# MHT CET 2024 3 May Shift 2 PCM Question Paper with Solutions

## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. This question booklet contains 150 Multiple Choice Questions (MCQs).
2. Section-A: Physics & Chemistry - 50 Questions each and Section-B: Mathematics - 50 Questions.
3. Choice and sequence for attempting questions will be as per the convenience of the candidate.
4. Read each question carefully.
5. Determine the one correct answer out of the four available options given for each question.
6. Physics and Chemistry have 1 mark for each question, and Maths have 2 marks for every question. There shall be no negative marking.
7. No mark shall be granted for marking two or more answers of the same question, scratching, or overwriting.
8. Duration of the paper is 3 Hours.

**1. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is:**

(1) $\frac{1}{2}$

(2) 1

(3) $\frac{1}{\sqrt{2}}$

(4) 2

**Correct Answer:** (3) $\frac{1}{\sqrt{2}}$.

**Solution:**

**Step 1:** Let $y = \sec(\tan^{-1} x)$.

Let $\theta = \tan^{-1}(x)$. Then, by the definition of the inverse tangent function, we have:

$$\tan(\theta) = x.$$

**Step 2:** Differentiate the expression for $y$. We know that:

$$\sec^2(\theta) = 1 + \tan^2(\theta) = 1 + x^2,$$

so:

$$\sec(\theta) = \sqrt{1 + x^2}.$$

Thus, $y = \sec(\theta) = \sqrt{1 + x^2}$.

**Step 3:** Differentiate $y$ with respect to $x$. Now, differentiate $y = \sqrt{1 + x^2}$ using the chain rule:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 + x^2}} \cdot 2x = \frac{x}{\sqrt{1 + x^2}}.$$

**Step 4:** Evaluate $\frac{dy}{dx}$ at $x = 1$. Substitute $x = 1$ into the derivative:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + 1^2}} = \frac{1}{\sqrt{2}}.$$

Thus, the correct answer is $\frac{1}{\sqrt{2}}$.

**Quick Tip**

When differentiating $\sec(\tan^{-1} x)$, use the fact that $\sec(\theta) = \sqrt{1 + \tan^2(\theta)}$ and apply the chain rule to differentiate the function.

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**2. If $y = \log_e \left[ e^{3x} \left( \frac{x-4}{x+3} \right)^{3/2} \right]$, then find $\frac{dy}{dx}$:**

(1) $3 + \frac{21}{2(x-4)(x+3)}$

(2) $3 + \frac{21}{(x-4)(x+3)}$
(3) $3 + \frac{21}{2(x+3)(x-4)}$
(4) $3 + \frac{7}{(x-4)(x+3)}$
**Correct Answer:** (1) $3 + \frac{21}{2(x-4)(x+3)}$.
**Solution:** We are given the function:

$$y = \log_e \left[ e^{3x} \left( \frac{x-4}{x+3} \right)^{3/2} \right].$$

**Step 1:** Simplify the logarithmic expression using properties of logarithms.

$$y = \log_e \left( e^{3x} \right) + \log_e \left( \left( \frac{x-4}{x+3} \right)^{3/2} \right).$$

Using the property of logarithms $\log_e(a^b) = b \log_e(a)$, we get:

$$y = 3x + \frac{3}{2} \log_e \left( \frac{x-4}{x+3} \right).$$

**Step 2:** Differentiate $y$ with respect to $x$. The derivative of $3x$ is 3, and for the second term, we apply the chain rule to differentiate the logarithmic expression:

$$\frac{dy}{dx} = 3 + \frac{3}{2} \cdot \frac{d}{dx} \left[ \log_e \left( \frac{x-4}{x+3} \right) \right].$$

Now, use the derivative of a logarithmic function $\frac{d}{dx} \left[ \log_e \left( \frac{u}{v} \right) \right] = \frac{1}{u/v} \cdot \frac{d}{dx} \left( \frac{u}{v} \right)$:

$$\frac{d}{dx} \left[ \log_e \left( \frac{x-4}{x+3} \right) \right] = \frac{1}{\frac{x-4}{x+3}} \cdot \frac{d}{dx} \left[ \frac{x-4}{x+3} \right].$$

Now differentiate $\frac{x-4}{x+3}$ using the quotient rule:

$$\frac{d}{dx} \left[ \frac{x-4}{x+3} \right] = \frac{(x+3)(1) - (x-4)(1)}{(x+3)^2} = \frac{7}{(x+3)^2}.$$

Substitute this into the derivative expression:

$$\frac{dy}{dx} = 3 + \frac{3}{2} \cdot \frac{7}{(x-4)(x+3)} = 3 + \frac{21}{2(x-4)(x+3)}.$$

Thus, the correct answer is $3 + \frac{21}{2(x-4)(x+3)}$.

**Quick Tip**

When differentiating logarithmic functions, first simplify the expression using logarithmic properties, then apply the chain rule to differentiate.

**3. Find the differential equation of the family of all circles, whose center lies on the x-axis and touches the y-axis at the origin.** (1) $2xy\frac{dy}{dx} = y^2 - x^2$

(2) $2xy\frac{dy}{dx} = x^2 - y^2$

(3) $x^2 + y^2 = 2xy\frac{dy}{dx}$

(4) $x^2 + y^2 = 2y\frac{dy}{dx}$

**Correct Answer:** (1) $2xy\frac{dy}{dx} = y^2 - x^2$.

**Solution:** The equation of a circle with center at $(h, 0)$ and radius $h$ is given by:

$$(x - h)^2 + y^2 = h^2.$$

$$x^2 + y^2 + h^2 - 2hx = h^2$$

$$x^2 + y^2 - 2hx = 0$$

$$2x + 2y\frac{dy}{dx} - 2h = 0$$

$$h = x + y\frac{dy}{dx}$$

$$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$$

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$

Thus, the equation of the family of circles is:

$$2xy\frac{dy}{dx} = y^2 - x^2.$$

**Quick Tip**

When differentiating implicit equations, apply the chain rule and remember to differentiate each term with respect to $x$. For a family of circles, center and radius conditions are key.

**4. If $f(x) = 3x + 6$, $g(x) = 4x + k$, and $f \circ g(x) = g \circ f(x)$, then find $k$:**

(1) 3

(2) 6

(3) 9

(4) 12

**Correct Answer:** (3) 9.

**Solution:** We are given the functions $f(x) = 3x + 6$ and $g(x) = 4x + k$. We are also given that $f \circ g(x) = g \circ f(x)$, meaning:

$$f(g(x)) = g(f(x)).$$

Let's first compute both compositions:

$$f(g(x)) = f(4x + k) = 3(4x + k) + 6 = 12x + 3k + 6.$$

$$g(f(x)) = g(3x + 6) = 4(3x + 6) + k = 12x + 24 + k.$$

Now, equate the two expressions:

$$12x + 3k + 6 = 12x + 24 + k.$$

Canceling out the $12x$ terms:

$$3k + 6 = 24 + k.$$

Solving for $k$:

$$3k - k = 24 - 6,$$

$$2k = 18,$$

$$k = 9.$$

Thus, the value of $k$ is 9.

**Quick Tip**

When solving problems with function compositions, equate the two expressions and solve for the unknowns. Pay attention to the structure of the functions to avoid mistakes.