Limit, Continuity, And Differentiability JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Limit, Continuity, And Differentiability

1. $\lim_{n\to\infty} \frac{3}{n} \left\{ 1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right\}$ **(+4, -1)**

a. does not exist

b. is 1

c. is 2

d. is 3

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2. The value of lim $_{0}$ $\frac{\int_0^{2} \sec^2}{\sin}$ is **(+4, -1)**

a. (A) 3

b. (B) 2

c. (C) 1

d. (D) 0

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3. $\lim_{n\to\infty} \left[\frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \dots + \frac{1}{2n} \right]$ is equal to **(+4, -1)**

a. $\log_e 2$

b. $\log_e \left(\frac{2}{3}\right)$

c. 0

d. $\log_e \left(\frac{3}{2}\right)$

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4. If $\int_{\frac{1}{3}}^{3} |\log_e x| \, dx = \frac{m}{n} \log e \left(\frac{n^2}{v}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to _____ **(+4, -1)**

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5. Let $a_n = \int\limits_{-1}^{n} \left(1 + \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{x^{n-1}}{n}\right) dx$ for $n \in N$ Then the sum of all the elements of the set $\{n \in N : a_n \in (2, 30)\}$ is ______ (+4, -1)

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6. $\lim\limits_{t \to 0} \left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to (+4, -1)

a. n^2

b. $n^2 + n$

c. n

d. $\frac{n(n+1)}{2}$

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7. Let $p = \lim\limits_{x \to 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ then $log\, p$ is equal to (+4, -1)

a. 2

b. 1

c. $\frac{1}{2}$

d. $\frac{1}{4}$

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8. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals : (+4, -1)

a. $\frac{1}{16}$

b. $\frac{1}{8}$

c. $\frac{1}{4}$

d. $\frac{1}{24}$

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9. $\lim\limits_{x \to 0} \left(\dfrac{3x^2 + 2}{7x^2 + 2}\right)^{\frac{1}{x^2}}$ is equal to : (+4, -1)

a. e

b. $\frac{1}{e^2}$

c. $\frac{1}{e}$

d. e^2

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10. $\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$ equals : **(+4, -1)**

a. $2\sqrt{2}$

b. $4\sqrt{2}$

c. $\sqrt{2}$

d. 4

Answers

1. Answer: c

Explanation:

$$\lim_{n\to\infty} \frac{3}{n} \sum_{r=0}^{n-1} \sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}$$
$$= 3\int_0^1 \frac{dx}{\sqrt{1+3x}} = 2\left[\sqrt{1+3x}\right]_0^1 = 2$$

Concepts:

1. Limits:

A function's limit is a number that a function reaches when its independent variable comes to a certain value. The value (say a) to which the function f(x) approaches casually as the independent variable x approaches casually a given value "A" denoted as f(x) = A.

If $\lim_{x\to a-} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the left of 'a'. This value is also called the left-hand limit of 'f' at a.

If $\lim_{x\to a+} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the right of 'a'. This value is also called the right-hand limit of f(x) at a.

If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a} f(x)$.

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2. Answer: c

Explanation:

Explanation:

Given:The expression $\lim_{\ \ 0} \frac{\int_0^{\ 2} \sec^2}{\sin}$ We have to evaluate the given expression.Here, we'll use fundamental integrals of trigonometric functions.

$\lim_{\ \ 0} \frac{\int_0^{\ 2} \sec^2}{\sin}$ $[\int \sec^2 \quad = \tan\ ]$ $\lim_{\ \ 0} \frac{[\tan\ ]^{\ 2}}{\sin} = \lim_{\ \ 0} \frac{\tan\ ^2}{\sin}$

= lim 0 $\frac{\frac{\tan^2}{2}}{2}$ [Dividing by ^{2} in Numerator & denominator] $\frac{\lim_{0} (\frac{\tan^2}{2})}{\sin}$ = $\frac{1}{1}$ = 1[Using limit of trigonometric functions]Hence, the correct option is (C).

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3. **Answer: a**

Explanation:

$$\lim_{n\to\infty}\left(\frac{1}{1+n}+\ldots+\frac{1}{n+n}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r}$$
$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$
$$=\int_0^1\frac{1}{1+x}dx=[\ell ln(1+x)]_0^1=\ell n2$$

Concepts:

1. **Limits:**

A function's limit is a number that a function reaches when its independent variable comes to a certain value. The value (say a) to which the function f(x) approaches casually as the independent variable x approaches casually a given value "A" denoted as f(x) = A.

If $\lim_{x\to a^-} f(x)$ is the expected value of f when x = a, given the values of ‘f’ near x to the left of ‘a’. This value is also called the left-hand limit of ‘f’ at a.

If $\lim_{x\to a+} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the right of 'a'. This value is also called the right-hand limit of f(x) at a.

If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a} f(x)$.

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4. **Answer: 20 - 20**

Explanation:

The correct answer is 20.

$$\int_{\frac{1}{3}}^{3} |\ell nx| dx = \int_{\frac{1}{3}}^{1} (-\ell nx) dx + \int_{1}^{3} (\ell nx) dx$$
$$= -[x\ell nx - x]_{1/3}^{1} + [x\ell nx - x]_{1}^{3}$$
$$= -\left[-1 - \left(\tfrac{1}{3}\ell n\tfrac{1}{3} - \tfrac{1}{3}\right)\right] + [3\ell n3 - 3 - (-1)]$$
$$= \left[-\tfrac{2}{3} - \tfrac{1}{3}\ell n\tfrac{1}{3}\right] + [3\ell n3 - 2]$$
$$= -\tfrac{4}{3} + \tfrac{8}{3}\ell n3$$
$$= \tfrac{4}{3}(2\ell n3 - 1)$$
$$= \tfrac{4}{3}\left(\ell n\tfrac{9}{e}\right)$$
$$\therefore m = 4, n = 3$$

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

Concepts:

1. Limits:

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If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a} f(x)$.

5. **Answer: 5 - 5**

Explanation:

The correct answer is 5.

$\int_{-1}^{n}(1+\frac{x}{2}+\frac{x^2}{3}+....+\frac{x^{n-1}}{n})dx$

$[x+\frac{x^2}{2}+\frac{x^3}{3^2}+...\frac{x^n}{n^2}]^n$

$(n+\frac{n^2}{2^2}+\frac{n^3}{3^2}+...+-\frac{n^n}{n^2})$

$-(-1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}+...+\frac{(-1)^n}{n^2})$

$a_n=(n+1)+\frac{1}{2^2}(n^2-1)+\frac{1}{3^2}(n^3+1)+...+\frac{1}{n^2}(n^n-(-1)^n)$

$If\ n=1\Rightarrow a_n=2\ \notin(2,30)$

If n = 2

$\Rightarrow a_n=(2+1)+\frac{1}{2^2}(2^2-1)=3+\frac{3}{4}<30$

If n = 3

$\Rightarrow a_n=(3+1)+\frac{1}{4}(8)+\frac{1}{9}(28)=11+\frac{28}{9}<30$

If n = 4

$\Rightarrow a_n=(4+1)+\frac{1}{4}(16-1)+\frac{1}{9}(64+1)+\frac{1}{16}$

$=5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16}>30$

Test {2,3} sum of elements 5

Concepts:

1. Limits:

A function's limit is a number that a function reaches when its independent variable comes to a certain value. The value (say a) to which the function f(x) approaches casually as the independent variable x approaches casually a given value "A" denoted as f(x) = A.

If $\lim_{x\to a-} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the left of 'a'. This value is also called the left-hand limit of 'f' at a.

If $\lim_{x\to a+} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the right of 'a'. This value is also called the right-hand limit of f(x) at a.

If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a} f(x)$.

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6. **Answer: c**

Explanation:

$$\lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+\ldots\ldots.+n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$$

$$=\lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosecs}^2 t}+\ldots\ldots.+1\right)^{\sin^2 t}$$

$$=n$$

Concepts:

1. Limits:

A function's limit is a number that a function reaches when its independent variable comes to a certain value. The value (say a) to which the function f(x) approaches casually as the independent variable x approaches casually a given value "A" denoted as f(x) = A.

If $\lim_{x\to a-} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the left of 'a'. This value is also called the left-hand limit of 'f' at a.

If $\lim_{x\to a+} f(x)$ is the expected value of f when x = a, given the values of 'f' near x to the right of 'a'. This value is also called the right-hand limit of f(x) at a.

If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a} f(x)$.

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7. **Answer: c**

Explanation:

$$p = \lim_{x \to 0^+} \{1 + \tan^2 \sqrt{x}\}^{\frac{1}{\tan^2 \sqrt{x}} \times \frac{\tan^2 \sqrt{x}}{2x}}$$

$$= e^{\lim_{x \to 0} \frac{\tan^2 \sqrt{x}}{(\sqrt{x})^2} \times \frac{1}{2}} = e^{\frac{1}{2}}$$

$$\log_e p = \frac{1}{2}$$

Concepts:

1. Limits And Derivatives:

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n \to c} f(n) = L$$

Limits Formula:

- $\lim_{x \to 0} sinx = 0$
- $\lim_{x \to 0} cosx = 1$
- $\lim_{x \to 0} \frac{1 - cosx}{x} = 0$
- $\lim_{x \to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{sinx}{x} = 1$
- $\lim_{x \to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Properties of Derivatives:

1. $\frac{d}{dx}[p(x)+q(x)] = \frac{d}{dx}(p(x)) + \frac{d}{dx}(q(x))$
2. $\frac{d}{dx}[p(x)-q(x)] = \frac{d}{dx}(p(x)) - \frac{d}{dx}(q(x))$
3. $\frac{d}{dx}[p(x)\times q(x)] = \frac{d}{dx}[p(x)]\, q(x) + p(x)\frac{d}{dx}[q(x)]$
4. $\frac{d}{dx}\left[\frac{p(x)}{q(x)}\right] = \frac{\frac{d}{dx}[p(x)]\, q(x) - p(x)\frac{d}{dx}[q(x)]}{(q(x))^2}$

Read More: Limits and Derivatives

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8. **Answer: a**

Explanation:

$\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Put, $\frac{\pi}{2} - x = t$

$\lim_{t\to 0} \frac{\tan t - \sin t}{8t^3}$

$= \lim_{t\to 0} \frac{\sin t \cdot 2\sin^2 \frac{t}{2}}{8t^3}$

$= \frac{1}{16}.$

Concepts:

1. Limits And Derivatives:

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n\to c} f(n) = L$$

Limits Formula:

- $\lim_{x\to 0} sinx = 0$
- $\lim_{x\to 0} cosx = 1$
- $\lim_{x\to 0} \frac{1-cosx}{x} = 0$
- $\lim_{x\to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x\to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x\to 0} \frac{sinx}{x} = 1$
- $\lim_{x\to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Properties of Derivatives:

1. $\frac{d}{dx}\ [p(x)+q(x)] = \frac{d}{dx}\ (p(x)) + \frac{d}{dx}\ (q(x))$

2. $\frac{d}{dx}\ [p(x)-q(x)] = \frac{d}{dx}\ (p(x)) - \frac{d}{dx}\ (q(x))$

3. $\frac{d}{dx}\ [p(x)\times q(x)] = \frac{d}{dx}\ [p(x)]\ q(x) + p(x)\ \frac{d}{dx}\ [q(x)]$

4. $\frac{d}{dx}\left[\frac{p(x)}{q(x)}\right] = \frac{\frac{d}{dx}\ [p(x)]\ q(x) - p(x)\ \frac{d}{dx}\ [q(x)]}{(q(x))^2}$

Read More: Limits and Derivatives

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9. **Answer: b**

Explanation:

Required limit $= e^{\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{-4}{7x^2+2}\right)} = \frac{1}{e^2}$

The correct option is (B): $\frac{1}{e^2}$.

Concepts:

1. Limits And Derivatives:

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n\to c} f(n) = L$$

Limits Formula:

- $\lim_{x \to 0} sinx = 0$
- $\lim_{x \to 0} cosx = 1$
- $\lim_{x \to 0} \frac{1 - cosx}{x} = 0$
- $\lim_{x \to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{sinx}{x} = 1$
- $\lim_{x \to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Properties of Derivatives:

$$1.\ \frac{d}{dx}\left[p(x)+q(x)\right]=\frac{d}{dx}\left(p(x)\right)+\frac{d}{dx}\left(q(x)\right)$$
$$2.\ \frac{d}{dx}\left[p(x)-q(x)\right]=\frac{d}{dx}\left(p(x)\right)-\frac{d}{dx}\left(q(x)\right)$$
$$3.\ \frac{d}{dx}\left[p(x)\times q(x)\right]=\frac{d}{dx}\left[p(x)\right]q(x)+p(x)\frac{d}{dx}\left[q(x)\right]$$
$$4.\ \frac{d}{dx}\left[\frac{p(x)}{q(x)}\right]=\frac{\frac{d}{dx}\left[p(x)\right]q(x)-p(x)\frac{d}{dx}\left[q(x)\right]}{\left(q(x)\right)^2}$$

Read More: Limits and Derivatives

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10. **Answer: b**

Explanation:

$$\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$
$$=\frac{(1)^2.\left(2\sqrt{2}\right)}{\frac{1}{2}}=4\sqrt{2}$$

Concepts:

1. **Limits And Derivatives:**

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n\to c} f(n) = L$$

Limits Formula:

- $\lim_{x \to 0} sinx = 0$
- $\lim_{x \to 0} cosx = 1$
- $\lim_{x \to 0} \frac{1-cosx}{x} = 0$
- $\lim_{x \to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{sinx}{x} = 1$
- $\lim_{x \to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Properties of Derivatives:

1. $\frac{d}{dx}\,[p(x)+q(x)] = \frac{d}{dx}\,(p(x)) + \frac{d}{dx}\,(q(x))$

2. $\frac{d}{dx}\,[p(x)-q(x)] = \frac{d}{dx}\,(p(x)) - \frac{d}{dx}\,(q(x))$

3. $\frac{d}{dx}\,[p(x)\times q(x)] = \frac{d}{dx}\,[p(x)]\;q(x) + p(x)\;\frac{d}{dx}\,[q(x)]$

4. $\frac{d}{dx}\left[\frac{p(x)}{q(x)}\right] = \frac{\frac{d}{dx}\,[p(x)]\;q(x) - p(x)\;\frac{d}{dx}\,[q(x)]}{(g(x))^2}$

Read More: Limits and Derivatives