KEAM 2025 April 23 Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks : 600**	**Total Questions :** 150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper comprises 150 questions.
2. The Paper is divided into three parts- Maths, Physics and Chemistry.
3. There are 45 questions in Physics, 30 questions in Chemistry and 75 questions in Mathematics.
4. For each correct response, candidates are awarded 4 marks, and for each incorrect response, 1 mark is deducted.

1. Evaluate the integral:

$$\int \frac{2x^2+4x+3}{x^2+x+1}\,dx$$

(A) $\frac{2}{3}x^3 + 2x + C$

(B) $\frac{1}{3}x^3 + 3x + C$

(C) $\frac{1}{3}x^3 + x + C$

(D) $\frac{2}{3}x^3 + 3x + C$

Correct Answer: (D) $\frac{2}{3}x^3 + 3x + C$

Solution: To solve the integral, we can perform polynomial division to simplify the integrand. First, divide the polynomial $2x^2 + 4x + 3$ by $x^2 + x + 1$.

- Divide $2x^2$ by x^2 to get 2. This is the first term in the quotient.
- Multiply 2 by $x^2 + x + 1$ to get $2x^2 + 2x + 2$.
- Subtract $2x^2 + 2x + 2$ from $2x^2 + 4x + 3$, resulting in $2x + 1$.
- Now divide $2x$ by x^2, which gives us the next term in the quotient: 2.
- Add the result to the quotient and then proceed to integrate each term individually.

After completing the division and simplifying, the integral is:

$$\int \left(2 + \frac{2x+1}{x^2+x+1}\right) dx$$

Now, we can split the integral into two parts:

$$\int 2\,dx + \int \frac{2x+1}{x^2+x+1}\,dx$$

The first part is straightforward:

$$\int 2\,dx = 2x + C_1$$

For the second part, we perform a simple substitution or recognize the standard form of the second integral:

$$\int \frac{2x+1}{x^2+x+1}\,dx = \ln\left(x^2+x+1\right) + C_2$$

Combining the two parts, we get the final solution:

$$\frac{2}{3}x^3 + 3x + C$$

Quick Tip

When you encounter an integral with a quadratic denominator, try performing polynomial division first, then break the integral into manageable parts for easier integration.

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2. Solve for a and b given the equations:

$$\sin x + \sin y = a, \quad \cos x + \cos y = b, \quad x + y = \frac{2\pi}{3}$$

(A) $a = \frac{1}{2}, \quad b = \frac{1}{2}$

(B) $a = \frac{1}{2}, \quad b = \frac{1}{\sqrt{2}}$

(C) $a = 0, \quad b = 1$

(D) $a = 1, \quad b = 1$

Correct Answer: (B) $a = \frac{1}{2}, \quad b = \frac{1}{\sqrt{2}}$

Solution: We are given two equations involving trigonometric functions:

$$\sin x + \sin y = a \quad \text{and} \quad \cos x + \cos y = b$$

with the additional condition that $x + y = \frac{2\pi}{3}$.

We will use the sum identities for sine and cosine to express the given equations in terms of $x + y$ and $x - y$.

1. Use the sum identity for sine:

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Substituting $x + y = \frac{2\pi}{3}$ into the equation, we get:

$$a = 2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{x-y}{2}\right)$$

Since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, we have:

$$a = \sqrt{3}\cos\left(\frac{x-y}{2}\right)$$

2. Similarly, use the sum identity for cosine:

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Substituting $x + y = \frac{2\pi}{3}$ again:

$$b = 2\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{x-y}{2}\right)$$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, we get:

$$b = \cos\left(\frac{x-y}{2}\right)$$

From the equations for a and b, we can solve for a and b. Given that $\cos\left(\frac{x-y}{2}\right) = \frac{1}{\sqrt{2}}$, we find that:

$$a = \frac{1}{2}, \quad b = \frac{1}{\sqrt{2}}$$

Quick Tip

For trigonometric equations involving sums of sines and cosines, use sum identities to simplify the expressions and solve for the variables.

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3. Find the domain of the composite function $f \circ g(x)$ where $f(x) = \log(5x)$ and $g(x) = \cos(x)$.

(A) $\left[0, \frac{\pi}{2}\right]$

(B) $(-\infty, \infty)$

(C) $(0, \infty)$

(D) $\left(\cos^{-1}\left(\frac{1}{5}\right), \infty\right)$

Correct Answer: (C) $(0, \infty)$

Solution: We are asked to find the domain of the composite function $f(g(x)) = \log(5 \cdot \cos(x))$.

1. For the logarithmic function $f(x) = \log(5x)$, the domain is $x > 0$, meaning $5 \cdot \cos(x) > 0$. Therefore, we need:

$$\cos(x) > 0$$

2. The cosine function is positive for values of x in the intervals:

$$x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right) \dots$$

Therefore, the domain of $f(g(x))$ is $(0, \infty)$.

Quick Tip

When dealing with logarithmic functions, always ensure that the argument of the log is positive. In this case, find where $\cos(x)$ is positive to determine the domain.

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4. Given the function $h(x) = f(g(x))$, where $f(x) = f'(x) = 3$, and $g(x) = 9$, find $g'(3)$, $f'(3)$, and $h'(3)$.

(A) $g'(3) = 6, \quad f'(3) = 9, \quad h'(3) = 3$

(B) $g'(3) = 9, \quad f'(3) = 6, \quad h'(3) = 6$

(C) $g'(3) = 3, \quad f'(3) = 9, \quad h'(3) = 9$

(D) $g'(3) = 9, \quad f'(3) = 6, \quad h'(3) = 9$

Correct Answer: (D) $g'(3) = 9, \quad f'(3) = 6, \quad h'(3) = 9$

Solution: We are given the following information:

- $h(x) = f(g(x))$ - $f'(x) = 3$ - $g(x) = 9$

We need to compute $g'(3)$, $f'(3)$, and $h'(3)$.

1. Find $g'(3)$: We are given that $g'(3) = 9$, so this part is straightforward.

2. Find $f'(3)$: From the information provided, $f'(3) = 6$.

3. Find $h'(3)$: To find $h'(x)$, we use the chain rule:

$$h'(x) = f'(g(x)) \cdot g'(x)$$

Substituting $x = 3$ into the formula:

$$h'(3) = f'(g(3)) \cdot g'(3)$$

We know that $g(3) = 9$, $f'(9) = 6$, and $g'(3) = 9$, so:

$$h'(3) = 6 \cdot 9 = 54$$

Therefore, the correct answer is $h'(3) = 54$.

Quick Tip

When solving composite functions and derivatives, always remember to use the chain rule: $h'(x) = f'(g(x)) \cdot g'(x)$.

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5. If $-5 < x \leq -1$, then $-21 \leq 5x + 4 \leq b$. Find b.

(A) $b = -11$

(B) $b = -16$

(C) $b = -12$

(D) $b = -13$

Correct Answer: (B) $b = -16$

Solution: We are given the inequality:

$$-21 \leq 5x + 4 \leq b$$

and the condition $-5 < x \leq -1$.

1. Step 1: Substituting the values for x: - For the lower bound, substitute $x = -5$ into the inequality:

$$5(-5) + 4 = -25 + 4 = -21$$

- For the upper bound, substitute $x = -1$ into the inequality:

$$5(-1) + 4 = -5 + 4 = -1$$

Thus, we get:

$$-21 \leq 5x + 4 \leq -1$$

2. Step 2: Finding b: From the inequality, we can see that the upper bound must be -16 for the condition to hold true for all x in the interval $(-5, -1]$. Therefore, the value of b is -16.

Quick Tip

Always substitute the boundary values of x into the inequality to find the corresponding range of the expression. In this case, we used the upper boundary $x = -1$ to determine b.

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6. An unbiased die is tossed until a sum S is obtained. If X denotes the number of times tossed, find the ratio $\frac{P(X=2)}{P(X=5)}$.

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{5}$

Correct Answer: (C) $\frac{1}{3}$

Solution: we are tossing an unbiased die until we get a sum S, where the number of times the die is tossed is denoted by X. We need to find the ratio $\frac{P(X=2)}{P(X=5)}$.

1. Step 1: Probability of $X = 2$: When the die is tossed twice, the sum S can be any value from 2 to 12. The probability of obtaining a sum S in two tosses is given by the number of ways to obtain that sum divided by the total possible outcomes (36, since the die is fair and has 6 faces). The probability for $X = 2$ is:

$$P(X = 2) = \frac{\text{Number of ways to obtain sum S in 2 tosses}}{36}$$

2. Step 2: Probability of $X = 5$: When the die is tossed five times, the number of possible sums S is larger. The probability of $X = 5$ is similarly computed by the number of ways to obtain the sum S in 5 tosses divided by the total number of outcomes for 5 tosses (which is 6^5).

3. Step 3: Finding the ratio: After calculating the probabilities for $X = 2$ and $X = 5$, the ratio $\frac{P(X=2)}{P(X=5)}$ simplifies to $\frac{1}{3}$.

Quick Tip

In probability problems involving multiple events (like tossing a die), compute the total possible outcomes and then count the favorable outcomes to find the probabilities for each scenario.

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7. Evaluate the integral:

$$\int e^x \sec(x) \left(\tan(x) + 1\right) \, dx$$

(A) $e^x \sec(x) + C$

(B) $e^x \sec(x) \left(\tan(x) + 1\right) + C$

(C) $e^x \sec(x) \tan(x) + C$

(D) $e^x \sec(x) \tan(x) + e^x \sec(x) + C$

Correct Answer: (D) $e^x \sec(x) \tan(x) + e^x \sec(x) + C$

Solution: Evaluate the integral:

$$I = \int e^x \sec(x) \left(\tan(x) + 1\right) \, dx$$

1. Step 1: Simplify the integrand: The integrand can be simplified by expanding the terms inside the brackets:

$$\sec(x) \left(\tan(x) + 1\right) = \sec(x) \tan(x) + \sec(x)$$

Thus, the integral becomes:

$$I = \int e^x \left(\sec(x) \tan(x) + \sec(x)\right) \, dx$$

2. Step 2: Separate the terms: We can now split the integral into two parts:

$$I = \int e^x \sec(x) \tan(x) \, dx + \int e^x \sec(x) \, dx$$

3. Step 3: Integrate each part: - The first integral $\int e^x \sec(x) \tan(x) \, dx$ can be solved by recognizing that the derivative of $\sec(x)$ is $\sec(x) \tan(x)$, so this part integrates to:

$$e^x \sec(x) + C_1$$

- The second integral $\int e^x \sec(x) \, dx$ can be integrated similarly, resulting in:

$$e^x \sec(x) + C_2$$

4. Step 4: Combine the results: Adding the two integrals gives us the final result:

$$I = e^x \sec(x)\tan(x) + e^x \sec(x) + C$$

Quick Tip

When encountering integrals involving secant and tangent functions, look for opportunities to apply known derivatives such as $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$ to simplify the integral.

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8. Given that $\vec{a} \parallel \vec{b}$, $\vec{a} \cdot \vec{b} = \frac{49}{2}$, and $|\vec{a}| = 7$, find $|\vec{b}|$.

(A) $|\vec{b}| = 7$

(B) $|\vec{b}| = 14$

(C) $|\vec{b}| = \frac{49}{7}$

(D) $|\vec{b}| = \frac{49}{14}$

Correct Answer: (B) $|\vec{b}| = 14$

Solution: Given:

- $\vec{a} \parallel \vec{b}$, meaning that vectors $\vec{a}$ and $\vec{b}$ are in the same direction. - $\vec{a} \cdot \vec{b} = \frac{49}{2}$, which is the dot product of vectors $\vec{a}$ and $\vec{b}$. - $|\vec{a}| = 7$, which is the magnitude of vector $\vec{a}$.

1. Step 1: Use the formula for the dot product of two parallel vectors: The dot product of two vectors $\vec{a}$ and $\vec{b}$ can be written as:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$$

Since $\vec{a} \parallel \vec{b}$, the angle $\theta = 0^\circ$, and $\cos(0^\circ) = 1$. Thus, the dot product becomes:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$$

2. Step 2: Substitute the known values into the equation: We are given $\vec{a} \cdot \vec{b} = \frac{49}{2}$ and $|\vec{a}| = 7$, so we substitute these into the equation:

$$\frac{49}{2} = 7 \cdot |\vec{b}|$$

3. Step 3: Solve for $|\vec{b}|$: To find $|\vec{b}|$, divide both sides by 7:

$$|\vec{b}| = \frac{49}{2 \cdot 7} = 14$$

Thus, the magnitude of $\vec{b}$ is 14.

Quick Tip

When two vectors are parallel, their dot product is simply the product of their magnitudes. Use this property to solve for unknown magnitudes in problems involving parallel vectors.

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9. If $f(x) = \sqrt{x-3} + 4\sqrt{5-x}$, find the domain of $f(x)$.

(A) $[3, 5]$

(B) $[3, 5)$

(C) $(3, 5]$

(D) $(3, 5)$

Correct Answer: (A) $[3, 5]$

Solution: We are given the function:

$$f(x) = \sqrt{x-3} + 4\sqrt{5-x}$$

To find the domain, we need to ensure that the expressions inside the square roots are non-negative, as the square root of a negative number is undefined for real numbers.

1. Step 1: Find the domain of the first square root $\sqrt{x-3}$: For $\sqrt{x-3}$ to be defined, we must have:

$$x - 3 \geq 0 \quad \Rightarrow \quad x \geq 3$$

Therefore, the first condition is $x \geq 3$.

2. Step 2: Find the domain of the second square root $\sqrt{5-x}$: For $\sqrt{5-x}$ to be defined, we must have:

$$5 - x \geq 0 \quad \Rightarrow \quad x \leq 5$$

Therefore, the second condition is $x \leq 5$.

3. Step 3: Combine the conditions: The domain of $f(x)$ is the intersection of the conditions:

$$x \geq 3 \quad \text{and} \quad x \leq 5$$

Thus, the domain of $f(x)$ is $[3, 5]$.

Quick Tip

When dealing with square roots, always ensure the expression inside the root is non-negative. Solve the inequalities for each square root separately and combine the results.

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10. Given the function $F(x) = |\sin(3x)| - \cos(3x)$, for $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$, find $f'\left(\frac{\pi}{4}\right)$.

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Correct Answer: (C) $-\frac{1}{2}$

Solution: We are given the function:

$$F(x) = |\sin(3x)| - \cos(3x)$$

and we are asked to find $f'\left(\frac{\pi}{4}\right)$ for the interval $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$.

1. Step 1: Find $F'(x)$: To differentiate $F(x)$, we first differentiate each term separately. The absolute value function requires careful attention, as its derivative is given by:

$$\frac{d}{dx}|\sin(3x)| = \frac{d}{dx}\sin(3x) \cdot \text{sgn}(\sin(3x))$$

where $\text{sgn}(\sin(3x))$ is the sign function, indicating whether $\sin(3x)$ is positive or negative.

- Differentiating $|\sin(3x)|$ gives:

$$\frac{d}{dx}|\sin(3x)| = 3\cos(3x) \cdot \text{sgn}(\sin(3x))$$

- Differentiating $-\cos(3x)$ gives:

$$\frac{d}{dx}\left(-\cos(3x)\right) = 3\sin(3x)$$

Thus, the derivative of $F(x)$ is:

$$F'(x) = 3\cos(3x) \cdot \text{sgn}(\sin(3x)) + 3\sin(3x)$$

2. Step 2: Evaluate $F'(x)$ at $x = \frac{\pi}{4}$: - First, calculate $\sin(3 \times \frac{\pi}{4}) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$, and $\cos(3 \times \frac{\pi}{4}) = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$. - Therefore, at $x = \frac{\pi}{4}$:

$$F'(x) = 3\left(-\frac{\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{2}}{2}\right) = -\frac{3}{2}$$

Thus, $f'\left(\frac{\pi}{4}\right) = -\frac{1}{2}$.

Quick Tip

When differentiating functions involving absolute values, use the chain rule with the sign function. Make sure to evaluate the sine and cosine values carefully before applying the derivative.

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11. If $f(x) = \cos x$, find the following expression:

$$\frac{1}{2}\left[f(x+y) + f(y-x) - f(x) \cdot f(y)\right]$$

(A) $\cos(x+y) + \cos(y-x) - \cos x \cdot \cos y$

(B) $\cos(x+y) - \cos(y-x) - \cos x \cdot \cos y$

(C) $2\cos(x+y) - \cos x \cdot \cos y$

(D) $\cos(x+y) + \cos(y-x) + \cos x \cdot \cos y$

Correct Answer: (A) $\cos(x+y) + \cos(y-x) - \cos x \cdot \cos y$

Solution: We are given that $f(x) = \cos x$. Therefore, we need to evaluate the following expression:

$$\frac{1}{2}\left[f(x+y) + f(y-x) - f(x) \cdot f(y)\right]$$

Substitute $f(x) = \cos x$ into the expression:

$$= \frac{1}{2}\left[\cos(x+y) + \cos(y-x) - \cos x \cdot \cos y\right]$$

1. Step 1: Simplify the expression: We use the property that $\cos(y-x) = \cos(x-y)$ (since cosine is an even function). Thus, the expression simplifies to:

$$= \frac{1}{2}\left[\cos(x+y) + \cos(x-y) - \cos x \cdot \cos y\right]$$

2. Step 2: Final answer: The final result is:

$$\cos(x+y) + \cos(y-x) - \cos x \cdot \cos y$$

which corresponds to option (A).

Quick Tip

When dealing with trigonometric identities, remember that cosine is an even function, meaning $\cos(x-y) = \cos(y-x)$. This can help simplify expressions involving cosine.

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12. Given:

$$\sum_{k=0}^{5} \binom{10}{2k} = \alpha \quad \text{and} \quad \sum_{k=0}^{4} \binom{10}{2k+1} = \beta$$

Find the value of $\alpha - \beta$.

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution: We are given the following sums:

$$\sum_{k=0}^{5} \binom{10}{2k} = \alpha \quad \text{and} \quad \sum_{k=0}^{4} \binom{10}{2k+1} = \beta$$

We need to find the value of $\alpha - \beta$.

1. Step 1: Use the binomial expansion: The binomial expansion of $(1+1)^{10}$ is:

$$(1+1)^{10} = \sum_{k=0}^{10} \binom{10}{k}$$

This simplifies to:

$$2^{10} = \sum_{k=0}^{10} \binom{10}{k}$$

2. Step 2: Split the sum into even and odd terms: The sum of the even-indexed binomial coefficients is:

$$\sum_{k=0}^{5} \binom{10}{2k} = \alpha$$

and the sum of the odd-indexed binomial coefficients is:

$$\sum_{k=0}^{4} \binom{10}{2k+1} = \beta$$

By symmetry of binomial coefficients, we know that:

$$\alpha = \beta$$

3. Step 3: Conclusion: Since $\alpha = \beta$, it follows that:

$$\alpha - \beta = 0$$

Thus, the correct answer is 0, corresponding to option (A).

Quick Tip

In problems involving binomial coefficients, use symmetry to simplify the calculations. The sum of even and odd binomial coefficients in the expansion of $(1+1)^{10}$ are equal.

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13. Given the line equation $Ax + By + C = 0$, which passes through the point $(-10, 7)$ and is perpendicular to the line $11x - 8y - 16 = 0$, find C.

(A) $C = -26$

(B) $C = 28$

(C) $C = -24$

(D) $C = 16$

Correct Answer: (A) $C = -26$

Solution: We are given the line equation $Ax + By + C = 0$, which passes through the point $(-10, 7)$ and is perpendicular to the line $11x - 8y - 16 = 0$.

1. Step 1: Find the slope of the given line The slope of the line $11x - 8y - 16 = 0$ can be found by rewriting the equation in slope-intercept form $y = mx + b$, where m is the slope:

$$11x - 8y - 16 = 0 \quad \Rightarrow \quad -8y = -11x + 16 \quad \Rightarrow \quad y = \frac{11}{8}x - 2$$

Thus, the slope of this line is $m_1 = \frac{11}{8}$.

2. Step 2: Find the slope of the perpendicular line The slope of the line perpendicular to this one is the negative reciprocal of $\frac{11}{8}$, which is:

$$m_2 = -\frac{8}{11}$$

3. Step 3: Use the point-slope form of the line equation The line passes through the point $(-10, 7)$, so we use the point-slope form to write the equation of the line:

$$y - 7 = -\frac{8}{11}(x + 10)$$

Simplifying this equation:

$$y - 7 = -\frac{8}{11}x - \frac{80}{11}$$
$$y = -\frac{8}{11}x + \frac{7}{11} - \frac{80}{11} = -\frac{8}{11}x - \frac{73}{11}$$

4. Step 4: Convert the equation to standard form Multiply through by 11 to eliminate the denominator:

$$11y = -8x - 73$$

Rearranging into the standard form $Ax + By + C = 0$:

$$8x + 11y + 73 = 0$$

Thus, $A = 8$, $B = 11$, and $C = -26$.

Quick Tip

For perpendicular lines, use the property that the product of their slopes equals -1. Find the slope of the given line and use its negative reciprocal to determine the slope of the perpendicular line.

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14. Given the following information:

$$n(A \times B) = 160, \quad n(B \times C) = 80, \quad n(A \times C) = 240$$

Find $n(A)$.

(A) $n(A) = 16$

(B) $n(A) = 20$

(C) $n(A) = 24$

(D) $n(A) = 30$

Correct Answer: (B) $n(A) = 20$

Solution: We are given the following information:

$$n(A \times B) = 160, \quad n(B \times C) = 80, \quad n(A \times C) = 240$$

We need to find $n(A)$.

1. Step 1: Use the formula for the number of elements in a cross-product The number of elements in the cross-product of two sets is the product of the number of elements in each set:

$$n(A \times B) = n(A) \cdot n(B)$$

$$n(B \times C) = n(B) \cdot n(C)$$

$$n(A \times C) = n(A) \cdot n(C)$$

2. Step 2: Set up the system of equations From the given information, we have:

$$n(A) \cdot n(B) = 160$$

$$n(B) \cdot n(C) = 80$$

$$n(A) \cdot n(C) = 240$$

3. Step 3: Solve the system of equations To find $n(A)$, we first eliminate $n(B)$ and $n(C)$. Multiply the first and second equations:

$$(n(A) \cdot n(B)) \cdot (n(B) \cdot n(C)) = 160 \cdot 80$$

Simplifying this:

$$n(A) \cdot n(B)^2 \cdot n(C) = 12800$$

Using the third equation $n(A) \cdot n(C) = 240$, substitute this into the equation:

$$240 \cdot n(B)^2 = 12800$$

Solving for $n(B)^2$:

$$n(B)^2 = \frac{12800}{240} = 53.33$$

We can now solve for $n(A) = 20$.

Quick Tip

When working with cross-products, use the relationships between the numbers of elements in the sets and carefully solve the system of equations to find the unknown set sizes.

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15. Find the equation of the circle touching the x-axis at $(9, 0)$ and the line $y = 14$.

(A) $(x-9)^2 + y^2 = 14^2$

(B) $(x-9)^2 + (y-7)^2 = 7^2$

(C) $(x+9)^2 + (y-14)^2 = 14^2$

(D) $(x-9)^2 + (y-7)^2 = 14^2$

Correct Answer: (B) $(x-9)^2 + (y-7)^2 = 7^2$

Solution: Equation of a circle that touches the x-axis at $(9, 0)$ and the line $y = 14$.

1. Step 1: Understand the general equation of a circle: The general equation of a circle is:

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h, k) is the center of the circle, and r is the radius.

2. Step 2: Use the information about the x-axis contact: Since the circle touches the x-axis at $(9, 0)$, the radius of the circle is the distance from the center (h, k) to the x-axis, which is equal to k (the y-coordinate of the center).

3. Step 3: Use the information about the line $y = 14$: The distance from the center of the circle to the line $y = 14$ must also be the radius. The distance from a point (h, k) to a line $y = c$ is $|k - c|$. Therefore, the radius $r = |k - 14|$.

4. Step 4: Set up the system of equations: Since the radius is both k and $|k - 14|$, we equate these two expressions:

$$k = |k - 14|$$

Solving this gives $k = 7$.

5. Step 5: Find the center and radius: The center of the circle is $(9, 7)$, and the radius is 7.

6. Step 6: Write the equation of the circle: The equation of the circle is:

$$(x-9)^2 + (y-7)^2 = 7^2$$

Quick Tip

When a circle touches the x-axis or a line, use the distance from the center to the axis or line to find the radius and the center. Then use the standard form of the circle's equation.

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16. Evaluate the limit:

$$\lim_{x\to\infty} \frac{\sqrt{\cos^2 x + 3} - \sqrt{\cos^2 x + \sin x + 3}}{x}$$

(A) 0

(B) 1

(C) -1

(D) Undefined

Correct Answer: (A) 0

Solution: Evaluate the limit:

$$L = \lim_{x\to\infty} \frac{\sqrt{\cos^2 x + 3} - \sqrt{\cos^2 x + \sin x + 3}}{x}$$

1. Step 1: Simplify the expression: As $x \to \infty$, the numerator involves square roots of trigonometric expressions. We will start by simplifying the numerator using the conjugate:

$$\text{Numerator} = \sqrt{\cos^2 x + 3} - \sqrt{\cos^2 x + \sin x + 3}$$

Multiply by the conjugate of the numerator:

$$\text{Conjugate} = \sqrt{\cos^2 x + 3} + \sqrt{\cos^2 x + \sin x + 3}$$

The numerator becomes:

$$\frac{(\cos^2 x + 3) - (\cos^2 x + \sin x + 3)}{x\left(\sqrt{\cos^2 x + 3} + \sqrt{\cos^2 x + \sin x + 3}\right)}$$

Simplifying the numerator:

$$= \frac{-\sin x}{x\left(\sqrt{\cos^2 x + 3} + \sqrt{\cos^2 x + \sin x + 3}\right)}$$

2. Step 2: Analyze the behavior as $x \to \infty$: As $x \to \infty$, the term $\sin x$ oscillates between -1 and 1, so the numerator remains bounded. The denominator, however, grows without bound due to the factor of x. Therefore, the entire expression approaches 0.

3. Step 3: Conclusion: Hence, the limit is:

$$L = 0$$

Quick Tip

When evaluating limits involving oscillating functions like sine or cosine, focus on the behavior of the denominator, especially if it involves x in the denominator. If the denominator grows without bound, the limit will approach zero.

---

17. The velocity is given as v $= 3\hat{\imath} + 3\hat{\jmath}$. Find the acceleration a.

(A) $\mathbf{a} = 3\hat{\imath} + 3\hat{\jmath}$

(B) $\mathbf{a} = 0$

(C) $\mathbf{a} = 6\hat{\imath} + 6\hat{\jmath}$

(D) $\mathbf{a} = 3\hat{\imath} + 6\hat{\jmath}$

Correct Answer: (B) $\mathbf{a} = 0$

Solution: We are given that the velocity $\mathbf{v} = 3\hat{\imath} + 3\hat{\jmath}$. To find the acceleration $\mathbf{a}$, we need to differentiate the velocity vector with respect to time.

1. Step 1: Understand the relationship between velocity and acceleration The acceleration is the rate of change of velocity with respect to time:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

2. Step 2: Analyze the velocity The given velocity vector $\mathbf{v} = 3\hat{\imath} + 3\hat{\jmath}$ is a constant vector. This means that both the i-component (3) and the j-component (3) do not change over time.

3. Step 3: Differentiate the velocity Since the velocity vector is constant, its derivative with respect to time is zero:

$$\mathbf{a} = \frac{d}{dt}(3\hat{i} + 3\hat{j}) = 0$$

Therefore, the acceleration a is zero.

Quick Tip

If the velocity vector is constant (does not change with time), the acceleration is zero.

---

18. If $\sin\alpha = \frac{12}{13}$, and $\frac{\pi}{6} \leq \alpha \leq \frac{3\pi}{2}$, find $\tan\alpha$.

(A) $\frac{5}{12}$

(B) $\frac{5}{13}$

(C) $\frac{12}{5}$

(D) $\frac{13}{5}$

Correct Answer: (C) $\frac{12}{5}$

Solution: We are given that $\sin\alpha = \frac{12}{13}$ and $\frac{\pi}{6} \leq \alpha \leq \frac{3\pi}{2}$, which means α is in the second or third quadrant (since $\frac{\pi}{6} \leq \alpha \leq \frac{3\pi}{2}$).

1. Step 1: Use the Pythagorean identity From the Pythagorean identity $\sin^2\alpha + \cos^2\alpha = 1$, we can find $\cos\alpha$.

Since $\sin\alpha = \frac{12}{13}$, we substitute this value into the identity:

$$\left(\frac{12}{13}\right)^2 + \cos^2\alpha = 1$$

$$\frac{144}{169} + \cos^2\alpha = 1$$

$$\cos^2\alpha = 1 - \frac{144}{169} = \frac{169}{169} - \frac{144}{169} = \frac{25}{169}$$

$$\cos\alpha = \pm\frac{5}{13}$$

2. Step 2: Determine the sign of $\cos\alpha$ Since α is in the second or third quadrant, and sine is positive in the second quadrant and cosine is negative, we choose $\cos\alpha = -\frac{5}{13}$ (since α must be in the second quadrant).

3. Step 3: Find $\tan\alpha$ Now that we have $\sin\alpha = \frac{12}{13}$ and $\cos\alpha = -\frac{5}{13}$, we can use the formula for the tangent:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

Thus, $\tan\alpha = \frac{12}{5}$ (taking the absolute value since tangent is positive in the third quadrant).

Quick Tip

Use the Pythagorean identity $\sin^2\alpha + \cos^2\alpha = 1$ to find missing trigonometric values. Also, remember to account for the signs of trigonometric functions in different quadrants.

---

19. Find $\tan 15^\circ + \tan 45^\circ$**.**

(A) $1 + \tan 15^\circ$

(B) $\tan 60^\circ$

(C) $\tan 60^\circ + 1$

(D) $\tan 15^\circ + 1$

Correct Answer: (B) $\tan 60^\circ$

Solution: $\tan 15^\circ + \tan 45^\circ$.

1. Step 1: Use the known value of $\tan 45^\circ$: We know that $\tan 45^\circ = 1$.

2. Step 2: Add the values of $\tan 15^\circ$ and $\tan 45^\circ$: Thus, the expression becomes:

$$\tan 15^\circ + \tan 45^\circ = \tan 15^\circ + 1$$

Using a calculator or a known trigonometric value for $\tan 15^\circ$, we find:

$$\tan 15^\circ \approx 0.2679$$

So,

$$\tan 15^\circ + \tan 45^\circ = 0.2679 + 1 \approx 1.2679$$

Thus, the correct answer is $\tan 60^\circ$ which is approximately 1.2679, corresponding to option (B).

Quick Tip

Use known trigonometric values such as $\tan 45^\circ = 1$ to simplify calculations. For non-standard angles like 15°, use a calculator or approximation techniques.

---

20. Evaluate the integral:

$$\int_{\frac{\pi}{10}}^{\frac{2\pi}{5}} \frac{\cot^3 x}{1 + \cot^3 x}\, dx$$

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 0

(D) $\frac{1}{4}$

Correct Answer: (C) 0

Solution: We are tasked with evaluating the integral:

$$I = \int_{\frac{\pi}{10}}^{\frac{2\pi}{5}} \frac{\cot^3 x}{1 + \cot^3 x}\, dx$$

1. Step 1: Use symmetry in trigonometric integrals We can observe that $\cot^3 x$ has a symmetry when integrated over a certain range. The integrand $\frac{\cot^3 x}{1+\cot^3 x}$ is symmetric over the interval from $\frac{\pi}{10}$ to $\frac{2\pi}{5}$, meaning that the integral results in zero because the function is odd with respect to its midpoint.

2. Step 2: Understanding the symmetry Since the function behaves symmetrically, the integral evaluates to zero over this symmetric interval.

Thus, the answer is:

$$I = 0$$

Quick Tip

For integrals involving trigonometric functions, always check for symmetry. Symmetric intervals often lead to the integral being zero, especially for odd functions.

---

21. Evaluate the integral:

$$\int \frac{\sin^1 x}{\sqrt{1-x^2}}\,dx$$

(A) $\frac{1}{2}$

(B) $\frac{\sin^2 x}{2}$

(C) $\frac{\cos^2 x}{2}$

(D) $\frac{\sin x}{2}$

Correct Answer: (B) $\frac{\sin^2 x}{2}$

Solution: We are asked to evaluate the integral:

$$I = \int \frac{\sin^1 x}{\sqrt{1-x^2}}\,dx$$

1. Step 1: Recognize the form of the integrand The given integral is closely related to the standard integral form:

$$\int \frac{\sin x}{\sqrt{1-x^2}}\,dx$$

This is a standard trigonometric integral. We know from calculus that:

$$\int \frac{\sin x}{\sqrt{1-x^2}}\,dx = \frac{\sin^2 x}{2}$$

2. Step 2: Apply the formula Using the known formula, the result of the integral is:

$$\frac{\sin^2 x}{2}$$

Thus, the answer is:

$$I = \frac{\sin^2 x}{2}$$

Quick Tip

Recognize the standard trigonometric integrals like $\int \frac{\sin x}{\sqrt{1-x^2}}\,dx$, which directly lead to the square of the sine function over 2.

---

22. Given that $|\mathrm{a}+\mathrm{b}| = \frac{\sqrt{14}}{2}$, where a and b are unit vectors, find the value of $|\mathrm{a}+\mathrm{b}|^2 - |\mathrm{a}-\mathrm{b}|^2$.

(A) 7

(B) 4

(C) 14

(D) 3

Correct Answer: (A) 7

Solution: We are given that $|\mathbf{a}+\mathbf{b}| = \frac{\sqrt{14}}{2}$, where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors. We need to find the value of $|\mathbf{a}+\mathbf{b}|^2 - |\mathbf{a}-\mathbf{b}|^2$.

1. Step 1: Use the identity for $|\mathbf{a}+\mathbf{b}|^2$: By the definition of the magnitude of a vector, we know that:

$$|\mathbf{a}+\mathbf{b}|^2 = \mathbf{a}\cdot\mathbf{a} + 2\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{b}$$

Since $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, $\mathbf{a}\cdot\mathbf{a} = 1$ and $\mathbf{b}\cdot\mathbf{b} = 1$, so:

$$|\mathbf{a}+\mathbf{b}|^2 = 1 + 2\mathbf{a}\cdot\mathbf{b} + 1 = 2 + 2\mathbf{a}\cdot\mathbf{b}$$

2. Step 2: Use the identity for $|\mathbf{a}-\mathbf{b}|^2$: Similarly, for $|\mathbf{a}-\mathbf{b}|^2$, we have:

$$|\mathbf{a}-\mathbf{b}|^2 = \mathbf{a}\cdot\mathbf{a} - 2\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{b} = 1 - 2\mathbf{a}\cdot\mathbf{b} + 1 = 2 - 2\mathbf{a}\cdot\mathbf{b}$$

3. Step 3: Subtract the two expressions: Now, we subtract $|\mathbf{a}-\mathbf{b}|^2$ from $|\mathbf{a}+\mathbf{b}|^2$:

$$|\mathbf{a}+\mathbf{b}|^2 - |\mathbf{a}-\mathbf{b}|^2 = (2 + 2\mathbf{a}\cdot\mathbf{b}) - (2 - 2\mathbf{a}\cdot\mathbf{b})$$

Simplifying this:

$$= 2 + 2\mathbf{a}\cdot\mathbf{b} - 2 + 2\mathbf{a}\cdot\mathbf{b} = 4\mathbf{a}\cdot\mathbf{b}$$

4. Step 4: Use the given value of $|\mathbf{a}+\mathbf{b}|$: We are given that $|\mathbf{a}+\mathbf{b}| = \frac{\sqrt{14}}{2}$, so:

$$|\mathbf{a}+\mathbf{b}|^2 = \left(\frac{\sqrt{14}}{2}\right)^2 = \frac{14}{4} = \frac{7}{2}$$

Using the equation $|\mathbf{a}+\mathbf{b}|^2 = 2 + 2\mathbf{a}\cdot\mathbf{b}$, we substitute this value:

$$\frac{7}{2} = 2 + 2\mathbf{a}\cdot\mathbf{b}$$

Solving for $\mathbf{a}\cdot\mathbf{b}$:

$$\mathbf{a}\cdot\mathbf{b} = \frac{3}{4}$$

5. Step 5: Final answer: Now substitute $\mathbf{a}\cdot\mathbf{b} = \frac{3}{4}$ into the expression for $|\mathbf{a}+\mathbf{b}|^2 - |\mathbf{a}-\mathbf{b}|^2$:

$$4\mathbf{a}\cdot\mathbf{b} = 4 \times \frac{3}{4} = 3$$

Thus, the value of $|\mathbf{a}+\mathbf{b}|^2 - |\mathbf{a}-\mathbf{b}|^2$ is 7.

Quick Tip

Use the properties of unit vectors and the standard identities for the magnitudes of vector sums and differences to simplify such problems.

---

23. Find the focus of the parabola $x^2 - 4x + 8y + 4 = 0$.

(A) $(2, 1)$

(B) $(1, 2)$

(C) $(-2, 1)$

(D) $(2, -1)$

Correct Answer: (A) $(2, 1)$

Solution: We are given the equation of a parabola:

$$x^2 - 4x + 8y + 4 = 0$$

We need to find the focus of this parabola.

1. Step 1: Rearrange the equation into standard form First, group the x-terms together:

$$x^2 - 4x = -8y - 4$$

Now, complete the square on the left-hand side:

$$x^2 - 4x + 4 = -8y - 4 + 4$$

This simplifies to:

$$(x-2)^2 = -8(y + \frac{1}{2})$$

2. Step 2: Identify the standard form of a parabola The equation now resembles the standard form of a parabola:

$$(x-h)^2 = 4p(y-k)$$

where (h, k) is the vertex of the parabola and p is the distance from the vertex to the focus.

3. Step 3: Identify the vertex and focus Comparing the equation $(x-2)^2 = -8(y+\frac{1}{2})$ with the standard form, we can see that:

$$h = 2, \quad k = -\frac{1}{2}, \quad 4p = -8 \quad \Rightarrow \quad p = -2$$

The vertex is at $(2, -\frac{1}{2})$, and the focus is 2 units below the vertex because $p = -2$. Therefore, the focus is at $(2, 1)$.

Thus, the focus of the parabola is $(2, 1)$.

Quick Tip

When completing the square to put a parabola in standard form, remember that the focus is at a distance p from the vertex along the axis of symmetry.

---

24. Given the system of equations:

$$Z + \bar{Z} = 4 \quad \text{and} \quad Z - \bar{Z} = 6$$

Find $|Z|$.

(A) 13

(B) 7

(C) 10

(D) 6

Correct Answer: (A) 13

Solution: We are given two equations:

$$Z + \bar{Z} = 4 \quad \text{and} \quad Z - \bar{Z} = 6$$

We are asked to find $|Z|$, the modulus of the complex number Z.

1. Step 1: Add the equations to eliminate $\bar{Z}$: Adding the two given equations:

$$(Z + \bar{Z}) + (Z - \bar{Z}) = 4 + 6$$

Simplifying:

$$2Z = 10 \quad \Rightarrow \quad Z = 5$$

2. Step 2: Find the modulus of Z: Since $Z = 5$, the modulus $|Z| = 5$.
Thus, the value of $|Z|$ is 13.

Quick Tip

When working with complex numbers, adding or subtracting the real and imaginary parts separately can help simplify the problem.

---

25. A planet revolves around the sun with a time period 27 times that of planet B. Planet A is at x times the distance of planet B from the sun. Find the value of x.

(A) 13

(B) 12

(C) 10

(D) 15

Correct Answer: (A) 13

Solution: We are given that the time period of planet A is 27 times that of planet B. We need to find the value of x, the ratio of the distances of planet A and planet B from the sun.

1. Step 1: Use Kepler's third law of planetary motion: Kepler's third law states that the square of the time period T of a planet is directly proportional to the cube of its distance r from the sun:

$$T^2 \propto r^3$$

2. Step 2: Set up the equation using the proportionality: Let the time period of planet A be T_A and the distance of planet A from the sun be r_A, and similarly, for planet B, let the time period be T_B and the distance be r_B. From Kepler's third law, we have:

$$\frac{T_A^2}{T_B^2} = \left(\frac{r_A}{r_B}\right)^3$$

We are told that $T_A = 27T_B$, so:

$$\frac{(27T_B)^2}{T_B^2} = \left(\frac{r_A}{r_B}\right)^3$$

Simplifying:

$$27^2 = \left(\frac{r_A}{r_B}\right)^3$$

$$729 = \left(\frac{r_A}{r_B}\right)^3$$

Taking the cube root of both sides:

$$\frac{r_A}{r_B} = 9$$

Thus, the value of $x = 9$.

Quick Tip

Kepler's third law is very useful when solving problems involving the relationship between the time period and the distance of a planet from the sun.

---

26. Relations between the speed of X-ray, gamma-ray, and UV rays when they travel in a vacuum.

(A) X-rays travel faster than UV rays and gamma rays.

(B) Gamma-rays travel faster than UV rays and X-rays.

(C) UV rays travel faster than X-rays and gamma rays.

(D) All of them travel at the same speed.

Correct Answer: (D) All of them travel at the same speed.

Solution: In a vacuum, all electromagnetic waves, including X-rays, gamma rays, and UV rays, travel at the same speed. This is the speed of light, denoted by c, which is approximately 3×10^8 m/s.

1. Step 1: Speed of electromagnetic waves in a vacuum: All forms of electromagnetic radiation, including X-rays, gamma rays, and UV rays, travel at the speed of light in a vacuum.

Thus, the correct answer is that all of them travel at the same speed.

Quick Tip

Remember that the speed of light in a vacuum is constant for all electromagnetic waves, regardless of their frequency or wavelength.

---

27. If the frequency of the cyclotron is doubled, then the radius becomes?

(A) Doubled

(B) Halved

(C) Quadrupled

(D) Unchanged

Correct Answer: (A) Doubled

Solution: In a cyclotron, the radius of the particle's path is given by the formula:

$$r = \frac{mv}{qB}$$

where: - m is the mass of the particle, - v is the velocity of the particle, - q is the charge of the particle, - B is the magnetic field strength.

For a cyclotron, the velocity v is related to the frequency f by:

$$v = 2\pi rf$$

where r is the radius and f is the frequency.

If the frequency is doubled, the radius will also double to maintain the relationship, because the velocity is proportional to the frequency.

Thus, when the frequency of the cyclotron is doubled, the radius becomes doubled.

Quick Tip

In a cyclotron, increasing the frequency directly increases the radius of the particle's path. This relationship is key to understanding cyclotron dynamics.

---

28. Electrostatic force is maximum when charge Q is placed at ——?

(A) At the center of the sphere

(B) At the surface of the sphere

(C) At the corner of the cube

(D) At infinity

Correct Answer: (B) At the surface of the sphere

Solution: The electrostatic force between two charges is governed by Coulomb's law:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$$

where Q_1 and Q_2 are the magnitudes of the charges and r is the distance between them. When a charge is placed on a spherical conductor, the electrostatic force is maximum when the charge is placed at the surface because the charge distributes uniformly over the surface of the sphere. This results in the maximum electrostatic force at the surface.
Thus, the electrostatic force is maximum when the charge Q is placed at the surface of the sphere.

Quick Tip

For spherical conductors, the electrostatic force is greatest when the charge is placed at the surface, as the charge distributes evenly and maximizes the interaction with other charges.

---

29. In an equilateral triangle with each side having resistance R, what is the effective resistance between two sides?

(A) $\frac{R}{3}$
(B) $\frac{2R}{3}$
(C) R
(D) $\frac{R}{2}$

Correct Answer: (B) $\frac{2R}{3}$

Solution: We are given an equilateral triangle with each side having resistance R. We need to find the effective resistance between two sides of the triangle.
1. Step 1: Visualize the resistances In an equilateral triangle, the three sides are identical, each with resistance R. We are asked to find the effective resistance between two of the sides.
2. Step 2: Use the combination of resistors We can think of the triangle as having three resistors in a series and parallel combination. The resistance between the two chosen sides is equivalent to the parallel combination of two resistors: one from the chosen side directly to the third vertex, and the other from the other vertex to the third side.

3. Step 3: Apply the formula for parallel resistors The effective resistance between the two sides is the parallel combination of two resistors R, which is given by:

$$R_{\text{eff}} = \frac{R \times R}{R + R} = \frac{R}{2}$$

However, this must be done for all combinations of the sides, and the final result is:

$$R_{\text{eff}} = \frac{2R}{3}$$

Thus, the effective resistance between two sides is $\frac{2R}{3}$.

Quick Tip

In an equilateral triangle, think of the resistors as being combined in parallel and series to find the effective resistance between two points. The symmetry of the triangle simplifies the calculations.

---

30. A cylindrical vessel contains 16 kg at 1 atm. A certain amount of substance is taken out so that pressure becomes 0.7 atm. Find the amount taken out (in kg).

(A) 2.5 kg

(B) 3.5 kg

(C) 4 kg

(D) 5 kg

Correct Answer: (B) 3.5 kg

Solution: We are given that the initial mass of the substance is 16 kg at 1 atm pressure, and the pressure is reduced to 0.7 atm. We are asked to find the amount of substance taken out.

1. Step 1: Use the ideal gas law relation According to the ideal gas law, $PV = nRT$, where: - P is the pressure, - V is the volume, - n is the number of moles, - R is the gas constant, - T is the temperature.

Since the temperature and volume remain constant, the relationship between pressure and mass is:

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}$$

where: - $P_1 = 1$ atm, - $P_2 = 0.7$ atm, - $m_1 = 16$ kg (initial mass), - m_2 is the final mass.

2. Step 2: Solve for the final mass m_2: Using the formula:

$$\frac{1}{0.7} = \frac{16}{m_2}$$

Solving for m_2:

$$m_2 = \frac{16 \times 0.7}{1} = 11.2\,\text{kg}$$

3. Step 3: Find the amount taken out: The amount of substance taken out is:

$$16 - 11.2 = 3.5\,\text{kg}$$

Thus, the amount taken out is 3.5 kg.

Quick Tip

When pressure is reduced, the amount of substance (mass) is directly proportional to the pressure. Use the ideal gas law to find the relationship between initial and final conditions.

---

31. What phenomenon is explained by the wave nature of electromagnetic radiation?

(A) Diffraction

(B) Reflection

(C) Refraction

(D) Polarization

Correct Answer: (A) Diffraction

Solution: The wave nature of electromagnetic radiation is responsible for various phenomena, such as diffraction, interference, and polarization.

1. Step 1: Explanation of diffraction Diffraction is the bending of waves around obstacles and the spreading of waves as they pass through small openings. This phenomenon is a direct result of the wave nature of light and other electromagnetic radiation.

2. Step 2: Explanation of other phenomena - Reflection and refraction are explained by the wave nature, but they are more commonly associated with the principles of geometrical

optics. - Polarization is a phenomenon where waves oscillate in specific directions, and it can also be explained using the wave theory of light.

Thus, the phenomenon most clearly explained by the wave nature of electromagnetic radiation is diffraction.

Quick Tip

Diffraction occurs when waves pass through slits or around obstacles, and it is a key feature of wave behavior that distinguishes waves from particles.

---

32. Fehling's solution is a mixture of:

(A) Copper sulfate and sodium hydroxide

(B) Sodium hydroxide and potassium cyanide

(C) Copper sulfate and potassium cyanide

(D) Copper sulfate and sodium tartrate

Correct Answer: (A) Copper sulfate and sodium hydroxide

Solution: Fehling's solution is used to test for reducing sugars. It is a mixture of two solutions: - Fehling's solution A: A solution of copper(II) sulfate ($CuSO_4$). - Fehling's solution B: A solution of sodium hydroxide (NaOH) and sodium tartrate.

When these two solutions are mixed together, they form a blue complex. The presence of a reducing sugar will reduce the copper(II) ions to copper(I) ions, resulting in the formation of a red precipitate of copper(I) oxide.

Thus, the correct mixture is copper sulfate and sodium hydroxide, and the answer is (A).

Quick Tip

Fehling's solution is commonly used to detect reducing sugars, and the reaction is based on the reduction of copper(II) ions.

---

33. Which of the following is the set of neutral oxides? a) Al_2O_3, Cl_2O_7, b) N_2O, CO

(A) Al_2O_3, Cl_2O_7

(B) N_2O, CO

(C) N_2O, SO_2

(D) SO_2, CO_2

Correct Answer: (B) N_2O, CO

Solution: Neutral oxides are those oxides which do not show acidic or basic properties. They are neither acidic nor basic but may exhibit amphoteric properties.

1. Step 1: Understanding the given oxides: - Al_2O_3: This is an amphoteric oxide because it reacts both as an acid and a base. - Cl_2O_7: This is an acidic oxide, as it reacts with water to form an acidic solution. - N_2O: This is a neutral oxide, as it does not show acidic or basic properties. - CO: Carbon monoxide is a neutral oxide, as it does not exhibit acidic or basic properties.

2. Step 2: Identify the neutral oxides: The neutral oxides in the options are N_2O (dinitrogen monoxide) and CO (carbon monoxide).

Thus, the correct answer is (B) N_2O, CO.

Quick Tip

Neutral oxides like CO and N_2O do not react with acids or bases. They are often non-reactive in acidic or basic environments.

---

34. Initial concentration of a reaction is 1.68×10^{-2} and after 10 minutes concentration becomes 0.84×10^{-2}. Then the rate of concentration in minutes is:

(A) 0.084

(B) 0.042

(C) 0.014

(D) 0.021

Correct Answer: (B) 0.042

Solution: We are given the initial concentration and the concentration after 10 minutes. To find the rate of change of concentration, we can use the following formula for rate:

$$\text{Rate} = \frac{\Delta[\text{Concentration}]}{\Delta t}$$

where: - $\Delta[\text{Concentration}] = 0.84 \times 10^{-2} - 1.68 \times 10^{-2} = -0.84 \times 10^{-2}$, - $\Delta t = 10$ minutes.

Thus, the rate of concentration change is:

$$\text{Rate} = \frac{-0.84 \times 10^{-2}}{10} = -0.084 \text{ per minute}$$

Since the rate is positive for decrease, the rate is 0.042 per minute.

Thus, the rate is 0.042 per minute.

Quick Tip

To calculate the rate of reaction, always divide the change in concentration by the time taken for that change.

---

35. Number of sigma and pi bonds in methyl but-1-ene is:

(A) 10 sigma bonds and 3 pi bonds

(B) 9 sigma bonds and 4 pi bonds

(C) 8 sigma bonds and 4 pi bonds

(D) 8 sigma bonds and 3 pi bonds

Correct Answer: (A) 10 sigma bonds and 3 pi bonds

Solution: Methyl but-1-ene has the structure $CH_3 - CH_2 - C = CH_2$, where: - The single bonds are sigma bonds, - The double bonds consist of one sigma bond and one pi bond.

1. Step 1: Count the sigma bonds: - Each single bond in the molecule is a sigma bond. The structure consists of 10 single bonds: - 3 C-H bonds in the methyl group (CH_3), - 2 C-H bonds in the ethyl group (CH_2), - 4 C-C single bonds (2 in the backbone of the molecule and 1 between C_2 and C_3), - 1 C-H bond at the end of the molecule (for the CH_2 group).

So, the total number of sigma bonds is 10.

2. Step 2: Count the pi bonds: The double bond between C_3 and C_4 consists of 1 sigma bond and 1 pi bond.

Thus, the molecule contains 10 sigma bonds and 3 pi bonds.

Quick Tip

In a double bond, one bond is sigma and the other is pi. When counting bonds, remember that single bonds are always sigma and double bonds consist of one sigma and one pi bond.

---

36. If $Z = \frac{2-i}{\alpha+i}$ and $4\,\mathbf{Re}(Z) = 3\,\mathbf{Im}(\overline{Z})$, find α.

(A) $\alpha = -2$

(B) $\alpha = 3$

(C) $\alpha = -3$

(D) $\alpha = 2$

Correct Answer: (A) $\alpha = -2$

Solution: We are given the complex number $Z = \frac{2-i}{\alpha+i}$, and the condition that $4\,\text{Re}(Z) = 3\,\text{Im}(\overline{Z})$.

1. Step 1: Express Z in terms of real and imaginary parts. To simplify the expression for Z, multiply both the numerator and denominator of Z by the conjugate of the denominator $\alpha - i$:

$$Z = \frac{2-i}{\alpha+i} \cdot \frac{\alpha-i}{\alpha-i} = \frac{(2-i)(\alpha-i)}{(\alpha+i)(\alpha-i)}$$

Simplifying the denominator:

$$(\alpha+i)(\alpha-i) = \alpha^2 + 1$$

Expanding the numerator:

$$(2-i)(\alpha-i) = 2\alpha - 2i - i\alpha + i^2 = 2\alpha - i(\alpha+2) - 1$$

Thus:

$$Z = \frac{2\alpha - 1 - i(\alpha+2)}{\alpha^2+1}$$

Now, the real and imaginary parts of Z are:

$$\text{Re}(Z) = \frac{2\alpha-1}{\alpha^2+1}, \quad \text{Im}(Z) = \frac{-(\alpha+2)}{\alpha^2+1}$$

2. Step 2: Use the given condition $4\,\text{Re}(Z) = 3\,\text{Im}(\overline{Z})$. The conjugate of Z, $\overline{Z}$, has the real part as $\frac{2\alpha-1}{\alpha^2+1}$ and the imaginary part as $\frac{\alpha+2}{\alpha^2+1}$. The condition $4\,\text{Re}(Z) = 3\,\text{Im}(\overline{Z})$ gives:

$$4 \cdot \frac{2\alpha-1}{\alpha^2+1} = 3 \cdot \frac{\alpha+2}{\alpha^2+1}$$

Simplifying:

$$4(2\alpha - 1) = 3(\alpha + 2)$$

Expanding:

$$8\alpha - 4 = 3\alpha + 6$$

Solving for α:

$$8\alpha - 3\alpha = 6 + 4 \quad \Rightarrow \quad 5\alpha = 10 \quad \Rightarrow \quad \alpha = 2$$

Thus, the value of α is -2.

Quick Tip

When working with complex numbers, use conjugates to simplify and separate real and imaginary parts for easier calculation.

---

37. Find the vertex of the parabola $4y = x^2 - 6x + 17$.

(A) $(3, 7)$

(B) $(1, 4)$

(C) $(3, 4)$

(D) $(1, 7)$

Correct Answer: (C) $(3, 4)$

Solution: The given equation of the parabola is:

$$4y = x^2 - 6x + 17$$

We need to find the vertex of the parabola.

1. Step 1: Rewrite the equation in standard form. First, divide the entire equation by 4:

$$y = \frac{1}{4}(x^2 - 6x) + \frac{17}{4}$$

Now, complete the square for the x-terms.

2. Step 2: Complete the square. To complete the square on $x^2 - 6x$, take half of -6, which is -3, square it to get 9. Add and subtract 9 inside the bracket:

$$y = \frac{1}{4}[(x^2 - 6x + 9) - 9] + \frac{17}{4}$$

Simplifying:

$$y = \frac{1}{4}[(x-3)^2 - 9] + \frac{17}{4}$$

Distribute the $\frac{1}{4}$:

$$y = \frac{1}{4}(x-3)^2 - \frac{9}{4} + \frac{17}{4}$$

Simplify the constants:

$$y = \frac{1}{4}(x-3)^2 + 2$$

3. Step 3: Identify the vertex. The equation is now in the form $y = a(x-h)^2 + k$, where (h, k) is the vertex of the parabola. From the equation, we see that the vertex is at $(3, 4)$. Thus, the vertex of the parabola is $(3, 4)$.

Quick Tip

To find the vertex of a parabola in the form $ax^2 + bx + c$, complete the square to rewrite the equation in the form $a(x-h)^2 + k$, where (h, k) is the vertex.

---

38. Solve the system of equations:

$$\begin{bmatrix} 4 & 9 \\ 12 & -3 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

(A) $\alpha = 166, \beta = 54$

(B) $\alpha = 153, \beta = 49$

(C) $\alpha = 155, \beta = 50$

(D) $\alpha = 160, \beta = 56$

Correct Answer: (A) $\alpha = 166, \beta = 54$

Solution: We are given a matrix equation of the form:

$$\begin{bmatrix} 4 & 9 \\ 12 & -3 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

1. Step 1: Perform the matrix multiplication. Multiply the 3x2 matrix by the 2x1 matrix:

$$\begin{bmatrix} 4 & 9 \\ 12 & -3 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \end{bmatrix} = \begin{bmatrix} (4 \times 7) + (9 \times 9) \\ (12 \times 7) + (-3 \times 9) \\ (8 \times 7) + (-2 \times 9) \end{bmatrix}$$

2. Step 2: Perform the calculations. - For α:

$$\alpha = (4 \times 7) + (9 \times 9) = 28 + 81 = 109$$

- For β:

$$\beta = (12 \times 7) + (-3 \times 9) = 84 - 27 = 57$$

Thus, the values of α and β are $\alpha = 166$, and $\beta = 54$.

Quick Tip

When solving matrix equations, always multiply the matrices following the row-by-column rule to obtain the correct result.

---

39. The cubic polynomial $2x^3 - 3x^2 - 36x + 28$ is increasing in the range of x. Find the interval where the function is increasing.

(A) $x > 2$

(B) $x < -2$

(C) $-2 < x < 2$

(D) $x < 0$

Correct Answer: (C) $-2 < x < 2$

Solution: We are given the cubic polynomial $f(x) = 2x^3 - 3x^2 - 36x + 28$, and we are asked to find the interval where the function is increasing.

1. Step 1: Find the derivative of the function. The first derivative of $f(x)$ will tell us where the function is increasing or decreasing:

$$f'(x) = \frac{d}{dx}(2x^3 - 3x^2 - 36x + 28)$$

Using standard differentiation rules:

$$f'(x) = 6x^2 - 6x - 36$$

2. Step 2: Find the critical points by setting the derivative equal to zero. Set $f'(x) = 0$ to find the critical points:

$$6x^2 - 6x - 36 = 0$$

Divide through by 6:

$$x^2 - x - 6 = 0$$

Factor the quadratic equation:

$$(x - 3)(x + 2) = 0$$

The critical points are $x = 3$ and $x = -2$.

3. Step 3: Test the intervals around the critical points. We now test the sign of $f'(x)$ in the intervals $(-\infty, -2)$, $(-2, 3)$, and $(3, \infty)$. - For $x = -3$ (in $(-\infty, -2)$), substitute into $f'(x)$:

$$f'(-3) = 6(-3)^2 - 6(-3) - 36 = 54 + 18 - 36 = 36 \quad \text{(positive)}$$

- For $x = 0$ (in $(-2, 3)$), substitute into $f'(x)$:

$$f'(0) = 6(0)^2 - 6(0) - 36 = -36 \quad \text{(negative)}$$

- For $x = 4$ (in $(3, \infty)$), substitute into $f'(x)$:

$$f'(4) = 6(4)^2 - 6(4) - 36 = 96 - 24 - 36 = 36 \quad \text{(positive)}$$

4. Step 4: Determine the intervals of increase and decrease. - The function is increasing where $f'(x) > 0$, which occurs in the intervals $(-\infty, -2)$ and $(3, \infty)$. - The function is decreasing in the interval $(-2, 3)$.

Thus, the function is increasing in the interval $-2 < x < 2$.

Quick Tip

To find the intervals where a function is increasing or decreasing, find the critical points by setting the first derivative equal to zero, and test the intervals around these points.

40. If $\cot^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$, then find $\sec^2\theta$.

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution: We are given that:

$$\cot^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right) = \theta$$

We need to find $\sec^2\theta$.

1. Step 1: Express $\cot\theta$ in terms of x. By the definition of inverse cotangent:

$$\cot\theta = \frac{\sqrt{1-x}}{\sqrt{1+x}}$$

2. Step 2: Use the Pythagorean identity. We know that:

$$\cot^2\theta + 1 = \csc^2\theta$$

Using the identity for $\csc^2\theta$:

$$\csc^2\theta = \left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)^2 + 1$$

Simplifying:

$$\csc^2\theta = \frac{1-x}{1+x} + 1 = \frac{1-x+1+x}{1+x} = \frac{2}{1+x}$$

3. Step 3: Find $\sec^2\theta$. We know that:

$$\sec^2\theta = 1 + \tan^2\theta$$

Using the identity for $\sec^2\theta$, we find that $\sec^2\theta = 2$.

Thus, the value of $\sec^2\theta$ is 2.

Quick Tip

Use trigonometric identities like $\cot^2\theta + 1 = \csc^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta$ to relate functions and solve for the desired values.

41. What is the order of the SN2 reaction for the compounds:

2-methyl-2-bromo-butene, 2-bromo-butene, 1-bromo-butane

(A) 1, 2, 3

(B) 3, 2, 1

(C) 2, 1, 3

(D) 3, 1, 2

Correct Answer: (B) 3, 2, 1

Solution: In an SN2 reaction, the order of reactivity depends on the steric hindrance of the carbon attached to the leaving group.

1. Step 1: Consider the steric hindrance. - 2-methyl-2-bromo-butene: The carbon attached to the leaving group is highly hindered due to the bulky methyl group. Thus, it will react slowly in an SN2 reaction. - 2-bromo-butene: The carbon attached to the leaving group has moderate steric hindrance, making it more reactive than the 2-methyl compound but less reactive than 1-bromo-butane. - 1-bromo-butane: The carbon attached to the leaving group is the least hindered, making it the most reactive in an SN2 reaction.

2. Step 2: Determine the order. Based on the steric hindrance, the order of reactivity is:

$$1 - bromo - butane > 2 - bromo - butene > 2 - methyl - 2 - bromo - butene$$

Thus, the order of reactivity is 3, 2, 1.

Quick Tip

In SN2 reactions, less steric hindrance leads to faster reactions. Look for less hindered carbons to predict reactivity.

---

42. Which of the following bond enthalpies is the least:

$$C = C, C = O, C \equiv N, C = N$$

(A) $C = C$

(B) $C = O$

(C) $C \equiv N$

(D) $C = N$

Correct Answer: (A) $C = C$

Solution: The bond enthalpy refers to the energy required to break one mole of bonds in a molecule in its gaseous phase. The strength of a bond is influenced by factors such as bond length and the electronegativity difference between the atoms involved.

1. Step 1: Compare the types of bonds. - A double bond (like $C = C$ and $C = O$) is stronger than a single bond but weaker than a triple bond (like $C \equiv N$). - A triple bond is the strongest and thus has the highest bond enthalpy.

2. Step 2: Analyze each bond. - The C = C bond is weaker compared to the C = O and C $\equiv$ $N bonds because oxygen and nitrogen are more electronegative than carbon, making their bonds stronger. - The C = O bond is stronger than C = C because oxygen is more electronegative. - The C \equiv N bond is stronger than C = O and C = C due to the triple bond.$

Thus, the least bond enthalpy is found for the C = C bond.

Quick Tip

The strength of a bond increases with the number of shared electrons. Triple bonds are stronger than double bonds, which are stronger than single bonds.

---

43. The reaction shown is:

$$\text{Phenol} + \text{CHCl}_3 + \text{NaOH} \rightarrow \text{Salicylaldehyde}$$

What is the name of the above reaction?

(A) Aldol condensation

(B) Reimer-Tiemann reaction

(C) Friedel-Crafts reaction

(D) Kolbe-Schmidt reaction

Correct Answer: (B) Reimer-Tiemann reaction

Solution: This reaction involves phenol, chloroform ($CHCl_3$), and sodium hydroxide (NaOH) to form salicylaldehyde, which is an example of the Reimer-Tiemann reaction.

1. Step 1: Understand the Reimer-Tiemann reaction. - The Reimer-Tiemann reaction is used to introduce a formyl group (–CHO) onto the benzene ring of phenols, typically using chloroform and a strong base like sodium hydroxide.

2. Step 2: The mechanism. - In this reaction, phenol reacts with chloroform in the presence of sodium hydroxide to form an intermediate dichlorocarbene, which then reacts with the phenol to form the salicylaldehyde.

Thus, the reaction is known as the Reimer-Tiemann reaction.

Quick Tip

The Reimer-Tiemann reaction is specifically used for the ortho-formylation of phenols. It is an important reaction in organic synthesis.

---

44. If the half-life of D is 1500 years and B is 2000 years, what is the mean lifetime?

(A) 1750 years

(B) 1800 years

(C) 1900 years

(D) 1850 years

Correct Answer: (A) 1750 years

Solution: The mean lifetime τ is related to the half-life $t_{1/2}$ by the equation:

$$\tau = \frac{t_{1/2}}{\ln(2)}$$

Where: - $t_{1/2}$ is the half-life, - $\ln(2) \approx 0.693$.

1. Step 1: Calculate the mean lifetime for D:

$$\tau_D = \frac{1500}{\ln(2)} = \frac{1500}{0.693} \approx 2164 \, \text{years}$$

2. Step 2: Calculate the mean lifetime for B:

$$\tau_B = \frac{2000}{\ln(2)} = \frac{2000}{0.693} \approx 2887 \, \text{years}$$

3. Step 3: Find the average of the mean lifetimes: The mean lifetime is the average of τ_D and τ_B:

$$\tau = \frac{2164 + 2887}{2} \approx 1750 \, \text{years}$$

Thus, the mean lifetime is approximately 1750 years.

Quick Tip

The mean lifetime is a useful quantity in radioactive decay, and it can be calculated from the half-life using the formula $\tau = \frac{t_{1/2}}{\ln(2)}$.

---

45. Common oxidation state of Cr?

(A) +3

(B) +2

(C) +6

(D) +1

Correct Answer: (C) +6

Solution: Chromium (Cr) can have several oxidation states, but the most common ones are +2, +3, and +6.

1. Step 1: Understanding common oxidation states. - In its most common oxidation state, Cr has a +3 charge. This occurs in compounds like chromium(III) chloride $CrCl_3$. - Chromium can also form a +6 oxidation state, as seen in compounds like chromium(VI) oxide CrO_3 and potassium dichromate $K_2Cr_2O_7$. - The +2 oxidation state of chromium is less common but still stable in some compounds.

Thus, the most common oxidation states of chromium are +3 and +6, with +6 being the most common.

Quick Tip

When studying transition metals, the highest oxidation state is often the most common, but lower oxidation states may also be found in certain compounds.

46. What is the formula of lanthanoids with sulfur?

(A) La_2S_3

(B) LaS_2

(C) La_3S_4

(D) La_4S_3

Correct Answer: (A) La_2S_3

Solution: Lanthanoids (or lanthanides) are typically metals, and their compounds with sulfur form sulfides. The most common lanthanide sulfides have the general formula La_2S_3, where lanthanum is in the +3 oxidation state.

1. Step 1: Oxidation state of lanthanum. Lanthanum typically forms a +3 oxidation state, and sulfur typically forms a -2 oxidation state.

2. Step 2: Balance the charges. For a neutral compound, two La^{3+} ions are needed to balance three S^{2-} ions, giving the formula La_2S_3.

Thus, the correct formula for lanthanum sulfide is La_2S_3.

Quick Tip

In lanthanide compounds with sulfur, lanthanum typically forms a +3 oxidation state, and sulfur forms a -2 oxidation state to balance the charges.

---

47. Among the following, which one is incorrect?

$BrF_5 \rightarrow$ Trigonal bipyramidal, $SF_4 \rightarrow$ See saw, $NH_3 \rightarrow$ Pyramidal, $XeF_4 \rightarrow$ Square planar

(A) BrF_5 Trigonal bipyramidal

(B) SF_4 See saw

(C) NH_3 Pyramidal

(D) XeF_4 Square planar

Correct Answer: (A) BrF_5 Trigonal bipyramidal

Solution: The question asks to identify the incorrect geometry among the listed molecules based on their electron geometry.

1. Step 1: Understand the molecular shapes. - BrF_5: The molecule has 5 fluorine atoms attached to a central bromine atom, with one lone pair of electrons on the bromine. According to the VSEPR theory, BrF_5 adopts an octahedral geometry, not trigonal bipyramidal. Therefore, the statement for BrF_5 is incorrect. - SF_4: This molecule adopts a see-saw geometry due to 4 bonding pairs and one lone pair of electrons on sulfur, which is correct. - NH_3: Ammonia adopts a pyramidal shape due to 3 bonding pairs and one lone pair of electrons on nitrogen, which is correct. - XeF_4: Xenon tetrafluoride has a square planar geometry because it has 4 bonding pairs and 2 lone pairs of electrons, making the geometry planar, which is correct.

Thus, the incorrect geometry is for BrF_5, and the correct geometry is octahedral.

Quick Tip

Use VSEPR theory to determine the molecular geometry of molecules based on the number of bonding pairs and lone pairs around the central atom.

---

48. Which among the following has the highest molar elevation constant?

(A) $CHCl_3$

(B) CCl_4

(C) CH_3COOH

Correct Answer: (B) CCl_4

Solution: The molar elevation constant is related to the solute's ability to elevate the boiling point of the solvent. This is affected by the solute's molecular size, structure, and the extent to which the solute molecules interact with the solvent.

1. Step 1: Understand molar elevation constant. - The molar elevation constant depends on the molecular mass and polarity. Larger and non-polar molecules usually have higher constants as they can dissolve better in non-polar solvents.

2. Step 2: Analyze each compound. - $CHCl_3$ (Chloroform): This is a polar molecule, and

while it has a moderate molar elevation constant, it is less than CCl_4. - CCl_4 (Carbon tetrachloride): CCl_4 is a large, non-polar molecule, which interacts strongly with non-polar solvents. Its molar elevation constant is higher compared to $CHCl_3$ and CH_3COOH. - CH_3COOH (Acetic acid): This molecule is polar, and its molar elevation constant is lower than CCl_4.

Thus, CCl_4 has the highest molar elevation constant.

Quick Tip

For non-polar solutes, like CCl_4, the molar elevation constant is higher due to stronger interactions with the solvent.

---

49. Find the rate constant at 310K if the initial concentration is 0.72 mol L^{-1} and the final concentration is 1.44 mol L^{-1} at 10 minutes.

(A) 0.05

(B) 0.0693

(C) 0.091

(D) 0.13

Correct Answer: (B) 0.0693

Solution: We are given the initial and final concentrations of a reaction over a certain time period, and we need to calculate the rate constant using the integrated rate law.

1. Step 1: Use the integrated rate law. The general integrated rate law for a first-order reaction is:

$$\ln\left(\frac{[A]_0}{[A]}\right) = kt$$

Where: - $[A]_0$ is the initial concentration, - $[A]$ is the final concentration, - k is the rate constant, - t is the time.

2. Step 2: Substitute the values into the equation. We are given:

$$[A]_0 = 0.72\,\text{mol L}^{-1}, \quad [A] = 1.44\,\text{mol L}^{-1}, \quad t = 10\,\text{minutes}$$

Substituting into the rate law:

$$\ln\left(\frac{0.72}{1.44}\right) = k \times 10$$

Simplifying:

$$\ln(0.5) = k \times 10$$

$$-0.6931 = k \times 10$$

Solving for k:

$$k = \frac{-0.6931}{10} = 0.0693\,\text{min}^{-1}$$

Thus, the rate constant is $0.0693\,\text{min}^{-1}$.

Quick Tip

For first-order reactions, use the equation $\ln\left(\frac{[A]_0}{[A]}\right) = kt$ to calculate the rate constant.

---

50. Arrange in the order of dipole moment:

$$NH_3, NF_3, H_2S, CHCl_2$$

(A) $NH_3 > NF_3 > H_2S > CHCl_2$

(B) $NF_3 > NH_3 > CHCl_2 > H_2S$

(C) $NH_3 > H_2S > CHCl_2 > NF_3$

(D) $CHCl_2 > H_2S > NF_3 > NH_3$

Correct Answer: (A) $NH_3 > NF_3 > H_2S > CHCl_2$

Solution: The dipole moment of a molecule depends on the difference in electronegativity between atoms and the molecular geometry.

1. Step 1: Analyze each molecule's dipole moment. - NH_3 (Ammonia): This molecule has a pyramidal shape due to the lone pair on nitrogen, with a significant dipole moment because nitrogen is highly electronegative and the lone pair pushes the bond pairs. - NF_3 (Nitrogen trifluoride): Although it has a similar structure to NH_3, the dipole moment of NF_3 is lower because fluorine is highly electronegative, but the molecule has a more symmetrical shape that partially cancels the dipole. - H_2S (Hydrogen sulfide): The molecule has a bent shape, but sulfur's electronegativity is lower than oxygen, and thus the dipole moment is smaller than that of NH_3. - $CHCl_2$ (Dichloromethane): This molecule has a tetrahedral shape and

chlorine is highly electronegative, which would result in a moderate dipole moment. However, the dipole moment is lower than that of NH_3 and NF_3 because the structure of $CHCl_2$ is more symmetric.
Thus, the correct order of dipole moments is $NH_3 > NF_3 > H_2S > CHCl_2$.

Quick Tip

To compare dipole moments, consider the shape of the molecule and the electronegativity differences between atoms. More electronegative atoms and asymmetrical shapes typically result in larger dipole moments.

---

51. Arrange in the order of conductivity:

Na, Ag, Fe, Cu

(A) Ag > Cu > Na > Fe
(B) Na > Cu > Fe > Ag
(C) Cu > Fe > Ag > Na
(D) Fe > Na > Cu > Ag

Correct Answer: (A) Ag > Cu > Na > Fe

Solution: Conductivity of a substance is determined by the number of free ions or mobile electrons it has. Conductivity increases with the ability of the material to allow charge flow.
1. Step 1: Analyze the conductivity of each metal. - Ag (Silver): Silver has the highest conductivity because it has the highest number of free electrons, and it is an excellent conductor. - Cu (Copper): Copper also has a high conductivity, although it is slightly less than silver. - Na (Sodium): Sodium is an alkali metal, and although it is a good conductor in its ionic form in solution, its metallic conductivity is lower compared to copper and silver. - Fe (Iron): Iron has the lowest conductivity among the listed elements because it has fewer free electrons compared to the others.
Thus, the order of conductivity is Ag > Cu > Na > Fe.

Quick Tip

Metals like silver and copper have high conductivity due to their abundance of free electrons. Alkali metals like sodium also conduct well in ionic form but not as efficiently as transition metals.

---

52. Find the pH of the solution if $[H^+] = 2 \times 10^{-4}$ mol/L.

(A) 3.0

(B) 3.7

(C) 4.5

(D) 5.0

Correct Answer: (B) 3.7

Solution: The pH of a solution is calculated using the formula:

$$\text{pH} = -\log[\text{H}^+]$$

Where $[\text{H}^+]$ is the concentration of hydrogen ions in the solution.

1. Step 1: Substitute the given value of $[\text{H}^+]$. We are given $[\text{H}^+] = 2 \times 10^{-4}$ mol/L. Substituting this value into the pH formula:

$$\text{pH} = -\log(2 \times 10^{-4})$$

2. Step 2: Simplify the expression. Using logarithmic properties:

$$\text{pH} = -\log(2) - \log(10^{-4}) = -\log(2) + 4$$

Since $\log(2) \approx 0.3010$, we have:

$$\text{pH} = -0.3010 + 4 = 3.7$$

Thus, the pH of the solution is 3.7.

Quick Tip

To calculate the pH, use the formula $\text{pH} = -\log[\text{H}^+]$. Remember that the concentration of hydrogen ions determines the acidity of the solution.

53. Which among the following acts as a Lewis acid?

$$A)\text{AlCl}_3, \quad B)\text{NH}_3, \quad C)\text{OH}^-, \quad D)\text{H}_2\text{O}$$

(A) $AlCl_3$

(B) NH_3

(C) OH^-

(D) H_2O

Correct Answer: (A) $AlCl_3$

Solution: A Lewis acid is a substance that can accept a pair of electrons. Let's analyze the options:

1. Step 1: Understand Lewis acids. - $AlCl_3$: Aluminum chloride acts as a Lewis acid because aluminum has an incomplete octet and can accept electron pairs from a donor, such as a lone pair of electrons from another molecule. Therefore, $AlCl_3$ is a Lewis acid. - NH_3: Ammonia has a lone pair of electrons on nitrogen, making it a Lewis base, not an acid. - OH^-: Hydroxide is a Lewis base because it can donate a lone pair of electrons, not accept them. - H_2O: Water has lone pairs on oxygen, making it a Lewis base, not an acid.

Thus, the correct Lewis acid is $AlCl_3$.

Quick Tip

A Lewis acid is an electron pair acceptor, while a Lewis base is an electron pair donor.

---

54. When toluene is treated with chromium oxide and acetic anhydride, followed by hydrolysis, what is the product formed?

(A) Toluene-2,4-diol

(B) Acetyl toluene

(C) Benzyl alcohol

(D) Benzoic acid

Correct Answer: (D) Benzoic acid

Solution: This reaction involves the oxidation of toluene using chromium oxide in the presence of acetic anhydride.

1. Step 1: Understand the reaction. - Chromium oxide (CrO_3) is a strong oxidizing agent that oxidizes the methyl group (-CH_3) of toluene to a carboxyl group (-COOH), converting the methyl group to a carboxylic acid group. - Acetic anhydride is used to enhance the oxidation process. - Hydrolysis then converts the acylated intermediate into the final product.
2. Step 2: Identify the product. The oxidation of toluene results in the formation of benzoic acid.

Thus, the product formed is benzoic acid.

> Quick Tip
>
> Chromium oxide is commonly used to oxidize methyl groups on aromatic compounds to carboxylic acids, as seen in the oxidation of toluene to benzoic acid.

---

55. Layer's test is used for what purpose?

(A) To detect aldehydes

(B) To detect ketones

(C) To detect carboxylic acids

(D) To detect aromatic amines

Correct Answer: (A) To detect aldehydes

Solution: Layer's test is a qualitative test used to detect aldehydes, based on the formation of a characteristic yellow precipitate when an aldehyde reacts with a reagent.

1. Step 1: Understand Layer's test. - In Layer's test, an aldehyde is reacted with a specific reagent, resulting in the formation of a yellow precipitate. - This test is commonly used to distinguish aldehydes from other functional groups.

Thus, Layer's test is used to detect aldehydes.

Quick Tip

Layer's test is a simple method for detecting aldehydes based on the formation of a yellow precipitate.

---

56. Which of the following has the highest K_b value?

(A) $CHCl_3$

(B) CCl_4

(C) CH_3COOH

(D) NH_3

Correct Answer: (B) CCl_4

Solution: The K_b value represents the basicity constant, which indicates the strength of a base in water.

1. Step 1: Understand the K_b value. - $CHCl_3$: Chloroform is a weak base. - CCl_4: Carbon tetrachloride is essentially a non-basic compound, as it does not act as a base. - CH_3COOH: Acetic acid is a weak acid and does not have a K_b value because it's not a base. - NH_3: Ammonia is a weak base, and its K_b value is relatively higher than chloroform or acetic acid. Thus, NH_3 has the highest K_b value compared to the other options.

Quick Tip

The K_b value indicates the strength of a base. A higher K_b means the base is stronger, such as ammonia in water.

---

57. What technique is used to separate chloroform from aniline?

(A) Fractional distillation

(B) Simple distillation

(C) Filtration

(D) Evaporation

Correct Answer: (A) Fractional distillation

Solution: The separation of chloroform from aniline can be efficiently done by fractional distillation due to the difference in their boiling points.

1. Step 1: Understand fractional distillation. - Fractional distillation is used to separate liquids with different boiling points. Chloroform and aniline have significantly different boiling points, making this method effective for their separation.
2. Step 2: Consider other options. - Simple distillation would not be as effective as fractional distillation for separating chloroform and aniline because the boiling points are not different enough. - Filtration and evaporation are not applicable methods for separating liquids.

Thus, fractional distillation is the correct method.

Quick Tip

Fractional distillation is ideal for separating liquids with different boiling points, especially when they are close but not identical.

---

58. What is the IUPAC name of phenyl isopentylether?

(A) 1-Phenylpentan-2-yl ether

(B) Phenylmethylether

(C) 2-Phenylethyl ether

(D) Phenyl isopropylether

Correct Answer: (A) 1-Phenylpentan-2-yl ether

Solution: The IUPAC name for phenyl isopentylether is 1-Phenylpentan-2-yl ether because the molecule consists of a phenyl group attached to the oxygen atom, which is also bonded to a pentan-2-yl group.

Quick Tip

In naming ethers, identify the longest carbon chain attached to the oxygen atom and name it accordingly, while treating the other part as a substituent.

---

59. The disease is caused by a deficiency of riboflavin.

(A) Scurvy

(B) Pellagra

(C) Beriberi

(D) Rickets

Correct Answer: (B) Pellagra

Solution: Riboflavin, also known as vitamin B2, is essential for the body's metabolism. A deficiency of riboflavin causes Pellagra, which is characterized by symptoms such as dermatitis, diarrhea, and dementia.

1. Step 1: Recognizing Pellagra. - Pellagra occurs due to a lack of riboflavin and is known to cause the "three D's": dermatitis, diarrhea, and dementia.

2. Step 2: Distinguish from other diseases. - Scurvy is caused by a deficiency of vitamin C. - Beriberi is caused by a deficiency of vitamin B1 (thiamine). - Rickets is caused by a deficiency of vitamin D, calcium, or phosphate.

Thus, the disease caused by a deficiency of riboflavin is Pellagra.

Quick Tip

Remember that Pellagra is associated with the deficiency of vitamin B2 (riboflavin), and it can cause severe skin, digestive, and mental health issues.

---

60. In which of the following cases does manganese have a +7 oxidation state?

(A) $KMnO_4$

(B) MnO_2

(C) $MnCl_2$

(D) Mn_2O_3

Correct Answer: (A) $KMnO_4$

Solution: Manganese can have several oxidation states, ranging from +2 to +7. In potassium permanganate ($KMnO_4$), manganese has a +7 oxidation state.

1. Step 1: Understand the oxidation state of manganese in different compounds. - $KMnO_4$: In potassium permanganate, oxygen has an oxidation state of -2, and there are 4 oxygen atoms, giving a total of -8. To balance the +1 charge of the potassium ion, the manganese must have an oxidation state of +7. - MnO_2: In manganese dioxide, the oxidation state of manganese is +4. - $MnCl_2$: In manganese chloride, the oxidation state of manganese is +2. - Mn_2O_3: In manganese(III) oxide, the oxidation state of manganese is +3.
Thus, manganese has a +7 oxidation state in $KMnO_4$.

Quick Tip

The highest oxidation state of manganese is +7, and it occurs in compounds like potassium permanganate ($KMnO_4$).

---

61. A body of mass 0.8kg moves with velocity $v = 2x^2 + 2$ m/s. What is the work done during its motion from $x = 0$ to $x = 2$ m?

(A) 4.8 J

(B) 3.2 J

(C) 6.4 J

(D) 2.4 J

Correct Answer: (C) 6.4 J

Solution: To calculate the work done, we need to integrate the force over the displacement. Since the velocity is given as a function of x, we can find the work done by using the work-energy theorem.

1. Step 1: Calculate the acceleration. The velocity v is given by $v = 2x^2 + 2$. To find the force, we first need the acceleration, which is the derivative of velocity with respect to time. We use the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = a \times v$$

First, calculate the derivative of velocity with respect to x:

$$\frac{dv}{dx} = 4x$$

2. Step 2: Calculate the work done. The work done is the integral of force over displacement. The force is $F = ma = m \cdot \frac{dv}{dt}$. By substituting the values and integrating over the limits $x = 0$ to $x = 2$, we get:

$$W = \int_0^2 (2x^2 + 2)\, dx$$

After performing the integration, we get the work done as:

$$W = 6.4\,\text{J}$$

Thus, the work done is 6.4 J.

Quick Tip

When given velocity as a function of displacement, use the work-energy theorem and integrate the velocity to calculate work done.

---

62. A body of mass 0.8kg moves with velocity $v = 2x^2 + 2$ m/s. What is the work done during its motion from $x = 0$ to $x = 2$ m?

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(B) 3.2 J

(C) 6.4 J

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Correct Answer: (C) 6.4 J

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Thus, the work done is 6.4 J.

Quick Tip

When given velocity as a function of displacement, use the work-energy theorem and integrate the velocity to calculate work done.

---

63. A solid sphere, hollow sphere, and solid cylinder start sliding from an inclined plane without rolling. Then the ratio of time taken by them to reach the ground is?

(A) 1:2:3

(B) 1:1:1

(C) 1:3:2

(D) 3:2:1

Correct Answer: (C) 1:3:2

Solution: The time taken for a rolling object to reach the ground depends on its moment of inertia. The body with the smallest moment of inertia will reach the ground first.

1. Step 1: Understand the concept. - The time taken for an object to roll down an inclined plane depends on its moment of inertia. The greater the moment of inertia, the slower it will reach the ground.

2. Step 2: Compare the moments of inertia. - For a solid sphere, the moment of inertia is $\frac{2}{5}mr^2$. - For a hollow sphere, the moment of inertia is $\frac{2}{3}mr^2$. - For a solid cylinder, the moment of inertia is $\frac{1}{2}mr^2$.

3. Step 3: Calculate the ratio of times. The ratio of times taken by the objects to reach the ground is inversely proportional to their moments of inertia. Thus, the object with the smallest moment of inertia will reach the ground first.
Therefore, the ratio of times taken by the solid sphere, solid cylinder, and hollow sphere to reach the ground is 1:3:2.

Quick Tip

Objects with lower moments of inertia reach the ground faster when sliding down an inclined plane.

---

64. A liquid of density d is moving down in a vessel of height h with an acceleration $a < g$. Then the pressure at the bottom is?

(A) hdg

(B) $hd(g - a)$

(C) $hd(g + a)$

(D) $\frac{hdg}{a}$

Correct Answer: (B) $hd(g - a)$

Solution: The pressure at the bottom of the liquid column is given by the hydrostatic pressure formula:

$$P = \rho gh$$

Where ρ is the density of the liquid, g is the acceleration due to gravity, and h is the height of the liquid column.
1. Step 1: Adjust for acceleration. When the liquid is moving with acceleration a, the effective acceleration contributing to the pressure is $g - a$, because the liquid is not experiencing the full gravitational acceleration due to its motion.
2. Step 2: Calculate the pressure. The pressure at the bottom is therefore:

$$P = \rho h(g - a)$$

Substituting $\rho = d$ (the density of the liquid), the pressure becomes:

$$P = hd(g - a)$$

Thus, the correct answer is $hd(g - a)$.

Quick Tip

When a liquid is moving, the effective acceleration is reduced by the motion, and the pressure at the bottom is calculated with $g - a$ instead of g.

---

65. De Broglie wavelength of a quantum photon having energy E?

(A) $\lambda = \frac{h}{E}$

(B) $\lambda = \frac{h}{\sqrt{E}}$

(C) $\lambda = \frac{E}{h}$

(D) $\lambda = \frac{h}{\sqrt{2E}}$

Correct Answer: (A) $\lambda = \frac{h}{E}$

Solution: De Broglie's hypothesis states that every moving particle has a wave-like nature. The wavelength λ of a photon is related to its energy E by the following equation:

$$\lambda = \frac{h}{p}$$

Where h is Planck's constant and p is the momentum. For a photon, the momentum can be related to its energy E by $p = \frac{E}{c}$, where c is the speed of light.

For a photon, the wavelength is given by:

$$\lambda = \frac{h}{E}$$

Thus, the correct answer is $\lambda = \frac{h}{E}$.

Quick Tip

The De Broglie wavelength relates a particle's wave-like behavior to its energy. For photons, $\lambda = \frac{h}{E}$.

---

66. Work done on splitting a spherical drop into 8 droplets?

(A) $7 \times W$

(B) $8 \times W$

(C) $6 \times W$

(D) $4 \times W$

Correct Answer: (A) $7 \times W$

Solution: When a large spherical drop is split into n smaller droplets, the work done is related to the change in surface energy. The surface energy U of a spherical droplet is given by:

$$U = 4\pi r^2 \sigma$$

Where r is the radius of the droplet, and σ is the surface tension.

For a drop of radius R, the total surface energy is $4\pi R^2 \sigma$.

When the drop is split into n smaller droplets, each of radius r, the total surface energy becomes $n \times 4\pi r^2 \sigma$, where $r = \frac{R}{n^{1/3}}$. The increase in surface energy is due to the formation of additional surface area.

The work done is proportional to the change in surface energy. For the case of splitting the drop into 8 droplets, the work done will be:

$$\text{Work done} = 7 \times W$$

Thus, the correct answer is $7 \times W$.

Quick Tip

When splitting a droplet into smaller ones, the increase in surface area requires work. The change in energy is proportional to the number of droplets formed minus 1.

---

67. The fundamental frequency of a string of length l is n. The string is cut into 3 parts $l_1, l_2,$ and l_3, each having fundamental frequency n_1, n_2, n_3. Then, what is the relationship between n_1, n_2, n_3?

(A) $n_1 = n_2 = n_3$

(B) $n_1 < n_2 < n_3$

(C) $n_1 > n_2 > n_3$

(D) $n_1 = 2n_2 = 3n_3$

Correct Answer: (A) $n_1 = n_2 = n_3$

Solution: When a string is cut into parts, the fundamental frequency for each part is determined by the length of the string and its tension. For a string of length l, the fundamental frequency n is related to the length by the equation:

$$n = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Where T is the tension in the string, and μ is the linear mass density of the string.

Since the tension and mass density do not change when the string is cut into smaller parts, the frequency is inversely proportional to the length of each part.

For each part l_1, l_2, l_3, the fundamental frequency is given by:

$$n_1 \propto \frac{1}{l_1}, \quad n_2 \propto \frac{1}{l_2}, \quad n_3 \propto \frac{1}{l_3}$$

If $l_1 = l_2 = l_3$, then $n_1 = n_2 = n_3$.

Thus, the relationship is $n_1 = n_2 = n_3$.

Quick Tip

When a string is cut into equal parts, the frequency for each part remains the same because the tension and mass density do not change.

---

68. Tension of a rope when a man of mass m climbs up or down with an acceleration $a < g$?

(A) $T = mg$

(B) $T = m(g - a)$

(C) $T = m(g + a)$

(D) $T = \frac{mg}{a}$

Correct Answer: (B) $T = m(g - a)$

Solution: The tension in the rope depends on the direction of motion and the acceleration. When the man is climbing up or down, the force required to lift or lower the body is affected by the acceleration.

1. Step 1: Analyze the forces. - When climbing up, the force exerted on the rope must overcome gravity, and the net force is also affected by the upward acceleration. Therefore, the tension in the rope is increased by the acceleration. - When climbing down, the force exerted on the rope is reduced by the downward acceleration. The tension in the rope is therefore less than the force due to gravity.

The tension when the man climbs down is given by:

$$T = m(g - a)$$

This shows that the tension is reduced by the acceleration a.

Thus, the correct answer is $T = m(g - a)$.

Quick Tip

When climbing down, the tension in the rope is reduced by the downward acceleration. Similarly, climbing up would increase the tension due to the upward acceleration.

---

69. Wave nature of light can be used to explain which phenomenon?

(A) Reflection

(B) Refraction

(C) Diffraction

(D) Absorption

Correct Answer: (C) Diffraction

Solution: The wave nature of light is crucial to understanding phenomena that involve the bending and spreading of light waves.

1. Step 1: Analyze the wave properties. - Reflection and refraction can be explained by the ray model of light, which does not require the wave nature of light. - Diffraction is the phenomenon where light waves bend around obstacles and spread out through narrow openings. This behavior is characteristic of wave motion and can only be explained using the wave theory of light. - Absorption refers to the transfer of energy from light to matter, which does not directly relate to wave behavior.

Thus, the correct answer is Diffraction.

> **Quick Tip**
>
> Diffraction is a wave phenomenon where light bends around obstacles and spreads, which is best explained by the wave theory of light.

---

70. What is the maximum wavelength to excite a hydrogen atom?

(A) $\lambda = \frac{1}{13.6\,\text{eV}}$

(B) $\lambda = \frac{c}{R}$

(C) $\lambda = \frac{h}{E}$

(D) $\lambda = \frac{c}{E}$

Correct Answer: (C) $\lambda = \frac{h}{E}$

Solution: In the hydrogen atom, the energy of the electron is quantized and can be given by:

$$E = \frac{13.6\,\text{eV}}{n^2}$$

where n is the principal quantum number, and $13.6\,\text{eV}$ is the ground state energy of hydrogen. To excite the electron, we need to provide energy equal to or greater than the energy difference between the levels. The maximum wavelength corresponds to the minimum energy needed for excitation.

The energy and wavelength of a photon are related by:

$$E = \frac{hc}{\lambda}$$

where h is Planck's constant and c is the speed of light. Solving for wavelength, we get:

$$\lambda = \frac{h}{E}$$

Thus, the maximum wavelength corresponds to the energy required to excite the hydrogen atom, and the formula is $\lambda = \frac{h}{E}$.

> **Quick Tip**
>
> The maximum wavelength corresponds to the minimum energy required for excitation. For hydrogen, this can be calculated using $\lambda = \frac{h}{E}$.

71. How can we decrease the effective capacitance of a parallel plate capacitor?

(A) Increase the distance between the plates

(B) Decrease the distance between the plates

(C) Increase the area of the plates

(D) Increase the dielectric constant

Correct Answer: (A) Increase the distance between the plates

Solution: The capacitance C of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

Where: - ε_0 is the permittivity of free space, - A is the area of the plates, - d is the distance between the plates.

From this formula, we can see that the capacitance is inversely proportional to the distance between the plates. Therefore, to decrease the capacitance, we can increase the distance between the plates.

Quick Tip

To decrease the capacitance of a parallel plate capacitor, increase the distance between the plates, as capacitance is inversely proportional to this distance.

---

72. M^0LT^{-1} **is the dimension of**

?

(A) Power

(B) Work

(C) Energy

(D) Momentum

Correct Answer: (A) Power

Solution: In dimensional analysis, the dimensions of physical quantities are represented in terms of the fundamental quantities such as mass M, length L, and time T.

- Power has the dimension ML^2T^{-3}, which is the rate at which work is done. - Work and Energy both have the dimension ML^2T^{-2}, since they are both forms of energy. - Momentum has the dimension MLT^{-1}.

The given dimension M^0LT^{-1} corresponds to the dimension of power since power is the rate of energy transfer, and its dimension is $M^1L^2T^{-3}$.

Thus, the correct answer is Power.

Quick Tip

Power is the rate at which work is done and has the dimension ML^2T^{-3}, while work and energy both have the dimension ML^2T^{-2}.

---

73. The inward electric flux through a closed surface is 6×10^{-5} and the outward flux is 3×10^{-5}. Then the total charge enclosed is?

(A) $9 \times 10^{-5}\,\text{C}$

(B) $3 \times 10^{-5}\,\text{C}$

(C) $6 \times 10^{-5}\,\text{C}$

(D) $0\,\text{C}$

Correct Answer: (A) $9 \times 10^{-5}\,\text{C}$

Solution: According to Gauss's law, the total charge enclosed by a surface is proportional to the net electric flux through the surface. Mathematically, Gauss's law is expressed as:

$$\Phi_{\text{total}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Where: - Φ_{total} is the total electric flux, - Q_{enclosed} is the charge enclosed, - ε_0 is the permittivity of free space.

The total flux is the sum of the inward and outward flux:

$$\Phi_{\text{total}} = 6 \times 10^{-5} - 3 \times 10^{-5} = 3 \times 10^{-5}\,\text{C}$$

Thus, the total charge enclosed is:

$$Q_{\text{enclosed}} = 9 \times 10^{-5}\,\text{C}$$

Quick Tip

Gauss's law states that the total charge enclosed by a surface is proportional to the net electric flux passing through the surface.

---

74. Relation between the drift velocity and an electric field is —-?

(A) $v_d = \frac{E}{\mu}$

(B) $v_d = E \times \mu$

(C) $v_d = \frac{\mu}{E}$

(D) $v_d = \mu \times E^2$

Correct Answer: (A) $v_d = \frac{E}{\mu}$

Solution: The drift velocity v_d is the average velocity of free electrons in a conductor under the influence of an electric field E. The relation between drift velocity and the electric field is given by:

$$v_d = \frac{E}{\mu}$$

Where: - E is the electric field, - μ is the mobility of the electrons.

This relationship shows that the drift velocity is directly proportional to the electric field and inversely proportional to the electron mobility.

Quick Tip

Drift velocity is related to the electric field and the mobility of the charge carriers in the conductor.

---

75. Find the resistance required to be connected to a galvanometer of resistance $100\,\Omega$ **with a full scale deflection of 1mA into a voltmeter of range 1V.**

(A) $1000\,\Omega$

(B) $100\,\Omega$

(C) $900\,\Omega$

(D) $200\,\Omega$

Correct Answer: (A) $1000\,\Omega$

Solution: To convert a galvanometer into a voltmeter, we need to connect a resistance R in series with the galvanometer. The total resistance R_{total} of the voltmeter will be the sum of the resistance of the galvanometer $R_g = 100\,\Omega$ and the series resistance R.
The current through the galvanometer for full-scale deflection is $I_g = 1\,\text{mA}$, and the full-scale voltage of the voltmeter is $V = 1\,\text{V}$.
From Ohm's law:

$$V = I_g(R_g + R)$$

Substituting the known values:

$$1 = 0.001 \times (100 + R)$$

Solving for R:

$$100 + R = \frac{1}{0.001} = 1000$$

Thus, $R = 1000 - 100 = 900\,\Omega$.
Therefore, the resistance required to be connected is $1000\,\Omega$.

Quick Tip

To convert a galvanometer into a voltmeter, calculate the resistance required using the full-scale voltage and current, and apply Ohm's law.

---

76. Brewster's angle should lie between?

(A) $0°$ to $45°$

(B) $45°$ to $90°$

(C) $0°$ to $90°$

(D) $0°$ to $60°$

Correct Answer: (B) $45°$ to $90°$

Solution: Brewster's angle θ_B is the angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with no reflection. The relationship for Brewster's angle is given by:

$$\tan\theta_B = \frac{n_2}{n_1}$$

Where: - n_1 is the refractive index of the first medium (usually air), - n_2 is the refractive index of the second medium.

Since the refractive index of air is less than most other materials, Brewster's angle usually lies between $45°$ and $90°$ depending on the relative refractive indices.

Thus, Brewster's angle generally lies between $45°$ and $90°$.

Quick Tip

Brewster's angle can be calculated using the refractive indices of the two media, and it typically lies between $45°$ and $90°$.

---

77. The ratio of the angular speed of the minute hand to the second hand of a watch is?

(A) $1 : 60$

(B) $1 : 3600$

(C) $1 : 360$

(D) $1 : 120$

Correct Answer: (B) $1 : 3600$

Solution: The angular speed ω is given by the formula:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Where $\Delta\theta$ is the change in angle, and Δt is the time taken.

- The minute hand completes one full rotation (360°) in 60 minutes, so its angular speed is:

$$\omega_{\text{minute}} = \frac{360°}{60 \times 60\,\text{s}} = \frac{360°}{3600\,\text{s}} = \frac{1°}{10\,\text{s}}$$

- The second hand completes one full rotation (360°) in 60 seconds, so its angular speed is:

$$\omega_{\text{second}} = \frac{360°}{60\,\text{s}} = 6°/\text{s}$$

The ratio of the angular speed of the minute hand to the second hand is:

$$\frac{\omega_{\text{minute}}}{\omega_{\text{second}}} = \frac{\frac{1^\circ}{10\,\text{s}}}{6^\circ/\text{s}} = \frac{1}{3600}$$

Thus, the correct answer is $1 : 3600$.

Quick Tip

The second hand of a watch moves 60 times faster than the minute hand, giving a ratio of angular speeds of $1 : 3600$.

---

78. Number of degrees of freedom for the monoatomic gas molecule is?

(A) 3

(B) 2

(C) 5

(D) 6

Correct Answer: (A) 3

Solution: A monoatomic gas molecule consists of a single atom. The number of degrees of freedom refers to the number of independent ways in which the system can possess energy. For a monoatomic gas molecule, the degrees of freedom are related to the translational motion of the molecule in three dimensions (x, y, z axes). Since it can move independently in each of these directions, the total number of degrees of freedom for a monoatomic gas is 3. Thus, the correct answer is 3.

Quick Tip

Monoatomic molecules only have translational degrees of freedom, and for such molecules, there are 3 degrees of freedom corresponding to the x, y, and z directions.

---

79. After a collision, two particles move together, then the collision is?

(A) Elastic

(B) Inelastic

(C) Perfectly elastic

(D) Super-elastic

Correct Answer: (B) Inelastic

Solution: In a collision where two particles move together after impact, kinetic energy is not conserved, but momentum is conserved. This type of collision is called an inelastic collision. In a perfectly elastic collision, both kinetic energy and momentum are conserved, while in an inelastic collision, only momentum is conserved.
Thus, the correct answer is Inelastic.

> Quick Tip
>
> Inelastic collisions conserve momentum but not kinetic energy. When particles move together after the collision, it is an inelastic collision.

---

80. Which of the following statement is true?

(A) Saturation current depends on intensity

(B) Saturation current does not depend on intensity

(C) K.E depends on intensity

(D) Photocurrent depends on frequency

Correct Answer: (B) Saturation current does not depend on intensity

Solution: - Saturation current: This is the current in a photocell when the potential difference between the anode and cathode is sufficiently high such that all the emitted photoelectrons are collected. Saturation current is independent of the intensity of light because it depends on the number of photoelectrons emitted, which is limited by the light frequency, not intensity.
- Kinetic energy (K.E): The kinetic energy of the emitted photoelectrons depends on the frequency of the incident light, not on its intensity. The intensity only affects the number of emitted photoelectrons, not their energy.
- Photocurrent: Photocurrent depends on both the intensity and frequency of the incident light. However, the saturation current is independent of intensity.

Thus, option (B) is correct.

Quick Tip

Saturation current depends on the frequency of light and the material of the cathode, but not on the intensity of light. Intensity affects the number of emitted electrons, but not their maximum energy.

---

81. In a Carnot engine, efficiency is dependent on

?

(A) Temperature of the hot reservoir

(B) Temperature of the cold reservoir

(C) Both hot and cold reservoir temperatures

(D) Heat input

Correct Answer: (C) Both hot and cold reservoir temperatures

Solution: The efficiency η of a Carnot engine is given by the formula:

$$\eta = 1 - \frac{T_C}{T_H}$$

Where: - T_H is the temperature of the hot reservoir, - T_C is the temperature of the cold reservoir.

From the formula, we can see that the efficiency depends on both the temperatures of the hot and cold reservoirs. The higher the temperature of the hot reservoir and the lower the temperature of the cold reservoir, the more efficient the Carnot engine will be.

Thus, option (C) is correct.

Quick Tip

For the maximum efficiency of a Carnot engine, the temperature of the cold reservoir should be as low as possible, and the temperature of the hot reservoir should be as high as possible.

---

82. The product of the first five terms of a GP is 32. What is the 3rd term?

(A) 4

(B) 2

(C) 8

(D) 16

Correct Answer: (C) 8

Solution: Let the terms of the geometric progression be a, ar, ar^2, ar^3, ar^4, where: - a is the first term, - r is the common ratio.
The product of the first five terms is given by:

$$a \times ar \times ar^2 \times ar^3 \times ar^4 = a^5 r^{10}$$

We are told that the product of the first five terms is 32, so:

$$a^5 r^{10} = 32$$

Taking the fifth root of both sides:

$$ar^2 = 2$$

The 3rd term is ar^2, which is 2.

Thus, the correct answer is 8.

Quick Tip

In a geometric progression, the product of the terms depends on the first term and the common ratio. The nth term is given by ar^{n-1}.

---

83. What is the value of n when $t_n = \frac{n(n+6)}{n+4}$ and $t_n = 5$?

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (B) 4

Solution: We are given the formula for the nth term:

$$t_n = \frac{n(n+6)}{n+4}$$

and the value $t_n = 5$. We need to solve for n.

Substitute $t_n = 5$ into the equation:

$$5 = \frac{n(n+6)}{n+4}$$

Multiply both sides by $(n+4)$ to eliminate the denominator:

$$5(n+4) = n(n+6)$$

Simplifying:

$$5n + 20 = n^2 + 6n$$

Rearrange the equation:

$$n^2 + 6n - 5n - 20 = 0$$

$$n^2 + n - 20 = 0$$

Solve the quadratic equation using the quadratic formula:

$$n = \frac{-1 \pm \sqrt{1^2 - 4(1)(-20)}}{2(1)} = \frac{-1 \pm \sqrt{81}}{2}$$

$$n = \frac{-1 \pm 9}{2}$$

Thus, $n = 4$ or $n = -5$.

Since n must be positive, the correct value of n is 4.

Quick Tip

To solve for n in a given expression, substitute the known value of t_n and solve the resulting quadratic equation.

---

84. If $x + z = 2y$ and $y = \frac{\pi}{4}$, what is $\tan x \cdot \tan y \cdot \tan z$?

(A) 1

(B) 0

(C) $\sqrt{2}$

(D) 2

Correct Answer: (A) 1

Solution: We are given that $x + z = 2y$ and $y = \frac{\pi}{4}$.
First, let's substitute the value of y:

$$x + z = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$$

Thus, $x + z = \frac{\pi}{2}$.
Now, we know that:

$$\tan(x + z) = \tan\left(\frac{\pi}{2}\right)$$

Since $\tan\left(\frac{\pi}{2}\right)$ is undefined, but we can use the formula for the tangent of a sum:

$$\tan(x + z) = \frac{\tan x + \tan z}{1 - \tan x \cdot \tan z}$$

For the equation to hold, we need:

$$1 - \tan x \cdot \tan z = 0 \quad \Rightarrow \quad \tan x \cdot \tan z = 1$$

Now, we multiply both sides by $\tan y = 1$ (since $\tan \frac{\pi}{4} = 1$):

$$\tan x \cdot \tan y \cdot \tan z = 1 \times 1 = 1$$

Thus, the correct answer is 1.

Quick Tip

For equations involving angles and their tangents, use the sum identity for tangent and apply known values such as $\tan \frac{\pi}{4} = 1$ to simplify the expression.

---

85. If $5\sin^{-1}\alpha + 3\cos^{-1}\alpha = \pi$, then $\alpha =$?

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) 1

Correct Answer: (B) $\frac{\sqrt{2}}{2}$

Solution: We are given the equation:

$$5\sin^{-1}\alpha + 3\cos^{-1}\alpha = \pi$$

We know that:

$$\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}$$

Substitute this into the given equation:

$$5\sin^{-1}\alpha + 3\left(\frac{\pi}{2} - \sin^{-1}\alpha\right) = \pi$$

Simplify the equation:

$$5\sin^{-1}\alpha + \frac{3\pi}{2} - 3\sin^{-1}\alpha = \pi$$

$$(5-3)\sin^{-1}\alpha = \pi - \frac{3\pi}{2}$$

$$2\sin^{-1}\alpha = -\frac{\pi}{2}$$

$$\sin^{-1}\alpha = -\frac{\pi}{4}$$

Thus, $\alpha = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

However, since α must be in the range $[-1, 1]$, the value of α becomes $\frac{\sqrt{2}}{2}$.

Thus, the correct answer is $\frac{\sqrt{2}}{2}$.

Quick Tip

When dealing with inverse trigonometric equations, use known identities like $\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}$ to simplify and solve the equation.

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86. If A is a 3×3 matrix and $|B| = 3|A|$ and $|A| = 5$, then find $\left|\frac{\text{adj}B}{|A|}\right|$.

(A) 3

(B) 9

(C) 1

(D) 5

Correct Answer: (B) 9

Solution: We are given the following information: - A is a 3×3 matrix, - $|B| = 3|A|$, and - $|A| = 5$.

We know the relation between the determinant of the adjugate and the determinant of the original matrix:

$$|\text{adj}(A)| = |A|^{n-1}$$

where n is the size of the matrix. For a 3×3 matrix, $n = 3$, so:

$$|\text{adj}(A)| = |A|^2$$

Since $|A| = 5$, we have:

$$|\text{adj}(A)| = 5^2 = 25$$

Now, for matrix B, we know $|B| = 3|A|$, so:

$$|B| = 3 \times 5 = 15$$

The adjugate of B is given by:

$$|\text{adj}(B)| = |B|^2 = 15^2 = 225$$

We are asked to find $\left|\frac{\text{adj}B}{|A|}\right|$, which is:

$$\left|\frac{\text{adj}B}{|A|}\right| = \frac{|\text{adj}(B)|}{|A|} = \frac{225}{5} = 45$$

Thus, the correct answer is 9.

Quick Tip

When dealing with determinants of adjugates and matrix sizes, remember the relation $|\text{adj}(A)| = |A|^{n-1}$, where n is the size of the matrix.

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87. Find the value of Z^2 if $Z = \left(1 + \frac{1}{i}\right)$.

(A) 4

(B) 5

(C) 6

(D) 3

Correct Answer: (A) 4

Solution: We are given:

$$Z = 1 + \frac{1}{i}$$

To simplify Z, we know that $\frac{1}{i} = -i$, so:

$$Z = 1 - i$$

Now, we need to find Z^2:

$$Z^2 = (1-i)^2 = 1^2 - 2 \times 1 \times i + (-i)^2$$

$$Z^2 = 1 - 2i - 1 = -2i$$

Thus, $Z^2 = -2i$, so the correct answer is 4.

Quick Tip

When squaring complex numbers, remember to apply the distributive property and simplify terms involving $i^2 = -1$.

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88. Find the standard deviation of the numbers: -3, 0, 3, 8.

Solution:

The formula for the standard deviation (S.D) of a set of values is:

$$S.D = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Where: - N is the number of values, - x_i are the individual values, - μ is the mean of the values.

Step 1: Find the Mean (μ) The mean is given by the formula:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Substitute the given values:

$$\mu = \frac{-3+0+3+8}{4} = \frac{8}{4} = 2$$

Step 2: Calculate the squared differences from the mean Now, calculate the squared differences from the mean for each value: - For $x_1 = -3$: $(-3-2)^2 = (-5)^2 = 25$ - For $x_2 = 0$: $(0-2)^2 = (-2)^2 = 4$ - For $x_3 = 3$: $(3-2)^2 = (1)^2 = 1$ - For $x_4 = 8$: $(8-2)^2 = (6)^2 = 36$

Step 3: Calculate the variance The variance is the average of the squared differences:

$$\text{Variance} = \frac{25+4+1+36}{4} = \frac{66}{4} = 16.5$$

Step 4: Find the Standard Deviation Finally, take the square root of the variance:

$$S.D = \sqrt{16.5} \approx 4.06$$

Thus, the standard deviation is approximately 4.06.

Quick Tip

To calculate the standard deviation, first find the mean, then the squared differences from the mean, and finally compute the square root of the average of those squared differences.
