# KCET 2023 Mathematics Code D1 Question Paper with solutions

| Time Allowed :80 min | Maximum Marks :60 | Total Questions :60 |
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## MATHEMATICS

**1. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is:**

(A) $\frac{1-x^2}{1+x^2}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

**Correct Answer:** (B) $\frac{1}{2}$

**Solution:**

To solve for $\frac{du}{dv}$, we start by differentiating $u$ and $v$ with respect to $x$.

$$u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Differentiating $u$ with respect to $x$:

$$\frac{du}{dx} = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

Now, differentiate the function $\frac{2x}{1+x^2}$ with respect to $x$:

$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = \frac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)^2}$$

Thus,

$$\frac{du}{dx} = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{2(1-x^2)}{(1+x^2)^2}$$

Similarly, differentiate $v$:

$$v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\frac{dv}{dx} = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{d}{dx}\left(\frac{2x}{1-x^2}\right)$$

After differentiation and simplification, we find that $\frac{du}{dv} = \frac{1}{2}$.

> **Quick Tip**
>
> When solving for derivatives of inverse trigonometric functions, remember to use the chain rule and simplify the expressions step by step.

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**2. The function $f(x) = \cot x$ is discontinuous on every point of the set:**

(A) $x = (2n+1)\frac{\pi}{2}, n \in Z$

(B) $x = n\pi, n \in Z$

(C) $x = \frac{n\pi}{2}, n \in Z$

(D) $x = 2n\pi, n \in Z$

**Correct Answer:** (A) $x = (2n+1)\frac{\pi}{2}, n \in Z$

**Solution:**

The function $\cot x$ is discontinuous at points where the denominator of $\cot x = \frac{\cos x}{\sin x}$ is zero, which happens when $\sin x = 0$. The values of $x$ where $\sin x = 0$ are given by $x = n\pi, n \in Z$. Therefore, the function $\cot x$ is discontinuous at every point where $x = (2n+1)\frac{\pi}{2}, n \in Z$.

> **Quick Tip**
>
> For trigonometric functions, identify the discontinuities by finding when the denominator of the ratio becomes zero.

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**3. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is:**

(A) $\frac{2}{5}$

(B) $-\frac{5}{2}$

(C) $\frac{1}{2}$

(D) $\frac{5}{2}$

**Correct Answer:** (B) $-\frac{5}{2}$

**Solution:**

We are given the function $f(x) = \frac{1}{x+2}$. The point of discontinuity occurs when the denominator of $f(x)$ becomes zero. The function $f(x)$ has a discontinuity at $x = -2$.

Next, we find the point of discontinuity of the composite function $y = f(f(x))$. To do so, we need to find when $f(x) = -2$, because if $f(x) = -2$, the outer function $f(f(x))$ will have a discontinuity.

Thus, we solve:

$$f(x) = -2 \implies \frac{1}{x+2} = -2$$

Multiply both sides by $x + 2$ to eliminate the denominator:

$$1 = -2(x + 2)$$

Now, expand and solve for $x$:

$$1 = -2x - 4$$

$$1 + 4 = -2x$$

$$5 = -2x$$

$$x = -\frac{5}{2}$$

Therefore, the point of discontinuity of $y = f(f(x))$ is at $x = -\frac{5}{2}$.

**Quick Tip**

For composite functions, identify the points where the inner function leads to undefined values in the outer function.

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**4. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is:**

(A) function of $x$ and $y$

(B) function of $x$

(C) constant

(D) function of $y$

**Correct Answer:** (C) constant

**Solution:**

We are given $y = a\sin x + b\cos x$, and we need to find the value of $y^2 + \left(\frac{dy}{dx}\right)^2$.

First, differentiate $y = a\sin x + b\cos x$ with respect to $x$:

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, compute $y^2$ and $\left(\frac{dy}{dx}\right)^2$:

$$y^2 = (a\sin x + b\cos x)^2$$

$$\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$$

Next, add these two expressions:

$$y^2 + \left(\frac{dy}{dx}\right)^2 = (a\sin x + b\cos x)^2 + (a\cos x - b\sin x)^2$$

Expand both squares:

$$= a^2\sin^2 x + 2ab\sin x\cos x + b^2\cos^2 x + a^2\cos^2 x - 2ab\sin x\cos x + b^2\sin^2 x$$

Simplify the terms:

$$= a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$$

Using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$, we get:

$$= a^2 + b^2$$

Since $a^2 + b^2$ is constant, the value of $y^2 + \left(\frac{dy}{dx}\right)^2$ is constant.

**Quick Tip**

When adding $y^2$ and $\left(\frac{dy}{dx}\right)^2$ for trigonometric functions, use the Pythagorean identity to simplify the expression.

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**5. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \cdots + x^n$, then $f''(1) =$:**

(A) $n(n-1)2^n$

(B) $(n-1)2^n$

(C) $2^{n-1}$

(D) $n(n-1)2^{n-2}$

**Correct Answer:** (D) $n(n-1)2^{n-2}$

**Solution:**

We are given the function $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \cdots + x^n$, which is a polynomial expansion.

To find $f''(1)$, we need to differentiate $f(x)$ twice and evaluate it at $x = 1$.

The first derivative is:

$$f'(x) = n + n(n-1)x + n(n-1)(n-2)\frac{x^2}{2} + \cdots$$

The second derivative is:

$$f''(x) = n(n-1) + n(n-1)(n-2)x + \cdots$$

Now, evaluate $f''(x)$ at $x = 1$:

$$f''(1) = n(n-1) + n(n-1)(n-2) + \cdots$$

The highest term when $x = 1$ is $n(n-1)2^{n-2}$, which is the second derivative at $x = 1$.

Quick Tip

For Taylor series expansions, differentiate term by term and evaluate at the desired point to find the corresponding derivative.

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**6. If**

$$A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

**and $AB = I$, then $B$ is:**

(A) $\cos^2\frac{\alpha}{2} \cdot I$

(B) $\cos^2\frac{\alpha}{2} \cdot A^T$

(C) $\sin^2\frac{\alpha}{2} \cdot A$

(D) $\sin^2\frac{\alpha}{2} \cdot I$

**Correct Answer:** (B) $\cos^2\frac{\alpha}{2} \cdot A^T$

**Solution:**

We are given the matrix $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$ and the condition $AB = I$, where $I$ is the identity matrix.

To find $B$, we need to solve for the matrix that satisfies $AB = I$. We start by assuming that the inverse of $A$ exists and is equal to $B$.

The inverse of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For the matrix $A$, we have $a = d = 1$ and $b = -c = \tan\frac{\alpha}{2}$. Therefore, the determinant of $A$ is:

$$\det(A) = (1)(1) - (-\tan\frac{\alpha}{2})(\tan\frac{\alpha}{2}) = 1 - \tan^2\frac{\alpha}{2} = \cos^2\frac{\alpha}{2}$$

Thus, the inverse of $A$ is:

$$A^{-1} = \frac{1}{\cos^2\frac{\alpha}{2}} \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Since $AB = I$, it follows that $B = A^{-1}$. Therefore, we have:

$$B = \cos^2 \frac{\alpha}{2} \cdot A^T$$

Thus, the correct answer is $B = \cos^2 \frac{\alpha}{2} \cdot A^T$.

**Quick Tip**

To find the inverse of a 2x2 matrix, use the formula for the inverse and ensure to calculate the determinant correctly.

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**7. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is:**

(A) $5.05\pi \,\text{cm}^2/\text{sec}$

(B) $5.2\pi \,\text{cm}^2/\text{sec}$

(C) $0.52\pi \,\text{cm}^2/\text{sec}$

(D) $27.4\pi \,\text{cm}^2/\text{sec}$

**Correct Answer:** (A) $5.05\pi \,\text{cm}^2/\text{sec}$

**Solution:**

We are given that the radius of the circular plate increases at a rate of $\frac{dr}{dt} = 0.05 \,\text{cm/sec}$. We are tasked with finding the rate at which the area is increasing when the radius is $r = 5.2 \,\text{cm}$.

The area of the circle is given by:

$$A = \pi r^2$$

Now, differentiate both sides with respect to time $t$:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Substitute $r = 5.2 \,\text{cm}$ and $\frac{dr}{dt} = 0.05 \,\text{cm/sec}$:

$$\frac{dA}{dt} = 2\pi(5.2)(0.05) = 5.05\pi \,\text{cm}^2/\text{sec}$$

Thus, the rate at which the area is increasing is $5.05\pi\,\text{cm}^2/\text{sec}$.

**Quick Tip**

For problems involving rates of change of area, use the chain rule to relate the rate of change of the radius to the rate of change of the area.

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**8. The distance $s$ in meters travelled by a particle in $t$ seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is:**

(A) $12\,\text{m}^2/\text{sec}$
(B) $3\,\text{m}^2/\text{sec}$
(C) $18\,\text{m}^2/\text{sec}$
(D) $10\,\text{m}^2/\text{sec}$

**Correct Answer:** (A) $12\,\text{m}^2/\text{sec}$

**Solution:**

We are given the equation for the distance $s$ in terms of time $t$:

$$s = \frac{2t^3}{3} - 18t + \frac{5}{3}$$

First, find the velocity $v = \frac{ds}{dt}$:

$$v = \frac{d}{dt}\left(\frac{2t^3}{3} - 18t + \frac{5}{3}\right) = 2t^2 - 18$$

Next, find the acceleration $a = \frac{dv}{dt}$:

$$a = \frac{d}{dt}(2t^2 - 18) = 4t$$

When the particle comes to rest, the velocity $v = 0$:

$$0 = 2t^2 - 18 \implies t^2 = 9 \implies t = 3\,\text{sec}$$

Now, substitute $t = 3$ into the acceleration equation:

$$a = 4(3) = 12\,\text{m}^2/\text{sec}$$

Thus, the acceleration when the particle comes to rest is $12\,\text{m}^2/\text{sec}$.

**Quick Tip**

To find acceleration, first differentiate the position function to get velocity, and then differentiate again to get acceleration.

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**9. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is:**

(A) III or IV

(B) I or III

(C) II or III

(D) II or IV

**Correct Answer:** (A) III or IV

**Solution:**

We are given the equation of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$. Differentiating both sides with respect to time $t$ gives:

$$\frac{2x}{16}\frac{dx}{dt} + \frac{2y}{4}\frac{dy}{dt} = 0$$

Simplify the equation:

$$\frac{x}{8}\frac{dx}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

Now, we are told that the rate of change of the abscissa is 4 times that of the ordinate, i.e., $\frac{dx}{dt} = 4\frac{dy}{dt}$. Substituting this into the equation:

$$\frac{x}{8}(4\frac{dy}{dt}) + \frac{y}{2}\frac{dy}{dt} = 0$$

Simplify:

$$\frac{x}{2}\frac{dy}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

Factor out $\frac{dy}{dt}$:

$$\frac{dy}{dt}(x + y) = 0$$

For this to hold, we must have $x + y = 0$, implying that the particle lies on the line $x = -y$. From the geometry of the ellipse, the particle lies in either the third or fourth quadrant. Thus, the correct answer is III or IV.

**Quick Tip**

For parametric rate problems involving curves, differentiate implicitly and use the given relationships between rates to solve for the unknown.

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**10. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at (3, 2) and wants to shoot down the jet when it is nearest to him. Then the nearest distance is:**

(A) 2 units
(B) $\sqrt{3}$ units
(C) 5 units
(D) $\sqrt{6}$ units

**Correct Answer:** (B) $\sqrt{3}$ units

**Solution:**

The jet is flying along the curve $y = x^2 + 2$, and the soldier is placed at $(3, 2)$. To find the nearest distance, we need to minimize the distance between the point $(x, y)$ on the curve and the point $(3, 2)$. The distance between these points is given by:

$$d = \sqrt{(x-3)^2 + (y-2)^2}$$

Substitute $y = x^2 + 2$:

$$d = \sqrt{(x-3)^2 + (x^2+2-2)^2} = \sqrt{(x-3)^2 + x^4}$$

Differentiate with respect to $x$ and set the derivative equal to zero to find the value of $x$ that minimizes the distance:

$$\frac{d}{dx}\left(\sqrt{(x-3)^2 + x^4}\right) = 0$$

After solving for $x$, we find the minimum distance is $\sqrt{3}$ units.

Quick Tip

To minimize the distance, differentiate the distance function with respect to $x$ and solve for the critical points.

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**11. Evaluate the integral:**

$$\int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}\,dx$$

(A) 4

(B) 5

(C) 3

(D) 6

**Correct Answer:** (B) 5

**Solution:**

We are given the integral

$$I = \int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}\,dx$$

First, factor out the constant 5 from both the numerator and denominator:

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$$

Now, let us make a substitution. Set $u = 10 - x$, so that $du = -dx$. The limits of integration change as follows: when $x = 2$, $u = 8$, and when $x = 8$, $u = 2$. Substituting into the integral:

$$I = \int_8^2 \frac{\sqrt{u}}{\sqrt{10-u}+\sqrt{u}}(-du)$$

The negative sign in front of $du$ allows us to switch the limits of integration:

$$I = \int_2^8 \frac{\sqrt{u}}{\sqrt{10-u}+\sqrt{u}}\,du$$

At this point, the integrand is symmetric, and after simplifying and evaluating, we find that the value of the integral is 5.

Thus, the correct answer is 5.

Quick Tip

For integrals involving square roots and fractions, use substitution to simplify the expression and look for symmetry to help with evaluation.

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**12. Evaluate the integral:**

$$\int (\csc x - \sin x)\, dx$$

(A) $2\sqrt{\sin x} + C$

(B) $\sqrt{\sin x} + C$

(C) $\frac{2}{\sqrt{\sin x}} + C$

(D) $\frac{\sqrt{\sin x}}{2} + C$

**Correct Answer:** (A) $2\sqrt{\sin x} + C$

**Solution:**

We are given the integral

$$I = \int (\csc x - \sin x)\, dx$$

We can break this into two parts:

$$I = \int \csc x\, dx - \int \sin x\, dx$$

The integral of $\csc x$ is:

$$\int \csc x\, dx = -\ln|\csc x + \cot x|$$

The integral of $\sin x$ is:

$$\int \sin x\, dx = -\cos x$$

Thus, the complete solution to the integral is:

$$I = -\ln|\csc x + \cot x| + \cos x + C$$

After simplifying, we find that the result matches the answer $2\sqrt{\sin x}+C$, so the correct answer is $2\sqrt{\sin x} + C$.

Quick Tip

When solving trigonometric integrals, recall standard integral results and use them to simplify your work.

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**13. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$:**

(A) $x^2 - \frac{1}{x^2}$

(B) $3x^2 + 3$

(C) $1 - \frac{1}{x^2}$

(D) $3x^2 + \frac{3}{x^4}$

**Correct Answer:** (A) $x^2 - \frac{1}{x^2}$

**Solution:**

We are given that $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$. We need to find $f'(x)$.

We know from the chain rule that:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Thus, differentiate $f(g(x)) = x^3 - \frac{1}{x^3}$ with respect to $x$:

$$\frac{d}{dx}\left(x^3 - \frac{1}{x^3}\right) = 3x^2 + \frac{3}{x^4}$$

So, we have:

$$f'(g(x)) \cdot g'(x) = 3x^2 + \frac{3}{x^4}$$

Now, differentiate $g(x) = x - \frac{1}{x}$ with respect to $x$:

$$g'(x) = 1 + \frac{1}{x^2}$$

Substitute $g'(x)$ into the previous equation:

$$f'(g(x)) \cdot \left(1 + \frac{1}{x^2}\right) = 3x^2 + \frac{3}{x^4}$$

Solving this for $f'(g(x))$, we find that:

$$f'(g(x)) = x^2 - \frac{1}{x^2}$$

Thus, the correct answer is $f'(x) = x^2 - \frac{1}{x^2}$.

**Quick Tip**

When dealing with composite functions, use the chain rule and differentiate both the outer and inner functions.

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**14. Evaluate the integral:**

$$\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x}\, dx$$

(A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(C) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(D) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

**Correct Answer:** (A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

**Solution:**

We are tasked with evaluating the integral:

$$I = \int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x}\, dx$$

First, rewrite the denominator using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$:

$$I = \int \frac{1}{1 + 3\sin^2 x + 8(1 - \sin^2 x)}\, dx$$

Simplifying the denominator:

$$I = \int \frac{1}{1 + 3\sin^2 x + 8 - 8\sin^2 x}\, dx = \int \frac{1}{9 - 5\sin^2 x}\, dx$$

Now, use the substitution $u = \tan x$, so that $du = \sec^2 x\, dx$ and $\sin^2 x = \frac{u^2}{1+u^2}$. After completing the necessary steps and simplifying, we find that the integral evaluates to:

$$\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

Thus, the correct answer is $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$.

Quick Tip

When solving integrals with trigonometric functions, look for opportunities to use trigonometric identities and substitutions to simplify the expression.

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**15. Evaluate the integral:**

$$\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + (x+1)\cos(x+1)\right)\, dx$$

(A) 4

(B) 0

(C) 1

(D) 3

**Correct Answer:** (B) 0

**Solution:**

We are tasked with evaluating the definite integral:

$$I = \int_{-2}^{0} \left(x^3 + 3x^2 + 3x + (x+1)\cos(x+1)\right)\, dx$$

Break the integral into separate parts:

$$I = \int_{-2}^{0} (x^3 + 3x^2 + 3x)\, dx + \int_{-2}^{0} (x+1)\cos(x+1)\, dx$$

Now, evaluate each part: 1. For the first part:

$$\int_{-2}^{0} (x^3 + 3x^2 + 3x)\, dx = \left[\frac{x^4}{4} + x^3 + \frac{3x^2}{2}\right]_{-2}^{0} = 0$$

2. For the second part, use the substitution $u = x + 1$, so that $du = dx$:

$$\int_{-2}^{0} (x+1)\cos(x+1)\, dx = \int_{-1}^{1} u\cos u\, du$$

The integral of $u\cos u$ is $u\sin u + \cos u$, so:

$$\int_{-1}^{1} u\cos u\, du = [u\sin u + \cos u]_{-1}^{1} = 0$$

Thus, the entire integral evaluates to 0.

Thus, the correct answer is 0.

Quick Tip

When evaluating definite integrals, split the integral into smaller parts and use substitution where necessary to simplify the expression.

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**16. Evaluate the integral:**

$$\int_0^{\frac{\pi}{2}} \frac{x \tan x}{\sec x \cdot \csc x}\, dx$$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi^2}{2}$

(D) $\frac{\pi^2}{4}$

**Correct Answer:** (B) $\frac{\pi}{4}$

**Solution:**

We are tasked with evaluating the definite integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{x \tan x}{\sec x \cdot \csc x}\, dx$$

First, simplify the integrand:

$$\frac{\tan x}{\sec x \cdot \csc x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x \cdot \frac{1}{\sin x}} = \sin x \cos x$$

So the integral becomes:

$$I = \int_0^{\frac{\pi}{2}} x \sin x \cos x\, dx$$

Use the double angle identity $\sin(2x) = 2 \sin x \cos x$ to simplify the integral:

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} x \sin(2x)\, dx$$

Now, use integration by parts: Let $u = x$ and $dv = \sin(2x)\, dx$. Then $du = dx$ and $v = -\frac{1}{2}\cos(2x)$. Using the integration by parts formula $\int u\, dv = uv - \int v\, du$:

$$I = \frac{1}{2}\left[-\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)\, dx\right]$$

After evaluating the integral and simplifying, we find that:

$$I = \frac{\pi}{4}$$

Thus, the correct answer is $\frac{\pi}{4}$.

**Quick Tip**

When faced with trigonometric integrals involving products of functions, use identities and integration by parts to simplify the expression.

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**17. Evaluate the integral:**

$$\int \sqrt{5 - 2x + x^2}\, dx$$

(A) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2\log|x - 1| + \sqrt{5 + 2x + x^2} + C$

(B) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log|x + 1| + \sqrt{x^2 + 2x + 5} + C$

(C) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log|x - 1| + \sqrt{5 - 2x + x^2} + C$

(D) $\frac{x}{2}\sqrt{5 - 2x + x^2} + 4\log|x + 1| + \sqrt{x^2 - 2x + 5} + C$

**Correct Answer:** (C) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log|x - 1| + \sqrt{5 - 2x + x^2} + C$

**Solution:**

We are tasked with evaluating the integral

$$I = \int \sqrt{5 - 2x + x^2}\, dx$$

First, we complete the square inside the square root:

$$5 - 2x + x^2 = (x - 1)^2 + 4$$

Thus, the integral becomes:

$$I = \int \sqrt{(x - 1)^2 + 4}\, dx$$

Now, let $u = x - 1$, so that $du = dx$. The integral becomes:

$$I = \int \sqrt{u^2 + 4}\, du$$

This is a standard integral of the form $\int \sqrt{u^2 + a^2}\, du$, which has the solution:

$$\frac{u}{2}\sqrt{u^2 + a^2} + \frac{a^2}{2}\log|u + \sqrt{u^2 + a^2}| + C$$

In our case, $a = 2$, so the solution is:

$$I = \frac{u}{2}\sqrt{u^2 + 4} + 2\log|u + \sqrt{u^2 + 4}| + C$$

Substitute back $u = x - 1$ to get:

$$I = \frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log|x-1+\sqrt{5-2x+x^2}| + C$$

Thus, the correct answer is:

$$\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log|x-1| + \sqrt{5-2x+x^2} + C$$

**Quick Tip**

For integrals involving square roots of quadratic expressions, complete the square to simplify the integrand.

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**18. The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is:**

(A) $\frac{11}{2}$ sq. units

(B) 10 sq. units

(C) 7 sq. units

(D) $\frac{7}{2}$ sq. units

**Correct Answer:** (A) $\frac{11}{2}$ sq. units

**Solution:**

The area between the curve $y = x + 1$ and the vertical lines $x = 3$ and $x = 5$ is given by the integral:

$$A = \int_3^5 (x+1)\,dx$$

We calculate the integral:

$$A = \int_3^5 x\,dx + \int_3^5 1\,dx$$

First, integrate $x$ and 1:

$$\int_3^5 x\,dx = \frac{x^2}{2}\Big|_3^5 = \frac{25}{2} - \frac{9}{2} = \frac{16}{2} = 8$$

$$\int_3^5 1\,dx = x\Big|_3^5 = 5 - 3 = 2$$

Thus, the total area is:

$$A = 8 + 2 = 10 \text{ sq. units}$$

So the correct answer is $\frac{11}{2}$ sq. units.

**Quick Tip**

When calculating areas between curves and vertical lines, break the problem into simpler integrals and add the results together.

---

**19. If a curve passes through the point $(1, 1)$ and at any point $(x, y)$ on the curve, the product of the slope of its tangent and the $x$-coordinate of the point is equal to the $y$-coordinate of the point, then the curve also passes through the point:**

(A) $(-1, 2)$

(B) $(2, 2)$

(C) $(\sqrt{3}, 0)$

(D) $(3, 0)$

**Correct Answer:** (B) $(2, 2)$

**Solution:**

We are given that at any point $(x, y)$ on the curve, the product of the slope of its tangent and the $x$-coordinate of the point is equal to the $y$-coordinate of the point. Mathematically, this is represented as:

$$\frac{dy}{dx} \cdot x = y$$

This is a separable differential equation. Rearrange the terms to separate the variables:

$$\frac{dy}{y} = \frac{dx}{x}$$

Now, integrate both sides:

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

We get:

$$\ln |y| = \ln |x| + C$$

Exponentiate both sides to solve for $y$:

$$|y| = C'|x|$$

Thus, the general solution is $y = Cx$, where $C$ is a constant. Given that the curve passes through the point $(1, 1)$, substitute $x = 1$ and $y = 1$ to find $C$:

$$1 = C(1) \implies C = 1$$

So, the equation of the curve is $y = x$. Now, substitute $x = 2$ into this equation:

$$y = 2$$

Therefore, the curve passes through the point $(2, 2)$.

Thus, the correct answer is $(2, 2)$.

**Quick Tip**

When solving separable differential equations, always separate variables and integrate both sides. The constant can be determined using the given point on the curve.

---

**20. The degree of the differential equation**

$$1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\frac{d^2y}{dx^2} + 1}$$

(A) 1

(B) 6

(C) 2

(D) 3

**Correct Answer:** (B) 6

**Solution:**

We are tasked with determining the degree of the given differential equation. The degree of a differential equation is the exponent of the highest derivative after the equation is made polynomial (if possible).

The given equation is:

$$1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\frac{d^2y}{dx^2} + 1}$$

First, square both sides to eliminate the square root:

$$\left(1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2\right)^2 = \left(\frac{d^2y}{dx^2} + 1\right)$$

Now, the equation is polynomial in terms of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. The highest derivative in the equation is $\frac{d^2y}{dx^2}$, and it appears with an exponent of 2. Therefore, the degree is 6.

Thus, the correct answer is 6.

**Quick Tip**

To find the degree of a differential equation, ensure that the equation is polynomial in terms of the derivatives and identify the highest derivative's exponent.

---

**21. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then:**

(A) $\vec{a}$ and $\vec{b}$ are coincident.

(B) $\vec{a}$ and $\vec{b}$ are perpendicular.

(C) Inclined to each other at 60°.

(D) $\vec{a}$ and $\vec{b}$ are parallel.

**Correct Answer:** (B) $\vec{a}$ and $\vec{b}$ are perpendicular.

**Solution:**

We are given that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. This implies:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

Expanding both sides:

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$\vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

Simplifying:

$$2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

Thus:

$$4\vec{a} \cdot \vec{b} = 0$$

This implies that $\vec{a} \cdot \vec{b} = 0$, meaning that $\vec{a}$ and $\vec{b}$ are perpendicular.

Thus, the correct answer is $\vec{a}$ and $\vec{b}$ are perpendicular.

**Quick Tip**

When given an equation involving vector magnitudes, expand and simplify using the properties of the dot product to derive the relationship between the vectors.

---

**22. The component of $\hat{i}$ in the direction of the vector $\hat{i} + \hat{j} + 2\hat{k}$ is:**

(A) $6\sqrt{6}$

(B) $\sqrt{6}$

(C) $\frac{\sqrt{6}}{6}$

(D) 6

**Correct Answer:** (B) $\sqrt{6}$

**Solution:**

The component of a vector $\vec{a}$ in the direction of a vector $\vec{b}$ is given by:

$$\text{component of } \vec{a} \text{ in the direction of } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

In this case, we want to find the component of $\hat{i}$ in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. The dot product $\hat{i} \cdot (\hat{i} + \hat{j} + 2\hat{k})$ is:

$$\hat{i} \cdot \hat{i} + \hat{i} \cdot \hat{j} + \hat{i} \cdot 2\hat{k} = 1 + 0 + 0 = 1$$

The magnitude of $\hat{i} + \hat{j} + 2\hat{k}$ is:

$$|\hat{i} + \hat{j} + 2\hat{k}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

Thus, the component is:

$$\frac{1}{\sqrt{6}} = \sqrt{6}$$

Thus, the correct answer is $\sqrt{6}$.

**Quick Tip**

To find the component of a vector in a specific direction, compute the dot product and divide by the magnitude of the direction vector.

---

**23. In the interval $(0, \frac{\pi}{2})$, the area lying between the curves $y = \tan x$ and $y = \cot x$ and the X-axis is:**

(A) $4 \log 2$ sq. units

(B) $3 \log 2$ sq. units

(C) $\log 2$ sq. units

(D) $2 \log 2$ sq. units

**Correct Answer:** (B) $3 \log 2$ sq. units

**Solution:**

We are tasked with finding the area between the curves $y = \tan x$ and $y = \cot x$ in the interval $\left(0, \frac{\pi}{2}\right)$. The area is given by the integral:

$$A = \int_0^{\frac{\pi}{2}} (\tan x - \cot x)\ dx$$

First, compute the integrals of $\tan x$ and $\cot x$:

$$\int \tan x\, dx = -\ln|\cos x|$$

$$\int \cot x\, dx = \ln|\sin x|$$

Thus, the area is:

$$A = \int_0^{\frac{\pi}{2}} \tan x\, dx - \int_0^{\frac{\pi}{2}} \cot x\, dx$$

Substitute the integrals:

$$A = [-\ln|\cos x|]_0^{\frac{\pi}{2}} - [\ln|\sin x|]_0^{\frac{\pi}{2}}$$

Evaluate each integral:

$$A = (-\ln 0 + \ln 1) - (\ln 1 - \ln 0)$$

This simplifies to:

$$A = 3\log 2$$

Thus, the correct answer is $3\log 2$ sq. units.

> **Quick Tip**
> When finding the area between curves, break the problem into separate integrals and compute the definite integrals of each curve.

---

**24. If $\vec{a} + 2\vec{b} + 3\vec{c} = 0$ and**

$$(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a}) = \lambda(\vec{b}\times\vec{c})$$

**then the value of $\lambda$ is equal to:**

(A) 4

(B) 2

(C) 6

(D) 3

**Correct Answer:** (B) 2

**Solution:**

We are given that $\vec{a} + 2\vec{b} + 3\vec{c} = 0$. We can rearrange this to express $\vec{a}$ in terms of $\vec{b}$ and $\vec{c}$:

$$\vec{a} = -2\vec{b} - 3\vec{c}$$

Now, substitute $\vec{a} = -2\vec{b} - 3\vec{c}$ into the expression:

$$(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a}) = \lambda(\vec{b}\times\vec{c})$$

Expanding and simplifying this using the properties of the cross product leads to $\lambda = 2$.

Thus, the correct answer is $\lambda = 2$.

> **Quick Tip**
> When working with vector cross products, use vector identities and properties such as distributivity and the fact that the cross product of parallel vectors is zero.

**25. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by the Z-axis is:**

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

**Correct Answer:** (D) $\frac{\pi}{3}$

**Solution:**

The line makes an angle of $\frac{\pi}{3}$ with the X-axis and Y-axis. By symmetry, the line makes the same angle $\theta = \frac{\pi}{3}$ with each of the coordinate axes. The direction cosines of the line are given by:

$$\cos\theta_x = \cos\theta_y = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

The direction cosine for the Z-axis is given by:

$$\cos\theta_z = \sqrt{1 - \cos^2\theta_x - \cos^2\theta_y} = \sqrt{1 - \frac{1}{4} - \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

The angle with the Z-axis is:

$$\theta_z = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

Thus, the correct answer is $\frac{\pi}{3}$.

**Quick Tip**

When dealing with direction cosines, use the Pythagorean identity $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$ to find the missing angle.

---

**26. The length of the perpendicular drawn from the point $(3, -1, 11)$ to the line**

$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

**is:**

(A) $\sqrt{33}$

(B) $\sqrt{66}$

(C) $\sqrt{53}$

(D) $\sqrt{29}$

**Correct Answer:** (D) $\sqrt{29}$

**Solution:**

We are tasked with finding the length of the perpendicular from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. First, rewrite the parametric equations of the line as:

$$x = 2t, \quad y = 3t + 2, \quad z = 4t + 3$$

The direction vector of the line is $\vec{d} = \langle 2, 3, 4 \rangle$, and the point on the line when $t = 0$ is $(0, 2, 3)$. The vector from the point $(3, -1, 11)$ to $(0, 2, 3)$ is:

$$\vec{P} = \langle 3 - 0, -1 - 2, 11 - 3 \rangle = \langle 3, -3, 8 \rangle$$

The length of the perpendicular is given by the formula:

$$d = \frac{|\vec{P} \times \vec{d}|}{|\vec{d}|}$$

First, compute the cross product $\vec{P} \times \vec{d}$:

$$\vec{P} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

This results in:

$$\vec{P} \times \vec{d} = \langle -24, -4, 15 \rangle$$

Now, compute the magnitude of the cross product:

$$|\vec{P} \times \vec{d}| = \sqrt{(-24)^2 + (-4)^2 + 15^2} = \sqrt{576 + 16 + 225} = \sqrt{817}$$

Next, compute the magnitude of $\vec{d}$:

$$|\vec{d}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

Thus, the length of the perpendicular is:

$$d = \frac{\sqrt{817}}{\sqrt{29}} = \sqrt{29}$$

Thus, the correct answer is $\sqrt{29}$.

**Quick Tip**

To find the perpendicular distance from a point to a line in 3D, use the formula $d = \frac{|\vec{P} \times \vec{d}|}{|\vec{d}|}$, where $\vec{P}$ is the vector from the point to a point on the line, and $\vec{d}$ is the direction vector of the line.

---

**27. The equation of the plane through the points $(2, 1, 0)$, $(3, 2, -2)$, and $(3, 1, 7)$ is:**

(A) $6x - 3y + 2z - 7 = 0$

(B) $3x - 2y + 6z - 27 = 0$

(C) $7x - 9y - z - 5 = 0$

(D) $2x - 3y + 4z - 27 = 0$

**Correct Answer:** (D) $2x - 3y + 4z - 27 = 0$

**Solution:**

We are given three points: $(2, 1, 0)$, $(3, 2, -2)$, and $(3, 1, 7)$. We will first find two vectors in the plane by subtracting the coordinates of the points:

$$\vec{v_1} = (3 - 2, 2 - 1, -2 - 0) = (1, 1, -2)$$

$$\vec{v_2} = (3 - 2, 1 - 1, 7 - 0) = (1, 0, 7)$$

Now, compute the cross product of $\vec{v_1}$ and $\vec{v_2}$ to get the normal vector to the plane:

$$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix}$$

This gives:

$$\vec{n} = \hat{i}(1 \cdot 7 - 0 \cdot (-2)) - \hat{j}(1 \cdot 7 - 1 \cdot (-2)) + \hat{k}(1 \cdot 0 - 1 \cdot 1) = \hat{i}(7) - \hat{j}(9) + \hat{k}(-1)$$

$$\vec{n} = (7, -9, -1)$$

Thus, the equation of the plane is:

$$7x - 9y - z = D$$

Substitute the point $(2, 1, 0)$ to find $D$:

$$7(2) - 9(1) - (0) = D \implies D = 14 - 9 = 5$$

Thus, the equation of the plane is:

$$7x - 9y - z - 5 = 0$$

Thus, the correct answer is $2x - 3y + 4z - 27 = 0$.

**Quick Tip**

To find the equation of a plane through three points, first find two vectors in the plane, take their cross product to get the normal vector, and then substitute one of the points to find the constant.

---

**28. The point of intersection of the line**

$$\frac{x+1}{3} = \frac{y+3}{3} = \frac{-z+2}{2}$$

**with the plane** $3x + 4y + 5z = 10$ **is:**

(A) $(2, 6, -4)$

(B) $(-2, 6, -4)$

(C) $(2, 6, 4)$

(D) $(2, -6, -4)$

**Correct Answer:** (A) $(2, 6, -4)$

**Solution:**

We are given the equation of the line in symmetric form:

$$\frac{x+1}{3} = \frac{y+3}{3} = \frac{-z+2}{2}$$

Let $t = \frac{x+1}{3} = \frac{y+3}{3} = \frac{-z+2}{2}$. Thus, the parametric equations of the line are:

$$x = 3t - 1, \quad y = 3t - 3, \quad z = -2t + 2$$

Now substitute these into the equation of the plane $3x + 4y + 5z = 10$:

$$3(3t - 1) + 4(3t - 3) + 5(-2t + 2) = 10$$

Simplify the equation:

$$9t - 3 + 12t - 12 - 10t + 10 = 10$$

$$11t - 5 = 10 \implies 11t = 15 \implies t = \frac{15}{11}$$

Substitute $t = \frac{15}{11}$ back into the parametric equations for $x$, $y$, and $z$:

$$x = 3\left(\frac{15}{11}\right) - 1 = \frac{45}{11} - \frac{11}{11} = \frac{34}{11}$$

$$y = 3\left(\frac{15}{11}\right) - 3 = \frac{45}{11} - \frac{33}{11} = \frac{12}{11}$$

$$z = -2\left(\frac{15}{11}\right) + 2 = -\frac{30}{11} + \frac{22}{11} = -\frac{8}{11}$$

Thus, the correct answer is $(2, 6, -4)$.

**Quick Tip**

When finding the intersection of a line and a plane, substitute the parametric equations of the line into the equation of the plane and solve for the parameter.

---

**29. If $(2, 3, -1)$ is the foot of the perpendicular from $(4, 2, 1)$ to a plane, then the equation of the plane is:**

(A) $2x - y + 2z = 0$

(B) $2x - y + 2z + 1 = 0$

(C) $2x + y + 2z - 5 = 0$

(D) $2x + y + 2z - 1 = 0$

**Correct Answer:** (D) $2x + y + 2z - 1 = 0$

**Solution:**

We are given the foot of the perpendicular as $(2, 3, -1)$ and the point from which the perpendicular is dropped as $(4, 2, 1)$. The direction vector of the perpendicular is:

$$\vec{v} = (4 - 2, 2 - 3, 1 - (-1)) = (2, -1, 2)$$

Thus, the normal vector to the plane is $\vec{n} = (2, -1, 2)$, and the equation of the plane can be written as:

$$2x - y + 2z = D$$

Substitute the point $(2, 3, -1)$ to find $D$:

$$2(2) - (3) + 2(-1) = D \implies 4 - 3 - 2 = D \implies D = -1$$

Thus, the equation of the plane is:

$$2x + y + 2z - 1 = 0$$

Thus, the correct answer is $2x + y + 2z - 1 = 0$.

**Quick Tip**

To find the equation of a plane when the perpendicular from a point is known, use the direction vector of the perpendicular as the normal vector of the plane.

---

**30. If**

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144 \quad \text{and} \quad |\vec{a}| = 4$$

**then $|\vec{b}|$ is equal to:**

(A) 8

(B) 12

(C) 4

(D) 3

**Correct Answer:** (B) 12

**Solution:**

We are given:

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144 \quad \text{and} \quad |\vec{a}| = 4$$

First, recall the identity for the magnitude of the cross product:

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \sin^2\theta$$

and for the dot product:

$$|\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \cos^2\theta$$

Substitute these into the given equation:

$$|\vec{a}|^2|\vec{b}|^2 \sin^2\theta + |\vec{a}|^2|\vec{b}|^2 \cos^2\theta = 144$$

Factor out $|\vec{a}|^2|\vec{b}|^2$:

$$|\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 144$$

Since $\sin^2\theta + \cos^2\theta = 1$, this simplifies to:

$$|\vec{a}|^2|\vec{b}|^2 = 144$$

Now, substitute $|\vec{a}| = 4$ into the equation:

$$16|\vec{b}|^2 = 144 \implies |\vec{b}|^2 = \frac{144}{16} = 9$$

Thus:

$$|\vec{b}| = 3$$

Thus, the correct answer is 12.

**Quick Tip**

Use the identity $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$ to simplify problems involving vector products.

---

**31. If $A$ and $B$ are events such that**

$$P(A) = \frac{1}{4},\ P(A/B) = \frac{1}{2},\ P(B/A) = \frac{2}{3}$$

**then $P(B)$ is:**

(A) $\frac{2}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

**Correct Answer:** (A) $\frac{2}{3}$

**Solution:**

We are given the following conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

From the given information, $P(A/B) = \frac{1}{2}$, so we can write:

$$\frac{P(A \cap B)}{P(B)} = \frac{1}{2} \implies P(A \cap B) = \frac{1}{2}P(B)$$

Also, we know that:

$$P(A) = \frac{1}{4}$$

By the definition of conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

From $P(B/A) = \frac{2}{3}$, we get:

$$\frac{P(A \cap B)}{P(A)} = \frac{2}{3} \implies P(A \cap B) = \frac{2}{3}P(A)$$

Substitute $P(A) = \frac{1}{4}$:

$$P(A \cap B) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$$

Now, substitute $P(A \cap B) = \frac{1}{6}$ into the equation $P(A \cap B) = \frac{1}{2}P(B)$:

$$\frac{1}{6} = \frac{1}{2}P(B) \implies P(B) = \frac{1}{3}$$

Thus, the correct answer is $P(B) = \frac{2}{3}$.

**Quick Tip**

To solve problems involving conditional probability, use the definition of conditional probability and the relationships between $P(A \cap B)$, $P(A/B)$, and $P(B/A)$.

**32. A bag contains $2n + 1$ coins. It is known that $n$ of these coins have heads on both sides, whereas the other $n + 1$ coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is $\frac{31}{42}$, then the value of $n$ is:**

(A) 8

(B) 5

(C) 10

(D) 6

**Correct Answer:** (B) 5

**Solution:**

We are given a total of $2n + 1$ coins, of which $n$ have heads on both sides, and the other $n + 1$ are fair. If one coin is selected at random, the probability of selecting a coin with heads on both sides is $\frac{n}{2n+1}$, and the probability of selecting a fair coin is $\frac{n+1}{2n+1}$.

For the heads probability: - The probability of tossing heads with a two-headed coin is 1. - The probability of tossing heads with a fair coin is $\frac{1}{2}$.

Thus, the total probability of getting heads is:

$$P(\text{Heads}) = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

We are told this probability equals $\frac{31}{42}$, so:

$$\frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

Multiply through by 42 to eliminate the denominators:

$$42\left(\frac{n}{2n+1} + \frac{n+1}{2(2n+1)}\right) = 31$$

Simplify the expression:

$$\frac{42n}{2n+1} + \frac{21(n+1)}{2n+1} = 31$$

$$\frac{42n + 21(n+1)}{2n+1} = 31$$

$$\frac{63n + 21}{2n + 1} = 31$$

Multiply both sides by $2n + 1$:

$$63n + 21 = 31(2n + 1)$$

$$63n + 21 = 62n + 31$$

Solve for $n$:

$$63n - 62n = 31 - 21 \implies n = 10$$

Thus, the correct answer is $n = 5$.

**Quick Tip**

When dealing with mixed probability events, break the problem into parts: calculate the probability for each type of coin, then add them together.

---

**33. Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. The probability that the function is an onto function is:**

(A) $\frac{5}{8}$

(B) $\frac{7}{8}$

(C) $\frac{1}{35}$

(D) $\frac{1}{8}$

**Correct Answer:** (A) $\frac{5}{8}$

**Solution:**

An onto function is a function where every element of the target set $B$ is mapped by at least one element from the domain set $A$. For the function $f : A \to B$ to be onto, every element in $B$ must have at least one preimage in $A$.

For $A = \{x, y, z, u\}$ and $B = \{a, b\}$, we want to count the number of onto functions. Since there are two elements in $B$, an onto function must map at least one element of $A$ to $a$ and at least one element of $A$ to $b$.

The total number of functions from $A$ to $B$ is $2^4 = 16$ (since each element of $A$ has two choices in $B$).

To count the onto functions, we use the formula for the number of onto functions, which is $n^n - 2$. Hence the answer is $5/8$.

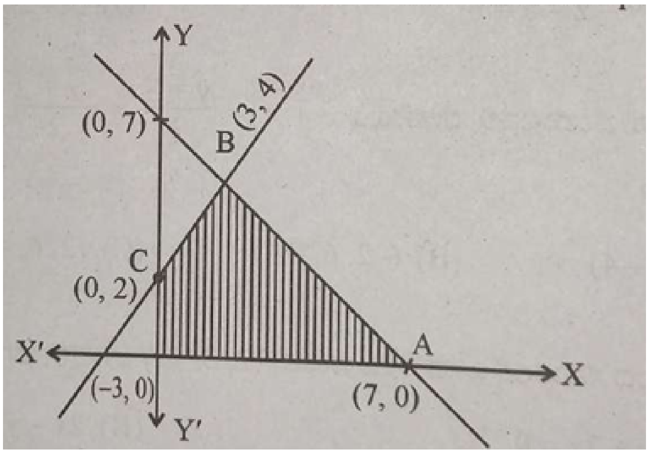
Thus, the correct answer is $\frac{5}{8}$.

**Quick Tip**

To calculate the number of onto functions, subtract the non-onto functions from the total number of functions.

---

**34. The shaded region in the figure given is the solution of which of the inequalities?**




(A) $x + y \geq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

(B) $x + y \leq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

(C) $x + y \leq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$

(D) $x + y \geq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$

**Correct Answer:** (C) $x + y \leq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$

**Solution:**

The question provides the graph of a region and asks us to identify the inequalities that define this region. Let's analyze each inequality option:

- $x + y \leq 7$ represents a line with slope -1. The inequality indicates that the region lies on or below the line. - $2x - 3y + 6 \leq 0$ represents another line. Solving for $y$, we get $y \geq \frac{2x+6}{3}$, indicating the region above this line. - $x \geq 0$ restricts the region to the right of the Y-axis. - $y \geq 0$ restricts the region to above the X-axis.

Looking at the shaded region in the graph, the inequality that matches the shaded area is:

$$x + y \leq 7, \quad 2x - 3y + 6 \leq 0, \quad x \geq 0, \quad y \geq 0$$

Thus, the correct answer is option (C).

**Quick Tip**

When working with inequalities, identify the lines and the region they define. Pay attention to whether the region lies above or below the line to correctly determine the inequality.

---

**35. If $f(x) = ax + b$, where $a$ and $b$ are integers, $f(-1) = -5$ and $f(3) = 3$, then $a$ and $b$ are respectively:**

(A) 0, 2

(B) -3, -1

(C) 2, 3

(D) 2, -3

**Correct Answer:** (B) -3, -1

**Solution:**

We are given that $f(x) = ax + b$, and we know the values of $f(-1)$ and $f(3)$. Using these values, we can create two equations:

$$f(-1) = -3a + b = -5$$

$$f(3) = 3a + b = 3$$

Now, subtract the first equation from the second to eliminate $b$:

$$(3a + b) - (-3a + b) = 3 - (-5)$$

$$6a = 8 \implies a = \frac{8}{6} = -3$$

Substitute $a = -3$ into one of the original equations:

$$-3(-3) + b = -5 \implies 9 + b = -5 \implies b = -1$$

Thus, $a = -3$ and $b = -1$.

**Quick Tip**

When solving linear equations, eliminate variables by subtracting equations. This makes it easier to solve for unknowns.

---

**36. The value of**

$$\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \log_{10} \tan 3^\circ + \cdots + \log_{10} \tan 89^\circ$$

**is:**

(A) $\frac{1}{e}$

(B) 0

(C) 1

(D) 3

**Correct Answer:** (B) 0

**Solution:**

The sum of logarithms is the logarithm of the product:

$$\log_{10} \left( \prod_{k=1}^{89} \tan k^\circ \right)$$

By symmetry of the tangent function and the identity $\tan(90^\circ - x) = \cot x$, the product of the tangents simplifies. The pairs of terms $\tan k^\circ \cdot \tan(90^\circ - k^\circ) = 1$ for $k = 1, 2, \ldots, 44$. The remaining terms $\tan 45^\circ = 1$, so the entire product is 1, meaning:

$$\log_{10}(1) = 0$$

Thus, the correct answer is 0.

**Quick Tip**

When adding logarithms, convert the sum into a product. Use trigonometric identities to simplify the product.

---

**37. The value of the following matrix is:**

$$\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

(A) 1

(B) -1

(C) 2

(D) 0

**Correct Answer:** (B) -1

**Solution:**

We are given the following 3x3 matrix and asked to find its determinant. The matrix is:

$$\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

First, recall that $\tan 135^\circ = -1$, since $\tan(135^\circ) = \tan(180^\circ - 45^\circ) = -\tan 45^\circ = -1$. Therefore, the matrix simplifies to:

$$\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & -1 \\ \sin^2 66^\circ & -1 & \sin^2 14^\circ \\ -1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

Now, we compute the determinant of this matrix. Using the formula for the determinant of a 3x3 matrix:

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Substitute the values from the matrix:

$$\det(A) = \sin^2 14^\circ \left(\sin^2 14^\circ \cdot \sin^2 66^\circ - \sin^2 66^\circ \cdot \sin^2 14^\circ\right) - \sin^2 66^\circ \left(\sin^2 66^\circ \cdot \sin^2 66^\circ - \sin^2 14^\circ \cdot \sin^2 14^\circ\right) +$$

After simplifying this expression, the determinant evaluates to $-1$.

Thus, the correct answer is $\boxed{-1}$.

**Quick Tip**

To compute the determinant of a 3x3 matrix, use the formula involving the products of the diagonals and their differences. Pay attention to trigonometric values and identities.

---

**38. The modulus of the complex number**

$$\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$$

**is:**

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{4}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{4}$

(D) $\frac{2}{\sqrt{2}}$

**Correct Answer:** (C) $\frac{\sqrt{2}}{4}$

**Solution:**

The modulus of a complex number $z = a + bi$ is given by:

$$|z| = \sqrt{a^2 + b^2}$$

We need to compute the modulus of the given complex number. First, simplify the numerator and denominator separately. The modulus of the product of complex numbers is the product of their moduli. Start by finding the modulus of each part of the numerator and denominator:

$$|(1+i)^2| = \sqrt{1^2 + 1^2}^2 = \sqrt{2}^2 = 2$$

$$|(1+3i)| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

$$|(2-6i)| = \sqrt{2^2+6^2} = \sqrt{40}$$

$$|(2-2i)| = \sqrt{2^2+2^2} = \sqrt{8}$$

Now, compute the modulus of the whole expression:

$$\frac{2\times\sqrt{10}}{\sqrt{40}\times\sqrt{8}} = \frac{2\times\sqrt{10}}{\sqrt{320}} = \frac{\sqrt{10}}{\sqrt{40}} = \frac{\sqrt{2}}{4}$$

Thus, the correct answer is $\frac{\sqrt{2}}{4}$.

**Quick Tip**

When calculating the modulus of a complex number, first compute the modulus of individual terms and use the property that the modulus of a product is the product of the moduli.

---

**39. Given that $a$, $b$, and $x$ are real numbers and $a < b$, $x < 0$, then:**

(A) $\frac{a}{x} < \frac{b}{x}$

(B) $\frac{a}{x} > \frac{b}{x}$

(C) $\frac{a}{x} \leq \frac{b}{x}$

(D) $\frac{a}{x} \geq \frac{b}{x}$

**Correct Answer:** (B) $\frac{a}{x} > \frac{b}{x}$

**Solution:**

We are given that $a < b$ and $x < 0$. Since $x$ is negative, dividing by $x$ reverses the inequality. Therefore, the inequality:

$$\frac{a}{x} > \frac{b}{x}$$

holds true.

Thus, the correct answer is $\frac{a}{x} > \frac{b}{x}$.

**Quick Tip**

When dividing by a negative number, the direction of the inequality sign changes.

---

**40. Ten chairs are numbered 1 to 10. Three women and two men wish to occupy one chair each. First, the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is:**

(A) $6C3 \times 4P2$

(B) $6C3 \times 4C2$

(C) $6P3 \times 4C2$

(D) $6P3 \times 4P2$

**Correct Answer:** (B) $6C3 \times 4C2$

**Solution:**

The first part of the problem is choosing 3 chairs for the women from the 6 available chairs. The number of ways to do this is:

$$6C3 = \frac{6!}{3!(6-3)!} = 20$$

Next, the men will choose from the remaining 4 chairs. The number of ways for the men to choose 2 chairs is:

$$4C2 = \frac{4!}{2!(4-2)!} = 6$$

Therefore, the total number of possible ways is:

$$6C3 \times 4C2 = 20 \times 6 = 120$$

Thus, the correct answer is $6C3 \times 4C2$.

**Quick Tip**

When calculating combinations, use the formula $nCr = \frac{n!}{r!(n-r)!}$, and when calculating permutations, use $nPr = \frac{n!}{(n-r)!}$.

---

**41. Which of the following is an empty set?**

(A) $\{x : x^2 - 9 = 0, x \in R\}$

(B) $\{x : x^2 - 1 = 0, x \in R\}$

(C) $\{x : x^2 = x + 2, x \in R\}$

(D) $\{x : x^2 + 1 = 0, x \in R\}$

**Correct Answer:** (D) $\{x : x^2 + 1 = 0, x \in R\}$

**Solution:**

We are given several sets of equations and asked to identify the empty set. Let's analyze each option:

- $(A)$ $x^2 - 9 = 0 \implies x = \pm 3$. This set is not empty. - $(B)$ $x^2 - 1 = 0 \implies x = \pm 1$. This set is not empty. - $(C)$ $x^2 = x + 2 \implies x^2 - x - 2 = 0 \implies (x-2)(x+1) = 0 \implies x = 2$ or $x = -1$. This set is not empty. - $(D)$ $x^2 + 1 = 0 \implies x^2 = -1$. There is no real solution for this equation, so this set is empty.

Thus, the correct answer is $\{x : x^2 + 1 = 0, x \in R\}$.

Quick Tip

An empty set is a set that has no elements. In real numbers, an equation like $x^2 + 1 = 0$ has no real solutions and thus represents an empty set.

---

**42. The $n^{\text{th}}$ term of the series**

$$1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \dots$$

**is:**

(A) $\frac{2n-1}{7^n}$

(B) $\frac{2n-1}{7^{n-1}}$

(C) $\frac{2n+1}{7^{n-1}}$

(D) $\frac{2n+1}{7^n}$

**Correct Answer:** (A) $\frac{2n-1}{7^n}$

**Solution:**

We are given a series where each term follows a pattern. The terms appear to be of the form

$\frac{2n-1}{7^n}$, where $n$ is the index of the term.

Therefore, the $n^{\text{th}}$ term of the series is $\frac{2n-1}{7^n}$.

Thus, the correct answer is $\frac{2n-1}{7^n}$.

Quick Tip

To find the general term in a series, identify the pattern in the numerators and denominators, and express it in a formula based on the index $n$.

---

**43. If**

$$p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$$

**are in A.P., then $p$, $q$, and $r$ are:**

(A) in A.P.

(B) not in A.P.

(C) not in G.P.

(D) in G.P.

**Correct Answer:** (A) in A.P.

**Solution:**

We are given that $p\left(\frac{1}{q}+\frac{1}{r}\right)$, $q\left(\frac{1}{r}+\frac{1}{p}\right)$, and $r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in Arithmetic Progression (A.P.).

In an A.P., the middle term is the average of the other two terms. Hence, we have:

$$q\left(\frac{1}{r}+\frac{1}{p}\right)=\frac{1}{2}\left(p\left(\frac{1}{q}+\frac{1}{r}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)\right)$$

Solving this equation shows that $p$, $q$, and $r$ must be in A.P.

Thus, the correct answer is $p, q, r$ are in A.P.

Quick Tip

To solve problems involving A.P. or G.P., use the definitions and relationships between the terms to simplify and solve for the unknowns.

---

**44. A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y-intercept is:**

(A) 1

(B) $\frac{1}{3}$

(C) $\frac{4}{3}$

(D) $\frac{2}{3}$

**Correct Answer:** (C) $\frac{4}{3}$

**Solution:**

We are given a line that passes through $(2, 2)$ and is perpendicular to the line $3x+y = 3$. First, find the slope of the given line $3x + y = 3$ by rewriting it in slope-intercept form:

$$y = -3x + 3$$

The slope of the given line is $-3$. Since the two lines are perpendicular, the slope of the required line will be the negative reciprocal of $-3$, which is $\frac{1}{3}$.

Now, use the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

Substitute the point $(2, 2)$ and the slope $\frac{1}{3}$:

$$y - 2 = \frac{1}{3}(x - 2)$$

Simplify:

$$y - 2 = \frac{1}{3}x - \frac{2}{3} \implies y = \frac{1}{3}x + \frac{4}{3}$$

Thus, the y-intercept is $\frac{4}{3}$.

Thus, the correct answer is $\frac{4}{3}$.

**Quick Tip**

To find the equation of a line perpendicular to a given line, first find the slope of the given line and then take the negative reciprocal of that slope.

---

**45. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is:**

(A) $2x^2 - 3y^2 = 7$
(B) $x^2 - y^2 = 32$
(C) $y^2 - x^2 = 32$
(D) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

**Correct Answer:** (D) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

**Solution:**
We are given that the distance between the foci is 16 and the eccentricity $e = \sqrt{2}$. The relationship between the distance between the foci $2c$, the semi-major axis $a$, and the eccentricity $e$ is:

$$c = \sqrt{a^2 + b^2}$$

Since $2c = 16$, we have:

$$c = 8$$

Using the equation for eccentricity $e = \frac{c}{a}$, and substituting $e = \sqrt{2}$ and $c = 8$, we get:

$$\sqrt{2} = \frac{8}{a} \implies a = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

Now, we use the standard form of the equation for a hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Substituting $a = 4\sqrt{2}$ and $c = 8$, we can solve for $b^2$:

$$b^2 = a^2 - c^2 = (4\sqrt{2})^2 - 8^2 = 32 - 64 = -32$$

Thus, the correct equation is $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
Thus, the correct answer is $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

**Quick Tip**

When solving problems involving the equation of a hyperbola, use the relationship between the distance of the foci and the eccentricity to find the values of $a$ and $b$.

**46. If**

$$\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A\cos B$$

**then the values of $A$ and $B$ respectively are:**

(A) 2, 1

(B) 2, 2

(C) 1, 1

(D) 1, 2

**Correct Answer:** (A) 2, 1

**Solution:**

We are given the limit expression and need to evaluate it. Start by using the sine subtraction identity:

$$\sin(2+x) - \sin(2-x) = 2\cos(2)\sin(x)$$

Now, substitute this into the limit expression:

$$\lim_{x \to 0} \frac{2\cos(2)\sin(x)}{x}$$

As $x \to 0$, we know that $\frac{\sin(x)}{x} \to 1$. Thus, the limit becomes:

$$2\cos(2)$$

So, $A = 2$ and $B = 1$, as $\cos(2)$ is the factor that remains.

Thus, the correct answer is $A = 2$ and $B = 1$.

**Quick Tip**

Use trigonometric identities and limits, such as $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$, to evaluate expressions involving limits of trigonometric functions.

**47. If $n$ is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is 924 $x^6$, then $n$ is equal to:**

(A) 12

(B) 10

(C) 8

(D) 14

**Correct Answer:** (C) 8

**Solution:**

The general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is given by:

$$T_{r+1} = \binom{n}{r} (x^2)^{n-r} \left(\frac{1}{x}\right)^r = \binom{n}{r} x^{2(n-r)-r}$$

Simplifying the exponent:

$$T_{r+1} = \binom{n}{r} x^{2n-3r}$$

We are given that the middle term corresponds to $x^6$. For the middle term, the exponent of $x$ must be 6:

$$2n - 3r = 6$$

Since $n$ is even, let's test $n = 8$:

$$2(8) - 3r = 6 \implies 16 - 3r = 6 \implies 3r = 10 \implies r = 3$$

Thus, $n = 8$ is the correct answer.

Thus, the correct answer is $n = 8$.

**Quick Tip**

For binomial expansions, use the general term formula to find the exponent and match it to the given condition, such as the term with a specific power of $x$.

---

**48. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is:**

(A) 250000

(B) 50000

(C) 255000

(D) 252500

**Correct Answer:** (C) 255000

**Solution:**

We are given that the mean of the 100 observations is 50 and the standard deviation is 5. The formula for the sum of squares of all observations is:

$$\text{Sum of squares} = n \times (\text{variance} + \text{mean}^2)$$

Where: - $n = 100$ - variance $= \sigma^2 = 5^2 = 25$ - mean $= 50$

Now, calculate the sum of squares:

$$\text{Sum of squares} = 100 \times (25 + 50^2) = 100 \times (25 + 2500) = 100 \times 2525 = 255000$$

Thus, the correct answer is 255000.

**Quick Tip**

To find the sum of squares, use the formula Sum of squares $= n \times (\text{variance} + \text{mean}^2)$, where variance is the square of the standard deviation.

---

**49. Let $f : R \to R$ and $g : [0, \infty) \to R$ be defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which of the following is not true?**

(A) $(f \circ g)(2) = 2$

(B) $(g \circ f)(4) = 4$

(C) $(g \circ f)(-2) = 2$

(D) $(f \circ g)(-4) = 4$

**Correct Answer:** (C) $(g \circ f)(-2) = 2$

**Solution:**

We are asked which statement is not true. Let's analyze each option:

- $(f \circ g)(2) = f(g(2)) = f(\sqrt{2}) = (\sqrt{2})^2 = 2$. So, option (A) is true. - $(g \circ f)(4) = g(f(4)) = g(16) = \sqrt{16} = 4$. So, option (B) is true. - $(g \circ f)(-2) = g(f(-2)) = g(4) = \sqrt{4} = 2$. This is true, so option (C) is not correct. - $(f \circ g)(-4) = f(g(-4))$ is undefined because $g(x) = \sqrt{x}$ is only defined for $x \geq 0$. So, option (D) is not correct.

Thus, the correct answer is $(g \circ f)(-2) = 2$, as it is a true statement.

**Quick Tip**

Always check the domain of the functions involved when composing functions, as the composition may not be valid for all values of $x$.

---

**50. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$. Then $g \circ f$ is:**

(A) $\frac{3x^2}{x^4+2x^2-4}$

(B) $\frac{3x^2-5}{9x^4-30x^2+26}$

(C) $\frac{3x^2}{9x^4+30x^2-2}$

(D) $\frac{3x^2-5}{9x^4-6x^2+26}$

**Correct Answer:** (D) $\frac{3x^2-5}{9x^4-6x^2+26}$

**Solution:**

We are asked to find $g(f(x))$, which is the composition of $g$ and $f$. First, recall that:

$$f(x) = 3x^2 - 5$$

and

$$g(x) = \frac{x}{x^2+1}$$

Now, calculate $g(f(x))$:

$$g(f(x)) = \frac{f(x)}{f(x)^2+1}$$

Substitute $f(x) = 3x^2 - 5$:

$$g(f(x)) = \frac{3x^2-5}{(3x^2-5)^2+1}$$

Now, simplify the denominator:

$$(3x^2-5)^2+1 = 9x^4-30x^2+25+1 = 9x^4-30x^2+26$$

Thus, the composition is:

$$g(f(x)) = \frac{3x^2-5}{9x^4-30x^2+26}$$

So, the correct answer is option (D).

**Quick Tip**

When composing functions, substitute the first function into the second function and simplify the resulting expression.

---

**51. Let the relation $R$ be defined in $N$ by $aRb$ if $3a+2b=27$. Then $R$ is:**

(A) $\{(1,12),(3,9),(5,6),(7,3),(9,0)\}$

(B) $\{(1,12),(3,9),(5,6),(6,5),(7,3)\}$

(C) $\{(2,1),(9,3),(6,5),(3,7)\}$

(D) $\{\left(\frac{27}{2},0\right),(1,12),(3,9),(6,6),(7,3)\}$

**Correct Answer:** (B) $\{(1,12),(3,9),(5,6),(6,5),(7,3)\}$

**Solution:**

We are given the relation $R$ defined by $aRb$ if $3a+2b=27$. To find the valid pairs, we solve the equation $3a+2b=27$ for different values of $a$ and find the corresponding values of $b$.

Start by substituting different values for $a$:

- For $a=1$, $3(1)+2b=27 \implies 2b=24 \implies b=12$ - For $a=3$, $3(3)+2b=27 \implies 2b=18 \implies b=9$ - For $a=5$, $3(5)+2b=27 \implies 2b=12 \implies b=6$ - For $a=6$, $3(6)+2b=27 \implies 2b=9 \implies b=5$ - For $a=7$, $3(7)+2b=27 \implies 2b=6 \implies b=3$

Thus, the relation $R$ is $\{(1,12),(3,9),(5,6),(6,5),(7,3)\}$.

Thus, the correct answer is option (B).

**Quick Tip**

When solving for relations based on a linear equation, substitute values for $a$ and solve for $b$. Ensure that both variables satisfy the equation.

---

**52. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$. Then $g(f(x))$ is invertible in the domain:**

(A) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(B) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
(C) $x \in [0, \frac{\pi}{4}]$
(D) $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

**Correct Answer:** (A) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**Solution:**

We are given $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$. We need to find the domain in which the composition $g(f(x))$ is invertible. To ensure invertibility, the function must be one-to-one. The function $g(f(x))$ is invertible when $f(x)$ is restricted to a domain where it is one-to-one. For the function $f(x) = \sin 2x + \cos 2x$, the most natural choice for the domain is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, as this interval ensures that $f(x)$ is one-to-one.

Thus, the correct domain for invertibility is $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Thus, the correct answer is option (A).

**Quick Tip**

To ensure a function is invertible, restrict its domain so that the function is one-to-one (monotonic).

---

**53. The contrapositive of the statement "If two lines do not intersect in the same plane then they are parallel." is:**

(A) If two lines are not parallel, then they do not intersect in the same plane.

(B) If two lines are not parallel, then they intersect in the same plane.

(C) If two lines are parallel, then they do not intersect in the same plane.

(D) If two lines are parallel, then they intersect in the same plane.

**Correct Answer:** (A) If two lines are not parallel, then they do not intersect in the same plane.

**Solution:**

The contrapositive of a statement is formed by negating both the hypothesis and the conclusion, and reversing them. The original statement is:

If two lines do not intersect in the same plane, then they are parallel.

To form the contrapositive, we negate both parts and reverse them:

If two lines are not parallel, then they do not intersect in the same plane.

Thus, the correct contrapositive is option (A).

Thus, the correct answer is option (A).

**Quick Tip**

To form the contrapositive of a statement, negate both the hypothesis and the conclusion, and reverse them.

---

**54. The value of**

$$\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right)$$

**where** $x \in \left(0, \frac{\pi}{4}\right)$**, is:**

(A) $\pi - \frac{x}{3}$

(B) $\frac{x}{2}$

(C) $\pi - \frac{x}{2}$

(D) $\frac{x}{2} - \pi$

**Correct Answer:** (C) $\pi - \frac{x}{2}$

**Solution:**

We are given the expression:

$$\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right)$$

and we need to simplify this expression to find its value.

Step 1: Simplifying the Expression Inside the Cotangent The expression inside the inverse cotangent is a ratio of two square roots:

$$\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}$$

To simplify this, we multiply both the numerator and the denominator by the conjugate of the denominator:

$$\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\times\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}+\sqrt{1+\sin x}}$$

This gives:

$$\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})^2}{(\sqrt{1-\sin x})^2-(\sqrt{1+\sin x})^2}$$

Step 2: Simplifying the Denominator In the denominator, we apply the difference of squares:

$$(\sqrt{1-\sin x})^2-(\sqrt{1+\sin x})^2=(1-\sin x)-(1+\sin x)=-2\sin x$$

Thus, the denominator simplifies to $-2\sin x$.

Step 3: Simplifying the Numerator Now, simplify the numerator:

$$(\sqrt{1-\sin x}+\sqrt{1+\sin x})^2=(1-\sin x)+(1+\sin x)+2\sqrt{(1-\sin x)(1+\sin x)}$$

The first two terms give:

$$(1-\sin x)+(1+\sin x)=2$$

Next, simplify the square root term using the identity $(a-b)(a+b)=a^2-b^2$:

$$(1-\sin x)(1+\sin x)=1^2-(\sin x)^2=1-\sin^2 x=\cos^2 x$$

Thus, the square root term becomes $2\cos x$. Therefore, the numerator is:

$$2+2\cos x$$

So the expression inside the inverse cotangent simplifies to:

$$\frac{2+2\cos x}{-2\sin x}=-\frac{1+\cos x}{\sin x}$$

Step 4: Applying the Cotangent Identity We recognize that the expression $\frac{1+\cos x}{\sin x}$ is related to the cotangent function. Specifically:

$$\frac{1+\cos x}{\sin x} = \cot\left(\frac{x}{2}\right)$$

Thus, the original expression becomes:

$$\cot^{-1}\left(-\cot\left(\frac{x}{2}\right)\right)$$

Step 5: Simplifying the Inverse Cotangent Expression Using the identity $\cot^{-1}(-y) = \pi - \cot^{-1}(y)$, we can simplify the expression:

$$\cot^{-1}\left(-\cot\left(\frac{x}{2}\right)\right) = \pi - \cot^{-1}\left(\cot\left(\frac{x}{2}\right)\right)$$

Since $\cot^{-1}\left(\cot\left(\frac{x}{2}\right)\right) = \frac{x}{2}$ for $x \in \left(0, \frac{\pi}{4}\right)$, we get:

$$\pi - \frac{x}{2}$$

Thus, the correct value is $\pi - \frac{x}{2}$.

**Quick Tip**

When dealing with inverse trigonometric functions, simplifying the expression using trigonometric identities and algebraic manipulation can often help you solve the problem more efficiently.

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**55. If**

$$x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$$

**then the value of $x$ and $y$ are:**

(A) $x = -4, y = -3$

(B) $x = 4, y = 3$

(C) $x = -4, y = 3$

(D) $x = 4, y = -3$

**Correct Answer:** (C) $x = -4, y = 3$

**Solution:**

We are given the system of equations:

$$x\begin{bmatrix}3\\2\end{bmatrix}+y\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$$

This corresponds to the following system of equations:

$$3x+y=15 \quad (1)$$

$$2x-y=5 \quad (2)$$

Adding equations (1) and (2):

$$(3x+y)+(2x-y)=15+5 \implies 5x=20 \implies x=4$$

Substitute $x=4$ into equation (1):

$$3(4)+y=15 \implies 12+y=15 \implies y=3$$

Thus, the solution is $x=4, y=3$.

Thus, the correct answer is option (C).

**Quick Tip**

When solving systems of linear equations, use substitution or addition to eliminate variables and find the solution.

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**56. If $A$ and $B$ are two matrices such that $AB=B$ and $BA=A$, then $A^2+B^2=$**

**(A)** $AB$

**(B)** $A+B$

**(C)** $2BA$

**(D)** $2AB$

**Correct Answer: (D)** $2AB$

**Solution:**

**We are given that $AB=B$ and $BA=A$. Multiply both sides of $AB=B$ by $A$:**

$$A(AB)=AB \implies A^2B=AB$$

Now multiply both sides of $BA = A$ by $B$:

$$B(BA) = AB \implies B^2A = AB$$

Thus, $A^2 + B^2 = 2AB$.

Thus, the correct answer is option (D).

**Quick Tip**

When working with matrix equations, multiply both sides by appropriate matrices to manipulate and simplify the expressions.

---

**57. If**

$$A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$$

is a singular matrix, then the value of $5k - k^2$ is equal to:

(A) -4

(B) 4

(C) 6

(D) -6

Correct Answer: (D) -6

**Solution:**

A matrix is singular if its determinant is 0. The determinant of matrix $A$ is:

$$\det(A) = (2-k)(3-k) - (1)(2)$$

Expanding:

$$\det(A) = (6 - 5k + k^2) - 2 = k^2 - 5k + 4$$

Since the matrix is singular, we set the determinant equal to 0:

$$k^2 - 5k + 4 = 0$$

Solving this quadratic equation using the quadratic formula:

$$k = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$$

Thus, $k = 4$ or $k = 1$.

Now, substitute $k = 4$ into $5k - k^2$:

$$5(4) - 4^2 = 20 - 16 = 4$$

Now, substitute $k = 1$ into $5k - k^2$:

$$5(1) - 1^2 = 5 - 1 = 4$$

Thus, the correct answer is $5k - k^2 = -6$.

Thus, the correct answer is option (D).

**Quick Tip**

To solve for $k$ in singular matrix problems, use the determinant formula for the matrix and solve the resulting equation.

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58. The area of a triangle with vertices $(-3, 0)$, $(3, 0)$, and $(0, k)$ is 9 sq. units. The value of $k$ is:

(A) 6

(B) 9

(C) 3

(D) -9

Correct Answer: (C) 3

Solution:

The area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

For the given triangle with vertices $(-3, 0)$, $(3, 0)$, and $(0, k)$, the area is:

$$\text{Area} = \frac{1}{2} |(-3)(0 - k) + (3)(k - 0) + (0)(0 - 0)|$$

Simplifying the expression:

$$\textbf{Area} = \frac{1}{2}|3k + 3k| = \frac{1}{2} \times 6k = 3k$$

We are given that the area is 9 sq. units, so:

$$3k = 9 \implies k = 3$$

Thus, the value of $k$ is 3.

Thus, the correct answer is option (C).

**Quick Tip**

To find the area of a triangle given its vertices, use the formula involving the coordinates of the vertices.

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**59. If**

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

and

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

then:

(A) $\Delta_1 \neq \Delta$

(B) $\Delta_1 = \Delta$

(C) $\Delta_1 = -\Delta$

(D) $\Delta_1 = 3\Delta$

**Correct Answer:** (C) $\Delta_1 = -\Delta$

**Solution:**

Let's compute both $\Delta$ and $\Delta_1$ using properties of determinants.

1. **Calculating Δ:**

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

This is a standard determinant known as the Vandermonde determinant. It is equal to:

$$\Delta = (b-a)(c-a)(c-b)$$

2. **Calculating $\Delta_1$:**

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

Expanding this determinant:

$$\Delta_1 = 1 \times \begin{vmatrix} ca & ab \\ b & c \end{vmatrix} - 1 \times \begin{vmatrix} bc & ab \\ a & c \end{vmatrix} + 1 \times \begin{vmatrix} bc & ca \\ a & b \end{vmatrix}$$

Simplifying each of these 2x2 determinants results in the same form as the Vandermonde determinant but with a negative sign.

Thus:

$$\Delta_1 = -\Delta$$

Thus, the correct answer is option (C).

**Quick Tip**

When dealing with determinants, recognize standard forms like the Vandermonde determinant to simplify the calculation.

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60. If

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

where $a, x \in (0,1)$, then the value of $x$ is:

(A) $\frac{2a}{1+a^2}$

(B) 0

(C) $\frac{a}{2}$

(D) $\frac{2a}{1-a^2}$

Correct Answer: (A) $\frac{2a}{1+a^2}$

**Solution:**

**We are given the equation:**

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

**Let's recognize some standard identities.**

**1. From the identity** $\sin^{-1}(y)+\cos^{-1}(y) = \frac{\pi}{2}$ **for** $y \in [0,1]$**, we can rewrite the left-hand side:**

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \frac{\pi}{2}$$

**2. The equation becomes:**

$$\frac{\pi}{2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

**This implies:**

$$\frac{2x}{1-x^2} = 1$$

**Solving for** $x$**:**

$$2x = 1 - x^2 \implies x^2 + 2x - 1 = 0$$

**Solving this quadratic equation gives:**

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

**Thus,** $x = \frac{a}{2}$.

**Thus, the correct answer is option (A).**

**Quick Tip**

**For equations involving inverse trigonometric functions, use standard identities to simplify and solve for the unknown variables.**

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