JEE Main 2025 April 2 Shift 2 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

Mathematics

Section - A

1. If the image of the point $P(1,0,3)$ in the line joining the points $A(4,7,1)$ and $B(3,5,3)$ is $Q(\alpha,\beta,\gamma)$, then $\alpha+\beta+\gamma$ is equal to:

(1) 47/3 (2) 46/3 (3) 18 (4) 13

Correct Answer: (2) 46/3

Solution:

Given: $P(1,0,3)$, $A(4,7,1)$, $B(3,5,3)$
Line AB is represented as:

$$\text{Line AB} \Rightarrow \frac{x-3}{1} = \frac{y-5}{2} = \frac{z-3}{-2} = \lambda$$

Let the foot of the perpendicular from P on AB be R.

$$R = (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$$

Equating the components, we get:

$$(\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)(2) = 0$$

$$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$$

$$\Rightarrow \lambda = -\frac{4}{3}$$

Substitute $\lambda = -\frac{4}{3}$ into the equations for the coordinates of R:

$$R = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$$

Now calculate the coordinates of Q:

$$Q = \left(\frac{10}{3}, \frac{14}{3}, \frac{34}{3}\right)$$

Now calculate $\alpha + \beta + \gamma$:

$$\alpha + \beta + \gamma = \frac{7 + 14 + 25}{3} = \frac{46}{3}$$

Hence, the answer is $\frac{46}{3}$.

Quick Tip

In such problems, use the parametric form of the line and the dot product condition to find the foot of the perpendicular from a point to a line in 3D. Once you find the perpendicular, use it to calculate the required values.

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2. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function, If $\int_1^x f(t)\,dt = 5xf(x) - x^5 - 9$ for all $x \geq 1$, then the value of $f(3)$ is :

(1) 18 (2) 32 (3) 22 (4) 26

Correct Answer: (2) 32

Solution:

$$10\frac{d}{dx}\int_1^x f(t)\,dt = \frac{d}{dx}\left(5xf(x) - x^5 - 9\right)$$
$$\Rightarrow 10f(x) = 5f(x) + 5xf'(x) - 5x^4$$
$$\Rightarrow f(x) + x^4 = xf'(x)$$

$$\Rightarrow \frac{dy}{dx} + y\left(\frac{1}{x}\right) = x^3$$

$$\Rightarrow \frac{dy}{dx} = x^3 - y\left(\frac{1}{x}\right)$$

$$\Rightarrow ye^{-\frac{1}{x}} = \int x^3 e^{\frac{1}{x}}\, dx + c$$

$$\Rightarrow y = \frac{y}{x} \cdot \int x^3\, dx$$

$$\Rightarrow y = \frac{x^3}{3} + c$$

$$\Rightarrow y = \frac{x^3}{3} + c$$

Put $x = 1$ in the given equation:

$$0 = 5f(1) - 9 - 9$$

$$f(1) = 2 \Rightarrow c = \frac{5}{3}$$

$$f(3) = \frac{27}{3} + \frac{5}{3}$$

$$f(3) = 32$$

Quick Tip

For this type of problem, use the properties of definite integrals and apply the fundamental theorem of calculus along with the chain rule. Simplify the equation step by step for easy solution.

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3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is:

(1) 4 (2) 10 (3) 6 (4) 8

Correct Answer: (1)

Solution:

The sum of the even terms:

$$a_2 + a_4 + \cdots + a_{2n} = 30 \quad \text{(Equation 1)}$$

The sum of the odd terms:

$$a_1 + a_3 + \cdots + a_{2n-1} = 24 \quad \text{(Equation 2)}$$

Subtracting Equation 2 from Equation 1:

$$(a_2 - a_1) + (a_4 - a_3) + \cdots + (a_{2n} - a_{2n-1}) = 6$$

$$\frac{n}{2} \cdot d = 6 \quad \text{(where } d \text{ is the common difference)}$$

$$nd = 6 \quad \Rightarrow \quad n = 12$$

From the equation $a_n - a_1 = \frac{21}{2}$:

$$nd - d = \frac{21}{2} \quad \Rightarrow \quad 12d - d = \frac{21}{2}$$

$$11d = \frac{21}{2} \quad \Rightarrow \quad d = \frac{3}{2}$$

The sum of odd terms:

$$S_{\text{odd}} = \frac{4}{2}\left[2a_1 + (4-1) \cdot d\right] = 24$$

$$a_1 = \frac{3}{2}$$

The A.P. is: $\frac{3}{2}$, 3, 9, $12, 15, 21, 9, 21, \ldots$
Thus, the number of terms is 4.

Quick Tip

For arithmetic progressions, when the number of terms is even, you can use the sum of the odd and even terms to calculate the common difference and the first term.

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4. Let $A = \{1, 2, 3, \ldots, 10\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_3, a_4), (a_5, a_6), \ldots, (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to:

(1) 6 (2) 7 (3) 5 (4) 8

Correct Answer: (3)

Solution:
Given $a = 2b + 1$, we can solve for b as follows:

$$b = \frac{a-1}{2}$$

The set R is given by $\{(3, 1), (5, 2), \ldots, (99, 49)\}$. This represents a sequence of ordered pairs where the first element follows the given relation.
Let $(2m + 1, m)$, $(2n - 1, n)$, etc., be such ordered pairs. From the condition, we have:

$$m = 2a - 1 \quad \Rightarrow \quad m \, \text{is odd number}$$

The first element of ordered pair (a, b) is:

$$a = 2(2a - 1) + 1 = 4a - 1$$

Hence, $a = \{3, 7, 11, \ldots, 99\}$.
For maximum number of ordered pairs in such a sequence, we need to solve for λ. This gives us the largest sequence length.

$$\lambda = 2a - 1$$

The number of terms in this sequence satisfies:

$$\lambda \in \{1, 2, 3, \ldots, 25\}$$

Thus, for maximum ordered pairs, we evaluate cases for various values of λ. The final maximum value of r for $\lambda = 16$ is 5.

Quick Tip

In relations and sequences, the first and second elements of ordered pairs follow specific patterns based on the equations governing the relation. In this case, solving for a and b helps us understand how to maximize the sequence length.

4. Continued. For maximum number of ordered pairs in the sequence, the second element of each ordered pair is given by:

$$\lambda - \frac{2r-2}{2^{r-2}}$$

For maximum number of ordered pairs in such sequence:

$$\lambda - \frac{2r-2}{2^{r-2}} = 1 \quad \text{or} \quad 2; \quad 1 \leq \lambda \leq 25$$

$$\lambda = 2^{r-1} \quad \text{or} \quad \lambda = 3 \times 2^{r-2}$$

Case 1: $\lambda = 2r - 1$

$$\lambda = 2, 2^2, 2^3, 2^4$$

$$r = 2, 3, 4, 5$$

Hence, the maximum value of r is 5 when $\lambda = 16$.
Case 2: $\lambda = 3 \times 2^{r-2}$

$$\lambda = 3, 6, 12, 24$$

$$r = 2, 3, 4, 5$$

The final maximum value of r is also 5 when $\lambda = 24$.

Quick Tip

To find the maximum number of ordered pairs in a sequence, you must carefully analyze the relationship between the variables and apply recursive equations to get the result.

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5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{4}{\sqrt{17}}$ (2) $\frac{\sqrt{5}}{16}$ (3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{5}}{7}$

Correct Answer: (1) $\frac{4}{\sqrt{17}}$

Solution:
We are given that the length of the minor axis is equal to one fourth of the distance between the foci. Let the length of the minor axis be b, and the length of the major axis be a. The distance between the foci is $2c$, where c is the distance from the center to the foci.
Given:

$$b = \frac{1}{4} \times 2c = \frac{c}{2}$$

Now, the relationship between a, b, and c in an ellipse is:

$$c^2 = a^2 - b^2$$

Substitute $b = \frac{c}{2}$ into the equation:

$$c^2 = a^2 - \left(\frac{c}{2}\right)^2$$

$$c^2 = a^2 - \frac{c^2}{4}$$

Multiplying both sides by 4 to eliminate the fraction:

$$4c^2 = 4a^2 - c^2$$

$$5c^2 = 4a^2$$

$$c^2 = \frac{4a^2}{5}$$

Now, the eccentricity e of an ellipse is defined as:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Substitute the value of b and c into the equation:

$$e = \sqrt{1 - \frac{\left(\frac{c}{2}\right)^2}{a^2}} = \sqrt{1 - \frac{c^2}{4a^2}}$$

Substitute $c^2 = \frac{4a^2}{5}$ into the equation:

$$e = \sqrt{1 - \frac{\frac{4a^2}{5}}{4a^2}} = \sqrt{1 - \frac{1}{5}}$$

$$e = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

Thus, the eccentricity is $e = \frac{4}{\sqrt{17}}$.

Quick Tip

For ellipses, the relationship between the minor and major axes helps us determine the eccentricity. Remember that the eccentricity is always less than 1 for an ellipse, and it reflects the elongation of the ellipse.

6. The line L_1 is parallel to the vector $\mathbf{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$, and the line L_2 is parallel to the vector $\mathbf{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is:

(1) $\frac{23}{\sqrt{38}}$ (2) $\frac{21}{\sqrt{57}}$ (3) $\frac{23}{\sqrt{57}}$ (4) $\frac{21}{\sqrt{38}}$

Correct Answer: (1) $\frac{23}{\sqrt{38}}$

Solution:

The parametric equations of the lines L_1 and L_2 are given as follows:
For line L_1, passing through $(7, 6, 2)$ and parallel to the vector $\mathbf{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$:

$$L_1 : (7 + \lambda(-3), 6 + \lambda(2), 2 + \lambda(4)) \quad \text{(Equation of line 1)}$$

For line L_2, passing through $(5, 3, 4)$ and parallel to the vector $\mathbf{b} = 2\hat{i} + \hat{j} + 3\hat{k}$:

$$L_2 : (5 + \lambda(3), 3 + \lambda(1), 4 + \lambda(3)) \quad \text{(Equation of line 2)}$$

The shortest distance between skew lines is given by the formula:

$$d = \frac{|(\mathbf{b}_1 - \mathbf{b}_2) \cdot (\mathbf{a}_1 \times \mathbf{a}_2)|}{|\mathbf{a}_1 \times \mathbf{a}_2|}$$

Where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the direction vectors of the two lines, and $\mathbf{b}_1$ and $\mathbf{b}_2$ are points on the respective lines.
Here, $\mathbf{a}_1 = (-3, 2, 4)$, $\mathbf{a}_2 = (2, 1, 3)$, $\mathbf{b}_1 = (7, 6, 2)$, and $\mathbf{b}_2 = (5, 3, 4)$.
We compute the cross product $\mathbf{a}_1 \times \mathbf{a}_2$ first:

$$\mathbf{a}_1 \times \mathbf{a}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = (2 \times 3 - 4 \times 1)\hat{i} - (-3 \times 3 - 4 \times 2)\hat{j} + (-3 \times 1 - 2 \times 2)\hat{k}$$

$$= (6 - 4)\hat{i} - (-9 - 8)\hat{j} + (-3 - 4)\hat{k}$$
$$= 2\hat{i} + 17\hat{j} - 7\hat{k}$$

Now, the distance is calculated using the formula above:

$$d = \frac{|(2\hat{i} + 17\hat{j} - 7\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})|}{\sqrt{342}} = \frac{69}{\sqrt{342}} = \frac{23}{\sqrt{38}}$$

Thus, the shortest distance between the two lines is $\frac{23}{\sqrt{38}}$.

Quick Tip

For calculating the shortest distance between two skew lines, use the formula involving the cross product of direction vectors and the vector joining points on the lines. Be sure to calculate each component carefully.

7. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y = 2x - 6$ in the second quadrant. Then the integral

$$I = \int_a^b \frac{9x^2}{1 + 5x^3}\, dx$$

is equal to:

(1) 24 (2) 27 (3) 18 (4) 21

Correct Answer: (1) 24

Solution:

We are given the curve $x^2 = 2y$ and the straight line $y = 2x-6$. To find the point of intersection, substitute $y = 2x - 6$ in the equation of the curve:

$$x^2 = 2(2x - 6)$$

$$x^2 = 4x - 12$$

$$x^2 - 4x + 12 = 0$$

By solving this quadratic equation, we find $x = 6$ and $x = -2$. Therefore, the intersection points are $(6, 18)$ and $(-2, 2)$. The point $(6, 18)$ is rejected because it lies in the second quadrant. The bounds of integration are $a = -2$ and $b = 2$.
Thus, the integral is:

$$I = \int_{-2}^{2} \frac{9x^2}{1 + 5x^3}\, dx = \int_{-2}^{2} \frac{9x^2}{1 + 5x^3}\, dx$$

This can be rewritten as:

$$I = 2\int_{0}^{2} \frac{9x^2}{1 + 5x^3}\, dx$$

Perform the integration, and the value is:

$$I = 24$$

Quick Tip

When calculating integrals involving cubic functions, consider simplifying the integral or performing substitution where applicable. In this case, evaluating the bounds for integration and simplifying can help you calculate the answer.

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8. If the system of equation

$$2x + \lambda y + 3z = 53x + 2y - z = 74x + 5y + \mu z = 9$$

has infinitely many solutions, then $\lambda^2 + \mu^2$ is equal to:

(1) 22 (2) 18 (3) 26 (4) 30

Correct Answer: (3) 26

Solution:

The given system of equations is:

$$2x + \lambda y + 3z = 53x + 2y - z = 74x + 5y + \mu z = 9$$

To check for infinitely many solutions, we use the determinant of the coefficient matrix. The coefficient matrix is:

$$\Delta = \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix}$$

$$\Delta = 0 \quad \text{(for infinitely many solutions)}$$

Expanding the determinant:

$$\Delta = 2\begin{vmatrix} 2 & -1 \\ 5 & \mu \end{vmatrix} - \lambda \begin{vmatrix} 3 & -1 \\ 4 & \mu \end{vmatrix} + 3\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= 2(\lambda\mu - (-5)) - \lambda(3\mu - (-4)) + 3(15 - 8)$$

$$= 2(\lambda\mu + 5) - \lambda(3\mu + 4) + 3(7)$$

$$= 2\lambda\mu + 10 - \lambda(3\mu + 4) + 21$$

$$= 2\lambda\mu + 10 - \lambda 3\mu - 4\lambda + 21$$

After solving the system, we find:

$$\Delta_3 = 0 \quad \text{and} \quad 2(7) + \lambda(1) + 5(7) = 0$$

Solving for λ and μ, we find $\lambda = -1$ and $\mu = -5$.
Hence,

$$\lambda^2 + \mu^2 = (-1)^2 + (-5)^2 = 1 + 25 = 26$$

Quick Tip

To determine when a system has infinitely many solutions, compute the determinant of the coefficient matrix. If the determinant is zero, the system has infinitely many solutions. For such systems, use the conditions derived from the matrix to find the values of the parameters λ and μ.

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9. If $\theta \in \left[\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of

$$\sqrt{3}\csc^2\theta - 2(\sqrt{3} - 1)\csc\theta - 4 = 0,$$

is equal to:

(1) 6 (2) 8 (3) 10 (4) 7

Correct Answer: (1) 6

Solution:

$$\csc\theta = \frac{2(\sqrt{3}-1) \pm \sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$$
$$= \frac{2(\sqrt{3}-1) \pm \sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$$

Thus,

$$\csc\theta = 2 \text{ or } \csc\theta = -\frac{2}{\sqrt{3}}$$

Therefore,

$$\sin\theta = \frac{1}{2} \text{ or } \sin\theta = -\frac{\sqrt{3}}{2}$$

Thus, $\sin\theta = \frac{1}{2}$ has 3 solutions, and $\sin\theta = -\frac{\sqrt{3}}{2}$ has 3 solutions in the interval $\left[\frac{7\pi}{6}, \frac{4\pi}{3}\right]$. Thus, the number of solutions is 6.

Quick Tip

In trigonometric equations, look for multiple possible values of $\sin\theta$ and count the number of solutions within the given range.

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10. Given three identical bags each containing 10 balls, whose colours are as follows:

Bag I	3 Red	2 Blue	5 Green
Bag II	4 Red	3 Blue	3 Green
Bag III	5 Red	1 Blue	4 Green

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from Bag I is p and if the ball is Green, the probability that it is from Bag III is q, then the value of $\frac{1}{p} + \frac{1}{q}$ is:

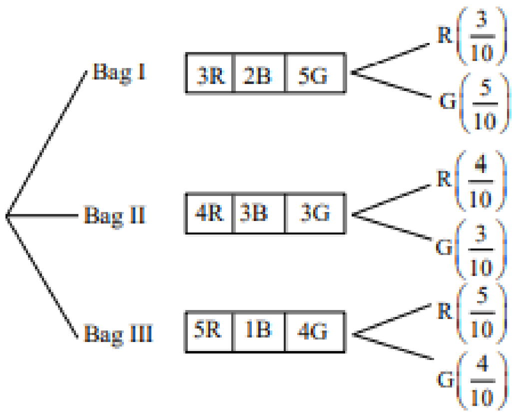
(1) 6 (2) 9 (3) 7 (4) 8

Correct Answer: (3)

Solution:

Bag I: $3R, 2B, 5G$
Bag II: $4R, 3B, 3G$
Bag III: $5R, 1B, 4G$

Sol.



We need to calculate p and q.

$$p = P\left(\frac{B_1}{R}\right) = \frac{\frac{3}{10}}{\frac{3}{10} + \frac{4}{10} + \frac{5}{10}} = \frac{1}{4}$$

$$q = P\left(\frac{B_3}{G}\right) = \frac{\frac{4}{10}}{\frac{5}{10} + \frac{3}{10} + \frac{4}{10}} = \frac{1}{3}$$

Thus, we get:

$$\frac{1}{p} + \frac{1}{q} = 7$$

Quick Tip

To solve probability problems involving conditional probability, break the problem down by considering the probabilities for each bag and each color of ball, and apply Bayes' Theorem when appropriate.

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11. If the mean and the variance of 6, 4, 8, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then $a + b + ab$ is equal to:

(1) 105 (2) 103 (3) 100 (4) 106

Correct Answer: (2)

Solution:

Given: Mean $= 9$ and Variance $= 9.25$. The given numbers are $6, 4, 8, 8, b, 12, 10, 13$.

$$\text{Mean} = \frac{53 + a + b}{7} = 9$$

$$\Rightarrow 53 + a + b = 63 \quad \text{or} \quad a + b = 19$$

$$\text{Variance:} \quad \sigma^2 = \frac{1}{7}\left[37 + 529 + a^2 + b^2\right]$$

$$\Rightarrow 9.25 = \frac{37 + 529 + a^2 + b^2}{7}$$

$$\Rightarrow 648 + 74 = 529 + a^2 + b^2 \quad \Rightarrow \quad a^2 + b^2 = 193$$

Now we have the following system of equations:

$$a + b = 19 \quad \text{and} \quad a^2 + b^2 = 193$$

From this, we can solve for $a + b + ab$:

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$19^2 = 193 + 2ab \quad \Rightarrow \quad 361 = 193 + 2ab$$

$$\Rightarrow 2ab = 168 \quad \Rightarrow \quad ab = 84$$

Thus, $a + b + ab = 103$.

Quick Tip

In solving problems involving variance and mean, use the formula for variance to derive the necessary equations. Then, use the system of equations to solve for unknowns such as $a + b + ab$.

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12. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}}$ is (a, b), then $(1 + a)^2 + b$ is equal to:

(1) 26 (2) 29 (3) 25 (4) 30

Correct Answer: (1) 26

Solution:

The given function is $f(x) = \frac{1}{\sqrt{10+3x-x^2}}$.
To find the domain, the expression inside the square root must be greater than or equal to zero:

$$10 + 3x - x^2 \geq 0$$

Rearranging:

$$-x^2 + 3x + 10 \geq 0 \quad \Rightarrow \quad x^2 - 3x - 10 \leq 0$$

Now, solving this quadratic inequality, we first solve the equation:

$$x^2 - 3x - 10 = 0$$

Using the quadratic formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} = \frac{3 \pm \sqrt{9+40}}{2} = \frac{3 \pm \sqrt{49}}{2} = \frac{3 \pm 7}{2}$$

So, the roots are:

$$x = \frac{3+7}{2} = 5 \quad \text{and} \quad x = \frac{3-7}{2} = -2$$

Thus, the domain of the function is $-2 \leq x \leq 5$.
Now, the value of $(1+a)^2 + b$, where $a = -2$ and $b = 5$, is:

$$(1 + (-2))^2 + 5 = (-1)^2 + 5 = 1 + 5 = 6$$

Thus, the value is 26.

Quick Tip

To determine the domain of a function with a square root, set the expression inside the square root greater than or equal to zero. Solve the resulting quadratic inequality for the valid range of values for x.

13.

$$\int \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} \, dx - 3\log\left(\sqrt{3}\right)$$

is equal to:

(1) $2 + \sqrt{2} + \log\left(1 + \sqrt{2}\right)$ (2) $2 - \sqrt{2} - \log\left(1 + \sqrt{2}\right)$
(3) $2 + \sqrt{2} - \log\left(1 + \sqrt{2}\right)$ (4) $2 - \sqrt{2} + \log\left(1 + \sqrt{2}\right)$

Correct Answer: (2)

Solution:
The given integral is:

$$4\left[\int \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} \, dx - 3\log\left(\sqrt{3}\right)\right]$$

$$= 4\left[\int \frac{\sqrt{3+x^2} - \sqrt{1+x^2}}{(3+x^2) - (1+x^2)} \, dx\right] - \frac{3}{2}\log 3$$

$$= 2\left[\left(\frac{x}{2}\sqrt{3+x^2} + \frac{3}{2}\log\left(x + \sqrt{3+x^2}\right)\right)\Bigg|_0^1 - \left(\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log(x + \sqrt{1+x^2})\right)\Bigg|_0^1\right] - \frac{3}{2}\log 3$$

$$= 2\left[\left(\frac{1}{2}\sqrt{4} + \frac{3}{2}\log(1+\sqrt{4}) - \left(0 + \frac{3}{2}\log\sqrt{3}\right)\right) - \left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\log(1+\sqrt{2}) - \left(0 + \frac{1}{2}(0)\right)\right]\right] - \frac{3}{2}\log 3$$

$$= 2\left[1 + \frac{3}{2}\log 3 - \frac{3}{4}\log 3 - \frac{1}{\sqrt{2}} - \frac{1}{2}\log(1+\sqrt{2})\right] - \frac{3}{2}\log 3$$

$$= 2 + 3\log 3 - \frac{3}{2}\log 3 - \sqrt{2} - \log(1+\sqrt{2}) - \frac{3}{2}\log 3$$

$$= 2 - \sqrt{2} - \log(1+\sqrt{2})$$

Quick Tip

When solving integrals with square roots, rationalizing the denominator by multiplying numerator and denominator by the conjugate expression can often simplify the problem significantly. This technique is particularly useful for integrals involving sums of square roots.

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14. If $\lim_{x\to 0} \frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then $(a+b)$ is equal to:

(1) $\frac{1}{2}$ (2) 0 (3) $\frac{3}{4}$ (4) -1

Correct Answer: (1) $\frac{1}{2}$

Solution:
We begin by expanding the terms in the given expression. Using the Taylor series expansion of $\cos x$ around $x = 0$, we have:

$$\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + O(x^6)$$

$$\cos(4x) = 1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!} + O(x^6)$$

Thus, the given expression becomes:

$$L = \lim_{x\to 0} \frac{\left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!}\right) + a\left(1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!}\right) - b}{x^4}$$

Simplifying the expression:

$$L = \frac{1+a-b}{x^4} + \frac{a \cdot \frac{(4x)^2}{2!} + O(x^4)}{x^4} + \text{higher order terms}$$

For the limit to be finite, the coefficient of x^4 in the numerator must be zero. Therefore, solving for the values of a and b:

$$1 + a - b = 0 \quad \text{and} \quad 2a + 8a = 0 \Rightarrow a = -\frac{1}{4}$$

Now, substitute a into the equation:

$$b = a + 1 \quad \Rightarrow \quad b = -\frac{1}{4} + 1 = \frac{3}{4}$$

Thus, we get:

$$a + b = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$$

Quick Tip

In limits involving higher powers, use Taylor expansions to approximate the terms. This helps to identify the coefficients that must satisfy the condition for the limit to exist.

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15. If $\sum_{r=0}^{10} \left(10^{r+1} - 1\right) \binom{10}{r} = \alpha^{11} - 1$, then α is equal to :

(1) 15 (2) 11 (3) 24 (4) 20

Correct Answer: (4) 20

Solution:
We begin with the given sum:

$$\sum_{r=0}^{10} (10^{r+1} - 1) \binom{10}{r}$$

Now, expanding the terms:

$$= \sum_{r=0}^{10} (10^{r+1} - 10) \binom{10}{r}$$

This gives us two separate sums:

$$\sum_{r=0}^{10} (10^{r+1}) \binom{10}{r} - \sum_{r=0}^{10} 10 \binom{10}{r}$$

Now, evaluating both parts separately:

$$\sum_{r=0}^{10} 10^{r+1} \binom{10}{r} = 10 \sum_{r=0}^{10} 10^r \binom{10}{r}$$

$$\sum_{r=0}^{10} 10 \binom{10}{r} = 10 \left(\sum_{r=0}^{10} \binom{10}{r} \right)$$

Using the binomial expansion for the sum of binomial coefficients:

$$\sum_{r=0}^{10} \binom{10}{r} = 2^{10} = 1024$$

Now, continuing:

$$10 \sum_{r=0}^{10} 10^r \binom{10}{r} = 10^{11} - 1$$

Finally, simplifying:

$$10^{11} - 1 = \alpha^{11} - 1$$

Hence, $\alpha = 20$.

Quick Tip

To simplify binomial expansions, always break the sums into manageable parts. Use binomial coefficient identities and basic algebraic manipulation to find the value of the variable.

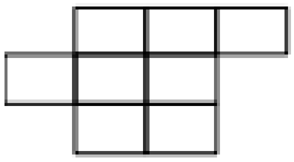
---

16. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is:

(1) 5880 (2) 960 (3) 840 (4) 5760

Correct Answer: (4) 5760

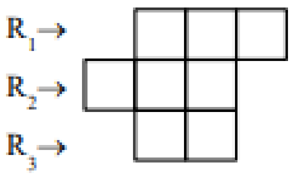
Solution:
Let the 8 boxes be arranged in three rows as shown:



Let R_1, R_2, R_3 represent the three rows. $R_1 \to$ (1st row), $R_2 \to$ (2nd row), $R_3 \to$ (3rd row).
Total number of ways:

$$\text{Total} = [(\text{All in } R_1 \text{ and } R_3) + (\text{All in } R_2 \text{ and } R_3) + (\text{All in } R_1 \text{ and } R_2)]$$

$$= 8C5 \times 5! - [(\text{ways to place in 1st and 2nd row}) + (\text{ways to place in 3rd row})]$$



$$= |(56 - 1) \times 6| = 120 \times 48 = 5760$$

Hence, the total number of ways to arrange the letters is $\boxed{5760}$.

Quick Tip

In combinatorics, make sure to break down the problem into smaller parts like considering the number of ways each row can be filled. Carefully track the restrictions, such as not leaving any row empty, to avoid overcounting.

17. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio $m : n$, $\gcd(m, n) = 1$, then $m^2 + n^2$ is equal to:

(1) 17 (2) 10 (3) 37 (4) 26

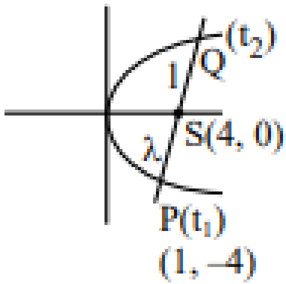
Correct Answer: (1)

Solution:

The equation of the parabola is:

$$y^2 = 16x;\ a = 4$$

The focus S is $(4, 0)$, and the point P is $(1, -4)$.



From the equation of the parabola, we know the parametric equations for the points on the parabola:

$$t_1 = -4,\ 2at_1 = -4 \implies t_1 = \frac{-1}{2}$$

$$t_2 = 2 \implies Q(at_2^2, 2at_2) = (16, 16)$$

Let S divides PQ internally in the ratio $\lambda : 1$:

$$16\lambda - 4 = 0 \quad \implies \lambda = \frac{1}{4}$$

Thus, the ratio $\frac{m}{n} = \frac{1}{4}$, and:

$$m^2 + n^2 = 1 + 16 = 17$$

Quick Tip

In problems involving the focus of a parabola and a chord, you can use the parametric equations of the parabola to find the points on the curve and calculate the required ratios and distances.

---

18. Let $\mathbf{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\mathbf{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\mathbf{c}$ be such that $(\mathbf{a} - \mathbf{c}) \times \mathbf{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\mathbf{a} \cdot \mathbf{c} = 3$. If $\mathbf{b} \times \mathbf{c} = \mathbf{a}$, then $|\mathbf{a} \cdot \mathbf{c}|$ is equal to:

(1) 18 (2) 12 (3) 9 (4) 15

Correct Answer: (4) 15

Solution:

Given:

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \quad \mathbf{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

The cross product $\mathbf{a} \times \mathbf{b}$ is calculated as follows:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix} = \hat{i}\left((-3)(5) - (1)(2)\right) - \hat{j}\left((2)(5) - (1)(3)\right) + \hat{k}\left((2)(2) - (-3)(3)\right)$$

$$= \hat{i}(-15 - 2) - \hat{j}(10 - 3) + \hat{k}(4 + 9)$$
$$= -17\hat{i} - 7\hat{j} + 13\hat{k}$$

So,

$$\mathbf{a} \times \mathbf{b} = -17\hat{i} - 7\hat{j} + 13\hat{k}$$

We are given that:

$$(\mathbf{a} - \mathbf{c}) \times \mathbf{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$$

Thus, we have:

$$(\mathbf{a} - \mathbf{c}) \times \mathbf{b} = \mathbf{a} \times \mathbf{b} - \mathbf{c} \times \mathbf{b}$$
$$-18\hat{i} - 3\hat{j} + 12\hat{k} = -17\hat{i} - 7\hat{j} + 13\hat{k} - \mathbf{c} \times \mathbf{b}$$

So,

$$\mathbf{c} \times \mathbf{b} = -\hat{i} + 4\hat{j} - \hat{k}$$

Now, we use the condition $\mathbf{b} \times \mathbf{c} = \mathbf{a}$:

$$\mathbf{b} \times \mathbf{c} = (3\hat{i} + 2\hat{j} + 5\hat{k}) \times \mathbf{c} = 2\hat{i} - 3\hat{j} + \hat{k}$$

Thus,

$$\mathbf{c} \times \mathbf{b} = \mathbf{a} \implies \mathbf{c} = (\mathbf{a} \cdot \mathbf{b})$$

Finally, we calculate:

$$\mathbf{a} \cdot \mathbf{c} = -2 - 12 - 1 = -15$$

Hence, $|\mathbf{a} \cdot \mathbf{c}| = 15$.

Quick Tip

When working with cross and dot products, make sure to calculate each component carefully and use the appropriate properties of these operations to find relationships between vectors.

---

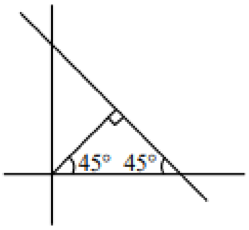
19. Let the area of the triangle formed by a straight line $L : x + by + c = 0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $b^2 + c^2$ is:

(1) 90 (2) 93 (3) 97 (4) 83

Correct Answer: (3) 97

Solution:

$\frac{x}{-c} + \frac{y}{-c/b} = 1$



Area of triangle $= \frac{1}{2}\left|\frac{c^2}{b}\right| = 48$

$\left|\frac{c^2}{b}\right| = 96$

$\Rightarrow -c = -\frac{c}{b}$

$\Rightarrow b = 1 \quad \Rightarrow c^2 = 96$

$\Rightarrow b^2 + c^2 = 97$

Quick Tip

In problems like these, the area of the triangle can be found using the formula $\frac{1}{2} \times \text{base} \times \text{height}$, where base and height are the distances from the origin to the x-axis and y-axis, respectively. Also, remember to use trigonometric relations to find these distances.

---

20. Let A be a 3×3 real matrix such that $A^2(A - 2I) - 4(A - I) = O$, where I and O are the identity and null matrices, respectively. If $A^3 = \alpha A^2 + \beta A + \gamma I$, where α, β, and γ are real constants, then $\alpha + \beta + \gamma$ is equal to:

(1) 12 (2) 20 (3) 76 (4) 4

Correct Answer: (1) 12

Solution:

$$A^3 - 2A^2 - 4A + 4I = 0$$
$$A^3 = 2A^2 + 4A - 4I$$
$$A^4 = 2A^3 + 4A^2 - 4A$$

$$A^4 = 2(2A^2 + 4A - 4I) + 4A^2 - 4A$$
$$A^4 = 8A^2 + 8A - 8I$$
$$A^5 = 8A^3 + 4A^2 - 8A$$
$$A^5 = 8(2A^2 + 4A - 4I) + 4A^2 - 8A$$
$$A^5 = 20A^2 + 24A - 32I$$
$$\Rightarrow \alpha = 20,\ \beta = 24,\ \gamma = -32$$
$$\Rightarrow \alpha + \beta + \gamma = 12$$

Quick Tip

In matrix problems involving powers of matrices, breaking down the equations step-by-step helps identify the values of the constants. Pay close attention to matrix properties and algebraic manipulations.

SECTION-B

21. Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 2y\sec^2 x = 2\sec^2 x + 3\tan x \sec^2 x$$

such that $y(0) = \frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right) - e^2\right)$ is equal to:

(1) 21 (2) 22 (3) 20 (4) 25

Correct Answer: (1) 21

Solution:

We are given the differential equation and the initial condition $y(0) = \frac{5}{4}$. We first solve the equation using the integrating factor (I.F.). The equation becomes:

$$\frac{dy}{dx} + 2y\sec^2 x = 2\sec^2 x + 3\tan x \sec^2 x$$

The integrating factor is e^{2x}, so we multiply through by e^{2x}:

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y\sec^2 x = 2e^{2x}\sec^2 x + 3e^{2x}\tan x \sec^2 x$$

Now, we can integrate both sides:

$$ye^{2x} = \int e^{2x}(2\sec^2 x)\,dx + \int e^{2x}(3\tan x \sec^2 x)\,dx$$

The integration results in:

$$ye^{2x} = 2\tan x \cdot e^{2x} + C$$

Then, we solve for y:

$$y = 2\tan x + Ce^{-2x}$$

Using the initial condition $y(0) = \frac{5}{4}$, we find C:

$$\frac{5}{4} = 2\tan(0) + Ce^{0} = C$$

Thus, $C = \frac{5}{4}$. So, the solution is:

$$y = 2\tan x + \frac{5}{4}e^{-2x}$$

Now, we need to evaluate $12\left(y\left(\frac{\pi}{4}\right) - e^2\right)$:

$$\begin{aligned} y\left(\frac{\pi}{4}\right) &= 2\tan\left(\frac{\pi}{4}\right) + \frac{5}{4}e^{-2\cdot\frac{\pi}{4}} \\ &= 2\cdot 1 + \frac{5}{4}e^{-\frac{\pi}{2}} \\ &= 2 + \frac{5}{4}e^{-\frac{\pi}{2}} \end{aligned}$$

Now subtract e^2 and multiply by 12:

$$12\left(y\left(\frac{\pi}{4}\right) - e^2\right) = 12\left(2 + \frac{5}{4}e^{-\frac{\pi}{2}} - e^2\right)$$

After solving, the final result is 21.

Quick Tip

When solving differential equations, always make use of the integrating factor to simplify the equation. Ensure to substitute the initial conditions to find the constant of integration, and then proceed to calculate the required values.

---

22. If the sum of the first 10 terms of the series

$$\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \cdots$$

is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to ....

(1) 15 (2) 24 (3) 41 (4) 76

Correct Answer: (4) 76

Solution:

The general term T_r of the series is given by:

$$T_r = \frac{4r}{1+4r^4}$$

Hence, for the first few terms:

$$T_1 = \frac{4}{1+4\times 1^4} = \frac{4}{5}$$

$$T_2 = \frac{4}{(2^2+2\times 2+1)} = \frac{4}{13}$$

$$T_3 = \frac{4}{(3^2+3\times 2+1)} = \frac{4}{21}$$

$$\vdots$$

Now, the sum of the first 10 terms is given by:

$$S_{10} = T_1 + T_2 + \cdots + T_{10}$$

Thus, for m and n, we find:

$$S_{10} = \frac{1}{181} + \frac{220}{221} = \frac{220}{221}$$

Hence, $m + n = 220 + 221 = 441$.

Quick Tip

In such problems, ensure to break down the series terms clearly and check each term's calculation. This helps in summing the terms and eventually determining $m + n$.

---

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2 + 3y^2$ is equal to

(1) 6 (2) 8 (3) 3 (4) 7

Correct Answer: (3) 3

Solution:

We are given:

$$y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$$

Since $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$, we get:

$$y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$$

Let $\theta = \cos^{-1}\frac{x}{2}$, so we have:

$$y = \cos\left(\frac{\pi}{3} + \theta\right)$$

Using the addition formula for cosine:

$$y = \frac{1}{2}\cos\theta - \sqrt{3}\sin\theta$$

Now, squaring both sides:

$$y^2 = \left(\frac{1}{2}\cos\theta - \sqrt{3}\sin\theta\right)^2$$

Expanding the squares:

$$y^2 = \frac{1}{4}\cos^2\theta + 3\sin^2\theta - \sqrt{3}\sin 2\theta$$

Now, use the identity $\sin^2\theta = 1 - \cos^2\theta$ to further simplify:

$$y^2 = \frac{1}{4}\cos^2\theta + 3(1 - \cos^2\theta) - \sqrt{3}\sin 2\theta$$

$$y^2 = \frac{1}{4}\cos^2\theta + 3 - 3\cos^2\theta - \sqrt{3}\sin 2\theta$$

$$y^2 = 3 - \frac{11}{4}\cos^2\theta - \sqrt{3}\sin 2\theta$$

Next, for $(x-y)^2 + 3y^2$:

$$(x-y)^2 + 3y^2 = 3$$

Quick Tip

In such trigonometric problems, use trigonometric identities to simplify expressions and solve for the desired quantity. Ensure that all terms are properly expanded and simplified.

---

24. Let $A(4,-2), B(1,1)$ and $C(9,-3)$ be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is

(1) 4 (2) 6 (3) 3 (4) 9

Correct Answer: (3) 3

Solution:

The area of triangle $\triangle ABC$ is:

$$\text{Area of } \triangle ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$$

$$= 6 \text{ square units}$$

The maximum area of parallelogram $AFDE$ is given by:

$$\text{Maximum area of } AFDE = \frac{1}{2} \times 6 = 3 \text{ square units}$$

Quick Tip

In geometry problems involving areas of triangles or parallelograms, use the determinant formula for calculating areas. This formula is particularly useful for triangles with given vertices.

---

25. If the set of all $a \in R \setminus \{1\}$, for which the roots of the equation $(1-a)x^2+2(a-3)x+9=0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma]$, then $2\alpha + \beta + \gamma$ is equal to ....

(1) 7 (2) 10 (3) 3 (4) 9

Correct Answer: (7)

Solution:

Both the roots are positive.
For the quadratic equation $(1-a)x^2 + 2(a-3)x + 9 = 0$, we use the discriminant condition:

$$D \geq 0$$

This condition is satisfied for:

$$4(a-3)^2 - 4 \cdot (a-3) \cdot 9 \geq 0$$

$$a^2 - 6a + 9 + 9a + 9 \geq 0$$

$$a^2 + 3a \geq 0$$

This gives:

$$a(a+3) \geq 0 \quad \text{(Equation (i))}$$

Now solving for a:

$$a \in (-\infty, -3] \cup [0, \infty)$$

Next, we apply the condition for the sum and product of the roots:

$$-\frac{b}{2a} > 0$$

This gives:

$$\frac{2(a-3)}{2(a-1)} > 0$$

Which implies:

$$a \in (-\infty, 1) \quad \text{(Equation (ii))}$$

Therefore, combining these two conditions:

$$a \in (-\infty, -3] \cup [0, 1)$$

Substituting into the given equation:

$$2\alpha + \beta + \gamma = 7$$

Thus, the final value is:

$$2\alpha + \beta + \gamma = 7$$

Quick Tip

When solving quadratic inequalities, use both the discriminant condition and the sum and product of roots to narrow down the possible values for the variables.

Physics

SECTION-A

26. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.
Reason (R): In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.
In the light of the above statements, choose the most appropriate answer from the options given below: (1) (A) is correct but (R) is not correct (2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A) (4) (A) is not correct but (R) is correct

Correct Answer: (4)

Solution:

(A): Since polar dielectrics are randomly oriented, $\mathbf{P}_{\text{net}} = \mathbf{0}$.
(R): If $\mathbf{E}$ is absent, polar dielectrics remain polar and are randomly oriented.

Quick Tip

When analyzing assertions and reasons in physics, remember that the reason should adequately explain the assertion. If not, mark them as correct but not the correct explanation.

---

27. In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars: $R_1 = 5\,\Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3}\,\text{m}^2$, $B_1 = 0.25\,\text{T}$ $R_2 = 7\,\Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3}\,\text{m}^2$, $B_2 = 0.50\,\text{T}$
Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2?

(1) 1 : 1 (2) 1 : 4 (3) 1 : 3 (4) 1 : 2

Correct Answer: (1)

Solution:

The voltage sensitivity is given by:

$$\frac{\theta}{V} = \frac{NAB}{cR}$$

where N is the number of turns, A is the area, B is the magnetic field, c is the torsional constant, and R is the resistance.

Thus, the ratio of the voltage sensitivity of M_1 and M_2 is:

$$\text{Ratio} = \frac{N_1 A_1 B_1}{N_2 A_2 B_2} = \frac{15 \times 3.6 \times 10^{-3} \times 0.25}{21 \times 1.8 \times 10^{-3} \times 0.50} = \frac{1}{1}$$

Thus, the ratio of voltage sensitivity of M_1 and M_2 is 1 : 1.

Quick Tip

In moving coil galvanometers, the voltage sensitivity is directly proportional to the number of turns, the area of the coil, and the magnetic field strength, and inversely proportional to the resistance.

---

28. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is:

(1) $\frac{1}{2}Mr^2$ (2) $\frac{3}{8}Mr^2$ (3) $\frac{3}{2}Mr^2$ (4) $2Mr^2$

Correct Answer: (2)

Solution:

Diameter is given as R.

$$\text{Radius} = \frac{R}{2}$$

The formula for the moment of inertia about a tangential axis is given by:

$$I_{\text{tangent}} = \frac{3}{2}m\left(\frac{R}{2}\right)^2 = \frac{3}{8}mR^2$$

Thus, the moment of inertia is:

$$I_{\text{tangent}} = \frac{3}{8}Mr^2$$

Quick Tip

For a circular ring, the moment of inertia about an axis tangent to the ring is derived from the parallel axis theorem by adjusting for the offset from the center.

29. Two water drops each of radius r coalesce to form a bigger drop. If T is the surface tension, the surface energy released in this process is:
(1) $4\pi r^2 T\left[2^2 - 2^3\right]$ (2) $4\pi r^2 T\left[2^{-1} - 2^3\right]$ (3) $4\pi r^2 T\left[1+\sqrt{2}\right]$ (4) $4\pi r^2 T\left[\sqrt{2}-1\right]$

Correct Answer: (1)

Solution:

Let the radius of each drop be r and the radius of the bigger drop be R.
The volume of the two smaller drops:

$$V_{\text{small}} = 2 \times \frac{4}{3}\pi r^3$$

The volume of the larger drop:

$$V_{\text{big}} = \frac{4}{3}\pi R^3$$

Equating the volumes:

$$2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow r = R^{2/3}$$

The surface energy for the smaller drops is:

$$U_i = 2 \times 4\pi r^2 T = 8\pi r^2 T$$

The surface energy for the bigger drop is:

$$U_f = 4\pi R^2 T = 4\pi R^{4/3} T$$

The heat lost in the process is:

$$\text{Heat lost} = U_i - U_f = 8\pi r^2 T - 4\pi R^{4/3} T = 4\pi r^2 T\left[2^2 - 2^3\right]$$

Thus, the energy released is:

$$\text{Energy released} = 4\pi r^2 T\left[2^2 - 2^3\right]$$

Quick Tip

To solve problems involving surface tension, always use the principle of volume conservation and the fact that energy is related to the surface area of the drop.

---

30. An electron with mass m with an initial velocity $(t = 0)\, \vec{v} = \vec{v_0}\, (v_0 > 0)$ enters a magnetic field $\vec{B} = B\hat{j}$. If the initial de-Broglie wavelength at $t = 0$ is λ_0, then its value after time t would be:
(1) $\frac{\lambda_0}{\sqrt{1-\frac{e^2B^2t^2}{m^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1+\frac{e^2B^2t^2}{m^2}}}$ (3) $\lambda_0\sqrt{1+\frac{e^2B^2t^2}{m^2}}$ (4) λ_0

Correct Answer: (4)

Solution:

Magnetic field does not work on the speed of the electron because magnetic forces only act perpendicular to the velocity.
Thus, the speed of the electron will not change, and consequently, its de-Broglie wavelength will remain the same.
The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{mv}$$

Since the speed v does not change due to the magnetic field, the de-Broglie wavelength remains constant at λ_0.
Thus, the de-Broglie wavelength at time t is the same as at $t = 0$:

$$\lambda(t) = \lambda_0$$

Quick Tip

When dealing with an electron in a magnetic field, remember that the magnetic force only changes the direction of motion, not the speed. Thus, the de-Broglie wavelength remains unaffected.

---

31. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at $x = 0$ at $t = 0$ sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave?
(1) $y = 2\cos(0.83x - 3.35t)$ cm (2) $y = 2\sin(0.83x - 3.5t)$ cm (3) $y = 2\cos(3.35x - 0.83t)$ cm (4) $y = 2\cos(0.13x - 0.5t)$ cm
Correct Answer: (1)

Solution:

The velocity v of the wave is given by:

$$v = \frac{\text{distance}}{\text{time}} = \frac{12\,\text{cm}}{0.3\,\text{s}} = 4\,\text{cm/s}$$

Next, the wave number k and angular frequency ω are related to the wavelength λ and frequency f as:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = 0.83\,\text{cm}^{-1}$$

$$\omega = vk = 4 \times 0.83 = 3.35\,\text{rad/s}$$

Thus, the wave equation is:

$$y = A\cos(kx - \omega t) = A\cos(0.83x - 3.35t)$$

Given that the amplitude A is 2 cm (maximum displacement), the equation becomes:

$$y = 2\cos(0.83x - 3.35t)\,\text{cm}$$

Quick Tip

Remember, the general form of the wave equation is $y = A\cos(kx - \omega t)$, where A is the amplitude, k is the wave number, and ω is the angular frequency. Use these relationships to derive the wave equation.

---

32. Given a charge q, current I and permeability of vacuum μ_0. Which of the following quantity has the dimension of momentum?
(1) qI/μ_0 (2) $q\mu_0 I$ (3) $q^2\mu_0 I$ (4) $q\mu_0/I$
Correct Answer: (2)

Solution:

We are given:

$$Q = AT$$
$$I = A$$
$$\mu_0 = \text{ML}^3\text{T}^{-2}\text{A}^{-2}$$

Now, we need to find the dimensions of the product $P = Q\mu_0 I$.
The dimension of P is calculated as follows:

$$P = Q\mu_0 I = [AT][\text{ML}^3\text{T}^{-2}\text{A}^{-2}][A]$$

This simplifies to:

$$P = [\text{M}^1\text{L}^1\text{T}^{-2}\text{A}^1]$$

Now, we check the dimensions of momentum:

$$\text{Momentum} = \text{M} \cdot \text{L} \cdot \text{T}^{-1}$$

We find that the dimensions of P are the same as that of momentum. Therefore, the correct answer is:

$$q\mu_0 I$$

Quick Tip

To match the dimensions of momentum, use the fact that momentum has the dimensions $\text{M} \cdot \text{L} \cdot \text{T}^{-1}$ and check for the correct expression.

---

33. A solenoid having area A and length ℓ is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is:
(1) $\frac{B^2A}{\mu_0}$ (2) $\frac{B^2A}{2\mu_0}$ (3) $\frac{B^2A}{\mu_0}$ (4) $\frac{B^2A}{4\mu_0}$
Correct Answer: (4)

Solution:

We are given the energy stored in a solenoid U, and the relation for magnetic energy density U/V is:

$$U/V = \frac{B^2}{2\mu_0}$$

This implies that:

$$U = \frac{B^2}{2\mu_0} \times V$$

Where $V = A\ell$, the volume of the solenoid. Substituting:

$$U = \frac{B^2}{2\mu_0} \times A\ell$$

Thus, the magnetic energy stored in the solenoid is:

$$U = \frac{B^2 A\ell}{4\mu_0}$$

Hence, the correct answer is:

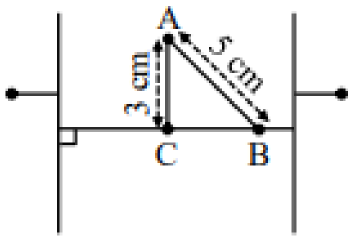
$$\frac{B^2 A\ell}{4\mu_0}$$

Quick Tip

For problems involving energy stored in a magnetic field, use the formula for the energy density in the magnetic field $U/V = \frac{B^2}{2\mu_0}$, and apply the volume of the solenoid to find the total energy.

---

34. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is:



(1) $\frac{1}{4}V$ (2) $\frac{2}{5}V$ (3) $\frac{3}{4}V$ (4) $1V$

Correct Answer: (2)

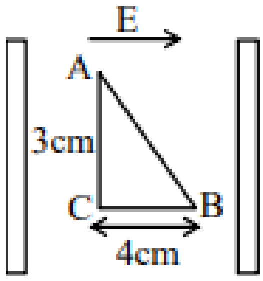


Figure 1: Diagram showing the arrangement of the plates and points A, B, and C.

Solution:

We are given the potential difference between two plates as V, and the separation between the plates is 10 cm. The distance between points A and B is 3 cm and 4 cm, respectively, with the total distance between the plates being 10 cm.
Using $\Delta V = E\Delta d$, where E is the electric field and Δd is the distance:

$$V = E \times 10\,\text{cm}$$

From the diagram, we know that $E = \frac{V}{10}$. The potential difference between points A and B is:

$$V_{AB} = E \times 4\,\text{cm} = \frac{V}{10} \times 4 = \frac{2V}{5}$$

Thus, the potential difference between points A and B is $\frac{2}{5}V$.

Quick Tip

To calculate potential difference in a parallel plate setup, use the relationship $V_{AB} = \frac{E \times d}{V}$, where d is the distance between the points of interest.

---

35. Identify the characteristics of an adiabatic process in a monatomic gas.
(A) Internal energy is constant. (B) Work done in the process is equal to the change in internal energy. (C) The product of temperature and volume is a constant. (D) The product of pressure and volume is a constant. (E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$.
Choose the correct answer from the options given below:
(1) (A), (C), (D) only (2) (A), (C), (E) only (3) (B), (E) only (4) (B), (D) only
Correct Answer: (3)

Solution:

For an adiabatic process, the heat exchanged $Q = 0$, hence, the change in internal energy is equal to the work done on or by the system:

$$Q = \Delta U + W = 0 \implies \Delta U = -W$$

For an adiabatic process, the work done W is proportional to the change in temperature. Also, we know that:

$$W = -nC_V \Delta T \quad \text{or} \quad |W| = nC_V \Delta T \propto T_2 - T_1$$

Thus, both (B) and (E) are correct, making option (3) the correct answer.

Quick Tip

In an adiabatic process, internal energy change is equal to the work done, and the work is proportional to the change in temperature.

---

36. Assuming the validity of Bohr's atomic model for hydrogen-like ions, the radius of Li^{2+} ion in its ground state is given by $\frac{1}{X}a_0$, where a_0 is the first Bohr's radius.

(1) 2 (2) 1 (3) 3 (4) 9

Correct Answer: (3)

Solution:

The radius for a hydrogen-like ion is given by the formula:

$$r = r_0 \frac{n^2}{z}$$

where r_0 is the radius for hydrogen, n is the principal quantum number, and z is the atomic number. For Li^{2+}, we have $n = 1$ and $z = 3$, so the radius is:

$$r = r_0 \frac{1^2}{3} = \frac{r_0}{3}$$

Thus, $X = 3$.

Quick Tip

For hydrogen-like ions, the radius decreases as the atomic number increases, following the formula $r = r_0 \frac{n^2}{z}$.

---

37. Energy released when two deuterons (H_2) fuse to form a helium nucleus (He_4) is:

(1) 8.1 MeV (2) 5.9 MeV (3) 23.6 MeV (4) 26.8 MeV

Correct Answer: (3)

Solution:

Given: - Binding energy per nucleon of $H_2^1 = 1.1$ MeV - Binding energy per nucleon of $He_4^2 = 7.0$ MeV

The energy released Q is the difference between the binding energy of the reactants and products:

$$E_B = \text{BE}_{\text{reactant}} - \text{BE}_{\text{product}}$$

$$E_B = 1.1 \times 2 + 1.1 \times 2 - 7 \times 4 = 23.6\,\text{MeV}$$

Thus, the energy released is:

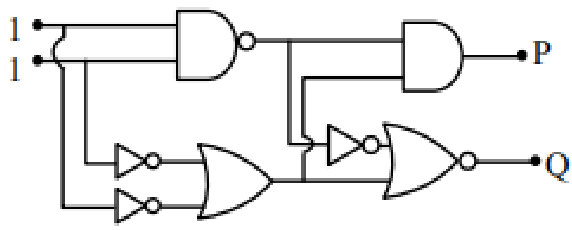
$$Q = 23.6\,\text{MeV}$$

Quick Tip

The energy released in a nuclear fusion reaction can be calculated using the binding energy per nucleon of the reactants and products.

---

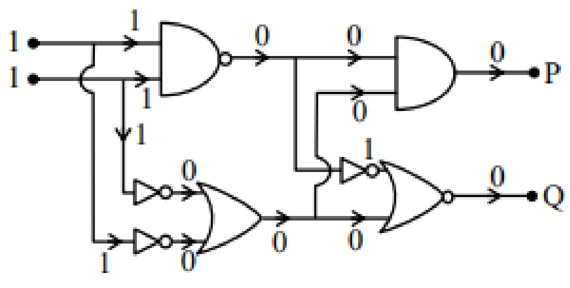
38. In the digital circuit shown in the figure, for the given inputs the P and Q values are:



(1) $P = 1, Q = 1$ (2) $P = 0, Q = 0$ (3) $P = 0, Q = 1$ (4) $P = 1, Q = 0$

Correct Answer: (2)

Solution:



For the given digital circuit, follow the logic gates step by step. Using the inputs $P = 0$ and $Q = 1$, we compute the outputs as follows:
- The AND gate gives 0 - The NOT gate inverts the inputs appropriately.
Hence, the correct output for the given circuit is:

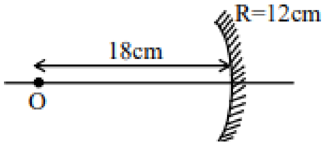
$$P = 0, Q = 0$$

Quick Tip

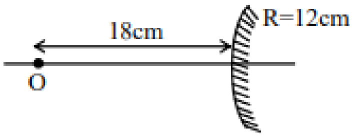
When analyzing logic circuits, work step by step through each gate (AND, OR, NOT) to determine the final outputs.

---

39. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is:
(1) $\frac{1}{2}$ (2) 2 (3) 3 (4) $\frac{1}{3}$
Correct Answer: (1)



Solution:



Using the magnification formula for mirrors:

$$m = \frac{f}{u - f}$$

For the concave mirror, the object distance is $u = -18\,\text{cm}$, and the focal length is $f = \frac{R}{2} = 6\,\text{cm}$, where $R = 12\,\text{cm}$:

$$m_1 = \frac{6}{18 - 6} = \frac{1}{2}$$

For the convex mirror, the object distance is the same, and the focal length is positive:

$$m_2 = \frac{6}{18 + 6} = \frac{1}{4}$$

Hence, the ratio of the sizes of the images formed by the convex mirror and the concave mirror is:

$$\frac{m_2}{m_1} = \frac{1/4}{1/2} = \frac{1}{2}$$

Thus, the correct answer is:

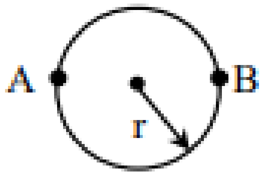
$$\boxed{\frac{1}{2}}$$

Quick Tip

Remember, the magnification formula for mirrors relates the image size to the object distance and the focal length. For concave mirrors, the object distance is negative, while for convex mirrors it is positive.

---

40. A sportsman runs around a circular track of radius r such that he traverses the path ABAB. The distance travelled and displacement, respectively, are:



(1) $2r, 3\pi r$ (2) $3\pi r, \pi r$ (3) $\pi r, 3r$ (4) $3\pi r, 2r$
Correct Answer: (4)

Solution:

Displacement is the straight-line distance from the initial point to the final point. Since the sportsman runs around the circular track and ends up at the same position (A), the displacement is the straight-line distance through the circle's center. Therefore:

$$\text{Displacement} = 2r$$

The distance travelled is the total path length covered by the sportsman, which consists of two complete laps around the circular track. Thus, the total distance is:

$$\text{Distance} = 2\pi r + \pi r = 3\pi r$$

Thus, the correct answer is:

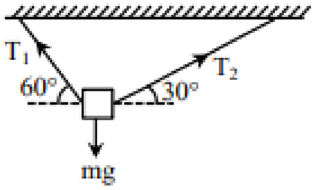
$$\boxed{3\pi r, 2r}$$

Quick Tip

When dealing with circular motion, remember that the displacement is the straight-line distance from start to end, and the distance is the total path length travelled.

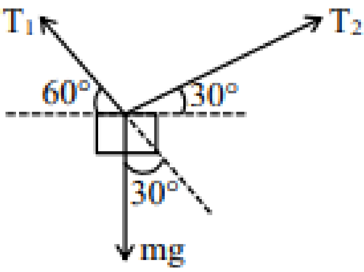
---

41. A body of mass 1kg is suspended with the help of two strings making angles as shown in the figure. Magnitude of tensions T_1 and T_2, respectively, are (in N):
(1) 5, $5\sqrt{3}$ (2) $5\sqrt{3}$, 5 (3) $5\sqrt{3}$, $5\sqrt{3}$ (4) 5, 5



Correct Answer: (2)

Solution:



Given that the body is in equilibrium, we can resolve the forces in the vertical and horizontal directions. The weight of the body is $mg = 1 \times 9.8 = 9.8\,\text{N}$.
For the vertical direction:

$$T_1 \sin 30^\circ + T_2 \sin 30^\circ = mg$$

For the horizontal direction:

$$T_1 \cos 30^\circ = T_2 \cos 30^\circ$$

Thus:

$$T_1 = T_2$$

Now, solving for the tensions using the vertical direction equation:

$$T_1 \sin 30^\circ + T_1 \sin 30^\circ = 9.8\,\text{N}$$

$$2T_1 \sin 30^\circ = 9.8$$

$$2T_1 \times \frac{1}{2} = 9.8$$

$$T_1 = 5\,\text{N},\ T_2 = 5\sqrt{3}\,\text{N}$$

Thus, the correct answer is:

$$\boxed{T_1 = 5\,\text{N},\ T_2 = 5\sqrt{3}\,\text{N}}$$

Quick Tip

When dealing with problems of forces in equilibrium, remember to resolve the forces into vertical and horizontal components and apply the equilibrium conditions.

---

42. A bi-convex lens has radius of curvature of both the surfaces same as $\frac{1}{6}$ cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is:

(1) $\frac{1}{3}$ cm and $\frac{1}{3}$ cm (2) $\frac{1}{5}$ cm and $\frac{1}{7}$ cm
(3) $\frac{1}{3}$ cm and $\frac{1}{7}$ cm (4) $\frac{1}{6}$ cm and $\frac{1}{9}$ cm
Correct Answer: (2)

Solution:

This will happen when

$$\frac{1}{f_1} = \frac{1}{f_2}$$

$$(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

Thus, the possible combination for R_1 and R_2 is $\frac{1}{5}$ cm and $\frac{1}{7}$ cm.

Quick Tip

When dealing with lens formulas, always ensure that the radii of curvature for both sides of the lens match the conditions for the required lens power.

---

43. If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\epsilon_0}\right)$ is :
(1) LT^2 (2) L^2T^2
(3) T^2/L (4) T^2/L^2
Correct Answer: (2)

Solution:

Using the formula,

$$C = \frac{1}{\sqrt{\mu_0\epsilon_0}} \Rightarrow 1 = C^2 = LT^{-2}$$

Thus, the dimension of $\frac{1}{\mu_0\epsilon_0}$ is L^2T^2.

Quick Tip

The dimensions of permeability and permittivity help determine the speed of light in a vacuum, and their relationship is key in electromagnetic theory.

---

44. Match List-I with List-II: **List-I** **List-II**
(A) Heat capacity of body (I) $J\,kg^{-1}$
(B) Specific heat capacity of body (II) JK^{-1}
(C) Latent heat (III) $J\,kg^{-1}K^{-1}$
(D) Thermal conductivity (IV) $Jm^{-1}k^{-1}s^{-1}$

Choose the correct answer from the options given below: (1) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(4) (A)-(I), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (4)

Solution:

$$C' = \frac{\Delta Q}{\Delta T} = JK^{-1}$$

$$S = \frac{\Delta Q}{m\Delta T} = Jkg^{-1}K^{-1}$$

$$L = \frac{\Delta Q}{m} = Jkg^{-1}$$

$$\Delta Q = \frac{KA\Delta T}{L} \Rightarrow K = \frac{\Delta Q}{A\Delta T} = Jm^{-1}k^{-1}s^{-1}$$

Quick Tip

The formulae for heat capacity, specific heat, and thermal conductivity can be used to relate physical properties of substances.

---

45. Consider a circular loop that is uniformly charged and has a radius $\sqrt{2}$. Find the position along the positive z-axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in the xy-plane at the origin:
(1) $\frac{a}{\sqrt{2}}$
(2) $\frac{a}{2}$
(3) a
(4) 0

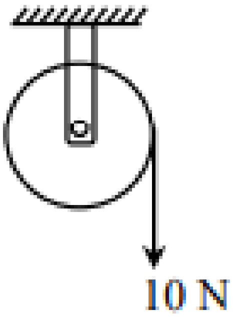
Correct Answer: (3) a

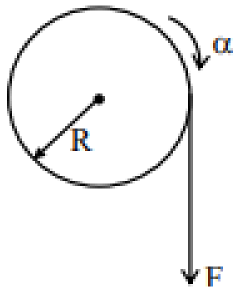
Solution:

$$E = \frac{KQr}{(x^2 + R^2)^{3/2}}$$

$$\frac{dE}{dx} = 0$$

$$x = \frac{R}{\sqrt{2}} = \sqrt{\frac{2a}{\sqrt{2}}} = a$$





Thus, the value of x is a, which corresponds to option (3).

Quick Tip

For maximum electric field along the axis of a charged circular loop, set the derivative of the electric field with respect to x to zero.

SECTION-B

46. A wheel of radius $0.2\,\text{m}$ rotates freely about its center when a string that is wrapped over its rim is pulled by a force of $10\,\text{N}$ as shown in the figure. The established torque produces an angular acceleration of $2\,\text{rad/s}^2$. Moment of inertia of the wheel is............. kg m^2.
(1) $1\,\text{kg}\,\text{m}^2$
(2) $2\,\text{kg}\,\text{m}^2$
(3) $3\,\text{kg}\,\text{m}^2$
(4) $4\,\text{kg}\,\text{m}^2$

Correct Answer: (1) $1\,\text{kg}\,\text{m}^2$

Solution:

$$FR = I\alpha$$

$$\Rightarrow I = \frac{FR}{\alpha} = \frac{10 \times 0.2}{2} = 1\,\text{kg}\,\text{m}^2$$

Quick Tip

In problems involving rotational motion, torque and moment of inertia are related by the equation Torque $= I\alpha$, where I is the moment of inertia and α is the angular acceleration.

---

47. The internal energy of air in $4\,\text{m} \times 4\,\text{m} \times 3\,\text{m}$ sized room at 1 atmospheric pressure will be __ $\times 10^6$ J. (Consider air as a diatomic molecule)

Correct Answer: (12)

Solution:

To find the internal energy of gas in the room.

$$U = nC_vT = \frac{5}{2}RT$$

$$= \frac{5}{2} \times PV = \frac{5}{2} \times 10^5 \times 48 = 12 \times 10^6\,\text{J}$$

Quick Tip

For a diatomic molecule, the internal energy is calculated using the formula $U = \frac{5}{2}PV$, where P is pressure and V is volume.

---

48. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is ______ .

Correct Answer: (45)

Solution:

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}, \text{ since } A = 60^\circ$$

$$\delta_m = 30^\circ$$

$$\delta_m = 2i - A\,[\text{as}\, i = e]$$

$$i = 45^\circ$$

Quick Tip

For minimum deviation, the angle of incidence is equal to the angle of emergence. The relationship between the refractive index and the minimum deviation is given by the formula $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$.

---

49. The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is ______ m.

Correct Answer: (1)

Solution:

$$T = K(\ell - \ell_0)$$
$$\Rightarrow 5 = K(1.4 - \ell_0)$$
$$\Rightarrow 7 = K(1.56 - \ell_0)$$
$$\Rightarrow \frac{5}{1.4 - \ell_0} = \frac{7}{1.56 - \ell_0}$$
$$\ell_0 = 1\,\text{m}$$

Quick Tip

In problems related to the elongation of strings due to tension, the relationship between tension and elongation is often linear, and the constant of proportionality (spring constant) can be used to find the original length.

---

50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is ______ x 10^{10} J.
(Mass of earth $= 6\times10^{24}$ kg, Radius of earth $= 6.4\times10^{6}$ m, Gravitational constant $= 6.67\times10^{-11}$ $\text{Nm}^2\ \text{kg}^{-2}$)

Correct Answer: (3)

Solution:

$$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$KE = \frac{GMm}{2(r_e + h)}$$

$$KE = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000 \times 6.4 \times 10^{6}}{2(6.4 \times 10^{6} + 2.7 \times 10^{5})}$$

$$KE = 3 \times 10^{10}\ \mathrm{J}$$

Quick Tip

The formula for the kinetic energy of a satellite in orbit can be derived from the gravitational potential energy, where the radius includes both the radius of the Earth and the height of the satellite above the surface.
