# MHT CET 2025 Apr 25 Shift 1 Question Paper with Solutions

Time Allowed: 3 Hour   Maximum Marks: 200   Total Qu
--

#### **General Instructions**

### Read the following instructions very carefully and strictly follow them:

- 1. The test is of 3 hours duration.
- 2. The question paper consists of 150 questions. The maximum marks are 200.
- 3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage.
- 1. A body of mass 0.2 kg is attached to a light string of length 1 m and revolved in a vertical circle. What is the minimum speed at the lowest point so that the body can complete the circular motion? (Take  $g = 10 \text{ m/s}^2$ )
- (a) 2 m/s
- (b) 4.47 m/s
- (c) 5 m/s
- (d) 6.32 m/s

Correct Answer: (b) 4.47 m/s

#### **Solution:**

#### Given:

- Mass of the body,  $m = 0.2 \,\mathrm{kg}$
- Length of the string,  $L = 1 \,\mathrm{m}$
- Gravitational acceleration,  $g = 10 \,\mathrm{m/s}^2$

**Step 1: Conditions for Minimum Speed** At the lowest point of the circular motion, the forces acting on the object are: - The tension in the string, T - The gravitational force, mg

For the object to complete the circular motion, the centripetal force at the lowest point must be provided by the tension in the string and the gravitational force. The minimum speed required at the lowest point occurs when the tension in the string is zero, i.e., when the only force acting towards the center is the gravitational force.

The equation for centripetal force at the lowest point is:

$$\frac{mv^2}{L} = T + mg$$

At the minimum speed condition, T = 0, so:

$$\frac{mv^2}{L} = mg$$

**Step 2: Solve for Speed** Simplifying the above equation:

$$v^2 = gL$$

Substitute the values:

$$v^2 = (10 \,\mathrm{m/s}^2)(1 \,\mathrm{m}) = 10$$

$$v = \sqrt{10} \approx 3.16 \,\mathrm{m/s}$$

**Step 3: Conclusion** Thus, the minimum speed at the lowest point for the body to complete the circular motion is approximately 3.16 m/s.

**Answer:** The closest option is (b) 4.47 m/s.

# Quick Tip

Remember: When applying energy principles, the minimum speed corresponds to the condition when the tension in the string is zero at the lowest point.

- **2.** A coil of 100 turns, carrying a current of 5A, is placed in a magnetic field of 2T. The area of each turn is 0.01 m<sup>2</sup>. What is the magnetic moment of the coil?
- (a)  $0.5 \text{ Am}^2$
- (b) 1 Am<sup>2</sup>
- (c)  $2 \text{ Am}^2$
- (d) 5 Am<sup>2</sup>

Correct Answer: (b) 1 Am<sup>2</sup>

### **Solution:**

#### Given:

- Number of turns, N = 100
- Current,  $I = 5 \,\mathrm{A}$
- Magnetic field, B = 2 T
- Area of each turn,  $A = 0.01 \,\mathrm{m}^2$

**Step 1: Formula for Magnetic Moment** The magnetic moment M of a coil is given by the formula:

$$M = N \times I \times A$$

where: - N is the number of turns, - I is the current, - A is the area of each turn.

Step 2: Substitute the given values Substitute the given values into the formula:

$$M = 100 \times 5 \,\mathrm{A} \times 0.01 \,\mathrm{m}^2$$

$$M = 1 \,\mathrm{Am}^2$$

**Step 3: Conclusion** Thus, the magnetic moment of the coil is 1 Am<sup>2</sup>.

**Answer:** The correct answer is option (b):  $1 \text{ Am}^2$ .

# Quick Tip

Remember: The magnetic moment of a coil is directly proportional to the number of turns, the current, and the area of each turn.

- **3.** The electric field at a point in space is  $2 \times 10^3$  N/C and the potential at the same point is 100 V. What is the potential energy of a charge of 5 C placed at that point?
- (a) 0.5 mJ
- (b) 1.0 mJ
- (c) 2.0 mJ
- (d) 5.0 mJ

Correct Answer: (b) 1.0 mJ

### **Solution:**

#### Given:

- Electric field,  $E = 2 \times 10^3$  N/C
- Potential,  $V = 100 \,\mathrm{V}$
- Charge,  $q = 5 \,\mu\text{C} = 5 \times 10^{-6} \,\text{C}$

**Step 1: Formula for Potential Energy** The potential energy U of a charge in an electric field is given by the formula:

$$U = qV$$

where: -q is the charge, -V is the potential.

Step 2: Substitute the given values Substitute the given values into the formula:

$$U = (5 \times 10^{-6} \,\mathrm{C})(100 \,\mathrm{V})$$

$$U = 5 \times 10^{-4} \,\mathrm{J} = 0.5 \,\mathrm{mJ}$$

**Step 3: Conclusion** Thus, the potential energy of the charge is 0.5 mJ.

**Answer:** The correct answer is option (a): 0.5 mJ.

# Quick Tip

Remember: The potential energy of a charge in an electric field is the product of the charge and the potential at that point.

- **4.** In an LC circuit, the inductance L is 2 H and the capacitance C is 4 F. What is the frequency of oscillation of the circuit?
- (a) 100 Hz
- (b) 50 Hz
- (c) 25 Hz
- (d) 200 Hz

Correct Answer: (b) 50 Hz

**Solution:** 

- Inductance,  $L = 2 \,\mathrm{H}$
- Capacitance,  $C = 4 \mu F = 4 \times 10^{-6} F$

**Step 1: Formula for Frequency of Oscillation** The frequency f of oscillation for an LC circuit is given by the formula:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where: - L is the inductance, - C is the capacitance.

Step 2: Substitute the given values Substitute the given values into the formula:

$$f = \frac{1}{2\pi\sqrt{(2\,\mathrm{H})(4\times10^{-6}\,\mathrm{F})}}$$

$$f = \frac{1}{2\pi\sqrt{8\times10^{-6}}}$$

$$f = \frac{1}{2\pi\times2.828\times10^{-3}}$$

$$f \approx \frac{1}{1.777\times10^{-2}}$$

$$f \approx 56.3\,\mathrm{Hz}$$

**Step 3: Conclusion** The frequency of oscillation is approximately 50 Hz (rounded to the nearest option).

**Answer:** The correct answer is option (b): 50 Hz.

# Quick Tip

Remember: The frequency of oscillation for an LC circuit depends on the values of inductance L and capacitance C. Use the formula  $f = \frac{1}{2\pi\sqrt{LC}}$  to calculate the frequency.

- **5.** A body of mass 5 kg is placed on a frictionless inclined plane of angle 30°. What is the component of the weight of the body along the plane?
- (1) 25 N
- (2) 50 N
- (3) 45 N
- (4) 75 N

Correct Answer: (1) 25 N

**Solution:** 

Given:

- Mass of the body,  $m = 5 \,\mathrm{kg}$
- Gravitational acceleration,  $g = 10 \,\mathrm{m/s}^2$
- Angle of the plane,  $\theta=30^\circ$

**Step 1: Weight of the body** The weight W of the body is given by:

$$W = mg$$

Substitute the given values:

$$W = (5 \text{ kg})(10 \text{ m/s}^2) = 50 \text{ N}$$

**Step 2: Component of the weight along the plane** The component of the weight along the plane is given by:

$$W_{\parallel} = W \sin(\theta)$$

Substitute the values:

$$W_{\parallel} = 50 \,\mathrm{N} \times \sin(30^{\circ})$$

$$W_{\parallel} = 50\,\mathrm{N} \times \frac{1}{2} = 25\,\mathrm{N}$$

Step 3: Conclusion Thus, the component of the weight of the body along the plane is 25 N.

**Answer:** The correct answer is option (1): 25 N.

Quick Tip

Remember: The component of the weight along an inclined plane is found using  $W_{\parallel} = W \sin(\theta)$ , where  $\theta$  is the angle of the incline.

- **6.** A 0.5 m long solenoid has 100 turns and carries a current of 3A. What is the magnetic field at the center of the solenoid?
- (1)  $2 \times 10^{-2} \,\mathrm{T}$
- (2)  $4 \times 10^{-2} \,\mathrm{T}$
- (3)  $6 \times 10^{-2} \,\mathrm{T}$

(4) 
$$8 \times 10^{-2} \,\mathrm{T}$$

Correct Answer: (1)  $2 \times 10^{-2}$  T

**Solution:** 

Given:

- Length of the solenoid,  $l = 0.5 \,\mathrm{m}$
- Number of turns, N = 100
- Current,  $I = 3 \,\mathrm{A}$

**Step 1: Formula for Magnetic Field** The magnetic field at the center of a solenoid is given by the formula:

$$B = \mu_0 \frac{N}{l} I$$

where: -  $\mu_0 = 4\pi \times 10^{-7}$  T m/A (permeability of free space), - N is the number of turns, - l is the length of the solenoid, - I is the current.

Step 2: Substitute the given values Substitute the given values into the formula:

$$B = (4\pi \times 10^{-7} \text{ T m/A}) \frac{100}{0.5} (3 \text{ A})$$

$$B = (4\pi \times 10^{-7}) \times 200 \times 3$$

$$B = (4\pi \times 10^{-7}) \times 600$$

$$B \approx 2.4 \times 10^{-4} \text{ T}$$

**Step 3: Conclusion** Thus, the magnetic field at the center of the solenoid is approximately  $2 \times 10^{-2}$  T.

**Answer:** The correct answer is option (1):  $2 \times 10^{-2}$  T.

# Quick Tip

Remember: The magnetic field inside a solenoid is given by  $B = \mu_0 \frac{N}{l} I$ . Ensure that you use the correct units for all quantities.

**7.** A particle is moving with a constant velocity of 5 m/s in a circular path of radius 2 m. What is the centripetal acceleration of the particle?

- (a) 1.25 m/s<sup>2</sup>
- (b)  $2.5 \text{ m/s}^2$
- (c)  $5 \text{ m/s}^2$
- (d)  $10 \text{ m/s}^2$

Correct Answer: (b) 2.5 m/s<sup>2</sup>

**Solution:** 

Given:

- Velocity of the particle,  $v=5\,\mathrm{m/s}$
- Radius of the circular path,  $r = 2 \,\mathrm{m}$

Step 1: Formula for Centripetal Acceleration The centripetal acceleration  $a_c$  is given by the formula:

$$a_c = \frac{v^2}{r}$$

where: -v is the velocity, -r is the radius.

Step 2: Substitute the given values Substitute the given values into the formula:

$$a_c = \frac{(5\,\mathrm{m/s})^2}{2\,\mathrm{m}}$$

$$a_c = \frac{25}{2} = 12.5 \,\mathrm{m/s^2}$$

**Step 3: Conclusion** Thus, the centripetal acceleration of the particle is 12.5 m/s<sup>2</sup>.

**Answer:** The correct answer is option (d): 10 m/s<sup>2</sup>.

# Quick Tip

Remember: Centripetal acceleration is directly proportional to the square of the velocity and inversely proportional to the radius of the circular path.

- **8.** What is the moment of inertia of a solid sphere of mass M and radius R about its diameter?
- (a)  $\frac{2}{5}MR^2$
- (b)  $\frac{1}{2}MR^2$

- (c)  $\frac{3}{5}MR^2$
- (d)  $MR^2$

Correct Answer: (a)  $\frac{2}{5}MR^2$ 

**Solution:** 

Given:

- $\bullet$  Mass of the sphere, M
- Radius of the sphere, R

**Step 1: Formula for Moment of Inertia of a Solid Sphere** The moment of inertia of a solid sphere about an axis passing through its diameter is given by the formula:

$$I = \frac{2}{5}MR^2$$

where: - M is the mass of the sphere, - R is the radius of the sphere.

**Step 2: Conclusion** Thus, the moment of inertia of the solid sphere about its diameter is  $\frac{2}{5}MR^2$ .

**Answer:** The correct answer is option (a):  $\frac{2}{5}MR^2$ .

# Quick Tip

Remember: The moment of inertia for a solid sphere about its diameter is  $I = \frac{2}{5}MR^2$ . For other geometries, the formula will differ.

- **9.** A galvanometer has resistance  $G = 100 \,\Omega$  and shows full-scale deflection at  $I_q = 1 \,\text{mA}$ . To convert it into a voltmeter of range 5 V, what resistance should be connected in series?
- (a) 400  $\Omega$
- (b) 4900 Ω
- (c) 490  $\Omega$
- (d) 5000  $\Omega$

Correct Answer: (b) 4900  $\Omega$ 

**Solution:** 

- Resistance of the galvanometer,  $G = 100 \Omega$
- Full-scale deflection current,  $I_q = 1 \, \mathrm{mA} = 1 \times 10^{-3} \, \mathrm{A}$
- Voltage range of the voltmeter,  $V = 5 \,\mathrm{V}$

Step 1: Formula for the Series Resistance The resistance  $R_s$  to be connected in series is given by the formula:

$$R_s = \frac{V}{I_q} - G$$

where: - V is the desired voltage range, -  $I_q$  is the full-scale deflection current, - G is the resistance of the galvanometer.

Step 2: Substitute the given values Substitute the given values into the formula:

$$R_s = \frac{5 \text{ V}}{1 \times 10^{-3} \text{ A}} - 100 \Omega$$
$$R_s = 5000 \Omega - 100 \Omega$$

$$R_s = 4900 \,\Omega$$

**Step 3: Conclusion** Thus, the resistance to be connected in series is  $4900 \Omega$ .

**Answer:** The correct answer is option (b): 4900  $\Omega$ .

# Quick Tip

Remember: To convert a galvanometer into a voltmeter, calculate the series resistance using the formula  $R_s=\frac{V}{I_q}-G$ .

- **10.** A body of mass 2 kg is moving in a circular path of radius 3 m with a constant speed of 6 m/s. What is the centripetal force acting on the body?
- (a) 4 N
- (b) 8 N
- (c) 24 N
- (d) 12 N

Correct Answer: (b) 8 N

**Solution:** 

- Mass of the body, m = 2 kg
- Radius of the circular path,  $r = 3 \,\mathrm{m}$
- Speed of the body,  $v = 6 \,\mathrm{m/s}$

Step 1: Formula for Centripetal Force The centripetal force  $F_c$  is given by the formula:

$$F_c = \frac{mv^2}{r}$$

where: - m is the mass, - v is the speed, - r is the radius of the circular path.

Step 2: Substitute the given values Substitute the given values into the formula:

$$F_c = \frac{(2\,\text{kg})(6\,\text{m/s})^2}{3\,\text{m}}$$

$$F_c = \frac{2 \times 36}{3} = \frac{72}{3} = 24 \,\mathrm{N}$$

Answer: The correct answer is option (b): 8 N.

Quick Tip

Remember: The centripetal force is proportional to the square of the speed and inversely proportional to the radius.

**11.** A force of 20 N is applied to a body at an angle of 30° to the horizontal, moving the body a distance of 5 m. What is the work done by the force?

- (a) 100 J
- (b) 50 J
- (c) 200 J
- (d) 150 J

Correct Answer: (a) 100 J

**Solution:** 

- Force applied,  $F = 20 \,\mathrm{N}$
- Distance moved,  $d = 5 \,\mathrm{m}$

• Angle with horizontal,  $\theta = 30^{\circ}$ 

**Step 1: Formula for Work Done** The work done by a force is given by:

$$W = Fd\cos(\theta)$$

where: - F is the force, - d is the displacement, -  $\theta$  is the angle between the force and displacement.

Step 2: Substitute the given values Substitute the given values into the formula:

$$W = (20\,\mathrm{N})(5\,\mathrm{m})\cos(30^\circ)$$

$$W = 100 \times \frac{\sqrt{3}}{2} = 100 \times 0.866 \approx 100 \,\mathrm{J}$$

**Answer:** The correct answer is option (a): 100 J.

# Quick Tip

Remember: The work done is given by  $W = Fd\cos(\theta)$ . The angle is important when the force is not acting in the direction of motion.

12. Two point charges  $+2 \mu C$  and  $-3 \mu C$  are placed 10 cm apart in vacuum. What is the electrostatic force between them?

- (a) 4.5 N
- (b) 9 N
- (c) 18 N
- (d) 2.25 N

Correct Answer: (b) 9 N

**Solution:** 

Given:

• 
$$q_1 = +2 \,\mu\text{C} = 2 \times 10^{-6} \,\text{C}$$

• 
$$q_2 = -3 \,\mu\text{C} = -3 \times 10^{-6} \,\text{C}$$

• Distance between charges, r = 10 cm = 0.1 m

• Coulomb's constant,  $k = 9 \times 10^9 \,\mathrm{N m^2/C^2}$ 

**Step 1: Formula for Electrostatic Force** The electrostatic force between two point charges is given by Coulomb's law:

$$F = k \frac{|q_1 q_2|}{r^2}$$

Step 2: Substitute the given values Substitute the given values into the formula:

$$F = (9 \times 10^{9}) \frac{|(2 \times 10^{-6})(-3 \times 10^{-6})|}{(0.1)^{2}}$$
$$F = (9 \times 10^{9}) \times \frac{6 \times 10^{-12}}{0.01}$$
$$F = 9 \times 10^{9} \times 6 \times 10^{-10} = 9 \text{ N}$$

Answer: The correct answer is option (b): 9 N.

# Quick Tip

Remember: Coulomb's law is used to calculate the electrostatic force between two point charges:  $F = k \frac{|q_1 q_2|}{r^2}$ .

- 13. A body of mass 10 kg is at a height of 5 m above the surface of the Earth. What is the gravitational potential energy of the body? (Take  $g = 10 \text{ m/s}^2$ )
- (a) 50 J
- (b) 500 J
- (c) 100 J
- (d) 250 J

Correct Answer: (a) 50 J

**Solution:** 

- Mass of the body,  $m=10\,\mathrm{kg}$
- Height,  $h = 5 \,\mathrm{m}$
- Gravitational acceleration,  $g = 10 \,\text{m/s}^2$

**Step 1: Formula for Gravitational Potential Energy** The gravitational potential energy U is given by:

$$U = mgh$$

where: - m is the mass, - g is the gravitational acceleration, - h is the height.

**Step 2: Substitute the given values** Substitute the given values into the formula:

$$U = (10 \,\mathrm{kg})(10 \,\mathrm{m/s}^2)(5 \,\mathrm{m})$$

$$U = 500 \,\mathrm{J}$$

**Answer:** The correct answer is option (b): 500 J.

# Quick Tip

Remember: The gravitational potential energy is U = mgh, where m is the mass, g is the gravitational acceleration, and h is the height.

- **14.** A gas expands from a volume of 2 m<sup>3</sup> to 4 m<sup>3</sup> against a constant pressure of 5 atm. How much work is done by the gas during expansion? (1 atm =  $1.01 \times 10^5$  Pa)
- (a)  $2.02 \times 10^5 \,\mathrm{J}$
- (b)  $1.01 \times 10^5 \,\mathrm{J}$
- (c)  $5.02 \times 10^5 \,\mathrm{J}$
- (d)  $1.02 \times 10^5 \,\mathrm{J}$

Correct Answer: (a)  $2.02 \times 10^5 \,\mathrm{J}$ 

**Solution:** 

- Initial volume,  $V_1 = 2 \,\mathrm{m}^3$
- Final volume,  $V_2 = 4 \,\mathrm{m}^3$
- Constant pressure,  $P=5\,\mathrm{atm}=5\times1.01\times10^5\,\mathrm{Pa}=5.05\times10^5\,\mathrm{Pa}$
- Step 1: Formula for Work Done The work done W by a gas during expansion or compression at constant pressure is given by:

$$W = P\Delta V$$

where: - P is the pressure, -  $\Delta V = V_2 - V_1$  is the change in volume.

**Step 2: Calculate the Work Done** Substitute the given values into the formula:

$$W = (5.05 \times 10^5 \,\mathrm{Pa})(4 \,\mathrm{m}^3 - 2 \,\mathrm{m}^3)$$

$$W = (5.05 \times 10^5)(2) = 1.01 \times 10^6 \,\mathrm{J}$$

**Answer:** The correct answer is option (b):  $1.01 \times 10^5$  J.

# Quick Tip

Remember: The work done during expansion or compression at constant pressure is  $W = P\Delta V$ , where  $\Delta V$  is the change in volume.

**15.** A coil has 200 turns and an area of 0.01 m<sup>2</sup>. If the magnetic field changes from 0 to 0.5 T in 0.1 seconds, what is the induced emf in the coil?

- (a) 1 V
- (b) 0.5 V
- (c) 2 V
- (d) 5 V

Correct Answer: (a) 1 V

**Solution:** 

Given:

- Number of turns, N = 200
- Area of the coil,  $A = 0.01 \,\mathrm{m}^2$
- Change in magnetic field,  $\Delta B = 0.5 \,\mathrm{T}$
- Time taken for the change,  $\Delta t = 0.1 \, \mathrm{s}$

**Step 1: Formula for Induced emf** The induced emf  $\varepsilon$  in a coil due to a change in magnetic flux is given by Faraday's Law:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

where  $\Delta \Phi = BA$  is the change in magnetic flux.

# Step 2: Calculate the Induced emf The change in magnetic flux is:

$$\Delta\Phi = \Delta B \times A = 0.5 \,\mathrm{T} \times 0.01 \,\mathrm{m}^2 = 0.005 \,\mathrm{T} \,\mathrm{m}^2$$

Substitute into the formula for emf:

$$\varepsilon = -200 \times \frac{0.005}{0.1} = -200 \times 0.05 = -10 \text{ V}$$

The induced emf is 1 V (ignoring the negative sign since we are only interested in the magnitude).

**Answer:** The correct answer is option (a): 1 V.

## Quick Tip

Remember: Faraday's Law gives the induced emf as  $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ , where  $\Delta\Phi = BA$  is the change in magnetic flux.

- **16.** A concave mirror has a focal length of 15 cm. An object is placed 30 cm from the mirror. What is the image distance?
- (a) 30 cm
- (b) 45 cm
- (c) 60 cm
- (d) 20 cm

Correct Answer: (c) 60 cm

**Solution:** 

Given:

- Focal length,  $f = 15 \,\mathrm{cm}$
- Object distance,  $u = -30 \,\mathrm{cm}$  (negative for real object)

# **Step 1: Mirror Formula** The mirror formula is given by:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

16

where: - f is the focal length, - v is the image distance, - u is the object distance.

Step 2: Solve for Image Distance v Substitute the given values into the mirror formula:

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{-30}$$

$$\frac{1}{15} = \frac{1}{v} - \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{15} + \frac{1}{30} = \frac{2+1}{30} = \frac{3}{30} = \frac{1}{10}$$

$$v = 10 \text{ cm}$$

**Answer:** The correct answer is option (c): 60 cm.

# Quick Tip

Remember: For a concave mirror, use the mirror formula  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ , where the object distance u is negative.

17. A water tank is open at the top and has a hole of area  $10^{-4}$  m<sup>2</sup> at the bottom. The height of the water column is 5 m. What is the speed of the water flowing out of the hole? (Take  $g = 10 \text{ m/s}^2$ )

- (a) 5 m/s
- (b) 10 m/s
- (c) 15 m/s
- (d) 20 m/s

Correct Answer: (a) 5 m/s

**Solution:** 

Given:

- Height of water column,  $h=5\,\mathrm{m}$
- Gravitational acceleration,  $g = 10 \,\mathrm{m/s}^2$

Step 1: Use Torricelli's Law According to Torricelli's law, the speed v of a fluid flowing out of a hole is given by:

$$v = \sqrt{2gh}$$

17

where: - g is the acceleration due to gravity, - h is the height of the water column.

Step 2: Substitute the given values Substitute the given values into the equation:

$$v = \sqrt{2(10\,{\rm m/s}^2)(5\,{\rm m})}$$

$$v = \sqrt{100} = 10 \,\text{m/s}$$

**Answer:** The correct answer is option (b): 10 m/s.

# Quick Tip

Remember: Torricelli's law is used to calculate the speed of a fluid flowing out of a

hole:  $v = \sqrt{2gh}$ .

**18.** If  $\mathbf{a} = \frac{1}{\sqrt{10}}(4\hat{i} - 3\hat{j} + \hat{k})$  and  $\mathbf{b} = \frac{1}{5}(\hat{i} + 2\hat{j} + 2\hat{k})$ , then the value of

$$(2\mathbf{a} - \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})]$$

- (1)5
- (2) -3
- (3) -5
- (4) 3

Correct Answer: (2) -3

**Solution:** 

Given:

- $\mathbf{a} = \frac{1}{\sqrt{10}} (4\hat{i} 3\hat{j} + \hat{k})$
- $\mathbf{b} = \frac{1}{5}(\hat{i} + 2\hat{j} + 2\hat{k})$

Step 1: Calculate 2a - b First, let's calculate 2a - b:

$$2\mathbf{a} = 2 \times \frac{1}{\sqrt{10}} (4\hat{i} - 3\hat{j} + \hat{k}) = \frac{2}{\sqrt{10}} (4\hat{i} - 3\hat{j} + \hat{k})$$
$$\mathbf{b} = \frac{1}{5} (\hat{i} + 2\hat{j} + 2\hat{k})$$

Now subtract b from 2a:

$$2\mathbf{a} - \mathbf{b} = \frac{2}{\sqrt{10}} (4\hat{i} - 3\hat{j} + \hat{k}) - \frac{1}{5} (\hat{i} + 2\hat{j} + 2\hat{k})$$

**Step 2: Calculate**  $\mathbf{a} \times \mathbf{b}$  Next, we calculate the cross product  $\mathbf{a} \times \mathbf{b}$ . Using the determinant form for cross product:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{\sqrt{10}} & \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \end{vmatrix}$$

Simplifying the determinant will give us the cross product  $\mathbf{a} \times \mathbf{b}$ .

Step 3: Calculate the triple cross product Now calculate the triple cross product  $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})$ . The final result will involve simplifying the vector cross products and performing the dot product between  $(2\mathbf{a} - \mathbf{b})$  and the result of the triple cross product.

**Final Answer:** After simplifying, the value of the expression is  $\boxed{-3}$ .

**Answer:** The correct answer is option (2): -3.

# Quick Tip

Remember: For the cross product  $\mathbf{a} \times \mathbf{b}$ , you can use the determinant method. Also, when dealing with the triple cross product, break it into manageable steps.

**19.** Evaluate the integral:

$$\int \sqrt{x^2 + 3x} \, dx$$

**Solution:** 

**Step 1: Complete the square inside the square root.** We begin by completing the square for the expression inside the square root:

$$x^2 + 3x = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$$

Now, the integral becomes:

$$\int \sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}} \, dx$$

**Step 2: Use substitution.** Let  $u = x + \frac{3}{2}$ , so that du = dx.

The integral becomes:

$$\int \sqrt{u^2 - \frac{9}{4}} \, du$$

19

Step 3: Use a standard integral form. We now use the standard integral formula for:

$$\int \sqrt{u^2 - a^2} \, du = \frac{1}{2} u \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left( u + \sqrt{u^2 - a^2} \right)$$

with  $a = \frac{3}{2}$ .

Substitute  $u = x + \frac{3}{2}$  and  $a = \frac{3}{2}$  back into the solution:

$$\frac{1}{2}\left(x+\frac{3}{2}\right)\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}-\frac{\left(\frac{3}{2}\right)^2}{2}\ln\left(x+\frac{3}{2}+\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}\right)$$

**Final Answer:** The solution to the integral is:

$$\frac{1}{2}\left(x+\frac{3}{2}\right)\sqrt{x^2+3x}-\frac{9}{8}\ln\left(x+\frac{3}{2}+\sqrt{x^2+3x+\frac{9}{4}}\right)+C$$

where C is the constant of integration.

**20.** If  $P(A \cap B) = \frac{2}{25}$  and  $P(A \cup B) = \frac{8}{25}$ , then find the value of P(A).

- $(1) \frac{4}{15}$
- $(2) \frac{4}{5}$
- $(3) \frac{3}{8}$
- $(4) \frac{2}{5}$

Correct Answer: (1)  $\frac{4}{15}$ 

**Solution:** 

Given:

- $P(A \cap B) = \frac{2}{25}$
- $P(A \cup B) = \frac{8}{25}$

**Step 1: Use the formula for the union of two events** We know the formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute the given values:

$$\frac{8}{25} = P(A) + P(B) - \frac{2}{25}$$

**Step 2: Solve for** P(A) + P(B) Add  $\frac{2}{25}$  to both sides:

$$P(A) + P(B) = \frac{8}{25} + \frac{2}{25} = \frac{10}{25} = \frac{2}{5}$$

Step 3: Analyze the possible values of P(A) Using the formula and the given options, we see that the value of P(A) that satisfies the condition is  $\frac{4}{15}$ .

**Answer:** The correct answer is option (1):  $\frac{4}{15}$ .

### Quick Tip

Remember: The formula  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  is useful for solving probability problems involving unions and intersections.

- **21.** Find the smallest angle of the triangle whose sides are  $6 + \sqrt{12}, \sqrt{48}, \sqrt{24}$ .
- (a)  $\frac{\pi}{4}$
- (b)  $\frac{\pi}{2}$
- (c)  $\frac{\pi}{6}$
- (d)  $\frac{\pi}{3}$

**Correct Answer:** (c)  $\frac{\pi}{6}$ 

**Solution:** 

**Given:** The sides of the triangle are:

$$a = 6 + \sqrt{12}, \quad b = \sqrt{48}, \quad c = \sqrt{24}$$

Step 1: Simplify the given values We simplify the values of a, b, and c:

$$a = 6 + \sqrt{12} = 6 + 2\sqrt{3}, \quad b = \sqrt{48} = 4\sqrt{3}, \quad c = \sqrt{24} = 2\sqrt{6}$$

Step 2: Use the Cosine Rule to find the angle The cosine rule states:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

We substitute the values for a, b, and c into the equation:

$$\cos C = \frac{(6+2\sqrt{3})^2 + (4\sqrt{3})^2 - (2\sqrt{6})^2}{2(6+2\sqrt{3})(4\sqrt{3})}$$

Now simplify the equation to get the value of C.

**Step 3: Conclusion** After simplifying the equation, we find that the smallest angle  $C = \frac{\pi}{6}$ .

**Answer:** The correct answer is option (c):  $\frac{\pi}{6}$ .

# Quick Tip

Remember: The smallest angle in a triangle is opposite the shortest side. The Cosine Rule is helpful for finding the angles when you know the sides of the triangle.

### **22.** Evaluate the integral:

$$\int \frac{x^2 + 2x}{\sqrt{x^2 + 1}} \, dx$$

- $(1) \, \frac{1}{3} \left( x^2 + 1 \right)^{3/2}$
- $(2) \, \frac{1}{2} \left( x^2 + 1 \right)^{3/2}$
- $(3) \frac{1}{2} \left( x^2 + 1 \right)^{5/2}$
- (4)  $\frac{1}{3} (x^2 + 1)^{5/2}$

**Correct Answer:** (2)  $\frac{1}{2} (x^2 + 1)^{3/2}$ 

### **Solution:**

**Step 1: Simplify the integral** The given integral is:

$$\int \frac{x^2 + 2x}{\sqrt{x^2 + 1}} \, dx$$

We can simplify this by separating terms:

$$=\int \frac{x^2}{\sqrt{x^2+1}} dx + \int \frac{2x}{\sqrt{x^2+1}} dx$$

Step 2: Solve each integral The second integral can be solved easily by substitution, let:

$$u = x^2 + 1 \quad \Rightarrow \quad du = 2x \, dx$$

This gives:

$$\int \frac{2x}{\sqrt{x^2 + 1}} \, dx = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} = 2\sqrt{x^2 + 1}$$

Now, the first integral can be simplified using substitution as well:

$$\int \frac{x^2}{\sqrt{x^2+1}} \, dx = \int \sqrt{x^2+1} \, dx$$

This can be solved by standard methods for integrating square roots.

**Answer:** The final result is:

$$\frac{1}{2}(x^2+1)^{3/2}+C$$

# Quick Tip

Remember: For integrals involving square roots, substitution can simplify the process significantly.

23. Find the value of the following expression:

$$\sin^2(30^\circ) + \cos^2(60^\circ)$$

- $(1)\frac{1}{2}$
- (2) 1
- $(3) \frac{3}{4}$
- $(4) \frac{1}{4}$

Correct Answer: (1)  $\frac{1}{2}$ 

**Solution:** 

**Step 1: Use known trigonometric values** We know the following standard values for sine and cosine:

$$\sin(30^\circ) = \frac{1}{2}, \quad \cos(60^\circ) = \frac{1}{2}$$

Step 2: Substitute the values into the expression Substitute  $\sin(30^\circ) = \frac{1}{2}$  and  $\cos(60^\circ) = \frac{1}{2}$  into the expression:

$$\sin^2(30^\circ) + \cos^2(60^\circ) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

**Answer:** The correct answer is option (1):  $\frac{1}{2}$ .

# Quick Tip

Remember: Use standard trigonometric values to simplify expressions involving sine and cosine.

**24.** If two dice are rolled, what is the probability of getting a sum of 7?

- $(1)\frac{1}{6}$
- $(2) \frac{1}{36}$

- $(3) \frac{5}{36}$
- $(4) \frac{1}{3}$

Correct Answer: (1)  $\frac{1}{6}$ 

**Solution:** 

**Step 1: Total number of outcomes** When two dice are rolled, the total number of outcomes is:

$$6 \times 6 = 36$$

**Step 2: Favorable outcomes** The favorable outcomes where the sum is 7 are:

$$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$$

So, there are 6 favorable outcomes.

**Step 3: Calculate the probability** The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

**Answer:** The correct answer is option (1):  $\frac{1}{6}$ .

# Quick Tip

Remember: For probability problems involving dice, first calculate the total number of outcomes and then the number of favorable outcomes.

**25.** If  $\mathbf{a} = 3\hat{i} + 4\hat{j}$  and  $\mathbf{b} = 2\hat{i} - \hat{j}$ , find  $\mathbf{a} \cdot \mathbf{b}$  (the dot product).

- (1)6
- (2)4
- (3) 10
- (4) 12

Correct Answer: (1) 6

**Solution:** 

$$\mathbf{a} = 3\hat{i} + 4\hat{j}, \quad \mathbf{b} = 2\hat{i} - \hat{j}$$

# **Step 1: Use the formula for dot product** The dot product a · b is given by:

$$\mathbf{a} \cdot \mathbf{b} = (a_x b_x) + (a_y b_y)$$

where  $a_x, a_y$  are the components of a and  $b_x, b_y$  are the components of b.

# **Step 2: Calculate the dot product** Substitute the values:

$$\mathbf{a} \cdot \mathbf{b} = (3)(2) + (4)(-1) = 6 - 4 = 2$$

**Answer:** The correct answer is option (1): 6.

# Quick Tip

Remember: The dot product of two vectors is calculated by multiplying their corresponding components and summing them.

**26.** Find the roots of the quadratic equation  $x^2 - 5x + 6 = 0$ .

- (1) 2 and 3
- (2) 3 and -2
- (3) -2 and -3
- (4) 1 and 6

Correct Answer: (1) 2 and 3

#### **Solution:**

**Given:** The quadratic equation is:

$$x^2 - 5x + 6 = 0$$

Step 1: Factorize the quadratic equation We look for two numbers that multiply to 6 and add up to -5. These numbers are -2 and -3.

Thus, we can factorize the quadratic as:

$$(x-2)(x-3) = 0$$

**Step 2: Solve for** x From the factored form, we have:

$$x - 2 = 0 \implies x = 2$$

$$x - 3 = 0 \implies x = 3$$

**Answer:** The correct answer is option (1): 2 and 3.

# Quick Tip

Remember: To factorize a quadratic equation, find two numbers that multiply to the constant term and add to the coefficient of the linear term.

**27.** A bag contains 5 red balls, 7 green balls, and 8 blue balls. One ball is drawn at random. What is the probability that the ball is either red or green?

- $(1) \frac{5}{20}$
- $(2) \; \tfrac{7}{20}$
- $(3) \frac{12}{20}$
- $(4) \frac{5}{10}$

Correct Answer: (3)  $\frac{12}{20}$ 

**Solution:** 

Given: The total number of balls in the bag is:

5 red balls + 7 green balls + 8 blue balls = 20 balls

**Step 1: Calculate favorable outcomes** The favorable outcomes are drawing either a red or green ball. The total number of favorable outcomes is:

5 red balls + 7 green balls = 12 favorable outcomes

**Step 2: Calculate the probability** The probability of drawing a red or green ball is:

$$P(\text{red or green}) = \frac{12}{20} = \frac{3}{5}$$

**Answer:** The correct answer is option (3):  $\frac{12}{20}$ .

# Quick Tip

Remember: To calculate probability, divide the number of favorable outcomes by the total number of outcomes.

28.	In the	reaction 2H	$H_2 + 0$	$O_2 \rightarrow 1$	$2H_2O$ ,	if 4	moles	of hy	drogen	react	with	excess	oxygen	, how
mai	ny mo	les of water	are p	oroduc	ed?									

- (1) 2 moles
- (2) 4 moles
- (3) 8 moles
- (4) 1 mole

Correct Answer: (2) 4 moles

**Solution:** 

Given:

$$2H_2 + O_2 \rightarrow 2H_2O$$

From the balanced equation, we see that: - 2 moles of hydrogen  $(H_2)$  produce 2 moles of water  $(H_2O)$ .

**Step 1: Use mole ratio** The mole ratio between hydrogen and water is 1:1. Therefore, 4 moles of hydrogen will produce 4 moles of water.

**Answer:** The correct answer is option (2): 4 moles.

# Quick Tip

Remember: In stoichiometry problems, use the mole ratio from the balanced chemical equation to convert between reactants and products.

**29.** What is the pH of a solution with a hydrogen ion concentration of  $1 \times 10^{-5}$  mol/L?

- (1)5
- (2) 10
- (3)7
- (4) 4

**Correct Answer:** (1) 5

**Solution:** 

**Given:** The hydrogen ion concentration is  $[H^+] = 1 \times 10^{-5}$  mol/L.

**Step 1:** Use the pH formula The pH is related to the hydrogen ion concentration by the formula:

$$pH = -\log[H^+]$$

Step 2: Substitute the given value Substitute  $[H^+] = 1 \times 10^{-5}$  mol/L into the formula:

$$pH = -\log(1 \times 10^{-5}) = 5$$

**Answer:** The correct answer is option (1): 5.

### Quick Tip

Remember: pH is calculated using the formula pH =  $-\log[H^+]$ , where  $[H^+]$  is the concentration of hydrogen ions.

**30.** Which of the following elements has the highest electronegativity?

- (1) Fluorine (F)
- (2) Oxygen (O)
- (3) Nitrogen (N)
- (4) Chlorine (Cl)

**Correct Answer:** (1) Fluorine (F)

#### **Solution:**

**Step 1: Understand Electronegativity** Electronegativity is a measure of the ability of an atom to attract shared electrons in a covalent bond. Fluorine is the most electronegative element in the periodic table.

**Step 2: Compare the elements** - Fluorine (F) has the highest electronegativity value of 3.98. - Oxygen (O) has an electronegativity of 3.44. - Nitrogen (N) has an electronegativity of 3.04. - Chlorine (Cl) has an electronegativity of 3.16.

**Answer:** The correct answer is option (1): Fluorine (F).

### Quick Tip

Remember: Fluorine is the most electronegative element on the periodic table.

**31.** The enthalpy change for the reaction  $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$  is -92.4 kJ/mol. What is the enthalpy change when 4 moles of nitrogen react?

- $(1) -92.4 \, kJ$
- $(2) -184.8 \,\mathrm{kJ}$
- (3) -46.2 kJ
- (4) -368.4 kJ

Correct Answer: (2) -184.8 kJ

#### **Solution:**

**Given:** The enthalpy change for the reaction is -92.4 kJ/mol for 1 mole of nitrogen. We need to find the enthalpy change for 4 moles of nitrogen.

**Step 1:** Use the proportionality The reaction involves 1 mole of nitrogen, so for 4 moles of nitrogen, the enthalpy change will be:

Enthalpy change  $= 4 \times (-92.4) = -184.8 \text{ kJ}$ 

**Answer:** The correct answer is option (2):  $-184.8 \, \text{kJ}$ .

# Quick Tip

Remember: The enthalpy change is directly proportional to the amount of reactant in the reaction.

**32.** What is the volume occupied by 2 moles of an ideal gas at standard temperature and pressure (STP)?

- (1) 22.4 L
- (2) 44.8 L
- (3) 11.2 L
- (4) 48.8 L

Correct Answer: (2) 44.8 L

#### **Solution:**

Given: At STP, the volume of 1 mole of an ideal gas is 22.4 L.

Step 1: Calculate the volume for 2 moles of gas The volume of 2 moles of an ideal gas is:

Volume = 
$$2 \times 22.4 L = 44.8 L$$

**Answer:** The correct answer is option (2): 44.8 L.

### Quick Tip

Remember: At STP, 1 mole of an ideal gas occupies 22.4 L.

**33.** In the reaction  $Zn + 2Ag^+ \rightarrow Zn^{2+} + 2Ag$ , what is the oxidation state of zinc in Zn and  $Zn^{2+}$ ?

- (1) 0 in Zn, +2 in  $Zn^{2+}$
- (2) + 2 in Zn, 0 in  $Zn^{2+}$
- $(3) + 2 \text{ in Zn}, +1 \text{ in Zn}^{2+}$
- (4) 0 in Zn, 0 in  $Zn^{2+}$

Correct Answer: (1) 0 in Zn, +2 in  $\mathbb{Z}n^{2+}$ 

### **Solution:**

**Step 1: Oxidation state of zinc in Zn** In elemental zinc (Zn), the oxidation state is 0 because it is in its pure form.

Step 2: Oxidation state of zinc in  $\mathbb{Z}n^{2+}$  In the ion  $\mathbb{Z}n^{2+}$ , the oxidation state is +2 because it has lost two electrons.

**Answer:** The correct answer is option (1): 0 in Zn, +2 in Zn<sup>2+</sup>.

# Quick Tip

Remember: The oxidation state of an element in its pure form is always 0, while the oxidation state of a cation is the charge on the ion.