CBSE Class X Mathematics (Basic) Set 1 (430/5/1)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each. Internal choice is provided in 2 marks questions in each case study.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
9. Draw neat diagrams wherever required. Take p = 7 22 wherever required, if not stated.
10. Use of calculator is not allowed.

Section A

Section A

1.HCF $\times$ LCM for the numbers 40 and 30 is:

(a) 12

(b) 120

(c) 1200

(d) 40

Correct Answer: (c) 1200

Solution:

1. The product of HCF and LCM of two numbers is equal to the product of the numbers:

$$\text{HCF} \times \text{LCM} = 40 \times 30 = 1200$$

Quick Tip

Use the formula HCF $\times$ LCM $=$ Product of the numbers to quickly calculate the answer.

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2. The roots of the quadratic equation $x^2 + 3x - 10 = 0$ are:

(a) 5, 2

(b) -5, 2

(c) 5, -2

(d) -5, -2

Correct Answer: (b) -5, 2

Solution:

1. Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $x^2 + 3x - 10 = 0$, $a = 1$, $b = 3$, $c = -10$. Substitute these values:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 40}}{2} = \frac{-3 \pm \sqrt{49}}{2}$$

2. Solve for the two roots:

$$x = \frac{-3+7}{2} = 2, \quad x = \frac{-3-7}{2} = -5$$

3. Therefore, the roots are -5 and 2.

Quick Tip

For quadratic equations, always use the quadratic formula and verify by substitution.

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3. The pair of linear equations $2kx + 5y = 7$, $6x + 5y = 11$ have a unique solution, if:

(a) $k \neq 3$

(b) $k \neq -3$

(c) $k \neq \frac{1}{3}$

(d) $k \neq -\frac{1}{3}$

Correct Answer: (a) $k \neq 3$

Solution:

1. For two linear equations to have a unique solution, their slopes must not be equal:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

2. Compare coefficients:

$$\frac{2k}{6} \neq \frac{5}{5} \quad \Rightarrow \quad \frac{k}{3} \neq 1 \quad \Rightarrow \quad k \neq 3$$

Quick Tip

For a unique solution, check that the ratios of coefficients of x and y are not equal.

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4. If the mean and mode of a frequency distribution are 28 and 16 respectively, then its median is:

(a) 22

(b) 23.5

(c) 24

(d) 24.5

Correct Answer: (c) 24

Solution:

1. Use the empirical relationship between mean, median, and mode:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

2. Substitute the given values:

$$28 - 16 = 3(28 - \text{Median}) \quad \Rightarrow \quad 12 = 84 - 3 \times \text{Median}$$

$$3 \times \text{Median} = 84 - 12 = 72 \quad \Rightarrow \quad \text{Median} = \frac{72}{3} = 24$$

Quick Tip

Remember the formula $\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$ for frequency distributions.

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5. A die is rolled once. What is the probability of getting an odd prime number?

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

Correct Answer: (b) $\frac{1}{3}$

Solution:

1. Odd prime numbers on a die are 3 and 5.

2. Total possible outcomes when rolling a die are 6.

3. Probability of getting an odd prime number:

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Quick Tip

Prime numbers on a die are 2, 3, and 5. For "odd primes," only 3 and 5 qualify.

6: If the area of a sector of a circle is $\frac{1}{8}$ of the area of the circle, then the central angle of the sector is:

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Correct Answer: (b) 45°

Solution:

1. The area of a sector is proportional to its central angle. Since the area of the sector is $\frac{1}{8}$ of the total circle:

$$\frac{\theta}{360^\circ} = \frac{1}{8}$$

2. Solving for θ:

$$\theta = \frac{360^\circ}{8} = 45^\circ$$

Quick Tip

For sectors, use the formula $\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\theta}{360^\circ}$ to relate area and central angle.

7: Prime factorization of 424 is:

(a) $2 \times 53 \times 4$

(b) $2 \times 53 \times 2$

(c) $2^3 \times 53$

(d) $2^4 \times 53$

Correct Answer: (c) $2^3 \times 53$

Solution:

1. Perform prime factorization of 424:

$$424 \div 2 = 212, \quad 212 \div 2 = 106, \quad 106 \div 2 = 53$$

2. Therefore:

$$424 = 2^3 \times 53$$

Quick Tip

Divide the number successively by the smallest primes (2, 3, 5, etc.) until only a prime factor remains.

8: If x is a whole number, then 8^x ends with an even digit, except for which value of x?

(a) 6

(b) 4

(c) 2

(d) 0

Correct Answer: (d) 0

Solution:

1. For $x = 0$, $8^x = 8^0 = 1$, which ends in an odd digit.

2. For all other values of x, 8^x results in powers of 8, which always end in even digits.

3. Hence, the exception is when $x = 0$.

Quick Tip

Check special cases like $x = 0$ in problems involving patterns of powers.

9: A tree casts a shadow 7 m long on the ground when the angle of elevation of the Sun is $45°$. The height of the tree is:

(a) $\frac{7}{\sqrt{3}}$ m

(b) $\frac{7\sqrt{3}}{3}$ m

(c) 7 m

(d) 3.5 m

Correct Answer: (c) 7 m

Solution:

1. For an angle of elevation of $45°$, the height of the tree is equal to the length of the shadow:

$$\text{Height} = \text{Shadow Length} = 7\,\text{m}$$

Quick Tip

For $45°$, height and shadow length are always equal in right triangles.

10: $\frac{5}{\cot^2\theta} - \frac{5}{\cos^2\theta}$ **is equal to:**

(a) 1

(b) 5

(c) -5

(d) 0

Correct Answer: (c) -5

Solution:

1. Rewrite $\cot^2\theta$ in terms of $\sin\theta$ and $\cos\theta$:

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

2. Substitute into the expression:

$$\frac{5}{\cot^2\theta} - \frac{5}{\cos^2\theta} = 5 \times \frac{\sin^2\theta}{\cos^2\theta} - \frac{5}{\cos^2\theta}$$

3. Factorize:

$$\frac{5(\sin^2\theta - 1)}{\cos^2\theta}$$

4. Using the identity $\sin^2\theta - 1 = -\cos^2\theta$:

$$\frac{5(-\cos^2\theta)}{\cos^2\theta} = -5$$

Quick Tip

Always use trigonometric identities like $\sin^2\theta + \cos^2\theta = 1$ to simplify expressions.

11: If $\tan^2\theta = 3$, where θ is an acute angle, then the value of θ is:

(a) $30°$

(b) 60°

(c) 0°

(d) 45°

Correct Answer: (b) 60°

Solution:

1. Given $\tan^2\theta = 3$, take the square root to find $\tan\theta$:

$$\tan\theta = \sqrt{3}$$

2. The angle whose tangent is $\sqrt{3}$ is $\theta = 60^\circ$ (since θ is acute).

3. Therefore, $\theta = 60^\circ$.

> **Quick Tip**
>
> Remember the standard trigonometric values: $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\tan 45^\circ = 1$, and $\tan 60^\circ = \sqrt{3}$.

12: The length of an arc of a circle with radius 12 cm is 10π cm. The central angle subtended by this arc at the centre, is:

(a) 120°

(b) 6°

(c) 75°

(d) 150°

Correct Answer: (d) 150°

Solution:

1. Use the formula for the length of an arc:

$$\text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

Substituting the given values:

$$10\pi = \frac{\theta}{360^\circ} \times 2\pi \times 12$$

2. Simplify:

$$10\pi = \frac{\theta}{360^\circ} \times 24\pi \quad \Rightarrow \quad 10 = \frac{\theta \times 24}{360}$$

$$\theta = \frac{10 \times 360}{24} = 150^\circ$$

Quick Tip

To calculate the central angle, rearrange the arc length formula and solve for θ.

13: If the angle between the two radii of a circle is 130°, then the angle between the tangents at the ends of these radii, is:

(a) 50°

(b) 60°

(c) 90°

(d) 130°

Correct Answer: (a) 50°

Solution:

1. The angle between the tangents is supplementary to the angle between the radii:

$$\text{Angle between tangents} = 180^\circ - \text{Angle between radii}$$

2. Substituting the given angle:

$$\text{Angle between tangents} = 180^\circ - 130^\circ = 50^\circ$$

Quick Tip

The angle between tangents at the endpoints of two radii is always the supplement of the angle between the radii.

14: The discriminant of the quadratic equation $x^2 - 4x + 3 = 0$ is:

(a) 28

(b) -8

(c) 4

(d) 2

Correct Answer: (c) 4

Solution:

1. The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$D = b^2 - 4ac$$

2. Substituting $a = 1$, $b = -4$, $c = 3$:

$$D = (-4)^2 - 4(1)(3) = 16 - 12 = 4$$

Quick Tip

The discriminant determines the nature of roots: $D > 0$ (real and distinct), $D = 0$ (real and equal), $D < 0$ (complex roots).

15: The mid-point of the line segment AB joining $A(-2, 8)$ and $B(-6, 4)$ is:

(a) $(2, 6)$

(b) $(-4, 12)$

(c) $(-4, 6)$

(d) $(4, -6)$

Correct Answer: (c) $(-4, 6)$

Solution:

1. The formula for the mid-point of a line segment joining two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

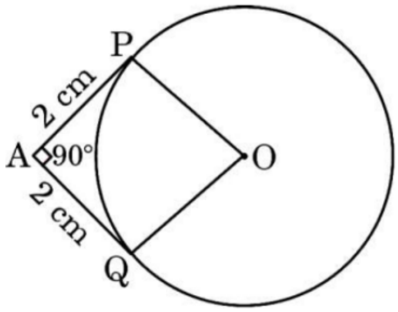
2. Substituting $A(-2, 8)$ and $B(-6, 4)$:

$$\text{Mid-point} = \left(\frac{-2 + (-6)}{2}, \frac{8 + 4}{2}\right) = \left(\frac{-8}{2}, \frac{12}{2}\right) = (-4, 6)$$

Quick Tip

For mid-point calculations, take the average of the x-coordinates and the y-coordinates separately.

16: AP and AQ are tangents drawn from an external point A to a circle with centre O and inclined to each other at an angle of $90°$. If the length of each tangent is 2 cm, then the radius of the circle is:



(a) 4 cm

(b) 2 cm

(c) $2\sqrt{2}$ cm

(d) 1 cm

Correct Answer: (b) 2 cm

Solution:

Step 1: Draw the figure and note that $\triangle OAP$ is a right triangle. Here: - OA is the hypotenuse, - OP is the radius, and - $AP = AQ = 2$ cm is the tangent.

Step 2: Apply the Pythagoras theorem:

$$OA^2 = OP^2 + AP^2$$

Substitute $AP = 2$:

$$OA^2 = OP^2 + 2^2$$

Step 3: Express OA in terms of OP. Since $OA = OP + 2$, let $OP = r$. Then:

$$(r + 2)^2 = r^2 + 4$$

Step 4: Expand and simplify:

$$r^2 + 4r + 4 = r^2 + 4 \quad \Rightarrow \quad 4r = 0 \quad \Rightarrow \quad r = 2$$

Step 5: Therefore, the radius is 2 cm.

Quick Tip

For tangents meeting at a 90° angle, use the Pythagoras theorem with radius and tangent length.

17: It is given that $\triangle ABC \sim \triangle DEF$. If $\angle A = 55^\circ$, $\angle E = 45^\circ$, then $\angle C$ is:

(a) 80°

(b) 90°

(c) 55°

(d) 45°

Correct Answer: (a) 80°

Solution:

Step 1: In $\triangle ABC$, the sum of angles is 180°. Using $\triangle ABC \sim \triangle DEF$, corresponding angles are equal.

Step 2: Write the angle sum property:

$$\angle A + \angle B + \angle C = 180^\circ$$

Step 3: Substitute $\angle A = 55^\circ$ and $\angle B = \angle E = 45^\circ$:

$$55^\circ + 45^\circ + \angle C = 180^\circ$$

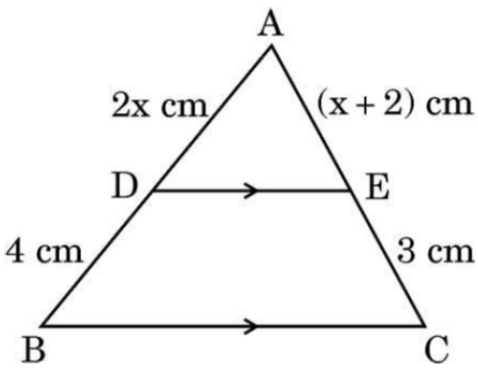
Step 4: Simplify for $\angle C$:

$$\angle C = 180^\circ - (55^\circ + 45^\circ) = 80^\circ$$

Quick Tip

For similar triangles, use the angle sum property and corresponding angle relationships.

18: In the given figure, in $\triangle ABC$, $DE \parallel BC$. If $AD = 2x$ cm, $AE = (x + 2)$ cm, $DB = 4$ cm, $EC = 3$ cm, then the value of x is:



(a) 3

(b) 2

(c) 6

(d) 4

Correct Answer: (d) 4

Solution:

Step 1: Since $DE \parallel BC$, apply the Basic Proportionality Theorem (BPT):

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 2: Substitute the given values:

$$\frac{2x}{4} = \frac{x+2}{3}$$

Step 3: Cross-multiply:

$$3(2x) = 4(x+2)$$

Step 4: Expand and simplify:

$$6x = 4x + 8 \quad \Rightarrow \quad 2x = 8 \quad \Rightarrow \quad x = 4$$

Quick Tip

When parallel lines divide triangles, apply the Basic Proportionality Theorem (BPT).

19: Assertion (A): The probability of getting number 8 on rolling a die is zero (0).

Reason (R): The probability of an impossible event is zero (0).

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Step 1: Rolling a die produces outcomes $1, 2, 3, 4, 5, 6$. Since 8 is not a possible outcome, the probability is zero.

Step 2: The Reason (R) correctly explains why the Assertion (A) is true: The probability of an impossible event is zero.

Quick Tip

The probability of an impossible event is always zero.

20: Assertion (A): Common difference of the A.P. $5, 1, -3, -7, \ldots$ is 4.

Reason (R): Common difference of the A.P. $a_1, a_2, a_3, \ldots, a_n$ is obtained by $d = a_n - a_{n-1}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution:

Step 1: The common difference is given by:

$$d = a_2 - a_1 = 1 - 5 = -4$$

Hence, Assertion (A) is false.

Step 2: The formula $d = a_n - a_{n-1}$ is correct, making Reason (R) true.

Quick Tip

Verify assertions using the definitions or formulas directly to validate statements.

Section B

21: Evaluate: $\sin^2 30^\circ + \cos^2 45^\circ - \cos 0^\circ \cdot \tan 45^\circ$

Solution:

Step 1: Use the standard trigonometric values:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 0^\circ = 1, \quad \tan 45^\circ = 1$$

Step 2: Substitute these values into the given expression:

$$\sin^2 30^\circ + \cos^2 45^\circ - \cos 0^\circ \cdot \tan 45^\circ$$
$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 \times 1$$

Step 3: Simplify each term:

$$\sin^2 30^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \quad \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}, \quad \cos 0^\circ \cdot \tan 45^\circ = 1$$

Step 4: Add and subtract:

$$\sin^2 30^\circ + \cos^2 45^\circ - \cos 0^\circ \cdot \tan 45^\circ = \frac{1}{4} + \frac{1}{2} - 1$$
$$= \frac{1}{4} + \frac{2}{4} - \frac{4}{4} = \frac{3}{4} - \frac{4}{4} = -\frac{1}{4}$$

Final Answer: $-\frac{1}{4}$

Quick Tip

Always substitute the standard values for trigonometric functions before simplifying. Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ when needed.

22(a): Find the ratio in which the point $(3, y)$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Also, find the value of y.

Solution:

Step 1: Let the point $(3, y)$ divide the line segment in the ratio $k : 1$. Using the section formula:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}$$

Here, $(x_1, y_1) = (-2, -5)$ and $(x_2, y_2) = (6, 3)$.

Step 2: Substitute $x = 3$ in the formula for x:

$$3 = \frac{k(6) + (-2)}{k+1}$$

Simplify:

$$3(k+1) = 6k - 2 \quad \Rightarrow \quad 3k + 3 = 6k - 2 \quad \Rightarrow \quad 3k = 5 \quad \Rightarrow \quad k = \frac{5}{3}$$

Step 3: Substitute $k = \frac{5}{3}$ in the formula for y:

$$y = \frac{k(3) + (-5)}{k+1} = \frac{\frac{5}{3}(3) - 5}{\frac{5}{3} + 1}$$

Simplify the numerator and denominator:

$$y = \frac{5 - 5}{\frac{5}{3} + 1} = \frac{0}{\frac{8}{3}} = 0$$

Final Answer: The ratio is $5 : 3$ and $y = 0$.

Quick Tip

Use the section formula:

$$(x, y) = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right)$$

to divide line segments in a given ratio.

22(b): Find a point on the y-axis which is equidistant from the points $A(6, 5)$ and $B(-4, 3)$.

Solution:

Step 1: Let the point on the y-axis be $P(0, y)$. Since P is equidistant from A and B, we have:

$$PA = PB$$

$$PA^2 = PB^2$$

Step 2: Write the distance formulas:

$$PA^2 = (0-6)^2 + (y-5)^2, \quad PB^2 = (0-(-4))^2 + (y-3)^2$$

Simplify:

$$PA^2 = 6^2 + (y-5)^2 = 36 + (y^2 - 10y + 25)$$

$$PB^2 = 4^2 + (y-3)^2 = 16 + (y^2 - 6y + 9)$$

Step 3: Set $PA^2 = PB^2$:

$$36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

Step 4: Simplify and solve for y:

$$61 - 10y = 25 - 6y \quad \Rightarrow \quad 61 - 25 = 10y - 6y \quad \Rightarrow \quad 36 = 4y \quad \Rightarrow \quad y = 9$$

Final Answer: The point is $P(0, 9)$.

Quick Tip

To find equidistant points, equate the square of the distances from the point to each given point to eliminate square roots.

23: If $\frac{2}{3}$ is a root of the quadratic equation $kx^2 - x - 2 = 0$, then find the value of k.

Solution:

Step 1: Substitute $x = \frac{2}{3}$ into the quadratic equation $kx^2 - x - 2 = 0$:

$$k\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2 = 0$$

Step 2: Simplify each term:

$$k\left(\frac{4}{9}\right) - \frac{2}{3} - 2 = 0 \quad \Rightarrow \quad \frac{4k}{9} - \frac{2}{3} - 2 = 0$$

Step 3: Eliminate fractions by multiplying through by 9:

$$4k - 6 - 18 = 0 \quad \Rightarrow \quad 4k = 24 \quad \Rightarrow \quad k = 6$$

Final Answer: $k = 6$

> Quick Tip
>
> Substitute the root into the quadratic equation and simplify to find the unknown coefficient.

24: The length of a tangent drawn to a circle from a point A, at a distance of 10 cm from the centre of the circle, is 6 cm. Find the radius of the circle.

Solution:

Step 1: Use the Pythagoras theorem for the right triangle formed by the radius, tangent, and distance from the centre to the external point:

$$OA^2 = OP^2 + AP^2$$

Here, OA is the distance from the centre to the external point ($OA = 10$ cm), AP is the tangent length ($AP = 6$ cm), and OP is the radius.

Step 2: Substitute the known values:

$$10^2 = OP^2 + 6^2$$

Step 3: Simplify:

$$100 = OP^2 + 36 \quad \Rightarrow \quad OP^2 = 100 - 36 = 64 \quad \Rightarrow \quad OP = \sqrt{64} = 8$$

Final Answer: Radius = 8 cm

> Quick Tip
>
> For tangents drawn from an external point, apply the Pythagoras theorem: (Distance from centre)^2 $=$ (Radius)^2 $+$ (Tangent length)^2.

25(a): If α, β are zeroes of the quadratic polynomial $2x^2 + 7x + 5$, then find the value of $\alpha^2 + \beta^2 + \alpha\beta$.

Solution:

Step 1: From Vieta's formulas, for the polynomial $2x^2 + 7x + 5$:

$$\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{7}{2}, \quad \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{5}{2}$$

Step 2: Use the identity:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Step 3: Substitute the values of $\alpha + \beta$ and $\alpha\beta$:

$$\alpha^2 + \beta^2 = \left(-\frac{7}{2}\right)^2 - 2\left(\frac{5}{2}\right)$$

Simplify:

$$\alpha^2 + \beta^2 = \frac{49}{4} - \frac{10}{2} = \frac{49}{4} - \frac{20}{4} = \frac{29}{4}$$

Step 4: Add $\alpha\beta$ to the result:

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{29}{4} + \frac{5}{2} = \frac{29}{4} + \frac{10}{4} = \frac{39}{4}$$

Final Answer: $\frac{39}{4}$

Quick Tip

Use the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ to simplify expressions involving zeroes of a polynomial.

25(b): If one zero of the quadratic polynomial $6x^2 + 37x - (p - 2)$ is reciprocal of the other, then find the value of p.

Solution:

Step 1: Let the zeroes of the polynomial be α and $\frac{1}{\alpha}$. From Vieta's formulas:

$$\alpha + \frac{1}{\alpha} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{37}{6}, \quad \alpha \cdot \frac{1}{\alpha} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{p-2}{6}$$

Step 2: Simplify $\alpha \cdot \frac{1}{\alpha}$:

$$\alpha \cdot \frac{1}{\alpha} = 1 \quad \Rightarrow \quad \frac{p-2}{6} = 1$$

Step 3: Solve for p:

$$p - 2 = 6 \quad \Rightarrow \quad p = 8$$

Final Answer: $p = 8$

Quick Tip

For zeroes that are reciprocals, use the property $\alpha \cdot \frac{1}{\alpha} = 1$ and Vieta's formulas to solve for unknown coefficients.

OR

25(b): If one zero of the quadratic polynomial $6x^2 + 37x - (p-2)$ is reciprocal of the other, then find the value of p.

Solution:

Step 1: Let $p(x) = 6x^2 + 37x - (p-2)$, where $a = 6$, $b = 37$, and $c = -(p-2)$.

Let the zeroes of $p(x)$ be α and $\frac{1}{\alpha}$.

Step 2: From Vieta's formulas:

$$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$$

Substitute $a = 6$ and $c = -(p-2)$:

$$\alpha \cdot \frac{1}{\alpha} = \frac{-(p-2)}{6}$$

Step 3: Since $\alpha \cdot \frac{1}{\alpha} = 1$, equate:

$$1 = \frac{-(p-2)}{6}$$

Step 4: Solve for p:

$$6 \cdot 1 = -(p-2) \quad \Rightarrow \quad 6 = -p + 2 \quad \Rightarrow \quad p = 2 - 6 \quad \Rightarrow \quad p = -4$$

Final Answer: $p = -4$

Quick Tip

When zeroes are reciprocals, use the property $\alpha \cdot \frac{1}{\alpha} = 1$ and apply Vieta's formulas to relate coefficients.

Section C

26: Prove that $6 + 3\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number.

Solution:

Step 1: Assume that $6 + 3\sqrt{2}$ is a rational number. Let:

$$6 + 3\sqrt{2} = x \quad \text{where } x \text{ is a rational number.}$$

Step 2: Rearrange to isolate $\sqrt{2}$:

$$3\sqrt{2} = x - 6$$

$$\sqrt{2} = \frac{x - 6}{3}$$

Step 3: Analyze the result: - Since x is a rational number, and 6 and 3 are also rational numbers, $\frac{x-6}{3}$ must be a rational number. - Thus, $\sqrt{2}$ is a rational number.

Step 4: Contradiction: - This contradicts the given fact that $\sqrt{2}$ is an irrational number. - Therefore, our initial assumption that $6 + 3\sqrt{2}$ is rational must be incorrect.

Step 5: Conclusion:

$$6 + 3\sqrt{2} \quad \text{is an irrational number.}$$

Quick Tip

To prove a number is irrational, assume it is rational and derive a contradiction using the properties of rational and irrational numbers.

27: Find the zeroes of the quadratic polynomial $6x^2 - 7x - 3$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution:

Step 1: Write the quadratic polynomial:

$$p(x) = 6x^2 - 7x - 3$$

Step 2: Factorize the quadratic polynomial:

$$p(x) = 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$$

Step 3: Find the zeroes: - Set $2x - 3 = 0$:

$$x = \frac{3}{2}$$

- Set $3x + 1 = 0$:

$$x = -\frac{1}{3}$$

Thus, the zeroes are $\frac{3}{2}$ and $-\frac{1}{3}$.

Step 4: Verify the sum of the zeroes: - The sum of the zeroes is:

$$\frac{3}{2} + \left(-\frac{1}{3}\right) = \frac{9}{6} - \frac{2}{6} = \frac{7}{6}$$

- The coefficient of x is -7, and the coefficient of x^2 is 6. By Vieta's formula:

$$\text{Sum of zeroes} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-7}{6} = \frac{7}{6}$$

- The sum is verified.

Step 5: Verify the product of the zeroes: - The product of the zeroes is:

$$\frac{3}{2} \times \left(-\frac{1}{3}\right) = -\frac{3}{6} = -\frac{1}{2}$$

- The constant term is -3, and the coefficient of x^2 is 6. By Vieta's formula:

$$\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-3}{6} = -\frac{1}{2}$$

- The product is verified.

Step 6: Conclusion: - The sum and product of the zeroes match the relationships given by Vieta's formulas. Hence, the verification is complete.

Quick Tip

To verify zeroes of a quadratic polynomial, use Vieta's formulas:

$$\text{Sum of zeroes} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}, \quad \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

28: Prove that:

$$(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta.$$

Solution:

Step 1: Expand the left-hand side (LHS):

$$(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= \sin^2\theta + \csc^2\theta + 2\sin\theta\csc\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$$

Step 2: Use trigonometric identities:

$$\sin\theta\csc\theta = 1, \quad \cos\theta\sec\theta = 1, \quad \sin^2\theta + \cos^2\theta = 1$$

Substitute these into the equation:

$$LHS = (\sin^2\theta + \cos^2\theta) + \csc^2\theta + \sec^2\theta + 2(1) + \csc^2\theta + \sec^2\theta$$

$$LHS = 1 + \csc^2\theta + \sec^2\theta + 2$$

Step 3: Use the definitions of $\csc^2\theta$ and $\sec^2\theta$ in terms of $\tan^2\theta$ and $\cot^2\theta$:

$$\csc^2\theta = 1 + \cot^2\theta, \quad \sec^2\theta = 1 + \tan^2\theta$$

Substitute:

$$LHS = 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2$$

Step 4: Simplify:

$$LHS = 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 = 7 + \tan^2\theta + \cot^2\theta$$

Conclusion:

$$LHS = RHS$$

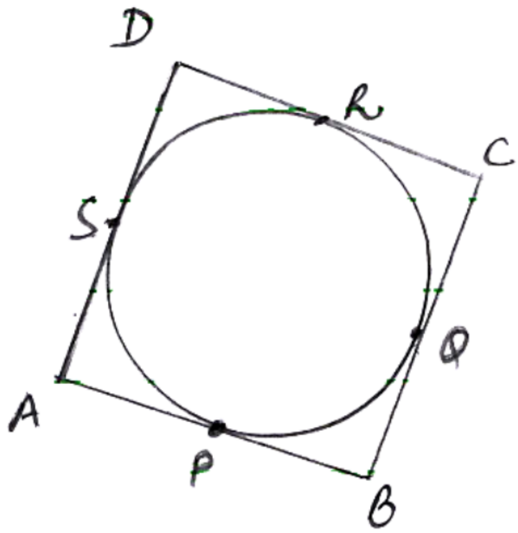
The given identity is proved.

Quick Tip

Use standard trigonometric identities like $\sin^2\theta + \cos^2\theta = 1$, $\csc^2\theta = 1 + \cot^2\theta$, and $\sec^2\theta = 1 + \tan^2\theta$ to simplify such expressions step by step.

29(a): Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:



Step 1: Recall the property of a circumscribed parallelogram: - A parallelogram circumscribes a circle if the sum of the lengths of opposite sides is equal.

Step 2: Let the parallelogram have sides AB, BC, CD, and DA. Then:

$$AB + CD = BC + DA$$

Step 3: In a parallelogram: - Opposite sides are equal: $AB = CD$ and $BC = DA$. - Substituting this property:

$$AB + AB = BC + BC \quad \Rightarrow \quad 2AB = 2BC \quad \Rightarrow \quad AB = BC$$

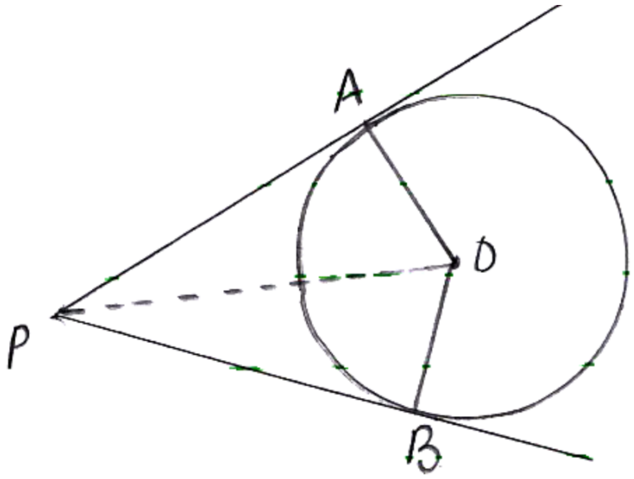
Step 4: Conclusion: - Since all sides are equal, the parallelogram is a rhombus.

Quick Tip

A parallelogram circumscribing a circle is always a rhombus because the tangential property ensures all sides are equal.

29(b): Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Solution:



Step 1: Consider a circle with centre O and an external point P. Let PA and PB be the tangents drawn from P to the circle, touching the circle at A and B.

Step 2: Join O to P, O to A, and O to B: - OA and OB are radii of the circle. - PA and PB are tangents.

Step 3: Use the tangential property: - The radius is perpendicular to the tangent at the point of contact. Hence:

$$\angle OAP = \angle OBP = 90^\circ$$

Step 4: In $\triangle OAP$ and $\triangle OBP$: - OP is common, - $OA = OB$ (radii), - $\angle OAP = \angle OBP = 90^\circ$.

Step 5: By RHS (Right angle-Hypotenuse-Side) congruence:

$$\triangle OAP \cong \triangle OBP$$

Step 6: Conclude that corresponding sides are equal:

$$PA = PB$$

Conclusion: The lengths of the tangents drawn from an external point to a circle are equal.

Quick Tip

For tangents drawn from an external point, use the RHS congruence criterion to prove equality of tangent lengths.

30(a): A horse is tied with a 14 m long rope at one corner of an equilateral triangular field having side 20 m. Find the area of the field where the horse cannot graze.

Solution:

Step 1: Given: - Length of the rope (radius of the grazing area), $r = 14\,\text{m}$, - Side of the equilateral triangle, $a = 20\,\text{m}$, - The angle subtended by each vertex in the equilateral triangle, $\theta = 60°$.

Step 2: Area grazed by the horse: The horse grazes a sector of the circle. The area of the sector is given by:

$$\text{Area of sector} = \frac{\theta}{360°} \cdot \pi r^2$$

Substitute the values:

$$\text{Area grazed} = \frac{60°}{360°} \cdot \frac{22}{7} \cdot 14 \cdot 14 = \frac{1}{6} \cdot \frac{22}{7} \cdot 196 = \frac{308}{3}\,\text{m}^2 \text{ or } 102.67\,\text{m}^2.$$

Step 3: Total area of the equilateral triangle: The area of an equilateral triangle is:

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

Substitute $a = 20$:

$$\text{Area of triangle} = \frac{\sqrt{3}}{4} \cdot 20^2 = 100\sqrt{3}\,\text{m}^2 \text{ or } 173\,\text{m}^2.$$

Step 4: Area where the horse cannot graze:

$$\text{Required area} = \text{Total area of triangle} - \text{Area grazed}$$

$$\text{Required area} = 100\sqrt{3} - 102.67\,\text{m}^2 \text{ or approximately } 70.33\,\text{m}^2.$$

Final Answer: $70.33\,\text{m}^2$.

Quick Tip

The area grazed by the horse is a sector of a circle. Use the sector area formula and subtract it from the total area of the triangle.

30(b): The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand between 8:00 am and 8:05 am.

Solution:

Step 1: Given: - Length of the minute hand, $r = 14\,\text{cm}$, - Angle swept by the minute hand in 60 minutes is $360°$.

Step 2: Calculate the angle swept in 5 minutes:

$$\text{Angle swept in 5 minutes} = \frac{5}{60} \cdot 360^\circ = 30^\circ$$

Step 3: Area swept by the minute hand: The area swept is a sector of a circle, given by:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \cdot \pi r^2$$

Substitute $\theta = 30^\circ$ and $r = 14$:

$$\text{Area swept} = \frac{30^\circ}{360^\circ} \cdot \frac{22}{7} \cdot 14 \cdot 14 = \frac{1}{12} \cdot \frac{22}{7} \cdot 196 = \frac{154}{3}\,\text{cm}^2 \text{ or approximately } 51.33\,\text{cm}^2.$$

Final Answer: $51.33\,\text{cm}^2$.

Quick Tip

To find the area swept by the clock's hand, first calculate the angle swept for the given time interval and use the sector area formula.

31: Find the coordinates of the points of trisection of the line segment joining the points $A(5, -3)$ and $B(-4, 3)$.

Solution:

Step 1: Let the points of trisection be P and Q, such that $AP : PB = 1 : 2$ and $AQ : QB = 2 : 1$.

Step 2: Use the section formula:

$$(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2} \right)$$

Here, $(x_1, y_1) = (5, -3)$ and $(x_2, y_2) = (-4, 3)$.

Step 3: Find the coordinates of P: - For P, $AP : PB = 1 : 2$, so $m_1 = 1$, $m_2 = 2$:

$$P = \left(\frac{2(5) + 1(-4)}{1 + 2}, \frac{2(-3) + 1(3)}{1 + 2} \right)$$

Simplify:

$$P = \left(\frac{10 - 4}{3}, \frac{-6 + 3}{3} \right) = \left(\frac{6}{3}, \frac{-3}{3} \right) = (2, -1)$$

Step 4: Find the coordinates of Q: - For Q, $AQ : QB = 2 : 1$, so $m_1 = 2$, $m_2 = 1$:

$$Q = \left(\frac{1(5) + 2(-4)}{2 + 1}, \frac{1(-3) + 2(3)}{2 + 1} \right)$$

Simplify:

$$Q = \left(\frac{5-8}{3}, \frac{-3+6}{3}\right) = \left(\frac{-3}{3}, \frac{3}{3}\right) = (-1, 1)$$

Final Answer: The points of trisection are $P(2, -1)$ and $Q(-1, 1)$.

Quick Tip

For dividing a line segment in a ratio, use the section formula:

$$(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right)$$

Section D

32(a): A toy is in the form of a cone of radius 7 cm mounted on a hemisphere of the same radius. The total height of the toy is 31 cm. Find the surface area of the toy.

Solution:

Step 1: Given:

- Radius of the cone and hemisphere, $r = 7\,\text{cm}$,

- Total height of the toy, $h_{\text{total}} = 31\,\text{cm}$.

The height of the cone is:

$$h_{\text{cone}} = h_{\text{total}} - r = 31 - 7 = 24\,\text{cm}.$$

Step 2: Find the slant height (l) of the cone: The slant height is given by:

$$l = \sqrt{r^2 + h_{\text{cone}}^2}$$

Substitute $r = 7$ and $h_{\text{cone}} = 24$:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}.$$

Step 3: Surface area of the toy: - Curved surface area of the cone:

$$\text{CSA}_{\text{cone}} = \pi r l = \frac{22}{7} \cdot 7 \cdot 25 = 550\,\text{cm}^2$$

- Curved surface area of the hemisphere:

$$\text{CSA}_{\text{hemisphere}} = 2\pi r^2 = 2 \cdot \frac{22}{7} \cdot 7 \cdot 7 = 308\,\text{cm}^2$$

- Total surface area:

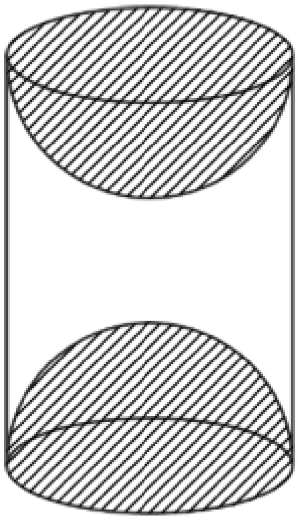
$$\text{Total surface area} = \text{CSA}_{\text{cone}} + \text{CSA}_{\text{hemisphere}} = 550 + 308 = 858\,\text{cm}^2.$$

Final Answer: Total surface area = $858\,\text{cm}^2$.

Quick Tip

For combined solids, calculate the individual surface areas and add them appropriately, avoiding double-counting of shared surfaces.

32(b): A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 15 cm and its base radius is 4.2 cm, find the total surface area of the article.



Solution:

Step 1: Given:

- Radius of the cylinder and hemisphere, $r = 4.2\,\text{cm}$

- Height of the cylinder, $h = 15\,\text{cm}$.

Step 2: Surface area of the article: - Curved surface area of the cylinder:

$$\text{CSA}_{\text{cylinder}} = 2\pi r h$$

Substitute $r = 4.2$ and $h = 15$:

$$\text{CSA}_{\text{cylinder}} = 2 \cdot \frac{22}{7} \cdot 4.2 \cdot 15 = 2 \cdot 22 \cdot 15 = 660\,\text{cm}^2.$$

- Surface area of the two hemispheres:

$$\text{CSA}_{\text{hemispheres}} = 2 \cdot 2\pi r^2 = 4\pi r^2$$

Substitute $r = 4.2$:

$$\text{CSA}_{\text{hemispheres}} = 4 \cdot \frac{22}{7} \cdot 4.2 \cdot 4.2 = 4 \cdot 22 \cdot 2.52 = 221.76\,\text{cm}^2.$$

- Total surface area:

$$\text{Total surface area} = \text{CSA}_{\text{cylinder}} + \text{CSA}_{\text{hemispheres}} = 660 + 221.76 = 881.76\,\text{cm}^2.$$

Final Answer: Total surface area = $881.76\,\text{cm}^2$.

Quick Tip

For objects with scooped-out portions, subtract overlapping areas and add only the visible curved surfaces.

33: The ratio of monthly incomes of two persons is $9 : 7$ and the ratio of their expenditures is $4 : 3$. If each of them manages to save ₹ 2000 per month, then find their monthly incomes.

Solution:

Step 1: Let the monthly incomes of the two persons be $9x$ and $7x$, respectively.

Let their monthly expenditures be $4y$ and $3y$, respectively.

Step 2: Use the savings relation:

$$\text{Income} - \text{Expenditure} = \text{Savings}$$

For the first person:

$$9x - 4y = 2000 \quad \text{(i)}$$

For the second person:

$$7x - 3y = 2000 \quad \text{(ii)}$$

Step 3: Solve the two equations: From equation (i):

$$4y = 9x - 2000 \quad \Rightarrow \quad y = \frac{9x - 2000}{4}$$

Substitute this into equation (ii):

$$7x - 3\left(\frac{9x - 2000}{4}\right) = 2000$$

Simplify:

$$7x - \frac{27x - 6000}{4} = 2000$$
$$\frac{28x - 27x + 6000}{4} = 2000$$
$$x = 2000$$

Step 4: Calculate the incomes: - For the first person:

$$\text{Income} = 9x = 9 \cdot 2000 = |\ 18,000$$

- For the second person:

$$\text{Income} = 7x = 7 \cdot 2000 = |\ 14,000$$

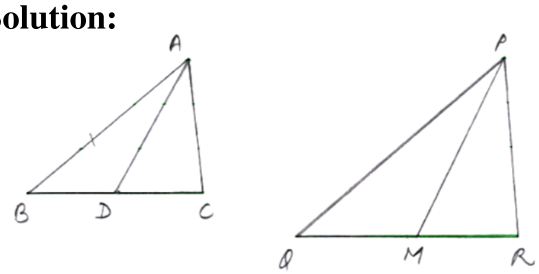
Final Answer: The monthly incomes are ₹ 18,000 and ₹ 14,000, respectively.

Quick Tip

Use ratios and savings relations to form linear equations, then solve step-by-step for the variables.

34(a): Sides AB, BC, and median AD of $\triangle ABC$ are respectively proportional to sides PQ, QR, and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution:



Step 1: Given: - $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, where AD and PM are medians.
Step 2: By the property of medians: The medians of two triangles divide them into two smaller triangles of equal area. Hence, the corresponding sides of $\triangle ABC$ and $\triangle PQR$ are proportional.

Step 3: Use the Side-Side-Side (SSS) similarity criterion: Since:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR},$$

$\triangle ABC \sim \triangle PQR$ by the SSS similarity criterion.

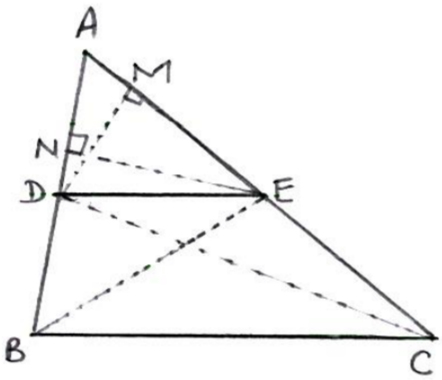
Final Answer: $\triangle ABC \sim \triangle PQR$.

Quick Tip

To prove similarity using SSS, ensure that all corresponding sides of the two triangles are proportional.

34(b): Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.

Solution:



Step 1: Consider $\triangle ABC$ with a line $DE \parallel BC$, intersecting AB at D and AC at E.

Step 2: By the Basic Proportionality Theorem (BPT): If a line is parallel to one side of a triangle, it divides the other two sides in the same ratio:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 3: Proof: - In $\triangle ABC$, since $DE \parallel BC$, $\triangle ADE \sim \triangle ABC$ (by AA similarity criterion).
- From the similarity:

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}.$$

Conclusion: The line DE divides the two sides AB and AC in the same ratio.

Quick Tip

Use the Basic Proportionality Theorem (BPT) whenever a line is drawn parallel to one side of a triangle.

35: The following frequency distribution table gives the monthly consumption of electricity of 70 consumers of a locality. Find the median of the data.

Monthly Consumption (in units)	Number of Consumers (f)	Cumulative Frequency (CF)
$65 - 85$	7	7
$85 - 105$	8	15
$105 - 125$	7	22
$125 - 145$	20	42
$145 - 165$	14	56
$165 - 185$	9	65
$185 - 205$	5	70

Solution:

Step 1: Identify the total number of consumers (N) and the median class.

- Total number of consumers:

$$N = 70$$

- Median class corresponds to the cumulative frequency just greater than $\frac{N}{2} = \frac{70}{2} = 35$. From the table, the median class is $125 - 145$.

Step 2: Note down the values: - Lower boundary of the median class (l): 125, - Frequency of the median class (f): 20, - Cumulative frequency of the class before the median class (CF): 22, - Class width (h): 20.

Step 3: Use the median formula:

$$\text{Median} = l + \left(\frac{\frac{N}{2} - CF}{f}\right) \cdot h$$

Step 4: Substitute the values:

$$\text{Median} = 125 + \left(\frac{35 - 22}{20}\right) \cdot 20$$

Simplify:

$$\text{Median} = 125 + \left(\frac{13}{20}\right) \cdot 20 = 125 + 13 = 138$$

Final Answer: The median of the data is 138 units.

Quick Tip

To find the median in grouped data, always identify the class interval where the cumulative frequency exceeds $\frac{N}{2}$, then apply the median formula step by step.

Section E

36: Family structure: In a recent survey of this year, 51% of the families in the United States of America had no children, 20% had one child, 19% had two children, 7% had three children, and 3% had four or more children. A family is selected at random. Based on this information, answer the following questions:

(i) Find the probability that the selected family has two or three children.

(ii) Find the probability that the selected family has more than one child.

(iii)(a) Find the probability that the selected family has less than three children.

(iii)(b) Find the probability that the selected family has more than two children.

Solution:

Step 1: Given probabilities:

- Probability of no children: $P(\text{no children}) = 51\% = \frac{51}{100} = 0.51$,
- Probability of one child: $P(\text{one child}) = 20\% = \frac{20}{100} = 0.20$,
- Probability of two children: $P(\text{two children}) = 19\% = \frac{19}{100} = 0.19$,
- Probability of three children: $P(\text{three children}) = 7\% = \frac{7}{100} = 0.07$,
- Probability of four or more children: $P(\text{four or more children}) = 3\% = \frac{3}{100} = 0.03$.

Step 2: Solve each sub-question:

(i) Probability of two or three children:

$$P(\text{two or three children}) = P(\text{two children}) + P(\text{three children})$$

Substitute values:

$$P(\text{two or three children}) = 0.19 + 0.07 = 0.26$$

Final Answer: $P(\text{two or three children}) = 0.26$.

(ii) Probability of more than one child:

Families with more than one child have either two children, three children, or four or more children. Thus:

$$P(\text{more than one child}) = P(\text{two children}) + P(\text{three children}) + P(\text{four or more children})$$

Substitute values:

$$P(\text{more than one child}) = 0.19 + 0.07 + 0.03 = 0.29$$

Final Answer: $P(\text{more than one child}) = 0.29$.

(iii)(a) Probability of less than three children:

Families with less than three children have either no children, one child, or two children. Thus:

$$P(\text{less than three children}) = P(\text{no children}) + P(\text{one child}) + P(\text{two children})$$

Substitute values:

$$P(\text{less than three children}) = 0.51 + 0.20 + 0.19 = 0.90$$

Final Answer: $P(\text{less than three children}) = 0.90$.

(iii)(b) Probability of more than two children:

Families with more than two children have either three children or four or more children. Thus:

$$P(\text{more than two children}) = P(\text{three children}) + P(\text{four or more children})$$

Substitute values:

$$P(\text{more than two children}) = 0.07 + 0.03 = 0.10$$

Final Answer: $P(\text{more than two children}) = 0.10$.

Quick Tip

To calculate probabilities, add the probabilities of all relevant events. Ensure that the total probability of all events sums to 1.

37: Sumant's mother started a new shoe shop. To display the shoes, she put 3 pairs of shoes in the 1st row, 5 pairs in the 2nd row, 7 pairs in the 3rd row, and so on. Based on the above information, answer the following questions: (i) How many pairs of shoes are displayed in the 6th row?

(ii) What is the difference of pairs of shoes in the 1st row and the 6th row?

(iii)(a) Find the total number of pairs of shoes displayed in the first 15 rows.

(iii)(b) If the pairs of shoes displayed in the 4th row are 'on sale' at a price of ₹ 500 for each pair, then find the total amount (money) earned by Sumant's mother if all shoes displayed in the 4th row are sold out.

Solution:

Step 1: Observe the pattern: The number of pairs of shoes in each row forms an arithmetic progression (AP):

$$3, 5, 7, 9, \ldots$$

Here, the first term (a) is 3, and the common difference (d) is $5 - 3 = 2$.

(i) Number of pairs of shoes in the 6th row:

The general term of an AP is given by:

$$a_n = a + (n-1)d$$

Substitute $a = 3$, $d = 2$, and $n = 6$:

$$a_6 = 3 + (6-1) \cdot 2 = 3 + 10 = 13$$

Final Answer: 13 pairs of shoes are displayed in the 6th row.

(ii) Difference of pairs of shoes in the 1st row and the 6th row:

From part (i), we know: - Number of pairs in the 1st row (a_1) = 3, - Number of pairs in the 6th row (a_6) = 13.

The difference is:

$$a_6 - a_1 = 13 - 3 = 10$$

Final Answer: The difference is 10 pairs.

(iii)(a) Total number of pairs of shoes displayed in the first 15 rows:

The sum of the first n terms of an AP is given by:

$$S_n = \frac{n}{2} \cdot [2a + (n-1)d]$$

Substitute $n = 15$, $a = 3$, and $d = 2$:

$$S_{15} = \frac{15}{2} \cdot [2(3) + (15-1) \cdot 2]$$

Simplify:

$$S_{15} = \frac{15}{2} \cdot [6 + 28] = \frac{15}{2} \cdot 34 = 15 \cdot 17 = 255$$

Final Answer: The total number of pairs displayed in the first 15 rows is 255.

(iii)(b) Total amount earned if all shoes in the 4th row are sold:

From the AP, the number of pairs of shoes in the 4th row (a_4) is:

$$a_4 = a + (4-1)d = 3 + 3 \cdot 2 = 3 + 6 = 9$$

Price of each pair = ₹ 500. Total amount earned:

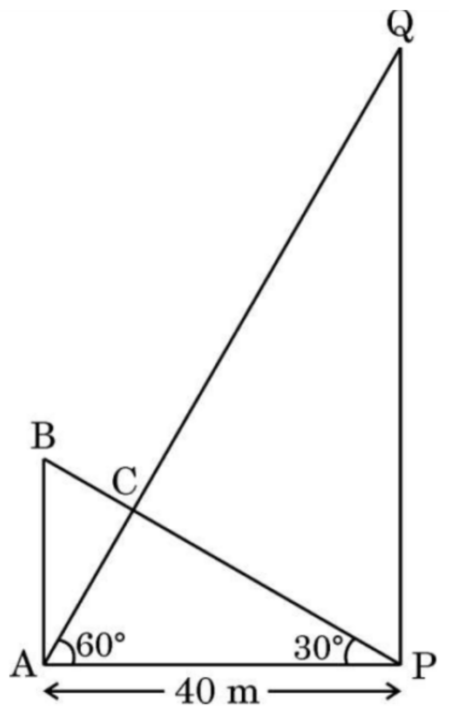
$$\text{Total amount} = a_4 \cdot \text{price per pair} = 9 \cdot 500 = |\ 4500$$

Final Answer: Total amount earned is ₹ 4500.

Quick Tip

For AP problems: 1. Use the general term formula $a_n = a + (n-1)d$ to find individual terms. 2. Use the sum formula $S_n = \frac{n}{2}[2a + (n-1)d]$ to calculate the total of the first n terms.

38: Two poles of different heights stand on level ground and at a distance of 40 m. Both poles are supported by wires attached from the top of each pole to the bottom of the other. A coupling is placed at point C, where the two wires cross (as shown in the figure).



(i) Find the height of pole AB.

(ii) Find the height of pole PQ.

(iii)(a) If the angle of elevation of the top of pole PQ from the top of the pole AB is $30°$, find the distance BQ.

(iii)(b) If the coupling is at a height of 20 m from the ground, how far down the wire from the smaller pole AB is the coupling?

Solution:

Step 1: Set up the known values: - Distance between poles AB and PQ = 40 m, - Height of pole AB = h_1, - Height of pole PQ = h_2.

(i) Find the height of pole AB:

From the geometry of the problem and using trigonometric ratios:

$$\tan 60° = \frac{h_1}{40} \quad \Rightarrow \quad h_1 = 40 \cdot \sqrt{3} \quad \Rightarrow \quad h_1 = 40 \cdot 1.732 = 69.28\,\text{m}.$$

Final Answer: The height of pole AB is 69.28 m.

(ii) Find the height of pole PQ:

Using the same approach:

$$\tan 30° = \frac{h_2}{40} \quad \Rightarrow \quad h_2 = 40 \cdot \frac{1}{\sqrt{3}} \quad \Rightarrow \quad h_2 = \frac{40}{1.732} = 23.09\,\text{m}.$$

Final Answer: The height of pole PQ is 23.09 m.

(iii)(a) Find the distance BQ:

The height difference between the poles is:

$$h_1 - h_2 = 69.28 - 23.09 = 46.19\,\text{m}.$$

Using the given angle of elevation ($30°$):

$$\tan 30° = \frac{h_1 - h_2}{BQ} \quad \Rightarrow \quad BQ = \frac{h_1 - h_2}{\tan 30°}$$

$$BQ = \frac{46.19}{\frac{1}{\sqrt{3}}} = 46.19 \cdot \sqrt{3} = 46.19 \cdot 1.732 = 80.01\,\text{m}.$$

Final Answer: The distance BQ is 80.01 m.

(iii)(b) How far down the wire from pole AB is the coupling?

The coupling is at a height of 20 m from the ground. The height of pole AB is 69.28 m. The length of the wire from A to C forms part of the triangle with base 40 m.

Using similar triangles:

$$\text{Fraction of wire down} = \frac{\text{Height from the top of } AB \text{ to the coupling}}{\text{Total height of } AB} = \frac{69.28 - 20}{69.28} = \frac{49.28}{69.28}.$$

The length of the wire from A to C is:

$$\text{Wire length} = \sqrt{40^2 + 69.28^2} = \sqrt{1600 + 4799.84} = \sqrt{6399.84} = 80\,\text{m}.$$

The distance from A to the coupling is:

$$\text{Distance down} = \frac{49.28}{69.28} \cdot 80 = 56.91\,\text{m}.$$

Final Answer: The coupling is 56.91 m down the wire from A.

Quick Tip

For problems involving poles and angles of elevation, always use trigonometric ratios like $\tan\theta = \frac{\text{height}}{\text{base}}$. For coupling points, use proportionality with similar triangles.