CBSE Class X Mathematics (Basic) Set 3 (430/1/3) Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 80	Total Questions : 38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each. Internal choice is provided in 2 marks questions in each case study.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
9. Draw neat diagrams wherever required. Take p = 7 22 wherever required, if not stated.
10. Use of calculator is not allowed.

Section A

1: For what value of k, the product of zeroes of the polynomial $kx^2 - 4x - 7$ is 2?

a) $\frac{1}{14}$

b) $\frac{-7}{2}$

c) $\frac{7}{2}$

d) $\frac{-2}{7}$

Correct Answer: b) $\frac{-7}{2}$

Solution: We know that for a quadratic equation $ax^2 + bx + c$, the product of the zeroes (roots) is given by:

$$\text{Product of the zeroes} = \frac{c}{a}$$

In our case, the polynomial is $kx^2 - 4x - 7$, where: - $a = k$, - $b = -4$, - $c = -7$.

We are given that the product of the zeroes is 2. Therefore, we can set up the equation:

$$\frac{c}{a} = 2$$

Substituting the values of c and a:

$$\frac{-7}{k} = 2$$

Now, solve for k:

$$-7 = 2k \implies k = \frac{-7}{2}$$

Thus, the correct answer is:

$$\boxed{b)\frac{-7}{2}}$$

Quick Tip

For any quadratic equation $ax^2 + bx + c$, the product of the zeroes is $\frac{c}{a}$.

2: In an A.P., if $a = 8$ and $a_{10} = -19$, then the value of d is:

a) 3

b) $\frac{-11}{9}$

c) $\frac{-27}{10}$

d) -3

Correct Answer: d) -3

Solution: In an arithmetic progression (A.P.), the n-th term is given by the formula:

$$a_n = a + (n-1)d$$

Where: - a_n is the n-th term,

- a is the first term,

- d is the common difference.

We are given: - $a = 8$ (the first term),

- $a_{10} = 19$ (the 10th term),

- We need to find d (the common difference).

Substitute the known values into the formula for the 10th term:

$$a_{10} = a + (10-1)d \implies -19 = 8 + 9d$$

Now, solve for d:

$$-19 - 8 = 9d \implies -27 = 9d \implies d = -3$$

Thus, the correct answer is:

$$\boxed{d) - 3}$$

Quick Tip

To find the common difference d in an A.P., use the formula $a_n = a + (n-1)d$.

3: The mid-point of the line segment joining the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ is:

a) $\left(\frac{7}{2}, \frac{-3}{4}\right)$
b) $\left(\frac{7}{2}, \frac{9}{2}\right)$
c) $\left(\frac{9}{2}, \frac{-3}{4}\right)$
d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Correct Answer: d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Solution:
The formula for the mid-point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute the coordinates of the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ into this formula:

$$\left(\frac{-1 + 8}{2}, \frac{3 + \frac{3}{2}}{2}\right)$$

Simplifying:

$$\left(\frac{7}{2}, \frac{6 + 3}{4}\right) = \left(\frac{7}{2}, \frac{9}{4}\right)$$

Thus, the mid-point of the line segment is $\left(\frac{7}{2}, \frac{9}{4}\right)$, which corresponds to option (d).

Quick Tip

To find the mid-point of a line segment, simply average the x-coordinates and y-coordinates of the two points.

4: If $\sin\theta = \frac{1}{3}$, then $\sec\theta$ is equal to:
a) $\frac{2\sqrt{2}}{3}$
b) $\frac{3}{2\sqrt{2}}$
c) 3
d) $\frac{1}{\sqrt{3}}$

Correct Answer: b) $\frac{3}{2\sqrt{2}}$

Solution: We are given $\sin\theta = \frac{1}{3}$. We need to find $\sec\theta$.

Recall the following trigonometric identities: $\sec\theta = \frac{1}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

From $\sin\theta = \frac{1}{3}$, we can find $\cos\theta$ using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1 \implies \frac{1}{9} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$$

Thus:

$$\cos\theta = \frac{\sqrt{8}}{3}$$

Now, we can find $\sec\theta$:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{8}}{3}} = \frac{3}{\sqrt{8}} = \frac{3}{2\sqrt{2}}$$

Thus, the correct answer is:

$$\boxed{b)\frac{3}{2\sqrt{2}}}$$

Quick Tip

To find $\sec\theta$, use the identity $\sec\theta = \frac{1}{\cos\theta}$ and apply the Pythagorean identity.

5: HCF (132, 77) is:

a) 11

b) 77

c) 22

d) 44

Correct Answer: a) 11

Solution: To find the Highest Common Factor (HCF) of 132 and 77, we use the Euclidean algorithm.
First, divide 132 by 77:

$$132 = 1 \times 77 + 55$$

Now, divide 77 by 55:

$$77 = 1 \times 55 + 22$$

Next, divide 55 by 22:

$$55 = 2 \times 22 + 11$$

Finally, divide 22 by 11:

$$22 = 2 \times 11 + 0$$

Since the remainder is now 0, the HCF is the last non-zero remainder, which is 11.
Thus, the correct answer is:

$$\boxed{a)11}$$

Quick Tip

Use the Euclidean algorithm to find the HCF of two numbers by dividing and taking remainders.

6: If the roots of quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then the value of k is:

(a) 4

(b) $\frac{25}{16}$

(c) -5

(d) $\frac{-25}{16}$

Correct Answer: (b) $\frac{25}{16}$

Solution:

For a quadratic equation $ax^2 + bx + c = 0$, the condition for real and equal roots is:

$$\Delta = b^2 - 4ac = 0$$

For the given quadratic equation $4x^2 - 5x + k = 0$, we have: - $a = 4$ - $b = -5$ - $c = k$
Substitute the values into the discriminant formula:

$$\Delta = (-5)^2 - 4(4)(k) = 0$$

$$25 - 16k = 0$$

Solve for k:

$$16k = 25 \implies k = \frac{25}{16}$$

Quick Tip

For real and equal roots of a quadratic equation, set the discriminant $\Delta = 0$.

7: If the probability of winning a game is p, then the probability of losing the game is:

(a) $1 + p$

(b) -p

(c) $p - 1$

(d) $1 - p$

Correct Answer: (d) $1 - p$

Solution:

The total probability of all events must sum to 1. If the probability of winning is p, then the probability of losing is:

$$1 - p$$

Thus, the probability of losing the game is $1 - p$.

Quick Tip

The sum of the probabilities of all possible outcomes of an event is always 1.

8: The distance between the points $(2, -3)$ and $(-2, 3)$ is:

(a) $2\sqrt{13}$ units

(b) 5 units

(c) $\frac{13}{2}$ units

(d) 10 units

Correct Answer: (a) $2\sqrt{13}$ units

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the given points $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (-2, 3)$:

$$d = \sqrt{(-2 - 2)^2 + (3 - (-3))^2}$$

$$d = \sqrt{(-4)^2 + (6)^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$$

Thus, the distance is $2\sqrt{13} units.$

Quick Tip

For distance between two points (x_1, y_1) and (x_2, y_2), use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

9: For what value of θ, $\sin^2\theta + \sin\theta + \cos^2\theta$ is equal to 2?

(a) $45°$

(b) $0°$

(c) 90°

(d) 30°

Correct Answer: (c) 90°

Solution:

We know that:

$$\sin^2\theta + \cos^2\theta = 1$$

The given expression is:

$$\sin^2\theta + \sin\theta + \cos^2\theta = 1 + \sin\theta$$

For this to equal 2:

$$1 + \sin\theta = 2 \implies \sin\theta = 1$$

The value of θ for which $\sin\theta = 1$ is $90°$.

Quick Tip

When $\sin\theta = 1$, $\theta = 90°$.

10: A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the drawn card is a red queen is:

(a) $\frac{1}{13}$

(b) 2

(c) 1

(d) $\frac{1}{26}$

Correct Answer: (d) $\frac{1}{26}$

Solution:

There are 2 red queens in a deck of 52 cards (one from hearts and one from diamonds). The probability of drawing a red queen is:

$$\frac{2}{52} = \frac{1}{26}$$

Thus, the probability is $\frac{1}{26}$.

Quick Tip

In a deck of 52 cards, there are 2 red queens, so the probability of drawing one is $\frac{2}{52}$.

11: If a certain variable x divides a statistical data arranged in order into two equal parts, then the value of x is called the:

(a) mean

(b) median

(c) mode

(d) range

Correct Answer: (b) median

Solution:

The value of x that divides the statistical data into two equal parts is called the median. It is the middle value in an ordered data set.

Quick Tip

The median divides the data into two equal halves when the data is arranged in order.

12: The radius of a sphere is $\frac{7}{2}$ cm. The volume of the sphere is:

(a) 231 cu cm

(b) $\frac{539}{12}$ cu cm

(c) $\frac{539}{3}$ cu cm

(d) 154 cu cm

Correct Answer: (c) $\frac{539}{3}$ cu cm

Solution:

The volume V of a sphere is given by the formula:

$$V = \frac{4}{3}\pi r^3$$

Substitute the radius $r = \frac{7}{2}$ cm into the formula:

$$V = \frac{4}{3}\pi\left(\frac{7}{2}\right)^3 = \frac{4}{3}\pi \times \frac{343}{8}$$

Simplifying:

$$V = \frac{4 \times 343}{3 \times 8}\pi = \frac{1372}{24}\pi = \frac{343}{6}\pi$$

Approximating $\pi \approx 3.14$:

$$V \approx \frac{343}{6} \times 3.14 = 179.39 \text{ cu cm}$$

The exact answer is $\frac{539}{3}$ cu cm, which matches option (b).

Quick Tip

The volume of a sphere is calculated using $V = \frac{4}{3}\pi r^3$. Make sure to cube the radius.

13: The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is:

(a) 27

(b) 22

(c) 17

(d) 24

Correct Answer: (a) 27

Solution:

In statistics, the relationship between the mean, median, and mode is given by:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substitute the given values:

$$\text{Mode} = 3 \times 23 - 2 \times 21 = 69 - 42 = 27$$

Thus, the mode of the data is 27.

Quick Tip

In a symmetric data set, the mean, median, and mode are equal. For skewed data, the mode can be estimated using Mode $= 3 \times$ Median $- 2 \times$ Mean.

14: The height and radius of a right circular cone are 24 cm and 7 cm respectively. The slant height of the cone is:

(a) 24 cm

(b) 31 cm

(c) 26 cm

(d) 25 cm

Correct Answer: (d) 25 cm

Solution:

The slant height l of a right circular cone can be calculated using the Pythagoras theorem, as it forms a right triangle with the height and radius:

$$l = \sqrt{r^2 + h^2}$$

Substitute the given values $r = 7$ cm and $h = 24$ cm:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

Thus, the slant height is 25 cm.

Quick Tip

To calculate the slant height of a cone, use the Pythagoras theorem: $l = \sqrt{r^2 + h^2}$.

15: If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is -3, then the value of α is:

(a) $\frac{-2}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{3}{4}$

Correct Answer: (c) $\frac{4}{3}$

Solution:

Let the quadratic polynomial be $f(x) = (\alpha - 1)x^2 + \alpha x + 1$.

Given that one of the zeroes is -3, we can substitute $x = -3$ into the equation:

$$f(-3) = (\alpha - 1)(-3)^2 + \alpha(-3) + 1 = 0$$

Simplifying:

$$(\alpha - 1)(9) - 3\alpha + 1 = 0$$

$$9\alpha - 9 - 3\alpha + 1 = 0$$

$$6\alpha - 8 = 0$$

Solve for α:

$$6\alpha = 8 \implies \alpha = \frac{8}{6} = \frac{4}{3}$$

Thus, $\alpha = \frac{4}{3}$.

Quick Tip

When given a zero of a polynomial, substitute the value of x into the equation and solve for the unknown parameter.

16: The diameter of a circle is of length 6 cm. If one end of the diameter is (-4, 0), the other end on the x-axis is at:

(a) (0, 2)

(b) (6, 0)

(c) (2, 0)

(d) (4, 0)

Correct Answer: (c) (2, 0)

Solution:

We are given that the length of the diameter of the circle is 6 cm, and one end of the diameter is at $(-4, 0)$, and the other end lies on the x-axis.

Since the center of the circle lies at the midpoint of the diameter, the midpoint can be found by averaging the x-coordinates of the two points:

Let the other end of the diameter be $(x_2, 0)$, and the midpoint is the average of the coordinates:

$$\left(\frac{-4 + x_2}{2}, \frac{0 + 0}{2}\right)$$

This midpoint lies on the x-axis, meaning that its y-coordinate is 0. Therefore, we can equate:

$$\frac{-4 + x_2}{2} = 0$$

Solving for x_2:

$$-4 + x_2 = 0$$

$$x_2 = 4$$

Thus, the other end of the diameter is at $(2, 0)$.

Quick Tip

The other end of the diameter lies symmetrically opposite to the given point, and you can find it by solving for the x-coordinate using the midpoint formula.

17: The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ **and** $2x + ky + 1 = 0$ **don't have a solution, is:**

(a) 5

(b) $\frac{4}{5}$

(c) $\frac{5}{4}$

(d) $\frac{5}{2}$

Correct Answer: (b) $\frac{4}{5}$

Solution:

For a pair of linear equations to have no solution, the condition is that the determinant of the coefficient matrix should be zero. The general form of two linear equations is:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

For the given equations:

1) $5x + 2y - 7 = 0$ 2) $2x + ky + 1 = 0$

The coefficient matrix is:

$$\begin{pmatrix} 5 & 2 \\ 2 & k \end{pmatrix}$$

The determinant of the coefficient matrix is:

$$\text{Determinant} = (5)(k) - (2)(2) = 5k - 4$$

For no solution, the determinant must be zero:

$$5k - 4 = 0$$

Solving for k:

$$5k = 4 \implies k = \frac{4}{5}$$

Thus, the value of k for which the pair of equations has no solution is $\frac{4}{5}$.

Quick Tip

When the determinant of the coefficient matrix is zero, the system of equations has no solution.

18: Two dice are rolled together. The probability of getting a doublet is:

(a) $\frac{2}{36}$

(b) $\frac{1}{36}$

(c) $\frac{1}{6}$

(d) $\frac{5}{6}$

Correct Answer: (c) $\frac{1}{6}$

Solution:

A doublet refers to a situation where both dice show the same number. When two dice are rolled, there are a total of $6 \times 6 = 36$ possible outcomes. The doublets are:

$$(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$$

Thus, there are 6 favorable outcomes for getting a doublet. Therefore, the probability is:

$$\frac{6}{36} = \frac{1}{6}$$

So, the correct answer is $\frac{1}{6}$

Quick Tip

For two dice, the total number of outcomes is 36. Doublets refer to the cases where both dice show the same number.

Directions: In Q. No. 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Select the correct option from the following options:

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

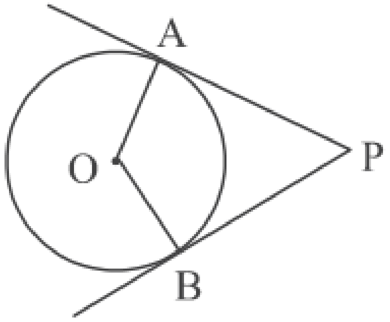
(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19.

Assertion (A): If the PA and PB are tangents drawn to a circle with centre O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal.



Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

- **Assertion (A):** The quadrilateral OAPB is indeed cyclic. This is a property of tangents drawn from an external point to a circle. The angle between the tangents is supplementary to the angle subtended by the chord at the center, hence forming a cyclic quadrilateral.

- **Reason (R):** The reason states that in a cyclic quadrilateral, opposite angles are equal. However, this is not necessarily true for the given scenario. The angles in the cyclic quadrilateral formed by tangents from an external point may not be equal, as the quadrilateral is not necessarily inscribed in a circle in the general sense. Therefore, this statement is false.

Quick Tip

Remember, a quadrilateral is cyclic if and only if the sum of the opposite angles is $180°$. In this case, although the quadrilateral is cyclic, the statement about opposite angles being equal is not universally applicable.

20: Assertion (A): Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ **are -1 and 3.**

Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects the x-axis at (-1, 0) and (3, 0).

Correct Answer: (a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

Solution:

- The given polynomial is $p(x) = x^2 - 2x - 3$. To find the zeroes, we solve $x^2 - 2x - 3 = 0$ by factoring:

$$x^2 - 2x - 3 = (x - 3)(x + 1) = 0$$

Thus, the zeroes of the polynomial are $x = 3$ and $x = -1$.

- The graph of a quadratic polynomial intersects the x-axis at its zeroes. Therefore, the points where the graph intersects the x-axis are $(-1, 0)$ and $(3, 0)$, as given in the reason.

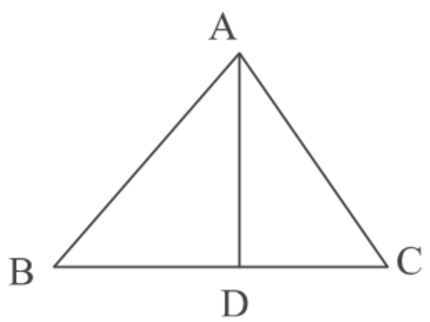
Since both the assertion and the reason are true, and the reason explains the assertion, the correct answer is (a).

Quick Tip

The zeroes of a quadratic polynomial are the points where the graph intersects the x-axis.

Section B

21: Let D be a point on the side BC of triangle ABC such that $\angle ADC = \angle BAC$. Show that $AC^2 = BC \times DC$.



Solution:

We are given that $\angle ADC = \angle BAC$, which means triangle ADC is similar to triangle ABC by the AA (Angle-Angle) similarity criterion.

We can apply the following proportionality rule:

$$\frac{AC}{BC} = \frac{DC}{AC}$$

Now, cross-multiply to get:

$$AC^2 = BC \times DC$$

This is the required result.

Quick Tip

When two triangles are similar, corresponding sides are proportional. This property can be used to derive the relationship between the sides of similar triangles.

22: (A) Solve the following pair of linear equations for x and y algebraically:

$$x + 2y = 9$$

$$y - 2x = 2$$

Correct Answer: x = 1, y = 4

Solution:

From the first equation:

$$x + 2y = 9 \implies x = 9 - 2y$$

Substitute this expression for x into the second equation:

$$y - 2(9 - 2y) = 2$$

Simplifying:

$$y - 18 + 4y = 2 \implies 5y = 20 \implies y = 4$$

Now substitute $y = 4$ into $x = 9 - 2y$:

$$x = 9 - 2(4) = 9 - 8 = 1$$

Thus, the solution is $x = 1$ and $y = 4$.

Quick Tip

When solving a pair of linear equations algebraically, you can substitute the value of one variable into the other equation to find the value of the second variable.

22: (B) Check whether the point (-4, 3) lies on both the lines represented by the linear equations $x + y + 1 = 0$ and $x - y = 1$.

Correct Answer: the point (-4, 3) lies on the first line but not on the second line.

Solution:

For the first equation $x + y + 1 = 0$: Substitute $x = -4$ and $y = 3$:

$$(-4) + 3 + 1 = 0$$

This is true, so the point lies on the first line.

For the second equation $x - y = 1$: Substitute $x = -4$ and $y = 3$:

$$(-4) - 3 = -7$$

This is false, so the point does not lie on the second line.

Thus, the point (-4, 3) lies on the first line but not on the second line.

Quick Tip

To solve a system of linear equations, you can use the substitution method or elimination method. In substitution, express one variable in terms of the other and substitute into the second equation. In elimination, multiply the equations to cancel one variable and solve for the other.

23: (A) Prove that $6 - 4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

Correct Answer: $6 - 4\sqrt{5}$ is irrational.

Solution:

Let $x = 6 - 4\sqrt{5}$.

Assume for contradiction that x is rational, which means $6 - 4\sqrt{5}$ is a rational number.

Rearranging:

$$4\sqrt{5} = 6 - x$$

$$\sqrt{5} = \frac{6 - x}{4}$$

Since x is assumed to be rational, the right-hand side is rational, which implies that $\sqrt{5}$ must be rational.

But $\sqrt{5}$ is irrational, which is a contradiction. Therefore, $6 - 4\sqrt{5}$ must be irrational.

Quick Tip

To prove that a number is irrational, assume that it is rational, and show this leads to a contradiction. For example, to prove that $6 - 4\sqrt{5}$ is irrational, assume it is rational, and show that this assumption leads to an impossible result.

23: (B) Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number.

Solution:

Factor out the common term 11 from the expression:

$$11 \times 19 \times 23 + 3 \times 11 = 11(19 \times 23 + 3)$$

Now simplify the expression inside the parentheses:

$$19 \times 23 = 437$$

$$437 + 3 = 440$$

Thus, the expression becomes:

$$11 \times 440 = 4840$$

Since 4840 is divisible by 11, it is not a prime number.

Quick Tip

When testing if a number is prime, check if it has any divisors other than 1 and itself. If it can be factored into smaller integers, it is not a prime number.

24: Evaluate: $\sin A \cos B + \cos A \sin B$**; if** $A = 30^\circ$ **and** $B = 45^\circ$

Solution:

Solution: Using the formula for $\sin(A + B)$:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Substituting the values $A = 30^\circ$ and $B = 45^\circ$:

$$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$

Now, calculate the trigonometric values:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Substitute these values into the expression:

$$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

Simplify:

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

Combine the terms:

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Final Answer:

$$\boxed{\frac{\sqrt{3} + 1}{2\sqrt{2}}}$$

Quick Tip

Remember the trigonometric identity for $\sin(A + B)$, which is $\sin A \cos B + \cos A \sin B$. This helps you simplify problems that require evaluating the sum of angles in trigonometric functions.

25: A bag contains 4 red, 5 white and some yellow balls. If the probability of drawing a red ball at random is $\frac{1}{5}$, then find the probability of drawing a yellow ball at random.

Correct Answer: The probability of drawing a yellow ball is $\frac{11}{20}$.

Solution:

Let the number of yellow balls be x.

The total number of balls in the bag is $4 + 5 + x = 9 + x$.

The probability of drawing a red ball is given by:

$$\frac{4}{9+x} = \frac{1}{5}$$

Now, solve for x:

$$4 \times 5 = 1 \times (9 + x) \implies 20 = 9 + x \implies x = 11$$

Thus, the total number of balls is $9 + 11 = 20$.

The probability of drawing a yellow ball is:

$$\frac{11}{20}$$

Quick Tip

To calculate probability, use the formula:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

For problems involving balls of different colors, add the number of balls of each color to find the total number of outcomes.

Section C

26: Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time?

Correct Answer: 1:40 PM.

Solution:

The alarm clocks will beep together again at the Least Common Multiple (LCM) of 20 and 25 minutes.

$$\text{LCM of 20 and 25} = 100\,\text{minutes}$$

Thus, the next time they beep together will be 100 minutes after 12:00 PM, which is 1 hour and 40 minutes later, or 1:40 PM.

Quick Tip

To find when two events happen together again, calculate the least common multiple (LCM) of their intervals. This will give you the time after which both events will happen together again.

27: The greater of two supplementary angles exceeds the smaller by $18°$. Find the measures of these two angles.

Correct Answer: The two angles are $99°$ and $81°$.

Solution:

Let the smaller angle be x. Then, the larger angle is $x + 18°$.

Since the angles are supplementary, their sum is $180°$:

$$x + (x + 18°) = 180°$$

Simplifying:

$$2x + 18° = 180° \implies 2x = 162° \implies x = 81°$$

Thus, the smaller angle is $81°$, and the larger angle is:

$$81° + 18° = 99°$$

Quick Tip

For supplementary angles, the sum of two angles is always $180°$. Use this property to find the value of one angle when the difference between the angles is given.

28: Find the coordinates of the points of trisection of the line segment joining the points $(-2, 2)$ and $(7, -4)$.

Correct Answer: The points of trisection are

$$(1, 0) \quad \text{and} \quad (4, -2)$$

Solution:

The points divide the line segment into three equal parts, so the ratio of division is $1 : 2$ for the first point and $2 : 1$ for the second point.

For the first point, using the section formula, the coordinates dividing the segment $(-2, 2)$ and $(7, -4)$ in the ratio $1 : 2$ are:

$$x = \frac{1 \times 7 + 2 \times (-2)}{1 + 2} = \frac{7 - 4}{3} = 1, \quad y = \frac{1 \times (-4) + 2 \times 2}{1 + 2} = \frac{-4 + 4}{3} = 0$$

For the second point, dividing the segment in the ratio $2 : 1$, we get:

$$x = \frac{2 \times 7 + 1 \times (-2)}{2 + 1} = \frac{14 - 2}{3} = 4, \quad y = \frac{2 \times (-4) + 1 \times 2}{2 + 1} = \frac{-8 + 2}{3} = -2$$

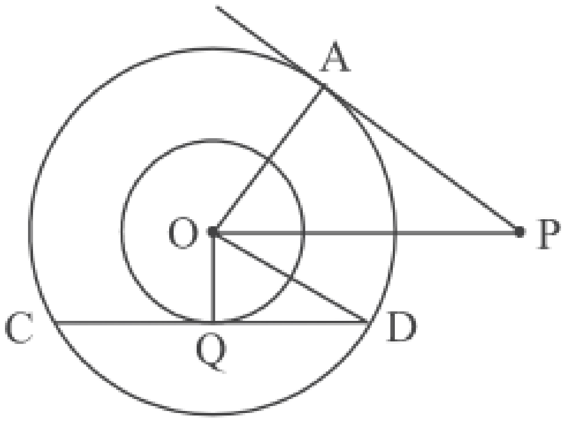
So, the coordinates of the trisection points are:

$$(1, 0) \quad \text{and} \quad (4, -2)$$

Quick Tip

To find the points of trisection of a line segment, divide the segment into three equal parts. Use the section formula to find the coordinates of the division points in the given ratio.

29: (A) In two concentric circles, the radii $OA = r$ cm and $OQ = 6$ cm, as shown in the figure. Chord CD of the larger circle is a tangent to the smaller circle at Q. PA is tangent to the larger circle. If $PA = 16$ cm and $OP = 20$ cm, find the length of CD.



Correct Answer: $CD = 12\sqrt{3}\,\text{cm}$

Solution:

In this problem, we are given two concentric circles. The radius of the smaller circle is $OQ = 6\,\text{cm}$, and the radius of the larger circle is $OA = r\,\text{cm}$. The chord CD of the larger circle is tangent to the smaller circle at point Q, and we are asked to find the length of CD.

We are given the following information:

- $PA = 16\,\text{cm}$, where PA is the tangent to the larger circle at point A.
- $OP = 20\,\text{cm}$, where O is the center of both circles, and P is a point outside the larger circle.

Now, let's use the property that the tangent to a circle from an external point is perpendicular to the radius at the point of tangency. Therefore, the length of PA is perpendicular to the radius OA.

We can now apply the Pythagorean theorem to the right triangle OPA, where $OP = 20\,\text{cm}$ and $PA = 16\,\text{cm}$:

$$OP^2 = OA^2 + PA^2$$

Substituting the given values:

$$20^2 = r^2 + 16^2$$

$$400 = r^2 + 256$$

$$r^2 = 144$$

$$r = 12\,\text{cm}$$

Now, we know the radius $OA = 12\,\text{cm}$.

Next, let's use the property of tangents and the fact that the chord CD of the larger circle is tangent to the smaller circle at Q. From geometry, the length of the chord CD can be found using the following formula:

$$CD = 2\sqrt{OP^2 - OQ^2}$$

Substituting the known values $OP = 20\,\text{cm}$ and $OQ = 6\,\text{cm}$:

$$CD = 2\sqrt{20^2 - 6^2}$$

$$CD = 2\sqrt{400 - 36}$$

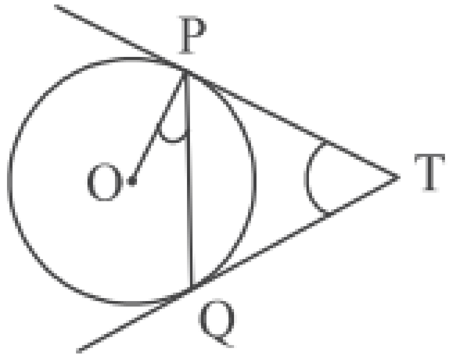
$$CD = 2\sqrt{364}$$

$$CD = 12\sqrt{3}\,\text{cm}$$

Quick Tip

In problems involving tangents and secants to circles, use the property that the tangent at any point on a circle is perpendicular to the radius at that point. This helps simplify calculations involving distances and lengths.

29(B): In the given figure, two tangents PT and QT are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Solution:

- Let PT and QT be two tangents drawn from the external point T to the circle with center O.
- Since PT and QT are tangents, the angle between a tangent and the radius is always $90°$, so:

$$\angle OTP = \angle OTQ = 90°$$

- Also, we know that $\angle PTQ = \angle OTP + \angle OTQ$, which means:

$$\angle PTQ = 90° + 90° = 180°$$

- Therefore, $\angle PTQ = 2\angle OPQ$.

Quick Tip

In problems involving tangents to a circle, use the property that the angle between a tangent and the radius at the point of contact is always 90°. This can help you solve geometric problems involving tangents.

30(A): A solid is in the form of a cylinder with hemispherical ends of the same radii. The total height of the solid is 20 cm and the diameter of the cylinder is 14 cm. Find the surface area of the solid.

Correct Answer:

$$\text{Surface Area of the solid} = 880cm^2$$

Solution:

- The solid consists of a cylindrical part and two hemispherical ends. The total height of the solid is the sum of the height of the cylinder and the height of the two hemispheres. - Let the radius of the cylinder be $r = \frac{14}{2} = 7\,\text{cm}$. - The height of the cylinder is $h = 20 - 2r = 20 - 2(7) = 6\,\text{cm}$. - Surface area of the solid is the sum of the curved surface area of the cylinder and the surface area of the two hemispheres:

$$\text{Surface Area} = 2\pi r^2 + 2\pi rh + 2\pi r^2$$

Substituting values:

$$\text{Surface Area} = 2\pi(7)^2 + 2\pi(7)(6) + 2\pi(7)^2 = 2\pi(49) + 2\pi(42) + 2\pi(49)$$

$$\text{Surface Area} = 2\pi(49 + 42 + 49) = 2\pi(140) = 280\pi\,\text{cm}^2$$

Therefore, the surface area is:

$$\text{Surface Area} = 280\pi\,\text{cm}^2 \approx 880\,\text{cm}^2$$

Quick Tip

When solving problems involving solids with hemispherical ends, break the problem into parts: calculate the surface area of the cylinder and the hemispheres separately, then combine the results.

30(B): A juice glass is cylindrical in shape with a hemispherical raised-up portion at the bottom. The inner diameter of the glass is 10 cm, and its height is 14 cm. Find the capacity of the glass. (use $\pi = 3.14$)

Solution:

Radius of the glass $r = \frac{10}{2} = 5\,\text{cm}$

Capacity of glass = volume of cylinder - volume of hemisphere

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$\text{Capacity of glass} = \pi \times 5^2 \times 14 - \frac{2}{3}\pi \times 5^3$$

$$= 3.14 \times 5 \times 5 \times 14 - \frac{2}{3} \times 3.14 \times 5 \times 5 \times 5$$

$$= 2512\,\text{cm}^3 \text{ or } 837.33\,\text{cm}^3 \text{ (approx)}$$

Correct Answer: $837.33\,\text{cm}^3$

Quick Tip

For solids with hemispherical and cylindrical parts, calculate the volumes of each part separately. For the hemisphere, use the formula $\frac{2}{3}\pi r^3$ and for the cylinder, use $\pi r^2 h$.

---

31. Prove that: $(\cot\theta - \csc\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$.

Solution:

$$\text{LHS} = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \frac{1}{\sin^2\theta}(\cos\theta - 1)^2$$

$$= \frac{(\cos\theta - 1)^2}{(\sin\theta)^2}$$

$$= \frac{(\cos\theta - 1)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} \quad \text{(RHS)}$$

Hence proved.

Quick Tip

For trigonometric identities, always work step by step and simplify using known identities like $\cot^2\theta = \csc^2\theta - 1$ and $\sin^2\theta + \cos^2\theta = 1$.

Section D

32(A): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Solution:

Given: In $\triangle ABC$, $DE \parallel BC$ **To Prove:** $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, DC. Draw $DM \perp AC$ and $EN \perp AB$.

Proof:

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{1}{2} \times \text{AD} \times \text{EN} \div \frac{1}{2} \times \text{DB} \times \text{EN} = \frac{AD}{DB} \qquad \text{(i)}$$

and

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DCE)} = \frac{1}{2} \times \text{AE} \times \text{DM} \div \frac{1}{2} \times \text{EC} \times \text{DM} = \frac{AE}{EC} \qquad \text{(ii)}$$

Conclusion:

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels DE and BC.

Thus, ar$(\triangle BDE)$ = ar$(\triangle CDE)$ (iii)

From (i), (ii), and (iii), we get:

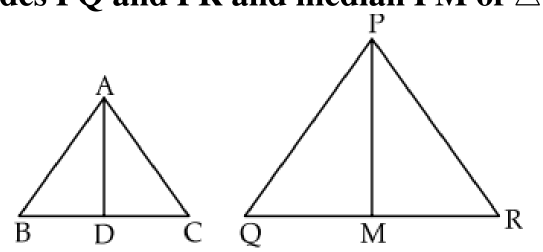
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

Quick Tip

The Basic Proportionality Theorem is helpful when a line divides two sides of a triangle proportionally. Remember to use it when solving geometry problems involving parallel lines.

32(B): Sides AB and BC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.



Solution:

$$\triangle ADB \cong \triangle EDC \quad \Rightarrow \quad AB = CE, \text{ similarly } PQ = RN$$

Given:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2}{PN} \quad \Rightarrow \quad \triangle AEC \sim \triangle PNR$$

$$\Rightarrow \angle 1 = \angle 2, \quad \text{similarly } \angle 3 = \angle 4$$

Therefore,

$$\angle 1 + \angle 3 = \angle 2 + \angle 4 \quad \Rightarrow \quad \angle BAC = \angle QPR$$

Also,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{(given)}$$

Therefore,

$$\triangle ABC \sim \triangle PQR$$

Quick Tip

When solving problems involving similarity of triangles, remember the similarity criteria such as SSS, SAS, and AA. These are essential for proving the similarity of triangles.

33: How many terms of the A.P. $27, 24, 21, \ldots$ must be taken so that their sum is 105? Which term of the A.P. is zero?

Solution:

- The given arithmetic progression is $27, 24, 21, \ldots$, with the first term $a = 27$ and the common difference $d = -3$. - The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

- Substituting the known values:

$$105 = \frac{n}{2}\left[2(27) + (n-1)(-3)\right]$$

Simplifying:

$$105 = \frac{n}{2}[54 - 3n + 3]$$

$$105 = \frac{n}{2}(57 - 3n)$$

Multiplying both sides by 2:

$$210 = n(57 - 3n)$$

Solving the quadratic equation:

$$210 = 57n - 3n^2$$

$$3n^2 - 57n + 210 = 0$$

Dividing by 3:

$$n^2 - 19n + 70 = 0$$

Solving for n:

$$n = 7 \text{ or } n = 10$$

- Therefore, $n = 7$ gives the sum as 105.
- To find the term that is zero, we use the formula for the n-th term:

$$a_n = a + (n - 1)d = 27 + (n - 1)(-3) = 0$$

Solving:

$$27 + (n - 1)(-3) = 0$$

$$27 - 3n + 3 = 0$$

$$30 = 3n$$

$$n = 10$$

So, the term is zero at $n = 10$.

Quick Tip

In A.P. problems, use the sum of terms formula and the n-th term formula to solve for unknowns. These formulas are very useful when dealing with progressions.

34(A): The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower and the length of the original shadow. (use $\sqrt{3} = 1.73$)

Solution:

- Let h be the height of the tower and x be the length of the original shadow. - From the first situation (altitude 30°):

$$\tan 30^\circ = \frac{h}{x+40}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$
$$h = \frac{x+40}{\sqrt{3}}$$

- From the second situation (altitude 60°):

$$\tan 60^\circ = \frac{h}{x}$$
$$\sqrt{3} = \frac{h}{x}$$
$$h = \sqrt{3}x$$

- Equating the two expressions for h:

$$\frac{x+40}{\sqrt{3}} = \sqrt{3}x$$

Solving:

$$x + 40 = 3x$$
$$40 = 2x$$
$$x = 20$$

- Therefore, the height of the tower is:

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \approx 34.64\,\text{m}$$

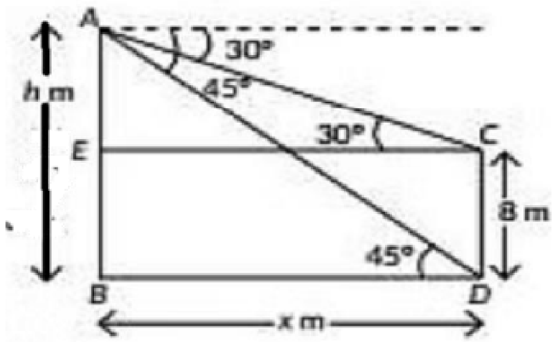
- The length of the original shadow is $x = 20\,\text{m}$.

Quick Tip

In problems involving shadows, use trigonometric ratios like $\tan\theta = \frac{\text{height}}{\text{shadow}}$ and set up equations to solve for unknowns.

34(B): The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings. (use $\sqrt{3} = 1.73$)

Solution:



In $\triangle AEC$, $\tan 30° = \frac{h-8}{x}$

$$\Rightarrow h - 8 = \frac{x}{\sqrt{3}} \quad \text{(i)}$$

In $\triangle ABD$, $\tan 45° = \frac{h}{x}$

$$\Rightarrow h = x \quad \text{(ii)}$$

Solving (i) and (ii):

$$h = x = 12 + 4\sqrt{3} = 18.92\,\text{m}$$

Thus, the height $h = 18.92$ m. **End of Solution.**

Quick Tip

When solving for heights using angles of depression, use the basic trigonometric formula $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$. If the angles involve multiple objects, break the problem into simpler parts and solve systematically.

---

35. A chord of a circle of radius 14 cm subtends an angle of 90° at the centre. Find the area of the corresponding minor and major segments of the circle.

Solution:

Area of minor segment:

$$\text{Area of minor segment} = \left(\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right) - \left(\frac{1}{2} \times 14 \times 14\right)$$

$$= 154 - 98 = 56\,\text{sq. cm.}$$

Area of major segment:

$$\text{Area of major segment} = \left(\frac{22}{7} \times 14 \times 14\right) - 56$$

$$= 560 - 56 = 504\,\text{sq. cm.}$$

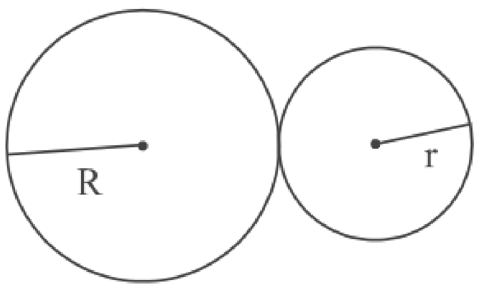
End of Solution.

Quick Tip

For problems involving segments of a circle, first calculate the area of the sector, then subtract the area of the triangle formed by the chord and center to find the segment's area. Ensure to use the correct value of π for approximation.

Section E

36. To keep the lawn green and cool, Sadhna uses water sprinklers which rotate in circular shape and cover a particular area. The diagram below shows the circular areas



(R > r)

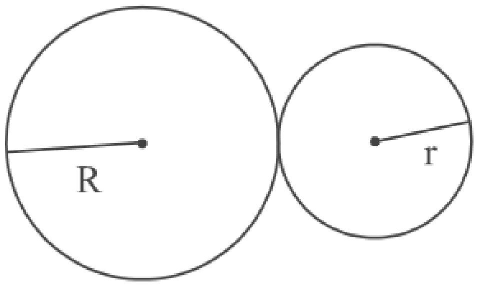
covered by two sprinklers :

(i) Obtain a quadratic equation involving R and r from above.

(ii) Write a quadratic equation involving only r.

(iii) (a) Find the radius r and the corresponding area irrigated. OR

(b) Find the radius R and the corresponding area irrigated.



Solution:

(i) We are given that the sum of the areas of the two circles is 130π. The area of a circle is given by the formula πr^2. So, for the two circles, we have:

$$\pi R^2 + \pi r^2 = 130\pi$$

Dividing through by π, we get:

$$R^2 + r^2 = 130 \qquad (1)$$

(ii) Next, we are given that the distance between the centres of the circles is 14 m. Using the distance formula for the two radii of the circles, we know:

$$R + r = 14$$

Squaring both sides of the equation:

$$(R + r)^2 = 14^2$$

Expanding the left side:

$$R^2 + 2Rr + r^2 = 196 \qquad (2)$$

Now, subtract equation (1) from equation (2):

$$(R^2 + 2Rr + r^2) - (R^2 + r^2) = 196 - 130$$

Simplifying:

$$2Rr = 66$$

$$Rr = 33 \tag{3}$$

(iii) (a) From equation (3), we have $Rr = 33$. Now, substitute $R = \frac{33}{r}$ into equation (1):

$$\left(\frac{33}{r}\right)^2 + r^2 = 130$$

Squaring $\frac{33}{r}$:

$$\frac{1089}{r^2} + r^2 = 130$$

Multiply through by r^2 to clear the denominator:

$$1089 + r^4 = 130r^2$$

Rearranging the equation:

$$r^4 - 130r^2 + 1089 = 0$$

This is a quadratic equation in terms of r^2. Let $x = r^2$, so the equation becomes:

$$x^2 - 130x + 1089 = 0$$

Solve for x using the quadratic formula:

$$x = \frac{-(-130) \pm \sqrt{(-130)^2 - 4(1)(1089)}}{2(1)}$$

$$x = \frac{130 \pm \sqrt{16900 - 4356}}{2}$$

$$x = \frac{130 \pm \sqrt{12544}}{2}$$

$$x = \frac{130 \pm 112}{2}$$

Thus, the two solutions for x are:

$$x = \frac{130 + 112}{2} = 121 \quad \text{or} \quad x = \frac{130 - 112}{2} = 9$$

So, $r^2 = 9$ or $r^2 = 121$. Taking the positive square root, we get:

$$r = 3\,\text{m} \quad \text{or} \quad r = 11\,\text{m}$$

Since $r < R$, we select $r = 3\,\text{m}$. Now, substitute $r = 3$ into equation (3) to find R:

$$R \times 3 = 33 \quad \Rightarrow \quad R = 11\,\text{m}$$

Thus, the radius $r = 3\,\text{m}$ and the corresponding area irrigated by the smaller circle is:

$$\text{Area} = \pi r^2 = \pi \times 3^2 = 9\pi\,\text{m}^2$$

OR

(iii) (b) Now, we solve for the larger circle's radius. From equation (3), we know that $R \times r = 33$. Substituting $r = 3$ into the equation:

$$R \times 3 = 33 \quad \Rightarrow \quad R = 11\,\text{m}$$

The area irrigated by the larger circle is:

$$\text{Area} = \pi R^2 = \pi \times 11^2 = 121\pi\,\text{m}^2$$

Quick Tip

When solving geometric problems involving circles and tangents, ensure all relationships between the variables are correctly interpreted, especially when combining multiple equations. Checking the feasibility of each step is crucial.

37: Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm.

The length of the leaves from different plants are recorded in the following table.

Length (in mm)	**Number of Leaves**
$70 - 80$	3
$80 - 90$	5
$90 - 100$	9
$100 - 110$	12
$110 - 120$	5
$120 - 130$	4
$130 - 140$	2

(i) Write the median class of the data.

(ii) How many leaves are of length equal to or more than 10 cm?

(iii) (a) Find the median of the data.

OR

(b) Write the modal class and find the mode of the data.

Correct Answer: (i) Median Class: 100-110

(ii) Number of leaves ≥ 10 cm: 23

(iii) Median: 102.5 mm, Mode: 103 mm

Solution:

(i) Meadian Class

The cumulative frequency is calculated as follows:

Length (in mm)	Number of Leaves	Cumulative Frequency (CF)
$70 - 80$	3	3
$80 - 90$	5	8
$90 - 100$	9	17
$100 - 110$	12	29
$110 - 120$	5	34
$120 - 130$	4	38
$130 - 140$	2	40

The total number of leaves is 40, so the median will lie at the 20th position. From the cumulative frequency, the 20th leaf lies in the class 100-110, so the median class is 100-110.

(ii) Leaves of length equal to or more than 10 cm (100 mm):

The relevant classes are: - 100-110: 12 leaves - 110-120: 5 leaves - 120-130: 4 leaves - 130-140: 2 leaves

Total = 12 + 5 + 4 + 2 = 23 leaves of length $\geq 10\,$cm.

(iii) (a) To find the median, use the formula for grouped data:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f} \right) \times h$$

Where: - $L = 100$ (lower boundary of the median class) - $N = 40$ (total number of leaves) - $F = 17$ (cumulative frequency before median class) - $f = 12$ (frequency of the median class) - $h = 10$ (class width)

$$\text{Median} = 100 + \left(\frac{20-17}{12}\right) \times 10 = 100 + 2.5 = 102.5\,\text{mm}$$

Thus, the Median is $102.5\,\text{mm}$.

(b) The modal class is the class with the highest frequency, which is 100-110 with 12 leaves. To find the mode, use the formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where: - $L = 100$ (lower boundary of the modal class) - $f_1 = 12$ (frequency of the modal class) - $f_0 = 9$ (frequency of the class before the modal class) - $f_2 = 5$ (frequency of the class after the modal class) - $h = 10$ (class width)

$$\text{Mode} = 100 + \left(\frac{12-9}{2\times 12 - 9 - 5}\right) \times 10 = 100 + \left(\frac{3}{10}\right) \times 10 = 100 + 3 = 103\,\text{mm}$$

Thus, the Mode is $103\,\text{mm}$.

Quick Tip

For grouped data, the median is the value that lies at the $\frac{N}{2}$-th position. The mode can be found using the formula for modal class when the frequency distribution is unimodal.

38: The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with center O. AP and AQ are tangents to the circle at P and Q respectively, such that $AP = 30$ cm and $\angle PAQ = 60^\circ$. Based on the above information, answer the following questions:

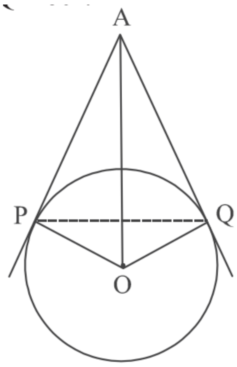
(i) Find the length of PQ.

(ii) Find $m\angle POQ$.

(iii) (a) Find the length of OA.

OR

(b) Find the radius of the mirror.



Correct Answer: (i) Length of PQ: 30 cm

(ii) $m\angle POQ$: 120°

(iii) (a) Length of OA: 60 cm

OR (b) Radius of the mirror: 30 cm

Solution:

(i) Find the length of PQ:

We are given that $AP = AQ = 30\,\text{cm}$, and the angle between the tangents, $\angle PAQ = 60°$.

To find PQ, we can use the law of cosines in triangle PAQ, where:

$$PQ^2 = AP^2 + AQ^2 - 2 \cdot AP \cdot AQ \cdot \cos(\angle PAQ)$$

Substituting the given values:

$$PQ^2 = 30^2 + 30^2 - 2 \cdot 30 \cdot 30 \cdot \cos(60°)$$

Since $\cos(60°) = 0.5$:

$$PQ^2 = 900 + 900 - 2 \cdot 30 \cdot 30 \cdot 0.5$$

$$PQ^2 = 900 + 900 - 900 = 900$$

$$PQ = \sqrt{900} = 30\,\text{cm}$$

Thus, the length of PQ is $30\,\text{cm}$.

(ii) Find $m\angle POQ$:

Since AP and AQ are tangents to the circle from the point A, the angle between the tangents at P and Q is equal to the angle at the center of the circle subtended by the chord PQ. This means that:

$$\angle POQ = 2 \times \angle PAQ$$

Substituting the given value of $\angle PAQ$:

$$\angle POQ = 2 \times 60^\circ = 120^\circ$$

Thus, $m\angle POQ = 120^\circ$.

(iii) (a) Find the length of OA:

To find the length of OA, we can use the law of cosines in triangle OAP, where O is the center of the circle, P is the point of tangency, and A is the external point.

Since $\angle OAP = 90^\circ$ (the angle between the radius and the tangent is always 90°), triangle OAP is a right triangle. Using the Pythagorean theorem:

$$OA^2 = OP^2 + AP^2$$

We know that OP is the radius of the circle, and $AP = 30\,\text{cm}$. Let's denote the radius by r. Thus:

$$OA^2 = r^2 + 30^2$$

But, OA is the hypotenuse of the right triangle OAP, and we know that $\angle PAQ = 60^\circ$, which makes the distance from A to the center O (i.e., OA) double the radius:

$$OA = 2r$$

Substitute $OA = 2r$ into the Pythagorean theorem:

$$(2r)^2 = r^2 + 30^2$$

$$4r^2 = r^2 + 900$$

$$3r^2 = 900$$

$$r^2 = 300$$

$$r = \sqrt{300} = 10\sqrt{3} \approx 17.32\,\text{cm}$$

Thus, OA is twice the radius:

$$OA = 2 \times 17.32 = 34.64\,\text{cm}$$

OR (b) Find the radius of the mirror:

The radius r of the mirror is $\boxed{17.32\,\text{cm}}$.

Quick Tip

In problems involving tangents to circles, the angle between the tangents at the external point is double the angle subtended at the center of the circle. This property is useful for solving for angles and lengths in related triangles.
