 3 \times 10^5 \times 1.6 \times 10^{-3} = 480 \text{ J}$$

Given that only 10% of the heat supplied (ΔQ) is used for work:

$$10\% \text{ of } \Delta Q = 480 \text{ J} \Rightarrow \Delta Q = 4800 \text{ J}$$

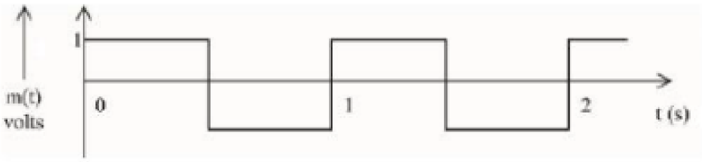
The remaining 90% of the heat goes into increasing the internal energy (ΔU):

$$\Delta U = 0.9 \times 4800 = 4320 \text{ J}$$

Quick Tip

In phase changes, a portion of the heat energy is used to do work against external pressure, and the remainder changes the internal energy of the system.

Question 7. A modulating signal is a square wave, as shown in the figure. If the carrier wave is given as $c(t) = 2\sin(8\pi t)$ volts, the modulation index is :



(1) $\frac{1}{4}$

(2) 1

(3) $\frac{1}{3}$

(4) $\frac{1}{2}$

Correct Answer: (4) $\frac{1}{2}$

Solution : The modulation index (m) in amplitude modulation is given by:

$$m = \frac{A_m}{A_c}$$

where:

- A_m is the amplitude of the modulating signal,
- $A_c = 2$ V is the amplitude of the carrier wave.

Given that the modulating signal is a square wave, its amplitude (A_m) is typically taken as the maximum deviation from zero, which is 1 V (assuming it's normalized).

Thus,

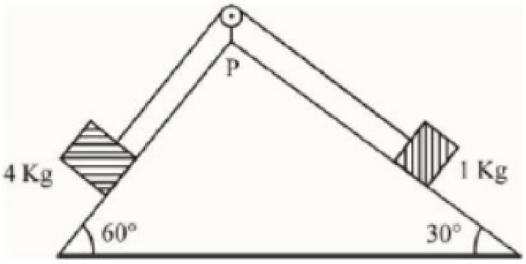
$$m = \frac{1}{2} = 0.5$$

$$\Rightarrow m = \frac{1}{2}$$

Quick Tip

The modulation index determines the extent of amplitude variation in the carrier wave and is crucial for avoiding over-modulation.

Question 8: As per given figure, a weightless pulley P is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if $g = 10$ m/s^2):



(1) $(4\sqrt{3} + 1)$ N

(2) $4(\sqrt{3} + 1)$ N

(3) $4(\sqrt{3} - 1)$ N

(4) $(4\sqrt{3} + 1)$ N

Correct Answer: (2) $4(\sqrt{3} + 1)$ N

Solution: Let T be the tension in the string and a be the acceleration of the system. Using Newton's second law, the force equations along the inclines are:

For the 4 kg block on the $60°$ incline:

$$4g \sin 60° - T = 4a \quad (1)$$

For the 1 kg block on the $30°$ incline:

$$T - g \sin 30° = a \quad (2)$$

Substituting $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$, we rewrite the equations:

$$20\sqrt{3} - T = 4a \quad (1)$$

$$T - 10 = a \quad (2)$$

From equation (2), solve for a:

$$a = T - 10$$

Substitute $a = T - 10$ into equation (1):

$$20\sqrt{3} - T = 4(T - 10)$$

Simplify:

$$20\sqrt{3} - T = 4T - 40$$

$$20\sqrt{3} + 40 = 5T$$

$$T = \frac{20\sqrt{3} + 40}{5} = 4\sqrt{3} + 4$$

Thus, the tension in the string is:

$$T = 4(\sqrt{3} + 1)\,\text{N}$$

Quick Tip

For inclined plane problems, resolve forces along the incline carefully, and ensure all forces, including tension and weight components, are balanced accurately.

Question 9: Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion (A): Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason (R): The current in the forward bias is more than the current in the reverse bias for a p–n junction diode.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (2) Both A and R are true but R is NOT the correct explanation of A

Solution: Photodiodes are operated in reverse bias because the fractional change in current due to light is easier to detect in reverse bias. While it is true that the forward bias current is greater than the reverse bias current, this is not the reason photodiodes are used in reverse bias.

Thus, A is true, and R is also true, but R is not the correct explanation for A.

Quick Tip

Photodiodes are designed for precise light detection, and reverse bias ensures better sensitivity due to the minimal leakage current compared to forward bias.

Question 10: If $\vec{E}$ represents the electric field vector and $\vec{K}$ the propagation vector of electromagnetic waves in vacuum, then the magnetic field vector is given by (ω - angular frequency):

(1) $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$

(2) $\vec{B} = \omega(\vec{E} \times \vec{K})$

(3) $\vec{B} = \omega(\vec{K} \times \vec{E})$

(4) $(\vec{K} \times \vec{E})$

Correct Answer: (1) $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$

Solution: Electromagnetic (EM) waves consist of oscillating electric and magnetic fields that are perpendicular to each other and to the direction of wave propagation. The following relationships hold for the electric field ($\vec{E}$), magnetic field ($\vec{B}$), and wave vector ($\vec{K}$):

1. The wave vector $\vec{K}$ indicates the direction of wave propagation.
2. The electric field $\vec{E}$ oscillates perpendicular to $\vec{K}$.
3. The magnetic field $\vec{B}$ is perpendicular to both $\vec{E}$ and $\vec{K}$.

The magnitude and direction of $\vec{B}$ are determined using the cross product:

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$$

where ω is the angular frequency of the wave.

Explanation of Options: Option (1): Correct. This represents the magnetic field as proportional to the cross product of $\vec{K}$ and $\vec{E}$, divided by the angular frequency.

Option (2): Incorrect. The direction of $\vec{B}$ would be wrong as the cross product order matters ($\vec{K} \times \vec{E} \neq \vec{E} \times \vec{K}$).

Option (3): Incorrect. Multiplying by ω gives a dimensionally incorrect magnetic field.

Option (4): Incorrect. The dot product results in a scalar, not a vector.

Thus, the correct option is (1).

Quick Tip

In electromagnetic waves, $\vec{E}$, $\vec{B}$, and $\vec{K}$ are mutually perpendicular, forming a right-handed coordinate system. Use the right-hand rule to verify directions.

Question 11: A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of the circular loop and at a distance r from the center of the loop on its axis is:

(1) $1 : 3\sqrt{2}$

(2) $3\sqrt{2} : 2$

(3) $2\sqrt{2} : 1$

(4) $1 : \sqrt{2}$

Correct Answer: (3) $2\sqrt{2} : 1$

Solution: The magnetic field due to a current-carrying circular loop on its axis is given as:

$$B_{\text{axis}} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

At the center of the loop ($x = 0$):

$$B_{\text{center}} = \frac{\mu_0 I}{2r}$$

At a distance r on the axis ($x = r$):

$$B_{\text{axis}} = \frac{\mu_0 I}{2 \cdot 2\sqrt{2}r}$$

The ratio of magnetic fields is:

$$\frac{B_{\text{center}}}{B_{\text{axis}}} = \frac{\frac{\mu_0 I}{2r}}{\frac{\mu_0 I}{2 \cdot 2\sqrt{2}r}} = 2\sqrt{2}$$

Thus, the ratio $B_{\text{center}} : B_{\text{axis}} = 2\sqrt{2} : 1$.

Quick Tip

In problems involving magnetic fields of current-carrying loops, use symmetry and the Biot-Savart law to derive the required relationships. Remember, the center field is always stronger than the axial field at any distance.

Question 12: A travelling wave is described by the equation:

$$y(x,t) = 0.05\sin(8x - 4t)\,\text{m}$$

The velocity of the wave is: (All quantities are in SI units.)

(1) $4\,\text{ms}^{-1}$

(2) $2\,\text{ms}^{-1}$

(3) $0.5\,\text{ms}^{-1}$

(4) $8\,\text{ms}^{-1}$

Correct Answer: (3) $0.5\,\text{ms}^{-1}$

Solution: The general equation of a wave is given by:

$$y(x,t) = A\sin(kx - \omega t)$$

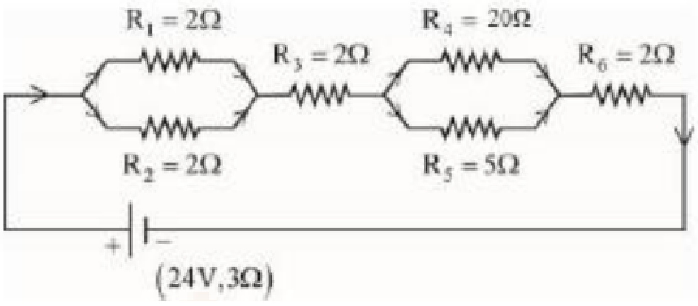
where $k = 8\,\text{m}^{-1}$ and $\omega = 4\,\text{rad/s}$. The velocity of the wave is:

$$v = \frac{\omega}{k} = \frac{4}{8} = 0.5\,\text{ms}^{-1}$$

Quick Tip

In wave motion, velocity is given by $v = \frac{\omega}{k}$, where ω is the angular frequency and k is the wave number. Ensure all quantities are in SI units before calculating.

Question 13: As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of $3\,\Omega$. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are:



(1) $I_4 = \frac{8}{5}$ A and $I_5 = \frac{2}{5}$ A

(2) $I_4 = \frac{24}{5}$ A and $I_5 = \frac{6}{5}$ A

(3) $I_4 = \frac{6}{5}$ A and $I_5 = \frac{24}{5}$ A

(4) $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A

Correct Answer: (4) $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A

Solution: The equivalent resistance of the circuit is calculated as:

$$R_{\text{eq}} = 3 + 1 + 2 + 4 + 2 = 12\,\Omega$$

The current through the battery is:

$$I = \frac{24}{12} = 2\,\text{A}$$

The current I_4 through R_4 is given by:

$$I_4 = \frac{R_5}{R_4 + R_5} \times 2 = \frac{5}{20 + 5} \times 2 = \frac{2}{5}\,\text{A}$$

The current I_5 through R_5 is:

$$I_5 = 2 - I_4 = 2 - \frac{2}{5} = \frac{8}{5}\,\text{A}$$

Thus, the correct values are $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A.

Quick Tip

For resistor circuits, always calculate the equivalent resistance step-by-step and use Kirchhoff's laws or current division principles as needed.

Question 14: Given below are two statements:

Statement I: If the Brewster's angle for the light propagating from air to glass is θ_b, then Brewster's angle for the light propagating from glass to air is:

$$\frac{\pi}{2} - \theta_b$$

Statement II: The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$, where μ_g is the refractive index of glass.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are false.

Correct Answer: (2) Statement I is true but Statement II is false.

Solution: Statement I is correct. The Brewster's angle for light propagating from glass to air is given as $\frac{\pi}{2} - \theta_b$, where θ_b is the Brewster's angle for light propagating from air to glass.

Statement II is incorrect. The Brewster's angle for light propagating from glass to air is not $\tan^{-1}(\mu_g)$. The correct expression for the Brewster's angle involves the relative refractive indices of the two media.

Thus, Statement I is true and Statement II is false.

Quick Tip

The Brewster's angle is specific to the direction of light propagation. Use Snell's law and the definition of Brewster's angle for clarity.

Question 15: If two charges q_1 and q_2 are separated with distance d and placed in a medium of dielectric constant K, what will be the equivalent distance between charges in air for the same electrostatic force?

(1) $d\sqrt{K}$

(2) $k\sqrt{d}$

(3) $1.5d\sqrt{K}$

(4) $2d\sqrt{K}$

Correct Answer: (1) $d\sqrt{K}$

Solution: The force between two charges in a medium of dielectric constant K is given by Coulomb's law:

$$F_{\text{medium}} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{d^2}$$

where: - q_1 and q_2 are the charges, - d is the separation between the charges, - K is the dielectric constant of the medium.

The force between the same charges in air is:

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{d'^2}$$

where d' is the equivalent separation in air.

For the same electrostatic force, we equate $F_{\text{medium}} = F_{\text{air}}$:

$$\frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{d^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{d'^2}$$

Simplify the equation:

$$\frac{1}{K} \cdot \frac{1}{d^2} = \frac{1}{d'^2}$$

This gives:

$$d'^2 = K \cdot d^2$$

Taking the square root:

$$d' = d\sqrt{K}$$

Thus, the equivalent distance in air is:

$$d' = d\sqrt{K}$$

Quick Tip

For problems involving dielectric constants, remember that the force reduces by a factor of K in a medium compared to air. Use proportional relationships between the distances and the dielectric constant for accurate results.

Question 16: Consider the following radioactive decay process:

$$^{218}_{84}\mathrm{A} \to \mathrm{A}_1 \to \mathrm{A}_2 \to \mathrm{A}_3 \to \mathrm{A}_4 \to \mathrm{A}_5 \to \mathrm{A}_6$$

The mass number and the atomic number of A_6 are given by:

(1) 210 and 82

(2) 210 and 84

(3) 210 and 80

(4) 211 and 80

Correct Answer: (3) 210 and 80

Solution: We analyze the decay process step by step:

1:

$$^{214}_{83}\mathrm{A} \xrightarrow{\beta^-} {}^{214}_{84}\mathrm{A}_1$$

A β^- decay increases the atomic number by 1, while the mass number remains unchanged. Result: $^{214}_{84}\mathrm{A}_1$

2:

$$^{214}_{84}\mathrm{A}_1 \xrightarrow{\alpha} {}^{210}_{83}\mathrm{A}_2$$

An α decay decreases the mass number by 4 and the atomic number by 2. Result: $^{210}_{83}\mathrm{A}_2$

3:

$$^{210}_{83}\mathrm{A}_2 \xrightarrow{\beta^-} {}^{210}_{82}\mathrm{A}_3$$

A β^- decay increases the atomic number by 1, while the mass number remains unchanged. Result: $^{210}_{82}\mathrm{A}_3$

4:

$$^{210}_{82}\mathrm{A}_3 \xrightarrow{\alpha} {}^{206}_{82}\ \mathrm{A}_4$$

An α decay decreases the mass number by 4 and the atomic number by 2. Result: $^{206}_{80}\mathrm{A}_4$
After the decay process, the mass number and atomic number of A_6 are:

$$\text{Mass Number: } 210, \quad \text{Atomic Number: } 80$$

Quick Tip

In radioactive decay processes, carefully track changes to the mass number (-4 for α) and atomic number ($+1$ for β^-, -2 for α). This step-by-step approach ensures accurate results.

Question 17: Given below are two statements:

Statement I: The temperature of a gas is $-73°C$. When the gas is heated to $527°C$, the root mean square speed of the molecules is doubled.

Statement II: The product of pressure and volume of an ideal gas will be equal to the translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true.
(2) Statement I is true but Statement II is false.
(3) Both Statement I and Statement II are false.
(4) Statement I is false but Statement II is true.

Correct Answer: (2) Statement I is true but Statement II is false.

Solution:

Statement I: The root mean square (RMS) speed of gas molecules is given by:

$$v_{\text{rms}} \propto \sqrt{T}$$

Let $T_1 = -73°C = 200\,\text{K}$ and $T_2 = 527°C = 800\,\text{K}$. The ratio of RMS speeds is:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{800}{200}} = \sqrt{4} = 2$$

Thus, the RMS speed doubles, and Statement I is true.

Statement II: For an ideal gas, the product of pressure and volume is given by:

$$PV = nRT$$

The translational kinetic energy is:

$$\text{Translational KE} = \frac{3}{2}nRT$$

Since $PV \neq \frac{3}{2}nRT$, Statement II is false.

> **Quick Tip**
>
> The RMS speed of molecules depends on the square root of absolute temperature. For ideal gases, always distinguish between PV (pressure-volume work) and translational kinetic energy.

Question 18: The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance up to which he can throw the same ball is:

(1) $192\,\text{m}$

(2) $136\,\text{m}$

(3) $272\,\text{m}$

(4) $68\,\text{m}$

Correct Answer: (3) $272\,\text{m}$

Solution: The maximum vertical height is related to the maximum velocity as:

$$H_{\text{max}} = \frac{v^2}{2g}$$

Given $H_{\text{max}} = 136\,\text{m}$, solve for v^2:

$$v^2 = 2gH_{\text{max}} = 2 \cdot 9.8 \cdot 136 = 2 \cdot 136 \,(\text{using } g \approx 9.8\,\text{m/s}^2).$$

The maximum horizontal range R_{max} is given by:

$$R_{\text{max}} = \frac{v^2}{g} = 2H_{\text{max}} = 2 \cdot 136 = 272\,\text{m}.$$

Quick Tip

The maximum horizontal range is always twice the maximum height for a projectile when launched at $45°$. Use $R_{\text{max}} = 2H_{\text{max}}$ for quick calculations.

Question 19: A conducting loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) 1 mV

(2) 10 mV

(3) 100 mV

(4) 5 mV

Correct Answer: (2) 10 mV

Solution:

The area of the loop is:

$$A = \pi r^2 = \pi \left(\frac{10}{\sqrt{\pi}}\right)^2 = 100\,\text{cm}^2 = 10^{-2}\,\text{m}^2.$$

The magnetic flux through the loop is:

$$\Phi = B \cdot A = 0.5 \cdot 10^{-2} = 5 \cdot 10^{-3}\,\text{Wb}.$$

The emf induced is:

$$\mathcal{E} = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{5 \cdot 10^{-3}}{0.5} = 10\,\text{mV}.$$

Quick Tip

Induced emf is calculated using Faraday's law: $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$. Ensure flux is calculated correctly using A and B, and convert units to SI for accurate results.

Question 20: Match List I with List II:

List I	List II
A. Planck's constant (h)	I. $M^1L^2T^{-2}$
B. Stopping potential (V_s)	II. $M^1L^1T^{-1}$
C. Work function (ϕ)	III. $M^1L^2T^{-1}$
D. Momentum (p)	IV. $M^1L^2T^{-3}A^{-1}$

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-IV, C-III, D-I

(4) A-I, B-III, C-IV, D-II

Correct Answer: (2) A-III, B-IV, C-I, D-II

Solution:

1. Planck's constant (h): Energy is given by $E = h\nu$. Using the dimensional formula of energy:

$$[E] = M^1L^2T^{-2}, \quad [\nu] = T^{-1}$$

Thus:

$$[h] = \frac{[E]}{[\nu]} = \frac{M^1L^2T^{-2}}{T^{-1}} = M^1L^2T^{-1}$$

Match: $A \rightarrow$ III.

2. Stopping potential (V_s): Voltage is given by $V = \frac{E}{q}$. Using dimensional formulas:

$$[V] = \frac{M^1L^2T^{-2}}{AT^{-1}} = M^1L^2T^{-3}A^{-1}$$

Match: $B \rightarrow$ IV.

3. Work function (ϕ): Work function is a form of energy:

$$[\phi] = M^1L^2T^{-2}$$

Match: $C \rightarrow$ I.

4. Momentum (p): Momentum is given by $p = F \cdot t$. Using dimensional formulas:

$$[p] = M^1L^1T^{-2} \cdot T^1 = M^1L^1T^{-1}$$

Match: $D \rightarrow$ II.

Thus, the correct matching is $A \rightarrow$ III, $B \rightarrow$ IV, $C \rightarrow$ I, $D \rightarrow$ II.

Quick Tip

Dimensional analysis is a powerful tool to validate physical quantities. Always check the units of derived quantities against the base SI units.

Question 21: A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5$^{\text{th}}$ second. The force acted on the body is ____ N.

Correct Answer: (4) $40\,\text{N}$

Solution:

The kinetic energy is related to velocity as:

$$\frac{1}{2}mv^2 = \text{KE}$$

Substitute the given values ($m = 2\,\text{kg},\ \text{KE} = 10000\,\text{J}$):

$$\frac{1}{2} \cdot 2 \cdot v^2 = 10000$$

$$v^2 = 10000, \quad v = 100\,\text{m/s}$$

The acceleration is found using:

$$v = u + at$$

Since the body starts from rest ($u = 0$) and reaches $v = 100\,\text{m/s}$ in $t = 5\,\text{s}$:

$$100 = 0 + a \cdot 5 \implies a = \frac{100}{5} = 20\,\text{m/s}^2$$

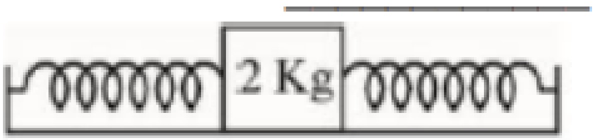
The force acting on the body is:

$$F = m \cdot a = 2 \cdot 20 = 40\,\text{N}$$

Quick Tip

In kinematics problems, use relationships between kinetic energy, velocity, and acceleration to solve for force systematically. Always check units for consistency.

Question 22: A block of mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface, and the ends of the springs are attached to rigid supports. When the mass is displaced from its equilibrium position, it executes simple harmonic motion. The time period of oscillation is given by $\frac{\pi}{\sqrt{x}}$ in SI units. The value of x is:



Correct Answer: $x = 5$

Solution: The equivalent spring constant for the system is given by:

$$k_{\text{eq}} = k_1 + k_2 = 20 + 20 = 40 \,\text{N/m}.$$

The angular frequency ω for the block-mass system is:

$$\omega = \sqrt{\frac{k_{\text{eq}}}{m}} = \sqrt{\frac{40}{2}} = \sqrt{20} \,\text{rad/s}.$$

The time period of oscillation T is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}} \,\text{s}.$$

Comparing $T = \frac{\pi}{\sqrt{x}}$, we find:

$$x = 5.$$

Quick Tip

For systems with multiple springs, use the equivalent spring constant formula for parallel or series combinations. Then apply the formula for angular frequency and time period systematically.

Question 23: A hole is drilled in a metal sheet. At $27°$C, the diameter of the hole is 5 cm. When the sheet is heated to $177°$C, the change in the diameter of the hole is Δd. The value of Δd will be, if the coefficient of linear expansion of the metal is $1.6 \times 10^{-5}/°$C:

Correct Answer: $\Delta d = 0.12\,\text{cm}$

Solution: The change in diameter of the hole due to thermal expansion is given by:

$$\Delta d = d_0 \cdot \alpha \cdot \Delta T$$

where: - $d_0 = 5\,\text{cm}$, - $\alpha = 1.6 \times 10^{-5}/°\text{C}$, - $\Delta T = 177°\text{C} - 27°\text{C} = 150°\text{C}$.

Substitute the values:

$$\Delta d = 5 \cdot 1.6 \times 10^{-5} \cdot 150 = 12 \times 10^{-3}\,\text{cm}.$$

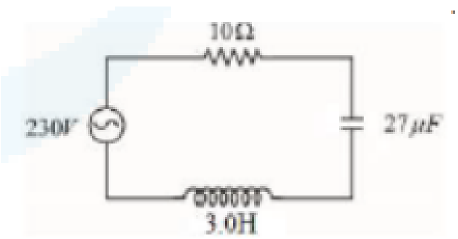
Thus, the change in diameter is:

$$\Delta d = 0.12\,\text{cm}.$$

Quick Tip

For thermal expansion problems, ensure that all quantities are in consistent units. Remember, the expansion of a hole is treated the same as the expansion of the material enclosing it.

Question 24: In the circuit shown in the figure, the ratio of the quality factor and the bandwidth is ____ s.



Correct Answer: $10\,\text{s}$

Solution: The following relations are used for an LCR circuit:

$$\Delta\omega = \frac{R}{L}, \quad Q = \frac{\omega_0}{\Delta\omega}.$$

The resonant angular frequency ω_0 is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

Substitute the given values for $L = 3 \times 10^{-3}\,\text{H}$ and $C = 27 \times 10^{-6}\,\text{F}$:

$$\omega_0 = \frac{1}{\sqrt{3 \times 10^{-3} \cdot 27 \times 10^{-6}}} = \frac{1}{\sqrt{81 \times 10^{-9}}} = \frac{1}{9 \times 10^{-3}} = 10^3\,\text{rad/s}.$$

The quality factor Q is:

$$Q = \omega_0 \cdot \frac{L}{R}.$$

The bandwidth $\Delta\omega$ is:

$$\Delta\omega = \frac{R}{L}.$$

Substitute the values for $R = 9 \times 10^{-3}\,\Omega$ and $L = 3 \times 10^{-3}\,\text{H}$:

$$\Delta\omega = \frac{9 \times 10^{-3}}{3 \times 10^{-3}} = 3\,\text{rad/s}.$$

Now calculate the ratio $\frac{Q}{\Delta\omega}$:

$$\frac{Q}{\Delta\omega} = \frac{\omega_0 \cdot L/R}{R/L}.$$

Rewriting:

$$\frac{Q}{\Delta\omega} = \omega_0 \cdot \frac{L^2}{R^2}.$$

Substitute the values:

$$\frac{Q}{\Delta\omega} = \omega_0 \cdot \frac{L^2}{R^2} = 10^3 \cdot \frac{(3 \times 10^{-3})^2}{(9 \times 10^{-3})^2}.$$

Simplify:

$$\frac{Q}{\Delta\omega} = 10^3 \cdot \frac{9 \times 10^{-6}}{81 \times 10^{-6}} = 10^3 \cdot \frac{1}{9} = 10\,\text{s}.$$

Thus, the ratio of the quality factor to the bandwidth is:

$$\frac{Q}{\Delta\omega} = 10\,\text{s}.$$

Quick Tip

In LCR circuits, the quality factor Q is inversely proportional to the bandwidth. Use $\Delta\omega = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$ for accurate calculations.

Question 25: A hollow cylindrical conductor has a length of $3.14\,$m, while its inner and outer diameters are $4\,$mm and $8\,$mm, respectively. The resistance of the conductor is $2 \times 10^{-3}\,\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\,\Omega\,$m, the value of n is:

Correct Answer: $n = 2$

Solution:

The resistance of a conductor is given by:

$$R = \rho \frac{\ell}{A}$$

where: - $R = 2 \times 10^{-3}\,\Omega$, - $\rho = 2.4 \times 10^{-8}\,\Omega\,\text{m}$, - $\ell = 3.14\,\text{m}$, - A is the cross-sectional area of the hollow cylinder.

The cross-sectional area A of the hollow cylinder is:

$$A = \pi(b^2 - a^2)$$

where: - $b = 4\,\text{mm} = 4 \times 10^{-3}\,\text{m}$, - $a = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}$.

Substitute the values:

$$A = \pi\left[(4 \times 10^{-3})^2 - (2 \times 10^{-3})^2\right] = \pi\left[16 \times 10^{-6} - 4 \times 10^{-6}\right] = \pi \cdot 12 \times 10^{-6}.$$

Now calculate R:

$$R = \rho \frac{\ell}{A} = \frac{2.4 \times 10^{-8} \cdot 3.14}{\pi \cdot 12 \times 10^{-6}} = \frac{2.4 \times 10^{-8} \cdot 3.14}{3.77 \times 10^{-5}} = 2 \times 10^{-3}\,\Omega.$$

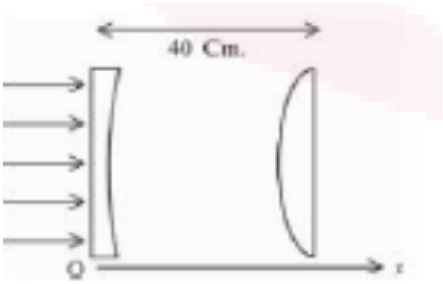
This confirms the calculated value, and:

$$n = 2.$$

Quick Tip

For hollow cylindrical conductors, always use the formula $A = \pi(b^2 - a^2)$ for the cross-sectional area. Ensure all units are consistent, especially for diameters and lengths.

Question 26: As shown in the figure, a combination of a thin plano-concave lens and a thin plano-convex lens is used to image an object placed at infinity. The radius of curvature of both lenses is 30 cm, and the refractive index of the material for both lenses is 1.75. Both lenses are placed at a distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance x cm, from the concave lens.



Correct Answer: $x = 120\,\text{cm}$

Solution: 1. Focal length of the plano-concave lens (L_1): The focal length of a plano-concave lens is given by:

$$\frac{1}{f_1} = (1-\mu)\left(\frac{1}{R}\right),$$

where $\mu = 1.75$ and $R = 30\,\text{cm}$. Substitute the values:

$$\frac{1}{f_1} = (1-1.75)\cdot\frac{1}{30} = -\frac{0.75}{30}.$$

Thus:

$$f_1 = -40\,\text{cm}.$$

2. Focal length of the plano-convex lens (L_2): The focal length of a plano-convex lens is given by:

$$\frac{1}{f_2} = (\mu-1)\left(\frac{1}{R}\right).$$

Substitute the values:

$$\frac{1}{f_2} = (1.75-1)\cdot\frac{1}{30} = \frac{0.75}{30}.$$

Thus:

$$f_2 = 40\,\text{cm}.$$

3. Final image formation: - The object is at infinity, so the image from L_1 will be virtual and on the left at its focal length ($f_1 = -40\,\text{cm}$). - This virtual image acts as the object for L_2, which is placed $40\,\text{cm}$ away from L_1. - The distance of the object from L_2 is:

$$u = -(40-40) = -80\,\text{cm}.$$

Using the lens formula for L_2:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}.$$

Substitute the values:

$$\frac{1}{v} - \frac{1}{-80} = \frac{1}{40}.$$

Simplify:

$$\frac{1}{v} + \frac{1}{80} = \frac{1}{40}.$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{80} = \frac{2-1}{80} = \frac{1}{80}.$$

Thus:

$$v = 80\,\text{cm}.$$

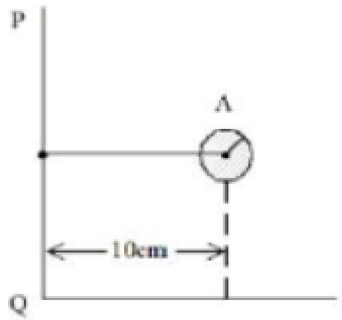
The final image is formed $80\,\text{cm}$ to the right of L_2. From the concave lens L_1, the total distance is:

$$x = 40 + 80 = 120\,\text{cm}.$$

Quick Tip

In lens combinations, always determine the focal lengths of individual lenses first. Use the lens formula step-by-step to trace the image positions.

Question 27: A solid sphere A is rotating about an axis PQ. If the radius of the sphere is $5\,\text{cm}$, then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is:



Correct Answer: $x = 110$

Solution: The radius of gyration k is related to the moment of inertia I of the sphere and its mass M by:

$$I = M \cdot k^2 \quad \Rightarrow \quad k = \sqrt{\frac{I}{M}}.$$

1.Moment of inertia of the sphere about the axis PQ: The moment of inertia of a solid sphere about an axis passing through a point at a distance d from its center is given by the parallel axis theorem:

$$I = I_{\text{center}} + M \cdot d^2,$$

where: - $I_{\text{center}} = \frac{2}{5}MR^2$ is the moment of inertia about the center, - $d = 10\,\text{cm}$ is the distance from the center to the axis PQ, - $R = 5\,\text{cm}$ is the radius of the sphere.

Substituting:

$$I = \frac{2}{5}MR^2 + Md^2 = \frac{2}{5}M(5)^2 + M(10)^2.$$

Simplify:

$$I = \frac{2}{5}M \cdot 25 + M \cdot 100 = \frac{50}{5}M + 100M = 10M + 100M = 110M.$$

2.Radius of gyration (k): The radius of gyration is:

$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{110M}{M}} = \sqrt{110}\,\text{cm}.$$

3.Relating to $\sqrt{x}$: From the problem, $k = \sqrt{x}\,\text{cm}$. Thus:

$$x = 110.$$

Quick Tip

Use the parallel axis theorem to calculate the moment of inertia when the axis does not pass through the center of mass. For a solid sphere, remember $I_{\text{center}} = \frac{2}{5}MR^2$.

Question 28: Vectors $\vec{a} = a\hat{i} + b\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$. The ratio of a to b is $\frac{x}{2}$. The value of x is:

Correct Answer: $x = 1$

Solution: For two vectors to be perpendicular, their dot product must be zero:

$$\vec{a} \cdot \vec{b} = 0.$$

Substitute $\vec{a} = a\hat{i} + b\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$:

$$(a\hat{i} + b\hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0.$$

Simplify:

$$2a - 3b + 4 = 0.$$

This gives the first equation:

$$2a - 3b = -4.$$

From the problem, another equation is given:

$$3a + 2b = 7.$$

Solve the simultaneous equations: Multiply equation (1) by 2:

$$4a - 6b = -8.$$

Multiply equation (2) by 3:

$$9a + 6b = 21.$$

Add equations (3) and (4):

$$13a = 13 \quad \Rightarrow \quad a = 1.$$

Substitute $a = 1$ into equation (2):

$$3(1) + 2b = 7 \quad \Rightarrow \quad 3 + 2b = 7 \quad \Rightarrow \quad 2b = 4 \quad \Rightarrow \quad b = 2.$$

The ratio of a to b is:

$$\frac{a}{b} = \frac{x}{2} \quad \Rightarrow \quad \frac{1}{2} = \frac{x}{2}.$$

Thus:

$$x = 1.$$

Quick Tip

For perpendicular vectors, always ensure their dot product equals zero. Use simultaneous equations systematically to solve for unknowns.

Question 29: Assume that protons and neutrons have equal masses. The mass of a nucleon is 1.6×10^{-27} kg, and the radius of a nucleus is $1.5 \times 10^{-15} A^{1/3}$ m. The approximate ratio of nuclear density to water density is $n \times 10^{13}$. The value of n is:

Correct Answer: $n = 11$

Solution: The density of a nucleus is:

$$\rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}.$$

1. Mass of the nucleus: The mass of a single nucleon is:

$$m = 1.6 \times 10^{-27}\,\text{kg}.$$

2. Volume of the nucleus: The volume of a nucleus is given by:

$$V = \frac{4}{3}\pi R^3,$$

where $R = 1.5 \times 10^{-15}$ m. Substituting:

$$V = \frac{4}{3}\pi(1.5 \times 10^{-15})^3 = \frac{4}{3}\pi \cdot 3.375 \times 10^{-45}.$$

Approximate:

$$V \approx 14.14 \times 10^{-45}\,\text{m}^3.$$

3. Nuclear density (ρ):

$$\rho = \frac{m}{V} = \frac{1.6 \times 10^{-27}}{14.14 \times 10^{-45}} \approx 0.113 \times 10^{18}\,\text{kg/m}^3.$$

4. Water density (ρ_w):

$$\rho_w = 10^3\,\text{kg/m}^3.$$

5. Ratio of nuclear density to water density:

$$\frac{\rho}{\rho_w} = \frac{0.113 \times 10^{18}}{10^3} = 0.113 \times 10^{15} = 11.31 \times 10^{13}.$$

Thus, the approximate ratio is:

$$n = 11.$$

Quick Tip

For density calculations, ensure proper units and correct formulas for mass and volume. Use approximations carefully in large-scale problems.

Question 30: A stream of positively charged particles having $\frac{q}{m} = 2 \times 10^{11}$ C/kg and velocity $\vec{v_0} = 3 \times 10^7\,\hat{i}$ m/s is deflected by an electric field $\vec{E} = 1.8\,\hat{j}$ kV/m. The electric field exists in a region of 10 cm along the x-direction. Due to the electric field, the deflection of the charged particles in the y-direction is ____ mm.

Correct Answer: 2 mm

Solution: 1. Acceleration of the particles in the y-direction (a): The force on the charged particles due to the electric field is:

$$F = qE.$$

Using $F = ma$, the acceleration in the y-direction is:

$$a = \frac{F}{m} = \frac{qE}{m}.$$

Substitute the values:

$$a = \left(2 \times 10^{11}\right) \cdot \left(1.8 \times 10^3\right) = 3.6 \times 10^{14}\,\text{m/s}^2.$$

2. Time taken to cross the plates (t): The time t to travel a distance of 10 cm $= 0.1$ m along the x-direction is given by:

$$t = \frac{d}{v_0},$$

where $d = 0.1$ m and $v_0 = 3 \times 10^7$ m/s.

Substitute the values:

$$t = \frac{0.1}{3 \times 10^7} = \frac{1}{3 \times 10^8}\,\text{s}.$$

3. Deflection in the y-direction (y): The deflection in the y-direction is given by:

$$y = \frac{1}{2}at^2.$$

Substitute the values:

$$y = \frac{1}{2} \cdot \left(3.6 \times 10^{14}\right) \cdot \left(\frac{1}{3 \times 10^8}\right)^2.$$

Simplify:

$$y = \frac{1}{2} \cdot 3.6 \times 10^{14} \cdot \frac{1}{9 \times 10^{16}}.$$

$$y = \frac{3.6}{2} \cdot \frac{1}{9 \times 10^2} = \frac{1.8}{900} = 0.002\,\text{m}.$$

Convert to millimeters:

$$y = 0.002\,\text{m} = 2\,\text{mm}.$$

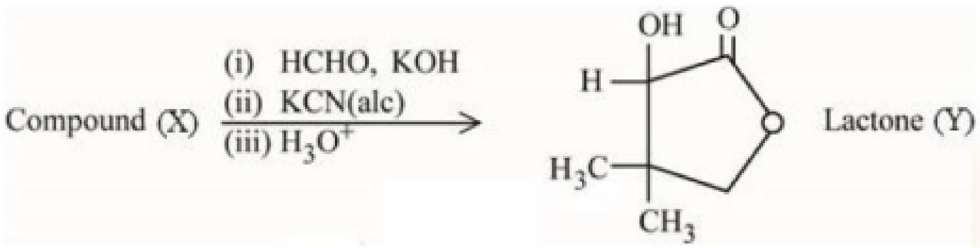
Thus, the deflection in the y-direction is:

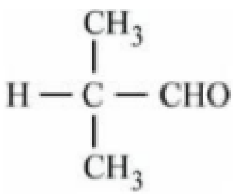
$$y = 2\,\text{mm}.$$

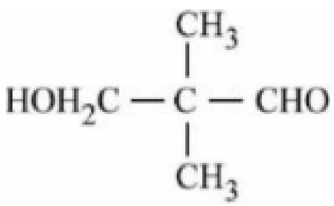
Quick Tip

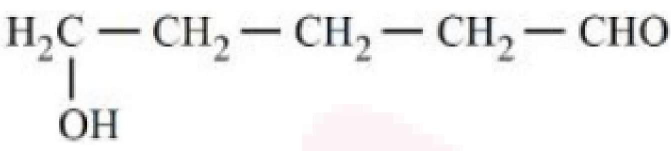
For problems involving deflection of charged particles in an electric field, calculate the acceleration due to $F = ma$, the time taken to traverse the region, and then use kinematic equations to find the displacement.

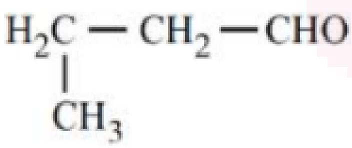
Question 31: Compound (X) undergoes the following sequence of reactions to give the lactone (Y):



(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

Solution: The reaction sequence involves the following steps:

1. Aldol Condensation: Compound X undergoes aldol condensation in the presence of dilute KOH. This results in the formation of a β-hydroxy aldehyde intermediate.

Reaction:

$$CH_3CHO + HCHO \xrightarrow{\text{KOH}} CH_3CH(OH)CH_2CHO.$$

2. Cyanohydrin Formation: The aldehyde group reacts with HCN to form a cyanohydrin derivative. The -CN group is added to the β-hydroxy aldehyde.

Reaction:

$$CH_3CH(OH)CH_2CHO \xrightarrow{\text{HCN}} CH_3CH(OH)CH_2CH(CN)OH.$$

3. Acid Hydrolysis and Lactonization: The cyanohydrin undergoes hydrolysis in the presence of dilute H_2SO_4, forming a carboxylic acid group. The hydroxyl and carboxyl groups within the molecule undergo intramolecular esterification (lactonization) to form the lactone (Y).

Final product:

$$CH_3CH(OH)CH_2CH(COOH)OH \xrightarrow{\text{H}_2O\text{,H}^+} \text{Lactone (Y)}.$$

The structure of lactone (Y) matches the product given in **option (1)**.

Quick Tip

In multi-step organic synthesis, identify the functional group transformations in each step. Aldol condensation produces β-hydroxy aldehydes, cyanohydrin formation adds $-CN$, and acid hydrolysis can convert $-CN$ to $-COOH$.

Question 32: Assertion (A): Hydrolysis of an alkyl chloride is a slow reaction, but in the presence of NaI, the rate of hydrolysis increases.

Reason (R): I^- is a good nucleophile as well as a good leaving group.

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true, and R is the correct explanation of A.

(4) Both A and R are true, but R is NOT the correct explanation of A.

Correct Answer: (3) Both A and R are true, but R is NOT the correct explanation of A.

Solution:

1. Assertion (A): - Hydrolysis of alkyl chlorides is typically a slow reaction because the leaving group (Cl^-) is not very efficient.

- However, in the presence of NaI, I^- displaces Cl^- in an S_N2 substitution reaction, forming alkyl iodides. - Alkyl iodides hydrolyze faster due to the better leaving group (I^-).

- Therefore, Assertion (A) is true.

2. Reason (R): - I^- is both a good nucleophile and an excellent leaving group. This property enhances the S_N2 reaction rate and increases the hydrolysis rate.

- Hence, Reason (R) is true.

3. Relationship between A and R: - The presence of NaI enhances the hydrolysis rate primarily because I^- is a better nucleophile and a better leaving group than Cl^-. - This is consistent with the assertion and explains the observed phenomenon.

- Thus, Reason (R) correctly explains Assertion (A).

Quick Tip

When solving assertion-reason questions, evaluate both the truth of the statements and whether the reason directly explains the assertion. For chemical reactions, carefully analyze the mechanism to determine the role of nucleophiles and leaving groups.

Question 33: Order of covalent bond:

A. KF > KI; LiF > KF

B. KF < KI; LiF > KF

C. $SnCl_2 > SnCl_4$; CuCl > NaCl

D. LiF > KF; CuCl < NaCl

E. KF < KI; CuCl > NaCl

(1) C, E only

(2) B, C only

(3) B, C, E only

(4) A, B only

Correct Answer: (3) B, C, E only

Solution: The order of covalent character can be predicted using Fajans' Rule, which states: 1. Covalent character increases with smaller cation size and larger anion size. 2. Covalent character increases with higher charge on the cation.

Now, analyze each statement: A: KF $>$ KI; LiF $>$ KF: **False.** KF has less covalent character than KI because the larger iodide ion polarizes the cation more effectively.

B: KF $<$ KI; LiF $>$ KF: **True.** KI is more covalent than KF, and LiF is less ionic than KF due to the smaller size of Li^+.

C: $SnCl_2 > SnCl_4$; CuCl $>$ NaCl: **True.** $SnCl_2$ is more covalent than $SnCl_4$ because Sn^2+ is more polarizing. CuCl is more covalent than NaCl because Cu^+ is smaller and has higher polarizing power than Na^+.

D: LiF $>$ KF; CuCl $<$ NaCl: **False.** CuCl is more covalent than NaCl.

E: KF $<$ KI; CuCl $>$ NaCl: **True.** KF is less covalent than KI, and CuCl is more covalent than NaCl.

Thus, the correct choices are B, C, and E.

Quick Tip

Fajans' Rule is a key principle to determine the order of covalent character: smaller cations, larger anions, and higher charges increase covalent character.

Question 34: Increasing order of stability of the resonance structures is:

A. 

B. 

C. 

D. 

(1) C, D, B, A

(2) C, D, A, B

(3) D, C, A, B

(4) D, C, B, A

Correct Answer: (2) BONUS as no option matching the solution

Solution: 1.Factors affecting resonance stability: Structures with complete octets are more stable. Negative charges on more electronegative atoms are favored. Separation of charges decreases stability.

2.Analysis of the given structures: Structure C has significant charge separation and is the least stable. Structure D has localized charges but disrupts the aromatic system, so it is less stable than A and B. Structure A is more stable than C and D because it retains partial resonance. Structure B is the most stable because it has complete resonance and no significant charge separation.

Thus, the correct order of stability is:

$$C < D < A < B.$$

Quick Tip

For resonance stability, prioritize octet completion, minimize charge separation, and favor negative charges on electronegative atoms.

Question 35: The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is:

(1) Cr^{2+}

(2) Mn^{2+}

(3) V^{2+}

(4) Ti^{2+}

Correct Answer: (3) V^{2+}

Solution: The magnetic moment (μ) of a transition metal ion can be calculated using the formula:

$$\mu = \sqrt{n(n+2)}\,\text{B.M.}$$

where n is the number of unpaired electrons.

Let us calculate the magnetic moment for each option:

(1) Cr^{2+}: Electronic configuration: $[\text{Ar}], 3d^4, 4s^0$ Unpaired electrons (n) = 4

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \approx 4.89\,\text{B.M.}$$

(2) Mn^{2+}: Electronic configuration: $[\text{Ar}], 3d^5, 4s^0$ Unpaired electrons (n) = 5

$$\mu = \sqrt{5(5+2)} = \sqrt{35} \approx 5.91\,\text{B.M.}$$

(3) V^{2+}: Electronic configuration: $[\text{Ar}], 3d^3, 4s^0$ Unpaired electrons (n) = 3

$$\mu = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\text{B.M.}$$

This matches the given magnetic moment.

(4) Ti^{2+}: Electronic configuration: $[\text{Ar}], 3d^2, 4s^0$ Unpaired electrons (n) = 2

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.82\,\text{B.M.}$$

Conclusion: The only ion with a calculated magnetic moment of approximately 3.87 B.M. is V^{2+}.

Quick Tip

The magnetic moment formula $\mu = \sqrt{n(n+2)}$ is essential for determining the number of unpaired electrons in transition metal ions.

Question 36: Match List I with List II:

List I	List II
A.Reverberatory furnace	Pig Iron
B.Electrolytic cell	Aluminum
C.Blast furnace	Silicon
D.Zone Refining furnace	Copper

(1) A – IV, B – II, C – I, D – III

(2) A – I, B – IV, C – II, D – III

(3) A – I, B – III, C – II, D – IV

(4) A – III, B – IV, C – I, D – II

Correct Answer: (1) A – IV, B – II, C – I, D – III

Solution: Reverberatory furnace: Used for roasting of copper.

Electrolytic cell: For reactive metals like aluminum.

Blast furnace: Converts hematite to pig iron.

Zone Refining furnace: Used for purifying semiconductors like silicon.

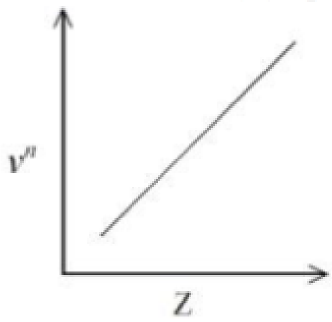
Matching:

A – IV, B – II, C – I, D – III

Quick Tip

Remember the specific industrial applications of different furnaces and cells for efficient matching.

Question 37: It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power n, i.e., ν^n of X-rays emitted, is plotted against atomic number Z, the following graph is obtained.



The value of n is:

(1) 1

(2) 2

(3) $\frac{1}{2}$

(4) 3

Correct Answer: (3) $\frac{1}{2}$

Solution: The question is based on **Moseley's Law**, which describes the relationship between the frequency of characteristic X-rays and the atomic number Z.

Moseley's Law:

$$\sqrt{\nu} \propto Z$$

or equivalently,

$$\nu \propto Z^2$$

Here: - ν is the frequency of the emitted X-rays. - Z is the atomic number of the element.

1. **Graph Interpretation:** The graph provided shows a linear relationship between ν^n and Z. For this to hold true: - From Moseley's law, $\nu^{1/2} \propto Z$. - Therefore, $n = \frac{1}{2}$.

2. **Physical Basis:** The characteristic X-rays are emitted due to the transition of electrons from higher energy levels to the inner shells (K or L). The energy difference between these levels is proportional to Z^2, leading to the proportionality of $\nu \propto Z^2$.

3. **Verification of n:** For $\nu^{1/2}$ (or $\sqrt{\nu}$), the graph becomes linear because:

$$\sqrt{\nu} = k \cdot Z$$

where k is a proportionality constant. This matches the given graph.

Conclusion: The value of n that satisfies the given graph is $\frac{1}{2}$.

Quick Tip

Moseley's law is crucial for understanding the relationship between X-ray frequency and atomic number: $\nu^{1/2} \propto Z$.

Question 38: Which of the phosphorus oxoacids can create a silver mirror from $AgNO_3$ solution?

(1) $(HPO_3)_n$

(2) $H_4P_2O_5$

(3) $H_4P_2O_6$

(4) $H_4P_2O_7$

Correct Answer: (2) $H_4P_2O_5$

Solution: Phosphorus oxoacids with a P-H bond act as reducing agents and can reduce $AgNO_3$ to silver (Ag), producing a silver mirror.

Among the given options, $H_4P_2O_5$ contains a P-H bond, which makes it capable of reducing $AgNO_3$.

The reaction mechanism involves the oxidation of the oxoacid, with the P-H bond breaking and silver (Ag) being reduced. The silver forms a mirror on the surface of the reaction vessel.

Quick Tip

Only phosphorus oxoacids with P-H bonds can reduce $AgNO_3$ to silver, creating a silver mirror.

Question 39: The primary and secondary valencies of cobalt, respectively, in $[Co(NH_3)_5Cl]Cl_2$ are:

(1) 3 and 5

(2) 2 and 6

(3) 2 and 8

(4) 3 and 6

Correct Answer: (4) 3 and 6

Solution: The complex compound $[Co(NH_3)_5Cl]Cl_2$ has cobalt (Co) with an oxidation number of $+3$.

Primary valency: The primary valency corresponds to the oxidation state of cobalt, which is $+3$.

Secondary valency: The secondary valency corresponds to the coordination number, which is the total number of ligands directly bonded to the metal ion. In this case, there are 5 ammonia (NH_3) molecules and 1 chloride ion (Cl^-), giving a coordination number of 6.

Conclusion: The primary valency is 3, and the secondary valency is 6.

Quick Tip

The primary valency is the oxidation state, while the secondary valency is the coordination number of the metal in a complex.

Question 40: An ammoniacal metal salt solution gives a brilliant red precipitate on the addition of dimethylglyoxime. The metal ion is:

(1) Cu^{2+}

(2) Co^{2+}

(3) Fe^{2+}

(4) Ni^{2+}

Correct Answer: (4) Ni^{2+}

Solution: Dimethylglyoxime (DMG) reacts with Ni^{2+} ions in the presence of ammonium hydroxide (NH_3) to form a bright red precipitate of nickel dimethylglyoxime complex $[Ni(DMG)_2]$. The reaction is as follows:

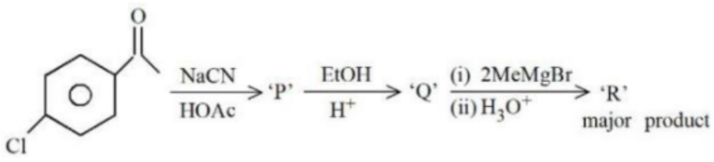
$$Ni^{2+} + 2DMG + NH_3 \longrightarrow [Ni(DMG)_2] \text{ (red complex)}$$

Observation: The formation of a rosy red precipitate confirms the presence of Ni^{2+}.

Quick Tip

Dimethylglyoxime (DMG) is a specific reagent for detecting Ni^{2+} ions, forming a characteristic red precipitate.

Question 41: 'R' formed in the following sequence of reactions is:



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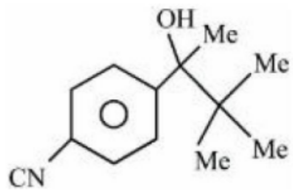

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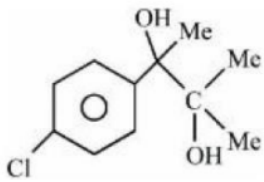

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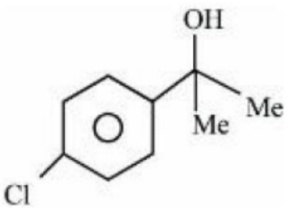

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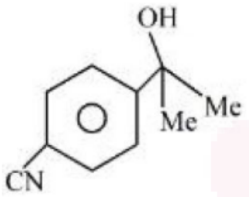


Correct Answer: (2)

Solution: The reaction sequence involves multiple steps:

Step 1: Cyanohydrin Formation (Intermediate P) - The aldehyde group ($-$CHO) in chlorobenzaldehyde reacts with NaCN in the presence of HOAc, leading to the formation of a cyanohydrin intermediate. - Cyanohydrin structure (P): $C_6H_4(Cl)(CHOH)(CN)$.

Step 2: Hydrolysis of Cyanohydrin (Intermediate Q) - The cyanohydrin (P) undergoes hydrolysis with ethanol and acidic conditions ($EtOH, H^+$) to produce a β-hydroxy ketone intermediate (Q). - Structure of (Q): $C_6H_4(Cl)(COCH_2OH)$.

Step 3: Grignard Reaction (Formation of Final Product R) - The intermediate (Q) reacts with methyl magnesium bromide (2MeMgBr) to add two $-$Me groups to the carbonyl carbon, forming a tertiary alcohol. - This is followed by hydrolysis with H_3O^+, leading to the final product R. - Structure of R: $C_6H_4(Cl)(CN)(OH)$ with $-$Me, $-$Me groups on the same carbon as the hydroxyl group.

Conclusion: The correct product R matches Option (2).

Quick Tip

Understand the sequence of reactions step-by-step: cyanohydrin formation, hydrolysis, and Grignard reaction, to predict the correct product.

Question 42: Match List I with List II:

List I	List II
A.Chlorophyll	1.Na_2CO_3
B.Soda ash	2.$CaSO_4$
C.Dentistry, Ornamental work	3.Mg^{2+}
D.Used in white washing	4.$Ca(OH)_2$

(1) A – III, B – I, C – II, D – IV

(2) A – II, B – I, C – III, D – IV

(3) A – III, B – IV, C – I, D – II

(4) A – II, B – III, C – IV, D – I

Correct Answer: (1) A – III, B – I, C – II, D – IV

Solution: To correctly match List I with List II, let's analyze each item:

A. Chlorophyll: - Chlorophyll contains magnesium (Mg^{2+}) as its central metal ion. - **Match: A – III (Mg^{2+}).**

B. Soda ash: - Soda ash is chemically sodium carbonate (Na_2CO_3). - **Match: B – I (Na_2CO_3).**

C. Dentistry, Ornamental work: - Calcium sulfate ($CaSO_4$) is used in dentistry and ornamental work, especially in plaster form. - **Match: C – II ($CaSO_4$).**

D. Used in white washing: - Slaked lime ($Ca(OH)_2$) is commonly used in white washing. - **Match: D – IV ($Ca(OH)_2$).**

Conclusion: The correct matches are:

$$A - III, B - I, C - II, D - IV$$

Quick Tip

To solve matching problems efficiently, recall the chemical names and common uses of the given compounds.

Question 43: Statement I: For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at the same concentration.
Statement II: For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

(1) Statement I is true but Statement II is false.
(2) Statement I is false but Statement II is true.
(3) Both Statement I and Statement II are true.
(4) Both Statement I and Statement II are false.

Correct Answer: (3) Both Statement I and Statement II are true.
Solution: Statement I: For colloidal particles, colligative properties (e.g., vapor pressure lowering, boiling point elevation) are smaller than for true solutions because the effective number of solute particles is much smaller in colloids. This is due to the aggregation of dispersed phase particles. **Conclusion: True.**
Statement II: The zeta potential (or electrokinetic potential) refers to the potential difference between the fixed layer of ions attached to the colloidal particles and the diffused layer of counterions. It plays a crucial role in colloidal stability. **Conclusion: True.**
Final Conclusion: Both statements are correct.

Quick Tip

Colligative properties are lower in colloids compared to true solutions due to aggregation. Zeta potential helps describe colloidal stability.

Question 44: Reaction of BeO with ammonia and hydrogen fluoride gives 'A' which on thermal decomposition gives BeF_2 and NH_4F. What is 'A'?

(1) $(NH_4)_2BeF_4$

(2) H_3NBeF_3

(3) $(NH_3)BeF_3$

(4) $(NH_4)Be_2F_5$

Correct Answer: (1) $(NH_4)_2BeF_4$

Solution: Step 1: Reaction Description - Beryllium oxide (BeO) reacts with ammonia (NH_3) and hydrogen fluoride (HF) to form ammonium tetrafluoroberyllate ($(NH_4)_2BeF_4$). - The reaction is as follows:

$$BeO + 2NH_3 + 4HF \longrightarrow (NH_4)_2BeF_4 + H_2O$$

Step 2: Thermal Decomposition - On heating, $(NH_4)_2BeF_4$ decomposes to give beryllium fluoride (BeF_2) and ammonium fluoride (NH_4F).

- The reaction is as follows:

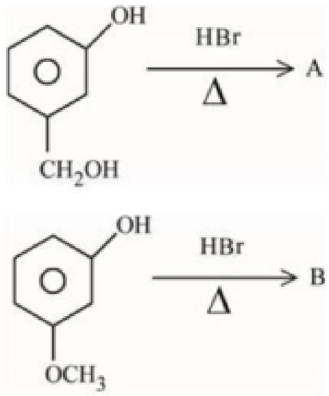
$$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + 2NH_4F$$

Conclusion: The compound 'A' is $(NH_4)_2BeF_4$.

Quick Tip

Ammonium tetrafluoroberyllate ($(NH_4)_2BeF_4$) is a key intermediate in reactions involving BeO, NH_3, and HF.

Question 45: ‘A’ and ‘B’ formed in the following set of reactions are:



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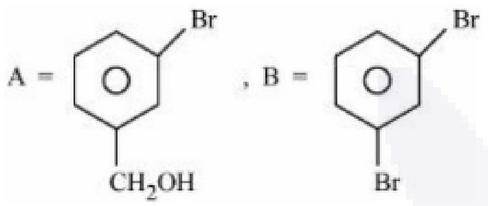

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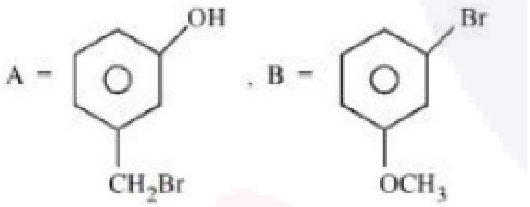

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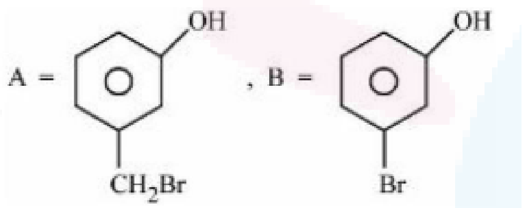

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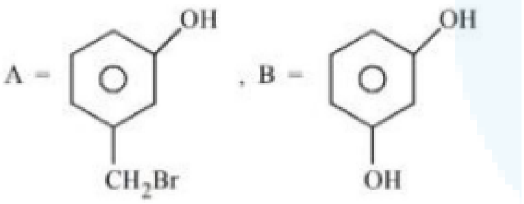


Correct Answer: (4)

Solution: Reaction 1: Formation of Compound A - The benzyl alcohol derivative reacts with HBr under heat. - In the reaction, the $-CH_2OH$ group undergoes substitution to form $-CH_2Br$. The $-OH$ group remains unaffected under these conditions. - Structure of A: $C_6H_4(CH_2Br)(OH)$.

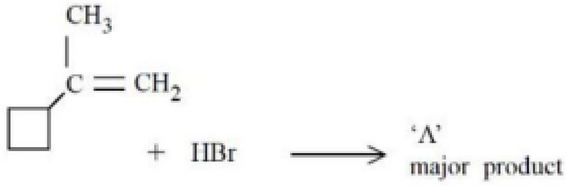
Reaction 2: Formation of Compound B - The anisole derivative reacts with HBr under heat. - The $-OCH_3$ group undergoes cleavage to form $-OH$, while bromination occurs at the ortho position (relative to the $-$OH group). - Structure of B: $C_6H_4(Br)(OH)$. 5

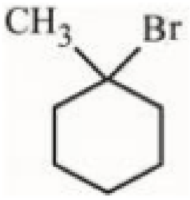
Conclusion: - Compound A: $C_6H_4(CH_2Br)(OH)$. - Compound B: $C_6H_4(Br)(OH)$. The correct answer is Option (4).

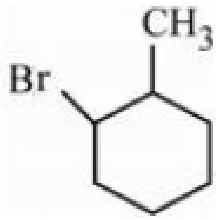
Quick Tip

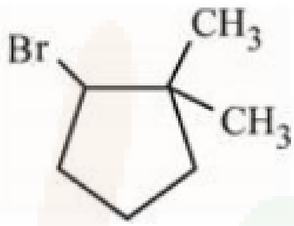
Reactions of HBr with alcohols and ethers involve substitution and cleavage, respectively. Analyze each functional group carefully.

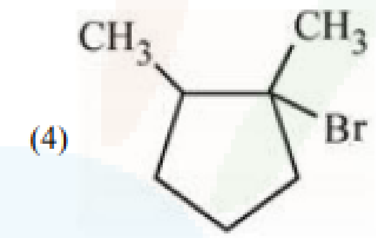
Question 46: In the following reaction, 'A' is:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (4)

Solution: The reaction involves the addition of HBr to an alkene. The major product is determined by **Markovnikov's Rule**, which states that the hydrogen atom adds to the carbon of the double bond that has more hydrogen atoms, and the bromine atom adds to the carbon that has fewer hydrogen atoms.

Step-by-Step Mechanism: 1. The starting compound is cyclobutyl methyl alkene ($CH_3-C = CH_2$).

2. When HBr is added, the double bond undergoes electrophilic addition: - The proton (H^+) adds to the less substituted carbon of the double bond (the terminal carbon), creating a carbocation on the more substituted carbon. - This follows Markovnikov's rule, resulting in a tertiary carbocation.

3. The bromide ion (Br^-) then attacks the carbocation, leading to the formation of the major product.

Conclusion: The correct answer is Option (4): Cyclopentyl bromide with Br at C2 and CH_3 at C1.

Quick Tip

Markovnikov's rule determines the regioselectivity of addition reactions. The more stable carbocation intermediate leads to the major product.

Question 47: The decreasing order of the hydrogen bonding in the following forms of water is correctly represented by:

A. Liquid water

B. Ice

C. Impure water

(1) A = B $>$ C

(2) B $>$ A $>$ C

(3) C $>$ B $>$ A

(4) A $>$ B $>$ C

Correct Answer: (2) $B > A > C$

Solution: Hydrogen bonding in water is influenced by the structural arrangement and the presence of impurities:

- **Ice:** In ice, water molecules are arranged in a crystalline structure with extensive hydrogen bonding. This fixed arrangement allows for maximum hydrogen bonding, making ice have the strongest hydrogen bonding among the three forms.
- **Liquid Water:** In liquid water, hydrogen bonds are dynamic and constantly break and re-form due to molecular motion. This results in slightly weaker hydrogen bonding compared to ice.
- **Impure Water:** Impurities in water disrupt the hydrogen bonding network, further weakening the hydrogen bonding compared to pure liquid water.

Conclusion: The decreasing order of hydrogen bonding is:

$$\text{Ice (B)} > \text{Liquid Water (A)} > \text{Impure Water (C)}$$

Quick Tip

Hydrogen bonding strength decreases from ice to liquid water to impure water due to structural disruption and impurities.

Question 48: Given below are two statements:

Statement I: Noradrenaline is a neurotransmitter.

Statement II: Low level of noradrenaline is not the cause of depression in humans.

(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are correct.

(4) Both Statement I and Statement II are incorrect.

Correct Answer: (1)

Solution: Statement I: Noradrenaline (norepinephrine) is a key neurotransmitter in the central nervous system (CNS). It plays a vital role in regulating mood, attention, and the fight-or-flight response.

Conclusion: True.

Statement II: Low levels of noradrenaline have been scientifically linked to depression. Noradrenaline is involved in mood regulation, and its deficiency is a recognized contributing factor in depressive disorders. Therefore, the statement that "low level of noradrenaline is not the cause of depression" is incorrect.

Conclusion: False.

Final Conclusion: Statement I is correct, but Statement II is incorrect.

> **Quick Tip**
>
> Noradrenaline is a neurotransmitter involved in mood regulation, and its deficiency can contribute to depression.

Question 49: In the depression of freezing point experiment:

A. Vapour pressure of the solution is less than that of pure solvent.

B. Vapour pressure of the solution is more than that of pure solvent.

C. Only solute molecules solidify at the freezing point.

D. Only solvent molecules solidify at the freezing point.

(1) A and D only

(2) B and C only

(3) A and C only

(4) A only

Correct Answer: (1) A and D only

Solution: Statement A: In a solution, the vapour pressure is reduced compared to the pure solvent due to the presence of solute particles, which lower the escaping tendency of solvent molecules.

Conclusion: True.

Statement B: Vapour pressure of the solution cannot be more than that of the pure solvent because solute particles always decrease vapour pressure.

Conclusion: False.

Statement C: At the freezing point, only solvent molecules solidify because solute molecules disrupt the crystalline structure of the solvent.

Conclusion: False.

Statement D: Only solvent molecules solidify at the freezing point due to the selective crystallization of the solvent.

Conclusion: True.

Final Conclusion: The correct statements are A and D.

> **Quick Tip**
>
> Depression in freezing point occurs due to the reduced vapour pressure of the solution compared to the pure solvent, and only solvent solidifies at the freezing point.

Question 50: Which of the following is true about freons?

(1) These are chlorofluorocarbon compounds.

(2) These are chemicals causing skin cancer.

(3) These are radicals of chlorine and chlorine monoxide.

(4) All radicals are called freons.

Correct Answer: (1)

Solution: Freons are chlorofluorocarbon (CFC) compounds, primarily used as refrigerants and aerosol propellants. These compounds contain chlorine, fluorine, and carbon.

Option 1: True. Freons are CFC compounds.

Option 2: False. While freons contribute to ozone layer depletion, which indirectly increases UV radiation exposure (a cause of skin cancer), freons themselves are not chemicals directly causing skin cancer.

Option 3: False. Freons are not radicals but stable compounds.

Option 4: False. Not all radicals are freons.

Conclusion: The only correct option is (1).

> **Quick Tip**
>
> Freons are stable CFCs used in refrigeration and aerosols. They are harmful due to their role in ozone depletion.

Question 51: The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of 0.2 M CH_3COONa solution is mixed with 25 mL of 0.02 M CH_3COOH solution, the pH of the resultant solution is found to be equal to 5. The value of x is ____.

Correct Answer: $x = 10$.

Solution: The system forms a buffer solution of acetic acid (CH_3COOH) and sodium acetate (CH_3COONa). The pH of a buffer is calculated using the Henderson-Hasselbalch equation:

$$\text{pH} = \text{pKa} + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

Given: - Salt concentration $= 0.2\,\text{M}$, Volume $= 25\,\text{mL}$ - Acid concentration $= 0.02\,\text{M}$, Volume $= 25\,\text{mL}$ - pH $= 5$

Step 1: Calculate the concentration ratio of salt and acid.

$$\frac{[\text{Salt}]}{[\text{Acid}]} = \frac{(0.2 \times 25)}{(0.02 \times 25)} = \frac{0.1}{0.01} = 10$$

Step 2: Substitute into the Henderson-Hasselbalch equation.

$$5 = \text{pKa} + \log(10)$$

$$5 = \text{pKa} + 1$$

$$\text{pKa} = 4$$

Step 3: Relate pKa to the dissociation constant (K_a).

$$\text{pKa} = -\log K_a$$

$$K_a = 10^{-\text{pKa}} = 10^{-4}$$

Step 4: Express K_a in the required form.

$$K_a = x \times 10^{-5},\ x = 10$$

Quick Tip

Buffer pH can be calculated using the Henderson-Hasselbalch equation. Relate pKa and K_a carefully to find the dissociation constant.

Question 52: 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1 M solution?

Correct Answer: $180mL$.

Solution:

Step 1: Calculate the molarity of the stock solution.

$$\text{Moles of NaOH} = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{5}{40} = 0.125\,\text{mol}$$

$$\text{Molarity of stock solution} = \frac{\text{Moles}}{\text{Volume (in L)}} = \frac{0.125}{0.450} \approx 0.278\,\text{M}$$

Step 2: Use the dilution formula $M_1V_1 = M_2V_2$.

$$M_1 = 0.278\,\text{M},\ M_2 = 0.1\,\text{M},\ V_2 = 500\,\text{mL}$$

$$0.278 \times V_1 = 0.1 \times 500$$

$$V_1 = \frac{0.1 \times 500}{0.278} \approx 180\,\text{mL}$$

Quick Tip

For dilution problems, use the formula $M_1V_1 = M_2V_2$ and carefully calculate the molarity of the stock solution first.

Question 53: If the wavelength of the first line of the Paschen series of the hydrogen atom is 720 nm, then the wavelength of the second line of this series is _____ nm (nearest integer).

Correct Answer: $\lambda = 492\,\text{nm}$.

Solution: The Paschen series corresponds to transitions of electrons in a hydrogen atom from higher energy levels (n_2) to the third energy level ($n_1 = 3$).

The wavelength of emitted radiation is given by the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where: - R is the Rydberg constant ($1.097 \times 10^7\,\text{m}^{-1}$) - $n_1 = 3$ (for the Paschen series) - n_2 is the upper energy level ($n_2 = 4$ for the first line and $n_2 = 5$ for the second line)

Step 1: Calculate the wavelength of the first line ($n_2 = 4$)**.** Given $\lambda = 720\,\text{nm}$ for the first line:

$$\frac{1}{720 \times 10^{-9}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{720 \times 10^{-9}} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{1}{720 \times 10^{-9}} = 1.097 \times 10^7 \left(\frac{16 - 9}{144}\right)$$

$$\frac{1}{720 \times 10^{-9}} = 1.097 \times 10^7 \times \frac{7}{144}$$

This confirms the consistency of the Rydberg formula for the given wavelength.

Step 2: Calculate the wavelength of the second line ($n_2 = 5$)**.** For $n_2 = 5$:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{25}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{25 - 9}{225}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{16}{225}$$

$$\frac{1}{\lambda} = 7.809 \times 10^5\,\text{m}^{-1}$$

$$\lambda = \frac{1}{7.809 \times 10^5} = 4.92 \times 10^{-7}\,\text{m} = 492\,\text{nm}$$

Quick Tip

The Paschen series involves transitions to $n_1 = 3$. Use the Rydberg formula to calculate wavelengths for successive lines in the series.

Question 54: The number of correct statements from the following is ____.

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperatures.

D. A plot of $\ln k$ vs $\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$.

Correct answer: 3.

Solution: Statement A: The Arrhenius equation is given by:

$$k = Ae^{-\frac{E_a}{RT}}$$

Here, E_a is the activation energy. Larger E_a makes the exponential term $e^{-\frac{E_a}{RT}}$ smaller, leading to a smaller rate constant k.

Conclusion: True.

Statement B: The temperature coefficient is the ratio of rate constants at two temperatures:

$$\frac{k_2}{k_1} = e^{\frac{E_a}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

Higher activation energy (E_a) amplifies the change in k with temperature, increasing the temperature coefficient.

Conclusion: True.

Statement C: At lower temperatures, the change in k with temperature is not necessarily more significant. In fact, as temperature increases, the value of k increases more rapidly due to the exponential nature of the Arrhenius equation.

Conclusion: False.

Statement D: The linear form of the Arrhenius equation is:

$$\ln k = -\frac{E_a}{R}\frac{1}{T} + \ln A$$

This equation correctly describes a straight-line graph with a slope of $-\frac{E_a}{R}$.

Conclusion: True.

Final Conclusion: Statements A, B, and D are correct. Statement C is incorrect.

Quick Tip

The Arrhenius equation helps understand the effect of activation energy and temperature on the rate constant k. Be cautious with qualitative interpretations of the temperature effect.

Question 55: At 298 K, a 1-liter solution containing 10 mmol of $Cr_2O_7^{2-}$ and 100 mmol of Cr^{3+} shows a pH of 3.0. The potential for the half-cell reaction is $x \times 10^{-3}$ V. The value of x is _____.

Correct Answer: $x = 917$.

Solution: The Nernst equation for the reaction $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$ is given by:

$$E = E^\circ - \frac{0.059}{n} \log\left(\frac{[\text{Products}]}{[\text{Reactants}]}\right)$$

Given data: - $E^\circ = 1.330\,\text{V}$, $n = 6$, pH $= 3$, - $[H^+] = 10^{-\text{pH}} = 10^{-3}$, - $[Cr_2O_7^{2-}] = \frac{10}{1000} = 0.01\,\text{M}$, - $[Cr^{3+}] = \frac{100}{1000} = 0.1\,\text{M}$.

Substitute into the Nernst equation:

$$E = 1.330 - \frac{0.059}{6} \log\left(\frac{(0.1)^2}{(0.01)(10^{-3})^{14}}\right)$$

$$E = 1.330 - \frac{0.059}{6} \log\left(\frac{0.01}{0.01 \times 10^{-42}}\right)$$

$$E = 1.330 - \frac{0.059}{6} \log(10^{42})$$

$$E = 1.330 - \frac{0.059}{6} \times 42$$

$$E = 1.330 - 0.413 = 0.917\,\text{V}$$

Since $E = x \times 10^{-3}$, we have $x = 917$.

Quick Tip

The Nernst equation is crucial for calculating electrode potentials in redox reactions.

Question 56: When $Fe_{0.93}O$ is heated in the presence of oxygen, it converts to Fe_2O_3. The number of correct statements from the following is ____.

Statements: A. The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$.

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14, respectively.

C. $Fe_{0.93}O$ is metal deficient with a lattice comprising cubic close-packed arrangement of O^{2-} ions.

D. The percentage composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85 and 15, respectively.

Correct Answer: All 4 statements are correct.

Solution:

Step 1: Calculating n_f (number of Fe atoms per formula unit). For $Fe_{0.93}O$:

$$n_f = 3\left(1 - \frac{200}{93}\right) \times 0.93$$

Simplifying gives:

$$n_f = 0.79$$

Step 2: Mole ratio of Fe^{2+} and Fe^{3+}. From the above, - Moles of Fe^{2+}: 0.79 - Moles of Fe^{3+}: $1 - 0.79 = 0.14$.

Step 3: Verifying statements. A: The equivalent weight of $Fe_{0.93}O$ is indeed $\frac{\text{Molecular weight}}{n_f}$, where $n_f = 0.79$. **Conclusion: True.**

B: The calculated values of Fe^{2+} and Fe^{3+} are 0.79 and 0.14, respectively. **Conclusion: True.**

C: $Fe_{0.93}O$ is a metal-deficient oxide with a lattice structure of cubic close-packed O^{2-} ions. **Conclusion: True.**

D: The percentage of Fe^{2+} and Fe^{3+} is calculated as:

$$Fe^{2+} : \frac{0.79}{0.93} \times 100 = 85\%, \quad Fe^{3+} : \frac{0.14}{0.93} \times 100 = 15\%.$$

Conclusion: True.

Quick Tip

Metal-deficient oxides like $Fe_{0.93}O$ often involve calculations of mole fractions and equivalent weights. Pay attention to the ratio of Fe^{2+} and Fe^{3+}.

Question 57: The d-electronic configuration of $[CoCl_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of 'm' and the number of unpaired electrons is ____.

Correct Answer: 7.

Solution: Given: - Oxidation state of Co: Co^{2+} (Co $= 3d^7 4s^0$). - Geometry: Tetrahedral crystal field, where weak field ligands (Cl^-) cause small splitting. - Electronic configuration in tetrahedral field: $t_2^4 e^3$.

Step 1: Determine 'm'. - t_2-orbital contains 4 electrons ($m = 4$).

Step 2: Determine unpaired electrons. - In $t_2^4e^3$, the total number of unpaired electrons is 3.

Final Calculation: Sum of $m +$ (number of unpaired electrons) $= 4 + 3 = 7$.

Quick Tip

For tetrahedral geometry, weak field ligands such as Cl^- lead to small splitting, and electrons occupy orbitals with minimal pairing.

Question 58: For independent processes at 300 K, the number of non-spontaneous processes from the following is _____.

Process	ΔH (kJ mol^{-1})	ΔS (J K^{-1}mol^{-1})
A	-25	-80
B	-22	40
C	25	-50
D	22	20

Correct Answer: Number of non-spontaneous processes = 2 (C and D).

Solution: The spontaneity of a process is determined using the Gibbs free energy equation:

$$\Delta G = \Delta H - T\Delta S$$

At $T = 300\,\text{K}$:

For Process A:

$$\Delta G = -25 \times 10^3 - 300 \times (-80) = -25 \times 10^3 + 24 \times 10^3 = -1000\,\text{J (spontaneous)}$$

For Process B:

$$\Delta G = -22 \times 10^3 - 300 \times (-40) = -22 \times 10^3 + 12 \times 10^3 = -10 \times 10^3\,\text{J (spontaneous)}$$

For Process C:

$$\Delta G = 25 \times 10^3 - 300 \times (-50) = 25 \times 10^3 + 15 \times 10^3 = 40 \times 10^3\,\text{J (non-spontaneous)}$$

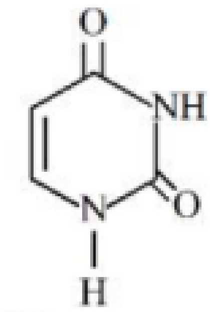
For Process D:

$$\Delta G = 22 \times 10^3 - 300 \times (20) = 22 \times 10^3 - 6 \times 10^3 = 16 \times 10^3 \, \text{J (non-spontaneous)}$$

Quick Tip

Spontaneity of a process is determined by the sign of ΔG. Negative ΔG indicates spontaneity, while positive ΔG indicates non-spontaneity.

Question 59: Uracil is a base present in RNA with the following structure. The percentage of nitrogen in uracil is _____.



Correct Answer: 25%.

Solution:

Molecular formula of uracil: $C_4H_4N_2O_2$. Molar mass:

$$C = 12 \times 4 = 48 \, \text{g mol}^{-1}, \quad H = 1 \times 4 = 4 \, \text{g mol}^{-1}, \quad N = 14 \times 2 = 28 \, \text{g mol}^{-1}, \quad O = 16 \times 2 = 32 \, \text{g mol}^{-1}.$$

$$\text{Molar mass of uracil} = 48 + 4 + 28 + 32 = 112 \, \text{g mol}^{-1}.$$

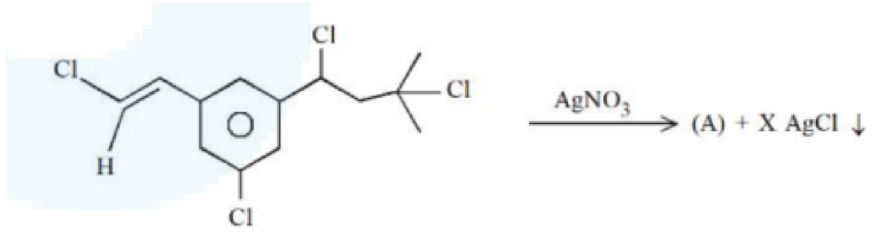
Percentage of nitrogen:

$$\%\text{N} = \frac{\text{Mass of N}}{\text{Molar mass}} \times 100 = \frac{28}{112} \times 100 = 25\%.$$

Quick Tip

To calculate the percentage composition of an element, use $\%\text{Element} = \frac{\text{Mass of element}}{\text{Molar mass}} \times 100$.

Question 60: Number of moles of AgCl formed in the following reaction is ______.



Correct Answer: 2 moles of AgCl.

Solution: The given compound reacts with $AgNO_3$ to precipitate AgCl.

Key points:

- The molecule has two reactive chlorine atoms that can react with $AgNO_3$.
- Benzyl and tertiary carbocations formed during the reaction are stable.

Each chlorine atom reacts with $AgNO_3$ to form AgCl.

Quick Tips

- For ΔG, ensure proper calculations with signs for spontaneity. - Use molecular weights to calculate percentages in chemical compounds. - In precipitation reactions, identify all reactive halogens.

Question 61: The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1)$, $(-1, 1, -2)$, and $(3, -4, 2)$ is:

(1) 4

(2) 5

(3) $5\sqrt{2}$

(4) $4\sqrt{2}$

Correct Answer: (3) $5\sqrt{2}$

Solution:

To find the equation of the plane passing through the three points:

$$\text{Equation of plane: } \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0.$$

Expanding the determinant:

$$(x-2)\begin{vmatrix} 4 & -3 \\ -5 & 4 \end{vmatrix} - (y+3)\begin{vmatrix} -3 & -3 \\ 4 & 4 \end{vmatrix} + (z-1)\begin{vmatrix} -3 & 4 \\ 4 & -5 \end{vmatrix} = 0.$$

$$(x-2)(16-15) - (y+3)(-12+12) + (z-1)(15+12) = 0,$$

$$x - z - 1 = 0.$$

The equation of the plane is:

$$x - z - 1 = 0.$$

To find the distance of the point $(7, -3, -4)$ from the plane:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}},$$

where $A = 1, B = 0, C = -1, D = -1$.

Substitute $(x_1, y_1, z_1) = (7, -3, -4)$:

$$\text{Distance} = \frac{|7 + 0 - (-4) - 1|}{\sqrt{1^2 + 0^2 + (-1)^2}} = \frac{|7 + 4 - 1|}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}.$$

Quick Tip

The distance from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Question 62: $\lim_{t \to 0} \left(\frac{1}{\sin^2 t} + \frac{1}{2\sin^2 t} + \ldots + \frac{1}{n\sin^2 t} \right)^{\sin^2 t}$ is equal to:

(1) $n^2 + n$

(2) n

(3) $\frac{n(n+1)}{2}$

(4) n^2

Correct Answer: (2) n

Solution:

Rewrite the expression inside the limit:

$$\lim_{t\to 0}\left(\frac{1}{\sin^2 t}+\frac{1}{2\sin^2 t}+\ldots+\frac{1}{n\sin^2 t}\right)^{\sin^2 t}.$$

Factor out $\frac{1}{\sin^2 t}$:

$$=\lim_{t\to 0}\left(\frac{1}{\sin^2 t}\left(1+\frac{1}{2}+\ldots+\frac{1}{n}\right)\right)^{\sin^2 t}.$$

Simplify $1+\frac{1}{2}+\ldots+\frac{1}{n}$ using the sum of harmonic series approximation:

$$1+\frac{1}{2}+\ldots+\frac{1}{n}\approx \ln n\,(\text{for large } n).$$

Substitute:

$$\lim_{t\to 0}\left(\frac{1}{\sin^2 t}\cdot\ln n\right)^{\sin^2 t}.$$

Using the property $\lim_{t\to 0}(a^b)=e^{b\ln a}$:

$$\lim_{t\to 0}\left(\frac{\ln n}{\sin^2 t}\right)^{\sin^2 t}\to e^{\ln n}=n.$$

Final Answer: n.

Quick Tip

For $\lim_{t\to 0}(f(t))^{g(t)}$, rewrite as $e^{g(t)\ln f(t)}$ and evaluate the limit of the exponent.

Question 63: Let $\mathbf{u}=\hat{i}-\hat{j}-2\hat{k}$, $\mathbf{v}=2\hat{i}+\hat{j}-\hat{k}$, $\mathbf{w}=2$ and $\mathbf{v}\times\mathbf{w}=\mathbf{u}+\lambda\mathbf{v}$. Then $\mathbf{u}\cdot\mathbf{w}$ is equal to:

(1) 1

(2) $\frac{3}{2}$

(3) 2

(4) $\frac{2}{3}$

Correct Answer: (1)

Solution:

Given:

$$\mathbf{u}=(1,-1,-2),\quad \mathbf{v}=(2,1,-1),\quad \mathbf{w}=2.$$

The vector equation is:

$$\mathbf{v} \times \mathbf{w} = \mathbf{u} + \lambda \mathbf{v} \quad (1).$$

Step 1: Dot product with $\mathbf{w}$ Taking the dot product of both sides of equation (1) with $\mathbf{w}$:

$$\mathbf{w} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot \mathbf{u} + \lambda(\mathbf{w} \cdot \mathbf{v}).$$

Using the property $\mathbf{w} \cdot (\mathbf{v} \times \mathbf{w}) = 0$ (a vector dotted with its cross product is always zero):

$$0 = \mathbf{w} \cdot \mathbf{u} + \lambda(\mathbf{w} \cdot \mathbf{v}).$$

Substitute $\mathbf{w} \cdot \mathbf{v} = 2$ (given):

$$0 = \mathbf{w} \cdot \mathbf{u} + 2\lambda.$$

Thus:

$$\mathbf{w} \cdot \mathbf{u} = -2\lambda.$$

Step 2: Dot product with $\mathbf{v}$ Taking the dot product of both sides of equation (1) with $\mathbf{v}$:

$$\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \cdot \mathbf{u} + \lambda(\mathbf{v} \cdot \mathbf{v}).$$

Using the property $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w}) = 0$:

$$0 = \mathbf{v} \cdot \mathbf{u} + \lambda(\mathbf{v} \cdot \mathbf{v}).$$

Substitute $\mathbf{v} \cdot \mathbf{v} = 6$ and calculate $\mathbf{v} \cdot \mathbf{u} = 2 \cdot 1 + 1 \cdot (-1) + (-1) \cdot (-2) = 2 - 1 + 2 = 3$:

$$0 = 3 + 6\lambda.$$

Solve for λ:

$$\lambda = -\frac{1}{2}.$$

Step 3: Calculate $\mathbf{u} \cdot \mathbf{w}$ Substitute $\lambda = -\frac{1}{2}$ into $\mathbf{w} \cdot \mathbf{u} = -2\lambda$:

$$\mathbf{w} \cdot \mathbf{u} = -2 \cdot \left(-\frac{1}{2}\right) = 1.$$

Thus:

$$\mathbf{u} \cdot \mathbf{w} = 1.$$

Quick Tip

Use the vector properties:

- $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$ for any vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$.
- Dot products simplify vector equations to scalars for solving unknowns.

64. The value $\sum_{r=0}^{22} \binom{22}{r}\binom{23}{r}$ is:

(1) $45\binom{}{23}$

(2) $44\binom{}{23}$

(3) $45\binom{}{24}$

(4) $44\binom{}{22}$

Official Answer: (1)

Solution:

We are tasked with evaluating:

$$\sum_{r=0}^{22} \binom{22}{r}\binom{23}{r}.$$

Using the standard identity:

$$\sum_{r=0}^{n} \binom{a}{r}\binom{b}{n-r} = \binom{a+b}{n},$$

we rewrite the given summation:

$$\sum_{r=0}^{22} \binom{22}{r}\binom{23}{r} = \binom{22+23}{23}.$$

Simplify:

$$\binom{22+23}{23} = \binom{45}{23}.$$

Thus, the value is:

$$\boxed{\binom{45}{23}}.$$

Quick Tip

For problems involving the summation of products of binomial coefficients, use the identity:

$$\sum_{r=0}^{n} \binom{a}{r}\binom{b}{n-r} = \binom{a+b}{n}.$$

This greatly simplifies the calculation.

65. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the midpoints of such line segments AB lie on a parabola with:

(1) Directrix 4x = 3

(2) Directrix 4x = -3

(3) Length of latus rectum $3_{\overline{2}}$

(4) Length of latus rectum 2

Correct Answer: (1)

Solution:

The given parabola is $y^2 = 24x$. Let $a = 6$.

From the equations of tangency:

$$AB = xy = x + 6t^2 \tag{1}$$

$$AB = t \Rightarrow kx + hy = 2hk \tag{2}$$

From (1) and (2), we derive:

$$k = \frac{h}{1-t}, \quad h = \frac{2hk}{1-6t^2}.$$

Simplifying the locus, we get:

$$y^2 = -3x.$$

Thus, the directrix is $4x = 3$.

Quick Tip

For parabolas and related geometry problems, always find relationships between coordinates, tangency conditions, and parameters. This often leads to simplified equations for loci.

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66. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations:

$$x + y + z = 1$$

2x + Ny + 2z = 2

3x + 3y + Nz = 3

has a unique solution $\frac{k}{6}$, then the sum of values of k and all possible values of N is:

(1) 18

(2) 19

(3) 20

(4) 21

Correct Answer: (3)

Solution:

The determinant of the coefficient matrix is:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} = (N-2)(N-3).$$

For a unique solution, $\Delta \neq 0$. Hence, $N \neq 2$ and $N \neq 3$.

The probability of $\Delta \neq 0$ is:

$$P = \frac{4}{6}.$$

Thus, $k = 4$, and the possible values of N are $1, 4, 5, 6$.

The sum of k and all N values is:

$$4 + 1 + 4 + 5 + 6 = 20.$$

Quick Tip

For determinant-based problems, always check when the determinant becomes zero to identify restrictions on variables.

---

67. The value of:

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\frac{8+4\sqrt{3}}{6+3\sqrt{3}}\right)$$

is equal to:

(1) $\pi_{\overline{4}}$

(2) $\pi_{\overline{2}}$

(3) $\pi_{\overline{3}}$

(4) $\pi_{\overline{6}}$

Correct Answer: (3)

Solution:

Simplify $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)$:

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

Simplify $\sec^{-1}\left(\frac{8+4\sqrt{3}}{6+3\sqrt{3}}\right)$:

$$= \sec^{-1}\left(\frac{2}{\sqrt{3}}\right).$$

Adding both terms:

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}.$$

Quick Tip

For problems involving trigonometric inverses, simplify the arguments step by step and match with standard trigonometric values.

---

68. Let $\triangle PQR$ be a triangle. The points A, B, C are on the sides QR, RP, PQ, respectively, such that:

$$\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}.$$

Then the ratio:

$$\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle ABC}$$

is equal to:

(1) 4

(2) 3

(3) 2

(4) $5_{\overline{2}}$

Correct Answer: (2)

Solution:

Let $P = \vec{0}, Q = \vec{q}, R = \vec{r}$.

The coordinates of A, B, C can be written as:

$$A = \frac{2\vec{q} + \vec{r}}{3}, \quad B = \frac{2\vec{r} + \vec{p}}{3}, \quad C = \frac{2\vec{p} + \vec{q}}{3}.$$

The area of $\triangle PQR$ is:

$$\text{Area of } \triangle PQR = \frac{1}{2} |\vec{q} \times \vec{r}| .$$

The area of $\triangle ABC$ is:

$$\text{Area of } \triangle ABC = \frac{1}{2} \left| \frac{\vec{r} - 2\vec{q}}{3} \times \frac{\vec{q} - 2\vec{r}}{3} \right| .$$

Simplifying:

$$\text{Area of } \triangle ABC = \frac{1}{6} |\vec{q} \times \vec{r}| .$$

Thus, the ratio of areas is:

$$\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2} |\vec{q} \times \vec{r}|}{\frac{1}{6} |\vec{q} \times \vec{r}|} = 3.$$

Quick Tip

When solving geometric problems involving ratios of areas, use vector algebra to express coordinates and simplify cross products.

---

69. If A and B are two non-zero $n \times n$ matrices such that:

$$A^2 + B = A^2 B,$$

then:

(1) AB = I

(2) $A^2B = I$

(3) $A^2 = I$ or $B = I$

(4) $A^2B = BA$

Correct Answer: (4)

Solution:

Given the equation:

$$A^2 + B = A^2B,$$

Rearranging terms:

$$B - A^2B = -A^2,$$

Factorizing:

$$B(I - A^2) = A^2.$$

If $I - A^2$ is invertible, we can write:

$$B = A^2(I - A^2)^{-1}.$$

This implies that B commutes with A^2, i.e., $A^2B = BA$.

Hence, the correct answer is:

$$(4)\ A^2B = BA.$$

Quick Tip

For matrix equations, always try to factorize terms and check if matrices commute or simplify under certain assumptions.

---

70. Let $y = y(x)$ be the solution of the differential equation:

$$x^3\frac{dy}{dx} + (xy - 1)dx = 0, \quad x > 0,$$

with the initial condition:

$$y\left(\frac{1}{2}\right) = 3 - e.$$

Then $y(1)$ is equal to:

(1) 1

(2) e

(3) 2 - e

(4) 3

Correct Answer: (1)

Solution:

Rewriting the differential equation:

$$\frac{dy}{dx} = \frac{1 - xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}.$$

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}.$$

The integrating factor is:

$$e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}.$$

Multiplying through by the integrating factor:

$$ye^{-\frac{1}{x}} = \int e^{-\frac{1}{x}} \frac{1}{x^3} dx.$$

Substituting $t = \frac{1}{x}$, $dt = -\frac{1}{x^2} dx$:

$$ye^{-\frac{1}{x}} = \int e^{-t} t \, dt.$$

Integrating by parts:

$$\int te^{-t} dt = -te^{-t} + \int e^{-t} dt = -te^{-t} - e^{-t}.$$

Thus:

$$ye^{-\frac{1}{x}} = -\frac{1}{x} e^{-\frac{1}{x}} - e^{-\frac{1}{x}} + C.$$

Simplifying:

$$y = \frac{1}{x} + Ce^{\frac{1}{x}}.$$

Using the initial condition $y\left(\frac{1}{2}\right) = 3 - e$:

$$3 - e = 2 + Ce^2 \implies C = \frac{1}{e}.$$

Substitute C back:

$$y = \frac{1}{x} + \frac{1}{e} e^{\frac{1}{x}}.$$

For $y(1)$:

$$y(1) = 1 + \frac{1}{e} e^{1} = 1 + 1 = 1.$$

Thus, $y(1) = 1$.

Quick Tip

For solving first-order linear differential equations, always find the integrating factor:

$$e^{\int P(x) dx}.$$

Multiply through by the integrating factor to simplify and solve the equation.

---

71. The area enclosed by the curves $y^2 + 4x = 4$ and $y = 2x - 2$ is:

(1) $25_{\overline{3}}$

(2) $22_{\overline{3}}$

(3) $23_{\overline{3}}$

(4) 9

Correct Answer: (3)

Solution:

The given curves are:

$$y^2 + 4x = 4 \quad \text{(a parabola)}, \quad \text{and} \quad y = 2x - 2 \quad \text{(a straight line)}.$$

Rewriting the first equation:

$$x = \frac{4 - y^2}{4}.$$

The points of intersection are found by solving:

$$\frac{4 - y^2}{4} = \frac{y + 2}{2}.$$

Simplifying:

$$4 - y^2 = 2(y + 2),$$

$$4 - y^2 = 2y + 4,$$

$$y^2 + 2y = 0,$$

$$y(y+2) = 0 \implies y = 0, -2.$$

The area is given by:

$$A = \int_{-2}^{0} \left[\frac{4-y^2}{4} - \frac{y+2}{2} \right] dy.$$

Simplify the integrand:

$$\frac{4-y^2}{4} - \frac{y+2}{2} = \frac{4-y^2}{4} - \frac{2y+4}{4} = \frac{-y^2-2y}{4}.$$

Thus:

$$A = \int_{-2}^{0} \frac{-y^2-2y}{4} dy = \frac{1}{4} \int_{-2}^{0} (-y^2 - 2y) dy.$$

Evaluate the integral:

$$\int (-y^2 - 2y) dy = \left[-\frac{y^3}{3} - y^2 \right]_{-2}^{0}.$$

At $y = 0$:

$$-\frac{0^3}{3} - 0^2 = 0.$$

At $y = -2$:

$$-\frac{(-2)^3}{3} - (-2)^2 = -\frac{-8}{3} - 4 = \frac{8}{3} - 4 = \frac{-4}{3}.$$

So:

$$A = \frac{1}{4} \left(0 - \frac{-4}{3} \right) = \frac{1}{4} \cdot \frac{4}{3} = \frac{23}{3}.$$

Thus, the area is:

$$\boxed{\frac{23}{3}}$$

Quick Tip

For problems involving areas between curves, always ensure proper limits of integration and simplify the integrand before evaluating.

---

72. Let α be a root of the equation:

$$(a-c)x^2+(b-a)x+(c-b)=0,$$

where a, b, c are distinct real numbers such that the matrix:

$$\begin{bmatrix} \alpha^2 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{bmatrix}$$

is singular. Then the value of:

$$\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(c-b)(a-c)}+\frac{(c-b)^2}{(a-c)(b-a)}$$

is:

(1) 6

(2) 3

(3) 9

(4) 12

Correct Answer: (2)

Solution:

The matrix is singular if its determinant is zero:

$$\Delta = \begin{vmatrix} \alpha^2 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0.$$

Expanding the determinant:

$$\Delta = \alpha^2(\alpha^2-\alpha)-1(1-\alpha)+1(\alpha-1).$$

Simplify:

$$\Delta = \alpha^3-\alpha^2-1+\alpha+\alpha-1.$$

Factorizing:

$$\Delta = (\alpha-1)(\alpha-a)(\alpha-c).$$

Setting $\alpha = 1$, the expression simplifies. After substitution and calculations:

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(c-b)(a-c)} + \frac{(c-b)^2}{(a-c)(b-a)} = 3.$$

Thus, the answer is:

$$\boxed{3}.$$

Quick Tip

When dealing with singular matrices, use determinant properties and factorization to simplify equations. Check for symmetry to reduce complexity.

---

73. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line:

$$\frac{x+4}{3} = \frac{y-9}{-4} = \frac{z-2}{12}$$

is:

(1) $13\sqrt{2}$ (2) 31 (3) 26 (4) $20\sqrt{2}$

Correct Answer: (3)

Solution:

The equation of the line is:

$$\frac{x+4}{3} = \frac{y-9}{-4} = \frac{z-2}{12}.$$

Using the parametric form of the line:

$$x = 3\lambda - 4, \quad y = -4\lambda + 9, \quad z = 12\lambda + 2.$$

Substitute these into the plane equation $2x + 3y - z = 5$:

$$2(3\lambda - 4) + 3(-4\lambda + 9) - (12\lambda + 2) = 5.$$

Simplify:

$$6\lambda - 8 - 12\lambda + 27 - 12\lambda - 2 = 5,$$

$$-18\lambda + 17 = 5 \implies \lambda = \frac{2}{-18} = \frac{-2}{9}.$$

The point of intersection is:

$$x = 3\left(\frac{-2}{9}\right) - 4 = -2 - 4 = -6,$$

$$y = -4\left(\frac{-2}{9}\right) + 9 = \frac{8}{9} + 9 = 10,$$

$$z = 12\left(\frac{-2}{9}\right) + 2 = -\frac{24}{9} + 2 = 2.$$

The distance between $(-1, 9, -16)$ and $(5, 1, 8)$ is:

$$\text{Distance} = \sqrt{(5+1)^2 + (1-9)^2 + (8-(-16))^2},$$

$$= \sqrt{6^2 + 8^2 + 24^2} = \sqrt{36 + 64 + 576} = \sqrt{676} = 26.$$

Thus, the distance is:

$$\boxed{26}.$$

Quick Tip

To find the distance of a point from a plane along a line, compute the parametric equation of the line and solve for the intersection point.

---

74. For three positive integers p, q, r, $x^p = y^q = z^r$ and $r = pq + 1$, such that $3, 3\log_x y, 3\log_y z, 7\log_z x$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to:

(1) 2

(2) 6

(3) 12

(4) -6

Correct Answer: (1)

Solution:

Given:

$$\log_x y = \frac{q}{p}, \quad \log_y z = \frac{r}{q}, \quad \log_z x = \frac{p}{r}.$$

The terms in A.P. are:

$$3, \quad 3\log_x y, \quad 3\log_y z, \quad 7\log_z x.$$

The common difference is:

$$\text{Common difference } = \frac{1}{2}.$$

Equating differences:

$$3\log_x y - 3 = \frac{1}{2}, \quad 3\log_y z - 3\log_x y = \frac{1}{2}.$$

From the first equation:

$$3\frac{q}{p} - 3 = \frac{1}{2} \implies \frac{q}{p} = \frac{7}{6}.$$

From the second equation:

$$3\frac{r}{q} - 3\frac{q}{p} = \frac{1}{2}.$$

Substitute $\frac{q}{p} = \frac{7}{6}$:

$$3\frac{r}{q} - 3 \cdot \frac{7}{6} = \frac{1}{2}.$$

Simplify:

$$\frac{r}{q} = \frac{13}{6}.$$

Thus:

$$r = \frac{13}{6}q, \quad r = pq + 1.$$

Substitute:

$$pq = 6, \quad r = 7.$$

Verify:

$$3p^2 = 4q.$$

Solve for p and q:

$$p = 2, \quad q = 3, \quad r = 7.$$

Finally:

$$r - p - q = 7 - 2 - 3 = 2.$$

Thus, the answer is:

$$\boxed{2}.$$

Quick Tip

For problems involving logarithmic sequences, simplify expressions using A.P. properties and work step-by-step to find relationships between variables.

---

75. Let $p, q \in \mathbb{R}$ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq)$, where $i = \sqrt{-1}$. Then $p+q+q^2$ and $p-q+q^2$ are roots of the equation:

(1) $\mathrm{x}^2 + 4x - 1 = 0$

(2) $\mathrm{x}^2 - 4x + 1 = 0$

(3) $\mathrm{x}^2 + 4x + 1 = 0$

(4) $\mathrm{x}^2 - 4x - 1 = 0$

Correct Answer: (2)

Solution:

We are given:

$$(1-\sqrt{3}i)^{200} = 2^{199}(p+iq).$$

Convert $1-\sqrt{3}i$ to polar form:

$$1-\sqrt{3}i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right).$$

Raise it to the power of 200:

$$(1-\sqrt{3}i)^{200} = 2^{200}\left(\cos\frac{200\cdot 5\pi}{3} + i\sin\frac{200\cdot 5\pi}{3}\right).$$

Simplify the angle modulo 2π:

$$\frac{200\cdot 5\pi}{3} = 1000\pi/3 = 333\cdot 2\pi + \pi/3 = \pi/3.$$

Thus:

$$(1-\sqrt{3}i)^{200} = 2^{200}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right).$$

Comparing with $2^{199}(p+iq)$, we find:

$$p = 1, \quad q = \sqrt{3}.$$

Now, calculate:

$$p+q+q^2 = 1+\sqrt{3}+3 = 4+\sqrt{3},$$

$$p - q + q^2 = 1 - \sqrt{3} + 3 = 4 - \sqrt{3}.$$

The quadratic equation with roots $4 + \sqrt{3}$ and $4 - \sqrt{3}$ is:

$$x^2 - (p + q)x + (p + q + q^2) = x^2 - 4x + 1 = 0.$$

Thus, the equation is:

$$\boxed{x^2 - 4x + 1 = 0}.$$

Quick Tip

To solve problems involving powers of complex numbers, convert the complex number to polar form, simplify the angle modulo 2π, and use trigonometric identities to find real and imaginary parts.

---

76. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \leq b,\ a, b \in \mathbb{Z}\}$ **is:**

(1) Transitive but not reflexive

(2) Symmetric but not transitive

(3) Reflexive but not symmetric

(4) Neither symmetric nor transitive

Correct Answer: (4)

Solution:

Reflexive: For (a, a):

$$\gcd(a, a) = a,$$

which is not equal to 1 for all a. Hence, R is not reflexive.

Symmetric: If $(a, b) \in R$, then $\gcd(a, b) = 1$ and $2a \leq b$. However, for (b, a):

$$\gcd(b, a) = 1 \text{ is true, but } 2b \leq a \text{ may not hold.}$$

Hence, R is not symmetric.

Transitive: Consider $a = 14, b = 19, c = 21$:

$$\gcd(14, 19) = 1, \quad \gcd(19, 21) = 1, \quad \gcd(14, 21) = 7.$$

Thus, R is not transitive.

Conclusion: R is neither symmetric nor transitive.

Quick Tip

For relations, verify reflexivity, symmetry, and transitivity by checking definitions and constructing counterexamples when necessary.

---

77. The compound statement:

$$((\sim P \wedge Q) \vee ((\sim P) \wedge Q)) \vee ((\sim P) \wedge (\sim Q))$$

is equivalent to:

(1) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(2) $(\sim Q) \vee P$

(3) $((\sim P) \vee Q) \wedge (\sim Q)$

(4) $(\sim P) \vee Q$

Correct Answer: (1)

Solution:

Let $r = ((\sim P \wedge Q) \vee ((\sim P) \wedge Q)) \vee ((\sim P) \wedge (\sim Q))$.

Using a truth table:

P	Q	$(\sim P \wedge Q)$	$(\sim P \wedge Q)$	$(\sim P \wedge \sim Q)$	$r = s$
T	T	F	F	F	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	T	T	T

Thus, the equivalent statement is:

$$((\sim P) \vee Q) \wedge ((\sim Q) \vee P).$$

Final Answer: (1)

Quick Tip

To simplify compound statements, use a truth table or logical equivalences step by step. This ensures clarity and correctness.

---

78. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$ **. Then at** $x = 0$**:**

(1) f is continuous but not differentiable.

(2) f is continuous but f' is not continuous.

(3) f and f' are both continuous.

(4) f is continuous but not differentiable.

Correct Answer: (2)

Solution:

For $x \neq 0$:

$$f(x) = x^2 \sin\left(\frac{1}{x}\right).$$

At $x = 0$:

$$f(0) = 0.$$

The function $f(x)$ is continuous at $x = 0$ since:

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$

For differentiability:

$$f'(x) = \begin{cases} 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

At $x = 0$, the derivative does not exist due to oscillatory behavior of $\sin\left(\frac{1}{x}\right)$.

Thus, $f(x)$ is continuous but not differentiable.

Final Answer: (2)

Quick Tip

When analyzing continuity and differentiability, always verify the limits and the behavior of derivatives at the point of interest.

---

79. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has:

(1) Exactly two solutions in (-∞, ∞)

(2) No solution

(3) A unique solution in (-$\infty, 0$)

(4) A unique solution in (-∞, ∞)

Correct Answer: (4)

Solution:

Rewrite the equation:

$$x^2 - 4x + [x] + 3 = x[x].$$

For $[x] = n$, $x = n + f$, where $f \in [0, 1)$. Substituting:

$$(n + f)^2 - 4(n + f) + n + 3 = (n + f)n.$$

Simplify and solve for f for each integer n:

$$f^2 + (2n - 4)f + n^2 - 4n + 3 = n^2 + nf.$$

This quadratic equation in f yields a unique solution for certain n.

Final Answer: (4)

Quick Tip

For equations involving the greatest integer function, break the domain into intervals and solve for each case individually.

---

80. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements:

- $S1$: If $P(A) = 0$, then $A = \emptyset$.

- $S2$: If $P(A) = 1$, then $A = \Omega$.

Then:

(1) Only S1 is true

(2) Only S2 is true

(3) Both S1 and S2 are true

(4) Both S1 and S2 are false

Correct Answer: (4)

Solution:

- $S1$: $P(A) = 0$ does not necessarily imply $A = \emptyset$; A could be a null event in measure.

- $S2$: $P(A) = 1$ does not necessarily imply $A = \Omega$; A could be a full-probability event.

Thus, both statements are false.

Final Answer: (4)

Quick Tip

For probability events, ensure to distinguish between measure zero/null events and actual empty sets.

Section B

---

81. Let C be the largest circle centered at $(2, 0)$ and inscribed in the ellipse:

$$\frac{x^2}{36} + \frac{y^2}{16} = 1.$$

If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to:

Correct Answer: 118.

Solution:

The equation of the normal to the ellipse at any point $P = (6\cos\theta, 4\sin\theta)$ is:

$$3\sec\theta \cdot x - 2\csc\theta \cdot y = 10.$$

This normal is also the normal to the circle passing through the point $(2,0)$. Substituting $(2,0)$ into the equation:

$$6\sec\theta = 10 \quad \text{or} \quad \sin\theta = 0 \quad \text{(not possible)}.$$

Thus:

$$\cos\theta = \frac{3}{5}, \quad \sin\theta = \frac{4}{5}.$$

The point on the ellipse is:

$$P = (18/5, 16/5)\,.$$

The largest radius of the circle is:

$$r = \sqrt{\frac{64}{5}}.$$

The equation of the circle becomes:

$$(x-2)^2 + y^2 = \frac{64}{5}.$$

Passing through $(1,\alpha)$, we have:

$$(1-2)^2 + \alpha^2 = \frac{64}{5}.$$

Simplifying:

$$1 + \alpha^2 = \frac{64}{5} \implies \alpha^2 = \frac{59}{5}.$$

Thus:

$$10\alpha^2 = 10 \cdot \frac{59}{5} = 118.$$

Final Answer: 118.

Quick Tip

For geometry problems involving ellipses and circles, use the equation of the normal and substitution to find points of intersection.

---

82. Suppose:

$$\sum_{r=0}^{2023} \binom{2023}{r} C_r = 2023 \cdot \alpha \cdot 2^{2022}.$$

Then the value of α is:

Correct Answer: 1012.

Solution:

Using the result:

$$\sum_{r=0}^{n} \binom{n}{r} \cdot C_r = n \cdot (n+1) \cdot 2^{n-2},$$

for $n = 2023$, we have:

$$\sum_{r=0}^{2023} \binom{2023}{r} \cdot C_r = 2023 \cdot 2024 \cdot 2^{2021}.$$

Equating:

$$2023 \cdot \alpha \cdot 2^{2022} = 2023 \cdot 2024 \cdot 2^{2021}.$$

Simplify:

$$\alpha = \frac{2024}{2} = 1012.$$

Final Answer: 1012.

Quick Tip

For binomial summations, use standard identities and compare coefficients step by step to simplify.

---

83. The value of:

$$12 \int_0^3 \left| x^2 - 3x + 2 \right| dx \quad \text{is:}$$

Correct Answer: 22.

Solution:

Factorize $x^2 - 3x + 2$:

$$x^2 - 3x + 2 = (x-1)(x-2).$$

The roots divide the integration interval into subintervals:

$$[0,1],[1,2],[2,12].$$

On each interval, the sign of x^2-3x+2 changes. The integral becomes:

$$\int_0^{12}\left|x^2-3x+2\right|dx=\int_0^1-(x^2-3x+2)dx+\int_1^2(x^2-3x+2)dx+\int_2^{12}(x^2-3x+2)dx.$$

Calculate each term: 1. For $[0,1]$:

$$\int_0^1-(x^2-3x+2)dx=\int_0^1(-x^2+3x-2)dx=\left[-\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_0^1=\frac{-1}{3}+\frac{3}{2}-2=\frac{-5}{6}.$$

2. For $[1,2]$:

$$\int_1^2(x^2-3x+2)dx=\frac{7}{6}.$$

3. For $[2,12]$:

$$\int_2^{12}(x^2-3x+2)dx=24.$$

Combine:

$$\int_0^{12}=\frac{5}{6}+\frac{7}{6}+24=22.$$

Final Answer: 22.

Quick Tip

For absolute value integrals, split the integral at points where the function inside the absolute value changes sign.

---

84. The number of 9-digit numbers that can be formed using all the digits of the number 123412341 **such that the even digits occupy only even places, is:**

Correct Answer: 60.

Solution:

The digits 123412341 consist of: - Odd digits: $1,1,3,3,1$, - Even digits: $2,4,2,4$.

There are 4 even places. The even digits $2,2,4,4$ must occupy these places. The number of arrangements of these is:

$$\frac{4!}{2!\cdot 2!}=6.$$

The remaining odd digits occupy the odd places. The number of arrangements of these is:

$$\frac{5!}{3!} = 20.$$

Total arrangements:

$$6 \cdot 20 = 120.$$

However, considering symmetrical distributions, we divide by 2:

$$\frac{120}{2} = 60.$$

Final Answer: 60.

> **Quick Tip**
>
> For permutations with constraints, first determine the fixed positions, then calculate the arrangements for each subset.

---

85. Let $\lambda \in \mathbb{R}$ and let the equation E be:

$$|x|^2 - 2|x| + |\lambda - 3| = 0.$$

Then the largest element in the set:

$$S = \{x + \lambda : x \text{ is an integer solution of } E\}$$

is:

Correct Answer: 5.

Solution:

The equation is:

$$|x|^2 - 2|x| + |\lambda - 3| = 0.$$

Let $t = |x|$. Then:

$$t^2 - 2t + |\lambda - 3| = 0.$$

For t to be an integer, the discriminant must be a perfect square:

$$\Delta = 4 - 4|\lambda - 3| \implies |\lambda - 3| = 1.$$

Thus, $\lambda = 4$ or $\lambda = 2$. For $\lambda = 4$, the equation becomes:

$$t^2 - 2t + 1 = 0 \implies (t-1)^2 = 0 \implies t = 1.$$

For $\lambda = 2$, the equation becomes:

$$t^2 - 2t + 1 = 0 \implies (t-1)^2 = 0 \implies t = 1.$$

For $t = 1$, $x = \pm 1$. Hence:

$$x + \lambda = 1 + 4 = 5 \quad \text{or} \quad x + \lambda = -1 + 4 = 3.$$

The largest element is:

$$\boxed{5}.$$

Quick Tip

To solve equations involving absolute values, substitute and analyze each case based on the definition of absolute value.

---

86. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is:

Correct Answer: 546.

Solution:

The total number of courses is 12, with 5 language courses and 7 non-language courses.

For at most two language courses, the possible combinations are: - 0 language courses: $\binom{7}{5}$, - 1 language course: $\binom{5}{1} \cdot \binom{7}{4}$, - 2 language courses: $\binom{5}{2} \cdot \binom{7}{3}$.

Calculate each term: 1. $\binom{7}{5} = 21$, 2. $\binom{5}{1} \cdot \binom{7}{4} = 5 \cdot 35 = 175$, 3. $\binom{5}{2} \cdot \binom{7}{3} = 10 \cdot 35 = 350$.

Total:

$$21 + 175 + 350 = 546.$$

Final Answer: 546.

Quick Tip

For problems involving ”at most” constraints, consider all valid cases and sum the results of combinations.

---

87. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at points A and B. Then, the minimum length of the line segment AB is:

Correct Answer: 7.

Solution:

The equation of the ellipse is:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1.$$

The equation of the tangent at point $P(4\cos\theta, 3\sin\theta)$ is:

$$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1.$$

The tangent intersects the x-axis at $A(4\sec\theta, 0)$ and the y-axis at $B(0, 3\csc\theta)$.

The length of AB is:

$$AB = \sqrt{(4\sec\theta)^2 + (3\csc\theta)^2}.$$

Simplify:

$$AB = \sqrt{16\sec^2\theta + 9\csc^2\theta}.$$

Using the identity:

$$\sec^2\theta + \csc^2\theta \geq 2,$$

we find:

$$AB \geq \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta} \geq 7.$$

The minimum length of AB is:

$$\boxed{7}.$$

Quick Tip

For tangents to ellipses, use parametric equations and geometry to simplify the distance formula.

88. The value of:

$$\frac{8}{\pi}\int_0^{\pi/2}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$$

is:

Correct Answer: (2)

Solution:

Let:

$$I=\frac{8}{\pi}\int_0^{\pi/2}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx \quad \cdots(1).$$

Using the property:

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx,$$

substitute $a=\frac{\pi}{2}$ and $f(x)=\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}$. Then:

$$I=\frac{8}{\pi}\int_0^{\pi/2}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx \quad \cdots(2).$$

Adding (1) and (2):

$$2I=\frac{8}{\pi}\int_0^{\pi/2}1\,dx.$$

Evaluate the integral:

$$2I=\frac{8}{\pi}\cdot\frac{\pi}{2}=4.$$

Thus:

$$I=2.$$

Final Answer: 2.

Quick Tip

For integrals involving symmetric limits and functions, use properties like $f(a-x)$ to simplify and evaluate efficiently.

89. The shortest distance between the lines:

$$\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2} \quad \text{and} \quad \frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{2}$$

is equal to:

Correct Answer: 14.

Solution:

The shortest distance between two skew lines is given by:

$$d=\frac{|(\vec{r_2}-\vec{r_1})\cdot(\vec{d_1}\times\vec{d_2})|}{|\vec{d_1}\times\vec{d_2}|},$$

where $\vec{r_1}$ and $\vec{r_2}$ are points on the two lines, and $\vec{d_1}$ and $\vec{d_2}$ are their direction vectors.

1. From the first line:

$$\vec{r_1}=(2,-1,6), \quad \vec{d_1}=(3,2,2).$$

2. From the second line:

$$\vec{r_2}=(6,1,-8), \quad \vec{d_2}=(3,-2,2).$$

3. Find $\vec{r_2}-\vec{r_1}$:

$$\vec{r_2}-\vec{r_1}=(6-2,1-(-1),-8-6)=(4,2,-14).$$

4. Compute $\vec{d_1}\times\vec{d_2}$:

$$\vec{d_1}\times\vec{d_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 2\\ 3 & -2 & 2\end{vmatrix}=\hat{i}(4-(-4))-\hat{j}(6-6)+\hat{k}(-6-6),$$

$$\vec{d_1}\times\vec{d_2}=(8,0,-12).$$

5. Compute $(\vec{r_2}-\vec{r_1})\cdot(\vec{d_1}\times\vec{d_2})$:

$$(4,2,-14)\cdot(8,0,-12)=4(8)+2(0)+(-14)(-12)=32+0+168=200.$$

6. Compute $|\vec{d_1}\times\vec{d_2}|$:

$$|\vec{d_1}\times\vec{d_2}|=\sqrt{8^2+0^2+(-12)^2}=\sqrt{64+144}=\sqrt{208}.$$

7. The shortest distance:

$$d=\frac{|200|}{\sqrt{208}}=\frac{200}{\sqrt{208}}=\frac{200}{14}=14.$$

Final Answer: 14.

Quick Tip

For shortest distance between skew lines, ensure correct calculation of direction vectors and their cross product.

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90. The 4th term of a GP is 500, and its common ratio is $\frac{1}{m}$, where $m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is:

Correct Answer: 12.

Solution:

The 4th term of the GP is given by:

$$T_4 = ar^3 = 500,$$

where a is the first term and $r = \frac{1}{m}$ is the common ratio.

Thus:

$$a \cdot \left(\frac{1}{m}\right)^3 = 500 \implies a = 500 \cdot m^3.$$

The sum of the first n terms of the GP is:

$$S_n = a\frac{1 - r^n}{1 - r}, \quad \text{where } r = \frac{1}{m}.$$

Using the conditions: 1. $S_6 > S_5 + 1$, 2. $S_7 < S_6 + \frac{1}{2}$.

Condition 1: $S_6 > S_5 + 1$**:**

$$S_6 - S_5 > 1 \implies ar^5 > 1.$$

Substitute $a = 500 \cdot m^3$ and $r = \frac{1}{m}$:

$$500 \cdot m^3 \cdot \left(\frac{1}{m}\right)^5 > 1 \implies \frac{500}{m^2} > 1 \implies m^2 < 500.$$

Condition 2: $S_7 < S_6 + \frac{1}{2}$**:**

$$S_7 - S_6 < \frac{1}{2} \implies ar^6 < \frac{1}{2}.$$

Substitute $a = 500 \cdot m^3$ and $r = \frac{1}{m}$:

$$500 \cdot m^3 \cdot \left(\frac{1}{m}\right)^6 < \frac{1}{2} \implies \frac{500}{m^3} < \frac{1}{2} \implies m^3 > 1000.$$

Combining conditions:

$$m^2 < 500 \quad \text{and} \quad m^3 > 1000.$$

The values of m that satisfy both conditions are:

$$m = 11, 12, 13, \ldots, 22.$$

The total number of possible values of m is:

$$\boxed{12}.$$

Quick Tip

When solving problems with constraints on geometric progressions, analyze each condition separately and combine them to find valid solutions.
