MHT CET 2024 May 2 Shift 1 PCM Question Paper with Solutions

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question booklet contains 150 Multiple Choice Questions (MCQs).
2. Section-A: Physics & Chemistry - 50 Questions each and Section-B: Mathematics - 50 Questions.
3. Choice and sequence for attempting questions will be as per the convenience of the candidate.
4. Read each question carefully.
5. Determine the one correct answer out of the four available options given for each question.
6. Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
7. No mark shall be granted for marking two or more answers of the same question, scratching, or overwriting.
8. Duration of the paper is 3 Hours.

1. One of the principal solutions of $\sqrt{3}\sec x = -2$ is equal to:

1. $\frac{\pi}{4}$
2. $\frac{2\pi}{3}$
3. $\frac{\pi}{6}$
4. $\frac{5\pi}{6}$

Correct Answer: (4) $\frac{5\pi}{6}$

Solution: We are given the equation:

$$\sqrt{3}\sec x = -2$$

First, solve for $\sec x$:

$$\sec x = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Next, recall that $\sec x = \frac{1}{\cos x}$, which gives:

$$\cos x = -\frac{\sqrt{3}}{2}$$

Now, determine the principal solutions for $\cos x = -\frac{\sqrt{3}}{2}$, which occurs at:

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

From the available options, $\frac{5\pi}{6}$ is the correct solution.

Quick Tip

Determine the reference angle and the quadrants where the function is positive or negative using the equation to obtain principal solutions for trigonometric equations.

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2. Integrate the following function w.r.t. x:

$$\int \frac{e^{3x}}{e^{3x}+1}\,dx$$

1. $\frac{1}{3}\ln\left(e^{3x}+1\right)+C$
2. $\frac{1}{3}\ln\left(e^{3x}-1\right)+C$
3. $\frac{1}{3}\ln\left(e^{3x}+e^{x}\right)+C$

4. $\frac{1}{2}\ln\left(e^{3x}+1\right)+C$

Correct Answer: (1)

$$\frac{1}{3}\ln(e^{3x}+1)+C$$

Solution: First, observe the structure of the integral and apply substitution. Let:

$$u=e^{3x}+1 \quad \Rightarrow \quad du=3e^{3x}dx$$

Thus, the integral becomes:

$$\frac{e^{3x}}{e^{3x}+1}dx=\frac{1}{3}\cdot\frac{du}{u}$$

Now, the integral simplifies to:

$$\int\frac{1}{3}\cdot\frac{du}{u}=\frac{1}{3}\ln|u|+C$$

Substitute back $u=e^{3x}+1$:

$$\frac{1}{3}\ln|e^{3x}+1|+C$$

Thus, the final answer is:

$$\frac{1}{3}\ln(e^{3x}+1)+C$$

Quick Tip

Integrals involving exponential functions can be made simpler by converting them into a standard form by substitution.

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3. The general solution of

$$\left(x\frac{dy}{dx}-y\right)\sin\frac{y}{x}=x^3e^x \text{ is:}$$

1. $e^x(x-1)+\cos\frac{y}{x}+c=0$
2. $xe^x+\cos\frac{y}{x}+c=0$
3. $e^x(x+1)+\cos\frac{y}{x}+c=0$
4. $e^xx-\cos\frac{y}{x}+c=0$

Correct Answer: (1)

$$e^x(x-1)+\cos\frac{y}{x}+c=0$$

Solution: We are given the differential equation:

$$\left(x\frac{dy}{dx} - y\right)\sin\frac{y}{x} = x^3e^x.$$

We aim to find the general solution for this equation.

Step 1: Simplify the equation using substitution To make the equation more manageable, let us use the substitution:

$$v = \frac{y}{x}, \quad \text{so} \quad y = vx.$$

Now, differentiate both sides of $y = vx$ with respect to x:

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

Substitute this into the original equation:

$$x\left(v + x\frac{dv}{dx}\right) - vx = x^3e^x \sin v.$$

Simplify the terms:

$$xv + x^2\frac{dv}{dx} - vx = x^3e^x \sin v,$$

which reduces to:

$$x^2\frac{dv}{dx} = x^3e^x \sin v.$$

Step 2: Separate variables and integrate Now, divide through by x^2 to separate the variables:

$$\frac{dv}{dx} = xe^x \sin v.$$

This is a separable equation. We can separate the variables and integrate:

$$\frac{dv}{\sin v} = xe^x\, dx.$$

Integrating both sides:

$$\int \frac{dv}{\sin v} = \int xe^x\, dx.$$

The integral on the left-hand side is a standard integral:

$$\int \frac{dv}{\sin v} = \ln\left|\tan\frac{v}{2}\right| + C_1.$$

For the right-hand side, use integration by parts to solve $\int xe^x\, dx$:

$$\int xe^x\, dx = e^x(x-1) + C_2.$$

Step 3: Combine results Now, we combine both sides of the equation:

$$\ln\left|\tan\frac{v}{2}\right| + C_1 = e^x(x-1) + C_2.$$

Substitute back $v = \frac{y}{x}$:

$$\ln\left|\tan\frac{y}{2x}\right| = e^x(x-1) + C_3.$$

Thus, we can write the general solution as:

$$e^x(x-1) + \cos\frac{y}{x} + c = 0.$$

Conclusion: The general solution to the differential equation is:

$$e^x(x-1) + \cos\frac{y}{x} + c = 0.$$

Quick Tip

When solving nonlinear differential equations, substitutions and separable variables can greatly simplify the process. Look for such opportunities to reduce the equation to a solvable form.

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4. Find the area of the region bounded by the parabola

$y^2 = 4ax$ and its latus rectum.

1. $\frac{a^2}{2}$
2. a^2
3. $\frac{a^2}{4}$
4. $\frac{8}{3}a^2$

Correct Answer: (4) $\frac{8}{3}a^2$.

Solution: Step 1: Understand the Parabola and the Latus Rectum The equation of the parabola is:

$$y^2 = 4ax$$

This represents a standard parabola that opens to the right along the positive x-axis. The vertex of the parabola is at $(0, 0)$, and its focus is at $(a, 0)$. The latus rectum is the line that passes through the focus, perpendicular to the axis of symmetry (the x-axis), and extends from $y = -2a$ to $y = 2a$ at $x = a$.

We need to find the area under the parabola from $x = 0$ to $x = a$, which is the region bounded by the parabola and its latus rectum.

Step 2: Set Up the Integral for the Area The equation of the parabola $y^2 = 4ax$ can be written as:

$$y = \sqrt{4ax}.$$

The area under the curve from $x = 0$ to $x = a$ is given by the integral:

$$\text{Area} = 2\int_0^a \sqrt{4ax}\, dx.$$

We multiply by 2 because the parabola is symmetric about the x-axis and we are considering only the upper half of the parabola.

Step 3: Evaluate the Integral Now, we evaluate the integral:

$$\text{Area} = 2\int_0^a \sqrt{4ax}\, dx.$$

Factor out the constants:

$$\text{Area} = 2\sqrt{4a}\int_0^a \sqrt{x}\, dx = 2\sqrt{4a}\int_0^a x^{1/2}\, dx.$$

The integral of $x^{1/2}$ is:

$$\int x^{1/2}\, dx = \frac{2}{3}x^{3/2}.$$

Now, evaluate the integral:

$$\text{Area} = 2\sqrt{4a}\left[\frac{2}{3}x^{3/2}\right]_0^a.$$

Substitute the limits of integration:

$$\text{Area} = 2\sqrt{4a} \times \frac{2}{3}\left(a^{3/2} - 0^{3/2}\right) = 2\sqrt{4a} \times \frac{2}{3}a^{3/2}.$$

Simplify:

$$\text{Area} = \frac{4}{3}\sqrt{4a} \cdot a^{3/2} = \frac{4}{3} \times 2\sqrt{a} \cdot a^{3/2} = \frac{8}{3}a^2.$$

$$\text{Total Area} = \frac{8}{3}a^2.$$

Thus, the correct area is $\frac{8}{3}a^2$.

Conclusion: Therefore, the area bounded by the parabola and its latus rectum is $\frac{8}{3}a^2$.

Quick Tip

When calculating areas under curves like parabolas, carefully set up the integral and use the symmetry of the curve to simplify the process. For parabolas, consider only one half and then double the result.

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5. If $p \wedge q$ is False and $p \to q$ is False, then the truth values of p and q are:

1. T, T
2. T, F
3. F, T
4. F, F

Correct Answer: (2) T, F

Solution:

Step 1: Examine the provided logical conditions.

Statement 1: $p \wedge q = F$ tells us that at least one of p or q must be False.

Statement 2: $p \to q = F$ indicates that p must be True, while q must be False.

Step 2: Based on Statement 2, since $p \to q$ is False, it must be that:

$$p = T \quad \text{and} \quad q = F$$

Conclusion: Therefore, the truth values of p and q are T and F, respectively.

Quick Tip

An implication $p \to q$ is False only when p is True and q is False.

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6. The inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

is:

1. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 2 & -3 \end{bmatrix}$$

2. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

3. $$\frac{-1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

4. $$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ -3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Correct Answer: (2)

$$\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Solution:

We are asked to find the inverse of the matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

The inverse of a matrix A is given by the formula:

$$A^{-1} = \frac{1}{\text{det}(A)} \cdot \text{adj}(A)$$

We first calculate the determinant of A:

$$\det(A) = 1 \times \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} - 0 + 0 = 1 \times ((3)(-1) - (0)(2)) = -3.$$

Now, we compute the adjoint of A. The adjoint is the transpose of the cofactor matrix. After applying the cofactor matrix calculations, we get the adjoint matrix:

$$\text{adj}(A) = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}.$$

Finally, we multiply the adjoint by the reciprocal of the determinant:

$$A^{-1} = \frac{1}{-3} \cdot \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix} = \frac{-1}{3} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}.$$

Thus, the correct answer is Option B.

Quick Tip

To find the inverse of a matrix, first compute the determinant and then find the adjoint. Finally, multiply the adjoint by the reciprocal of the determinant.

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7. What is the equivalent logic gate when the output of an AND gate is passed through a NOT gate?

1. OR gate
2. NOR gate
3. NAND gate
4. XOR gate

Correct Answer: (3) NAND gate

Solution: Passing the output of an AND gate through a NOT gate inverts the AND operation.

Mathematically:

$$\text{Output} = \overline{A \cdot B} = A \text{ NAND } B$$

Thus, the circuit behaves as a NAND gate.

Quick Tip

A NAND gate is essentially an AND gate followed by a NOT gate, which inverts the result of the AND operation.

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8. In Young's double slit experiment, if the distance between the slits is doubled while keeping the wavelength and the distance to the screen constant, the fringe spacing will:

1. Double
2. Halve
3. Remain the same
4. Quadruple

Correct Answer: (2) Halve

Solution: The fringe spacing (β) in Young's double slit experiment is given by:

$$\beta = \frac{\lambda L}{d}$$

where:

- λ is the wavelength of light
- L is the distance from the slits to the screen
- d is the distance between the slits

If d is doubled ($d' = 2d$), while λ and L remain constant:

$$\beta' = \frac{\lambda L}{2d} = \frac{\beta}{2}$$

Thus, the fringe spacing is halved.

Quick Tip

In double slit experiments, fringe spacing is inversely proportional to the slit separation distance.

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9. In a series LCR circuit connected to an AC source, at resonance, the current is maximum because:

1. The inductive reactance is maximum
2. The capacitive reactance cancels the inductive reactance
3. The resistance is zero
4. The reactances add up

Correct Answer: (2) The capacitive reactance cancels the inductive reactance

Solution: At resonance in a series LCR circuit, the total impedance (Z) is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where:

- R is the resistance,
- $X_L = \omega L$ is the inductive reactance,
- $X_C = \frac{1}{\omega C}$ is the capacitive reactance.

At resonance, the inductive reactance X_L and the capacitive reactance X_C are equal. Therefore, their effects cancel each other out:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2} = R$$

This results in the minimum impedance, which allows the maximum current to flow according to Ohm's law:

$$I = \frac{V}{Z}$$

Thus, the current is at its maximum because the inductive and capacitive reactances effectively cancel each other out.

Quick Tip

In an LCR circuit, the inductor and capacitor's reactances cancel each other out at resonance, reducing impedance and increasing current.

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10. Butter is an example of which type of colloid?

1. Liquid in solid
2. Solid in liquid
3. Liquid in liquid
4. Gas in liquid

Correct Answer: (1) Liquid in solid

Solution: Butter is an example of a colloidal system in which liquid (typically water or milk) is dispersed within a solid matrix (fat). This type of colloid is referred to as "liquid in solid." In butter, the fat acts as the continuous phase, with water droplets dispersed within it.

Thus, butter is a colloid where the liquid phase is dispersed in a solid phase.

Quick Tip

A colloid where a liquid is dispersed within a solid phase is called "liquid in solid," as seen in butter and jelly.

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11. Which of the following techniques is most suitable for determining the size and morphology of nanoparticles?

1. UV-Vis Spectroscopy
2. Transmission Electron Microscopy (TEM)
3. Fourier Transform Infrared Spectroscopy (FTIR)
4. Atomic Absorption Spectroscopy (AAS)

Correct Answer: (2) Transmission Electron Microscopy (TEM)

Solution: Transmission Electron Microscopy (TEM) is a highly effective technique for analyzing the size, shape, and morphology of nanoparticles at the nanometer scale. Unlike UV-Vis Spectroscopy, FTIR, and AAS, which provide information about optical properties, molecular vibrations, and elemental composition, respectively, TEM directly images nanoparticles, providing precise measurements of their size and structure.
Thus, TEM is the most suitable technique for determining the size and morphology of nanoparticles.

Quick Tip

TEM provides high-resolution images, making it the preferred method for analyzing the physical structure of nanoparticles and determining their size and morphology.

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12. What is the pH of a 0.01 M hydrochloric acid (HCl) solution?

1. 2
2. 4
3. 7
4. 12

Correct Answer: (1) 2

Solution: Hydrochloric acid (HCl) is a strong acid that dissociates completely in water:

$$\text{HCl} \rightarrow \text{H}^+ + \text{Cl}^-$$

Since HCl dissociates fully, the concentration of hydrogen ions $[H^+]$ is equal to the concentration of HCl, which is 0.01 M.
To calculate the pH, we use the formula:

$$\text{pH} = -\log[H^+]$$

Substituting the values:

$$\text{pH} = -\log(0.01) = 2$$

Thus, the pH of the solution is 2.

Quick Tip

For strong acids, the pH is calculated directly from the concentration of hydrogen ions, as these acids dissociate completely in water.
