JEE Main 2023 24 Jan Shift 2 Mathematics Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them: (A) The Duration of test is 3 Hours.

(B) This paper consists of 90 Questions.

(C) There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..

(E) (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

61: Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P., and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to:

(1) 220

(2) 210

(3) 200

(4) 105

62: Let $f(x)$ be a function such that $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is:

(1) 6

(2) 8

(3) 7

(4) 9

63: The number of real solutions of the equation $3(x^2 + \frac{1}{x^2}) - 2(x + \frac{1}{x})$, is:

(1) 4

(2) 0

(3) 3

(4) 2

64: If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots + f\left(\frac{2022}{2023}\right)$ is equal to:

(1) 2011

(2) 1010

(3) 2010

(4) 1011

65: If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$, $x \in \mathbb{R}$, then:

(1) $3f(1) + f(2) = f(3)$

(2) $f(3) - f(2) = f(1)$

(3) $2f(0) - f(1) + f(3) = f(2)$

(4) $f(1) + f(2) + f(3) = f(0)$

66: The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is:

(1) 120

(2) 168

(3) 220

(4) 48

67: If the system of equations

$$x + 2y + 3z = 3 \quad \cdots (i)$$

$$4x + 3y - 4z = 4 \quad \cdots (ii)$$

$$8x + 4y - z = 9 + \mu \quad \cdots (iii)$$

has infinitely many solutions, then the ordered pair (λ, μ) **is equal to:**

(1) $\left(\frac{72}{5}, \frac{21}{5}\right)$

(2) $\left(\frac{-72}{5}, \frac{-21}{5}\right)$

(3) $\left(\frac{72}{5}, \frac{-21}{5}\right)$

(4) $\left(\frac{-72}{5}, \frac{21}{5}\right)$

68: The value of

$$\left(\frac{1 + \sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1 + \sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)^3$$

is:

(1) $-\frac{1}{2}(1 - i\sqrt{3})$

(2) $\frac{1}{2}(1 - i\sqrt{3})$

(3) $-\frac{1}{2}(\sqrt{3} - i)$

(4) $\frac{1}{2}(\sqrt{3} + i)$

69: The equations of the sides AB and AC of a triangle ABC are:

$$(\lambda + 1)x + \lambda y = 4 \quad \text{and} \quad \lambda x + (1 - \lambda)y + \lambda = 0,$$

respectively. Its vertex A is on the y-axis and its orthocentre is $(1, 2)$. The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is:

(1) $\sqrt{6}$

(2) $2\sqrt{2}$

(3) 2

(4) 4

70: The set of all values of a for which

$$\lim_{x \to a} ([\![x - 5]\!] - [\![2x + 2]\!]) = 0,$$

where $[\![x]\!]$ denotes the greatest integer less than or equal to x, is equal to:

(1) $(-7.5, -6.5)$

(2) $(-7.5, -6.5]$

(3) $[-7.5, -6.5]$

(4) $[-7.5, -6.5)$

71: If

$$({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_2)^2 + \ldots + 30({}^{30}C_{30})^2 = \frac{\alpha \cdot 60!}{(30!)^2},$$

then α is equal to:

(1) 30

(2) 60

(3) 15

(4) 10

72: Let the plane containing the line of intersection of the planes

$$P_1 : x + (\lambda + 4)y + z = 1 \quad \text{and} \quad P_2 : 2x + y + z = 2$$

pass through the points $(0, 1, 0)$ and $(1, 0, 1)$. Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is:

(1) $5\sqrt{6}$

(2) $4\sqrt{6}$

(3) $2\sqrt{6}$

(4) $3\sqrt{6}$

73: Let $\vec{\alpha} = 4\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{\beta} = \hat{i}+2\hat{j}-4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i}+\hat{j}+\hat{k})$ is:

(1) 6

(2) 11

(3) 7

(4) 9

74: The locus of the midpoints of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$, which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_1. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$, and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to:

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

75: If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to:

(1) -1

(2) 3

(3) 1

(4) 5

76: Let $y = y(x)$ be the solution of the differential equation

$$(x^2 - 3y^2)dx + 3xy\,dy = 0, \quad y(1) = 1.$$

Then $6y^2(e)$ is equal to:

(1) $3e^2$

(2) e^2

(3) $2e^2$

(4) $\frac{3e^2}{2}$

77: Let p and q be two statements. Then $\sim (p \wedge (p \rightarrow \sim q))$ is equivalent to:

(1) $p \vee (p \wedge \sim q)$

(2) $p \vee (\sim p \wedge q)$

(3) $\sim p \vee q$

(4) $p \vee (p \wedge q)$

78: The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is:

(1) 225

(2) 120

(3) 150

(4) 125

79: The value of the integral:

$$\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx \quad \text{is equal to:}$$

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$

(4) 2π

80: Let A be a 3×3 matrix such that $|\mathbf{adj}(\mathbf{adj}(A))| = 12^4$. Then $|\mathbf{A}^{-1}\mathbf{adj}(A)|$ is equal to:

(1) $2\sqrt{3}$

(2) $\sqrt{6}$

(3) 12

(4) 1

81: The urns A, B, and C contain 4 red, 6 black; 5 red, 5 black, and λ red; 4 black balls respectively. One of the urns is selected at random, and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4, then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is:

82: If the area of the region bounded by the curves $y^2 - 2y = -x$ and $x + y = 0$ is A, then $8A$ is equal to:

83: If

$$\frac{1^3 + 2^3 + 3^3 + \dots \textbf{ (up to } n \textbf{ terms)}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots \textbf{ (up to } n \textbf{ terms)}} = \frac{9}{5},$$

then the value of n is:

84: Let f be a differentiable function defined on $\left(0, \frac{\pi}{2}\right)$ such that $f(x) > 0$ and

$$f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\, dt = e, \quad \forall x \in \left[0, \frac{\pi}{2}\right].$$

Then $\left(6 \log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to:

85: The minimum number of elements that must be added to the relation $R = \{(a,b),(b,c),(b,d)\}$ on the set $\{a,b,c,d\}$ so that it is an equivalence relation, is:

86: Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \lambda\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 5\mathbf{j} - \lambda\mathbf{k}$, $\mathbf{a} \cdot \mathbf{c} = 7$, $2\mathbf{b} \cdot \mathbf{c} + 43 = 0$, $\mathbf{a} \times \mathbf{c} = \mathbf{b} \times \mathbf{c}$. Then $|\mathbf{a} \cdot \mathbf{b}|$ is equal to:

87: Let the sum of the coefficients of the first three terms in the expansion of

$$\left(x - \frac{3}{x^2}\right)^n, \quad x \neq 0,\ n \in \mathbb{N},$$

be 376. Then the coefficient of x^4 is:

88: If the shortest distance between the lines

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{4} = \frac{z}{5}, \quad \frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$$

is 6, then the square of the sum of all possible values of λ is:

89: Let $S = \{\theta \in [0, 2\pi) : \tan(\cos\theta) + \tan(\sin\theta) = 0\}$. Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to:

90: The equations of the sides AB, BC, and CA of a triangle ΔABC are:

$$2x + y = 0, \quad x + py = 21a\,(a \neq 0), \quad x - y = 3,$$

and $P(2, a)$ is the centroid of ΔABC. Then $(BC)^2$ is equal to:
