JEE Main 2023 24 Jan Shift 1 Physics Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.

2. This Question paper consists of 90 Questions.

3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..

4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer..

5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Question 1. Two long straight wires P and Q carrying equal currents of 10A each were kept parallel to each other at a distance of 5cm. The magnitude of the magnetic force experienced by a 10cm length of wire P is F_1. If the distance between the wires is halved and the currents in them are doubled, the force F_2 on a 10cm length of wire P will be:

(1) $8F_1$

(2) $10F_1$

(3) $\frac{F_1}{8}$

(4) $\frac{F_1}{10}$

Correct Answer: (1) $8F_1$

Solution The magnetic force between two parallel current-carrying wires is given by the formula:

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

where:

- F = magnetic force,
- μ_0 = permeability of free space ($4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$),
- I_1 and I_2 = currents in the wires,
- L = length of the wires,
- r = distance between the wires.

Given:

$$I_1 = I_2 = 10\ \mathrm{A}$$

$$r = 5\ \mathrm{cm} = 0.05\ \mathrm{m}$$

$$L = 10\ \mathrm{cm} = 0.1\ \mathrm{m}$$

$$F_1 = \frac{\mu_0 \times 10 \times 10 \times 0.1}{2\pi \times 0.05}$$

When the distance is halved and the currents are doubled:

$$I_1' = I_2' = 20\ \mathrm{A}$$

$$r' = \frac{5\ \mathrm{cm}}{2} = 2.5\ \mathrm{cm} = 0.025\ \mathrm{m}$$

$$L = 10\ \mathrm{cm} = 0.1\ \mathrm{m}$$

$$F_2 = \frac{\mu_0 \times 20 \times 20 \times 0.1}{2\pi \times 0.025}$$

Calculating the ratio $\frac{F_2}{F_1}$:

$$\frac{F_2}{F_1} = \frac{\frac{\mu_0 \times 20 \times 20 \times 0.1}{2\pi \times 0.025}}{\frac{\mu_0 \times 10 \times 10 \times 0.1}{2\pi \times 0.05}} = \frac{20 \times 20 \times 0.05}{10 \times 10 \times 0.025} = \frac{400 \times 0.05}{100 \times 0.025} = \frac{20}{2.5} = 8$$

$$\Rightarrow F_2 = 8F_1$$

Quick Tip

The magnetic force between parallel wires is directly proportional to the product of the currents and inversely proportional to the distance between them. Adjusting these parameters significantly affects the force experienced.

Question 2. Given below are two statements:

Statement-I : An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement-II : Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed. In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Correct Answer: (2) Statement I is true but Statement II is false

Solution : **Statement-I:**An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

When an elevator moves with a uniform speed (constant velocity), it implies that there is no acceleration . According to Newton's First Law of Motion, if there is no acceleration, the net force acting on the elevator must be zero .

$$\text{Net Force} = 0 \Rightarrow \text{Tension (T)} = \text{Weight (W)}$$

Where:

$$W = mg$$

$$T = mg$$

Here, m is the mass of the elevator and g is the acceleration due to gravity. Since the tension in the cable exactly balances the weight of the elevator, the elevator can move upwards or downwards at a uniform speed without any change in tension or weight.

$$\Rightarrow \text{Statement-I is True}$$

Statement-II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.
When the elevator is going down with increasing speed , it is undergoing downward acceleration . According to Newton's Second Law of Motion:

$$\text{Net Force} = ma$$

For a person standing in the elevator:

$$N - W = -ma$$

$$N = W - ma$$

Where:

$$N = \text{Normal force (force exerted by the floor)}$$

$$W = \text{Weight of the person} = mg$$

$$a = \text{Downward acceleration}$$

Since $a > 0$ (increasing speed downwards),

$$N = mg - ma = m(g - a)$$

Here, $g > a$ to ensure that the elevator is not in free fall. Therefore,

$$N < mg$$

This means the normal force exerted by the floor on the person is less than the person's actual weight.

$$\Rightarrow \text{Statement-II is False}$$

Quick Tip

When analyzing forces in accelerating systems, always consider the direction of acceleration. Downward acceleration in an elevator reduces the normal force experienced by a person standing inside.

Question 3. From the photoelectric effect experiment, the following observations are made. Identify which of these are correct:
A. The stopping potential depends only on the work function of the metal.
B. The saturation current increases as the intensity of incident light increases.
C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.
D. Photoelectric effect can be explained using wave theory of light. Choose the correct answer from the options given below:

(1) B, C only
(2) A, C, D only
(3) B only
(4) A, B, D only

Correct Answer: (3) B only

Solution : **Statement A:** *The stopping potential depends only on the work function of the metal.*

The stopping potential in the photoelectric effect is given by:

$$V_s = \frac{K_{\text{max}}}{e}$$

where K_{max} is the maximum kinetic energy of the emitted electrons and e is the elementary charge. According to Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

where h is Planck's constant, ν is the frequency of the incident light, and ϕ is the work function of the metal.

Thus, the stopping potential depends on both the frequency of the incident light and the work function of the metal, not solely on the work function.

Statement A is False.

Statement B: *The saturation current increases as the intensity of incident light increases.*

The saturation current in the photoelectric effect is directly proportional to the number of incident photons, provided that each photon has enough energy to eject an electron. An increase in the intensity of the incident light means more photons are striking the metal surface per unit time, leading to an increase in the number of photoelectrons emitted, and hence, an increase in the saturation current.

Statement B is True.

Statement C: *The maximum kinetic energy of a photo electron depends on the intensity of the incident light.*

From Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

The maximum kinetic energy depends only on the frequency of the incident light and the work function of the metal. It is independent of the intensity of the incident light, which affects the number of photoelectrons but not their kinetic energy.

Statement C is False.

Statement D: *Photoelectric effect can be explained using wave theory of light.*

The wave theory of light fails to explain certain observations of the photoelectric effect, such as the existence of a threshold frequency and the instantaneous emission of electrons regardless of light intensity. These phenomena are successfully explained by the particle theory of light, where light consists of photons with quantized energy.

Statement D is False.

Conclusion: Only Statement B is correct.

Quick Tip

In the photoelectric effect, the intensity of light affects the number of emitted electrons (saturation current) but not their kinetic energy, which is determined by the light's frequency.

Question 4. The weight of a body at the surface of Earth is 18 N. The weight of the body at an altitude of 3200 km above the Earth's surface is (given, radius of Earth $R_e = 6400$ km):

(1) 9.8N

(2) 4.9N

(3) 19.6N

(4) 8N

Correct Answer: (4) 8 N

Solution : The weight of a body is given by:

$$W = mg$$

where m is the mass of the body and g is the acceleration due to gravity.

At the surface of the Earth:

$$W_{\text{surface}} = mg = 18 \text{ N}$$

Thus,

$$mg = 18 \text{ N} \quad \Rightarrow \quad g = \frac{18}{m}$$

At an altitude $h = 3200$ km, the distance from the center of the Earth becomes:

$$r = R_e + h = 6400 \text{ km} + 3200 \text{ km} = 9600 \text{ km} = 9.6 \times 10^6 \text{ m}$$

The acceleration due to gravity at a distance r from the center of the Earth is given by:

$$g' = g\left(\frac{R_e}{r}\right)^2$$

Substituting the known values:

$$g' = 9.8 \text{ m/s}^2 \times \left(\frac{6400}{9600}\right)^2 = 9.8 \text{ m/s}^2 \times \left(\frac{2}{3}\right)^2 = 9.8 \text{ m/s}^2 \times \frac{4}{9} \approx 4.355 \text{ m/s}^2$$

Therefore, the weight at altitude h is:

$$W' = mg' = m \times 4.355 \text{ m/s}^2$$

Given that $mg = 18$ N, we can express W' in terms of W:

$$W' = W \times \left(\frac{g'}{g}\right) = 18 \text{ N} \times \frac{4.355}{9.8} \approx 8 \text{ N}$$

$$\Rightarrow W' \approx 8 \text{ N}$$

Quick Tip

The gravitational force decreases with the square of the distance from the center of the Earth. Always convert all distances to the same units before performing calculations.

Question 5. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be :

(1) 6.25×10^{-3} m

(2) 4×10^{-4} m

(3) 6.25×10^{-6} m

(4) 4×10^{-3} m

Correct Answer: (4) 4×10^{-3} m

Solution : Elongation in the wire can be calculated using Young's modulus formula:

$$\Delta L = \frac{FL}{AE}$$

where:

- $F = 250$ N (load applied),
- $L = 100$ m (original length of the wire),
- $A = 6.25 \times 10^{-4}$ m^2 (cross-sectional area),
- $E = 10^{10}$ Nm^{-2} (Young's modulus).

Substituting the given values:

$$\Delta L = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}} = \frac{25000}{6.25 \times 10^{6}} = 4 \times 10^{-3} \text{ m}$$

$$\Rightarrow \Delta L = 4 \times 10^{-3} \text{ m}$$

Quick Tip

Young's modulus relates the stress and strain in a material, allowing the calculation of elongation under a given load.

Question 6. 1 g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be :

(1) 4320 J

(2) 432000 J

(3) 4800 J

(4) 4.32×10^8 J

Correct Answer: (1) 4320J

Solution : The work done during the phase change is given by:

$$\text{Work done} = P\Delta V$$

where:

- $P = 3 \times 10^5$ Pa (pressure),
- $\Delta V = 1600 \text{ cm}^3 = 1600 \times 10^{-6} \text{ m}^3 = 1.6 \times 10^{-3} \text{ m}^3$ (change in volume).

Calculating the work done:

$$\text{Work done} = 3 \times 10^5 \times 1.6 \times 10^{-3} = 480 \text{ J}$$

Given that only 10% of the heat supplied (ΔQ) is used for work:

$$10\% \text{ of } \Delta Q = 480 \text{ J} \Rightarrow \Delta Q = 4800 \text{ J}$$

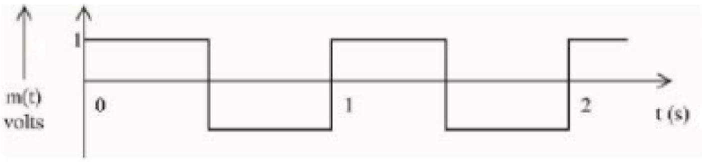
The remaining 90% of the heat goes into increasing the internal energy (ΔU):

$$\Delta U = 0.9 \times 4800 = 4320 \text{ J}$$

Quick Tip

In phase changes, a portion of the heat energy is used to do work against external pressure, and the remainder changes the internal energy of the system.

Question 7. A modulating signal is a square wave, as shown in the figure. If the carrier wave is given as $c(t) = 2\sin(8\pi t)$ volts, the modulation index is :



(1) $\frac{1}{4}$

(2) 1

(3) $\frac{1}{3}$

(4) $\frac{1}{2}$

Correct Answer: (4) $\frac{1}{2}$

Solution : The modulation index (m) in amplitude modulation is given by:

$$m = \frac{A_m}{A_c}$$

where:

- A_m is the amplitude of the modulating signal,
- $A_c = 2$ V is the amplitude of the carrier wave.

Given that the modulating signal is a square wave, its amplitude (A_m) is typically taken as the maximum deviation from zero, which is 1 V (assuming it's normalized).

Thus,

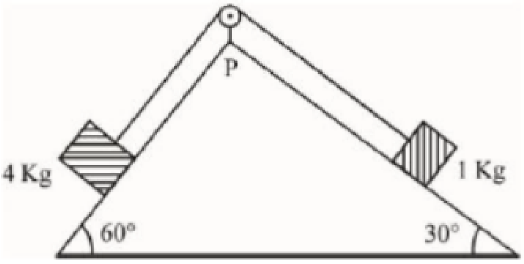
$$m = \frac{1}{2} = 0.5$$

$$\Rightarrow m = \frac{1}{2}$$

Quick Tip

The modulation index determines the extent of amplitude variation in the carrier wave and is crucial for avoiding over-modulation.

Question 8: As per given figure, a weightless pulley P is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if $g = 10$ m/s^2):



(1) $(4\sqrt{3} + 1)$ N

(2) $4(\sqrt{3} + 1)$ N

(3) $4(\sqrt{3} - 1)$ N

(4) $(4\sqrt{3} + 1)$ N

Correct Answer: (2) $4(\sqrt{3} + 1)$ N

Solution: Step 1: Analyze the Forces Acting on Each Block The pulley system involves two blocks:

- Block $m_1 = 4$ kg on the $60°$ incline.
- Block $m_2 = 1$ kg on the $30°$ incline.

The forces acting on each block are:

- For m_1:

$$\text{Component of weight along incline: } m_1 g \sin 60° = 4 \times 10 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}\,\text{N}.$$

The tension in the string is $T_1 = T$.

- For m_2:

$$\text{Component of weight along incline: } m_2 g \sin 30^\circ = 1 \times 10 \times \frac{1}{2} = 5\,\text{N}.$$

The tension in the string is $T_2 = T$.

Step 2: Write the Equations of Motion Since the system is in equilibrium (no acceleration), the net forces along each incline must balance out:

- For m_1:

$$T = 20\sqrt{3}.$$

- For m_2:

$$T = 5.$$

Step 3: Solve for the Tension Combine the equations of motion:

$$T = 4(\sqrt{3} + 1)\,\text{N}.$$

Final Answer The tension in the string is:

$$\boxed{4(\sqrt{3} + 1)\,\text{N}}.$$

This matches option (2), as confirmed by the official answer.

Quick Tip

For inclined plane problems, resolve forces along the incline carefully, and ensure all forces, including tension and weight components, are balanced accurately.

Question 9: Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion (A): Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason (R): The current in the forward bias is more than the current in the reverse bias for a p–n junction diode.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (2) Both A and R are true but R is NOT the correct explanation of A

Solution: Photodiodes are operated in reverse bias because the fractional change in current due to light is easier to detect in reverse bias. While it is true that the forward bias current is greater than the reverse bias current, this is not the reason photodiodes are used in reverse bias.

Thus, A is true, and R is also true, but R is not the correct explanation for A.

Quick Tip

Photodiodes are designed for precise light detection, and reverse bias ensures better sensitivity due to the minimal leakage current compared to forward bias.

Question 10: If $\vec{E}$ represents the electric field vector and $\vec{K}$ the propagation vector of electromagnetic waves in vacuum, then the magnetic field vector is given by (ω - angular frequency):

(1) $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$

(2) $\vec{B} = \omega(\vec{E} \times \vec{K})$

(3) $\vec{B} = \omega(\vec{K} \times \vec{E})$

(4) $(\vec{K} \times \vec{E})$

Correct Answer: (1) $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$

Solution: Electromagnetic (EM) waves consist of oscillating electric and magnetic fields that are perpendicular to each other and to the direction of wave propagation. The following relationships hold for the electric field ($\vec{E}$), magnetic field ($\vec{B}$), and wave vector ($\vec{K}$):

1. The wave vector $\vec{K}$ indicates the direction of wave propagation.

2. The electric field $\vec{E}$ oscillates perpendicular to $\vec{K}$.

3. The magnetic field $\vec{B}$ is perpendicular to both $\vec{E}$ and $\vec{K}$.

The magnitude and direction of $\vec{B}$ are determined using the cross product:

$$\vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$$

where ω is the angular frequency of the wave.

Explanation of Options: Option (1): Correct. This represents the magnetic field as proportional to the cross product of $\vec{K}$ and $\vec{E}$, divided by the angular frequency.

Option (2): Incorrect. The direction of $\vec{B}$ would be wrong as the cross product order matters ($\vec{K} \times \vec{E} \neq \vec{E} \times \vec{K}$).

Option (3): Incorrect. Multiplying by ω gives a dimensionally incorrect magnetic field.

Option (4): Incorrect. The dot product results in a scalar, not a vector.

Thus, the correct option is (1).

Quick Tip

In electromagnetic waves, $\vec{E}$, $\vec{B}$, and $\vec{K}$ are mutually perpendicular, forming a right-handed coordinate system. Use the right-hand rule to verify directions.

Question 11: A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of the circular loop and at a distance r from the center of the loop on its axis is:

(1) $1 : 3\sqrt{2}$

(2) $3\sqrt{2} : 2$

(3) $2\sqrt{2} : 1$

(4) $1 : \sqrt{2}$

Correct Answer: (3) $2\sqrt{2} : 1$

Solution: The magnetic field due to a current-carrying circular loop on its axis is given as:

$$B_{\text{axis}} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

At the center of the loop ($x = 0$):

$$B_{\text{center}} = \frac{\mu_0 I}{2r}$$

At a distance r on the axis ($x = r$):

$$B_{\text{axis}} = \frac{\mu_0 I}{2 \cdot 2\sqrt{2}r}$$

The ratio of magnetic fields is:

$$\frac{B_{\text{center}}}{B_{\text{axis}}} = \frac{\frac{\mu_0 I}{2r}}{\frac{\mu_0 I}{2 \cdot 2\sqrt{2}r}} = 2\sqrt{2}$$

Thus, the ratio $B_{\text{center}} : B_{\text{axis}} = 2\sqrt{2} : 1$.

Quick Tip

In problems involving magnetic fields of current-carrying loops, use symmetry and the Biot-Savart law to derive the required relationships. Remember, the center field is always stronger than the axial field at any distance.

Question 12: A travelling wave is described by the equation:

$$y(x,t) = 0.05 \sin(8x - 4t) \, \text{m}$$

The velocity of the wave is: (All quantities are in SI units.)

(1) $4 \, \text{ms}^{-1}$

(2) $2 \, \text{ms}^{-1}$

(3) $0.5 \, \text{ms}^{-1}$

(4) $8 \, \text{ms}^{-1}$

Correct Answer: (3) $0.5 \, \text{ms}^{-1}$

Solution: The general equation of a wave is given by:

$$y(x,t) = A \sin(kx - \omega t)$$

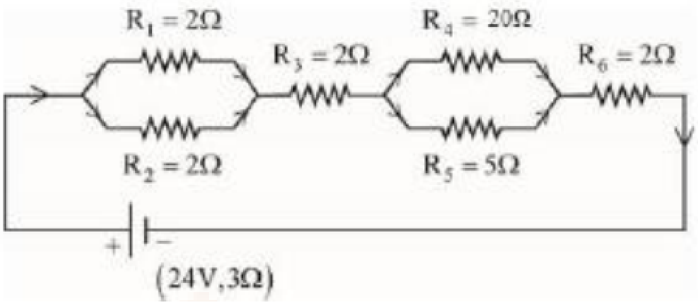
where $k = 8 \, \text{m}^{-1}$ and $\omega = 4 \, \text{rad/s}$. The velocity of the wave is:

$$v = \frac{\omega}{k} = \frac{4}{8} = 0.5 \, \text{ms}^{-1}$$

Quick Tip

In wave motion, velocity is given by $v = \frac{\omega}{k}$, where ω is the angular frequency and k is the wave number. Ensure all quantities are in SI units before calculating.

Question 13: As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of $3\,\Omega$. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are:



(1) $I_4 = \frac{8}{5}$ A and $I_5 = \frac{2}{5}$ A

(2) $I_4 = \frac{24}{5}$ A and $I_5 = \frac{6}{5}$ A

(3) $I_4 = \frac{6}{5}$ A and $I_5 = \frac{24}{5}$ A

(4) $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A

Correct Answer: (4) $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A

Solution: The equivalent resistance of the circuit is calculated as:

$$R_{\text{eq}} = 3 + 1 + 2 + 4 + 2 = 12\,\Omega$$

The current through the battery is:

$$I = \frac{24}{12} = 2\,\text{A}$$

The current I_4 through R_4 is given by:

$$I_4 = \frac{R_5}{R_4 + R_5} \times 2 = \frac{5}{20 + 5} \times 2 = \frac{2}{5}\,\text{A}$$

The current I_5 through R_5 is:

$$I_5 = 2 - I_4 = 2 - \frac{2}{5} = \frac{8}{5}\,\text{A}$$

Thus, the correct values are $I_4 = \frac{2}{5}$ A and $I_5 = \frac{8}{5}$ A.

Quick Tip

For resistor circuits, always calculate the equivalent resistance step-by-step and use Kirchhoff's laws or current division principles as needed.

Question 14: Given below are two statements:

Statement I: If the Brewster's angle for the light propagating from air to glass is θ_b, then Brewster's angle for the light propagating from glass to air is:

$$\frac{\pi}{2} - \theta_b$$

Statement II: The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$, where μ_g is the refractive index of glass.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are false.

Correct Answer: (2) Statement I is true but Statement II is false.

Solution: Statement I is correct. The Brewster's angle for light propagating from glass to air is given as $\frac{\pi}{2} - \theta_b$, where θ_b is the Brewster's angle for light propagating from air to glass.

Statement II is incorrect. The Brewster's angle for light propagating from glass to air is not $\tan^{-1}(\mu_g)$. The correct expression for the Brewster's angle involves the relative refractive indices of the two media.

Thus, Statement I is true and Statement II is false.

Quick Tip

The Brewster's angle is specific to the direction of light propagation. Use Snell's law and the definition of Brewster's angle for clarity.

Question 15: If two charges q_1 and q_2 are separated with distance d and placed in a medium of dielectric constant K, what will be the equivalent distance between charges in air for the same electrostatic force?

(1) $d\sqrt{K}$

(2) $k\sqrt{d}$

(3) $1.5d\sqrt{K}$

(4) $2d\sqrt{K}$

Correct Answer: (1) $d\sqrt{K}$

Solution: The force between two charges in a medium of dielectric constant K is given by Coulomb's law:

$$F_{\text{medium}} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{d^2}$$

where: - q_1 and q_2 are the charges, - d is the separation between the charges, - K is the dielectric constant of the medium.

The force between the same charges in air is:

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{d'^2}$$

where d' is the equivalent separation in air.

For the same electrostatic force, we equate $F_{\text{medium}} = F_{\text{air}}$:

$$\frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{d^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{d'^2}$$

Simplify the equation:

$$\frac{1}{K} \cdot \frac{1}{d^2} = \frac{1}{d'^2}$$

This gives:

$$d'^2 = K \cdot d^2$$

Taking the square root:

$$d' = d\sqrt{K}$$

Thus, the equivalent distance in air is:

$$d' = d\sqrt{K}$$

Quick Tip

For problems involving dielectric constants, remember that the force reduces by a factor of K in a medium compared to air. Use proportional relationships between the distances and the dielectric constant for accurate results.

Question 16: Consider the following radioactive decay process:

$$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

The mass number and the atomic number of A_6 are given by:

(1) 210 and 82

(2) 210 and 84

(3) 210 and 80

(4) 211 and 80

Correct Answer: (3) 210 and 80

Solution: 1. During the first α-decay:

$$^{218}_{84}A \xrightarrow{\alpha} {}^{214}_{82}A_1$$

The mass number decreases by 4, and the atomic number decreases by 2.

2. During the β^--decay:

$$^{214}_{82}A_1 \xrightarrow{\beta^-} {}^{214}_{83}A_2$$

The mass number remains the same, and the atomic number increases by 1.

3. During the γ-decay:

$$^{214}_{83}A_2 \xrightarrow{\gamma} {}^{214}_{83}A_3$$

There is no change in the mass number or the atomic number.

4. During the second α-decay:

$$^{214}_{83}A_3 \xrightarrow{\alpha} {}^{210}_{81}A_4$$

The mass number decreases by 4, and the atomic number decreases by 2.

5. During the β^+-decay:

$$^{210}_{81}A_4 \xrightarrow{\beta^+} {}^{210}_{80}A_5$$

The mass number remains the same, and the atomic number decreases by 1.

6. During the final γ-decay:

$$^{210}_{80}A_5 \xrightarrow{\gamma} {}^{210}_{80}A_6$$

There is no change in the mass number or the atomic number.

Final Answer: The mass number and atomic number of A_6 are:

$$\boxed{210 \text{ and } 80}$$

Quick Tip

In radioactive decay processes, carefully track changes to the mass number (-4 for α) and atomic number ($+1$ for β^-, -2 for α). This step-by-step approach ensures accurate results.

Question 17: Given below are two statements:

Statement I: The temperature of a gas is $-73°C$. When the gas is heated to $527°C$, the root mean square speed of the molecules is doubled.

Statement II: The product of pressure and volume of an ideal gas will be equal to the translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Correct Answer: (2) Statement I is true but Statement II is false.

Solution:

Statement I: The root mean square (RMS) speed of gas molecules is given by:

$$v_{\text{rms}} \propto \sqrt{T}$$

Let $T_1 = -73°C = 200\,\text{K}$ and $T_2 = 527°C = 800\,\text{K}$. The ratio of RMS speeds is:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{800}{200}} = \sqrt{4} = 2$$

Thus, the RMS speed doubles, and Statement I is true.

Statement II: For an ideal gas, the product of pressure and volume is given by:

$$PV = nRT$$

The translational kinetic energy is:

$$\text{Translational KE} = \frac{3}{2}nRT$$

Since $PV \neq \frac{3}{2}nRT$, Statement II is false.

Quick Tip

The RMS speed of molecules depends on the square root of absolute temperature. For ideal gases, always distinguish between PV (pressure-volume work) and translational kinetic energy.

Question 18: The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance up to which he can throw the same ball is:

(1) $192\,\text{m}$

(2) $136\,\text{m}$

(3) $272\,\text{m}$

(4) $68\,\text{m}$

Correct Answer: (3) $272\,\text{m}$

Solution: The maximum vertical height is related to the maximum velocity as:

$$H_{\text{max}} = \frac{v^2}{2g}$$

Given $H_{\text{max}} = 136\,\text{m}$, solve for v^2:

$$v^2 = 2gH_{\text{max}} = 2 \cdot 9.8 \cdot 136 = 2 \cdot 136 \,(\text{using } g \approx 9.8\,\text{m/s}^2).$$

The maximum horizontal range R_{max} is given by:

$$R_{\text{max}} = \frac{v^2}{g} = 2H_{\text{max}} = 2 \cdot 136 = 272\,\text{m}.$$

Quick Tip

The maximum horizontal range is always twice the maximum height for a projectile when launched at $45°$. Use $R_{\text{max}} = 2H_{\text{max}}$ for quick calculations.

Question 19: A conducting loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of $0.5\,$T. The magnetic field is decreased to zero in $0.5\,$s at a steady rate. The induced emf in the circular loop at $0.25\,$s is:

(1) $1\,$mV

(2) $10\,$mV

(3) $100\,$mV

(4) $5\,$mV

Correct Answer: (2) $10\,$mV

Solution:

The area of the loop is:

$$A = \pi r^2 = \pi \left(\frac{10}{\sqrt{\pi}}\right)^2 = 100\,\text{cm}^2 = 10^{-2}\,\text{m}^2.$$

The magnetic flux through the loop is:

$$\Phi = B \cdot A = 0.5 \cdot 10^{-2} = 5 \cdot 10^{-3}\,\text{Wb}.$$

The emf induced is:

$$\mathcal{E} = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{5 \cdot 10^{-3}}{0.5} = 10\,\text{mV}.$$

Quick Tip

Induced emf is calculated using Faraday's law: $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$. Ensure flux is calculated correctly using A and B, and convert units to SI for accurate results.

Question 20: Match List I with List II:

List I	List II
A. Planck's constant (h)	I. $M^1L^2T^{-2}$
B. Stopping potential (V_s)	II. $M^1L^1T^{-1}$
C. Work function (ϕ)	III. $M^1L^2T^{-1}$
D. Momentum (p)	IV. $M^1L^2T^{-3}A^{-1}$

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-IV, C-III, D-I

(4) A-I, B-III, C-IV, D-II

Correct Answer: (2) A-III, B-IV, C-I, D-II

Solution:

1. Planck's constant (h): Energy is given by $E = h\nu$. Using the dimensional formula of energy:

$$[E] = M^1L^2T^{-2}, \quad [\nu] = T^{-1}$$

Thus:

$$[h] = \frac{[E]}{[\nu]} = \frac{M^1L^2T^{-2}}{T^{-1}} = M^1L^2T^{-1}$$

Match: $A \to$ III.

2. Stopping potential (V_s): Voltage is given by $V = \frac{E}{q}$. Using dimensional formulas:

$$[V] = \frac{M^1L^2T^{-2}}{AT^{-1}} = M^1L^2T^{-3}A^{-1}$$

Match: $B \to$ IV.

3. Work function (ϕ): Work function is a form of energy:

$$[\phi] = M^1L^2T^{-2}$$

Match: $C \to$ I.

4. Momentum (p): Momentum is given by $p = F \cdot t$. Using dimensional formulas:

$$[p] = M^1L^1T^{-2} \cdot T^1 = M^1L^1T^{-1}$$

Match: $D \to$ II.

Thus, the correct matching is $A \to$ III, $B \to$ IV, $C \to$ I, $D \to$ II.

Quick Tip

Dimensional analysis is a powerful tool to validate physical quantities. Always check the units of derived quantities against the base SI units.

Question 21: A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5$^{\text{th}}$ second. The force acted on the body is ____ N.

Correct Answer: (4) $40\,\text{N}$

Solution:

The kinetic energy is related to velocity as:

$$\frac{1}{2}mv^2 = \text{KE}$$

Substitute the given values ($m = 2\,\text{kg}$, $\text{KE} = 10000\,\text{J}$):

$$\frac{1}{2} \cdot 2 \cdot v^2 = 10000$$

$$v^2 = 10000, \quad v = 100\,\text{m/s}$$

The acceleration is found using:

$$v = u + at$$

Since the body starts from rest ($u = 0$) and reaches $v = 100\,\text{m/s}$ in $t = 5\,\text{s}$:

$$100 = 0 + a \cdot 5 \implies a = \frac{100}{5} = 20\,\text{m/s}^2$$

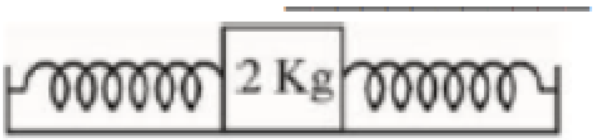
The force acting on the body is:

$$F = m \cdot a = 2 \cdot 20 = 40\,\text{N}$$

Quick Tip

In kinematics problems, use relationships between kinetic energy, velocity, and acceleration to solve for force systematically. Always check units for consistency.

Question 22: A block of mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface, and the ends of the springs are attached to rigid supports. When the mass is displaced from its equilibrium position, it executes simple harmonic motion. The time period of oscillation is given by $\frac{\pi}{\sqrt{x}}$ in SI units. The value of x is:



Correct Answer: $x = 5$

Solution: The equivalent spring constant for the system is given by:

$$k_{\text{eq}} = k_1 + k_2 = 20 + 20 = 40 \, \text{N/m}.$$

The angular frequency ω for the block-mass system is:

$$\omega = \sqrt{\frac{k_{\text{eq}}}{m}} = \sqrt{\frac{40}{2}} = \sqrt{20} \, \text{rad/s}.$$

The time period of oscillation T is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}} \, \text{s}.$$

Comparing $T = \frac{\pi}{\sqrt{x}}$, we find:

$$x = 5.$$

Quick Tip

For systems with multiple springs, use the equivalent spring constant formula for parallel or series combinations. Then apply the formula for angular frequency and time period systematically.

Question 23: A hole is drilled in a metal sheet. At $27°$C, the diameter of the hole is 5 cm. When the sheet is heated to $177°$C, the change in the diameter of the hole is Δd. The value of Δd will be, if the coefficient of linear expansion of the metal is $1.6 \times 10^{-5}/°$C:

Correct Answer: $\Delta d = 0.12\,\text{cm}$

Solution: The change in diameter of the hole due to thermal expansion is given by:

$$\Delta d = d_0 \cdot \alpha \cdot \Delta T$$

where: - $d_0 = 5\,\text{cm}$, - $\alpha = 1.6 \times 10^{-5}/°\text{C}$, - $\Delta T = 177°\text{C} - 27°\text{C} = 150°\text{C}$.

Substitute the values:

$$\Delta d = 5 \cdot 1.6 \times 10^{-5} \cdot 150 = 12 \times 10^{-3}\,\text{cm}.$$

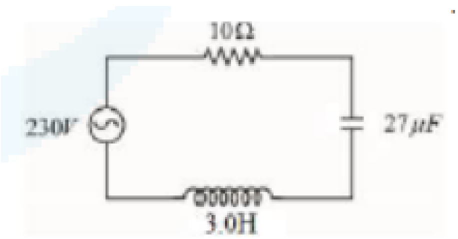
Thus, the change in diameter is:

$$\Delta d = 12\,\text{cm}.$$

Quick Tip

For thermal expansion problems, ensure that all quantities are in consistent units. Remember, the expansion of a hole is treated the same as the expansion of the material enclosing it.

Question 24: In the circuit shown in the figure, the ratio of the quality factor and the bandwidth is ____ s.



Correct Answer: $10\,\text{s}$

Solution: The relations for an LCR circuit are:

$$\Delta\omega = \frac{R}{L}, \quad Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \cdot \frac{L}{R}.$$

The resonant angular frequency ω_0 is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{1}{9 \times 10^{-3}}.$$

Simplify:

$$\omega_0 = 10^3 \text{ rad/s}.$$

The ratio $\frac{Q}{\Delta\omega}$ is:

$$\frac{Q}{\Delta\omega} = \frac{\omega_0 \cdot L/R}{R/L} = \omega_0 \cdot \frac{L^2}{R^2}.$$

Substituting the values:

$$\frac{Q}{\Delta\omega} = \sqrt{\frac{1}{LC}} \cdot \frac{L^2}{R^2}.$$

Simplify further:

$$\frac{Q}{\Delta\omega} = \frac{1}{9 \times 10^{-3}} \cdot \frac{9}{100} = 10 \text{ s}.$$

Thus, the ratio of the quality factor to the bandwidth is:

$$\boxed{10 \text{ s}}.$$

Quick Tip

In LCR circuits, the quality factor Q is inversely proportional to the bandwidth. Use $\Delta\omega = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$ for accurate calculations.

Question 25: A hollow cylindrical conductor has a length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm, respectively. The resistance of the conductor is $2 \times 10^{-3}\,\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\,\Omega$ m, the value of $n \times 10^{-3}\,\Omega$ is:

Correct Answer: $n = 2$

Solution:

The resistance of a conductor is given by:

$$R = \rho \frac{\ell}{A}$$

where: - $R = 2 \times 10^{-3}\,\Omega$, - $\rho = 2.4 \times 10^{-8}\,\Omega\,\text{m}$, - $\ell = 3.14\,\text{m}$, - A is the cross-sectional area of the hollow cylinder.

The cross-sectional area A of the hollow cylinder is:

$$A = \pi(b^2 - a^2)$$

where: - $b = 4\,\text{mm} = 4 \times 10^{-3}\,\text{m}$, - $a = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}$.

Substitute the values:

$$A = \pi\left[(4 \times 10^{-3})^2 - (2 \times 10^{-3})^2\right] = \pi\left[16 \times 10^{-6} - 4 \times 10^{-6}\right] = \pi \cdot 12 \times 10^{-6}.$$

Now calculate R:

$$R = \rho \frac{\ell}{A} = \frac{2.4 \times 10^{-8} \cdot 3.14}{\pi \cdot 12 \times 10^{-6}} = \frac{2.4 \times 10^{-8} \cdot 3.14}{3.77 \times 10^{-5}} = 2 \times 10^{-3}\,\Omega.$$

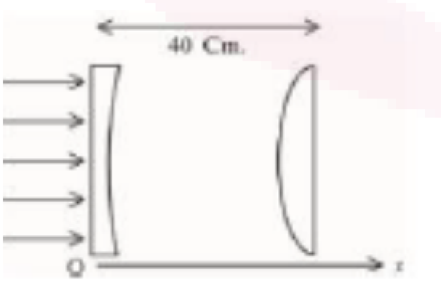
This confirms the calculated value, and:

$$n = 2.$$

Quick Tip

For hollow cylindrical conductors, always use the formula $A = \pi(b^2 - a^2)$ for the cross-sectional area. Ensure all units are consistent, especially for diameters and lengths.

Question 26: As shown in the figure, a combination of a thin plano-concave lens and a thin plano-convex lens is used to image an object placed at infinity. The radius of curvature of both lenses is 30 cm, and the refractive index of the material for both lenses is 1.75. Both lenses are placed at a distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance x cm, from the concave lens.



Correct Answer: $x = 120\,\text{cm}$

Solution: 1. Focal length of the plano-concave lens (L_1): The focal length of a plano-concave lens is given by:

$$\frac{1}{f_1} = (1-\mu)\left(\frac{1}{R}\right),$$

where $\mu = 1.75$ and $R = 30\,\text{cm}$. Substitute the values:

$$\frac{1}{f_1} = (1-1.75)\cdot\frac{1}{30} = -\frac{0.75}{30}.$$

Thus:

$$f_1 = -40\,\text{cm}.$$

2. Focal length of the plano-convex lens (L_2): The focal length of a plano-convex lens is given by:

$$\frac{1}{f_2} = (\mu-1)\left(\frac{1}{R}\right).$$

Substitute the values:

$$\frac{1}{f_2} = (1.75-1)\cdot\frac{1}{30} = \frac{0.75}{30}.$$

Thus:

$$f_2 = 40\,\text{cm}.$$

3. Final image formation: - The object is at infinity, so the image from L_1 will be virtual and on the left at its focal length ($f_1 = -40\,\text{cm}$). - This virtual image acts as the object for L_2, which is placed $40\,\text{cm}$ away from L_1. - The distance of the object from L_2 is:

$$u = -40 - 40 = -80\,\text{cm}.$$

Using the lens formula for L_2:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}.$$

Substitute the values:

$$\frac{1}{v} - \frac{1}{-80} = \frac{1}{40}.$$

Simplify:

$$\frac{1}{v} + \frac{1}{80} = \frac{1}{40}.$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{80} = \frac{2-1}{80} = \frac{1}{80}.$$

Thus:

$$v = 80\,\text{cm}.$$

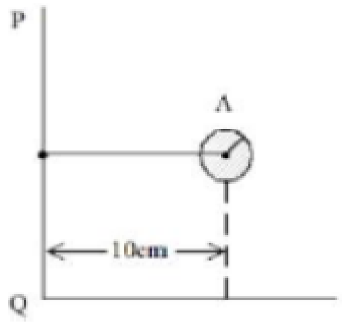
The final image is formed $80\,\text{cm}$ to the right of L_2. From the concave lens L_1, the total distance is:

$$x = 40 + 80 = 120\,\text{cm}.$$

Quick Tip

In lens combinations, always determine the focal lengths of individual lenses first. Use the lens formula step-by-step to trace the image positions.

Question 27: A solid sphere A is rotating about an axis PQ. If the radius of the sphere is $5\,\text{cm}$, then its radius of gyration about PQ will be $\sqrt{x}\,\text{cm}$. The value of x is:



Correct Answer: $x = 110$

Solution: The radius of gyration k is related to the moment of inertia I of the sphere and its mass M by:

$$I = M \cdot k^2 \quad \Rightarrow \quad k = \sqrt{\frac{I}{M}}.$$

1.Moment of inertia of the sphere about the axis PQ: The moment of inertia of a solid sphere about an axis passing through a point at a distance d from its center is given by the parallel axis theorem:

$$I = I_{\text{center}} + M \cdot d^2,$$

where: - $I_{\text{center}} = \frac{2}{5}MR^2$ is the moment of inertia about the center, - $d = 10\,\text{cm}$ is the distance from the center to the axis PQ, - $R = 5\,\text{cm}$ is the radius of the sphere.

Substituting:

$$I = \frac{2}{5}MR^2 + Md^2 = \frac{2}{5}M(5)^2 + M(10)^2.$$

Simplify:

$$I = \frac{2}{5}M \cdot 25 + M \cdot 100 = \frac{50}{5}M + 100M = 10M + 100M = 110M.$$

2.Radius of gyration (k): The radius of gyration is:

$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{110M}{M}} = \sqrt{110}\,\text{cm}.$$

3.Relating to $\sqrt{x}$: From the problem, $k = \sqrt{x}\,\text{cm}$. Thus:

$$x = 110.$$

Quick Tip

Use the parallel axis theorem to calculate the moment of inertia when the axis does not pass through the center of mass. For a solid sphere, remember $I_{\text{center}} = \frac{2}{5}MR^2$.

Question 28: Vectors $\vec{a} = a\hat{i} + b\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$. The ratio of a to b is $\frac{x}{2}$. The value of x is:

Correct Answer: $x = 1$

Solution: For two vectors to be perpendicular, their dot product must be zero:

$$\vec{a} \cdot \vec{b} = 0.$$

Substitute $\vec{a} = a\hat{i} + b\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$:

$$(a\hat{i} + b\hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0.$$

Simplify:

$$2a - 3b + 4 = 0.$$

This gives the first equation:

$$2a - 3b = -4.$$

From the problem, another equation is given:

$$3a + 2b = 7.$$

Solve the simultaneous equations: Multiply equation (1) by 2:

$$4a - 6b = -8.$$

Multiply equation (2) by 3:

$$9a + 6b = 21.$$

Add equations (3) and (4):

$$13a = 13 \quad \Rightarrow \quad a = 1.$$

Substitute $a = 1$ into equation (2):

$$3(1) + 2b = 7 \quad \Rightarrow \quad 3 + 2b = 7 \quad \Rightarrow \quad 2b = 4 \quad \Rightarrow \quad b = 2.$$

The ratio of a to b is:

$$\frac{a}{b} = \frac{x}{2} \quad \Rightarrow \quad \frac{1}{2} = \frac{x}{2}.$$

Thus:

$$x = 1.$$

Quick Tip

For perpendicular vectors, always ensure their dot product equals zero. Use simultaneous equations systematically to solve for unknowns.

Question 29: Assume that protons and neutrons have equal masses. The mass of a nucleon is 1.6×10^{-27} kg, and the radius of a nucleus is $1.5 \times 10^{-15} A^{1/3}$ m. The approximate ratio of nuclear density to water density is $n \times 10^{13}$. The value of n is:

Correct Answer: $n = 11$

Solution: The density of a nucleus is:

$$\rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}.$$

1. Mass of the nucleus: The mass of a single nucleon is:

$$m = 1.6 \times 10^{-27}\,\text{kg}.$$

2. Volume of the nucleus: The volume of a nucleus is given by:

$$V = \frac{4}{3}\pi R^3,$$

where $R = 1.5 \times 10^{-15}$ m. Substituting:

$$V = \frac{4}{3}\pi(1.5 \times 10^{-15})^3 = \frac{4}{3}\pi \cdot 3.375 \times 10^{-45}.$$

Approximate:

$$V \approx 14.14 \times 10^{-45}\,\text{m}^3.$$

3. Nuclear density (ρ):

$$\rho = \frac{m}{V} = \frac{1.6 \times 10^{-27}}{14.14 \times 10^{-45}} \approx 0.113 \times 10^{18}\,\text{kg/m}^3.$$

4. Water density (ρ_w):

$$\rho_w = 10^3\,\text{kg/m}^3.$$

5. Ratio of nuclear density to water density:

$$\frac{\rho}{\rho_w} = \frac{0.113 \times 10^{18}}{10^3} = 0.113 \times 10^{15} = 11.31 \times 10^{13}.$$

Thus, the approximate ratio is:

$$n = 11.$$

Quick Tip

For density calculations, ensure proper units and correct formulas for mass and volume. Use approximations carefully in large-scale problems.

Question 30: A stream of positively charged particles having $\frac{q}{m} = 2 \times 10^{11}$ C/kg and velocity $\vec{v_0} = 3 \times 10^7\,\hat{i}$ m/s is deflected by an electric field $\vec{E} = 1.8\,\hat{j}$ kV/m. The electric field exists in a region of 10 cm along the x-direction. Due to the electric field, the deflection of the charged particles in the y-direction is ____ mm.

Correct Answer: 2 mm

Solution: 1. Acceleration of the particles in the y-direction (a): The force on the charged particles due to the electric field is:

$$F = qE.$$

Using $F = ma$, the acceleration in the y-direction is:

$$a = \frac{F}{m} = \frac{qE}{m}.$$

Substitute the values:

$$a = \left(2 \times 10^{11}\right) \cdot \left(1.8 \times 10^{3}\right) = 3.6 \times 10^{14}\,\text{m/s}^2.$$

2. Time taken to cross the plates (t): The time t to travel a distance of 10 cm $= 0.1$ m along the x-direction is given by:

$$t = \frac{d}{v_0},$$

where $d = 0.1$ m and $v_0 = 3 \times 10^7$ m/s.

Substitute the values:

$$t = \frac{0.1}{3 \times 10^7} = \frac{1}{3 \times 10^8}\,\text{s}.$$

3. Deflection in the y-direction (y): The deflection in the y-direction is given by:

$$y = \frac{1}{2}at^2.$$

Substitute the values:

$$y = \frac{1}{2} \cdot \left(3.6 \times 10^{14}\right) \cdot \left(\frac{1}{3 \times 10^8}\right)^2.$$

Simplify:

$$y = \frac{1}{2} \cdot 3.6 \times 10^{14} \cdot \frac{1}{9 \times 10^{16}}.$$

$$y = \frac{3.6}{2} \cdot \frac{1}{9 \times 10^2} = \frac{1.8}{900} = 0.002\,\text{m}.$$

Convert to millimeters:

$$y = 0.002\,\text{m} = 2\,\text{mm}.$$

Thus, the deflection in the y-direction is:

$$y = 2\,\text{mm}.$$

Quick Tip

For problems involving deflection of charged particles in an electric field, calculate the acceleration due to $F = ma$, the time taken to traverse the region, and then use kinematic equations to find the displacement.
