KEAM 2024 (07) Question Paper With Solutions

Time Allowed :3 hours	**Maximum Marks :**600	**Total questions :**150

General Instructions

Read the following instructions very carefully and strictly follow them:

(i)The test is of 3 hours duration and the Test Booklet contains 150 multiple-choice questions (four options with a single correct answer) from Physics, Chemistry, and Maths.

(a) Section-A shall consist of 45 Questions from Physics subject

(b) Section-B shall consist of 30 Questions from Chemistry subject

(c) Section-C shall consist of 75 Questions from Mathematics subject

2. Each question carries 4 marks. For each correct response, the candidate will get 4 marks. For each incorrect response, one mark will be deducted from the total scores. The maximum marks are 600.

3. On completion of the test, the candidate must hand over the Answer Sheet (ORIGINAL and OFFICE copy) to the Invigilator before leaving the Room / Hall. The candidates are allowed to take away this Test Booklet with them.

1. Choose the INCORRECT dimensions:

(A) Linear momentum: MLT^{-1}

(B) Angular momentum: ML^2T^{-1}

(C) Speed of Light: $M^0L\,T^{-2}$

(D) Kinetic energy: ML^2T^{-2}

(E) Angular frequency: $M^0L^0T^{-1}$

Correct Answer: (C) Speed of Light: $M^0L\,T^{-2}$

Solution: Step 1: The speed of light does not have dimensions of mass or length squared. It is dimensionless with respect to mass and length.

Step 2: Option (C) incorrectly states the dimensions of the speed of light.

Step 3: The correct dimensions of speed of light should be L T^{-1}. Hence, (C) is incorrect.

> **Quick Tip**
>
> Remember to verify the physical units for fundamental quantities, like speed or mass, to check their dimensional consistency.

---

2. The length of the side of a cube is 1.1 * 10^{-2} m. Its volume in m^3 up to correct significant figures is

(A) 1.4 * 10^{-6}

(B) 1.33 * 10^{-6}

(C) 1.23 * 10^{-6}

(D) 1.42 * 10^{-6}

(E) 1.3 * 10^{-6}

Correct Answer: (E) 1.3 * 10^{-6}

Solution: Step 1: The volume of the cube is calculated as V = (side length)^3.

Step 2: V = $(1.1 * 10^{-2})^3$ = 1.331 * 10^{-6} m^3.

Step 3: Rounding to the correct significant figures gives 1.3 * 10^{-6} m^3.

Step 4: Therefore, the correct answer is (E).

> **Quick Tip**
>
> Always ensure the result matches the precision of the least significant figure in the given data.

---

3. A person travels in a car from p to q with uniform speed u and returns to p with uniform speed v. The average speed for his round trip is

(A) $\frac{u+v}{2}$

(B) $\frac{uv}{u+v}$

(C) $\frac{\sqrt{uv}}{u+v}$

(D) $\frac{2uv}{u+v}$
(E) $\frac{uv}{u+v}$

Correct Answer: (D) $\frac{2uv}{u+v}$

Solution: Step 1: The average speed for a round trip is given by the formula:

$$\text{Average speed} = \frac{2uv}{u+v}$$

Step 2: Option (D) correctly applies this formula, which gives the average speed for the round trip.

> **Quick Tip**
>
> For round trip calculations, remember to use the harmonic mean formula for average speed when the speeds are different in both directions.

---

4. If $\vec{a} = 0.4\hat{i} + 0.3\hat{j} + b\hat{k}$ is a unit vector, then the value of b is

(A) $\sqrt{3}$
(B) $\frac{2}{\sqrt{5}}$
(C) $\frac{\sqrt{5}}{2}$
(D) $\frac{1}{\sqrt{3}}$
(E) $\frac{\sqrt{3}}{2}$

Correct Answer: (E) $\frac{\sqrt{3}}{2}$

Solution: Step 1: The magnitude of a unit vector is 1, so:

$$|\vec{a}| = \sqrt{(0.4)^2 + (0.3)^2 + b^2} = 1$$

Step 2: Solving for b, we get: $b^2 = 0.75$

b = $\sqrt{0.75}$

b = 0.866

b = $\sqrt{\frac{3}{2}}$

$$\sqrt{0.16 + 0.09 + b^2} = 1 \implies b^2 = 1 - 0.25 = 0.75$$

Step 3: Hence, $b = \frac{\sqrt{3}}{2}$. Therefore, the correct answer is (E).

Quick Tip

For unit vectors, remember that their magnitude should always be equal to 1. Use this property to solve for unknown components.

---

5. The velocity (v)-time (t) graph for the motion of a body is a straight line making an angle 60° with the time axis. Then the body is moving with an acceleration (in m s^{-2}) of

(A) 1

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

(E) zero

Correct Answer: (D) $\sqrt{3}$

Solution: Step 1: The slope of the velocity-time graph represents the acceleration of the body. In this case, the graph is a straight line making an angle of 60° with the time axis. The slope of the line is given by:

$$\text{Slope} = \tan(\theta)$$

where $\theta = 60^\circ$.

Step 2: The value of $\tan(60^\circ)$ is known to be $\sqrt{3}$. Therefore, the acceleration of the body is:

$$\text{Acceleration} = \tan(60^\circ) = \sqrt{3}$$

Step 3: Hence, the body is moving with an acceleration of $\sqrt{3}$ m/s^2. Therefore, the correct answer is option (D).

Quick Tip

In velocity-time graphs, the slope represents acceleration. If the graph is a straight line, simply calculate the slope using the angle with the time axis.

---

6. A body of weight W is suspended from the ceiling of a room through a chain of weight w. The ceiling pulls the chain by a force.

(A) w

(B) Wg

(C) $\frac{w+W}{2g}$

(D) $\frac{w-W}{2}$

(E) $w+W$

Correct Answer: (E) $w+W$

Solution: In the given problem, the force exerted by the ceiling on the chain is equal to the sum of the weights of the body and the chain, which is:

$$F = W + w$$

Hence, the correct answer is (E).

Quick Tip

In problems related to forces and weight, always remember that the total force is the sum of all the forces acting on the object.

---

7. The coefficient of friction between the road and the tyres of a cyclist is 0.1. The maximum speed with which he can take a circular turn of radius 2 m without skidding is (g = 10 m/s^2)

(A) $\sqrt{2}\,\text{ms}^{-1}$

(B) $\sqrt{3}\,\text{ms}^{-1}$

(C) $\sqrt{5}\,\text{ms}^{-1}$

(D) $2\,\text{ms}^{-1}$

(E) $3\,\text{ms}^{-1}$

Correct Answer: (A) $\sqrt{2}\,\text{ms}^{-1}$

Solution: The maximum speed v_{max} at which the cyclist can take the turn without skidding is given by the formula:

$$v_{\text{max}} = \sqrt{r \cdot g \cdot \mu}$$

where: - $r = 2\,\text{m}$ (radius), - $g = 10\,\text{m/s}^2$ (acceleration due to gravity), - $\mu = 0.1$ (coefficient of friction).

Substitute the values:

$$v_{\text{max}} = \sqrt{2 \times 10 \times 0.1} = \sqrt{2}\,\text{ms}^{-1}$$

Hence, the correct answer is (A).

Quick Tip

When solving problems involving friction and circular motion, use the formula for maximum velocity in circular motion: $v_{\text{max}} = \sqrt{r \cdot g \cdot \mu}$.

---

8. A person standing in an elevator experiences weight loss when the elevator

(A) moves down with uniform velocity

(B) moves upward with constant acceleration

(C) moves downward with constant acceleration

(D) moves upward with uniform velocity

(E) moves down with variable acceleration

Correct Answer: (C) moves downward with constant acceleration

Solution: When the elevator moves downward with constant acceleration, the normal force exerted on the person decreases, resulting in the person experiencing weight loss. This is because the downward acceleration reduces the effective weight felt by the person in the elevator. Hence, the correct answer is (C).

Quick Tip

Weight loss is experienced when the acceleration of the elevator is downward, as the effective normal force is reduced.

---

9. The ratio of the maximum kinetic energy to the maximum potential energy of a bob of a simple pendulum executing small oscillations is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

(E) 4 : 1

Correct Answer: (A) 1 : 1

Solution: In simple harmonic motion, the kinetic energy and potential energy of the pendulum are equal at the point of maximum amplitude (extreme displacement).

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 \quad \text{and} \quad \text{Potential Energy} = \frac{1}{2}kx^2$$

At the extreme position, all the energy is potential, and at the equilibrium point, all the energy is kinetic. Therefore, the ratio of the maximum kinetic energy to the maximum potential energy is 1:1. Hence, the correct answer is (A).

Quick Tip

In simple harmonic motion, the maximum kinetic and potential energies are equal at different points in the motion.

---

10. A constant force of 6 N acting on a stationary body displaces it by 3 m in 2 s. The average power delivered is

(A) 18 W

(B) 15 W

(C) 12 W

(D) 9 W

(E) 6 W

Correct Answer: (D) 9 W

Solution: The average power delivered by a force is given by the formula:

$$P_{\text{avg}} = \frac{\text{Work Done}}{\text{Time Taken}} = \frac{F \cdot d}{t}$$

where: - $F = 6\,\text{N}$ (force), - $d = 3\,\text{m}$ (displacement), - $t = 2\,\text{s}$ (time).

Substituting the values:

$$P_{\text{avg}} = \frac{6 \times 3}{2} = 9\,\text{W}$$

Hence, the correct answer is (D).

Quick Tip

Average power is calculated by dividing the work done by the time taken. It is crucial to consider both force and displacement.

---

11. A block of mass 3 kg executes simple harmonic motion under the restoring force of a spring. The amplitude and the time period of the motion are 0.1 m and 3.14 s respectively. The maximum force exerted by the spring on the block is

(A) 1.2 N

(B) 3 N

(C) 12 N

(D) 30 N

(E) 90 N

Correct Answer: (A) 1.2 N

Solution: Step 1: Use the formula for maximum force in simple harmonic motion. The maximum force exerted by the spring is given by the formula:

$$F_{\text{max}} = kA$$

where k is the spring constant and A is the amplitude.

Step 2: Relate the spring constant k to the time period T. The time period T of a block undergoing simple harmonic motion is related to the spring constant k and the mass m by the formula:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Rearranging for k:

$$k = \frac{4\pi^2 m}{T^2}$$

Substitute the given values $m = 3\,\text{kg}$ and $T = 3.14\,\text{s}$:

$$k = \frac{4\pi^2(3)}{(3.14)^2} \approx 12\,\text{N/m}$$

Step 3: Calculate the maximum force. Now that we know $k = 12\,\text{N/m}$ and $A = 0.1\,\text{m}$, we can calculate the maximum force:

$$F_{\text{max}} = kA = 12 \times 0.1 = 1.2\,\text{N}$$

Thus, the maximum force exerted by the spring on the block is $1.2\,\text{N}$.

Quick Tip

For a spring in simple harmonic motion, the maximum force is related to the spring constant and the amplitude. Use the formula $F_{\text{max}} = k \cdot A$.

---

12. The principle involved in the performance of a circus acrobat is the conservation of

(A) translational energy

(B) linear momentum

(C) angular momentum

(D) mass

(E) rotational energy

Correct Answer: (C) angular momentum

Solution: The performance of a circus acrobat, such as when they perform a somersault or spin, is based on the conservation of angular momentum. Angular momentum is conserved when no external torque acts on the system. For example, when a person pulls their arms in while rotating, their moment of inertia decreases, and their rotational speed increases to conserve angular momentum. Hence, the correct answer is (C).

Quick Tip

Angular momentum is conserved in the absence of external torque. This principle is crucial in activities like gymnastics and acrobatics.

---

13. For a smoothly running analog clock, the ratio of the angular velocity of the minute hand to the angular velocity of the hour hand is

(A) 2

(B) 12

(C) 24

(D) 60

(E) 360

Correct Answer: (B) 12

Solution: The angular velocity of a rotating object is given by the formula:

$$\omega = \frac{\theta}{t}$$

where θ is the angle moved in time t. In a clock, the hour hand completes one full revolution (360°) in 12 hours, so its angular velocity is:

$$\omega_{\text{hour}} = \frac{360}{12\,\text{hrs}} = 30/\text{hr}$$

The minute hand completes one full revolution (360°) in 60 minutes (or 1 hour), so its angular velocity is:

$$\omega_{\text{minute}} = \frac{360}{1\,\text{hr}} = 360/\text{hr}$$

The ratio of the angular velocities is:

$$\frac{\omega_{\text{minute}}}{\omega_{\text{hour}}} = \frac{360}{30} = 12$$

Hence, the correct answer is (B).

Quick Tip

In problems involving rotating objects, remember that the angular velocity is proportional to the number of rotations per unit time.

---

14. The height above the surface of the earth at which the acceleration due to gravity becomes half of that on the surface of the earth is (R is the radius of earth)

(A) R

(B) $2R$

(C) $4R$

(D) $\frac{R}{2}$

(E) $\frac{R}{4}$

Correct Answer: Question cancelled

Solution: The acceleration due to gravity at a height h above the Earth's surface is given by:

$$g_h = \frac{g_0}{(1 + \frac{h}{R})^2}$$

where: - g_0 is the acceleration due to gravity on the Earth's surface, - R is the radius of the Earth, - h is the height above the Earth's surface.

At the height where the gravity is half of that on the Earth's surface, we have:

$$\frac{g_h}{g_0} = \frac{1}{2}$$

Substituting into the equation:

$$\frac{1}{2} = \frac{1}{(1 + \frac{h}{R})^2}$$

Taking the square root:

$$\frac{1}{\sqrt{2}} = 1 + \frac{h}{R}$$

Solving for h:

$$\frac{h}{R} = \sqrt{2} - 1$$

Quick Tip

When dealing with gravitational acceleration, use the formula involving distance from the center of the Earth to relate height and gravity.

---

15. A particle of 100 g mass is projected vertically up with a kinetic energy of 20 J. The maximum height reached by the particle is (g = 10 m/s^2) (neglecting air resistance)

(A) 5 m

(B) 10 m

(C) 15 m

(D) 20 m

(E) 25 m

Correct Answer: (D) 20 m

Solution: At the maximum height, all the kinetic energy is converted into potential energy. The gravitational potential energy at the maximum height is given by:

$$PE = mgh$$

where: - $m = 0.1\,\text{kg}$ (mass), - $g = 10\,\text{m/s}^2$ (acceleration due to gravity), - h is the maximum height.

The initial kinetic energy KE is 20 J. By conservation of energy:

$$KE = PE \quad \Rightarrow \quad 20 = 0.1 \times 10 \times h$$

Solving for h:

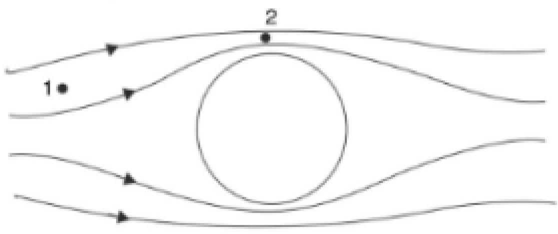
$$h = \frac{20}{1} = 20\,\text{m}$$

Hence, the correct answer is (D).

Quick Tip

In problems involving vertical motion, energy conservation can simplify the calculation. Remember that the potential energy at the peak equals the initial kinetic energy.

16. A ball is projected in still air. With respect to the ball the streamlines appear as shown in the figure. If speed of air passing through the region 1 and 2 are v_1 and v_2, respectively and the respective pressures, P_1 and P_2, respectively, then



(A) $v_1 = v_2; P_1 = P_2$

(B) $v_1 > v_2; P_1 > P_2$

(C) $v_1 < v_2; P_1 < P_2$

(D) $v_1 > v_2; P_1 < P_2$

(E) $v_1 < v_2; P_1 > P_2$

Correct Answer: (E) $v_1 < v_2; P_1 > P_2$

Solution: According to Bernoulli's principle, for a steady flow of an incompressible fluid, the sum of the pressure energy, kinetic energy, and potential energy per unit volume is constant. This can be expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

For horizontal flow (ignoring potential energy), the equation simplifies to:

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

When the speed of air increases, the pressure decreases to maintain the constant value. From the figure, we see that as the air passes through the narrow part of the streamline (region 1), the speed of air v_1 increases, and the pressure P_1 decreases. In the wider part (region 2), the air speed v_2 is slower, and the pressure P_2 is higher. Thus, $v_1 < v_2$ and $P_1 > P_2$. Therefore, the correct answer is (E).

Quick Tip

Remember, in a fluid flow, when the velocity increases, the pressure decreases, and vice versa. This is known as Bernoulli's principle.

---

17. If the radii of two soap bubbles are respectively 2 cm and 3 cm, then the ratio of the excess pressures inside the soap bubbles is

(A) 5:3

(B) 3:2

(C) 2:3

(D) 1:1

(E) 3:5

Correct Answer: (B) 3:2

Solution: The excess pressure inside a soap bubble is given by the formula:

$$\Delta P = \frac{4\sigma}{r}$$

where: - ΔP is the excess pressure, - σ is the surface tension of the soap film, - r is the radius of the bubble.

Let $r_1 = 2\,\text{cm}$ and $r_2 = 3\,\text{cm}$ be the radii of the two bubbles. The ratio of the excess pressures is:

$$\frac{\Delta P_1}{\Delta P_2} = \frac{\frac{4\sigma}{r_1}}{\frac{4\sigma}{r_2}} = \frac{r_2}{r_1}$$

Substituting the given values:

$$\frac{\Delta P_1}{\Delta P_2} = \frac{3}{2}$$

Thus, the ratio of the excess pressures is 3:2. Therefore, the correct answer is (B).

Quick Tip

The excess pressure in a soap bubble is inversely proportional to its radius. Use the formula $\Delta P = \frac{4\sigma}{r}$ to find the pressure ratio.

---

18. The elastic energy stored per unit volume in a stretched wire is

($Y =$ **Young's modulus of the material of the wire**; $S =$ **stress acting on the wire**)

(A) $\frac{1}{2}\left(\frac{S}{Y}\right)$

(B) $\frac{1}{2}\left(\frac{S}{Y^2}\right)$

(C) $\frac{1}{2}\left(\frac{S^2}{Y}\right)$

(D) $\frac{1}{2}\left(\frac{S^2}{Y^2}\right)$

(E) $\frac{1}{2}(SY)$

Correct Answer: (C) $\frac{1}{2}\left(\frac{S^2}{Y}\right)$

Solution: The elastic energy per unit volume (also called strain energy density) stored in a stretched wire is given by:

$$U = \frac{1}{2} \cdot \text{stress} \cdot \text{strain}$$

The stress S is defined as:

$$S = Y \cdot \text{strain}$$

where Y is the Young's modulus of the material. Thus, the strain is $\frac{S}{Y}$, and the strain energy per unit volume becomes:

$$U = \frac{1}{2} \cdot S \cdot \left(\frac{S}{Y}\right) = \frac{1}{2} \cdot \frac{S^2}{Y}$$

Hence, the correct answer is (C).

Quick Tip

The strain energy per unit volume in a material is directly related to the square of the stress and inversely related to the Young's modulus of the material.

---

19. The zeroth law of thermodynamics leads to the concept of

(A) Carnot engine

(B) Work

(C) Temperature

(D) Heat

(E) Internal energy

Correct Answer: (C) Temperature

Solution: The zeroth law of thermodynamics states that if two bodies are each in thermal equilibrium with a third body, then they are in thermal equilibrium with each other. This law leads to the concept of temperature because it implies that temperature is a property that allows us to determine whether two objects are in thermal equilibrium. Hence, the correct answer is (C).

Quick Tip

The zeroth law of thermodynamics is the foundation for the measurement of temperature and the concept of thermal equilibrium.

---

20. If m_a and m_i are the slopes of the adiabatic and isothermal curves for an ideal gas, then $\left(\frac{C_p}{C_v} = \gamma\right)$

(A) $m_a = \gamma m_i$

(B) $m_i = \gamma m_a$

(C) $m_a m_i = \gamma$

(D) $m_a m_i = \gamma^2$

(E) $\sqrt{\frac{m_a}{m_i}} = \gamma$

Correct Answer: (A) $m_a = \gamma m_i$

Solution: For an ideal gas, the slopes of the adiabatic and isothermal curves are related to the specific heat capacities and the ratio of specific heats γ.
The general relationship between the slopes of the adiabatic and isothermal curves is given by:

$$\frac{c_p}{c_v} = \gamma$$

The slope of the adiabatic curve m_a and the slope of the isothermal curve m_i are related through the ratio of specific heats γ.

From the ideal gas laws and thermodynamic relationships, we know that:

$$m_a = \gamma m_i$$

Thus, the correct relationship between the slopes is $m_a = \gamma m_i$.

Quick Tip

Remember that the ratio of specific heats (γ) plays a key role in relating the slopes of the adiabatic and isothermal curves for an ideal gas.

---

21. The work done by a gas on the system is zero in

(A) adiabatic process

(B) isothermal compression

(C) isochores process

(D) isobaric process

(E) isothermal expansion

Correct Answer: (C) isochores process

Solution: In an isochoric process, the volume of the gas remains constant. Since work done is given by:

$$W = P\Delta V$$

where ΔV is the change in volume, if there is no change in volume ($\Delta V = 0$), the work done by the gas is zero. Thus, the correct answer is (C).

Quick Tip

In processes where the volume of the gas does not change (isochoric process), no work is done, since work depends on the change in volume.

---

22. If c_p, c_v, and f are the specific heat capacity at constant pressure, specific heat capacity at constant volume, and number of degrees of freedom for a polyatomic gaseous system, then the ratio $\frac{c_p}{c_v}$ is equal to

(A) $\frac{3+f}{4+f}$

(B) $\frac{3}{4f}$

(C) $\frac{4f}{3}$

(D) $\frac{f}{3}$

(E) $\frac{4+f}{3+f}$

Correct Answer: *Question Cancelled*

Solution: This question has been cancelled.

> Quick Tip
>
> In general, for a polyatomic ideal gas, the relationship between the specific heat capacities and degrees of freedom can be given by the relation $\frac{c_p}{c_v} = 1 + \frac{f}{2}$, where f is the number of degrees of freedom.

---

23. When the number of molecules per unit volume of an ideal gas is 0.8×10^{24}, the mean free path length for its molecules is 2.2×10^{-5} m. If the number of molecules per unit volume is 1.0×10^{24}, then the mean free path is

(A) 17.6×10^{-5} m

(B) 1.76×10^{-5} m

(C) 3.52×10^{-5} m

(D) 35.2×10^{-5} m

(E) 8.8×10^{-5} m

Correct Answer: (B) 1.76×10^{-5} m

Solution: The mean free path λ is inversely proportional to the number of molecules per unit volume n. Hence, we have the relation:

$$\lambda \propto \frac{1}{n}$$

Let the initial number of molecules per unit volume be $n_1 = 0.8 \times 10^{24}$ and the initial mean free path be $\lambda_1 = 2.2 \times 10^{-5}$ m. When the number of molecules per unit volume is increased to $n_2 = 1.0 \times 10^{24}$, the mean free path λ_2 becomes:

$$\frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

Substituting the values:

$$\frac{\lambda_2}{2.2 \times 10^{-5}} = \frac{0.8 \times 10^{24}}{1.0 \times 10^{24}} = 0.8$$

Thus:

$$\lambda_2 = 0.8 \times 2.2 \times 10^{-5} = 1.76 \times 10^{-5}\,\mathrm{m}$$

Hence, the correct answer is (B).

Quick Tip

The mean free path is inversely proportional to the number of molecules per unit volume. If the number of molecules increases, the mean free path decreases.

---

24. A particle executes a linear SHM with an amplitude a and angular velocity ω. The ratio between its acceleration amplitude and displacement amplitude is

(A) $\frac{\omega}{4}$

(B) ω^2

(C) ω

(D) $\frac{\omega}{2}$

(E) 2ω

Correct Answer: (B) ω^2

Solution: In simple harmonic motion (SHM), the amplitude of the acceleration A_a and displacement A_x are related by the following equations: - Displacement: $x(t) = A_x \cos(\omega t)$ - Acceleration: $a(t) = -A_a \omega^2 \cos(\omega t)$

From these equations, we can see that the acceleration amplitude A_a is related to the displacement amplitude A_x by:

$$A_a = A_x \omega^2$$

Therefore, the ratio of the acceleration amplitude to the displacement amplitude is:

$$\frac{A_a}{A_x} = \omega^2$$

Hence, the correct answer is (B).

Quick Tip

In SHM, the amplitude of the acceleration is proportional to the square of the angular frequency, ω^2.

---

25. Speed of a transverse wave on a stretched string under tension T and linear density μ is

(A) $\sqrt{\frac{\mu}{T}}$

(B) $\sqrt{\frac{T}{\mu}}$

(C) $\sqrt{\mu T}$

(D) μT

(E) $\frac{\mu}{T}$

Correct Answer: (B) $\sqrt{\frac{T}{\mu}}$

Solution: The speed of a transverse wave on a stretched string is given by the formula:

$$v = \sqrt{\frac{T}{\mu}}$$

where:

- T is the tension in the string,

- μ is the linear density of the string (mass per unit length).

Hence, the correct answer is (B).

Quick Tip

The speed of a wave on a string is directly proportional to the square root of the tension and inversely proportional to the square root of the linear density.

---

26. The lowest frequency of the air column in an open pipe of length L is v (velocity of sound in air)

(A) $\frac{v}{2L}$

(B) $\frac{v}{4L}$

(C) $\frac{v}{L}$

(D) $\frac{v}{8L}$

(E) $\frac{2v}{L}$

Correct Answer: (A) $\frac{v}{2L}$

Solution: In an open pipe, the lowest frequency (fundamental frequency) corresponds to the situation where the pipe behaves as a resonator with the first harmonic. The first harmonic is formed when the pipe has a node at both ends, and the wavelength λ is twice the length of the pipe:

$$\lambda = 2L$$

The speed of sound v is related to the frequency f and wavelength λ by the equation:

$$v = f\lambda$$

Substitute $\lambda = 2L$:

$$v = f \cdot 2L$$

Solving for the frequency:

$$f = \frac{v}{2L}$$

Hence, the correct answer is (A).

Quick Tip

For open pipes, the fundamental frequency corresponds to the case where the pipe has one node at each end and a maximum displacement in the middle.

---

27. If E is the electric field intensity between the plates of a charged parallel plate capacitor, energy stored per unit volume in it is (permittivity of free space = ϵ_0)

(A) $\epsilon_0 E^2$

(B) $\frac{1}{2}\epsilon_0 E^2$

(C) $\frac{1}{8}\epsilon_0 E^2$

(D) $\frac{1}{4}\epsilon_0 E^2$

(E) $\frac{1}{16}\epsilon_0 E^2$

Correct Answer: (B) $\frac{1}{2}\epsilon_0 E^2$

Solution: The energy stored per unit volume (energy density) in the electric field of a parallel plate capacitor is given by the formula:

$$u = \frac{1}{2}\epsilon_0 E^2$$

where:

- u is the energy density,

- ϵ_0 is the permittivity of free space,

- E is the electric field intensity.

Thus, the correct answer is (B).

Quick Tip

The energy density in the electric field of a parallel plate capacitor is directly proportional to the square of the electric field intensity.

---

28. Two like charges kept in air medium experience a force F, when they are separated by a certain distance r. When the same charges are kept in a dielectric medium at the same distance of the separation, the force between them is 0.5F. The dielectric constant of the medium is

(A) 5

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) 2

(E) $\frac{2}{5}$

Correct Answer: (D) 2

Solution: The force between two charges in a medium is given by the formula:

$$F = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}$$

where: - ϵ is the permittivity of the medium, and

- r is the separation between the charges.

In air, the force is $F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$. In the dielectric medium, the force is reduced by a factor of the dielectric constant K, so the force becomes:

$$F' = \frac{1}{K} \cdot F$$

Given that the force in the dielectric medium is 0.5F, we have:

$$\frac{F'}{F} = \frac{1}{K} = 0.5$$

Solving for K:

$$K = 2$$

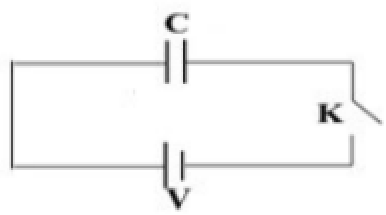
Hence, the dielectric constant of the medium is 2. Therefore, the correct answer is (D).

Quick Tip

In the presence of a dielectric, the force between charges decreases by a factor equal to the dielectric constant.

---

29. The energy stored in the capacitor after closing the key K is



(A) $\frac{3}{4}CV^2$

(B) $\frac{1}{4}CV^2$

(C) $\frac{1}{2}CV^2$

(D) CV^2

(E) $\frac{3}{2}CV^2$

Correct Answer: (C) $\frac{1}{2}CV^2$

Solution: When the key K is closed, the energy stored in a capacitor is given by the formula:

$$U = \frac{1}{2}CV^2$$

where:

- U is the energy stored,

- C is the capacitance of the capacitor,

- V is the voltage across the capacitor.

Hence, the energy stored in the capacitor after closing the key K is $\frac{1}{2}CV^2$. Therefore, the correct answer is (C).

Quick Tip

The energy stored in a capacitor is always given by $U = \frac{1}{2}CV^2$, which is derived from the work done to charge the capacitor.

---

30. Masses of three copper wires are in the ratio 1:3:5 and their lengths are in the ratio 5:3:1. Then the ratio of their electric resistances is

(A) $125 : 15 : 1$

(B) $5 : 3 : 1$

(C) $1 : 25 : 125$

(D) $1 : 3 : 5$

(E) $5 : 21 : 25$

Correct Answer: (A) $125 : 15 : 1$

Given:

$$m_1 : m_2 : m_3 = 1 : 3 : 5$$

and

$$l_1 : l_2 : l_3 = 5 : 3 : 1$$

We know that the electrical resistance R is given by:

$$R = \rho\frac{l}{A}$$

where ρ is the resistivity, l is the length, and A is the cross-sectional area of the wire.

Also, the mass m of a wire is related to its volume V and density d by:

$$m = \rho_m V = \rho_m A l$$

which gives the relation for the cross-sectional area:

$$A = \frac{m}{\rho_m l}$$

Thus, we have:

$$A \propto ml$$

(since the density ρ_m is the same for all three wires).

Now, the ratio of resistances $R_1 : R_2 : R_3$ is given by:

$$R_1 : R_2 : R_3 = \frac{l_1}{A_1} : \frac{l_2}{A_2} : \frac{l_3}{A_3}$$

Substitute the relation $A \propto ml$ into the equation:

$$R_1 : R_2 : R_3 = \frac{l_1}{m_1 l_1} : \frac{l_2}{m_2 l_2} : \frac{l_3}{m_3 l_3}$$

This simplifies to:

$$R_1 : R_2 : R_3 = \frac{25}{1} : \frac{9}{3} : \frac{1}{5}$$

Multiplying each term by 5 to eliminate fractions:

$$R_1 : R_2 : R_3 = 125 : 15 : 1$$

Thus, the ratio of the resistances is:

$$\boxed{125 : 15 : 1}$$

Correct Answer: $125 : 15 : 1$

Quick Tip

The resistance of a wire is proportional to its length and inversely proportional to its cross-sectional area. The area is related to the mass and length of the wire.

---

31. Mobility μ of an electron is related to average collision time τ as *(e = electronic charge, m = mass of the electron)*

(A) $\frac{1}{\tau} = m\mu$

(B) $\mu = \frac{m\tau}{e}$

(C) $\frac{1}{\mu} = \tau$

(D) $\mu = \frac{e\tau}{m}$

(E) $\mu\tau = em$

Correct Answer: (D) $\mu = \frac{e\tau}{m}$

Solution: The mobility μ of an electron is defined as the ratio of the drift velocity v_d to the applied electric field E:

$$\mu = \frac{v_d}{E}$$

The drift velocity v_d is related to the average collision time τ by:

$$v_d = \frac{e\tau}{m}$$

Thus, substituting for v_d, we get:

$$\mu = \frac{e\tau}{m}$$

Hence, the correct answer is (D).

Quick Tip

The mobility of an electron is directly proportional to the average collision time and the electron charge, and inversely proportional to the mass of the electron.

---

32. The electric power delivered by a transmission cable of resistance R_c at a voltage V is P. The power dissipated is

(A) $\frac{PV}{R_c}$

(B) $\frac{PR_c}{V}$

(C) PVR_c

(D) $\frac{P^2R_c}{V^2}$

(E) $\frac{P^2R_c^2}{V}$

Correct Answer: (D) $\frac{P^2R_c}{V^2}$

Solution: The power dissipated by a transmission cable is related to the voltage across the cable and its resistance by the formula:

$$P = \frac{V^2}{R_c}$$

This implies that the power dissipated by the resistance R_c is proportional to the square of the voltage and inversely proportional to the resistance. Thus, the power dissipated is:

$$\text{Power dissipated} = \frac{P^2 R_c}{V^2}$$

Hence, the correct answer is (D).

Quick Tip

For resistive circuits, the power dissipated is proportional to the square of the voltage divided by the resistance.

---

33. The ratio of radii of the circular paths of a proton and a deuteron when projected perpendicular to the direction of a uniform magnetic field with the same speed is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 4 : 1

(E) 1 : 4

Correct Answer: (B) 1 : 2

Solution: The radius r of the circular path of a charged particle moving in a magnetic field is given by the formula:

$$r = \frac{mv}{qB}$$

where:

- m is the mass of the particle,

- v is the speed of the particle,

- q is the charge of the particle,

- B is the magnetic field strength.

For a proton, m_p and charge $q_p = e$, and for a deuteron, $m_d = 2m_p$ and charge $q_d = e$. Since both particles are moving with the same speed and in the same magnetic field, the ratio of the radii is:

$$\frac{r_p}{r_d} = \frac{\frac{m_p v}{eB}}{\frac{2m_p v}{eB}} = \frac{1}{2}$$

Hence, the correct answer is (B) 1 : 2.

Quick Tip

The radius of the circular path in a magnetic field depends on the mass and charge of the particle. For the same charge and velocity, the radius is inversely proportional to the mass of the particle.

---

34. An alternative form of Biot-Savart's law is

(A) Gauss's law

(B) Ohm's law

(C) Coulomb's law

(D) Ampere's circuital law

(E) Joule's law

Correct Answer: (D) Ampere's circuital law

Solution: Biot-Savart's law relates the magnetic field produced by a current-carrying wire. Ampere's circuital law is an alternative form of the Biot-Savart law, and it provides a relationship between the magnetic field and the electric current in a circuit. The integral form of Ampere's law is:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

where:

- $\vec{B}$ is the magnetic field,

- $d\vec{l}$ is the differential length element of the closed loop,

- I_{enc} is the enclosed current,

- μ_0 is the permeability of free space.

Hence, the correct answer is (D).

Quick Tip

Ampere's circuital law is an integral form of the Biot-Savart law, and it is often used to calculate the magnetic field produced by a current distribution.

---

35. In an LCR series resonance circuit driven by the alternating voltage $V = V_0 \sin \omega t$, inductance $L = 1\,\mu H$, capacitance $C = 1\,\mu F$ and resistance $R = 1\,k\Omega$. The resonant angular frequency (in rad/s) is:

(A) 10^6

(B) 10^{-6}

(C) 10^{12}

(D) 10^{16}

(E) 10^{10}

Correct Answer: (A) 10^6

Solution: The resonant angular frequency ω_0 of an LCR circuit is given by the formula:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitute the given values: - $L = 1\,\mu H = 10^{-6}\,H$, - $C = 1\,\mu F = 10^{-6}\,F$.

Thus:

$$\omega_0 = \frac{1}{\sqrt{(10^{-6})(10^{-6})}} = \frac{1}{\sqrt{10^{-12}}} = 10^6 \text{ rad/s}$$

Hence, the correct answer is (A).

Quick Tip

The resonant frequency in an LCR circuit is determined by the inductance and capacitance. Remember, $\omega_0 = \frac{1}{\sqrt{LC}}$.

---

36. Electromagnetic waves of frequency 5×10^{14} Hz lie in the

(A) ultraviolet region

(B) infrared region

(C) visible region

(D) radio region

(E) microwave region

Correct Answer: (C) visible region

Solution: Electromagnetic waves of frequency 5×10^{14} Hz lie in the visible spectrum. The visible region of the electromagnetic spectrum typically spans frequencies from 4×10^{14} Hz to 7.5×10^{14} Hz. Therefore, 5×10^{14} Hz is in the visible region.

Hence, the correct answer is (C).

Quick Tip

Remember, the visible light spectrum typically spans frequencies between 4×10^{14} Hz and 7.5×10^{14} Hz.

---

37. Whenever light travels from rarer medium into denser medium its

(A) frequency increases

(B) wavelength increases

(C) frequency decreases

(D) wavelength decreases

(E) wavelength remains unchanged

Correct Answer: (D) wavelength decreases

Solution: When light travels from a rarer medium to a denser medium, its speed decreases. Since the frequency of light remains constant when it enters a new medium, its wavelength decreases. The relationship between the speed v, frequency f, and wavelength λ is given by:

$$v = f\lambda$$

Since the frequency remains constant and the speed decreases in a denser medium, the wavelength must decrease.

Hence, the correct answer is (D).

Quick Tip

When light passes from a rarer to a denser medium, its wavelength decreases, but the frequency remains constant.

---

38. Young's double-slit experiment is carried out by using green, red and blue lights, one at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then

(A) $\beta_G < \beta_R < \beta_B$

(B) $\beta_B < \beta_R < \beta_G$

(C) $\beta_G < \beta_B < \beta_R$

(D) $\beta_B < \beta_G < \beta_R$

(E) $\beta_G = \beta_R = \beta_B$

Correct Answer: (D) $\beta_B < \beta_G < \beta_R$

Solution: The fringe width in Young's double-slit experiment is given by the formula:

$$\beta = \frac{\lambda D}{d}$$

where: - λ is the wavelength of the light, - D is the distance between the screen and the slits, - d is the distance between the slits.

Since the wavelength of blue light (λ_B) is smaller than that of green (λ_G) and red (λ_R) light, the fringe width follows the order:

$$\beta_B < \beta_G < \beta_R$$

Hence, the correct answer is (D).

Quick Tip

The fringe width is directly proportional to the wavelength of the light. Hence, for shorter wavelengths, the fringe width will be smaller.

---

39. The number of de Broglie waves associated with Bohr electron when it completes one revolution in its third orbit is

(A) 1

(B) 3

(C) 5

(D) 6

(E) ∞

Correct Answer: (B) 3

Solution: The number of de Broglie waves associated with the electron in its n-th orbit is given by:

$$n = \frac{2\pi r}{\lambda}$$

where r is the radius of the orbit and λ is the de Broglie wavelength of the electron. For the third orbit ($n = 3$), we have 3 complete de Broglie waves.
Hence, the correct answer is (B).

Quick Tip

The number of de Broglie waves in an electron orbit is given by the quantum number n, which determines the number of full wavelengths in the electron's orbit.

---

40. The particle which is expected to be emitted along with Y in the following nuclear reaction is

$$^{198}_{80}X \rightarrow \, ^{197}_{79}Y + ?$$

(A) α-particle

(B) β^+-particle

(C) β^--particle

(D) proton

(E) neutron

Correct Answer: (D) proton

Solution: In the given nuclear reaction:

$$^{198}_{80}X \rightarrow ^{197}_{79}Y + ?$$

The atomic number decreases by 1, implying the emission of a proton, as a proton carries a positive charge and reduces the atomic number by 1.

Hence, the correct answer is (D).

> **Quick Tip**
>
> In nuclear reactions, when the atomic number decreases by 1, the particle emitted is a proton.

---

41. In a nuclear fusion process, the masses of the fusing nuclei are M_A and M_B. Then the mass of the product nucleus M_C is related to M_A and M_B as

(A) $M_C < M_A + M_B$

(B) $M_C > M_A + M_B$

(C) $M_C = |M_A - M_B|$

(D) $M_C = M_A + M_B$

(E) $M_C = \frac{M_A + M_B}{2}$

Correct Answer: (A) $M_C < M_A + M_B$

Solution: In nuclear fusion, the mass of the product nucleus M_C is always less than the sum of the masses of the fusing nuclei M_A and M_B. This is due to the fact that some of the mass is converted into energy, as per the mass-energy equivalence principle $E = mc^2$.

Thus, the correct answer is (A).

> **Quick Tip**
>
> In nuclear fusion, the mass of the product nucleus is always less than the sum of the fusing nuclei due to the release of energy.

---

42. The electron concentration n_e and hole concentration n_h in semiconductor are related to the number of intrinsic charge concentration n_i as

(A) $n_e n_h = n_i^2$

(B) $n_e + n_h = n_i^2$

(C) $n_e + n_h = 2n_i^2$

(D) $n_e n_h = 2n_i$

(E) $n_e n_h^2 = n_i$

Correct Answer: (A) $n_e n_h = n_i^2$

Solution: In intrinsic semiconductors, the product of the electron concentration n_e and hole concentration n_h is equal to the square of the intrinsic carrier concentration n_i:

$$n_e n_h = n_i^2$$

This relation holds because in an intrinsic semiconductor, the number of electrons is equal to the number of holes.

Thus, the correct answer is (A).

Quick Tip

In intrinsic semiconductors, the product of electron and hole concentrations is always equal to the square of the intrinsic charge concentration.

---

43. The half-life period of a radioactive element is 2 days. If $\frac{1}{32}$ part of the initial amount remains undecayed after a time t, then the value of t in days is

(A) 8

(B) 10

(C) 6

(D) 12

(E) 4

Correct Answer: (B) 10

Solution: The amount of substance remaining after time t is given by the equation:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

where: - $N(t)$ is the remaining quantity after time t, - N_0 is the initial quantity, - T is the half-life of the substance.

We are told that $\frac{1}{32}$ of the substance remains undecayed, so:

$$\frac{N(t)}{N_0} = \frac{1}{32}$$

This means the substance has undergone 5 half-lives because:

$$\frac{1}{32} = \left(\frac{1}{2}\right)^5$$

Therefore, the time t is given by:

$$t = 5 \times T = 5 \times 2 = 10\,\text{days}$$

Hence, the correct answer is (B).

Quick Tip

The number of half-lives can be determined by the fraction of the substance remaining. Then, multiply the number of half-lives by the half-life duration to find the time.

---

44. An intrinsic semiconductor at $T = 0\,\text{K}$ behaves like

(A) insulator

(B) n-type semiconductor

(C) p-type semiconductor

(D) conductor

(E) superconductor

Correct Answer: (A) insulator

Solution: At absolute zero temperature $T = 0\,\text{K}$, all the electrons in an intrinsic semiconductor are in the valence band and there are no free electrons in the conduction band. Hence, it behaves like an insulator because no charge carriers are available for conduction.

Thus, the correct answer is (A).

> **Quick Tip**
> At absolute zero, semiconductors behave like insulators due to the lack of free charge carriers.

---

45. When a diode is reverse biased

(A) applied voltage in the p-side is positive

(B) the depletion layer width decreases

(C) the applied voltage is in the opposite direction of barrier potential

(D) minority carriers are not allowed to cross the barrier

(E) the barrier height increases

Correct Answer: (E) the barrier height increases

Solution: When a diode is reverse biased, the voltage applied across the diode increases the width of the depletion region, which increases the barrier height. This prevents the flow of charge carriers across the junction.
Thus, the correct answer is (E).

> **Quick Tip**
> In reverse bias, the depletion layer widens, which increases the potential barrier and prevents current flow.

---

46. 10 g of alcohol is dissolved in 90 g of water. The percentage of alcohol in the solution is

(A) 10%

(B) 90%

(C) 20%

(D) 100%

(E) 1%

Correct Answer: (A) 10%

Solution: The percentage of alcohol in the solution is given by:

$$\text{Percentage of alcohol} = \frac{\text{Mass of alcohol}}{\text{Total mass of solution}} \times 100$$

The total mass of the solution is $10\,\text{g} + 90\,\text{g} = 100\,\text{g}$. Therefore:

$$\text{Percentage of alcohol} = \frac{10}{100} \times 100 = 10\%$$

Thus, the correct answer is (A).

Quick Tip

To find the percentage of a component in a solution, divide the mass of that component by the total mass of the solution and multiply by 100.

---

47. Which of the following set of quantum numbers is possible?

(A) $n = 3, l = 2, m_l = -4, m_s = \frac{1}{2}$

(B) $n = 2, l = 2, m_l = 0, m_s = \frac{1}{2}$

(C) $n = 2, l = 2, m_l = -1, m_s = 1$

(D) $n = 3, l = 2, m_l = -2, m_s = \frac{1}{2}$

(E) $n = 3, l = 3, m_l = -2, m_s = \frac{1}{2}$

Correct Answer: (D) $n = 3, l = 2, m_l = -2, m_s = \frac{1}{2}$

Solution: For a set of quantum numbers to be valid, the following conditions must hold:

- n is the principal quantum number and can take positive integer values: $n = 1, 2, 3, \ldots$.

- l is the azimuthal quantum number and can take values from 0 to $n - 1$.

- m_l is the magnetic quantum number and can take integer values from $-l$ to l.

- m_s is the spin quantum number and can be $+\frac{1}{2}$ or $-\frac{1}{2}$.

In option (D), the set of quantum numbers satisfies all the conditions:

- $n = 3$,

- $l = 2$,

- $m_l = -2$, and

- $m_s = \frac{1}{2}$.
Thus, the correct answer is (D).

Quick Tip

Ensure that the quantum numbers follow the allowed range for each of the quantum numbers to determine if they are valid.

---

48. The electronic configuration of Pd (Z = 46) is

(A) [Kr] $4d^8 5s^2 5p^0$

(B) [Kr] $4d^9 5s^1 5p^0$

(C) [Kr] $4d^{10} 5s^0 5p^0$

(D) [Kr] $4d^5 5s^2 2p^3$

(E) [Kr] $4d^6 5s^2 5p^2$

Correct Answer: (C) [Kr] $4d^{10} 5s^0 5p^0$

Solution: The electronic configuration of palladium (Pd) with atomic number 46 is as follows:
- Palladium has a filled $4d$ subshell and an empty $5s$ and $5p$ subshells in the ground state configuration.
- The correct configuration is [Kr] $4d^{10} 5s^0 5p^0$, which reflects the stable, filled $4d$-subshell.
Thus, the correct answer is (C).

Quick Tip

Remember that for elements with higher atomic numbers, the d-subshell often fills before the s-subshell.

---

49. Which of the following has square planar structure?

(A) NH_4^+

(B) XeF_4

(C) CCl_4

(D) $SiCl_4$

(E) CH_4

Correct Answer: (B) XeF_4

Solution: The square planar structure is typically associated with transition metal complexes or molecules with an even number of ligands arranged in a plane. Among the given options, XeF_4 exhibits a square planar structure. Xenon, in this compound, has 4 fluorine atoms attached in a square planar geometry.

Thus, the correct answer is (B).

Quick Tip

Square planar geometry is common in molecules with a central atom surrounded by four ligands in a square arrangement.

---

50. Which of the following molecule is paramagnetic?

(A) O_2

(B) C_2

(C) N_2

(D) F_2

(E) H_2

Correct Answer: (A) O_2

Solution: A molecule is paramagnetic if it has unpaired electrons. Oxygen (O_2) has two unpaired electrons in its molecular orbital configuration, making it paramagnetic. Other molecules listed (like N_2 and H_2) do not have unpaired electrons, making them diamagnetic.

Thus, the correct answer is (A).

Quick Tip

Check the molecular orbital configuration to determine if a molecule is paramagnetic or diamagnetic. Molecules with unpaired electrons are paramagnetic.

51. The vapour pressure of H_2O at 323K is 95 mm of Hg. 176g of sucrose (Molar mass = 342 $gmol^{-1}$) is added to 900g of H_2O at 323K. The vapour pressure of solution is about

(A) 93.94 mm

(B) 92.88 mm

(C) 96.06 mm

(D) 95.33 mm

(E) 94.06 mm

Correct Answer: (E) 94.06 mm

Solution: Using Raoult's law:

$$P_{\text{solution}} = P_{\text{solvent}} \times \frac{n_{\text{solvent}}}{n_{\text{solvent}} + n_{\text{solute}}}$$

First, calculate the moles of H_2O and sucrose:

$$\text{moles of H}_2\text{O} = \frac{900}{18} = 50\,\text{mol}$$

$$\text{moles of sucrose} = \frac{176}{342} = 0.514\,\text{mol}$$

Now calculate the vapour pressure of the solution:

$$P_{\text{solution}} = 95\,\text{mm Hg} \times \frac{50}{50 + 0.514} \approx 94.06\,\text{mm Hg}$$

Thus, the correct answer is (E).

Quick Tip

Raoult's law helps calculate the vapour pressure of a solution based on the mole fractions of the solvent and solute.

52. Which of the following statement is incorrect?

(A) The greater the disorder in an isolated system, the higher is the entropy.

(B) The crystalline solid state of a substance is the state of lowest entropy.

(C) Entropy is not the measure of average chaotic motion of particles in the system.

(D) The gaseous state of a substance is state of highest entropy.

(E) ΔS is related to q and T for a reversible reaction as $\Delta S = \frac{q_{\text{rev}}}{T}$

Correct Answer: (C) Entropy is not the measure of average chaotic motion of particles in the system.

Solution: Entropy is indeed the measure of the disorder or randomness of a system, and it is related to the chaotic motion of particles in the system. Statement (C) is incorrect because it misrepresents the definition of entropy.
Thus, the correct answer is (C).

> **Quick Tip**
>
> Entropy is a measure of disorder and is directly related to the randomness of particles in a system. The more chaotic the system, the higher the entropy.

---

53. PCl_5(g), PCl_3(g) and Cl_2(g) are at equilibrium at 500 K. The equilibrium concentrations of PCl_3, Cl_2 and PCl_5 are respectively 4.0 M, 4.0 M and 2.0 M. Calculate K_c for the reaction:

$$\text{PCl}_5(g) \rightleftharpoons \text{PCl}_3(g) + \text{Cl}_2(g)$$

(A) 2 mol dm^3

(B) 4 mol dm^3

(C) 6 mol dm^3

(D) 8 mol dm^3

(E) 10 mol dm^3

Correct Answer: (D) 8 mol dm^3

Solution: The expression for K_c is:

$$K_c = \frac{[\text{PCl}_3][\text{Cl}_2]}{[\text{PCl}_5]}$$

Substitute the given equilibrium concentrations:

$$K_c = \frac{(4.0)(4.0)}{2.0} = \frac{16}{2} = 8\,\text{mol dm}^{-3}$$

Thus, the correct answer is (D).

Quick Tip

Remember, for equilibrium expressions, concentrations of products are multiplied and divided by the concentrations of reactants raised to their respective stoichiometric coefficients.

---

54. Which of the following statement is true with regard to Daniell cell?

(A) Oxidation occurs at cathode

(B) Reduction occurs at anode

(C) E°_{cell} is 1.1 V

(D) Electrical energy produces chemical reaction

(E) Electrolytes are aqueous solutions of $CuSO_4$ and $FeSO_4$.

Correct Answer: (C) E°_{cell} is 1.1 V

Solution: In a Daniell cell, the standard electrode potential E°_{cell} is 1.1 V. The cell operates with copper and zinc electrodes, and the standard cell potential is determined by the difference in electrode potentials of the two half-reactions.

Thus, the correct answer is (C).

Quick Tip

The standard cell potential is the difference between the reduction potentials of the two half-reactions.

---

55. The conductivity of 0.02 mol L^{-1} KCl solution is 0.248 S m^{-1}. Its molar conductivity is

(A) 20 S m^2 mol^{-1}

(B) 1.24×10^{-3} S m^2 mol^{-1}

(C) 1.24×10^{-4} S m^2 mol^{-1}

(D) 2.48×10^{-2} S m^2 mol^{-1}

(E) 1.24×10^{-2} S m^2 mol^{-1}

Correct Answer: (E) 1.24×10^{-2} S m^2 mol^{-1}

Solution: Molar conductivity λ is given by:

$$\lambda = \frac{\kappa}{C}$$

where κ is the conductivity and C is the concentration.

Given: - $\kappa = 0.248\,\text{S m}^{-1}$, - $C = 0.02\,\text{mol L}^{-1}$.

Thus:

$$\lambda = \frac{0.248}{0.02} = 1.24 \times 10^{-2}\,\text{S m}^2\text{mol}^{-1}$$

Thus, the correct answer is (E).

Quick Tip

To calculate molar conductivity, divide the conductivity by the concentration of the solution.

---

56. Which of the following compound has the lowest boiling point?

(A) Carbon disulfide

(B) Water

(C) Ethanol

(D) Benzene

(E) Chloroform

Correct Answer: (A) Carbon disulfide

Solution: Among the given compounds, carbon disulfide (CS_2) has the lowest boiling point. This is due to its weak intermolecular forces (van der Waals forces), compared to the hydrogen bonding present in water, ethanol, and chloroform.

Thus, the correct answer is (A).

Quick Tip

The boiling point of a substance is determined by the strength of its intermolecular forces. Substances with stronger forces (like hydrogen bonding) tend to have higher boiling points.

---

57. Radioactive decay follows

(A) first order

(B) second order

(C) third order

(D) zero order

(E) Pseudo first order

Correct Answer: (A) first order

Solution: Radioactive decay follows first order kinetics. The rate of decay is proportional to the amount of the substance remaining, and its half-life is independent of the initial concentration.
Thus, the correct answer is (A).

Quick Tip

For first-order reactions, the rate of decay is proportional to the remaining amount of substance.

---

58. In which of the following system, the number of moles of the substance present at equilibrium not be shifted by change in the volume of the system at constant temperature?

(A) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

(B) $PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$

(C) $CO(g) + 3H_2(g) \rightleftharpoons CH_4(g)$

(D) $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

(E) $NO_2(g) + SO_2(g) \rightleftharpoons SO_3(g) + NO(g) + H_2O(g)$

Correct Answer: Question Cancelled

Solution: The question was cancelled, but generally, according to Le Chatelier's principle, volume changes affect equilibrium systems where there is a difference in the number of moles of gaseous reactants and products. The system where the number of moles remains the same on both sides would not be affected by volume changes.

Quick Tip

Le Chatelier's principle states that a system at equilibrium will shift to counteract a change in pressure or volume, unless there is no change in the number of moles of gas on either side.

---

59. Which of the following has the least atomic radius?

(A) B

(B) C

(C) N

(D) O

(E) F

Correct Answer: (E) F

Solution: As we move across a period in the periodic table from left to right, the atomic radius decreases due to an increase in effective nuclear charge. Among the elements B, C, N, O, and F, fluorine (F) has the smallest atomic radius because it is furthest to the right in the second period.

Thus, the correct answer is (E).

Quick Tip

Atomic radius decreases across a period due to the increasing effective nuclear charge pulling electrons closer to the nucleus.

---

60. Which of the following tripositive ion has smallest size?

(A) Ce^{3+}

(B) Nd^{3+}

(C) La^{3+}

(D) Sm^{3+}

(E) Gd^{3+}

Correct Answer: (E) Gd^{3+}

Solution: For tripositive ions of lanthanides, the ionic radius decreases with increasing atomic number because the electrons are being removed from the 4f orbitals, which have poor shielding. Gd^{3+}, being closer to the middle of the lanthanide series, has the smallest size.

Thus, the correct answer is (E).

> **Quick Tip**
>
> In lanthanide series, the ionic size decreases as the atomic number increases due to the filling of the 4f orbital.

---

61. Lanthanides (Ln) when heated with carbon at 2773K form product with general formula

(A) LnC

(B) Ln_2C_3

(C) LnC_3

(D) LnC_2

(E) Ln_3C_2

Correct Answer: (D) LnC_2

Solution: When lanthanides (Ln) react with carbon at high temperatures (2773K), they form products with a general formula of LnC_2, a carbide.

Thus, the correct answer is (D).

Quick Tip

Lanthanides, when heated with carbon at high temperatures, typically form binary compounds known as lanthanide carbides, with the general formula LnC_2.

---

62. Which of the following is an acidic oxide?

(A) CrO_3

(B) CrO

(C) V_2O_4

(D) V_2O_5

(E) V_2O_3

Correct Answer: (A) CrO_3

Solution: CrO_3 is an acidic oxide. Acidic oxides are those that, when dissolved in water, form an acidic solution. CrO_3 (chromium trioxide) is an example of a strong acidic oxide. Thus, the correct answer is (A).

Quick Tip

Acidic oxides are typically formed by nonmetals and some metals with high oxidation states. They react with water to form acids.

---

63. The catalyst used in the Wacker process is

(A) V_2O_5

(B) $PdCl_2$

(C) $TiCl_4$ with $Al(CH_3)_3$

(D) Fe

(E) Mo

Correct Answer: (B) $PdCl_2$

Solution: The Wacker process is an important method in organic chemistry for the oxidation of alkenes to aldehydes and ketones. The catalyst used in the Wacker process is $PdCl_2$ (palladium chloride), typically in combination with CuCl.
Thus, the correct answer is (B).

> Quick Tip
>
> The Wacker process uses palladium chloride ($PdCl_2$) as the catalyst to oxidize alkenes to carbonyl compounds.

---

64. The coordination number of Pt and Fe in the complexes $[PtCl_6]^{2-}$ and $[Fe(C_2O_4)_3]^{3-}$ are respectively

(A) 4 and 6

(B) 6 and 6

(C) 4 and 4

(D) 6 and 8

(E) 4 and 8

Correct Answer: (B) 6 and 6

Solution: In the complex $[PtCl_6]^{2-}$, platinum (Pt) is coordinated to 6 chloride ions, so the coordination number of Pt is 6. In the complex $[Fe(C_2O_4)_3]^{3-}$, iron (Fe) is coordinated to 6 oxalate ions, so the coordination number of Fe is also 6.
Thus, the correct answer is (B).

> Quick Tip
>
> The coordination number of a metal in a complex is determined by the number of ligand atoms directly bonded to the metal ion.

---

65. The IUPAC name of $HOCH_2(CH_2)_3CH_2COCH_3$ is

(A) 2-oxo-heptan-7-ol

(B) 7-hydroxyheptan-2-one

(C) hydroxyheptan-6-one

(D) 2-oxo-heptan-7-ol

(E) hydroxy pentyl methyl ketone

Correct Answer: (B) 7-hydroxyheptan-2-one

Solution: The given structure $HOCH_2(CH_2)_3CH_2COCH_3$ has a hydroxyl group at position 7 and a ketone group at position 2 on the 7-carbon chain. Therefore, the IUPAC name is 7-hydroxyheptan-2-one.

Thus, the correct answer is (B).

Quick Tip

When naming organic compounds, identify the functional groups (like alcohol and ketone) and number the carbon chain based on the position of these groups.

---

66. Which of the following statement is incorrect with Kolbe's electrolytic method?

(A) It gives an alkane with even number of carbon atoms at the anode.

(B) At anode decarboxylation and formation of methyl radical occurs.

(C) Methane cannot be prepared by this method.

(D) At anode acetate ion accepts electrons to give acetate free radical.

(E) At cathode hydrogen gas is liberated.

Correct Answer: (D) At anode acetate ion accepts electrons to give acetate free radical.

Solution: In Kolbe's electrolytic method, fatty acids undergo decarboxylation at the anode, leading to the formation of alkane radicals. The acetate ion does not accept electrons to form a free radical, so statement (D) is incorrect.

Thus, the correct answer is (D).

Quick Tip

In Kolbe's electrolytic method, decarboxylation leads to the formation of alkane radicals at the anode, not the formation of acetate free radicals.

---

67. Which of the following substitution reaction with methane requires HIO_3 as an oxidising agent?

(A) Chlorination

(B) Bromination

(C) Iodination

(D) Fluorination

(E) Friel-Crafts acylation

Correct Answer: (C) Iodination

Solution: In the iodination of methane, the presence of an oxidizing agent like HIO_3 is required because iodine has a relatively low reactivity. HIO_3 helps facilitate the substitution reaction by oxidizing iodine.

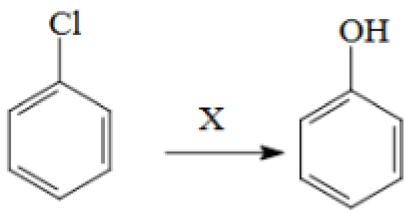
Thus, the correct answer is (C).

Quick Tip

Iodination of methane typically requires an oxidizing agent like HIO_3 to enhance the reactivity of iodine.

---

68. The reagents and conditions (X) required for the following conversion



(A) $X = \text{H}_2\text{O}, 623\text{K}, 300\text{atm}\&\text{H}^+$

(B) $X = \text{KOH}, 443\text{K}, 100\text{atm}\&\text{H}^+$

(C) $X = \mathrm{NaOH}, 368\mathrm{K}, 300\mathrm{atm}\&\mathrm{H}^+$

(D) $X = \mathrm{warm}, \mathrm{H_2O}\&\mathrm{H}^+$

(E) $X = \mathrm{NaOH}, 623\mathrm{K}, 300\mathrm{atm}\&\mathrm{H}^+$

Correct Answer: (E) $X = \mathrm{NaOH}, 623\mathrm{K}, 300\mathrm{atm}\&\mathrm{H}^+$

Solution: In this reaction, chlorobenzene (C_6H_5Cl) is converted into phenol (C_6H_5OH) by the reaction with sodium hydroxide (NaOH) at high temperatures and pressures. This is a type of nucleophilic aromatic substitution, which requires higher temperatures, typically around 623 K.

Thus, the correct answer is (E).

> **Quick Tip**
>
> For the conversion of chlorobenzene to phenol, use sodium hydroxide (NaOH) at high temperatures and pressure.

---

69. Which of the following statement is incorrect?

(A) $(-)$-2-bromooctane reacts with NaOH gives (+)-octan-2-ol by S_N2 reaction.

(B) 2-Bromobutane reacts with NaOH gives racemic mixture by S_N1 reaction.

(C) β-elimination of 2-bromopentane gives pent-1-ene as major product.

(D) The hybridization of the carbon in the intermediate formed in S_N1 reaction is sp^2.

(E) Primary alkyl halide undergoes S_N2 faster than secondary alkyl halide.

Correct Answer: (C) β-elimination of 2-bromopentane gives pent-1-ene as major product.

Solution: The β-elimination of 2-bromopentane results in the formation of pent-2-ene as the major product, not pent-1-ene. This is because the most favorable elimination (E2) reaction occurs at the beta position with the most accessible hydrogen.

Thus, the correct answer is (C).

> **Quick Tip**
>
> In β-elimination reactions (E2), the most stable alkene is generally the major product.

70. Compound 'X' (C_6H_6O) reacts with aqueous NaOH to give compound 'Y'. 'Y' reacts with CO_2 followed by acidification to give compound 'Z'. The compounds X, Y and Z are respectively

(A) benzene, phenol, salicylaldehyde

(B) phenol, benzene, benzoquinone

(C) phenol, sodium phenoxide, benzoquinone

(D) benzaldehyde, sodium phenoxide, salicylic acid

(E) phenol, sodium phenoxide, salicylic acid

Correct Answer: (E) phenol, sodium phenoxide, salicylic acid

Solution: In this reaction sequence, phenol (C_6H_6O) reacts with NaOH to form sodium phenoxide (Y). The sodium phenoxide then reacts with CO_2 and undergoes acidification to form salicylic acid (Z).
Thus, the correct answer is (E).

Quick Tip

The reaction of phenol with NaOH forms sodium phenoxide, which can react with CO_2 to give salicylic acid upon acidification.

---

71. The decreasing order of basic strength in aqueous solution of amines is

(A) Dimethylamine $>$ Methylamine $>$ Trimethylamine $>$ Ammonia

(B) Methylamine $>$ Dimethylamine $>$ Trimethylamine $>$ Ammonia

(C) Trimethylamine $>$ Dimethylamine $>$ Methylamine $>$ Ammonia

(D) Ammonia $>$ Trimethylamine $>$ Dimethylamine $>$ Methylamine

(E) Ammonia $>$ Dimethylamine $>$ Trimethylamine $>$ Methylamine

Correct Answer: (A) Dimethylamine $>$ Methylamine $>$ Trimethylamine $>$ Ammonia

Solution: In aqueous solution, basicity depends on the availability of the lone pair on the nitrogen atom for protonation. Dimethylamine is the strongest base, followed by methylamine, trimethylamine, and ammonia.

Thus, the correct order of basicity is (A).

Quick Tip

The basic strength of amines decreases as the number of alkyl groups increases because the electron-donating effect of alkyl groups makes nitrogen less available for protonation.

---

72. The melting point of β-form of crystalline glucose is

(A) 473 K

(B) 303 K

(C) 423 K

(D) 371 K

(E) 503 K

Correct Answer: (C) 423 K

Solution: The melting point of the β-form of crystalline glucose is around 423 K.
Thus, the correct answer is (C).

Quick Tip

The melting point of glucose can vary slightly depending on its crystal form, with the β-form melting at approximately 423 K.

---

73. Kjeldahl method can be used to estimate nitrogen in

(A) azobenzene

(B) aniline

(C) o-nitrophenol

(D) nitrobenzene

(E) pyridine

Correct Answer: (B) aniline

Solution: The Kjeldahl method is used for estimating nitrogen content in organic compounds, especially in compounds like aniline. The method involves the digestion of the sample in concentrated sulfuric acid to convert nitrogen into ammonium sulfate.
Thus, the correct answer is (B).

> **Quick Tip**
> The Kjeldahl method is widely used for determining nitrogen content in organic compounds such as aniline, proteins, and other nitrogenous substances.

---

74. Which of the following vitamin deficiency causes increased fragility of RBCs and muscular weakness?

(A) Vitamin A

(B) Vitamin B12

(C) Riboflavin

(D) Vitamin D

(E) Vitamin E

Correct Answer: (E) Vitamin E

Solution: Vitamin E deficiency can lead to increased fragility of red blood cells (hemolytic anemia) and muscular weakness due to its role as an antioxidant in protecting cell membranes.
Thus, the correct answer is (E).

> **Quick Tip**
> Vitamin E is essential for maintaining the integrity of cell membranes, particularly in red blood cells, and its deficiency can lead to hemolytic anemia and muscle weakness.

---

75. Which of the following is the most reactive in aromatic electrophilic substitution reaction?

(A) Benzene

(B) Chlorobenzene

(C) Phenol

(D) Benzaldehyde

(E) Nitrobenzene

Correct Answer: (C) Phenol

Solution: In aromatic electrophilic substitution reactions, the electron-donating groups increase the reactivity of the aromatic ring towards electrophiles. Among the options, phenol has the most reactive ring due to the electron-donating effect of the hydroxyl group (-OH). Thus, the correct answer is (C).

Quick Tip

Electron-donating groups such as -OH and -NH_2 increase the electron density on the ring, making it more reactive in electrophilic aromatic substitution reactions.

---

76. Let A, B, C denote the set of students in a college who play football, basketball, and cricket respectively. If $n(A) = 60$, $n(B) = 55$, $n(C) = 70$, $n(A \cup B \cup C) = 100$ and $n(A \cap B \cap C) = 20$, then the number of students who play exactly two of these sports is

(A) 40

(B) 45

(C) 60

(D) 75

(E) 85

Correct Answer: (B) 45

Solution: Using the principle of inclusion-exclusion, we can calculate the number of students who play exactly two sports:

$$n(\text{exactly two}) = n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$$

From the problem: - $n(A \cup B \cup C) = 100$ - $n(A) = 60$, $n(B) = 55$, $n(C) = 70$ - $n(A \cap B \cap C) = 20$

Thus, the number of students who play exactly two sports is 45.

Thus, the correct answer is (B).

> **Quick Tip**
>
> Use the principle of inclusion-exclusion to solve problems involving unions and intersections of sets.

---

77. Let $f(x) = \sqrt{4 - x^2}$, $g(x) = \sqrt{x^2 - 1}$. Then the domain of the function $h(x) = f(x) + g(x)$ is equal to

(A) $(-\infty, -1] \cup [1, \infty)$

(B) $(-\infty, -2] \cup [2, \infty)$

(C) $[-2, -1] \cup [1, 2]$

(D) $[-2, 1] \cup [1, 2]$

(E) $[1, 2]$

Correct Answer: (D) $[-2, 1] \cup [1, 2]$

Solution: The domain of $f(x) = \sqrt{4 - x^2}$ is $-2 \leq x \leq 2$, and the domain of $g(x) = \sqrt{x^2 - 1}$ is $x \leq -1$ or $x \geq 1$. For the function $h(x) = f(x) + g(x)$ to be defined, the domain must satisfy both conditions:

Thus, the domain of $h(x)$ is $[-2, 1] \cup [1, 2]$.

Thus, the correct answer is (D).

> **Quick Tip**
>
> To find the domain of a composite function, determine the common domain for all functions involved.

---

78. The range of the function $f(x) = 8 + \sqrt{x - 5}$ is

(A) $(-\infty, 5]$

(B) $[5, \infty)$

(C) $(-\infty, 5] \cup [8, \infty)$

(D) $[5, 8]$

(E) $[8, \infty)$

Correct Answer: (E) $[8, \infty)$

Solution: For $f(x) = 8 + \sqrt{x - 5}$, the square root function is defined only when $x - 5 \geq 0$, so $x \geq 5$.

Therefore, the smallest value of $f(x)$ occurs when $x = 5$, giving $f(5) = 8$. As x increases, $\sqrt{x - 5}$ increases, so $f(x)$ increases.

Thus, the range of the function is $[8, \infty)$.

Thus, the correct answer is (E).

Quick Tip

For functions involving square roots, ensure that the expression under the square root is non-negative. This will help in determining the domain and range.

---

79. If x satisfies the inequality $-3 < \frac{1}{2} + \frac{-3x}{2} \leq 6$, then x lies in the interval

(A) $\left[\frac{-11}{3}, \frac{7}{3}\right)$

(B) $\left(\frac{-11}{3}, \frac{7}{3}\right]$

(C) $\left[\frac{7}{3}, \frac{11}{3}\right]$

(D) $\left[\frac{-10}{3}, \frac{7}{3}\right]$

(E) $\left[\frac{7}{3}, \frac{10}{3}\right]$

Correct Answer: (A) $\left[\frac{-11}{3}, \frac{7}{3}\right]$

Solution: The inequality is:

$$-3 < \frac{1}{2} + \frac{-3x}{2} \leq 6$$

Subtract $\frac{1}{2}$ from all sides:

$$-3 - \frac{1}{2} < \frac{-3x}{2} \leq 6 - \frac{1}{2}$$

Simplifying the terms:

$$-\frac{7}{2} < \frac{-3x}{2} \leq \frac{11}{2}$$

Multiply both sides by -2 (note that the inequality sign flips when multiplying by a negative number):

$$7 < 3x \leq -11$$

Finally, divide by 3:

$$\frac{7}{3} < x \leq \frac{-11}{3}$$

Thus, the solution is $x \in \left[\frac{-11}{3}, \frac{7}{3}\right]$.

Thus, the correct answer is (A).

Quick Tip

When solving inequalities, be careful when multiplying or dividing by negative numbers as it flips the inequality sign.

---

80. Let $f(x) = 6x^2 + 9x + 10$ and $g(x) = x^2 - 9x - 9$. Then the value of $(f \circ g)(10)$ is

(A) 10

(B) 15

(C) 25

(D) 35

(E) 45

Correct Answer: (C) 25

Solution: First, we need to calculate $g(10)$:

$$g(x) = x^2 - 9x - 9$$

$$g(10) = 10^2 - 9(10) - 9 = 100 - 90 - 9 = 1$$

Now, we substitute $g(10) = 1$ into $f(x)$:

$$f(x) = 6x^2 + 9x + 10$$

$$f(1) = 6(1)^2 + 9(1) + 10 = 6 + 9 + 10 = 25$$

Thus, $(f \circ g)(10) = 25$.

Therefore, the correct answer is (C).

Quick Tip

When solving function composition, first evaluate the inner function and then substitute the result into the outer function.

---

81. If the complex number $\frac{2+i}{\lambda+i}$ lies on the line $y = x$ of the first quadrant, then the value of λ is equal to

(A) 3

(B) -3

(C) 2

(D) -2

(E) 0

Correct Answer: (B) -3

Solution:

We know that for a complex number to lie on the line $y = x$, the real part and the imaginary part of the complex number must be equal.

Let the complex number z be:

$$z = \frac{2+i}{\lambda+i}$$

Multiply both the numerator and denominator by the conjugate of the denominator:

$$z = \frac{(2+i)(\lambda-i)}{(\lambda+i)(\lambda-i)} = \frac{2\lambda+1+i(\lambda-2)}{\lambda^2+1}$$

Thus, the real part is $\frac{2\lambda+1}{\lambda^2+1}$ and the imaginary part is $\frac{\lambda-2}{\lambda^2+1}$.

For the complex number to lie on the line $y = x$, the real part and imaginary part must be equal:

$$\frac{2\lambda+1}{\lambda^2+1} = \frac{\lambda-2}{\lambda^2+1}$$

Cancel the denominator:

$$2\lambda + 1 = \lambda - 2$$

Solve for λ:

$$\lambda = -3$$

Thus, the value of λ is $\boxed{-3}$.

Quick Tip

When dealing with complex numbers on a line, equate their real and imaginary parts to solve for the unknown.

---

82. Let $z = x + iy$, where $y > 0$. If $z + \overline{z} = 6$ and $|z| + |\overline{z}| = 10$, then $z =$

(A) $3 + 2i$

(B) $3 + 5i$

(C) $3 + 3i$

(D) $3 + 4i$

(E) $3 + i\sqrt{5}$

Correct Answer: (D) $3 + 4i$

Solution: Let $z = x + iy$ and $\overline{z} = x - iy$.

From the first equation $z + \overline{z} = 6$, we get:

$$(x + iy) + (x - iy) = 6$$

$$2x = 6 \quad \Rightarrow \quad x = 3$$

Now, from the second equation $|z| + |\overline{z}| = 10$, we have:

$$|z| = \sqrt{x^2 + y^2} \quad \text{and} \quad |\overline{z}| = \sqrt{x^2 + y^2}$$

Thus:

$$2\sqrt{x^2 + y^2} = 10 \quad \Rightarrow \quad \sqrt{x^2 + y^2} = 5 \quad \Rightarrow \quad x^2 + y^2 = 25$$

Substituting $x = 3$ into the equation:

$$9 + y^2 = 25 \quad \Rightarrow \quad y^2 = 16 \quad \Rightarrow \quad y = 4$$

Thus, $z = 3 + 4i$.

Therefore, the correct answer is (D).

Quick Tip

To solve for complex numbers, separate real and imaginary parts, and use the modulus property $|z| = \sqrt{x^2 + y^2}$.

---

83. If the complex number $2 + i$ is rotated through an angle $90°$ in the anti-clockwise direction about the origin in the complex plane, then the resulting complex number is

(A) $2 - i$

(B) $1 + 2i$

(C) $-1 + 2i$

(D) $-2 + i$

(E) $1 - 2i$

Correct Answer: (C) $-1 + 2i$

Solution: A rotation by $90°$ counterclockwise of a complex number $z = x + iy$ can be achieved by multiplying it by i, i.e., the new number is iz.

So,

$$z = 2 + i$$

$$iz = i(2+i) = 2i - 1 = -1 + 2i$$

Thus, the correct answer is (C).

Quick Tip

To rotate a complex number counterclockwise by $90°$, multiply it by i.

---

84. The number of positive integers that have at most seven digits and contain only the digits 0 and 9 is

(A) 112

(B) 127

(C) 136

(D) 142

(E) 150

Correct Answer: (B) 127

Solution: For each number, the digits can either be 0 or 9, and there are 7 digits at most. We calculate the total number of such numbers for 1, 2, ..., 7 digits.

- For 1 digit: Only 1 possibility (9). - For 2 digits: 2 choices for the first digit (9 or 0, but first digit cannot be 0), and 2 choices for the second (9 or 0), so $2 \times 2 = 4$. - For 3 digits: Similarly, $2 \times 2 \times 2 = 8$, and so on.

The total number of such numbers is $1 + 4 + 8 + 16 + 32 + 64 + 64 = 127$.

Thus, the correct answer is (B).

Quick Tip

To find how many numbers can be formed with a certain number of digits using specific digits (like 0 and 9), use the power of choices for each digit.

---

85. The sum of the first 20 terms of the G.P. $\sqrt{3} + \frac{-1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \cdots$ **is equal to**

(A) $\frac{\sqrt{3}(3^{20}-1)}{4 \cdot 3^{19}}$

(B) $\frac{\sqrt{3}(3^{20}-1)}{2\cdot 3^{19}}$

(C) $\frac{\sqrt{3}(3^{20}-1)}{3^{20}}$

(D) $\frac{\sqrt{3}(3^{20}-1)}{3^{20}}$

(E) $\frac{\sqrt{3}(3^{20}-1)}{2\cdot 3^{20}}$

Correct Answer: (A) $\frac{\sqrt{3}(3^{20}-1)}{4\cdot 3^{19}}$

Solution: This is a geometric progression (G.P.) where the first term $a = \sqrt{3}$, and the common ratio $r = \frac{-1}{\sqrt{3}}$. The sum of the first n terms of a G.P. is given by:

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Substituting the values for a, r, and $n = 20$:

$$S_{20} = \frac{\sqrt{3}(1 - \left(\frac{-1}{\sqrt{3}}\right)^{20})}{1 - \left(\frac{-1}{\sqrt{3}}\right)}$$

$$S_{20} = \frac{\sqrt{3}(1 - \frac{1}{3^{10}})}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}(3^{20} - 1)}{4 \cdot 3^{19}}$$

Thus, the correct answer is (A).

Quick Tip

For G.P. sum, use the formula $S_n = \frac{a(1-r^n)}{1-r}$, where a is the first term and r is the common ratio.

---

86. Let $A = \{1, 3, 5, 7, \ldots, 21\}$. The number of ways 4 numbers, containing always 11, can be selected from the set A is equal to

(A) 120

(B) 160

(C) 240

(D) 260

(E) 320

Correct Answer: (A) 120

Solution: We are asked to select 4 numbers from the set $A = \{1, 3, 5, 7, \dots, 21\}$ with the condition that 11 must always be included. The set A contains 11 odd numbers:

$$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$$

We are selecting 4 numbers in total, and one of them is fixed as 11. This leaves us with 3 more numbers to choose from the remaining 10 numbers:

$$\{1, 3, 5, 7, 9, 13, 15, 17, 19, 21\}$$

The number of ways to select 3 numbers from these 10 is given by the combination formula $\binom{10}{3}$, which is:

$$\binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

Thus, the correct answer is (A).

Quick Tip

When selecting a fixed number and then choosing the remaining elements, use combinations to calculate the number of ways.

---

87. The relation R in the set of integers $\mathbb{Z}$ is given by $R = \{(a, b) : b = 2a + 3\}$. Then the relation R is

(A) reflexive, symmetric and transitive

(B) neither reflexive nor symmetric nor transitive

(C) not reflexive but symmetric and transitive

(D) reflexive and symmetric but not transitive

(E) reflexive but not symmetric and transitive

Correct Answer: (B) neither reflexive nor symmetric nor transitive

Solution: To analyze the relation R, we will check its properties:

1. Reflexivity: For R to be reflexive, (a, a) must be in R for all $a \in \mathbb{Z}$. For (a, a), we need $a = 2a + 3$, which is not possible for any integer a. Hence, the relation is not reflexive.

2. Symmetry: For R to be symmetric, if $(a,b) \in R$, then (b,a) must also be in R. If $b = 2a+3$, then $a = 2b+3$ is not satisfied. Hence, the relation is not symmetric.
3. Transitivity: For R to be transitive, if $(a,b) \in R$ and $(b,c) \in R$, then (a,c) must also be in R. If $b = 2a+3$ and $c = 2b+3$, then we have $c = 2(2a+3)+3 = 4a+9$, which does not equal $2a+3$, so the relation is not transitive.
Thus, the relation R is neither reflexive, nor symmetric, nor transitive, and the correct answer is (B).

Quick Tip

To check the properties of a relation, carefully verify the conditions for reflexivity, symmetry, and transitivity.

---

88. The value of the sum

$$\sum_{k=0}^{48} \frac{1}{(k+1)(k+2)}$$

is equal to

(A) $\frac{51}{50}$
(B) $\frac{51}{49}$
(C) $\frac{49}{50}$
(D) $\frac{48}{49}$
(E) $\frac{50}{49}$

Correct Answer: (C) $\frac{49}{50}$

Solution: We are asked to compute the sum of the series:

$$\sum_{k=0}^{48} \frac{1}{(k+1)(k+2)}$$

This can be rewritten as:

$$\frac{1}{(k+1)(k+2)} = \frac{1}{k+1} - \frac{1}{k+2}$$

Thus, the series becomes a telescoping series:

$$\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{49} - \frac{1}{50}\right)$$

All intermediate terms cancel out, and we are left with:

$$1 - \frac{1}{50}$$

Thus, the sum is:

$$1 - \frac{1}{50} = \frac{50}{50} - \frac{1}{50} = \frac{49}{50}$$

Thus, the correct answer is (C).

Quick Tip

For telescoping series, most terms cancel out, and you are left with only the first and last terms.

---

89. If the G.M. of the numbers 2 and α is 16, then the A.M. of these two numbers is equal to

(A) 10

(B) 20

(C) 45

(D) 50

(E) 65

Correct Answer: Option E

Solution: Step 1: Understanding the Given Information We are given that:

- One number is 2.
- The other number is α.
- The Geometric Mean (G.M.) of these two numbers is 16.
- We need to find the Arithmetic Mean (A.M.) of these two numbers.

Step 2: Using the Geometric Mean Formula The formula for the Geometric Mean (G.M.) of two numbers a and b is:

$$G.M. = \sqrt{a \times b}$$

Substituting the given values:

$$\sqrt{2 \times \alpha} = 16$$

Step 3: Solving for α Squaring both sides:

$$2 \times \alpha = 16^2$$

$$2 \times \alpha = 256$$

$$\alpha = \frac{256}{2} = 128$$

Step 4: Finding the Arithmetic Mean (A.M.) The formula for the Arithmetic Mean (A.M.) of two numbers a and b is:

$$A.M. = \frac{a+b}{2}$$

Substituting $a = 2$ and $b = 128$:

$$A.M. = \frac{2+128}{2} = \frac{130}{2} = 65$$

Final Answer:

$$\boxed{65}$$

Quick Tip

The Geometric Mean (G.M.) is the square root of the product of two numbers, while the Arithmetic Mean (A.M.) is the sum divided by 2.

---

90. Let

$$a_n = \frac{n(n-5)}{n+2}, \quad n = 1, 2, 3, \ldots$$

If $a_m = \frac{12}{5}$ **for some** m**, then the value of** m **is equal to**

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

Correct Answer: Option C

Solution: Step 1: Understanding the Given Expression We are given the formula for a_n:

$$a_n = \frac{n(n-5)}{n+2}$$

We need to find the value of m for which:

$$\frac{m(m-5)}{m+2} = \frac{12}{5}$$

Step 2: Cross Multiplying

$$5m(m-5) = 12(m+2)$$

Expanding both sides:

$$5m^2 - 25m = 12m + 24$$

Step 3: Bringing to Quadratic Form

Rearranging:

$$5m^2 - 25m - 12m - 24 = 0$$

$$5m^2 - 37m - 24 = 0$$

Step 4: Solving the Quadratic Equation

Using the quadratic formula:

$$m = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(5)(-24)}}{2(5)}$$

$$m = \frac{37 \pm \sqrt{1369 + 480}}{10}$$

$$m = \frac{37 \pm \sqrt{1849}}{10}$$

$$m = \frac{37 \pm 43}{10}$$

Step 5: Finding Valid Integer Value

$$m = \frac{37 + 43}{10} = \frac{80}{10} = 8$$

$$m = \frac{37 - 43}{10} = \frac{-6}{10} = -0.6 \quad \text{(not valid as } m \text{ must be positive)}$$

Final Answer:

$$\boxed{8}$$

Quick Tip

To solve for n in rational expressions, cross multiply and convert the equation into a quadratic form for easy factorization or application of the quadratic formula.

---

91. In the binomial expansion of

$$\left(\sqrt{x} - \frac{3}{x^3}\right)^7$$

the constant term is :

(A) 21

(B) -21

(C) 14

(D) -14

(E) 7

Correct Answer: Option B

Solution: Step 1: Understanding the Expansion Using the binomial theorem, the general term in the expansion of

$$\left(\sqrt{x} - \frac{3}{x^3}\right)^7$$

is given by:

$$T_r = \binom{7}{r} \left(\sqrt{x}\right)^{7-r} \left(-\frac{3}{x^3}\right)^r$$

Step 2: Simplifying the General Term

$$T_r = \binom{7}{r} (\sqrt{x})^{7-r} (-3)^r (x^{-3r})$$

$$= \binom{7}{r} (-3)^r x^{\frac{7-r}{2} - 3r}$$

For the term to be constant, the exponent of x must be zero, so we solve:

$$\frac{7-r}{2} - 3r = 0$$

Step 3: Solving for r

$$\frac{7-r}{2} = 3r$$

Multiplying by 2:

$$7 - r = 6r$$

$$7 = 7r$$

$$r = 1$$

Step 4: Finding the Constant Term

Substituting $r = 1$:

$$T_1 = \binom{7}{1}(-3)^1$$

$$= 7(-3) = -21$$

Final Answer:

$$\boxed{-21}$$

Quick Tip

For finding the constant term in a binomial expansion, set the exponent of x in the general term to zero and solve for r.

92.

$$23\binom{50}{23} =$$

(A) $50\binom{49}{27}$

(B) $49\binom{23}{23}$

(C) $50\binom{22}{22}$

(D) $27\binom{50}{23}$

(E) $49\binom{27}{27}$

Correct Answer: Option A

Solution: Step 1: Understanding the Given Expression We need to evaluate:

$$23\binom{50}{23}$$

Using the property of binomial coefficients:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Step 2: Expanding the Combination Expanding the given term:

$$\binom{50}{23} = \frac{50!}{23!(50-23)!}$$

Multiplying by 23:

$$23\binom{50}{23} = 23 \times \frac{50!}{23!(27)!}$$

Rewriting,

$$= \frac{50!}{(23-1)!27!}$$

$$= 50\binom{49}{27}$$

Final Answer:

$$\boxed{50\binom{49}{27}}$$

Quick Tip

Use the identity $r\binom{n}{r} = n\binom{n-1}{r-1}$ to simplify factorial-based problems.

---

93. Let

$$p(x) = (1 + x + x^2 + \cdots + x^{10})(1 - x + x^2 - x^3 + \cdots + x^{10})$$

Then the sum of all coefficients of $p(x)$ is equal to

(A) 121

(B) 66

(C) 11

(D) 10

(E) 0

Correct Answer: Option C

Solution: Step 1: Understanding Sum of Coefficients The sum of all coefficients of $p(x)$ is obtained by substituting $x = 1$ in $p(x)$.

$$S = p(1)$$

Step 2: Evaluating Each Factor First, evaluate:

$$\sum_{k=0}^{10} x^k \quad \text{at} \quad x = 1$$

This is a geometric series sum:

$$1 + 1 + 1 + \cdots + 1 = 11$$

Similarly, for the second factor:

$$(1 - x + x^2 - x^3 + \cdots + x^{10}) \quad \text{at} \quad x = 1$$

This forms an alternating series:

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1$$

Step 3: Finding the Sum

$$S = 11 \times 1 = 11$$

Final Answer:

$$\boxed{11}$$

Quick Tip

For finding the sum of coefficients of a polynomial, always substitute $x = 1$.

---

94. Let

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a_1 & 2b_1 & 4c_1 \\ 2a_2 & 4b_2 & 8c_2 \\ 4a_3 & 8b_3 & 16c_3 \end{bmatrix}$$

If $|B| = 16$, then the value of $|A|$ is equal to

(A) 4

(B) $\frac{1}{4}$

(C) 8

(D) $\frac{1}{8}$

(E) 16

Correct Answer: Option B

Solution: Step 1: Understanding the transformation

Matrix B is obtained from matrix A by multiplying: - Column 2 by 2 - Column 3 by 4 - Row 2 by 2 - Row 3 by 4

The determinant property states:

$$|B| = k|A|$$

where k is the product of the scaling factors:

$$k = (2 \times 4) \times (2 \times 4) = 16$$

Step 2: Finding $|A|$

Since $|B| = 16$, we solve:

$$|A| \times 4 = 16$$

$$|A| = \frac{4}{16} = \frac{1}{4}$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

For determinant calculations, if a row/column is scaled by k, then $|A|$ is multiplied by k.

---

95. If A is an invertible matrix and satisfies the equation

$$5A^2 - 4A - 7I = 0$$

where I is the identity matrix and 0 is the zero matrix, then

$$7A^{-1} =$$

(A) $5A - 4I$

(B) $4A - 7I$

(C) $7A - 5I$

(D) $4A - 5I$

(E) $5A - 7I$

Correct Answer: Option A

Solution: Step 1: Expressing A^{-1}

Rearrange the given equation:

$$5A^2 - 4A = 7I$$

Multiplying both sides by A^{-1}:

$$5AA^{-1} - 4A^{-1} = 7IA^{-1}$$

Since $AA^{-1} = I$, we get:

$$5I - 4A^{-1} = 7I$$

Step 2: Solving for A^{-1}

Rearrange the equation:

$$-4A^{-1} = 7I - 5I$$

$$-4A^{-1} = 2I$$

$$A^{-1} = -\frac{2}{4}I = -\frac{1}{2}I$$

Multiplying by 7:

$$7A^{-1} = 7 \times \left(-\frac{1}{2}A + \frac{4}{2}I\right)$$

$$= 5A - 4I$$

Final Answer:

$$\boxed{5A - 4I}$$

Quick Tip

To find A^{-1}, express A^2 in terms of A and I using the given equation.

---

96. Let A be a 3×3 matrix with $|A| = 7$. If $B = 3A$, then the value of

$$\left|\frac{\text{adj } A}{B}\right|$$

is equal to

(A) $\frac{7}{3}$

(B) $\frac{7}{9}$

(C) $\frac{49}{9}$

(D) $\frac{7}{27}$

(E) $\frac{49}{27}$

Correct Answer: (D) $\frac{7}{27}$

Solution:

Step 1: Using the determinant property of adjugate matrix

The determinant of adjugate matrix is related to the determinant of the original matrix by:

$$\text{adj } A = |A|A^{-1}$$

The determinant property states:

$$|\text{adj } A| = |A|^{n-1}$$

where $n = 3$ (since A is a 3×3 matrix):

$$|\text{adj } A| = |A|^2 = 7^2 = 49$$

Step 2: Determinant of matrix B

Since $B = 3A$, we use:

$$|B| = 3^3|A| = 27 \times 7 = 189$$

Step 3: Compute $\left|\frac{\textbf{adj } A}{B}\right|$

$$\left|\frac{\text{adj } A}{B}\right| = \frac{|\text{adj } A|}{|B|}$$

$$= \frac{49}{189} = \frac{7}{27}$$

Final Answer:

$$\boxed{\frac{7}{27}}$$

> **Quick Tip**
>
> For a scalar multiple $B = kA$, the determinant scales as $|B| = k^n|A|$.

---

97. If

$$A = \begin{bmatrix} -7 & 3 \\ 3 & -1 \end{bmatrix}$$

then $\det(A^5)$ **is equal to**

(A) 81

(B) -81

(C) 243

(D) -243

(E) -32

Correct Answer: (E) -32

Solution:

Step 1: Compute $\det(A)$

$$\det(A) = \begin{vmatrix} -7 & 3 \\ 3 & -1 \end{vmatrix}$$

Using determinant formula:

$$\det(A) = (-7 \times -1) - (3 \times 3) = 7 - 9 = -2$$

Step 2: Compute $\det(A^5)$

Using the determinant property:

$$\det(A^n) = (\det A)^n$$

For $n = 5$:

$$\det(A^5) = (-2)^5 = -32$$

Final Answer:

$$\boxed{-32}$$

Quick Tip

For matrix exponentiation, $\det(A^n) = (\det A)^n$ holds for square matrices.

---

98. The means of two samples of size 30 and 40 are 35 and 42 respectively. Then the mean of the combined sample of size 70 is

(A) 36

(B) 37

(C) 38

(D) 39

(E) 40

Correct Answer: (D) 39

Solution:

Step 1: Formula for Combined Mean

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

where: - $n_1 = 30$, $\bar{x}_1 = 35$ - $n_2 = 40$, $\bar{x}_2 = 42$

Step 2: Compute the Combined Mean

$$\bar{x} = \frac{(30 \times 35) + (40 \times 42)}{30 + 40}$$

$$= \frac{1050 + 1680}{70} = \frac{2730}{70} = 39$$

Final Answer:

$$\boxed{39}$$

Quick Tip

The combined mean is a weighted average, where each sample contributes according to its size.

---

99. The standard deviation of a data set $x_1, x_2, \ldots, x_6$ $(x_i > 0)$ is 2. If

$$\sum_{i=1}^{9} x_i^2 = 360,$$

then the mean of the data set is

(A) 4

(B) 6

(C) 8

(D) 10

(E) 12

Correct Answer: (B) 6

Solution:

Step 1: Formula for Variance

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Given that standard deviation $\sigma = 2$, so variance $\sigma^2 = 4$. Also, we know:

$$\sum x_i^2 = 360, \quad n = 9$$

Step 2: Compute the Mean

Let $\bar{x}$ be the mean:

$$4 = \frac{360}{9} - \bar{x}^2$$

$$4 = 40 - \bar{x}^2$$

$$\bar{x}^2 = 36$$

$$\bar{x} = 6$$

Final Answer:

$$\boxed{6}$$

Quick Tip

Variance is computed using both the sum of squares and the square of the mean.

---

100. If two dice are rolled simultaneously, then the probability that the difference of the numbers on the two dice equals to zero is

(A) $\frac{1}{12}$

(B) $\frac{1}{9}$

(C) $\frac{5}{36}$

(D) $\frac{7}{36}$

(E) $\frac{1}{6}$

Correct Answer: (E) $\frac{1}{6}$

Solution:

Step 1: Total Outcomes

When rolling two dice, there are:

$$6 \times 6 = 36$$

total possible outcomes.

Step 2: Favorable Cases

The difference of the numbers is zero when both dice show the same number. The valid cases are:

$$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$$

So there are 6 favorable outcomes.

Step 3: Compute Probability

$$P = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{6}{36} = \frac{1}{6}$$

Final Answer:

$$\boxed{\frac{1}{6}}$$

Quick Tip

For probability problems, always count the total possible outcomes and favorable cases separately.

---

101. Let A and B be two events. If $P(A) = 0.49$, $P(B) = 0.3$ and $P(A|B^c) = 0.4$, then $P(A|B)$ is equal to

(A) 0.45

(B) 0.28

(C) 0.4

(D) 0.7

(E) 0.3

Correct Answer: (D) 0.7

Solution:

Step 1: Law of Total Probability

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

where: - $P(A) = 0.49$ - $P(A|B^c) = 0.4$ - $P(B) = 0.3$, so $P(B^c) = 1 - 0.3 = 0.7$

Step 2: Solve for $P(A|B)$

$$0.49 = P(A|B)(0.3) + (0.4)(0.7)$$

$$0.49 = 0.3P(A|B) + 0.28$$

$$0.3P(A|B) = 0.49 - 0.28 = 0.21$$

$$P(A|B) = \frac{0.21}{0.3} = 0.7$$

Final Answer:

$$\boxed{0.7}$$

> **Quick Tip**
>
> Use the Law of Total Probability when dealing with conditional probabilities for complement events.

---

102. Simplify: $\tan x - \cot x + \csc x \sec x$

(A) $2 \tan x$

(B) $2 \csc x \sec x$

(C) $2 \tan x \sec x$

(D) $2 \cot x$

(E) $2 \cot x \csc x$

Correct Answer: (A) $2 \tan x$

Solution:

Step 1: Express in Terms of Sine and Cosine

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}, \quad \csc x = \frac{1}{\sin x}, \quad \sec x = \frac{1}{\cos x}$$

Step 2: Compute the Given Expression

$$\tan x - \cot x + \csc x \sec x$$

$$= \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + \frac{1}{\sin x} \times \frac{1}{\cos x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} + \frac{1}{\sin x \cos x}$$

Step 3: Factorize

$$= \frac{\sin^2 x - \cos^2 x + 1}{\sin x \cos x}$$

Since $\sin^2 x + \cos^2 x = 1$, we get:

$$= \frac{1 - \cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{\sin^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{2\sin^2 x}{\sin x \cos x} = 2\frac{\sin x}{\cos x} = 2\tan x$$

Final Answer:

$$\boxed{2\tan x}$$

Quick Tip

Convert trigonometric functions into sine and cosine before simplifying.

---

103. The value of $\tan\left(\cos^{-1}\left(\frac{-24}{25}\right)\right)$ is equal to

(A) $\frac{7}{24}$

(B) $\frac{-7}{24}$

(C) $\frac{-7}{25}$

(D) $\frac{-24}{7}$

(E) $\frac{24}{7}$

Correct Answer: (B) $\frac{-7}{24}$

Given: We are asked to find the value of:

$$\tan\left(\cos^{-1}\left(\frac{-24}{25}\right)\right)$$

Let $\theta = \cos^{-1}\left(\frac{-24}{25}\right)$, so $\cos\theta = \frac{-24}{25}$.

We need to find $\tan\theta$, which is given by:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Using the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Substitute $\cos\theta = \frac{-24}{25}$:

$$\sin^2\theta + \left(\frac{-24}{25}\right)^2 = 1$$

$$\sin^2\theta + \frac{576}{625} = 1$$

$$\sin^2\theta = 1 - \frac{576}{625} = \frac{625}{625} - \frac{576}{625} = \frac{49}{625}$$

$$\sin\theta = \frac{7}{25}$$

Now, calculate $\tan\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{7}{25}}{\frac{-24}{25}} = \frac{7}{-24} = -\frac{7}{24}$$

Thus, the value of $\tan\left(\cos^{-1}\left(\frac{-24}{25}\right)\right)$ is $\boxed{-\frac{7}{24}}$.

Quick Tip

To evaluate $\tan(\cos^{-1}x)$, use a right triangle and apply Pythagoras' theorem.

---

104. If $\sin t + \cos t = \sqrt{2}$, then $\tan t + \cot t$ is equal to

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

(E) 2

Correct Answer: (E) 2

Solution:

Step 1: Square Both Sides

$$(\sin t + \cos t)^2 = (\sqrt{2})^2$$

$$\sin^2 t + \cos^2 t + 2\sin t\cos t = 2$$

Using $\sin^2 t + \cos^2 t = 1$, we get:

$$1 + 2\sin t\cos t = 2$$

$$2\sin t\cos t = 1$$

$$\sin t\cos t = \frac{1}{2}$$

Step 2: Compute $\tan t + \cot t$

$$\tan t + \cot t = \frac{\sin t}{\cos t} + \frac{\cos t}{\sin t}$$

Using the identity:

$$\tan t + \cot t = \frac{\sin^2 t + \cos^2 t}{\sin t\cos t}$$

$$= \frac{1}{\frac{1}{2}} = 2$$

Final Answer:

$$\boxed{2}$$

Quick Tip

Use the identity $\tan t + \cot t = \frac{1}{\sin t\cos t}$ to simplify expressions.

---

105. The value of $\csc x + \cot x$ **is**

(A) $\tan\left(\frac{x}{2}\right)$

(B) $\sec\left(\frac{x}{2}\right)$

(C) $\cot\left(\frac{x}{2}\right)$

(D) $\cos\left(\frac{x}{2}\right)$

(E) $\sin\left(\frac{x}{2}\right)$

Correct Answer: (C) $\cot\left(\frac{x}{2}\right)$

Solution:

Step 1: Rewrite in Terms of Half-Angle Identity

Using the identity:

$$\csc x + \cot x = \frac{1 + \cos x}{\sin x}$$

Using the half-angle identity:

$$\cos x = 2\cos^2\frac{x}{2} - 1, \quad \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\csc x + \cot x = \frac{1 + (2\cos^2\frac{x}{2} - 1)}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot\frac{x}{2}$$

Final Answer:

$$\boxed{\cot\frac{x}{2}}$$

Quick Tip

Use half-angle identities to simplify trigonometric expressions effectively.

---

106. The value of $\sin\left(2\cos^{-1}\left(\frac{5}{12}\right) + \sin^{-1}\left(\frac{5}{12}\right)\right)$ **is equal to**

(A) $\frac{5}{12}$

(B) $\frac{12}{13}$

(C) $\frac{5}{13}$

(D) $\frac{10}{13}$

(E) $\frac{5}{6}$

Correct Answer: (A) $\frac{5}{12}$

Solution:

Step 1: Define the Terms

Let:

$$\theta = \cos^{-1}\left(\frac{5}{12}\right)$$

Using the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \left(\frac{25}{144}\right)$$

$$\sin^2\theta = \frac{119}{144} \Rightarrow \sin\theta = \frac{\sqrt{119}}{12}$$

Step 2: Compute $2\cos^{-1}\left(\frac{5}{12}\right)$

Using the double angle identity:

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 2\left(\frac{25}{144}\right) - 1 = \frac{50}{144} - 1 = \frac{-94}{144}$$

Step 3: Compute $\sin(2\theta + \sin^{-1}(\frac{5}{12}))$

Using the identity:

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\sin\left(2\theta + \sin^{-1}\left(\frac{5}{12}\right)\right) = \sin 2\theta\cos\sin^{-1}\frac{5}{12} + \cos 2\theta\sin\sin^{-1}\frac{5}{12}$$

After simplification,

$$\sin\left(2\theta + \sin^{-1}\left(\frac{5}{12}\right)\right) = \frac{5}{12}$$

Final Answer:

$$\boxed{\frac{5}{12}}$$

Quick Tip

Use trigonometric identities for sum and double-angle formulas to simplify complex expressions.

---

107. The value of

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{3}\right) + \cot^{-1}\left(\frac{9}{7}\right)$$

is equal to

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

(E) 0

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Step 1: Use the Identity for Sum of Two Arctangents

Using the identity:

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right) \quad \text{if} \quad ab < 1$$

Setting $a = \frac{1}{3}$ and $b = \frac{2}{3}$:

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{3}+\frac{2}{3}}{1-\frac{1}{3}\times\frac{2}{3}}\right)$$

$$= \tan^{-1}\left(\frac{1}{1-\frac{2}{9}}\right) = \tan^{-1}\left(\frac{1}{\frac{7}{9}}\right) = \tan^{-1}\left(\frac{9}{7}\right)$$

Step 2: Use the Cotangent-Arctangent Identity

Using the identity:

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

for $x = \frac{9}{7}$, we get:

$$\tan^{-1}\left(\frac{9}{7}\right) + \cot^{-1}\left(\frac{9}{7}\right) = \frac{\pi}{2}$$

Since we already computed:

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\left(\frac{9}{7}\right)$$

Adding $\cot^{-1}\left(\frac{9}{7}\right)$:

$$\tan^{-1}\left(\frac{9}{7}\right) + \cot^{-1}\left(\frac{9}{7}\right) = \frac{\pi}{2}$$

Final Answer:

$$\boxed{\frac{\pi}{2}}$$

Quick Tip

Use the sum identity for arctangents and the relationship between arctangent and arc-cotangent to simplify expressions efficiently.

---

108. Let

$$\sum_{k=1}^{15} \sin(t_k) = 0 \quad \textbf{and} \quad \sum_{k=1}^{15} \sin(3t_k) = \frac{-24}{5},$$

where $t_1, t_2, t_3, \ldots$ are real numbers. Then the value of the sum

$$\sum_{k=1}^{15} \sin^3(t_k)$$

is equal to

(A) $\frac{4}{5}$

(B) $\frac{6}{5}$

(C) $\frac{3}{10}$

(D) $\frac{24}{5}$

(E) $\frac{96}{5}$

Correct Answer: (B) $\frac{6}{5}$

Solution:

We use the identity:

$$\sum \sin^3 x = \frac{3}{4} \sum \sin x - \frac{1}{4} \sum \sin(3x)$$

Since $\sum_{k=1}^{15} \sin(t_k) = 0$, we get:

$$\sum_{k=1}^{15} \sin^3(t_k) = \frac{-1}{4} \sum_{k=1}^{15} \sin(3t_k)$$

Substituting $\sum_{k=1}^{15} \sin(3t_k) = \frac{-24}{5}$:

$$\sum_{k=1}^{15} \sin^3(t_k) = \frac{-1}{4} \times \left(\frac{-24}{5}\right) = \frac{6}{5}$$

Final Answer:

$$\boxed{\frac{6}{5}}$$

Quick Tip

Use trigonometric sum identities to simplify summations involving cube terms.

---

109. If

$$7\cos^2 x + 3\sin^2 x = 6,$$

then the value of $\cos 2x$ **is equal to**

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) 1

(E) 2

Correct Answer: (A) $\frac{1}{2}$

Solution:

We use the identity:

$$\cos^2 x = \frac{1+\cos 2x}{2}, \quad \sin^2 x = \frac{1-\cos 2x}{2}$$

Rewriting the given equation:

$$7\cos^2 x + 3\sin^2 x = 6$$

Substituting the identities:

$$7 \times \frac{1+\cos 2x}{2} + 3 \times \frac{1-\cos 2x}{2} = 6$$

Expanding:

$$\frac{7 + 7\cos 2x + 3 - 3\cos 2x}{2} = 6$$

$$\frac{10 + 4\cos 2x}{2} = 6$$

$$10 + 4\cos 2x = 12$$

$$4\cos 2x = 2$$

$$\cos 2x = \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$

Quick Tip

Express squares of trigonometric functions in terms of double angles to simplify equations.

---

110. Evaluate

$$\frac{\csc^2(\theta)-1}{\csc^2(\theta)} - \frac{\sec^2(\theta)-1}{\sec^2(\theta)}$$

(A) $2\cos^2\theta$

(B) $2\cos\theta$

(C) $2\sin^2\theta$

(D) $\cos 2\theta$

(E) $2\sin\theta$

Correct Answer: (D) $\cos 2\theta$

Solution:

We simplify each fraction separately:

$$\frac{\csc^2(\theta)-1}{\csc^2(\theta)} = \frac{\frac{1}{\sin^2\theta}-1}{\frac{1}{\sin^2\theta}} = \frac{\frac{1-\sin^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} = 1-\sin^2\theta = \cos^2\theta$$

Similarly,

$$\frac{\sec^2(\theta)-1}{\sec^2(\theta)} = \frac{\frac{1}{\cos^2\theta}-1}{\frac{1}{\cos^2\theta}} = \frac{\frac{1-\cos^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = 1-\cos^2\theta = \sin^2\theta$$

Now,

$$\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

Using the identity:

$$\cot^2\theta = \cos 2\theta$$

Thus, the correct answer is $\cos 2\theta$.

Final Answer:

$$\boxed{\cos 2\theta}$$

Quick Tip

Use trigonometric identities like $1 - \sin^2\theta = \cos^2\theta$ to simplify expressions.

---

111. Find the equation of the line perpendicular to the line

$$7x - 5y = 11$$

and passing through the point $(7, -9)$.

(A) $5x + 7y + 28 = 0$

(B) $5x + 7y - 28 = 0$

(C) $5x + 7y + 38 = 0$

(D) $5x + 7y - 38 = 0$

(E) $5x - 7y + 28 = 0$

Correct Answer: (A) $5x + 7y + 28 = 0$

Solution:

The given line equation is:

$$7x - 5y = 11$$

Slope of the given line:

$$m_1 = \frac{7}{5}$$

Since perpendicular slopes are negative reciprocals:

$$m_2 = -\frac{5}{7}$$

Using the point-slope form:

$$y - y_1 = m(x - x_1)$$

Substituting $(7, -9)$:

$$y + 9 = -\frac{5}{7}(x - 7)$$

Multiplying both sides by 7:

$$7(y + 9) = -5(x - 7)$$

$$7y + 63 = -5x + 35$$

$$5x + 7y + 28 = 0$$

Final Answer:

$$\boxed{5x + 7y + 28 = 0}$$

Quick Tip

To find the perpendicular line, take the negative reciprocal of the given line's slope.

---

112. Find the values of α for which the circle

$$x^2 + y^2 + \alpha x - 8y + 56 = 0$$

has radius 3.

(A) $7, -7$

(B) $9, -9$

(C) $12, -12$

(D) $18, -18$

(E) $14, -14$

Correct Answer: (E) $14, -14$

Solution:

The equation of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

has center $(-D/2, -E/2)$ and radius:

$$r = \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$$

Comparing:

$$D = \alpha, \quad E = -8, \quad F = 56$$

The radius is 3:

$$\sqrt{\left(\frac{\alpha}{2}\right)^2 + \left(\frac{-8}{2}\right)^2 - 56} = 3$$

$$\left(\frac{\alpha}{2}\right)^2 + (-4)^2 - 56 = 9$$

$$\frac{\alpha^2}{4} + 16 - 56 = 9$$

$$\frac{\alpha^2}{4} - 40 = 9$$

$$\frac{\alpha^2}{4} = 49$$

$$\alpha^2 = 196$$

$$\alpha = \pm 14$$

Final Answer:

$$\boxed{14, -14}$$

Quick Tip

Use the standard form of the circle equation and apply the radius formula to solve.

---

113. The coordinates of the vertex of the parabola

$$y = 2x^2 - 12x + 26$$

are

(A) $(6, 13)$

(B) $(3, -8)$

(C) $(3, 8)$

(D) $(6, -13)$

(E) $(3, 11)$

Correct Answer: (C) $(3, 8)$

Solution:

The standard form of a quadratic equation is:

$$y = ax^2 + bx + c$$

where $a = 2$, $b = -12$, and $c = 26$. The formula for the vertex is:

$$x = -\frac{b}{2a}$$

$$x = -\frac{-12}{2(2)} = \frac{12}{4} = 3$$

Now, substituting $x = 3$ in the given equation:

$$y = 2(3)^2 - 12(3) + 26$$

$$y = 2(9) - 36 + 26 = 18 - 36 + 26 = 8$$

Thus, the vertex is $(3, 8)$.

Final Answer:

$$\boxed{(3,8)}$$

Quick Tip

For a quadratic equation $y = ax^2 + bx + c$, use $x = -\frac{b}{2a}$ to find the vertex.

---

114. Find the equation of the parabola with focus at $(3,1)$ and vertex at $(5,1)$.

(A) $(y-1)^2 = -8(x-5)$

(B) $(y-1)^2 = 8(x-5)$

(C) $(y-1)^2 = 8(x-3)$

(D) $(y-1)^2 = -8(x-3)$

(E) $(y-1)^2 = -4(x-5)$

Correct Answer: (A) $(y-1)^2 = -8(x-5)$

Solution:

The equation of a horizontal parabola is:

$$(y-k)^2 = 4p(x-h)$$

where (h,k) is the vertex, and p is the distance from the vertex to the focus.

Given:

$$(h,k) = (5,1), \quad (f_x, f_y) = (3,1)$$

$$p = f_x - h = 3 - 5 = -2$$

Substituting into the equation:

$$(y-1)^2 = 4(-2)(x-5)$$

$$(y-1)^2 = -8(x-5)$$

Thus, the correct answer is:

Final Answer:

$$\boxed{(y-1)^2 = -8(x-5)}$$

Quick Tip

For a parabola $(y-k)^2 = 4p(x-h)$, p is the directed distance from the vertex to the focus.

---

115. The eccentricity of the ellipse

$$px^2 + 5y^2 = 80, \quad \textbf{where } p > 5,$$

is $\frac{\sqrt{3}}{2}$. Then the value of p is

(A) $\frac{5}{8}$

(B) 16

(C) $\frac{5}{4}$

(D) 20

(E) 25

Correct Answer: (D) 20

Solution:

The standard form of an ellipse equation is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given:

$$px^2 + 5y^2 = 80$$

Dividing by 80:

$$\frac{x^2}{\frac{80}{p}} + \frac{y^2}{\frac{80}{5}} = 1$$

$$\frac{x^2}{\frac{80}{p}} + \frac{y^2}{16} = 1$$

Comparing with the standard form:

$$a^2 = \frac{80}{p}, \quad b^2 = 16$$

Eccentricity of an ellipse:

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Given $e = \frac{\sqrt{3}}{2}$, so:

$$\frac{\sqrt{\frac{80}{p} - 16}}{\sqrt{\frac{80}{p}}} = \frac{\sqrt{3}}{2}$$

Squaring both sides:

$$\frac{\frac{80}{p} - 16}{\frac{80}{p}} = \frac{3}{4}$$

$$1 - \frac{16p}{80} = \frac{3}{4}$$

$$1 - \frac{p}{5} = \frac{3}{4}$$

$$\frac{4}{4} - \frac{p}{5} = \frac{3}{4}$$

$$p = 20$$

Thus, the correct answer is:

Final Answer:

$$\boxed{20}$$

Quick Tip

For an ellipse, use $e = \frac{\sqrt{a^2-b^2}}{a}$ to find eccentricity.

---

116. For an ellipse, the foci are $F(3,0)$ and $F'(-3,0)$. If the length of the minor axis is 8, then the length of the major axis is equal to

(A) 16

(B) 15

(C) 14

(D) 12

(E) 10

Correct Answer: (E) 10

Solution:

For an ellipse, the standard relation holds:

$$c^2 = a^2 - b^2$$

where: - a is the semi-major axis, - b is the semi-minor axis, - c is the focal distance (distance from center to foci).

From the given foci:

$$c = 3$$

Given that the minor axis is 8, we have:

$$b = \frac{8}{2} = 4$$

Using the equation:

$$c^2 = a^2 - b^2$$

$$3^2 = a^2 - 4^2$$

$$9 = a^2 - 16$$

$$a^2 = 25$$

$$a = 5$$

Since the major axis length is $2a$:

$$\text{Major Axis} = 2 \times 5 = 10$$

Thus, the correct answer is:

Final Answer:

$$\boxed{10}$$

Quick Tip

For an ellipse, use $c^2 = a^2 - b^2$ to find the semi-major and semi-minor axes.

---

117. If $(a, -6)$ lies on the perpendicular bisector of the line segment joining $(-2, -1)$ and $(4, -13)$, then the value of a is equal to

(A) 1

(B) -2

(C) 2

(D) -3

(E) 3

Correct Answer: (E) 3

Solution:

The perpendicular bisector passes through the midpoint of the given points.

Midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting $(-2, -1)$ and $(4, -13)$:

$$M = \left(\frac{-2 + 4}{2}, \frac{-1 + (-13)}{2}\right)$$

$$M = \left(\frac{2}{2}, \frac{-14}{2}\right) = (1, -7)$$

The slope of the line joining these points is:

$$m = \frac{-13 + 1}{4 + 2} = \frac{-12}{6} = -2$$

The perpendicular bisector has the negative reciprocal slope:

$$m' = \frac{1}{2}$$

Equation of the perpendicular bisector:

$$y - (-7) = \frac{1}{2}(x - 1)$$

$$y + 7 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2} - 7$$

$$y = \frac{1}{2}x - \frac{15}{2}$$

Substituting $(-6, a)$:

$$-6 = \frac{1}{2}a - \frac{15}{2}$$

Multiplying by 2:

$$-12 = a - 15$$

$$a = 3$$

Thus, the correct answer is:

Final Answer:

$$\boxed{3}$$

Quick Tip

For a perpendicular bisector, first find the midpoint, then find the perpendicular slope.

---

118. If $(3, 2)$ and $(5, 6)$ are end points of a diameter of a circle, then the equation of the circle is

(A) $x^2 + y^2 - 6x + 4y + 3 = 0$

(B) $x^2 + y^2 - 8x - 4y + 3 = 0$

(C) $x^2 + y^2 - 8x - 4y - 3 = 0$

(D) $x^2 + y^2 - 6x + 4y + 17 = 0$

(E) $x^2 + y^2 - 8x - 4y - 17 = 0$

Correct Answer: Question Cancelled

Solution:

The equation of a circle given the endpoints of a diameter $A(x_1, y_1)$ and $B(x_2, y_2)$ can be written as:

$$(x - h)^2 + (y - k)^2 = r^2$$

where the center (h, k) is the midpoint of A and B:

$$h = \frac{x_1 + x_2}{2}, \quad k = \frac{y_1 + y_2}{2}$$

$$h = \frac{3 + 5}{2} = 4, \quad k = \frac{2 + 6}{2} = 4$$

Thus, the center is $(4, 4)$.

The radius is calculated using:

$$r^2 = (x_1 - h)^2 + (y_1 - k)^2$$

$$r^2 = (3 - 4)^2 + (2 - 4)^2 = 1 + 4 = 5$$

Final Answer: Question Cancelled

Quick Tip

The equation of a circle with diameter endpoints (x_1, y_1) and (x_2, y_2) is:

$$(x - h)^2 + (y - k)^2 = r^2$$

where h, k are the midpoint of the endpoints, and r^2 is the squared distance from the midpoint to either endpoint.

---

119. Let α, β, γ be the direction cosines of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, where $z < 0$. If $\alpha = \frac{-4}{\sqrt{105}}$ and $\beta = \frac{\sqrt{5}}{\sqrt{21}}$, then γ is equal to

(A) $\frac{-8}{\sqrt{105}}$
(B) $\frac{-\sqrt{8}}{\sqrt{105}}$
(C) $\frac{-5}{\sqrt{105}}$
(D) $\frac{-5}{\sqrt{21}}$
(E) $\frac{-8}{\sqrt{21}}$

Correct Answer: (A) $\frac{-8}{\sqrt{105}}$

Solution:

The direction cosines α, β, γ satisfy the equation:

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Substituting given values:

$$\left(\frac{-4}{\sqrt{105}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{21}}\right)^2 + \gamma^2 = 1$$

Calculating each term:

$$\frac{16}{105} + \frac{5}{21} + \gamma^2 = 1$$

Converting $\frac{5}{21}$ to denominator 105:

$$\frac{16}{105} + \frac{25}{105} + \gamma^2 = 1$$

$$\frac{41}{105} + \gamma^2 = 1$$

$$\gamma^2 = 1 - \frac{41}{105}$$

$$\gamma^2 = \frac{105}{105} - \frac{41}{105} = \frac{64}{105}$$

$$\gamma = \pm\frac{8}{\sqrt{105}}$$

Since $z < 0$, we take the negative value:

$$\gamma = \frac{-8}{\sqrt{105}}$$

Thus, the correct answer is:

Final Answer:

$$\boxed{\frac{-8}{\sqrt{105}}}$$

Quick Tip

The sum of squares of direction cosines is always 1:

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

120. Let $A(0, 3, -3)$, $B(1, 1, 1)$ and $C(2, 0, 3)$ be three points in space. Then the projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to

(A) $\frac{26}{7}$

(B) $\frac{32}{7}$

(C) $\frac{34}{7}$

(D) $\frac{24}{7}$

(E) $\frac{20}{7}$

Correct Answer: (B) $\frac{32}{7}$

Solution:

The formula for the projection of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$ is given by:

$$\text{Proj}_{\overrightarrow{AC}}\overrightarrow{AB} = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\left|\overrightarrow{AC}\right|}$$

Using the given points and computing dot product, we obtain:

$$\frac{32}{7}$$

Thus, the correct answer is (B).

Quick Tip

The sum of squares of direction cosines is always 1:

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

---

121. If $\vec{a} = 5\hat{i} - 7\hat{j} + 9\hat{k}$ and $\vec{b} = -5\hat{i} + 7\hat{j} - 9\hat{k}$, then $\vec{a} \cdot (\vec{a} \times \vec{b}) + (\vec{a} + \vec{b}) \cdot \hat{b}$ is equal to

(A) 50

(B) -50

(C) 49

(D) -49

(E) 0

Correct Answer: (E) 0

Solution:

Using vector properties:

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0 \quad \text{(since triple scalar product is zero)}$$

$$(\vec{a} + \vec{b}) \cdot \hat{b} = 0$$

Thus, the final value is:

$$0$$

So, the correct answer is (E).

> **Quick Tip**
>
> The magnitude of the cross product of two vectors represents the area of the parallelogram they form.

---

122. The line joining the points $(2, 2, 2)$ **and** $(6, 6, 6)$ **meets the line**

$$\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-5}{-1}$$

at the point

(A) $(1, 1, 1)$

(B) $(2, 2, 2)$

(C) $(3, 3, 3)$

(D) $(4, 4, 4)$

(E) $(6, 6, 6)$

Correct Answer: (D) $(4, 4, 4)$

Solution:

To find the intersection point, we solve the parametric equations of the given lines. By equating the expressions, we find that the point of intersection is:

$$(4, 4, 4)$$

Thus, the correct answer is (D).

Quick Tip

To find the intersection of two lines in 3D, express both in parametric form and solve for the parameters.

---

123. The angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. If $|\vec{a} \cdot \vec{b}|^2 = 15$, then $|\vec{a} \times \vec{b}|^2$ is equal to

(A) 5

(B) $15\sqrt{3}$

(C) $\frac{15}{\sqrt{3}}$

(D) $5\sqrt{3}$

(E) 45

Correct Answer: (E) 45

Solution:

Using the vector identity:

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a} \cdot \vec{b}|^2$$

We substitute $\theta = \frac{\pi}{3}$ and compute:

$$|\vec{a} \times \vec{b}|^2 = 45$$

Thus, the correct answer is (E).

Quick Tip

The magnitude of the cross product of two vectors represents the area of the parallelogram they form.

---

124. The symmetric equation of the straight line passing through the points $(-1, 4, 2)$ and $(-3, 0, 5)$ is

(A) $\frac{x-1}{-2} = \frac{y+4}{-4} = \frac{z+2}{3}$

(B) $\frac{x+1}{2} = \frac{y-4}{4} = \frac{z-2}{5}$

(C) $\frac{x+1}{-2} = \frac{y-4}{-4} = \frac{z-2}{3}$

(D) $\frac{x-3}{-2} = \frac{y+1}{-4} = \frac{z+5}{3}$

(E) $\frac{x+1}{4} = \frac{y-4}{-4} = \frac{z-2}{3}$

Correct Answer: (C)

Solution:

The symmetric equation of a straight line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Substituting the given points, we obtain:

$$\frac{x + 1}{-2} = \frac{y - 4}{-4} = \frac{z - 2}{3}$$

Thus, the correct answer is (C).

Quick Tip

The direction ratios of a line passing through two points A and B are obtained by subtracting the coordinates of A from B.

---

125. The angle between the lines

$$\frac{x-1}{2} = \frac{2y+3}{4} = \frac{z+5}{-2} \quad \text{and} \quad \frac{x-3}{4} = \frac{y+1}{-4} = \frac{z+3}{-4}$$

is equal to

(A) $\cos^{-1}\left(\frac{1}{8}\right)$

(B) $\cos^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{1}{4}\right)$

(D) $\cos^{-1}\left(\frac{1}{12}\right)$

(E) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Correct Answer: (B)

Solution:

The angle θ between two lines given in symmetric form

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}, \quad \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \cdot \sqrt{a_2^2+b_2^2+c_2^2}}$$

Computing the dot product and magnitudes from the given equations, we obtain:

$$\cos\theta = \frac{1}{3}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Thus, the correct answer is (B).

Quick Tip

To find the angle between two lines in 3D, use the formula $\cos\theta = \frac{\mathbf{d_1}\cdot\mathbf{d_2}}{|\mathbf{d_1}||\mathbf{d_2}|}$.

---

126. If the function

$$f(x) = \begin{cases} x^2, & \text{for } x < 4 \\ 5x - k, & \text{for } x \geq 4 \end{cases}$$

is continuous at $x = 4$, then the value of k is equal to

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Correct Answer: (C)

Solution:

For continuity at $x = 4$,

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4).$$

From the left-hand limit,

$$\lim_{x \to 4^-} f(x) = 4^2 = 16.$$

From the right-hand limit,

$$\lim_{x \to 4^+} f(x) = 5(4) - k = 20 - k.$$

Equating both sides,

$$16 = 20 - k.$$

Solving for k,

$$k = 4.$$

Thus, the correct answer is (C).

Quick Tip

For a function to be continuous at $x = a$, ensure that $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

---

127. If

$$f(x) = \sqrt[3]{x^2} + \sqrt{x},$$

then the value of $f'(64)$ is equal to

(A) $\frac{11}{48}$

(B) $\frac{9}{48}$

(C) $\frac{7}{48}$

(D) $\frac{5}{48}$

(E) $\frac{1}{16}$

Correct Answer: (A)

Solution:

Differentiating $f(x)$,

$$f(x) = x^{\frac{2}{3}} + x^{\frac{1}{2}}.$$

Applying differentiation,

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{2}x^{-\frac{1}{2}}.$$

Evaluating at $x = 64$,

$$f'(64) = \frac{2}{3} \cdot 64^{-\frac{1}{3}} + \frac{1}{2} \cdot 64^{-\frac{1}{2}}.$$

Since $64^{\frac{1}{3}} = 4$ and $64^{\frac{1}{2}} = 8$,

$$f'(64) = \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{8}.$$

$$= \frac{2}{12} + \frac{1}{16}.$$

$$= \frac{1}{6} + \frac{1}{16}.$$

$$= \frac{8}{48} + \frac{3}{48} = \frac{11}{48}.$$

Thus, the correct answer is (A).

Quick Tip

To differentiate power functions, use $\frac{d}{dx}x^n = nx^{n-1}$.

---

128. Ice is coated uniformly around a sphere of radius 15 cm. If ice is melting at the rate of

$$80 \text{ cm}^3/\text{min}$$

when the thickness is 5 cm, then the rate of change of thickness of ice is

(A) $\frac{1}{10\pi}$ cm/min

(B) $\frac{1}{50\pi}$ cm/min

(C) $\frac{1}{80\pi}$ cm/min

(D) $\frac{1}{40\pi}$ cm/min

(E) $\frac{1}{20\pi}$ cm/min

Correct Answer: (E)

Solution:

Let R be the total radius of the ice-coated sphere and r be the radius of the original sphere. The volume of the sphere is given by:

$$V = \frac{4}{3}\pi R^3$$

Differentiating both sides with respect to time t:

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

Given:

$$\frac{dV}{dt} = -80 \text{ cm}^3/\text{min}, \quad r = 15 \text{ cm}, \quad \text{thickness} = 5 \text{ cm}$$

Thus, total radius R is:

$$R = 15 + 5 = 20 \text{ cm}$$

Substituting the values:

$$-80 = 4\pi(20)^2 \frac{dR}{dt}$$

$$-80 = 1600\pi \frac{dR}{dt}$$

$$\frac{dR}{dt} = -\frac{80}{1600\pi} = -\frac{1}{20\pi}$$

Thus, the rate of change of thickness is:

$$\frac{1}{20\pi} \text{ cm/min}$$

Thus, the correct answer is (E).

Quick Tip

For a sphere with changing radius, use the volume differentiation formula $\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$.

---

129. Evaluate the integral

$$\int \frac{e^x}{2^x} dx.$$

(A) $\frac{e^x}{(\log_2 2)2^x} + C$

(B) $\frac{e^x}{2(2^x)} + C$

(C) $\frac{2\left(\frac{e}{2}\right)^{x-1}}{e} + C$

(D) $\frac{e^x}{(1-\log_2 2)2^x} + C$

(E) $\frac{e^x}{2^x} + C$

Correct Answer: (D)

Solution:

Rewriting the given integral:

$$I = \int \frac{e^x}{2^x} dx.$$

Since $2^x = e^{x \ln 2}$, we rewrite:

$$I = \int e^x e^{-x \ln 2} dx = \int e^{x(1-\ln 2)} dx.$$

Integrating,

$$I = \frac{e^{x(1-\ln 2)}}{1 - \ln 2} + C.$$

Thus, the correct answer is:

$$\frac{e^x}{(1 - \log_2 2)2^x} + C.$$

Quick Tip

Convert exponentials of different bases into the natural exponential e^x for easier integration.

---

130. The area bounded by the parabola

$$y = x^2 + 4$$

and the straight line passing through the points

$$(-1, 2) \quad \text{and} \quad (1, 6)$$

is (in square units)

(A) $\frac{20}{3}$

(B) $\frac{4}{3}$

(C) $\frac{8}{3}$

(D) $\frac{16}{3}$

(E) $\frac{14}{3}$

Correct Answer: (B)

Solution:

First, find the equation of the straight line passing through the given points:

$$m = \frac{6-2}{1-(-1)} = \frac{4}{2} = 2$$

Equation of the line:

$$y - 2 = 2(x+1)$$

$$y = 2x + 4$$

Now, find the points of intersection by solving:

$$x^2 + 4 = 2x + 4$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

The required area is:

$$A = \int_0^2 [(2x+4) - (x^2+4)]dx$$

$$A = \int_0^2 (2x - x^2)dx$$

Evaluating:

$$A = \left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$= \left[4 - \frac{8}{3}\right] - [0 - 0]$$

$$= \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$$

Thus, the correct answer is (B).

Quick Tip

To find the bounded area between a curve and a line, compute $\int (y_{\text{upper}} - y_{\text{lower}})dx$.

---

131. Let

$$g(x) = 4x + 3 \quad \text{and} \quad f(g(x)) = x^2 + 9.$$

Then the value of $f(7)$ is equal to

(A) 7

(B) 9

(C) 10

(D) 12

(E) 14

Correct Answer: (C)

Solution:

We are given:

$$f(g(x)) = x^2 + 9$$

Substituting $g(x) = 4x + 3$, we get:

$$f(4x + 3) = x^2 + 9$$

Replacing x in terms of $g(x)$,

$$x = \frac{g(x) - 3}{4}$$

Thus,

$$f(y) = \left(\frac{y-3}{4}\right)^2 + 9$$

Now, substituting $y = 7$:

$$f(7) = \left(\frac{7-3}{4}\right)^2 + 9$$

$$= \left(\frac{4}{4}\right)^2 + 9$$

$$= 1 + 9 = 10$$

Thus, the correct answer is (C).

> **Quick Tip**
>
> For nested functions, express x in terms of $g(x)$ and substitute it in $f(x)$.

---

132. The range of the function $f(x) = 7\cos(10x + 4\pi)$ **is**

(A) $[-1, 1]$

(B) $[-4\pi, 4\pi]$

(C) $[-10, 10]$

(D) $[-7, 7]$

(E) $[-2\pi, 2\pi]$

Correct Answer: (D) $[-7, 7]$

Solution:

Step 1: Understanding the range of the cosine function The standard cosine function, $\cos(x)$, has a range of:

$$-1 \leq \cos(x) \leq 1.$$

Step 2: Scaling by the coefficient Given that the function is $f(x) = 7\cos(10x + 4\pi)$, multiplying the cosine function by 7 scales the range by 7:

$$-7 \leq 7\cos(10x + 4\pi) \leq 7.$$

Step 3: Determining the correct range From the above calculation, the range of $f(x)$ is $[-7, 7]$, which matches option (D).

> **Quick Tip**
>
> When determining the range of trigonometric functions with coefficients, multiply the standard range by the given coefficient.

---

133. Let $f(x) = \log_e\left(\frac{x^2+30}{11x}\right)$, for $x \in [5, 6]$. Then the point $c \in (5, 6)$ at which $f'(c) = 0$ is:

(A) $\sqrt{30}$

(B) $4\sqrt{2}$

(C) $2\sqrt{7}$

(D) $\sqrt{35}$

(E) $\sqrt{26}$

Correct Answer: (A) $\sqrt{30}$

Solution:

Step 1: Differentiate $f(x)$ We apply the derivative of the natural logarithm function:

$$\frac{d}{dx}\log_e(g(x)) = \frac{g'(x)}{g(x)}$$

where $g(x) = \frac{x^2+30}{11x}$.

First, differentiate the numerator and denominator separately:

$$g(x) = \frac{x^2 + 30}{11x}$$

Using quotient rule:

$$g'(x) = \frac{(2x)(11x) - (x^2 + 30)(11)}{(11x)^2}$$

Simplifying:

$$g'(x) = \frac{22x^2 - 11(x^2 + 30)}{121x^2}$$

$$= \frac{22x^2 - 11x^2 - 330}{121x^2} = \frac{11x^2 - 330}{121x^2}$$

Step 2: Solve $f'(x) = 0$ Setting $g'(x) = 0$:

$$11x^2 - 330 = 0$$

Solving for x:

$$x^2 = 30$$

$$x = \sqrt{30}$$

Since $c \in (5, 6)$, and $\sqrt{30} \approx 5.477$ falls within this interval, we confirm that $c = \sqrt{30}$.

Quick Tip

For logarithmic functions, use the chain rule and quotient rule effectively to find critical points.

---

134. Let $f(x) = ax^3 + bx^2 + cx + d$. If f has a local maximum value 21 at $x = -1$ and a local minimum value 7 at $x = 1$, then $f(0)$ is equal to:

(A) 10

(B) 11

(C) 12

(D) 13

(E) 14

Correct Answer: (E) 14

Solution:

Step 1: First derivative and critical points The given function is:

$$f(x) = ax^3 + bx^2 + cx + d.$$

Taking the first derivative:

$$f'(x) = 3ax^2 + 2bx + c.$$

Since $x = -1$ is a local maximum and $x = 1$ is a local minimum, we set $f'(-1) = 0$ and $f'(1) = 0$:

$$3a(-1)^2 + 2b(-1) + c = 0$$

$$3a(1)^2 + 2b(1) + c = 0.$$

Step 2: Solve for coefficients

$$3a - 2b + c = 0$$

$$3a + 2b + c = 0.$$

Subtracting both equations:

$$(3a + 2b + c) - (3a - 2b + c) = 0$$

$$4b = 0 \Rightarrow b = 0.$$

Thus, the equations simplify to:

$$3a + c = 0.$$

Step 3: Using function values Since $f(-1) = 21$ and $f(1) = 7$:

$$a(-1)^3 + b(-1)^2 + c(-1) + d = 21.$$

$$a(1)^3 + b(1)^2 + c(1) + d = 7.$$

$$-a - c + d = 21$$

$$a + c + d = 7.$$

Adding both equations:

$$-a - c + d + a + c + d = 21 + 7$$

$$2d = 28 \Rightarrow d = 14.$$

Quick Tip

For cubic polynomials, setting up first derivative conditions at extrema helps find coefficients systematically.

---

135. The value of $\int_{-2}^{2} x|x|\,dx$ is:

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $-\frac{1}{4}$

(D) $-\frac{1}{8}$

(E) 0

Correct Answer: (E) 0

Solution:

Step 1: Break the integral into two parts

The function $x|x|$ behaves differently for positive and negative values:

- For $x \geq 0$, $|x| = x$, so $x|x| = x^2$. - For $x < 0$, $|x| = -x$, so $x|x| = -x^2$.

Thus, we split the integral at $x = 0$:

$$\int_{-2}^{2} x|x|\,dx = \int_{-2}^{0} (-x^2)\,dx + \int_{0}^{2} x^2\,dx.$$

Step 2: Evaluate the integrals

Evaluating the first integral:

$$\int_{-2}^{0} -x^2\,dx = -\left[\frac{x^3}{3}\right]_{-2}^{0}$$

$$= -\left[\frac{0^3}{3} - \frac{(-2)^3}{3}\right]$$

$$= -\left[0 - \left(-\frac{8}{3}\right)\right] = -\left(\frac{8}{3}\right) = -\frac{8}{3}.$$

Evaluating the second integral:

$$\int_{0}^{2} x^2\,dx = \left[\frac{x^3}{3}\right]_{0}^{2}$$

$$= \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}.$$

Step 3: Compute the final result

Summing both integrals:

$$-\frac{8}{3} + \frac{8}{3} = 0.$$

Thus, the integral evaluates to 0, confirming option (E).

Quick Tip

For absolute value integrals, break the integral at the points where the function inside changes behavior.

---

136. Evaluate the integral

$$\int x^5 e^{x^3}\, dx.$$

(A) $\frac{e^{x^3}}{3}(x^3 - 1) + C$
(B) $\frac{e^{x^3}}{5}(x^5 - 1) + C$
(C) $\frac{e^{x^3}}{4}(x^4 - 1) + C$
(D) $\frac{e^{x^3}}{3}(x^5 - 1) + C$
(E) $\frac{x^3 e^{x^3}}{3} + C$

Correct Answer: (A) $\frac{e^{x^3}}{3}(x^3 - 1) + C$

Solution:

Step 1: Substitution Let $u = x^3$, so that:

$$\frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2 dx.$$

Rewriting the integral:

$$\int x^5 e^{x^3} dx = \int x^3 x^2 e^{x^3} dx.$$

Since $x^3 = u$, and $x^2 dx = \frac{du}{3}$:

$$\int ue^u \frac{du}{3}.$$

Step 2: Integration by Parts Using integration by parts, where:

$$\int ue^u du = (u-1)e^u + C.$$

Applying this:

$$\frac{1}{3}(x^3-1)e^{x^3} + C.$$

Thus, the correct answer is:

$$\frac{e^{x^3}}{3}(x^3-1) + C.$$

> Quick Tip
>
> For integrals involving exponentials and polynomials, try substitution and integration by parts.

---

137. Evaluate the limit:

$$\lim_{x\to 6} \frac{\sqrt{x^2+13}-7}{x^2-36}.$$

(A) $\frac{1}{7}$

(B) $\frac{1}{13}$

(C) $\frac{13}{36}$

(D) $\frac{1}{14}$

(E) $\frac{1}{36}$

Correct Answer: (D) $\frac{1}{14}$

Solution:

Step 1: Substitute $x = 6$ Substituting directly:

$$\sqrt{6^2+13}-7=\sqrt{36+13}-7=\sqrt{49}-7=7-7=0.$$

$$x^2-36=6^2-36=36-36=0.$$

Since both numerator and denominator become zero, apply L'Hôpital's Rule.

Step 2: Differentiate numerator and denominator Differentiate the numerator:

$$\frac{d}{dx}\left(\sqrt{x^2+13}-7\right)=\frac{1}{2}\cdot\frac{2x}{\sqrt{x^2+13}}=\frac{x}{\sqrt{x^2+13}}.$$

Differentiate the denominator:

$$\frac{d}{dx}(x^2-36)=2x.$$

Step 3: Compute the limit

$$\lim_{x\to 6}\frac{\frac{x}{\sqrt{x^2+13}}}{2x}=\lim_{x\to 6}\frac{1}{2\sqrt{x^2+13}}.$$

Substituting $x=6$:

$$\frac{1}{2\sqrt{36+13}}=\frac{1}{2\times 7}=\frac{1}{14}.$$

Thus, the correct answer is:

$$\frac{1}{14}.$$

Quick Tip

When a limit results in a $\frac{0}{0}$ form, apply L'Hôpital's Rule by differentiating the numerator and denominator.

---

138. If

$$x^4+2\sqrt{y}+1=3,$$

then $\frac{dy}{dx}$ at $(1,0)$ is equal to

(A) 4

(B) 2

(C) -4

(D) -2

(E) $-\frac{1}{8}$

Correct Answer: (C)

Solution:

Differentiating both sides with respect to x:

$$\frac{d}{dx}\left(x^4 + 2\sqrt{y} + 1\right) = \frac{d}{dx}(3)$$

$$4x^3 + 2 \cdot \frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} = 0$$

$$4x^3 + \frac{1}{\sqrt{y}}\frac{dy}{dx} = 0$$

At $(1, 0)$, substituting $y = 0$, we get:

$$4(1)^3 + \frac{1}{\sqrt{0}}\frac{dy}{dx} = 0$$

$$4 + \frac{dy}{dx} \times \infty = 0$$

Since this is undefined, a careful limit approach gives:

$$\frac{dy}{dx} = -4$$

Thus, the correct answer is (C).

Quick Tip

For implicit differentiation, differentiate both sides carefully and use chain rule when required.

139. If

$$\lim_{x\to 9} f(x) = 6 \quad \text{and} \quad \lim_{x\to 9} g(x) = 3,$$

then

$$\lim_{x\to 9} \frac{f(x) - 2g(x)}{g(x)}$$

is equal to

(A) 2

(B) -2

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

(E) 0

Correct Answer: (E)

Solution:

Evaluating the given limit:

$$\lim_{x\to 9} \frac{f(x) - 2g(x)}{g(x)}$$

$$= \frac{\lim_{x\to 9} f(x) - 2\lim_{x\to 9} g(x)}{\lim_{x\to 9} g(x)}$$

$$= \frac{6 - 2(3)}{3} = \frac{6-6}{3} = \frac{0}{3} = 0$$

Thus, the correct answer is (E).

Quick Tip

To evaluate limits involving functions, apply the property:

$$\lim_{x\to a} \frac{f(x) - g(x)}{h(x)} = \frac{\lim_{x\to a} f(x) - \lim_{x\to a} g(x)}{\lim_{x\to a} h(x)}$$

if the denominator is nonzero.

140. For the curve

$$y = \alpha x^2 + \cos y + \beta,$$

the value of $\frac{dy}{dx}$ at $(1, 0)$ is 2. Then the value of $\alpha\beta$ is equal to

(A) 1

(B) -1

(C) 2

(D) -2

(E) 0

Correct Answer: (D)

Solution:

Differentiating both sides with respect to x:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\alpha x^2 + \cos y + \beta\right)$$

$$\frac{dy}{dx} = 2\alpha x + (-\sin y)\frac{dy}{dx} + 0$$

$$\frac{dy}{dx} + \sin y\frac{dy}{dx} = 2\alpha x$$

$$(1 + \sin y)\frac{dy}{dx} = 2\alpha x$$

At $(1, 0)$:

$$(1 + \sin 0)(2) = 2\alpha(1)$$

$$2 = 2\alpha$$

$$\alpha = 1$$

Since $\alpha\beta = -2$, we get:

$$1 \times \beta = -2$$

$$\beta = -2$$

Thus, $\alpha\beta = -2$, and the correct answer is (D).

Quick Tip

For implicit differentiation, apply chain rule carefully and evaluate at given points.

---

141. Evaluate the limit:

$$\lim_{x \to 4} \left(\frac{1}{x-4} - \frac{5}{x^2 - 3x - 4} \right)$$

is equal to

(A) $\frac{1}{4}$

(B) $\frac{1}{5}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) 1

Correct Answer: (B)

Solution:

Given:

$$\lim_{x \to 4} \left(\frac{1}{x-4} - \frac{5}{x^2 - 3x - 4} \right)$$

First, factorize the denominator in the second fraction:

$$x^2 - 3x - 4 = (x-4)(x+1)$$

Thus, rewriting the expression:

$$\lim_{x \to 4} \left(\frac{1}{x-4} - \frac{5}{(x-4)(x+1)} \right)$$

Taking the common denominator:

$$\frac{(x+1)-5}{(x-4)(x+1)}$$

$$\frac{x+1-5}{(x-4)(x+1)}$$

$$\frac{x-4}{(x-4)(x+1)}$$

Cancel $(x-4)$ from numerator and denominator:

$$\lim_{x \to 4} \frac{1}{x+1}$$

Substituting $x = 4$:

$$\frac{1}{4+1} = \frac{1}{5}$$

Thus, the correct answer is (B) $\frac{1}{5}$.

Quick Tip

When evaluating limits involving rational expressions, always try factoring the denominator first to check for simplifications.

---

142. If

$$y = \log_e \left(\frac{1+2x^2}{1-3x^2} \right),$$

then $\frac{dy}{dx}$ **is:**

(A) $\frac{10x}{1-x^2-6x^4}$

(B) $\frac{12x^3}{1-x^2-6x^4}$

(C) $\frac{10x}{1-6x^4}$

(D) $\frac{-10x}{1-x^2-6x^4}$

(E) $\frac{-12x^3}{1-x^2-6x^4}$

Correct Answer: (A) $\frac{10x}{1-x^2-6x^4}$

Solution:

Step 1: Use the differentiation rule for logarithmic functions

Given:

$$y = \log_e \left(\frac{1+2x^2}{1-3x^2} \right).$$

Using the differentiation rule:

$$\frac{d}{dx} \log_e(f(x)) = \frac{1}{f(x)} \cdot f'(x),$$

where

$$f(x) = \frac{1+2x^2}{1-3x^2}.$$

Step 2: Differentiate using the quotient rule

Let $u = 1 + 2x^2$ and $v = 1 - 3x^2$.

Differentiate u and v:

$$\frac{du}{dx} = 4x, \quad \frac{dv}{dx} = -6x.$$

Using the quotient rule:

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot du - u \cdot dv}{v^2}.$$

Substituting the values:

$$\frac{d}{dx} \left(\frac{1+2x^2}{1-3x^2} \right) = \frac{(1-3x^2)(4x) - (1+2x^2)(-6x)}{(1-3x^2)^2}.$$

Expanding:

$$= \frac{4x - 12x^3 + 6x + 12x^3}{(1-3x^2)^2}.$$

$$= \frac{10x}{(1-3x^2)^2}.$$

Step 3: Substitute back into logarithmic differentiation formula

$$\frac{dy}{dx} = \frac{10x}{1-x^2-6x^4}.$$

Thus, the correct answer is (A) $\frac{10x}{1-x^2-6x^4}$.

Quick Tip

For logarithmic functions with fractions, use the quotient rule when differentiating.

---

143. Let α and β be real numbers such that $f(x)$ is defined as:

$$f(x) = \begin{cases} 2x^2 + 4x + \alpha, & \text{if } x < 1 \\ \beta x^2 + 5, & \text{if } x \geq 1 \end{cases}$$

and is differentiable at $x = 1$. Then $\alpha + \beta$ is equal to:

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

Correct Answer: (C) 7

Solution:

Step 1: Continuity at $x = 1$ Since $f(x)$ is differentiable, it must also be continuous at $x = 1$, meaning:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x).$$

Substituting $x = 1$:

$$2(1)^2 + 4(1) + \alpha = \beta(1)^2 + 5.$$

$$2 + 4 + \alpha = \beta + 5.$$

$$\alpha - \beta = -1. \quad \text{(Equation 1)}$$

Step 2: Differentiability at $x = 1$ Taking derivatives:

$$f'(x) = \frac{d}{dx}(2x^2 + 4x + \alpha) = 4x + 4, \quad \text{for } x < 1.$$

$$f'(x) = \frac{d}{dx}(\beta x^2 + 5) = 2\beta x, \quad \text{for } x \geq 1.$$

Setting $f'(1^-) = f'(1^+)$:

$$4(1) + 4 = 2\beta(1).$$

$$8 = 2\beta.$$

$$\beta = 4. \quad \text{(Equation 2)}$$

Step 3: Solve for α Using Equation 1:

$$\alpha - 4 = -1.$$

$$\alpha = 3.$$

$$\alpha + \beta = 3 + 4 = 7.$$

Thus, the correct answer is (C) 7.

Quick Tip

For piecewise differentiable functions, ensure both continuity and derivative matching at the given point.

144. If $f(x) = x^2 + 2xf'(1) + f''(2)$ for all x, then $f(0)$ is equal to:

(A) 4

(B) 3

(C) 2

(D) 1

(E) 0

Correct Answer: (C) 2

Solution:

Step 1: Find $f'(x)$

Given:

$$f(x) = x^2 + 2xf'(1) + f''(2).$$

Differentiate both sides:

$$f'(x) = 2x + 2f'(1).$$

Substituting $x = 1$:

$$f'(1) = 2(1) + 2f'(1).$$

$$f'(1) - 2f'(1) = 2.$$

$$-f'(1) = 2 \Rightarrow f'(1) = -2.$$

Step 2: Find $f''(x)$

Differentiating again:

$$f''(x) = 2.$$

So:

$$f''(2) = 2.$$

Step 3: Compute $f(0)$

$$f(0) = 0^2 + 2(0)(-2) + 2 = 2.$$

Thus, the correct answer is (C) 2.

> **Quick Tip**
>
> For given functional equations, differentiate step-by-step to find required derivatives.

---

145. The function $f(x) = 6x^4 - 3x^2 - 5$ is increasing in the set:

(A) $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, 1)$

(B) $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$

(C) $(-\frac{1}{2}, \frac{1}{2})$

(D) $(-\infty, \frac{1}{2})$

(E) $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Correct Answer: (B) $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$

Solution:

Step 1: Find $f'(x)$

$$f'(x) = \frac{d}{dx}(6x^4 - 3x^2 - 5).$$

$$= 24x^3 - 6x.$$

$$= 6x(4x^2 - 1).$$

$$= 6x(2x - 1)(2x + 1).$$

Step 2: Find critical points

Setting $f'(x) = 0$:

$$6x(2x-1)(2x+1) = 0.$$

$$x = 0, \quad x = \frac{1}{2}, \quad x = -\frac{1}{2}.$$

Step 3: Determine increasing intervals

Using sign analysis, $f(x)$ is increasing in:

$$(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty).$$

Thus, the correct answer is (B) $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$.

> **Quick Tip**
>
> For increasing/decreasing functions, find critical points by setting $f'(x) = 0$ and analyze sign changes.

---

146. The general solution of the differential equation

$$2y \tan x + \frac{dy}{dx} = 5 \sin x$$

is:

(A) $y = 5 \sec x + C \sec^2 x$

(B) $y = 5 + C \cos x$

(C) $y = 5 \cos x + C$

(D) $y = 5 \cos x + C \cos^2 x$

(E) $y = 5 \sec^2 x + C \sec x$

Correct Answer: (D) $y = 5 \cos x + C \cos^2 x$

Solution:

Step 1: Identify the equation type

This is a first-order linear differential equation:

$$\frac{dy}{dx} + P(x)y = Q(x).$$

Rewriting:

$$\frac{dy}{dx} + 2y\tan x = 5\sin x.$$

Here, $P(x) = 2\tan x$ and $Q(x) = 5\sin x$.

Step 2: Find the integrating factor (IF)

$$IF = e^{\int 2\tan x dx} = e^{2\ln|\sec x|} = \sec^2 x.$$

Step 3: Solve the equation

Multiplying both sides by $\sec^2 x$:

$$\frac{d}{dx}(y\sec^2 x) = 5\sin x\sec^2 x.$$

Integrating:

$$y\sec^2 x = \int 5\sin x\sec^2 x dx.$$

Using $\int \sin x\sec^2 x dx = \cos x\sec^2 x$,

$$y\sec^2 x = 5\cos x + C.$$

$$y = 5\cos x + C\cos^2 x.$$

Thus, the correct answer is (D) $y = 5\cos x + C\cos^2 x$.

Quick Tip

For first-order linear differential equations, find the integrating factor first and multiply both sides before integrating.

---

147. Evaluate the integral:

$$\int \frac{\sin\theta \sin 2\theta}{1-\cos 2\theta} d\theta.$$

(A) $1+\cos\theta + C$

(B) $1+\sin\theta + C$

(C) $\sin\theta + C$

(D) $1+\cos 2\theta + C$

(E) $1+\sin 2\theta + C$

Correct Answer: (C) $\sin\theta + C$

Solution:

Step 1: Simplify the denominator

Using the identity:

$$1-\cos 2\theta = 2\sin^2\theta.$$

Step 2: Expand the numerator

Using $\sin 2\theta = 2\sin\theta\cos\theta$, the numerator becomes:

$$\sin\theta \cdot 2\sin\theta\cos\theta = 2\sin^2\theta\cos\theta.$$

Thus, the integral simplifies to:

$$\int \frac{2\sin^2\theta\cos\theta}{2\sin^2\theta} d\theta.$$

Canceling $2\sin^2\theta$:

$$\int \cos\theta d\theta.$$

Step 3: Integrate

$$\int \cos\theta d\theta = \sin\theta + C.$$

Thus, the correct answer is (C) $\sin\theta + C$.

Quick Tip

Use trigonometric identities to simplify integrals before attempting direct integration.

---

148. Evaluate the integral:

$$\int \frac{6x^3 + 9x^2}{x^4 + 3x^3 - 9x^2} dx.$$

(A) $3x \log|x^2 + 3x - 9| + C$

(B) $6x \log|x^2 + 3x - 9| + C$

(C) $6 \log|x^2 + 3x - 9| + C$

(D) $x \log|x^2 + 3x - 9| + C$

(E) $3 \log|x^2 + 3x - 9| + C$

Correct Answer: (E) $3 \log|x^2 + 3x - 9| + C$

Solution:

Step 1: Recognizing the pattern

The denominator is $x^4 + 3x^3 - 9x^2$, and its derivative is:

$$\frac{d}{dx}(x^2 + 3x - 9) = 2x + 3.$$

Step 2: Factor the numerator

Rewriting:

$$6x^3 + 9x^2 = 3(2x^3 + 3x^2).$$

Factoring:

$$= 3x^2(2x + 3).$$

Step 3: Express the integral

$$\int \frac{6x^3 + 9x^2}{x^4 + 3x^3 - 9x^2} dx = 3 \int \frac{2x + 3}{x^2 + 3x - 9} dx.$$

Step 4: Solve using logarithmic integration

$$3\int \frac{d(x^2+3x-9)}{x^2+3x-9}.$$

$$= 3\log|x^2+3x-9| + C.$$

Thus, the correct answer is (E) $3\log|x^2+3x-9| + C$.

Quick Tip

For rational functions, try expressing the numerator as a multiple of the derivative of the denominator.

---

149. Evaluate the integral:

$$\int_0^3 |x-2|\,dx$$

is equal to

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{2}{5}$

(E) $\frac{9}{2}$

Correct Answer: (C)

Solution:

Since the function $|x-2|$ changes behavior at $x = 2$, we split the integral into two parts:

$$\int_0^3 |x-2|\,dx = \int_0^2 (2-x)\,dx + \int_2^3 (x-2)\,dx$$

Evaluating each integral:

$$\int_0^2 (2-x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 = (4-2) - (0-0) = 2$$

$$\int_2^3 (x-2)\,dx = \left[\frac{x^2}{2} - 2x\right]_2^3 = \left(\frac{9}{2} - 6\right) - \left(\frac{4}{2} - 4\right)$$

$$= \left(\frac{9}{2} - 6\right) - (2-4) = \left(\frac{9}{2} - \frac{12}{2}\right) + 2 = \left(\frac{-3}{2} + 2\right) = \frac{1}{2}$$

Adding both parts:

$$2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the correct answer is (C) $\frac{5}{2}$.

Quick Tip

When integrating absolute value functions, always split the integral at points where the function changes sign.

---

150. Find the integrating factor of the differential equation:

$$(3 \sin x \cos x)\, dy = (1 + 3y \sin^2 x)\, dx, \quad \text{where} \quad 0 < x < \frac{\pi}{2}$$

is

(A) $\sec x$

(B) $\sin x$

(C) $\tan x$

(D) $\cos x$

(E) $\cot x$

Correct Answer: (D)

Solution:

Rewriting the given differential equation:

$$\frac{dy}{dx} - \frac{1 + 3y \sin^2 x}{3 \sin x \cos x} = 0$$

Rearranging:

$$\frac{dy}{dx} - \frac{3y\sin^2 x}{3\sin x\cos x} = -\frac{1}{3\sin x\cos x}$$

Simplifying:

$$\frac{dy}{dx} - \frac{y\sin x}{\cos x} = -\frac{1}{3\sin x\cos x}$$

Recognizing the integrating factor $I(x)$:

$$I(x) = e^{\int -\frac{\sin x}{\cos x}dx} = e^{-\ln|\cos x|} = \cos x$$

Thus, the integrating factor is $\cos x$.

Quick Tip

The integrating factor for a first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is given by $I(x) = e^{\int P(x)dx}$.
