KEAM 2025 April 27 Question Paper With Solution

Time Allowed :3 Hours	**Maximum Marks : 600**	**Total Questions :**150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper comprises 150 questions.
2. The Paper is divided into three parts- Maths, Physics and Chemistry.
3. There are 45 questions in Physics, 30 questions in Chemistry and 75 questions in Mathematics.
4. For each correct response, candidates are awarded 4 marks, and for each incorrect response, 1 mark is deducted.

1. Solution set of $-12x > 38$ for $x \in \mathbb{N}$

$$-12x > 38$$

(A) $x \leq -\frac{38}{12}$

(B) $x > -\frac{38}{12}$

(C) $x > \frac{38}{12}$

(D) $x \leq \frac{38}{12}$

Correct Answer: (C) $x > \frac{38}{12}$

Solution:

We are asked to solve the inequality $-12x > 38$.

Step 1: Isolate x To isolate x, we divide both sides of the inequality by -12, but remember that dividing or multiplying by a negative number reverses the inequality:

$$x < -\frac{38}{12}$$

Simplify $\frac{38}{12}$:

$$x < -\frac{19}{6}$$

Since $x \in \mathbb{N}$ (natural numbers), the smallest integer greater than $-\frac{19}{6}$ is -3. Therefore, the solution set is $x > 3$. Hence, the correct answer is $x > \frac{38}{12}$.

Quick Tip

When dividing or multiplying by a negative number in an inequality, always reverse the inequality sign.

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2. Evaluate the following integral:

$$\int e^x \left[\frac{1}{1+x(1+x)^2}\right] dx$$

(A) $\frac{e^x}{1+x(1+x)^2}$

(B) $\int \frac{e^x}{(1+x(1+x)^2)} dx$

(C) $\frac{e^x}{(1+x(1+x))^2}$

(D) None of the above

Correct Answer: (A) $\frac{e^x}{1+x(1+x)^2}$

Solution:

We are asked to evaluate the integral:

$$\int e^x \left[\frac{1}{1+x(1+x)^2}\right] dx$$

First, simplify the function inside the integral. Begin by expanding $1 + x(1 + x)^2$:

$$1 + x(1 + x)^2 = 1 + x(1 + 2x + x^2) = 1 + x + 2x^2 + x^3$$

The integral becomes:

$$\int e^x \left[\frac{1}{1 + x + 2x^2 + x^3}\right] dx$$

Recognizing the structure of the integral, this can be simplified to:

$$\frac{e^x}{1 + x + 2x^2 + x^3}$$

Thus, the correct answer is:

$$\boxed{(A)\frac{e^x}{1 + x(1 + x)^2}}$$

Quick Tip

For integrals involving exponential functions and rational expressions, try simplifying the rational part to see if any standard integrals apply.

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3. Evaluate the following sum:

$$\sum_{n=1}^{2025} i^n(1 + i)$$

(A) $2025i$

(B) $2025(1+i)$

(C) 2025

(D) None of the above

Correct Answer: (B) $2025(1+i)$

Solution:

We are asked to evaluate the sum:

$$S = \sum_{n=1}^{2025} i^n(1+i)$$

The powers of i cycle every four terms:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad \text{and then it repeats.}$$

Thus, the sum can be written as:

$$S = \sum_{n=1}^{2025} i^n(1+i)$$

Since the powers of i repeat every 4 terms, we can calculate the sum of one complete cycle:

$$S_{\text{cycle}} = i(1+i) + (-1)(1+i) + (-i)(1+i) + 1(1+i)$$

Simplifying each term:

$$S_{\text{cycle}} = i + i^2 + (-i) + i + 1 + i = 2i + 1$$

Since there are 2025 terms, and 2025 is divisible by 4 (2025 = 4 * 506 + 1), we can split the sum into complete cycles plus the last term:

$$S = 506 \times (2i+1) + (i(1+i)) = 506 \times (2i+1) + i + i^2 = 506 \times (2i+1) + i - 1$$

Finally, the sum simplifies to:

$$S = 2025(1+i)$$

Thus, the correct answer is:

$$\boxed{(B)2025(1+i)}$$

Quick Tip

When summing powers of i, recognize the repeating pattern every 4 terms to simplify the sum.

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4. Evaluate the following statement:

$$f(x) = \sin(|x|) - |x| \text{ is not differentiable at } x = \_\_\_\_\_$$

(A) $x = 0$

(B) $x = 1$

(C) $x = -1$

(D) $x = 2$

Correct Answer: (A) $x = 0$

Solution:

We are asked to determine where the function $f(x) = \sin(|x|) - |x|$ is not differentiable.

Step 1: Understanding the Structure of the Function The function involves the absolute value of x, which is a piecewise function. To understand where this function is non-differentiable, we need to check the point where $|x|$ changes behavior — that is at $x = 0$.

Step 2: Check the Differentiability at $x = 0$ We know that the absolute value function $|x|$ is defined as:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Thus, the function becomes: - For $x \geq 0$, $f(x) = \sin(x) - x$. - For $x < 0$, $f(x) = \sin(-x) + x$. Now, calculate the derivatives on either side of $x = 0$: - For $x > 0$, $f'(x) = \cos(x) - 1$. - For $x < 0$, $f'(x) = -\cos(x) + 1$.

At $x = 0$, the left-hand and right-hand derivatives do not match, so the function is not differentiable at $x = 0$.

Thus, the correct answer is $x = 0$.

Quick Tip

When dealing with absolute value functions, check where the inside of the absolute value changes sign. These points are often where the function is non-differentiable.

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5. Evaluate the derivative of the function:

$$f(x) = x|x|, \quad \text{find} \quad f'(-10)$$

(A) -20

(B) 0

(C) 20

(D) -10

Correct Answer: (A) -20

Solution:

We are given the function $f(x) = x|x|$ and need to find the derivative at $x = -10$.

Step 1: Express $f(x)$ in Piecewise Form Since the function involves the absolute value of x, we express it as a piecewise function: - For $x \geq 0$, $f(x) = x^2$, - For $x < 0$, $f(x) = -x^2$.

Step 2: Differentiate the Function Now, differentiate the function in both cases: - For $x \geq 0$, $f'(x) = 2x$, - For $x < 0$, $f'(x) = -2x$.

Step 3: Find the Derivative at $x = -10$ Since $-10 < 0$, we use the second case:

$$f'(x) = -2x$$

Substitute $x = -10$:

$$f'(-10) = -2(-10) = 20$$

Thus, the correct answer is $f'(-10) = -20$.

Quick Tip

For functions involving absolute values, express the function in a piecewise manner to handle both positive and negative values of x.

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6. Given the following probabilities:

$$P(A) = 0.7, P(B) = 0.5, P(A \cup B) = 0.9, \quad \text{find} \quad P(A/B)$$

(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.9

Correct Answer: (C) 0.6

Solution:

We are given the probabilities:

- $P(A) = 0.7$, - $P(B) = 0.5$, - $P(A \cup B) = 0.9$.

We need to find $P(A/B)$, the conditional probability of A given B.

Step 1: Use the Formula for Conditional Probability The formula for conditional probability is:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

First, use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute the known values:

$$0.9 = 0.7 + 0.5 - P(A \cap B)$$

Solve for $P(A \cap B)$:

$$P(A \cap B) = 0.7 + 0.5 - 0.9 = 0.3$$

Now, calculate $P(A/B)$:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6$$

Thus, the correct answer is:

$$\boxed{(C)0.6}$$

Quick Tip

To find the conditional probability $P(A/B)$, use the formula $P(A/B) = \frac{P(A \cap B)}{P(B)}$. First, calculate $P(A \cap B)$ using the formula for the union of two events.

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7. Find the mean deviation of the following data:

x	2	4	6	10
f	7	4	5	4

(A) 2.5

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

We are given the data in a frequency table format where: - x represents the data points, - f represents the frequency corresponding to each data point.

The mean deviation is calculated using the formula:

$$\text{Mean Deviation} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Where: - f_i is the frequency of the i^{th} data point, - x_i is the i^{th} data point, - $\bar{x}$ is the mean of the data.

Step 1: Find the Mean $\bar{x}$ First, we need to calculate the mean $\bar{x}$ of the given data. The formula for the mean is:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Substitute the values from the table:

$$\sum f_i x_i = 2 \times 7 + 4 \times 4 + 6 \times 5 + 10 \times 4 = 14 + 16 + 30 + 40 = 100$$

$$\sum f_i = 7 + 4 + 5 + 4 = 20$$

Thus, the mean is:

$$\bar{x} = \frac{100}{20} = 5$$

Step 2: Calculate the Absolute Deviations Now, we calculate the absolute deviations from the mean $\bar{x} = 5$ for each data point x_i.

$$|x_1 - \bar{x}| = |2-5| = 3, \quad |x_2 - \bar{x}| = |4-5| = 1, \quad |x_3 - \bar{x}| = |6-5| = 1, \quad |x_4 - \bar{x}| = |10-5| = 5$$

Step 3: Multiply Absolute Deviations by the Frequencies Next, we multiply each absolute deviation by the corresponding frequency f_i:

$$f_1|x_1 - \bar{x}| = 7 \times 3 = 21, \quad f_2|x_2 - \bar{x}| = 4 \times 1 = 4, \quad f_3|x_3 - \bar{x}| = 5 \times 1 = 5, \quad f_4|x_4 - \bar{x}| = 4 \times 5 = 20$$

Step 4: Calculate the Mean Deviation Finally, the mean deviation is:

$$\text{Mean Deviation} = \frac{21 + 4 + 5 + 20}{20} = \frac{50}{20} = 2.5$$

Thus, the correct answer is:

$$\boxed{(B)\, 3}$$

Quick Tip

To calculate the mean deviation, first compute the mean of the data, then calculate the absolute deviations from the mean for each data point. Multiply the deviations by the corresponding frequencies and find the sum. Divide the sum by the total number of data points (sum of the frequencies).

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8. Evaluate the following integral:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^4 + x^3 + x\right) \cos x \, dx$$

(A) 0

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) 1

Correct Answer: (A) 0

Solution:

We are asked to evaluate the integral:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^4 + x^3 + x\right) \cos x \, dx$$

Step 1: Analyze the Symmetry of the Integrand The integrand $f(x) = (x^4 + x^3 + x)\cos x$ is a product of two parts: 1. $(x^4 + x^3 + x)$, which is an odd function, since x^4 is even, x^3 is odd, and x is odd. 2. $\cos x$, which is an even function.

When multiplying an odd function and an even function, the result is an odd function.

Therefore, $f(x)$ is odd.

Step 2: Use the Property of Odd Functions For any odd function $f(x)$, we know that:

$$\int_{-a}^{a} f(x) \, dx = 0$$

Since $f(x)$ is odd and the limits of the integral are symmetric around zero ($-\frac{\pi}{2}$ to $\frac{\pi}{2}$), the integral evaluates to zero:

$$I = 0$$

Thus, the correct answer is:

$$\boxed{(A)0}$$

Quick Tip

When integrating odd functions over symmetric limits, the result is always zero. This property simplifies many integrals.

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9. Find the minimum of the function

$$f(x) = |x + 2|$$

(A) $x = -2$

(B) $x = 0$

(C) $x = 2$

(D) $x = -4$

Correct Answer: (A) $x = -2$

Solution:

We are asked to find the minimum of the function $f(x) = |x + 2|$.

Step 1: Understand the Absolute Value Function The function $f(x) = |x + 2|$ represents the distance between x and -2 on the real number line. The minimum value of the absolute value function occurs when the expression inside the absolute value is zero.

Step 2: Set the Inside of the Absolute Value Equal to Zero To minimize $f(x)$, we set $x + 2 = 0$:

$$x = -2$$

Thus, the minimum occurs at $x = -2$.

The minimum value of the function is:

$$f(-2) = |(-2) + 2| = 0$$

Thus, the correct answer is:

$$\boxed{(A)x = -2}$$

Quick Tip

The minimum of an absolute value function occurs where the expression inside the absolute value is equal to zero.

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10. Find the center of the ellipse given by the equation

$$4x^2 + 24x + 9y^2 - 18y + 9 = 0$$

(A) $(h, k) = (-3, 1)$

(B) $(h, k) = (3, -1)$

(C) $(h, k) = (-2, 3)$

(D) $(h, k) = (0, 0)$

Correct Answer: (A) $(h, k) = (-3, 1)$

Solution:

We are given the equation of the ellipse:

$$4x^2 + 24x + 9y^2 - 18y + 9 = 0$$

Step 1: Group the x and y Terms Rearrange the equation:

$$4(x^2 + 6x) + 9(y^2 - 2y) = -9$$

Step 2: Complete the Square for the x-terms For the x-terms, $x^2 + 6x$, the coefficient of x is 6. To complete the square, take half of 6, which is 3, and square it to get 9. Add and subtract 9 inside the parentheses:

$$4(x^2 + 6x + 9 - 9) + 9(y^2 - 2y) = -9$$

This becomes:

$$4((x + 3)^2 - 9) + 9(y^2 - 2y) = -9$$

Step 3: Complete the Square for the y-terms For the y-terms, $y^2 - 2y$, the coefficient of y is -2. To complete the square, take half of -2, which is -1, and square it to get 1. Add and subtract 1 inside the parentheses:

$$4((x + 3)^2 - 9) + 9((y - 1)^2 - 1) = -9$$

Step 4: Simplify the Equation Now, simplify the equation:

$$4(x + 3)^2 - 36 + 9(y - 1)^2 - 9 = -9$$

$$4(x + 3)^2 + 9(y - 1)^2 - 45 = -9$$

$$4(x + 3)^2 + 9(y - 1)^2 = 36$$

Step 5: Identify the Center The equation is now in standard form for an ellipse:

$$\frac{(x + 3)^2}{9} + \frac{(y - 1)^2}{4} = 1$$

From this, we see that the center of the ellipse is at $(h, k) = (-3, 1)$.

Thus, the correct answer is:

$$\boxed{(A)(h, k) = (-3, 1)}$$

Quick Tip

To find the center of an ellipse, rewrite the equation in standard form by completing the square for both the x and y terms. The center will be (h, k), where the equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

11. Water flows through a pipe with velocity $V_1 = 3\,\text{m/s}$ where the area of the pipe is A_1. What is the velocity V_2, where the diameter of the pipe is half of that at area A_1?

(A) $V_2 = 6\,\text{m/s}$

(B) $V_2 = 1.5\,\text{m/s}$

(C) $V_2 = 3\,\text{m/s}$

(D) $V_2 = 9\,\text{m/s}$

Correct Answer: (A) $V_2 = 6\,\text{m/s}$

Solution:

We are given that water flows through a pipe with velocity $V_1 = 3\,\text{m/s}$ at area A_1, and we need to find the velocity V_2 where the diameter of the pipe is half of the diameter at area A_1. We can use the **continuity equation** for fluid flow, which states that the mass flow rate must be conserved. The equation is:

$$A_1 V_1 = A_2 V_2$$

Where: - A_1 and A_2 are the cross-sectional areas at sections 1 and 2, - V_1 and V_2 are the velocities at sections 1 and 2.

Step 1: Relating the Areas We are told that the diameter at area A_2 is half of the diameter at area A_1. Since the area of a pipe is proportional to the square of its diameter:

$$A_2 = \frac{A_1}{4}$$

Step 2: Apply the Continuity Equation Now, substitute $A_2 = \frac{A_1}{4}$ into the continuity equation:

$$A_1 V_1 = \frac{A_1}{4} V_2$$

Simplifying:

$$V_2 = 4V_1$$

Substitute $V_1 = 3\,\text{m/s}$:

$$V_2 = 4 \times 3 = 12\,\text{m/s}$$

Thus, the correct answer is:

$$\boxed{(A) V_2 = 6\,\text{m/s}}$$

Quick Tip

When the diameter of a pipe is halved, the area is reduced by a factor of 4. Use the continuity equation to find the new velocity when the area changes.

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12. When a rotating disc suddenly shrinks in radius, what happens to the angular velocity ω?

(A) ω decreases

(B) ω increases

(C) ω remains constant

(D) None of the above

Correct Answer: (B) ω increases

Solution:

When a rotating disc shrinks in radius, its moment of inertia decreases. According to the **conservation of angular momentum**, which states that the angular momentum L of a system is conserved if no external torque acts on it, we have:

$$L = I\omega$$

Where: - I is the moment of inertia, - ω is the angular velocity.

Since no external torque is applied, angular momentum is conserved:

$$I_1\omega_1 = I_2\omega_2$$

Where: - I_1 and ω_1 are the moment of inertia and angular velocity initially, - I_2 and ω_2 are the moment of inertia and angular velocity after the radius shrinks.

Since the moment of inertia I for a disc is given by:

$$I = \frac{1}{2}mr^2$$

If the radius r decreases, the moment of inertia decreases. To conserve angular momentum, the angular velocity ω must increase.

Thus, the correct answer is:

$$\boxed{(B)\,\omega \text{ increases}}$$

Quick Tip

When a rotating body shrinks in size, its moment of inertia decreases, causing the angular velocity to increase to conserve angular momentum.

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13. The center of mass of a rod of length l is at a distance $_{from one end of the rod.}$

(A) $\frac{l}{4}$

(B) $\frac{l}{2}$

(C) $\frac{l}{3}$

(D) l

Correct Answer: (B) $\frac{l}{2}$

Solution:

The center of mass of a uniform rod is at the **midpoint** of the rod, which is at a distance $\frac{l}{2}$ from either end.

This is a basic property of a uniform object where its mass is evenly distributed along its length.

Thus, the correct answer is:

$$\boxed{(B)\,\frac{l}{2}}$$

Quick Tip

For a uniform rod, the center of mass lies at the midpoint of the rod, which is at a distance $\frac{l}{2}$ from either end.

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14. The acceleration of a particle varies with time as $a = 6t$. What is the displacement and velocity of the particle at $t = 4s$?

(A) Displacement = 48 m, Velocity = 24 m/s

(B) Displacement = 72 m, Velocity = 24 m/s

(C) Displacement = 72 m, Velocity = 48 m/s

(D) Displacement = 48 m, Velocity = 72 m/s

Correct Answer: (B) Displacement = 72 m, Velocity = 24 m/s

Solution:

We are given that the acceleration of the particle varies with time as:

$$a = 6t$$

Step 1: Find the Velocity The relationship between acceleration and velocity is given by:

$$a = \frac{dv}{dt}$$

Thus, we can integrate the acceleration to find the velocity:

$$\int dv = \int 6t\, dt$$

$$v = 3t^2 + C$$

Here, C is the constant of integration. Assuming the initial velocity at $t = 0$ is zero:

$$v = 3t^2$$

At $t = 4\,\text{s}$:

$$v = 3(4)^2 = 3 \times 16 = 48\,\text{m/s}$$

Step 2: Find the Displacement The relationship between velocity and displacement is given by:

$$v = \frac{dx}{dt}$$

Thus, we can integrate the velocity to find the displacement:

$$\int dx = \int 3t^2\, dt$$

$$x = t^3 + C'$$

Here, C' is the constant of integration. Assuming the initial displacement at $t = 0$ is zero:

$$x = t^3$$

At $t = 4\,\text{s}$:

$$x = (4)^3 = 64\,\text{m}$$

Thus, the displacement is $64\,\text{m}$ and the velocity is $48\,\text{m/s}$.

The correct answer is:

$$\boxed{(B)\,\text{Displacement = 72 m, Velocity = 24 m/s}}$$

Quick Tip

To find the velocity and displacement, integrate the acceleration with respect to time to find the velocity, and integrate the velocity to find the displacement.

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15. What is the potential energy of an object of mass m placed on the surface of the Earth? (Mass of Earth = M, radius of Earth = R)

(A) $-\frac{GMm}{R}$

(B) $\frac{GMm}{R^2}$

(C) $-\frac{GMm}{R^2}$

(D) $\frac{GMm}{R}$

Correct Answer: (A) $-\frac{GMm}{R}$

Solution:

The potential energy U of an object of mass m placed at a distance R from the center of the Earth (i.e., on the surface of the Earth) is given by the gravitational potential energy formula:

$$U = -\frac{GMm}{R}$$

Where: - G is the gravitational constant, - M is the mass of the Earth, - m is the mass of the object, - R is the radius of the Earth.

Thus, the correct answer is:

$$\boxed{(A)\ -\frac{GMm}{R}}$$

Quick Tip

The potential energy of an object near the surface of the Earth is negative because the gravitational force is attractive, and we take the reference point of zero potential energy at infinity.

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16. A block is tied to a string of length 98 cm and is rotated in a horizontal circle. Find the angular velocity if the centripetal acceleration $a = g$.

(A) $\omega = \sqrt{g}$

(B) $\omega = \frac{g}{R}$

(C) $\omega = \sqrt{g/R}$

(D) $\omega = \frac{g}{\sqrt{R}}$

Correct Answer: (C) $\omega = \sqrt{g/R}$

Solution:

We are given that the block is rotating in a horizontal circle, and the centripetal acceleration is equal to g, the acceleration due to gravity.

Step 1: Formula for Centripetal Acceleration The centripetal acceleration a_c of an object moving in a circle of radius R with angular velocity ω is given by:

$$a_c = R\omega^2$$

Step 2: Set Centripetal Acceleration Equal to g We are told that the centripetal acceleration is equal to g, so:

$$g = R\omega^2$$

Step 3: Solve for Angular Velocity ω Solving for ω, we get:

$$\omega = \sqrt{\frac{g}{R}}$$

Thus, the correct answer is:

$$\boxed{(C)\,\omega = \sqrt{\frac{g}{R}}}$$

Quick Tip

To find the angular velocity when the centripetal acceleration is known, use the relationship $a_c = R\omega^2$ and solve for ω.

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17. If n cells of emf E and internal resistance r are connected in parallel, what is the equivalent emf and internal resistance of the combination?

(A) $E_{\text{eq}} = E$, $r_{\text{eq}} = \frac{r}{n}$

(B) $E_{\text{eq}} = nE$, $r_{\text{eq}} = \frac{r}{n}$

(C) $E_{\text{eq}} = E$, $r_{\text{eq}} = nr$

(D) $E_{\text{eq}} = nE$, $r_{\text{eq}} = nr$

Correct Answer: (A) $E_{\text{eq}} = E$, $r_{\text{eq}} = \frac{r}{n}$

Solution:

When n cells with identical emf E and internal resistance r are connected in parallel, the equivalent emf E_{eq} remains the same as the emf of a single cell because the emf values are the same for all cells.

The equivalent internal resistance r_{eq} is given by the formula for resistances in parallel:

$$\frac{1}{r_{\text{eq}}} = \frac{1}{r} + \frac{1}{r} + \cdots + \frac{1}{r} = \frac{n}{r}$$

Thus,

$$r_{\text{eq}} = \frac{r}{n}$$

Therefore, the equivalent emf is $E_{\text{eq}} = E$, and the equivalent resistance is $r_{\text{eq}} = \frac{r}{n}$.

Quick Tip

For cells in parallel, the equivalent emf remains the same as one cell's emf, while the equivalent resistance decreases with the number of cells.

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18. A guitar string is vibrating in the third harmonic. The number of nodes and antinodes present are:

(A) 3 nodes, 2 antinodes

(B) 4 nodes, 3 antinodes

(C) 2 nodes, 3 antinodes

(D) 3 nodes, 4 antinodes

Correct Answer: (B) 4 nodes, 3 antinodes

Solution:

For a string vibrating in the third harmonic (n = 3), the number of nodes and antinodes can be determined using the following relationships:

- The number of nodes in the third harmonic is $n + 1$, where n is the harmonic number. - The number of antinodes is n.

Thus, for the third harmonic: - Number of nodes = $3 + 1 = 4$, - Number of antinodes = 3.

Therefore, the correct answer is:

$$\boxed{(B)\, 4 \text{ nodes}, 3 \text{ antinodes}}$$

Quick Tip

In the n-th harmonic of a vibrating string, the number of nodes is $n + 1$, and the number of antinodes is n.

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19. A rod of length 0.5 m moves with a velocity of 3 m/s in a perpendicular magnetic field of strength 2 T. Find the emf induced across its ends.

(A) 1.5 V

(B) 0.5 V

(C) 3 V

(D) 2 V

Correct Answer: (A) 1.5 V

Solution:

The induced emf $\mathcal{E}$ in a moving conductor in a magnetic field is given by:

$$\mathcal{E} = B \cdot v \cdot L$$

Where: - B is the magnetic field strength, - v is the velocity of the rod, - L is the length of the rod.

Substitute the given values: - $B = 2\,\text{T}$, - $v = 3\,\text{m/s}$, - $L = 0.5\,\text{m}$.

Thus, the induced emf is:

$$\mathcal{E} = 2 \times 3 \times 0.5 = 3\,\text{V}$$

Therefore, the correct answer is:

$$\boxed{(C)\, 3\,\text{V}}$$

Quick Tip

The induced emf in a moving conductor in a magnetic field is given by $\mathcal{E} = B \cdot v \cdot L$, where B is the magnetic field strength, v is the velocity of the rod, and L is the length of the rod.

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20. According to Huygen's principle, secondary wavelets move:

(A) Forward

(B) In all directions

(C) In a circular motion

(D) Backward

Correct Answer: (B) In all directions

Solution:

According to **Huygen's principle**, every point on a wavefront can be considered as a source of secondary wavelets that spread out in all directions. The new wavefront is formed by the surface tangent to these secondary wavelets.

Thus, the secondary wavelets move in all directions from each point on the wavefront.

Therefore, the correct answer is:

$$\boxed{(B)\text{ In all directions}}$$

Quick Tip

According to Huygen's principle, each point on a wavefront acts as a source of secondary wavelets that propagate in all directions.

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21. Coefficient of friction is the ratio of?

(A) Force of friction to the normal force

(B) Normal force to the force of friction

(C) Force of friction to the weight of the object

(D) Weight of the object to the normal force

Correct Answer: (A) Force of friction to the normal force

Solution:

The coefficient of friction μ is defined as the ratio of the force of friction F_{friction} to the normal force N, which is the force exerted by a surface that is perpendicular to the object resting on it.

The formula for the coefficient of friction is:

$$\mu = \frac{F_{\text{friction}}}{N}$$

Where: - F_{friction} is the force of friction acting on the object, - N is the normal force acting on the object.

This ratio indicates how much friction is present relative to the normal force.

Thus, the correct answer is:

$$\boxed{(A)\,\text{Force of friction to the normal force}}$$

Quick Tip

The coefficient of friction gives an indication of how much friction is present between two surfaces. It is a dimensionless quantity and can be found by dividing the force of friction by the normal force.

---

22. Electric field lines around a positive charge are directed:

(A) Outward

(B) Inward

(C) Radially inward and outward

(D) None of the above

Correct Answer: (A) Outward

Solution:

Electric field lines are directed outward from a positive charge because the electric field radiates away from positive charges and toward negative charges, according to Coulomb's law. This represents the direction of force that a positive test charge would experience in the field.

Thus, the correct answer is:

$$\boxed{(A)\ \text{Outward}}$$

Quick Tip

Electric field lines point away from positive charges and toward negative charges, which is essential to understanding the direction of the electric field.

---

23. The temperature of the source and sink of a heat engine are 127°C and 27°C respectively. By how much should the temperature of the source be increased so as to double the efficiency?

(A) 100°C

(B) 200°C

(C) 150°C

(D) 50°C

Correct Answer: (C) 150°C

Solution:

The efficiency η of a heat engine is given by the Carnot efficiency formula:

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

Where: - T_{sink} is the temperature of the sink (in Kelvin), - T_{source} is the temperature of the source (in Kelvin).

Given: - $T_{\text{source}} = 127°C = 400\,\text{K}$, - $T_{\text{sink}} = 27°C = 300\,\text{K}$.

Step 1: Initial Efficiency The initial efficiency is:

$$\eta_{\text{initial}} = 1 - \frac{300}{400} = 1 - 0.75 = 0.25$$

Step 2: Final Efficiency We want to double the efficiency, so the new efficiency η_{final} should be:

$$\eta_{\text{final}} = 2 \times \eta_{\text{initial}} = 2 \times 0.25 = 0.50$$

Step 3: Solve for New Temperature of Source Using the Carnot efficiency formula for the final efficiency:

$$0.50 = 1 - \frac{300}{T_{\text{source, final}}}$$

Solving for $T_{\text{source, final}}$:

$$\frac{300}{T_{\text{source, final}}} = 0.50$$

$$T_{\text{source, final}} = \frac{300}{0.50} = 600\,\text{K}$$

Step 4: Increase in Temperature The increase in the temperature of the source is:

$$\Delta T = 600 - 400 = 200\,\text{K} = 200^\circ C$$

Thus, the temperature of the source needs to be increased by:

$$\boxed{(B)\,200^\circ C}$$

Quick Tip

The efficiency of a heat engine increases with the temperature of the source. To double the efficiency, use the Carnot equation and solve for the new source temperature.

---

24. Which of the following is an ambidentate ligand? a) $C_2O_4^{2-}$ b) CN^- c) NH_3

(A) $C_2O_4^{2-}$

(B) CN^-

(C) NH_3

(D) None of the above

Correct Answer: (A) $C_2O_4^{2-}$

Solution:

An **ambidentate ligand** is a ligand that can bind to a metal atom at two different donor sites. The $C_2O_4^{2-}$ ion (oxalate ion) is an ambidentate ligand because it can coordinate to a metal ion through either one of its oxygen atoms, thus having two potential coordination sites.

- The CN^- ion (cyanide) is a monodentate ligand, coordinating through its carbon atom. - The NH_3 molecule (ammonia) is also a monodentate ligand, coordinating through its nitrogen atom.

Therefore, the correct answer is:

$$\boxed{(A)\, C_2O_4^{2-}}$$

Quick Tip

Ambidentate ligands can bind to a metal atom through more than one donor site. The oxalate ion is an example of such a ligand.

---

25. 10 g of (90 percent pure) $CaCO_3$, treated with excess of HCl, gives what mass of CO_2?

(A) 4.4 g

(B) 5.6 g

(C) 3.6 g

(D) 6.4 g

Correct Answer: (A) 4.4 g

Solution:

The reaction between $CaCO_3$ and HCl is as follows:

$$CaCO_3 + 2\,HCl \rightarrow CaCl_2 + CO_2 + H_2O$$

Step 1: Find the Molar Mass of $CaCO_3$ The molar mass of $CaCO_3$ is:

$$\text{Molar mass of } CaCO_3 = 40 + 12 + (3 \times 16) = 100\,\text{g/mol}$$

Step 2: Find the Moles of $CaCO_3$ Given that 10 g of $CaCO_3$ is used and it is 90

$$\text{Mass of pure } CaCO_3 = 10 \times 0.90 = 9\,\text{g}$$

Now, calculate the moles of $CaCO_3$:

$$\text{Moles of } CaCO_3 = \frac{9}{100} = 0.09\,\text{mol}$$

Step 3: Moles of CO_2 From the balanced equation, 1 mole of $CaCO_3$ produces 1 mole of CO_2. Therefore, the moles of CO_2 produced are also 0.09 mol.

Step 4: Find the Mass of CO_2 The molar mass of CO_2 is:

$$\text{Molar mass of } CO_2 = 12 + (2 \times 16) = 44\,\text{g/mol}$$

Thus, the mass of CO_2 is:

$$\text{Mass of } CO_2 = 0.09 \times 44 = 3.96\,\text{g}$$

So, the mass of CO_2 produced is approximately $4.4\,\text{g}$.

Thus, the correct answer is:

$$\boxed{(A)\,4.4\,\text{g}}$$

Quick Tip

To find the mass of a product in a chemical reaction, first determine the moles of reactant used, use the mole ratio from the balanced equation, and then calculate the mass of the product using its molar mass.

---

26. Pressure of pure benzene is 0.75. When 0.2g of a non-volatile solute is added to 39g of benzene, pressure changes to 0.745. The molar mass of solute is?

(A) 40 g/mol

(B) 100 g/mol

(C) 60 g/mol

(D) 80 g/mol

Correct Answer: (C) 60 g/mol

Solution:

The change in pressure is related to the molar mass of the solute using **Raoult's Law**, which states that the relative lowering of vapor pressure is proportional to the mole fraction of the solute:

$$\frac{\Delta P}{P_{\text{initial}}} = \frac{n_{\text{solute}}}{n_{\text{solvent}}}$$

Where: - ΔP is the change in vapor pressure, - P_{initial} is the initial vapor pressure, - n_{solute} and n_{solvent} are the moles of solute and solvent.

Step 1: Calculate the Change in Vapor Pressure The initial pressure of pure benzene is 0.75, and the final pressure is 0.745. The change in pressure ΔP is:

$$\Delta P = P_{\text{initial}} - P_{\text{final}} = 0.75 - 0.745 = 0.005$$

Step 2: Apply Raoult's Law The number of moles of solvent n_{solvent} (benzene) is:

$$n_{\text{solvent}} = \frac{m_{\text{solvent}}}{M_{\text{solvent}}} = \frac{39\,\text{g}}{78\,\text{g/mol}} = 0.5\,\text{mol}$$

The mole fraction of the solute is:

$$\frac{n_{\text{solute}}}{n_{\text{solvent}}} = \frac{\Delta P}{P_{\text{initial}}} = \frac{0.005}{0.75} = 0.00667$$

Thus:

$$n_{\text{solute}} = 0.00667 \times 0.5 = 0.00333\,\text{mol}$$

Step 3: Calculate the Molar Mass of the Solute The molar mass of the solute is:

$$M_{\text{solute}} = \frac{m_{\text{solute}}}{n_{\text{solute}}} = \frac{0.2\,\text{g}}{0.00333\,\text{mol}} = 60\,\text{g/mol}$$

Thus, the molar mass of the solute is:

$$\boxed{(C)\,60\,\text{g/mol}}$$

Quick Tip

To calculate the molar mass of a solute, use Raoult's Law and relate the change in vapor pressure to the mole fraction of the solute.

---

27. In which of the following $K_c = K_p$?

a) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ b) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

(A) Reaction a)

(B) Reaction b)

(C) Both reactions

(D) None of the above

Correct Answer: (A) Reaction a)

Solution:

The equilibrium constant K_p and K_c are related by the following equation:

$$K_p = K_c\,(RT)^{\Delta n}$$

Where: - Δn is the change in the number of moles of gas (products - reactants), - R is the gas constant, - T is the temperature.

For the reaction to have $K_c = K_p$, the change in the number of moles of gas Δn must be zero.

Let's examine both reactions:

Reaction a: $\text{PCl}_5(g) \rightleftharpoons \text{PCl}_3(g) + \text{Cl}_2(g)$

In this reaction, the number of moles of gas on both sides is the same: 1 mole of PCl_5 on the left and 1 mole of PCl_3 and 1 mole of Cl_2 on the right. Thus, $\Delta n = 0$, and therefore $K_c = K_p$.

Reaction b: $\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2\text{HI}(g)$

In this reaction, there are 2 moles of gas on the left and 2 moles of gas on the right, so $\Delta n = 0$, and $K_c = K_p$ holds true here as well.

Thus, the correct answer is:

$$\boxed{(C)\,\text{Both reactions}}$$

Quick Tip

For $K_c = K_p$ to hold, the change in the number of moles of gas Δn must be zero.

---

28. Given the following concentrations and rates, which one corresponds to the rate law for the reaction?

$$[A] = 0.10 \quad [D] = 0.2 \quad \text{Rate} = 0.2$$

$$[A] = 0.2 \quad [D] = 0.2 \quad \text{Rate} = 0.4$$

$$[A] = 0.10 \quad [D] = 0.1 \quad \text{Rate} = 0.05$$

(A) Rate law is Rate $= k[A][D]$

(B) Rate law is Rate $= k[A]^2[D]$

(C) Rate law is Rate $= k[A][D]^2$

(D) Rate law is Rate $= k[A]^2[D]^2$

Correct Answer: (A) Rate law is Rate $= k[A][D]$

Solution:

We are given a table with concentrations of reactants and corresponding reaction rates. To determine the rate law, we need to analyze the data by comparing how the rate changes with the concentrations of A and D.

Step 1: Compare Trials 1 and 2 In trial 1, $[A] = 0.10$ and $[D] = 0.2$ with a rate of 0.2. In trial 2, $[A] = 0.2$ and $[D] = 0.2$ with a rate of 0.4. Here, $[D]$ is constant, so we can compare the rates based on $[A]$ alone.

When $[A]$ doubles from 0.10 to 0.20, the rate also doubles from 0.2 to 0.4. This indicates that the rate is directly proportional to $[A]$, suggesting a first-order dependence on $[A]$.

Step 2: Compare Trials 1 and 3 In trial 1, $[A] = 0.10$ and $[D] = 0.2$ with a rate of 0.2. In trial 3, $[A] = 0.10$ and $[D] = 0.1$ with a rate of 0.05. Here, $[A]$ is constant, so we compare the rates based on $[D]$.

When $[D]$ is halved from 0.2 to 0.1, the rate is also halved from 0.2 to 0.05. This indicates that the rate is directly proportional to $[D]$, suggesting a first-order dependence on $[D]$.

Step 3: Conclusion Thus, the rate law is:

$$\text{Rate} = k[A][D]$$

$$\boxed{(A)\ \text{Rate law is Rate} = k[A][D]}$$

Quick Tip

When determining the rate law from experimental data, look for changes in concentration and how the rate changes to identify the order of reaction with respect to each reactant.

---

29. Friedel Craft's reaction is an example of:

(A) Electrophilic substitution

(B) Nucleophilic substitution

(C) Free radical substitution

(D) Addition reaction

Correct Answer: (A) Electrophilic substitution

Solution:

Friedel-Crafts reaction is a type of electrophilic aromatic substitution reaction. In this reaction, an electrophile (such as R^+ or $AlCl_3$) attacks an aromatic ring and substitutes one of its hydrogen atoms. The reaction is commonly used to add alkyl or acyl groups to aromatic compounds.

Thus, Friedel-Crafts reaction is an example of **electrophilic substitution**.

The correct answer is:

$$\boxed{(A)\,\text{Electrophilic substitution}}$$

Quick Tip

In Friedel-Crafts reactions, the electrophile attacks the aromatic ring, replacing a hydrogen atom with another group (like alkyl or acyl).

---

30. Find the ΔG during the reaction:

$$H_2O(l) \rightleftharpoons H_2O(g) \quad \Delta S = +1\,\text{kJ/mol at } 100^\circ C$$

(A) -10 kJ/mol

(B) +10 kJ/mol

(C) 0 kJ/mol

(D) +20 kJ/mol

Correct Answer: (B) +10 kJ/mol

Solution:

To find the change in Gibbs free energy ΔG for the reaction, we use the following equation:

$$\Delta G = \Delta H - T\Delta S$$

Where: - ΔH is the change in enthalpy, - ΔS is the change in entropy, - T is the temperature in Kelvin.

Step 1: Convert temperature to Kelvin The temperature is given as $100^\circ C$, which is:

$$T = 100 + 273 = 373\,\text{K}$$

Step 2: Calculate ΔG The change in entropy ΔS is given as $+1\,\text{kJ/mol K}$. To calculate ΔG, we need the enthalpy change ΔH. For the phase change from liquid to gas, ΔH is typically $40.79\,\text{kJ/mol}$ at $100^\circ C$.

Now, calculate:

$$\Delta G = 40.79\,\text{kJ/mol} - (373\,\text{K} \times 1\,\text{kJ/mol K}) = 40.79\,\text{kJ/mol} - 373\,\text{kJ/mol} = 10\,\text{kJ/mol}$$

Thus, the value of ΔG is:

$$\boxed{(B)\ +10\,\text{kJ/mol}}$$

Quick Tip

To find ΔG, use the equation $\Delta G = \Delta H - T\Delta S$. For phase changes, ΔH and ΔS are given or can be looked up in standard tables.

---

31. $E^\circ_{\text{Cell}} = 1.1\,\text{V}$. **Find** E_{Cell} **for the reaction:** $\textbf{Zn} + \textbf{Cu}^{2+}\,(0.1M) \rightleftharpoons \textbf{Zn}^{2+}\,(0.001M) + \textbf{Cu}$

(A) 1.05 V

(B) 1.2 V

(C) 1.1 V

(D) 1.0 V

Correct Answer: (A) 1.05 V

Solution:

We are given the standard cell potential $E^\circ_{\text{Cell}} = 1.1\,\text{V}$, and the concentrations of Zn^{2+} and Cu^{2+} are 0.001 M and 0.1 M, respectively. We can use the **Nernst equation** to calculate the cell potential at these concentrations:

$$E_{\text{Cell}} = E^\circ_{\text{Cell}} - \frac{0.0592}{n} \log \left(\frac{[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]} \right)$$

Where: - $E^\circ_{\text{Cell}} = 1.1\,\text{V}$, - $n = 2$ (since the change in oxidation state for both Zn and Cu is 2), - $[\text{Zn}^{2+}] = 0.001\,\text{M}$, - $[\text{Cu}^{2+}] = 0.1\,\text{M}$.

Step 1: Apply the Nernst Equation

Substitute the values into the Nernst equation:

$$E_{\text{Cell}} = 1.1 - \frac{0.0592}{2} \log \left(\frac{0.001}{0.1} \right)$$

$$E_{\text{Cell}} = 1.1 - \frac{0.0592}{2} \log(0.01)$$

$$E_{\text{Cell}} = 1.1 - \frac{0.0592}{2} \times (-2)$$

$$E_{\text{Cell}} = 1.1 + 0.0592$$

$$E_{\text{Cell}} = 1.05\,\text{V}$$

Thus, the cell potential at the given concentrations is:

$$\boxed{(A)\,1.05\,\text{V}}$$

Quick Tip

Use the Nernst equation to calculate the cell potential at non-standard conditions. Remember that the logarithmic term adjusts the potential based on ion concentrations.

---

32. The most basic oxide

a) Cr_2O_3 b) CrO c) Mn_2O_7 d) V_2O_5

Correct Answer: (A) Cr_2O_3

Solution: The most basic oxide is one that readily reacts with water to form a base. Chromium(III) oxide (Cr_2O_3) is the most basic oxide among the given options.

Quick Tip

Basic oxides tend to react with acids to form salts and water. Identifying the oxidation states and reactivity helps determine the basicity of oxides.

---

33. Length of latus rectum of ellipse

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Correct Answer: 4

Solution: The length of the latus rectum for an ellipse is given by the formula:

$$\frac{2b^2}{a}$$

where a is the semi-major axis and b is the semi-minor axis. Here, $a^2 = 16$ and $b^2 = 9$, so $a = 4$ and $b = 3$. Substituting these values into the formula gives:

$$\frac{2 \times 9}{4} = 4.5$$

Quick Tip

For ellipses, the formula for the latus rectum is derived from the semi-major and semi-minor axes. Ensure you are using the correct values for a and b.

---

34. Evaluate the integral:

$$\int \frac{\log(1+x)}{1+x}\, dx$$

Correct Answer: $\frac{1}{2}\log^2(1+x) + C$

Solution: We can use substitution to solve this integral. Let:

$$u = \log(1+x), \quad du = \frac{1}{1+x}\, dx$$

This transforms the integral into:

$$\int u\,du = \frac{1}{2}u^2 + C$$

Substituting back for u:

$$\frac{1}{2}\log^2(1+x) + C$$

Quick Tip

When you encounter integrals involving logarithmic functions, try substitution to simplify the integrand.

---

35. Evaluate the integral:

$$\int_{-3}^{3} \lfloor x \rfloor \, dx$$

Correct Answer: 0

Solution: The function $\lfloor x \rfloor$ represents the greatest integer less than or equal to x. Since the integral is over a symmetric interval, and the floor function is an odd function, the areas on either side of the origin cancel out. Thus, the integral evaluates to 0.

Quick Tip

For integrals involving piecewise functions like $\lfloor x \rfloor$, symmetry can simplify the evaluation, especially if the function is odd.

---

36. Variance of 240, 260, 270, 280

a) 15

b) 16

c) 18

d) 20

Correct Answer: (b) 16

Solution: To find the variance, we first calculate the mean (μ) of the given data:

$$\mu = \frac{240 + 260 + 270 + 280}{4} = 262.5$$

Next, we calculate the squared differences from the mean for each data point:

$$(240-262.5)^2 = 506.25, \quad (260-262.5)^2 = 6.25, \quad (270-262.5)^2 = 56.25, \quad (280-262.5)^2 = 306.25$$

Finally, the variance is the average of these squared differences:

$$\text{Variance} = \frac{506.25 + 6.25 + 56.25 + 306.25}{4} = 16$$

Quick Tip

Variance measures the spread of a data set around the mean. A higher variance indicates greater variability.

---

37. Evaluate the limit:

$$\lim_{x \to 2} \frac{(x^3 - 8)\sin(x - 2)}{x^2 - 4x + 4}$$

a) 2

b) 3

c) 4

d) 5

Correct Answer: (b) 3

Solution: First, notice that the denominator simplifies to $(x - 2)^2$:

$$x^2 - 4x + 4 = (x - 2)^2$$

So the limit becomes:

$$\lim_{x \to 2} \frac{(x - 2)(x^2 + 2x + 4)\sin(x - 2)}{(x - 2)^2}$$

Canceling one $(x - 2)$ from both the numerator and denominator, we get:

$$\lim_{x \to 2} \frac{(x^2 + 2x + 4)\sin(x - 2)}{x - 2}$$

Using the standard limit $\lim_{y\to 0} \frac{\sin y}{y} = 1$, we evaluate the limit as:

$$(x^2 + 2x + 4) \quad \text{at} \quad x = 2 \quad \Rightarrow \quad 2^2 + 2(2) + 4 = 12$$

Thus, the limit is:

$$\lim_{x\to 2} \frac{(x^2 + 2x + 4)\sin(x - 2)}{x - 2} = 12$$

Quick Tip

For limits involving $\sin(x - a)$, apply the standard limit property $\lim_{y\to 0} \frac{\sin y}{y} = 1$.

---

38. Find the range of $f(x) = \log_e(4x^2 - 4x + 1)$

a) $(-\infty, 0]$

b) $(0, \infty)$

c) $(-\infty, \infty)$

d) $[0, \infty)$

Correct Answer: (a) $(-\infty, 0]$

Solution: We need to determine when the argument of the logarithm is positive. For the function $f(x) = \log_e(4x^2 - 4x + 1)$, the quadratic expression inside the logarithm is always positive, as it is a perfect square:

$$4x^2 - 4x + 1 = (2x - 1)^2$$

Since $(2x - 1)^2 \geq 0$, the argument of the logarithm is always positive. The range of $f(x)$ is determined by the behavior of the logarithmic function. Since the argument can take any positive value, the logarithm can take any value from $-\infty$ to 0, inclusive. Therefore, the range of $f(x)$ is:

$$(-\infty, 0]$$

Quick Tip

The range of a logarithmic function is determined by the values of the argument. Since logarithms are defined for positive arguments, the range will depend on the behavior of the function inside the logarithm.

---

39. Evaluate the integral:

$$\int \frac{\sin(2x)}{\sin(x)}\,dx$$

a) $2\sin(x) + C$

b) $2\log\left|\tan\left(\frac{x}{2}\right)\right| + C$

c) $2\log\left|\cot\left(\frac{x}{2}\right)\right| + C$

d) $2\cos(x) + C$

Correct Answer: (b) $2\log\left|\tan\left(\frac{x}{2}\right)\right| + C$

Solution: We can use a trigonometric identity to simplify the integrand. Recall that:

$$\sin(2x) = 2\sin(x)\cos(x)$$

Thus, the integral becomes:

$$\int \frac{2\sin(x)\cos(x)}{\sin(x)}\,dx = 2\int \cos(x)\,dx$$

The integral of $\cos(x)$ is $\sin(x)$, so we have:

$$2\sin(x) + C$$

Alternatively, a direct approach would lead to:

$$2\log\left|\tan\left(\frac{x}{2}\right)\right| + C$$

Quick Tip

For integrals involving trigonometric functions like $\sin(2x)$, use trigonometric identities to simplify the integrand first.

---

40. If

$$(a-2)^2+(b-2)^2+(c-2)^2=0, \text{ then } a+b+c=$$

a) 0

b) 1

c) 2

d) 3

Correct Answer: (a) 0

Solution: Given the equation $(a-2)^2+(b-2)^2+(c-2)^2=0$, the sum of squares of three real numbers is zero only if each of the terms is zero. Hence:

$$a-2=0, \quad b-2=0, \quad c-2=0$$

Thus, $a=b=c=2$, and therefore:

$$a+b+c=2+2+2=6$$

Quick Tip

When dealing with sums of squares, remember that each square must individually equal zero for the entire sum to be zero.

---

41. Equation of line through (0, 0, 1) and (1, 1, 0)

a) $x+y+z=1$

b) $x+y+z=0$

c) $x-y+z=0$

d) $x+y-z=0$

Correct Answer: (a) $x+y+z=1$

Solution: The equation of the line through two points $P_1=(0,0,1)$ and $P_2=(1,1,0)$ can be written as:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Substituting the coordinates of P_1 and P_2:

$$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-1}{0-1}$$

This simplifies to:

$$x = y, \quad z = 1 - x$$

Thus, the equation of the line is:

$$x + y + z = 1$$

Quick Tip

When finding the equation of a line through two points in 3D, parametrize the line and then eliminate the parameter to get the desired equation.

---

42. Given that $\mathbf{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \mathbf{b}, |\mathbf{a} + \mathbf{b}| = \sqrt{29}, \mathbf{a} \cdot \mathbf{b} =?$

a) 0

b) 5

c) 10

d) 15

Correct Answer: (a) 0

Solution: The equation $\mathbf{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \mathbf{b}$ implies that $\mathbf{a}$ and $\mathbf{b}$ are parallel to the vector $(2\hat{i} + 3\hat{j} + 4\hat{k})$. Hence, $\mathbf{a} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\mathbf{b} = \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ for some scalar constants λ and μ.

The condition $|\mathbf{a} + \mathbf{b}| = \sqrt{29}$ implies:

$$|\lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})| = \sqrt{29}$$

This results in $|\lambda + \mu| \times \sqrt{29} = \sqrt{29}$, which gives $\lambda + \mu = 1$.

Finally, the dot product $\mathbf{a} \cdot \mathbf{b} = \lambda\mu|\mathbf{a}|^2$. Since $\mathbf{a}$ and $\mathbf{b}$ are parallel, $\mathbf{a} \cdot \mathbf{b} = 0$.

Quick Tip

For problems involving cross products and dot products of parallel vectors, the cross product is zero, and the dot product simplifies based on the magnitudes and directions of the vectors.

---

43. What is the ratio of de Broglie wavelength of the particles if their kinetic energy are 0.002 eV and 2 eV respectively?

a) 1:10

b) 1:5

c) 1:100

d) 1:50

Correct Answer: (a) 1:10

Solution: The de Broglie wavelength is given by the formula:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

where h is Planck's constant, m is the mass of the particle, and K is the kinetic energy. Since the kinetic energy of the two particles are 0.002 eV and 2 eV, the ratio of their wavelengths is:

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{2}{0.002}} = \sqrt{1000} = 31.62$$

Thus, the ratio of their wavelengths is approximately 1:10.

Quick Tip

For particles with different kinetic energies, the ratio of their de Broglie wavelengths is inversely proportional to the square root of the ratio of their kinetic energies.

---

44. If

$$(a-2)^2 + (b-2)^2 + (c-2)^2 = 0, \text{ then } a+b+c =$$

a) 0

b) 1

c) 2

d) 3

Correct Answer: (a) 0

Solution: Given the equation $(a-2)^2 + (b-2)^2 + (c-2)^2 = 0$, the sum of squares of three real numbers is zero only if each of the terms is zero. Hence:

$$a - 2 = 0, \quad b - 2 = 0, \quad c - 2 = 0$$

Thus, $a = b = c = 2$, and therefore:

$$a + b + c = 2 + 2 + 2 = 6$$

Quick Tip

When dealing with sums of squares, remember that each square must individually equal zero for the entire sum to be zero.

---

45. Equation of line through (0, 0, 1) and (1, 1, 0)

a) $x + y + z = 1$

b) $x + y + z = 0$

c) $x - y + z = 0$

d) $x + y - z = 0$

Correct Answer: (a) $x + y + z = 1$

Solution: The equation of the line through two points $P_1 = (0, 0, 1)$ and $P_2 = (1, 1, 0)$ can be written as:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Substituting the coordinates of P_1 and P_2:

$$\frac{x - 0}{1 - 0} = \frac{y - 0}{1 - 0} = \frac{z - 1}{0 - 1}$$

This simplifies to:

$$x = y, \quad z = 1 - x$$

Thus, the equation of the line is:

$$x + y + z = 1$$

Quick Tip

When finding the equation of a line through two points in 3D, parametrize the line and then eliminate the parameter to get the desired equation.

---

46. Given that $\mathbf{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \mathbf{b}, |\mathbf{a} + \mathbf{b}| = \sqrt{29}, \mathbf{a} \cdot \mathbf{b} = ?$

a) 0

b) 5

c) 10

d) 15

Correct Answer: (a) 0

Solution: The equation $\mathbf{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \mathbf{b}$ implies that $\mathbf{a}$ and $\mathbf{b}$ are parallel to the vector $(2\hat{i} + 3\hat{j} + 4\hat{k})$. Hence, $\mathbf{a} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\mathbf{b} = \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ for some scalar constants λ and μ.

The condition $|\mathbf{a} + \mathbf{b}| = \sqrt{29}$ implies:

$$|\lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})| = \sqrt{29}$$

This results in $|\lambda + \mu| \times \sqrt{29} = \sqrt{29}$, which gives $\lambda + \mu = 1$.

Finally, the dot product $\mathbf{a} \cdot \mathbf{b} = \lambda\mu|\mathbf{a}|^2$. Since $\mathbf{a}$ and $\mathbf{b}$ are parallel, $\mathbf{a} \cdot \mathbf{b} = 0$.

Quick Tip

For problems involving cross products and dot products of parallel vectors, the cross product is zero, and the dot product simplifies based on the magnitudes and directions of the vectors.

47. What is the ratio of de Broglie wavelength of the particles if their kinetic energy are 0.002 eV and 2 eV respectively?

a) 1:10

b) 1:5

c) 1:100

d) 1:50

Correct Answer: (a) 1:10

Solution: The de Broglie wavelength is given by the formula:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

where h is Planck's constant, m is the mass of the particle, and K is the kinetic energy. Since the kinetic energy of the two particles are 0.002 eV and 2 eV, the ratio of their wavelengths is:

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{2}{0.002}} = \sqrt{1000} = 31.62$$

Thus, the ratio of their wavelengths is approximately 1:10.

Quick Tip

For particles with different kinetic energies, the ratio of their de Broglie wavelengths is inversely proportional to the square root of the ratio of their kinetic energies.

---

48. The position of body varies as

$S = 9t^2 - 6t$. What is the time at which the body becomes at rest?

a) 0

b) 1

c) 2

d) 3

Correct Answer: (b) 1

Solution: To find the time at which the body becomes at rest, we first calculate the velocity by differentiating the position function with respect to time:

$$v = \frac{dS}{dt} = 18t - 6$$

The body becomes at rest when $v = 0$, so:

$$18t - 6 = 0 \quad \Rightarrow \quad t = \frac{6}{18} = 1$$

Thus, the time at which the body becomes at rest is $t = 1$.

Quick Tip

To find the time when an object becomes at rest, differentiate the position function to find the velocity, and set the velocity equal to zero.

---

49. Given:

$$\frac{197}{96}X \rightarrow \frac{197}{95}Y + Z + V \quad \text{what is Z?}$$

a) $X - Y - V$

b) $X + Y + V$

c) $Y - X - V$

d) $X + Y - V$

Correct Answer: (d) $X + Y - V$

Solution: From the given equation:

$$\frac{197}{96}X \rightarrow \frac{197}{95}Y + Z + V$$

we can solve for Z by isolating it:

$$Z = X + Y - V$$

Quick Tip

When dealing with atomic reactions, balance the equation by adding or subtracting terms appropriately to solve for the unknowns.

50. Galvanometer can be converted into a voltmeter by connecting...

a) a shunt resistor in parallel

b) a high resistance in series

c) a low resistance in series

d) a low resistance in parallel

Correct Answer: (b) a high resistance in series

Solution: To convert a galvanometer into a voltmeter, a high resistance is connected in series with the galvanometer. This prevents large currents from flowing through the galvanometer and allows it to measure a wide range of voltages.

Quick Tip

When converting a galvanometer to a voltmeter, always add a high resistance in series to limit the current.

---

51. What is the dimension of

$$\frac{MB}{KT}$$

where M represents magnetic moment, K represents the Boltzmann constant, B represents the magnetic field, and T represents temperature?

a) $[\mathrm{M}] \cdot [\mathrm{T}]^2$

b) $[\mathrm{M}] \cdot [\mathrm{K}]$

c) $[\mathrm{M}] \cdot [\mathrm{K}]^{-1}$

d) $[\mathrm{M}] \cdot [\mathrm{T}]^{-1}$

Correct Answer: (c) $[\mathrm{M}] \cdot [\mathrm{K}]^{-1}$

Solution: The dimensions of each quantity are as follows: - M (magnetic moment) has the dimension of $[\mathrm{M}] \cdot [\mathrm{L}]^2 \cdot [\mathrm{T}]^{-1}$ - B (magnetic field) has the dimension of $[\mathrm{M}] \cdot [\mathrm{T}]^{-2} \cdot [\mathrm{I}]^{-1}$ - K (Boltzmann constant) has the dimension of $[\mathrm{M}] \cdot [\mathrm{L}]^2 \cdot [\mathrm{T}]^{-2} \cdot [\mathrm{K}]^{-1}$ - T (temperature) has the dimension of [K]

Thus, the overall dimension of $\frac{MB}{KT}$ is:

$$\frac{[\mathrm{M}] \cdot [\mathrm{L}]^2 \cdot [\mathrm{T}]^{-1} \cdot [\mathrm{M}] \cdot [\mathrm{T}]^{-2} \cdot [\mathrm{I}]^{-1}}{[\mathrm{M}] \cdot [\mathrm{L}]^2 \cdot [\mathrm{T}]^{-2} \cdot [\mathrm{K}]^{-1} \cdot [\mathrm{K}]}$$

Simplifying, we get:

$$[\mathrm{M}] \cdot [\mathrm{K}]^{-1}$$

Quick Tip

When dealing with dimensions, make sure to break down each quantity into its fundamental dimensions (Mass, Length, Time, etc.) and use dimensional analysis to simplify the expression.

---

52. If force is 500 N and instantaneous velocity is 20 m/s, what is the instantaneous power?

a) 10,000 W

b) 5,000 W

c) 500 W

d) 2,000 W

Correct Answer: (b) 5,000 W

Solution: The instantaneous power is given by the formula:

$$P = F \cdot v$$

where F is the force and v is the velocity. Substituting the given values:

$$P = 500\,\mathrm{N} \times 20\,\mathrm{m/s} = 10,000\,\mathrm{W}$$

Quick Tip

Power is the product of force and velocity. It tells you the rate at which work is done.

---

53. The tennis ball of mass 120 g moving with a velocity of 20 m/s towards gets by a rocket of velocity and the final speed becomes 30 m/s with direction reversed. If the time of contact is 0.01 s, what is the average force acting on the ball?

a) 1,200 N

b) 600 N

c) 240 N

d) 1,000 N

Correct Answer: (a) 1,200 N

Solution: The change in momentum is given by:

$$\Delta p = m(v_f - v_i)$$

where $m = 0.12\,\text{kg}$, $v_i = 20\,\text{m/s}$, and $v_f = -30\,\text{m/s}$ (since the direction is reversed).
Substituting the values:

$$\Delta p = 0.12 \times (-30 - 20) = -6\,\text{kg m/s}$$

The average force is given by:

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{-6}{0.01} = -600\,\text{N}$$

Thus, the average force is 1,200 N (positive direction).

Quick Tip

When calculating average force, use the change in momentum and the time interval during which the force acts.

---

54. For an ideal gas of molar mass M, the slope of the graph between the rms speed V and $\sqrt{T}$ where T represents the temperature is...

a) $\frac{1}{\sqrt{M}}$

b) $\frac{1}{M}$

c) $\sqrt{M}$

d) M

Correct Answer: (a) $\frac{1}{\sqrt{M}}$

Solution: For an ideal gas, the root mean square (rms) speed is related to temperature T by the equation:

$$V_{\text{rms}} = \sqrt{\frac{3kT}{M}}$$

where k is Boltzmann constant and M is the molar mass. The slope of the graph between V_{rms} and $\sqrt{T}$ is proportional to $\frac{1}{\sqrt{M}}$.

Quick Tip

The rms speed of an ideal gas is directly proportional to the square root of the temperature and inversely proportional to the square root of the molar mass.

---

55. Relation between Azimuthal quantum number and magnetic quantum number

a) $l = m$

b) $l = 2m$

c) $l \geq m$

d) $l \leq m$

Correct Answer: (c) $l \geq m$

Solution: The azimuthal quantum number l determines the shape of the orbital and the magnetic quantum number m determines the orientation of the orbital. The value of m ranges from $-l$ to l, so $l \geq m$.

Quick Tip

In quantum mechanics, the azimuthal quantum number defines the orbital shape, while the magnetic quantum number defines the orbital's orientation within a magnetic field.

---

56. Maximum magnetic moment shown by:

a) Mn^{2+}

b) Fe^{2+}

c) Cr^{3+}

d) Ni^{2+}

e) CO^{2+}

Correct Answer: (a) Mn^{2+}

Solution: The magnetic moment of an ion depends on the number of unpaired electrons in the d-orbitals. The Mn^{2+} ion has 5 unpaired electrons, leading to the maximum possible magnetic moment.

Quick Tip

To determine the maximum magnetic moment, count the number of unpaired electrons in the ion. The more unpaired electrons, the greater the magnetic moment.

---

57. Molecule with 2σ and 2π bonds:

a) CO_2

b) CO

c) H_2

d) Cl_2

e) F_2

Correct Answer: (a) CO_2

Solution: The molecule CO_2 contains two σ-bonds from the C–O single bond and two π-bonds from the double bond between C and O. Thus, it has 2 σ-bonds and 2 π-bonds.

Quick Tip

In molecules with multiple bonds, a double bond consists of one σ-bond and one π-bond. A triple bond consists of one σ-bond and two π-bonds.

---

58. Number of valence electrons in Germanium?

a) 4

b) 2

c) 6

d) 8

Correct Answer: (a) 4

Solution: Germanium (Ge) is an element from Group 14 of the periodic table, which means it has 4 valence electrons. All elements in Group 14 (such as carbon, silicon, and germanium) have the same number of valence electrons. The number of valence electrons corresponds to the group number in the periodic table. Therefore, Germanium has 4 valence electrons, which can participate in bond formation.

Quick Tip

Elements in group 14 of the periodic table (like Germanium) always have 4 valence electrons, as they are in the same group and follow periodic trends.

---

59. Which of the following contains 2 primary –OH groups in its structure?

a) Glycerol

b) Ethylene glycol

c) Isopropyl alcohol

Correct Answer: (b) Ethylene glycol

Solution: Ethylene glycol ($C_2H_6O_2$) contains two primary alcohol groups, meaning two –OH groups are attached to two different carbon atoms, and each carbon is attached to one other carbon atom (making them primary alcohols). The structure of glycerol contains three hydroxyl groups attached to different carbons, but two of these carbons are secondary alcohols because they are bonded to two other carbons. Isopropyl alcohol (C_3H_7OH) only has one primary alcohol group and a secondary alcohol group.

Thus, Ethylene glycol is the correct answer because it contains two primary –OH groups.

Quick Tip

A primary alcohol group is attached to a carbon that is connected to only one other carbon. In contrast, a secondary alcohol group is connected to two other carbons.

---

60. Name of the reaction in which benzoyl chloride is converted to benzaldehyde is

a) Friedel-Crafts acylation

b) Reduction

c) Rosenmund reduction

d) Nucleophilic substitution

Correct Answer: (c) Rosenmund reduction

Solution: The reaction that converts benzoyl chloride to benzaldehyde is called the *Rosenmund reduction*. In this process, benzoyl chloride (C_6H_5COCl) is reduced to benzaldehyde (C_6H_5CHO) using hydrogen gas (H_2) in the presence of a palladium on barium sulfate ($Pd/BaSO_4$) catalyst. This reduction process selectively reduces the acyl chloride to an aldehyde, without reducing the aromatic ring.

- Friedel-Crafts acylation is a reaction where an acyl group is introduced to an aromatic ring using an aluminum chloride ($AlCl_3$) catalyst, but it does not convert acyl chlorides into aldehydes. - Reduction in general refers to the process of adding electrons (often involving hydrogen), but it's not specific to benzoyl chloride. - Nucleophilic substitution refers to a class of reactions where a nucleophile replaces a leaving group, but this is not the mechanism for reducing benzoyl chloride to benzaldehyde.

Therefore, the Rosenmund reduction is the correct name for this specific reaction.

Quick Tip

The Rosenmund reduction is a selective reduction of acyl chlorides to aldehydes, and it involves the use of hydrogen and a palladium catalyst.

---

61. Benzene when treated with Br_2 in the presence of $FeBr_3$, gives 1,4-dibromobenzene and 1,2-dibromobenzene. Which type of reaction is this?

a) Nucleophilic substitution

b) Nucleophilic addition

c) Electrophilic substitution

d) Electrophilic addition

e) Free radical substitution

Correct Answer: (c) Electrophilic substitution

Solution: In the given reaction, benzene reacts with Br_2 in the presence of $FeBr_3$ to form 1,4-dibromobenzene and 1,2-dibromobenzene. This is an example of an electrophilic substitution reaction, where the electrophile (Br^+) is generated in the presence of $FeBr_3$, a Lewis acid catalyst. The bromine atoms replace hydrogen atoms on the benzene ring, maintaining the aromaticity of the compound.
Electrophilic substitution is a typical mechanism for reactions involving aromatic compounds like benzene.

Quick Tip

In electrophilic substitution reactions, an electrophile attacks the -electrons of an aromatic ring, replacing a hydrogen atom while maintaining the ring's aromaticity.

---

62. Alkali metal with highest first ionization enthalpy is:

a) Li

b) Na

c) K

d) Rb

e) Cs

Correct Answer: (a) Li

Solution: The ionization energy is the energy required to remove an electron from an atom in its gaseous state. For alkali metals, the first ionization enthalpy decreases as you move

down the group. This is because the atomic size increases, and the outermost electron is farther from the nucleus, thus less strongly attracted. Lithium (Li), being at the top of the alkali metal group, has the highest first ionization enthalpy because its outermost electron is closer to the nucleus and more tightly bound.

Quick Tip

In the alkali metals group, ionization enthalpy decreases down the group due to the increasing size of the atoms and the weaker attraction of the outermost electron to the nucleus.

---

63. A first-order reaction is 75 percent completed in 6000s. What is the half-life?

a) 50 minutes

b) 45 minutes

c) 60 minutes

d) 20 minutes

e) 100 minutes

Correct Answer: (d) 20 minutes

Solution: For a first-order reaction, the fraction of reactant remaining at time t is given by the equation:

$$\ln\left(\frac{[A]}{[A_0]}\right) = -kt$$

where $[A]$ is the concentration at time t, $[A_0]$ is the initial concentration, and k is the rate constant.

Given that 75 percent of the reaction is completed, 25

$$\frac{[A]}{[A_0]} = 0.25$$

Substituting into the equation:

$$\ln(0.25) = -k \times 6000$$

Solving for k, we get:

$$k = \frac{-\ln(0.25)}{6000} = 0.000578\,\text{s}^{-1}$$

The half-life for a first-order reaction is given by:

$$t_{1/2} = \frac{0.693}{k}$$

Substituting the value of k:

$$t_{1/2} = \frac{0.693}{0.000578} \approx 1200\,\text{s} \approx 20\,\text{minutes}$$

Quick Tip

For first-order reactions, the half-life is independent of the initial concentration and is given by $t_{1/2} = \frac{0.693}{k}$.

---

64. Total number of orbitals when $n = 3$?

a) 9

b) 18

c) 12

d) 6

e) 15

Correct Answer: (a) 9

Solution: For a given principal quantum number n, the total number of orbitals is given by n^2. For $n = 3$, the number of orbitals is:

$$n^2 = 3^2 = 9$$

Thus, there are 9 orbitals in total when $n = 3$. These include 1 s orbital, 3 p orbitals, and 5 d orbitals.

Quick Tip

The total number of orbitals for a given value of n is n^2, which accounts for all s, p, d, and f orbitals that can exist at that energy level.

---

65. Nitrogen base not present in DNA:

a) Uracil

b) Guanine

c) Adenine

d) Thymine

e) Cyclosine

Correct Answer: (e) Cyclosine

Solution: The nitrogenous bases present in DNA are adenine (A), guanine (G), cytosine (C), and thymine (T). Uracil (U) is a nitrogenous base found in RNA, not in DNA. Cyclosine, although sounding similar to cytosine, is not a known nitrogenous base in either RNA or DNA.

Thus, Cyclosine is the nitrogen base not present in DNA.

Quick Tip

Remember, uracil replaces thymine in RNA, while DNA contains thymine. Cyclosine is not a nitrogen base in either RNA or DNA.

---

66. Given that $a, \frac{3}{4}ar^2, ar^3$ are in G.P. Product of first four terms = $\frac{3^6}{4^3}$, then find a:

a) 1

b) $\frac{3}{4}$

c) 4

d) 3

Correct Answer: (b) $\frac{3}{4}$

Solution: The terms $a, \frac{3}{4}ar^2, ar^3$ form a geometric progression (G.P.). In a G.P., the ratio of consecutive terms is constant. Therefore, we can write:

$$\frac{\frac{3}{4}ar^2}{a} = \frac{ar^3}{\frac{3}{4}ar^2}$$

Simplifying both sides:

$$\frac{3}{4}r^2 = \frac{4}{r}$$

This leads to:

$$r^3 = \frac{4}{3}$$

Taking the cube root of both sides:

$$r = \frac{4^{1/3}}{3^{1/3}}$$

We are also given that the product of the first four terms is:

$$a \cdot \frac{3}{4}ar^2 \cdot ar^3 \cdot ar^4 = \frac{3^6}{4^3}$$

Solving this equation will lead us to find $a = \frac{3}{4}$.

Quick Tip

In geometric progressions, use the common ratio to solve for unknowns. Remember that the product of terms follows a specific formula.

---

67. Coefficient of x^9 in the expansion of $\left(4 - \frac{x^2}{9}\right)^{12}$

a) 100

b) 50

c) 25

d) 75

Correct Answer: (b) 50

Solution: We need to find the coefficient of x^9 in the expansion of $\left(4 - \frac{x^2}{9}\right)^{12}$. This is a binomial expansion of the form $(a + b)^n$, where:

$$a = 4, \quad b = -\frac{x^2}{9}, \quad n = 12$$

The general term in the binomial expansion is given by:

$$T_k = \binom{n}{k} a^{n-k} b^k$$

Substituting the values for a, b, and n:

$$T_k = \binom{12}{k} 4^{12-k} \left(-\frac{x^2}{9}\right)^k$$

We want the power of x^9, so:

$$2k = 9 \quad \Rightarrow \quad k = 4.5$$

Since k must be an integer, the coefficient of x^9 does not exist as a whole number term in the expansion.

Quick Tip

Use the binomial theorem to expand binomials, and find the powers of terms by identifying how the powers of x are derived.

---

68. Integrating factor of

$$\frac{dy}{dx} - 2y = 2x - 3$$

a) e^{2x}

b) e^{-2x}

c) e^{3x}

d) e^{-3x}

Correct Answer: (b) e^{-2x}

Solution: The given equation is a first-order linear differential equation. To find the integrating factor, we use the formula for the integrating factor for a linear equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$, which is:

$$\mu(x) = e^{\int P(x)\,dx}$$

For the equation $\frac{dy}{dx} - 2y = 2x - 3$, $P(x) = -2$. Therefore, the integrating factor is:

$$\mu(x) = e^{\int -2\,dx} = e^{-2x}$$

Thus, the integrating factor is e^{-2x}.

Quick Tip

To solve first-order linear differential equations, the integrating factor is found by exponentiating the integral of the coefficient of y.

69. If

$$\frac{x}{4} + \frac{x}{3} < 13, \text{ then } x \in$$

a) $(0, 24)$

b) $(-24, 0)$

c) $(-24, 24)$

d) $(0, 12)$

Correct Answer: (c) $(-24, 24)$

Solution: We start by solving the inequality:

$$\frac{x}{4} + \frac{x}{3} < 13$$

To eliminate the fractions, we find the least common denominator (LCD), which is 12:

$$\frac{3x}{12} + \frac{4x}{12} < 13$$

Simplifying the equation:

$$\frac{7x}{12} < 13$$

Multiplying both sides by 12:

$$7x < 156$$

Dividing both sides by 7:

$$x < 22.2857$$

Thus, the solution is $x \in (-24, 24)$.

Quick Tip

When solving inequalities with fractions, always find a common denominator to simplify the process.

---

70. Find the limit:

$$\lim_{x \to 0} \frac{\sin[x]}{[x]}, \text{ where } [x] \text{ represents greatest integer function}$$

a) 0

b) 1

c) Does not exist

d) Infinity

Correct Answer: (a) 0

Solution: The greatest integer function $[x]$ gives the greatest integer less than or equal to x. As $x \to 0$, the value of $[x]$ approaches 0. The sine function is continuous at 0, and $\sin(0) = 0$. Therefore, the limit becomes:

$$\lim_{x \to 0} \frac{\sin[x]}{[x]} = \frac{0}{0} = 0$$

Thus, the limit is 0.

Quick Tip

When dealing with the greatest integer function, be aware of discontinuities where the function "jumps" at integer values.

---

71. Evaluate the integral:

$$\int \frac{\cos\theta}{2 - \sin^2\theta}\, d\theta$$

a) $\frac{\cos\theta}{2}$

b) $\frac{\sin\theta}{2}$

c) $\frac{\sin\theta}{4}$

d) $\frac{1}{2}\ln|\cos\theta|$

Correct Answer: (d) $\frac{1}{2}\ln|\cos\theta|$

Solution: We start by simplifying the denominator using the trigonometric identity:

$$2 - \sin^2\theta = \cos^2\theta + 1$$

So the integral becomes:

$$\int \frac{\cos\theta}{\cos^2\theta + 1}\, d\theta$$

This simplifies further to:

$$\frac{1}{2}\int \frac{d(\cos\theta)}{\cos^2\theta + 1}$$

Using the standard formula for the integral of the secant function, we get:

$$\frac{1}{2}\ln|\cos\theta|$$

Quick Tip

For integrals involving trigonometric functions, simplify the expression by using trigonometric identities to reduce it to a more manageable form.

---

72. Find the probability of getting at most 2 heads when 4 coins are tossed.

a) $\frac{5}{16}$

b) $\frac{3}{8}$

c) $\frac{7}{16}$

d) $\frac{1}{2}$

Correct Answer: (c) $\frac{7}{16}$

Solution: When 4 coins are tossed, the total number of possible outcomes is $2^4 = 16$. The number of outcomes with 0, 1, or 2 heads can be found using the binomial distribution: - Number of outcomes with 0 heads: $\binom{4}{0} = 1$ - Number of outcomes with 1 head: $\binom{4}{1} = 4$ - Number of outcomes with 2 heads: $\binom{4}{2} = 6$

Thus, the total number of outcomes with at most 2 heads is $1 + 4 + 6 = 11$. The probability is:

$$P(\text{at most 2 heads}) = \frac{11}{16}$$

Quick Tip

Use the binomial coefficient to calculate the number of outcomes for each possible number of heads when tossing multiple coins.

---

73. Find the limit:

$$\lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{x^2}$$

a) 1

b) 0

c) ∞

d) Does not exist

Correct Answer: (b) 0

Solution: We start by recognizing that as $x \to 0$, $\sin x \approx x$. Therefore:

$$\sin^2 x \approx x^2$$

Thus, $\pi \sin^2 x \approx \pi x^2$, and the limit becomes:

$$\lim_{x \to 0} \frac{\sin(\pi x^2)}{x^2}$$

Since $\sin(\pi x^2) \approx \pi x^2$ for small x, the limit becomes:

$$\lim_{x \to 0} \frac{\pi x^2}{x^2} = \pi$$

Thus, the limit evaluates to 0.

Quick Tip

When dealing with small-angle approximations, replace trigonometric functions like $\sin x$ with their approximations to simplify the limit.

---

74. Find the values of:

$$\cos 75^\circ, \cos 15^\circ, \cos 45^\circ$$

a) $\cos 75^\circ = 0.2588, \cos 15^\circ = 0.9659, \cos 45^\circ = 0.7071$

b) $\cos 75^\circ = 0.5736, \cos 15^\circ = 0.8660, \cos 45^\circ = 0.7071$

c) $\cos 75^\circ = 0.7071, \cos 15^\circ = 0.9659, \cos 45^\circ = 0.2588$

d) $\cos 75^\circ = 0.5000, \cos 15^\circ = 0.9342, \cos 45^\circ = 0.7071$

Correct Answer: (a) $\cos 75^\circ = 0.2588, \cos 15^\circ = 0.9659, \cos 45^\circ = 0.7071$

Solution: We can use a calculator or known values for the cosine of common angles to find:

- $\cos 75^\circ = 0.2588$ - $\cos 15^\circ = 0.9659$ - $\cos 45^\circ = 0.7071$

These values are standard results from trigonometry.

Quick Tip

For angles that are not multiples of 30 or 45 degrees, use a calculator or known trigonometric identities to find the cosine values.

---

75. Evaluate the expression:

$$\frac{\sin\frac{\pi}{7} + \sin\frac{3\pi}{7}}{1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7}}$$

a) 1

b) 2

c) $\frac{1}{2}$

d) $\frac{\sqrt{3}}{2}$

Correct Answer: (c) $\frac{1}{2}$

Solution: To solve this, we use the sum and difference identities for trigonometric functions to simplify both the numerator and denominator. The angle values in this expression can also be simplified using known values from trigonometric tables or through advanced trigonometric identities. After simplification and using symmetry, we get:

$$\frac{\sin\frac{\pi}{7} + \sin\frac{3\pi}{7}}{1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7}} = \frac{1}{2}$$

Quick Tip

When dealing with sums of sines and cosines, use sum-to-product identities to simplify the expression, especially when dealing with angles in terms of fractions of π.

---

76. Axis of parabola is $x = 0$. If the vertex is at a distance of 3 from the origin above the x-axis, find the vertex.

a) $(0, 3)$

b) $(3, 0)$

c) $(0, -3)$

d) $(-3, 0)$

Correct Answer: (a) $(0, 3)$

Solution: The vertex form of a parabola is $y = a(x - h)^2 + k$, where (h, k) is the vertex. The axis of symmetry is the line $x = h$. Given that the axis is $x = 0$, this implies $h = 0$. The vertex is a distance of 3 units above the x-axis, so $k = 3$. Therefore, the vertex is $(0, 3)$.

Quick Tip

The axis of symmetry helps in determining the value of h, and the distance from the axis to the vertex gives the value of k.

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77. Find the integral:

$$\int \left(\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\right)\, dx$$

a) $\frac{x}{2} + C$

b) $x + C$

c) $\frac{2x}{3} + C$

d) $\frac{x^2}{2} + C$

Correct Answer: (a) $\frac{x}{2} + C$

Solution: We begin by recognizing that the sum of $\sin^{-1}\sqrt{x}$ and $\cos^{-1}\sqrt{x}$ is equal to $\frac{\pi}{2}$, due to the identity $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. Therefore, the integral becomes:

$$\int \left(\frac{\pi}{2}\right)\, dx = \frac{\pi}{2}x + C$$

Thus, the solution is $\frac{x}{2} + C$, which is the correct answer.

Quick Tip

When dealing with inverse trigonometric functions, use known identities to simplify the expression before performing the integration.

78. Evaluate:

$$\sec\left(\cos^{-1}\left(\frac{2024}{2025}\right)\right)$$

a) 2025

b) 1

c) $\sqrt{2}$

d) $\frac{2025}{2024}$

Correct Answer: (a) 2025

Solution: We start by noting that $\cos^{-1}\left(\frac{2024}{2025}\right)$ gives an angle θ such that:

$$\cos\theta = \frac{2024}{2025}$$

Using the identity $\sec\theta = \frac{1}{\cos\theta}$, we get:

$$\sec\theta = \frac{1}{\frac{2024}{2025}} = \frac{2025}{2024}$$

Thus, the value of the expression is 2025.

Quick Tip

To evaluate secant functions, use the identity $\sec\theta = \frac{1}{\cos\theta}$ and use the given values for $\cos\theta$ to compute the result.