CBSE Class X Mathematics (Basic) Set 2 (430/1/2) Question Paper with Solution

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each. Internal choice is provided in 2 marks questions in each case study.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
9. Draw neat diagrams wherever required. Take p = 7 22 wherever required, if not stated.
10. Use of calculator is not allowed.

Section A

1: LCM (850, 500) is:

(a) 850×50

(b) 17×500

(c) $17 \times 5^2 \times 2^2$

(d) $17 \times 5^3 \times 2$

Correct Answer: (b) 17×500

Solution:

Step 1: Perform the prime factorization of 850 and 500:

$$850 = 2 \times 5^2 \times 17, \quad 500 = 2^2 \times 5^3$$

Step 2: Find the LCM: The LCM is the product of the highest powers of all prime factors appearing in the numbers:

$$\text{LCM} = 2^2 \times 5^3 \times 17$$

Step 3: Simplify:

$$\text{LCM} = 17 \times 500$$

Correct Answer: The LCM is 17×500.

> **Quick Tip**
>
> To calculate the LCM, take the highest powers of all prime factors common to both numbers.

2: If the roots of quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then the value of k is:

(a) 4

(b) $\frac{25}{16}$

(c) -5

(d) $\frac{-25}{16}$

Correct Answer: (b) $\frac{25}{16}$

Solution:

For a quadratic equation $ax^2 + bx + c = 0$, the condition for real and equal roots is:

$$\Delta = b^2 - 4ac = 0$$

For the given quadratic equation $4x^2 - 5x + k = 0$, we have:

- $a = 4$
- $b = -5$
- $c = k$

Substitute the values into the discriminant formula:

$$\Delta = (-5)^2 - 4(4)(k) = 0$$

$$25 - 16k = 0$$

Solve for k:

$$16k = 25 \implies k = \frac{25}{16}$$

Quick Tip

For real and equal roots of a quadratic equation, set the discriminant $\Delta = 0$.

3: The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is:

(a) 27

(b) 22

(c) 17

(d) 24

Correct Answer: (a) 27

Solution:
In statistics, the relationship between the mean, median, and mode is given by:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substitute the given values:

$$\text{Mode} = 3 \times 23 - 2 \times 21 = 69 - 42 = 27$$

Thus, the mode of the data is 27.

Quick Tip

In a symmetric data set, the mean, median, and mode are equal. For skewed data, the mode can be estimated using $\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$.

4: The height and radius of a right circular cone are 24 cm and 7 cm respectively. The slant height of the cone is:
(a) 24 cm
(b) 31 cm
(c) 26 cm
(d) 25 cm

Correct Answer: (d) 25 cm

Solution:
The slant height l of a right circular cone can be calculated using the Pythagoras theorem, as it forms a right triangle with the height and radius:

$$l = \sqrt{r^2 + h^2}$$

Substitute the given values $r = 7$ cm and $h = 24$ cm:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

Thus, the slant height is 25 cm.

Quick Tip

To calculate the slant height of a cone, use the Pythagoras theorem: $l = \sqrt{r^2 + h^2}$.

5: If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is -3, then the value of α is:

(a) $\frac{-2}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{3}{4}$

Correct Answer: (c) $\frac{4}{3}$

Solution:

Let the quadratic polynomial be $f(x) = (\alpha - 1)x^2 + \alpha x + 1$.

Given that one of the zeroes is -3, we can substitute $x = -3$ into the equation:

$$f(-3) = (\alpha - 1)(-3)^2 + \alpha(-3) + 1 = 0$$

Simplifying:

$$(\alpha - 1)(9) - 3\alpha + 1 = 0$$

$$9\alpha - 9 - 3\alpha + 1 = 0$$

$$6\alpha - 8 = 0$$

Solve for α:

$$6\alpha = 8 \implies \alpha = \frac{8}{6} = \frac{4}{3}$$

Thus, $\alpha = \frac{4}{3}$.

Quick Tip

When given a zero of a polynomial, substitute the value of x into the equation and solve for the unknown parameter.

6: A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the drawn card is a red queen is:

(a) $\frac{1}{13}$

(b) 2

(c) 1

(d) $\frac{1}{26}$

Correct Answer: (d) $\frac{1}{26}$

Solution:

There are 2 red queens in a deck of 52 cards (one from hearts and one from diamonds). The probability of drawing a red queen is:

$$\frac{2}{52} = \frac{1}{26}$$

Thus, the probability is $\frac{1}{26}$.

Quick Tip

In a deck of 52 cards, there are 2 red queens, so the probability of drawing one is $\frac{2}{52}$.

7: If a certain variable x divides a statistical data arranged in order into two equal parts, then the value of x is called the:

(a) mean

(b) median

(c) mode

(d) range

Correct Answer: (b) median

Solution:

The value of x that divides the statistical data into two equal parts is called the median. It is the middle value in an ordered data set.

Quick Tip

The median divides the data into two equal halves when the data is arranged in order.

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8. Three coins are tossed together. The probability of getting exactly one tail is:

Options:

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) $\frac{7}{8}$

(d) $\frac{3}{8}$

Correct Answer: (d) $\frac{3}{8}$

Solution:

To find the probability of getting exactly one tail, we first look at all possible outcomes when three coins are tossed. The total number of outcomes is $2^3 = 8$, because each coin has two possible outcomes (heads or tails). The possible outcomes are:

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

We need to find the number of outcomes with exactly one tail. The outcomes with one tail are: HHT, HTH, and THH. So, there are 3 favorable outcomes.

Thus, the probability of getting exactly one tail is the ratio of favorable outcomes to total outcomes, which is:

$$P(\text{exactly one tail}) = \frac{3}{8}$$

Quick Tip

The total number of outcomes when tossing three coins is always $2^3 = 8$. Counting the outcomes with the desired number of tails will give the correct probability.

9: If $\sin\theta = \frac{1}{3}$, then $\sec\theta$ is equal to:

a) $\frac{2\sqrt{2}}{3}$

b) $\frac{3}{2\sqrt{2}}$

c) 3

d) $\frac{1}{\sqrt{3}}$

Correct Answer: b) $\frac{3}{2\sqrt{2}}$

Solution: We are given $\sin\theta = \frac{1}{3}$. We need to find $\sec\theta$.

Recall the following trigonometric identities: $\sec\theta = \frac{1}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

From $\sin\theta = \frac{1}{3}$, we can find $\cos\theta$ using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1 \implies \frac{1}{9} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$$

Thus:

$$\cos\theta = \frac{\sqrt{8}}{3}$$

Now, we can find $\sec\theta$:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{8}}{3}} = \frac{3}{\sqrt{8}} = \frac{3}{2\sqrt{2}}$$

Thus, the correct answer is:

$$\boxed{b)\frac{3}{2\sqrt{2}}}$$

Quick Tip

To find $\sec\theta$, use the identity $\sec\theta = \frac{1}{\cos\theta}$ and apply the Pythagorean identity.

10: Outer surface area of a cylindrical juice glass with radius 7 cm and height 10 cm, is:

(a) 440 sq m

(b) 594 sq m

(c) 748 sq m

(d) 1540 sq m

Correct Answer: (b) 594 sq m

Solution:

The formula for the outer surface area (curved surface area) of a cylinder is given by:

$$A = 2\pi rh$$

where: - r is the radius of the base of the cylinder, - h is the height of the cylinder.

Given: - Radius $r = 7\,\text{cm}$ - Height $h = 10\,\text{cm}$

Substituting the values into the formula:

$$A = 2\pi \times 7 \times 10 = 140\pi$$

Using $\pi = 3.14$:

$$A = 140 \times 3.14 = 439.6\,\text{sq cm}$$

Thus, the outer surface area is approximately $594\,\text{sq cm}$.

Therefore, the correct answer is option (b) $594\,\text{sq m}$.

Quick Tip

For the surface area of a cylinder, always remember the formula: $A = 2\pi r(h + r)$, where r is the radius and h is the height.

11: On a throw of a die, if getting 6 is considered success then the probability of losing the game is:

(a) 0

(b) 1

(c) 1/6

(d) 5/6

Correct Answer: (d) 5/6

Solution:

The probability of getting a 6 on a fair die is $\frac{1}{6}$.

Thus, the probability of losing the game (i.e., not getting a 6) is the complement of the probability of success:

$$P(\text{Losing}) = 1 - P(\text{Success}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Thus, the probability of losing the game is $\frac{5}{6}$.

Quick Tip

In probability, the sum of the probability of an event and its complement always equals 1.

12: The distance between the points $(2, -3)$ and $(-2, 3)$ is:

(a) $2\sqrt{13}$ units

(b) 5 units

(c) $13\sqrt{2}$ units

(d) 10 units

Correct Answer: (a) $2\sqrt{13}$ units

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute the coordinates $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (-2, 3)$:

$$d = \sqrt{((-2) - 2)^2 + (3 - (-3))^2} = \sqrt{(-4)^2 + (6)^2} = \sqrt{16 + 36} = \sqrt{52}$$

$$d = 2\sqrt{13}\,\text{units}$$

Thus, the distance is $2\sqrt{13}$ units.

Quick Tip

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

13: For what value of θ, $\sin^2\theta + \sin\theta + \cos^2\theta$ is equal to 2?

(a) 45°

(b) 0°

(c) 90°

(d) 30°

Correct Answer: (c) 90°

Solution:

We are given the equation:

$$\sin^2\theta + \sin\theta + \cos^2\theta = 2$$

Since $\cos^2\theta + \sin^2\theta = 1$ (the Pythagorean identity), the equation simplifies to:

$$1 + \sin\theta = 2$$

$$\sin\theta = 1$$

The sine of θ is 1 when $\theta = 90^\circ$.

Thus, the value of θ is 90°.

Quick Tip

Remember the identity $\sin^2\theta + \cos^2\theta = 1$, which simplifies trigonometric expressions.

14: The diameter of a circle is of length 6 cm. If one end of the diameter is $(-4, 0)$, the other end on the x-axis is at:

(a) $(0, 2)$

(b) $(6, 0)$

(c) $(2, 0)$

(d) $(4, 0)$

Correct Answer: (c) $(2, 0)$

Solution:

Step 1: Given the diameter length is 6 cm, the endpoints of the diameter are $(-4, 0)$ and $(x, 0)$. Since the second endpoint lies on the x-axis, its y-coordinate is 0.

Step 2: Distance formula for the diameter:

$$\text{Distance} = \sqrt{(x - (-4))^2 + (0 - 0)^2} = 6$$

$$\sqrt{(x + 4)^2} = 6 \quad \Rightarrow \quad x + 4 = \pm 6$$

Step 3: Solve for x:

$$x + 4 = 6 \quad \Rightarrow \quad x = 2 \quad \text{or} \quad x + 4 = -6 \quad \Rightarrow \quad x = -10$$

Step 4: Since the other endpoint is on the x-axis, it must satisfy the given distance. The valid point is $(2, 0)$.

Final Answer: The other endpoint is $(2, 0)$.

Quick Tip

When solving for endpoints of a diameter, use the distance formula and apply constraints of the given coordinates.

15: The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ and $2x + ky + 1 = 0$ don't have a solution, is:

(a) 5

(b) $\frac{4}{5}$

(c) $\frac{5}{4}$

(d) $\frac{5}{2}$

Correct Answer: (b) $\frac{4}{5}$

Solution:

Step 1: For the pair of equations to not have a solution, the lines must be parallel. This happens when:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Here:

$$a_1 = 5,\ b_1 = 2,\ c_1 = -7,\ a_2 = 2,\ b_2 = k,\ c_2 = 1$$

Step 2: Use the condition $\frac{a_1}{a_2} = \frac{b_1}{b_2}$:

$$\frac{5}{2} = \frac{2}{k} \quad \Rightarrow \quad 5k = 4 \quad \Rightarrow \quad k = \frac{4}{5}.$$

Final Answer: The value of k is $\frac{4}{5}$.

Quick Tip

For parallel lines, ensure that the ratios of the coefficients of x and y are equal, but not the ratio of the constants.

16: For what value of k, the product of zeroes of the polynomial $kx^2 - 4x - 7$ is 2?

a) $\frac{1}{14}$

b) $\frac{-7}{2}$

c) $\frac{7}{2}$

d) $\frac{-2}{7}$

Correct Answer:b) $\frac{-7}{2}$

Solution: We know that for a quadratic equation $ax^2 + bx + c$, the product of the zeroes (roots) is given by:

$$\text{Product of the zeroes} = \frac{c}{a}$$

In our case, the polynomial is $kx^2 - 4x - 7$, where: - $a = k$, - $b = -4$, - $c = -7$.

We are given that the product of the zeroes is 2. Therefore, we can set up the equation:

$$\frac{c}{a} = 2$$

Substituting the values of c and a:

$$\frac{-7}{k} = 2$$

Now, solve for k:

$$-7 = 2k \implies k = \frac{-7}{2}$$

Thus, the correct answer is: b) -7$_{\overline{2}}$

Quick Tip

For any quadratic equation $ax^2 + bx + c$, the product of the zeroes is $\frac{c}{a}$.

17: In an A.P., if $a = 8$ and $a_{10} = -19$, then the value of d is:

a) 3

b) $\frac{-11}{9}$

c) $\frac{-27}{10}$

d) -3

Correct Answer: d) -3

Solution: In an arithmetic progression (A.P.), the n-th term is given by the formula:

$$a_n = a + (n-1)d$$

Where: - a_n is the n-th term,

- a is the first term,

- d is the common difference.

We are given: - $a = 8$ (the first term),

- $a_{10} = 19$ (the 10th term),

- We need to find d (the common difference).

Substitute the known values into the formula for the 10th term:

$$a_{10} = a + (10 - 1)d \implies -19 = 8 + 9d$$

Now, solve for d:

$$-19 - 8 = 9d \implies -27 = 9d \implies d = -3$$

Thus, the correct answer is: d) -3

Quick Tip

To find the common difference d in an A.P., use the formula $a_n = a + (n - 1)d$.

18: The mid-point of the line segment joining the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ is:

a) $\left(\frac{7}{2}, \frac{-3}{4}\right)$

b) $\left(\frac{7}{2}, \frac{9}{2}\right)$

c) $\left(\frac{9}{2}, \frac{-3}{4}\right)$

d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Correct Answer: d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Solution:

The formula for the mid-point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute the coordinates of the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ into this formula:

$$\left(\frac{-1 + 8}{2}, \frac{3 + \frac{3}{2}}{2}\right)$$

Simplifying:

$$\left(\frac{7}{2}, \frac{6+3}{4}\right) = \left(\frac{7}{2}, \frac{9}{4}\right)$$

Thus, the mid-point of the line segment is $\left(\frac{7}{2}, \frac{9}{4}\right)$, which corresponds to option (d).

Quick Tip

To find the mid-point of a line segment, simply average the x-coordinates and y-coordinates of the two points.

Directions: In Q. No. 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Select the correct option from the following options:

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

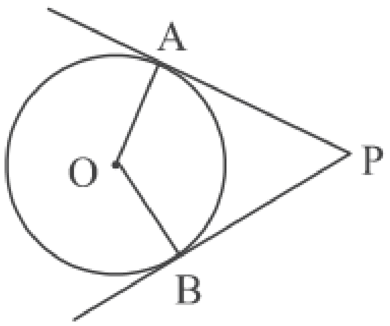
(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19: Assertion (A): If the PA and PB are tangents drawn to a circle with center O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal.



Correct Answer: (c) Assertion (A) is true but Reason (R) is false.

Solution:

Step 1: Analyze Assertion (A):

- PA and PB are tangents drawn to the circle. The angle between the tangents at the external point P and the angles subtended by the points of tangency form a quadrilateral $OAPB$. - A quadrilateral is cyclic if all its vertices lie on the circumference of a single circle. - Since $OAPB$ satisfies this condition, Assertion (A) is true.

Step 2: Analyze Reason (R):

- The given Reason states that in a cyclic quadrilateral, opposite angles are equal. This is incorrect. The correct property of a cyclic quadrilateral is that the sum of opposite angles is $180°$. - Hence, Reason (R) is false.

Conclusion: Assertion (A) is true, but Reason (R) is false.

> **Quick Tip**
>
> For cyclic quadrilaterals, remember the correct property: The sum of opposite angles is $180°$. Opposite angles being equal applies to rectangles or parallelograms, not cyclic quadrilaterals.

20: Assertion (A): Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are -1 and 3.
Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects the x-axis at (-1, 0) and (3, 0).

Correct Answer: (a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

Solution:

- The given polynomial is $p(x) = x^2 - 2x - 3$. To find the zeroes, we solve $x^2 - 2x - 3 = 0$ by factoring:

$$x^2 - 2x - 3 = (x - 3)(x + 1) = 0$$

Thus, the zeroes of the polynomial are $x = 3$ and $x = -1$.

- The graph of a quadratic polynomial intersects the x-axis at its zeroes. Therefore, the points where the graph intersects the x-axis are $(-1, 0)$ and $(3, 0)$, as given in the reason.

Since both the assertion and the reason are true, and the reason explains the assertion, the correct answer is (a).

Quick Tip

The zeroes of a quadratic polynomial are the points where the graph intersects the x-axis.

Section B

21: (A) Prove that $6 - 4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

Solution:

We are given that $\sqrt{5}$ is an irrational number. We need to prove that $6 - 4\sqrt{5}$ is also an irrational number.

Proof:

Assume, for the sake of contradiction, that $6 - 4\sqrt{5}$ is a rational number.

This means that $6 - 4\sqrt{5} = p/q$, where p and q are integers and $q \neq 0$.

Now, let's isolate $\sqrt{5}$:

$$6 - 4\sqrt{5} = \frac{p}{q}$$
$$4\sqrt{5} = 6 - \frac{p}{q}$$
$$\sqrt{5} = \frac{6q - p}{4q}$$

Thus, $\sqrt{5}$ is expressed as a ratio of two integers, which implies that $\sqrt{5}$ is a rational number.

But this contradicts the given information that $\sqrt{5}$ is irrational.

Therefore, our assumption that $6 - 4\sqrt{5}$ is rational must be false. Hence, $6 - 4\sqrt{5}$ is irrational.

Conclusion:

Thus, we have proved that $6 - 4\sqrt{5}$ is an irrational number.

Quick Tip

If the sum or difference of a rational and an irrational number is assumed to be rational, it leads to a contradiction, proving the irrationality.

(B) Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number. Solution:

First, simplify the given expression:

$$11 \times 19 \times 23 + 3 \times 11$$

Factor out 11 from both terms:

$$11\,(19 \times 23 + 3)$$

Now, calculate inside the parentheses:

$$19 \times 23 = 437$$

$$437 + 3 = 440$$

Thus, the expression becomes:

$$11 \times 440 = 4840$$

Since 4840 is divisible by 11, it is not a prime number.

Therefore, $11 \times 19 \times 23 + 3 \times 11 = 4840$ is not a prime number.

Quick Tip

A prime number has only two distinct divisors: 1 and itself. If a number is divisible by any other number, it cannot be prime.

22: A bag contains 4 red, 5 white and some yellow balls. If the probability of drawing a red ball at random is $\frac{1}{5}$, then find the probability of drawing a yellow ball at random.

Correct Answer: The probability of drawing a yellow ball is $\frac{11}{20}$.

Solution:

Let the number of yellow balls be x.

The total number of balls in the bag is $4 + 5 + x = 9 + x$.

The probability of drawing a red ball is given by:

$$\frac{4}{9+x} = \frac{1}{5}$$

Now, solve for x:

$$4 \times 5 = 1 \times (9+x) \implies 20 = 9 + x \implies x = 11$$

Thus, the total number of balls is $9 + 11 = 20$.

The probability of drawing a yellow ball is:

$$\frac{11}{20}$$

Quick Tip

To calculate probability, use the formula:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

For problems involving balls of different colors, add the number of balls of each color to find the total number of outcomes.

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23. In a ΔABC, $\angle A = 90°$. If $\tan C = \sqrt{3}$, then find the value of $\sin B + \cos C - \cos^2 B$.

Solution:

In the given right-angled triangle ΔABC, we have $\angle A = 90°$, so $\angle B + \angle C = 90°$. This implies that:

$$\angle B = 90° - \angle C$$

We are given that $\tan C = \sqrt{3}$. From the definition of tangent, we know:

$$\tan C = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{AC}$$

Thus, $\frac{AB}{AC} = \sqrt{3}$. This implies that:

$$AB = \sqrt{3} \times AC$$

Let $AC = x$, then $AB = \sqrt{3}x$.

Next, we use the Pythagorean theorem in ΔABC:

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = (\sqrt{3}x)^2 + x^2 = 3x^2 + x^2 = 4x^2$$

$$BC = 2x$$

Now, let's calculate the required expression $\sin B + \cos C - \cos^2 B$.

1. Finding $\sin B$:

Since $\angle B = 90° - \angle C$, we know:

$$\sin B = \cos C$$

2. Finding $\cos C$:

Using the definition of cosine:

$$\cos C = \frac{AC}{BC} = \frac{x}{2x} = \frac{1}{2}$$

3. Finding $\cos B$:

Since $\sin B = \cos C = \frac{1}{2}$, we use the Pythagorean identity to find $\cos B$:

$$\cos^2 B = 1 - \sin^2 B = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\cos B = \frac{\sqrt{3}}{2}$$

Now, we can substitute these values into the expression $\sin B + \cos C - \cos^2 B$:

$$\sin B + \cos C - \cos^2 B = \frac{1}{2} + \frac{1}{2} - \frac{3}{4}$$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

Thus, the value of $\sin B + \cos C - \cos^2 B$ is $\boxed{\frac{1}{4}}$.

Quick Tip

In right-angled triangles, use trigonometric identities and relationships like $\tan = \frac{\text{opposite}}{\text{adjacent}}$ and $\sin^2\theta + \cos^2\theta = 1$ to simplify expressions.

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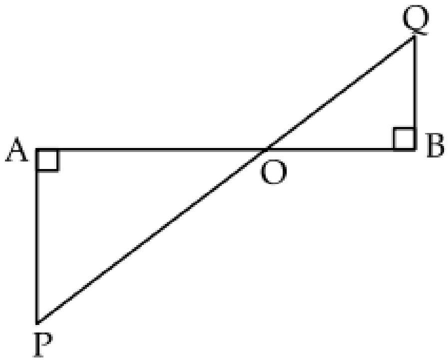
24. In the given figure, $AP \perp AB$ and $BQ \perp AB$. If $OA = 15$ cm, $BO = 12$ cm, and $AP = 10$ cm, then find the length of BQ.



Solution:

Since $\Delta OAP \sim \Delta OBQ$ (AA criterion), we can use the property of similar triangles:

$$\frac{OA}{OB} = \frac{AP}{BQ}$$

Substitute the given values:

$$\frac{15}{12} = \frac{10}{BQ}$$

Now solve for BQ:

$$BQ = \frac{10 \times 12}{15} = 8\,\text{cm}$$

Thus, the length of BQ is $8\,\text{cm}$.

Quick Tip

When dealing with right-angled triangles, the Pythagorean theorem $a^2 + b^2 = c^2$ is helpful for finding missing sides.

25: (A) Solve the following pair of linear equations for x and y algebraically:

$$x + 2y = 9$$

$$y - 2x = 2$$

Correct Answer: $x = -1, y = -4$

Solution:

From the first equation:

$$x + 2y = 9 \implies x = 9 - 2y$$

Substitute this expression for x into the second equation:

$$y - 2(9 - 2y) = 2$$

Simplifying:

$$y - 18 + 4y = 2 \implies 5y = 20 \implies y = 4$$

Now substitute $y = 4$ into $x = 9 - 2y$:

$$x = 9 - 2(4) = 9 - 8 = 1$$

Thus, the solution is $x = 1$ and $y = 4$.

Quick Tip

When solving a pair of linear equations algebraically, you can substitute the value of one variable into the other equation to find the value of the second variable.

25: (B) Check whether the point (-4, 3) lies on both the lines represented by the linear equations $x + y + 1 = 0$ and $x - y = 1$.

Correct Answer: The point (-4, 3) does not lie on both lines.

Solution:

For the first equation $x + y + 1 = 0$: Substitute $x = -4$ and $y = 3$:

$$(-4) + 3 + 1 = 0$$

This is true, so the point lies on the first line.

For the second equation $x - y = 1$: Substitute $x = -4$ and $y = 3$:

$$(-4) - 3 = -7$$

This is false, so the point does not lie on the second line.

Thus, the point (-4, 3) lies on the first line but not on the second line.

Quick Tip

To solve a system of linear equations, you can use the substitution method or elimination method. In substitution, express one variable in terms of the other and substitute into the second equation. In elimination, multiply the equations to cancel one variable and solve for the other.

Section C

26: Prove that:

$$\frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}} = 2\csc A$$

Solution:

Step 1: Start with the left-hand side (LHS):

$$\text{LHS} = \frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$$

Step 2: Simplify by taking a common denominator:

$$\text{LHS} = \frac{(\sec A - 1) + (\sec A + 1)}{\sqrt{\sec A + 1} \cdot \sqrt{\sec A - 1}}$$

Step 3: Simplify the numerator:

$$\text{Numerator} = (\sec A - 1) + (\sec A + 1) = 2\sec A$$

Step 4: Simplify the denominator using the identity ($\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$):

$$\text{Denominator} = \sqrt{(\sec A + 1)(\sec A - 1)} = \sqrt{\sec^2 A - 1}$$

Step 5: Use the trigonometric identity $\sec^2 A - 1 = \tan^2 A$:

$$\text{LHS} = \frac{2\sec A}{\sqrt{\tan^2 A}} = \frac{2\sec A}{\tan A}$$

Step 6: Simplify using the definition $\tan A = \frac{\sin A}{\cos A}$ and $\sec A = \frac{1}{\cos A}$:

$$\text{LHS} = \frac{2\sec A}{\tan A} = \frac{2 \cdot \frac{1}{\cos A}}{\frac{\sin A}{\cos A}} = \frac{2}{\sin A} = 2\csc A$$

Step 7: Conclude:

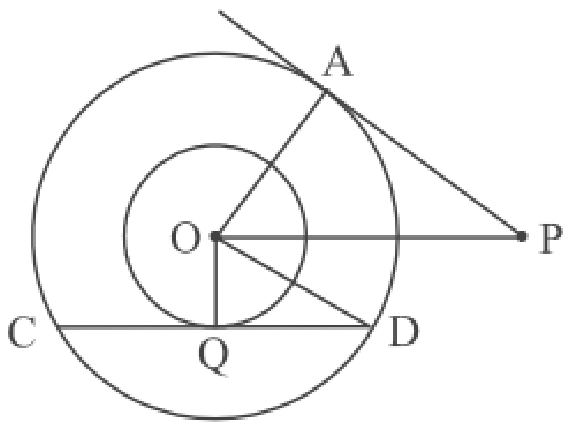
$$\text{LHS} = \text{RHS}$$

Final Answer: $\frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}} = 2 \csc A.$

Quick Tip

Use trigonometric identities like $\sec^2 A - 1 = \tan^2 A$ and simplify systematically to prove equations. Always simplify the numerator and denominator separately before substitution.

27: (A) In two concentric circles, the radii $OA = r$ cm and $OQ = 6$ cm, as shown in the figure. Chord CD of the larger circle is a tangent to the smaller circle at Q. PA is tangent to the larger circle. If $PA = 16$ cm and $OP = 20$ cm, find the length of CD.



Correct Answer: $CD = 12\sqrt{3}\,\text{cm}$

Solution:

In this problem, we are given two concentric circles. The radius of the smaller circle is $OQ = 6$ cm, and the radius of the larger circle is $OA = r$ cm. The chord CD of the larger circle is tangent to the smaller circle at point Q, and we are asked to find the length of CD.

We are given the following information:

- $PA = 16$ cm, where PA is the tangent to the larger circle at point A.
- $OP = 20$ cm, where O is the center of both circles, and P is a point outside the larger circle.

Now, let's use the property that the tangent to a circle from an external point is perpendicular to the radius at the point of tangency. Therefore, the length of PA is perpendicular to the radius OA.

We can now apply the Pythagorean theorem to the right triangle OPA, where $OP = 20\,\text{cm}$ and $PA = 16\,\text{cm}$:

$$OP^2 = OA^2 + PA^2$$

Substituting the given values:

$$20^2 = r^2 + 16^2$$

$$400 = r^2 + 256$$

$$r^2 = 144$$

$$r = 12\,\text{cm}$$

Now, we know the radius $OA = 12\,\text{cm}$.

Next, let's use the property of tangents and the fact that the chord CD of the larger circle is tangent to the smaller circle at Q. From geometry, the length of the chord CD can be found using the following formula:

$$CD = 2\sqrt{OP^2 - OQ^2}$$

Substituting the known values $OP = 20\,\text{cm}$ and $OQ = 6\,\text{cm}$:

$$CD = 2\sqrt{20^2 - 6^2}$$

$$CD = 2\sqrt{400 - 36}$$

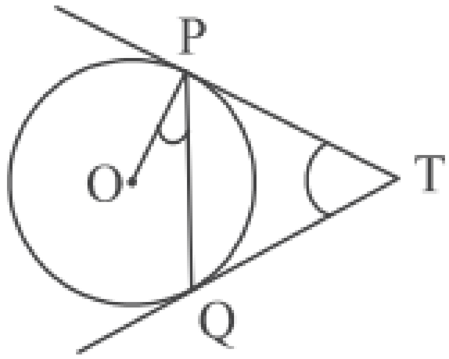
$$CD = 2\sqrt{364}$$

$$CD = 12\sqrt{3}\,\text{cm}$$

Quick Tip

In problems involving tangents and secants to circles, use the property that the tangent at any point on a circle is perpendicular to the radius at that point. This helps simplify calculations involving distances and lengths.

27(B): In the given figure, two tangents PT and QT are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Solution:

- Let PT and QT be two tangents drawn from the external point T to the circle with center O.
- Since PT and QT are tangents, the angle between a tangent and the radius is always $90°$, so:

$$\angle OTP = \angle OTQ = 90°$$

- Also, we know that $\angle PTQ = \angle OTP + \angle OTQ$, which means:

$$\angle PTQ = 90° + 90° = 180°$$

- Therefore, $\angle PTQ = 2\angle OPQ$.

Quick Tip

In problems involving tangents to a circle, use the property that the angle between a tangent and the radius at the point of contact is always 90°. This can help you solve geometric problems involving tangents.

28(A): A solid is in the form of a cylinder with hemispherical ends of the same radii. The total height of the solid is 20 cm and the diameter of the cylinder is 14 cm. Find the surface area of the solid.

Correct Answer: Surface Area of Solid is 880cm^2

Solution:

- The solid consists of a cylindrical part and two hemispherical ends. The total height of the solid is the sum of the height of the cylinder and the height of the two hemispheres. - Let the radius of the cylinder be $r = \frac{14}{2} = 7\,\text{cm}$. - The height of the cylinder is $h = 20 - 2r = 20 - 2(7) = 6\,\text{cm}$. - Surface area of the solid is the sum of the curved surface area of the cylinder and the surface area of the two hemispheres:

$$\text{Surface Area} = 2\pi r^2 + 2\pi rh + 2\pi r^2$$

Substituting values:

$$\text{Surface Area} = 2\pi(7)^2 + 2\pi(7)(6) + 2\pi(7)^2 = 2\pi(49) + 2\pi(42) + 2\pi(49)$$

$$\text{Surface Area} = 2\pi(49 + 42 + 49) = 2\pi(140) = 280\pi\,\text{cm}^2$$

Therefore, the surface area is:

$$\text{Surface Area} = 280\pi\,\text{cm}^2 \approx 880\,\text{cm}^2$$

Quick Tip

When solving problems involving solids with hemispherical ends, break the problem into parts: calculate the surface area of the cylinder and the hemispheres separately, then combine the results.

28(B): A juice glass is cylindrical in shape with a hemispherical raised-up portion at the bottom. The inner diameter of the glass is 10 cm and its height is 14 cm. Find the capacity of the glass. (use $\pi = 3.14$)

Solution:

Radius of the glass $r = \frac{10}{2} = 5\,\text{cm}$

Capacity of glass = volume of cylinder - volume of hemisphere

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$\text{Capacity of glass} = \pi \times 5^2 \times 14 - \frac{2}{3}\pi \times 5^3$$

$$= 3.14 \times 5 \times 5 \times 14 - \frac{2}{3} \times 3.14 \times 5 \times 5 \times 5$$

$$= 2512\,\text{cm}^3 \text{ or } 837.33\,\text{cm}^3 \text{ (approx)}$$

Correct Answer: $837.33\,\text{cm}^3$

Quick Tip

For solids with hemispherical and cylindrical parts, calculate the volumes of each part separately. For the hemisphere, use the formula $\frac{2}{3}\pi r^3$ and for the cylinder, use $\pi r^2 h$.

29: Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time?

Correct Answer: 1:40 PM.

Solution:

The alarm clocks will beep together again at the Least Common Multiple (LCM) of 20 and 25 minutes.

$$\text{LCM of 20 and 25} = 100\,\text{minutes}$$

Thus, the next time they beep together will be 100 minutes after 12:00 PM, which is 1 hour and 40 minutes later, or 1:40 PM.

Quick Tip

To find when two events happen together again, calculate the least common multiple (LCM) of their intervals. This will give you the time after which both events will happen together again.

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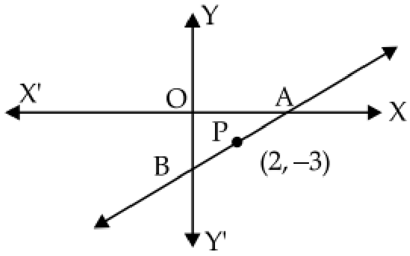
30. The line AB intersects the x-axis at A and the y-axis at B. The point $P(2, 3)$ lies on AB such that $AP : PB = 3 : 1$. Find the coordinates of A and B.



Solution:

Step 1: Assume the coordinates of A and B:

- Since A lies on the x-axis, its coordinates are $(x, 0)$. - Since B lies on the y-axis, its coordinates are $(0, y)$.

Step 2: Use the section formula:

The coordinates of $P(2, -3)$ divide AB in the ratio $AP : PB = 3 : 1$. By the section formula:

$$P(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2} \right)$$

Here: - $m_1 = 3, m_2 = 1$, - $A(x, 0)$, $B(0, y)$, - $P(2, -3)$.

Step 3: Substitute into the section formula: For the x-coordinate:

$$2 = \frac{1 \cdot x + 3 \cdot 0}{3 + 1} = \frac{x}{4} \quad \Rightarrow \quad x = 8$$

For the y-coordinate:

$$-3 = \frac{1 \cdot 0 + 3 \cdot y}{3 + 1} = \frac{3y}{4} \quad \Rightarrow \quad y = -4$$

Step 4: Write the coordinates of A and B:

- Coordinates of A are $(x, 0) = (8, 0)$, - Coordinates of B are $(0, y) = (0, -4)$.

Final Answer: The coordinates of A are $(8, 0)$, and the coordinates of B are $(0, -4)$.

Quick Tip

For problems involving division of a line segment, always use the section formula:

$$(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right)$$

31: The greater of two supplementary angles exceeds the smaller by $18°$. Find the measures of these two angles.

Correct Answer: The two angles are $99°$ and $81°$.

Solution:

Let the smaller angle be x. Then, the larger angle is $x + 18°$.

Since the angles are supplementary, their sum is $180°$:

$$x + (x + 18°) = 180°$$

Simplifying:

$$2x + 18° = 180° \implies 2x = 162° \implies x = 81°$$

Thus, the smaller angle is $81°$, and the larger angle is:

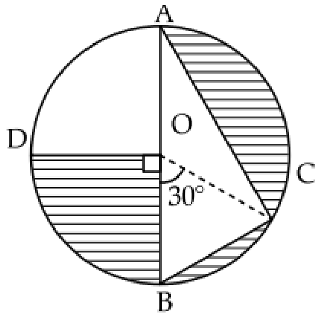
$$81° + 18° = 99°$$

Quick Tip

For supplementary angles, the sum of two angles is always $180°$. Use this property to find the value of one angle when the difference between the angles is given.

Section D

32. O is the center of the circle. If $AC = 28$ cm, $BC = 21$ cm, $\angle BOD = 90°$ and $\angle BOC = 30°$, then find the area of the shaded region given in the figure.



Solution:

Assuming AOB to be a straight line and hence the diameter of the circle.

$$\angle ACB = 90^\circ$$

Then in $\triangle ACB$,

$$AC^2 + BC^2 = 28^2 + 21^2 = (35)^2 = AB^2$$

Thus, $AB = 35\,\text{cm}$ is the diameter, and

$$r = \frac{35}{2}\,\text{cm}$$

Now, for the area of the shaded region:

$$\text{Area of shaded region} = \text{area of quadrant} + \left(\frac{1}{2} \times \pi r^2 - \text{area of } \triangle ACB\right)$$

Substitute the values:

$$= \left(\frac{3}{4} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) - \frac{1}{2} \times 28 \times 21$$

Simplifying this:

$$= 721.9 - 294 = 427.9\,\text{(approx.)}$$

Thus, the area of the shaded region is 427.9 sq. cm.

Quick Tip

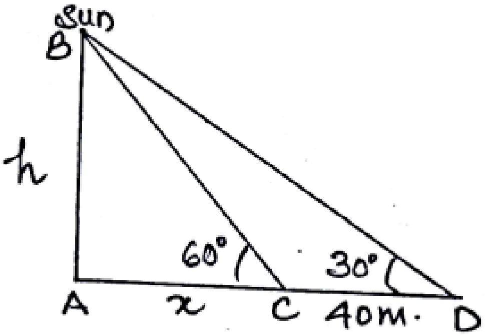
For problems involving circles, always simplify using πr^2 and remember geometric area formulas like triangles and sectors to calculate composite areas.

33(A): The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower and the length of the original shadow. (use $\sqrt{3} = 1.73$)

Correct Answer:

$$\text{Height of the tower} = 34.64\,\text{m}, \text{ Length of the shadow} = 20\,\text{m}$$

Solution:



- Let h be the height of the tower and x be the length of the original shadow. - From the first situation (altitude 30°):

$$\tan 30^\circ = \frac{h}{x+40}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$
$$h = \frac{x+40}{\sqrt{3}}$$

- From the second situation (altitude 60°):

$$\tan 60^\circ = \frac{h}{x}$$
$$\sqrt{3} = \frac{h}{x}$$
$$h = \sqrt{3}x$$

- Equating the two expressions for h:

$$\frac{x+40}{\sqrt{3}} = \sqrt{3}x$$

Solving:

$$x + 40 = 3x$$

$$40 = 2x$$

$$x = 20$$

- Therefore, the height of the tower is:

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \approx 34.64\,\text{m}$$

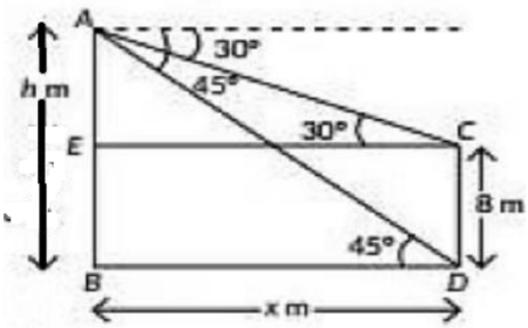
- The length of the original shadow is $x = 20\,\text{m}$.

Quick Tip

In problems involving shadows, use trigonometric ratios like $\tan\theta = \frac{\text{height}}{\text{shadow}}$ and set up equations to solve for unknowns.

33(B): The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings. (use $\sqrt{3} = 1.73$)

Solution:



Let the height of the multi-storeyed building be h, and the distance between the two buildings be d.

1. Using the angle of depression of 45°:

$$\tan(45^\circ) = \frac{8}{d_1}$$

Since $\tan(45^\circ) = 1$, we have:

$$d_1 = 8\,\text{m}$$

This is the horizontal distance from the base of the multi-storeyed building to the base of the 8 m tall building.

2. Using the angle of depression of 30°:

$$\tan(30^\circ) = \frac{h - 8}{d_1 + 14}$$

Substituting $\tan(30^\circ) = \frac{1}{\sqrt{3}}$ and $d_1 = 8$:

$$\frac{1}{\sqrt{3}} = \frac{h - 8}{8 + 14}$$

Simplifying:

$$\frac{1}{\sqrt{3}} = \frac{h-8}{22}$$

$$h - 8 = \frac{22}{\sqrt{3}} \approx 12.67$$

Therefore:

$$h = 8 + 12.67 = 20.67\,\text{m}$$

Thus, the height of the multi-storeyed building is approximately 20.67 m, and the distance between the two buildings is 14 m.

Quick Tip

When solving for heights using angles of depression, use the basic trigonometric formula $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$. If the angles involve multiple objects, break the problem into simpler parts and solve systematically.

35(A): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Solution:

Given: In $\triangle ABC$, $DE \parallel BC$ **To Prove:** $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, DC. Draw $DM \perp AC$ and $EN \perp AB$.

Proof:

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{1}{2} \times \text{AD} \times \text{EN} \div \frac{1}{2} \times \text{DB} \times \text{EN} = \frac{AD}{DB} \qquad \text{(i)}$$

and

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DCE)} = \frac{1}{2} \times \text{AE} \times \text{DM} \div \frac{1}{2} \times \text{EC} \times \text{DM} = \frac{AE}{EC} \qquad \text{(ii)}$$

Conclusion:

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels DE and BC.

Thus, $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$ (iii)

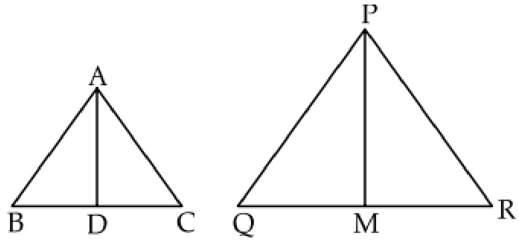
From (i), (ii), and (iii), we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

> **Quick Tip**
>
> The Basic Proportionality Theorem is helpful when a line divides two sides of a triangle proportionally. Remember to use it when solving geometry problems involving parallel lines.

34(B): Sides AB and BC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.



Solution:

$$\triangle ADB \cong \triangle EDC \quad \Rightarrow \quad AB = CE, \text{ similarly } PQ = RN$$

Given:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2}{PN} \quad \Rightarrow \quad \triangle AEC \sim \triangle PNR$$

$$\Rightarrow \angle 1 = \angle 2, \quad \text{similarly } \angle 3 = \angle 4$$

Therefore,

$$\angle 1 + \angle 3 = \angle 2 + \angle 4 \quad \Rightarrow \quad \angle BAC = \angle QPR$$

Also,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{(given)}$$

Therefore,

$$\triangle ABC \sim \triangle PQR$$

Quick Tip

When solving problems involving similarity of triangles, remember the similarity criteria such as SSS, SAS, and AA. These are essential for proving the similarity of triangles.

---

35. In an A.P. if $S_n = 4n^2 - n$, then:

(i) Find the first term and common difference.

(ii) Write the A.P.

(iii) Which term of the A.P. is 107?

Solution:

We are given that the n-th term sum of the A.P. is $S_n = 4n^2 - n$.

(i) Find the first term and common difference The first term of the A.P., a_1, is simply S_1, the sum of the first term. Thus:

$$S_1 = 4(1)^2 - 1 = 4 - 1 = 3.$$

So, the first term $a_1 = 3$.

To find the common difference, we use the fact that the difference between consecutive sums gives the n-th term:

$$a_n = S_n - S_{n-1}.$$

Let's find a_2 and a_3 to identify the common difference.

$$S_2 = 4(2)^2 - 2 = 16 - 2 = 14, \quad S_1 = 3.$$

Thus:

$$a_2 = S_2 - S_1 = 14 - 3 = 11.$$

The common difference d is the difference between consecutive terms:

$$d = a_2 - a_1 = 11 - 3 = 8.$$

Thus, the common difference is $d = 8$.

(ii) Write the A.P. The first term is $a_1 = 3$ and the common difference is $d = 8$. So, the A.P. is:

$$3, 11, 19, 27, 35, \ldots$$

(iii) Which term of the A.P. is 107? To find which term is 107, we use the formula for the n-th term of an A.P.:

$$a_n = a_1 + (n-1) \cdot d.$$

Substitute the known values $a_1 = 3$, $d = 8$, and $a_n = 107$:

$$107 = 3 + (n-1) \cdot 8.$$

Simplify and solve for n:

$$107 - 3 = (n-1) \cdot 8, \quad 104 = (n-1) \cdot 8,$$

$$n - 1 = \frac{104}{8} = 13, \quad n = 14.$$

Thus, the 14th term of the A.P. is 107.

Quick Tip

In an arithmetic progression, the common difference d can be found using the difference between consecutive terms. The n-th term can be found using the formula $a_n = a_1 + (n-1) \cdot d$.

Section E

36: Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm.

The length of the leaves from different plants are recorded in the following table.

Length (in mm)	**Number of Leaves**
$70 - 80$	3
$80 - 90$	5
$90 - 100$	9
$100 - 110$	12
$110 - 120$	5
$120 - 130$	4
$130 - 140$	2

(i) Write the median class of the data.

(ii) How many leaves are of length equal to or more than 10 cm?

(iii) (a) Find the median of the data.

OR

(b) Write the modal class and find the mode of the data.

Correct Answer: (i) Median Class: 100-110

(ii) Number of leaves ≥ 10 cm: 23

(iii) Median: 102.5 mm, Mode: 103 mm

Solution:

(i) Meadian Class

The cumulative frequency is calculated as follows:

Length (in mm)	Number of Leaves	Cumulative Frequency (CF)
$70-80$	3	3
$80-90$	5	8
$90-100$	9	17
$100-110$	12	29
$110-120$	5	34
$120-130$	4	38
$130-140$	2	40

The total number of leaves is 40, so the median will lie at the 20th position. From the cumulative frequency, the 20th leaf lies in the class 100-110, so the median class is 100-110.

(ii) Leaves of length equal to or more than 10 cm (100 mm):

The relevant classes are:

- 100-110: 12 leaves
- 110-120: 5 leaves
- 120-130: 4 leaves
- 130-140: 2 leaves

Total = 12 + 5 + 4 + 2 = 23 leaves of length ≥ 10 cm.

(iii) (a) To find the median, use the formula for grouped data:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

Where:

- $L = 100$ (lower boundary of the median class)
- $N = 40$ (total number of leaves)
- $F = 17$ (cumulative frequency before median class)
- $f = 12$ (frequency of the median class)
- $h = 10$ (class width)

$$\text{Median} = 100 + \left(\frac{20 - 17}{12}\right) \times 10 = 100 + 2.5 = 102.5\,\text{mm}$$

Thus, the Median is $102.5\,\text{mm}$.

(b) The modal class is the class with the highest frequency, which is 100-110 with 12 leaves. To find the mode, use the formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where:

- $L = 100$ (lower boundary of the modal class)
- $f_1 = 12$ (frequency of the modal class)
- $f_0 = 9$ (frequency of the class before the modal class)
- $f_2 = 5$ (frequency of the class after the modal class)
- $h = 10$ (class width)

$$\text{Mode} = 100 + \left(\frac{12 - 9}{2 \times 12 - 9 - 5}\right) \times 10 = 100 + \left(\frac{3}{10}\right) \times 10 = 100 + 3 = 103\,\text{mm}$$

Thus, the Mode is $103\,\text{mm}$.

Quick Tip

For grouped data, the median is the value that lies at the $\frac{N}{2}$-th position. The mode can be found using the formula for modal class when the frequency distribution is unimodal.

37: The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with center O. AP and AQ are tangents to the circle at P and Q respectively, such that $AP = 30$ cm and $\angle PAQ = 60°$. Based on the above information, answer the following questions:

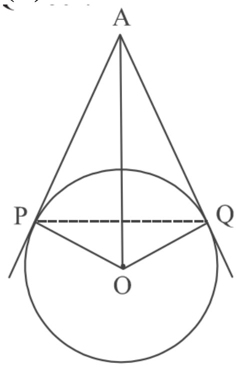
(i) Find the length of PQ.

(ii) Find $m\angle POQ$.

(iii) (a) Find the length of OA.

OR

(b) Find the radius of the mirror.



Correct Answer: (i) Length of PQ: 30 cm

(ii) $m\angle POQ$: 120°

(iii) (a) Length of OA: 60 cm

OR (b) Radius of the mirror: 30 cm

Solution:

(i) Find the length of PQ:

We are given that $AP = AQ = 30$ cm, and the angle between the tangents, $\angle PAQ = 60°$.

To find PQ, we can use the law of cosines in triangle PAQ, where:

$$PQ^2 = AP^2 + AQ^2 - 2 \cdot AP \cdot AQ \cdot \cos(\angle PAQ)$$

Substituting the given values:

$$PQ^2 = 30^2 + 30^2 - 2 \cdot 30 \cdot 30 \cdot \cos(60°)$$

Since $\cos(60°) = 0.5$:

$$PQ^2 = 900 + 900 - 2 \cdot 30 \cdot 30 \cdot 0.5$$

$$PQ^2 = 900 + 900 - 900 = 900$$

$$PQ = \sqrt{900} = 30\,\text{cm}$$

Thus, the length of PQ is $30\,$cm.

(ii) Find $m\angle POQ$:

Since AP and AQ are tangents to the circle from the point A, the angle between the tangents at P and Q is equal to the angle at the center of the circle subtended by the chord PQ. This means that:

$$\angle POQ = 2 \times \angle PAQ$$

Substituting the given value of $\angle PAQ$:

$$\angle POQ = 2 \times 60° = 120°$$

Thus, $m\angle POQ = 120°$.

(iii) (a) Find the length of OA:

To find the length of OA, we can use the law of cosines in triangle OAP, where O is the center of the circle, P is the point of tangency, and A is the external point.

Since $\angle OAP = 90°$ (the angle between the radius and the tangent is always 90°), triangle OAP is a right triangle. Using the Pythagorean theorem:

$$OA^2 = OP^2 + AP^2$$

We know that OP is the radius of the circle, and $AP = 30\,$cm. Let's denote the radius by r. Thus:

$$OA^2 = r^2 + 30^2$$

But, OA is the hypotenuse of the right triangle OAP, and we know that $\angle PAQ = 60°$, which makes the distance from A to the center O (i.e., OA) double the radius:

$$OA = 2r$$

Substitute $OA = 2r$ into the Pythagorean theorem:

$$(2r)^2 = r^2 + 30^2$$

$$4r^2 = r^2 + 900$$

$$3r^2 = 900$$

$$r^2 = 300$$

$$r = \sqrt{300} = 10\sqrt{3} \approx 17.32\,\text{cm}$$

Thus, OA is twice the radius:

$$OA = 2 \times 17.32 = 34.64\,\text{cm}$$

OR (b) Find the radius of the mirror:

The radius r of the mirror is $\boxed{17.32\,\text{cm}}$.

Quick Tip

In problems involving tangents to circles, the angle between the tangents at the external point is double the angle subtended at the center of the circle. This property is useful for solving for angles and lengths in related triangles.

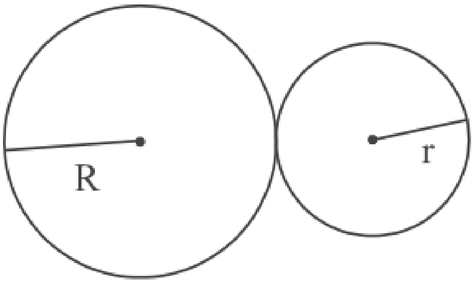
38: To keep the lawn green and cool, Sadhna uses water sprinklers which rotate in a circular shape and cover a particular area. The diagram below shows the circular areas

covered by two sprinklers: Two circles touch externally. The sum of their areas is 130 sq m and the distance between their centres is 14 m.

(i) Obtain a quadratic equation involving R and r from the above information.

(ii) Write a quadratic equation involving only r.

(iii)(a) Find the radius r and the corresponding area irrigated. (iii)(b) Find the radius R and the corresponding area irrigated.



(R > r)

Solution:

(i) From the given information:

$$\pi R^2 + \pi r^2 = 130\pi \quad \Rightarrow \quad R^2 + r^2 = 130$$

The distance between the centers is given as:

$$R + r = 14$$

(ii) Using $R + r = 14$, substitute $R = 14 - r$ into $R^2 + r^2 = 130$:

$$(14 - r)^2 + r^2 = 130$$

Expanding:

$$196 - 28r + r^2 + r^2 = 130$$

Combining like terms:

$$2r^2 - 28r + 196 = 130$$

Simplifying:

$$2r^2 - 28r + 66 = 0$$

Dividing through by 2:

$$r^2 - 14r + 33 = 0$$

(iii) (a) Solve the quadratic equation:

$$r^2 - 14r + 33 = 0 \quad \Rightarrow \quad (r-11)(r-3) = 0$$

$$\Rightarrow \quad r = 3\,(\text{as } r < R), \quad r \neq 11$$

The corresponding area irrigated is:

$$\pi r^2 = \pi(3^2) = 9\pi\,\text{m}^2$$

(iii) (b) Using $R + r = 14$, substitute $r = 3$ to find R:

$$R = 14 - 3 = 11$$

The corresponding area irrigated is:

$$\pi R^2 = \pi(11^2) = 121\pi\,\text{m}^2$$

Final Answer:

- Radius $r = 3$, corresponding area $9\pi\,\text{m}^2$.
- Radius $R = 11$, corresponding area $121\pi\,\text{m}^2$.

Quick Tip

When solving geometric problems involving circles and tangents, ensure all relationships between the variables are correctly interpreted, especially when combining multiple equations. Checking the feasibility of each step is crucial.
