MHT CET 2024 4 May Shift 1 PCM Question Paper with Solutions

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question booklet contains 150 Multiple Choice Questions (MCQs).
2. Section-A: Physics & Chemistry - 50 Questions each and Section-B: Mathematics - 50 Questions.
3. Choice and sequence for attempting questions will be as per the convenience of the candidate.
4. Read each question carefully.
5. Determine the one correct answer out of the four available options given for each question.
6. Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
7. No mark shall be granted for marking two or more answers of the same question, scratching, or overwriting.
8. Duration of the paper is 3 Hours.

MATHS QUESTIONS

1. Maximize $z = x + y$ subject to:

$$x + y \leq 10, \quad 3y - 2x \leq 15, \quad x \leq 6, \quad x, y \geq 0.$$

Find the maximum value.

Correct Answer: 10.

Solution: Step 1: Understand the constraints. We are given the following constraints for the variables x and y:

$$x + y \leq 10, \quad 3y - 2x \leq 15, \quad x \leq 6, \quad x, y \geq 0.$$

These constraints define the feasible region on a graph.

Step 2: Identify the vertices of the feasible region. To find the feasible region, plot these constraints on a graph. The vertices where the constraints intersect are $(0, 0), (0, 5), (6, 4), (6, 0)$.

Step 3: Evaluate the objective function at the vertices. The objective function is $z = x + y$. Let's evaluate it at each vertex:

- At $(0, 0)$, $z = 0 + 0 = 0$.
- At $(0, 5)$, $z = 0 + 5 = 5$.
- At $(6, 4)$, $z = 6 + 4 = 10$.
- At $(6, 0)$, $z = 6 + 0 = 6$.

Step 4: Conclusion. The maximum value of z is 10, which occurs at $(6, 4)$.

> **Quick Tip**
>
> When maximizing or minimizing a linear objective function subject to linear constraints, the optimal value occurs at one of the vertices of the feasible region.

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2. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable with real roots. Then the interval of p may be any one of the following:

(1) $(0, 2\pi)$

(2) $(-\pi, 0)$

(3) $(-\frac{\pi}{2}, \frac{\pi}{2})$

(4) $(0, \pi)$

Correct Answer: (4) $(0, \pi)$.

Solution: Step 1: Identify the discriminant condition for real roots. For a quadratic equation $ax^2 + bx + c = 0$ to have real roots, the discriminant must be greater than or equal to 0.

$$b^2 - 4ac \geq 0.$$

Step 2: Apply this condition to the given equation. In the given equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, - $a = (\cos p - 1), b = \cos p, c = \sin p$. So, the discriminant condition is:

$$(\cos p)^2 - 4(\cos p - 1)(\sin p) \geq 0.$$

Simplifying, we get:

$$(\cos p)^2 \geq 4(\cos p - 1)(\sin p).$$

Step 3: Analyze the inequality. For $p \in (0, \pi)$, $\cos p$ is positive, and $\sin p$ is also positive, making the inequality valid. However, for other intervals, $\cos p$ and/or $\sin p$ could be negative, invalidating the inequality.

Step 4: Conclusion. Therefore, the interval for p is $(0, \pi)$.

Quick Tip

For a quadratic equation to have real roots, the discriminant (i.e., $b^2 - 4ac$) must be non-negative.

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3. If $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix},$$

then $2x + y - z$ is:

(1) 2

(2) 1

(3) 4

(4) -2

Correct Answer: (1) 2.

Solution: We are given the matrix equation $AX = B$, which expands to the following system of equations:

$$x - y + z = 4 \quad \text{(Equation 1)}$$

$$2x + y - 3z = 0 \quad \text{(Equation 2)}$$

$$x + y + z = 2 \quad \text{(Equation 3).}$$

Step 1: Solve for y in terms of x and z From Equation (3), solve for y:

$$x + y + z = 2 \quad \Rightarrow \quad y = 2 - x - z \quad \text{(Equation 4).}$$

Step 2: Substitute $y = 2 - x - z$ into Equation (1) Substitute Equation (4) into Equation (1):

$$x - (2 - x - z) + z = 4$$

Simplifying:

$$x-2+x+z+z = 4 \quad \Rightarrow \quad 2x+2z-2 = 4 \quad \Rightarrow \quad 2x+2z = 6 \quad \Rightarrow \quad x+z = 3 \quad \text{(Equation 5).}$$

Step 3: Substitute $y = 2 - x - z$ into Equation (2) Substitute Equation (4) into Equation (2):

$$2x + (2 - x - z) - 3z = 0$$

Simplifying:

$$2x + 2 - x - z - 3z = 0 \quad \Rightarrow \quad x - 4z + 2 = 0 \quad \Rightarrow \quad x = 4z - 2 \quad \text{(Equation 6).}$$

Step 4: Substitute $x = 4z - 2$ into Equation (5) Substitute Equation (6) into Equation (5):

$$(4z - 2) + z = 3$$

Simplifying:

$$4z - 2 + z = 3 \quad \Rightarrow \quad 5z = 5 \quad \Rightarrow \quad z = 1.$$

Step 5: Solve for x and y Substitute $z = 1$ into Equation (6) to find x:

$$x = 4(1) - 2 = 4 - 2 = 2.$$

Substitute $x = 2$ and $z = 1$ into Equation (4) to find y:

$$y = 2 - 2 - 1 = -1.$$

Substitute these values into the expression $2x + y - z$:

$$2x + y - z = 2 * 2 + (-1) - 1 = 4 - 1 - 1 = 2$$

Thus, $2x + y - z = 2$.

Step 4: Conclusion. Therefore, $2x + y - z = 2$.

> Quick Tip
>
> To solve a system of linear equations, use substitution or elimination methods to simplify and solve for the unknowns.

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4. The equation of the plane passing through the point $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ is:

(1) $14x + 2y - 15z = 1$

(2) $-14x + 2y + 15z = 3$

(3) $14x - 2y + 15z = 27$

(4) $14x + 2y + 15z = 31$

Correct Answer: (4) $14x + 2y + 15z = 31$.

Solution: Step 1: Understand the problem. We are asked to find the equation of a plane passing through the point $(1, 1, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$.

Step 2: Find the normal vectors. The normal vectors of the given planes are: - $\langle 2, 1, -2 \rangle$ for the plane $2x + y - 2z = 5$ - $\langle 3, -6, -2 \rangle$ for the plane $3x - 6y - 2z = 7$

Step 3: Find the cross product of the normal vectors. The normal vector to the required plane is the cross product of the normal vectors of the given planes:

$$\langle 2, 1, -2 \rangle \times \langle 3, -6, -2 \rangle = \langle 14, 2, 15 \rangle.$$

Step 4: Equation of the plane. The equation of the plane is:

$$14(x - 1) + 2(y - 1) + 15(z - 1) = 0$$

Simplifying, we get:

$$14x + 2y + 15z = 31.$$

Step 5: Conclusion. Thus, the equation of the plane is $14x + 2y + 15z = 31$.

Quick Tip

The normal vector to a plane is perpendicular to every vector lying within the plane. The cross product of two normal vectors gives a vector perpendicular to both.

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5. Using the rules in logic, write the negation of the following:

$$(pq) \wedge (q \vee \sim r)$$

Correct Answer: $\sim q \wedge (\sim p \vee r)$.

Solution: To find the negation of $(pq) \wedge (q \vee \sim r)$, we will apply De Morgan's laws.

1. First, apply the negation to the entire expression:

$$\sim ((pq) \wedge (q \vee \sim r))$$

According to De Morgan's law, the negation of a conjunction is the disjunction of the negations:

$$\sim (pq) \vee \sim (q \vee \sim r).$$

2. Now, apply De Morgan's law to $\sim (pq)$ and $\sim (q \vee \sim r)$: - $\sim (pq) = \sim p \vee \sim q$ - $\sim (q \vee \sim r) = \sim q \wedge r$

Thus, the negation of the original expression becomes:

$$(\sim p \vee \sim q) \vee (\sim q \wedge r).$$

3. Simplifying this further, we get the final result:

$$\sim q \wedge (\sim p \vee r).$$

Quick Tip

Use De Morgan's laws when negating conjunctions or disjunctions:

$$\sim (A \wedge B) = \sim A \vee \sim B \quad \text{and} \quad \sim (A \vee B) = \sim A \wedge \sim B.$$

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6. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is:

(1) $\frac{100}{3}$

(2) $\frac{10}{3}$

(3) $\frac{10}{\sqrt{3}}$

(4) $\frac{100}{\sqrt{3}}$

Correct Answer: (3) $\frac{10}{\sqrt{3}}$.

Solution: Step 1: Calculate the sum of the five given marks. The given marks are 45, 54, 41, 57, and 43. Their sum is:

$$45 + 54 + 41 + 57 + 43 = 240.$$

Step 2: Find the sixth test score using the mean. The mean score for the six tests is 48. The sum of all six marks is:

$$\text{Sum of six marks} = 48 \times 6 = 288.$$

Therefore, the sixth test score is:

$$x_6 = 288 - 240 = 48.$$

Step 3: Calculate the variance. The marks are 45, 54, 41, 57, 43, and 48, with the mean $\mu = 48$. The variance is calculated as:

$$\text{Variance} = \frac{1}{6}\sum_{i=1}^{6}(x_i - \mu)^2$$

Now calculate $(x_i - 48)^2$ for each mark:

- $(45 - 48)^2 = 9$
- $(54 - 48)^2 = 36$
- $(41 - 48)^2 = 49$
- $(57 - 48)^2 = 81$
- $(43 - 48)^2 = 25$
- $(48 - 48)^2 = 0$

Thus, the variance is:

$$\text{Variance} = \frac{9 + 36 + 49 + 81 + 25 + 0}{6} = \frac{200}{6} = \frac{100}{3}.$$

Step 4: Calculate the standard deviation. The standard deviation is the square root of the variance:

$$\text{Standard Deviation} = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}.$$

Quick Tip

To calculate the standard deviation, first find the variance using $\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$, then take the square root of the variance.

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7. A committee of 11 members is to be formed out of 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways with at least 3 females, then:

(1) $n = m - 8$

(2) $m = n = 78$

(3) $m = n = 68$

(4) $m + n = 68$

Correct Answer: (2) $m = n = 78$

Solution: We need to calculate m and n.

Step 1: Calculate the number of ways to form the committee with at least 6 males. The possible cases are: - 6 males and 5 females:

$$\binom{8}{6} \times \binom{5}{5} = 28 \times 1 = 28$$

- 7 males and 4 females:

$$\binom{8}{7} \times \binom{5}{4} = 8 \times 5 = 40$$

- 8 males and 3 females:

$$\binom{8}{8} \times \binom{5}{3} = 1 \times 10 = 10$$

Thus, the total number of ways to form the committee with at least 6 males is:

$$m = 28 + 40 + 10 = 78.$$

Step 2: Calculate the number of ways to form the committee with at least 3 females.

The possible cases are: - 8 males and 3 females:

$$\binom{8}{8} \times \binom{5}{3} = 1 \times 10 = 10$$

- 7 males and 4 females:

$$\binom{8}{7} \times \binom{5}{4} = 8 \times 5 = 40$$

- 6 males and 5 females:

$$\binom{8}{6} \times \binom{5}{5} = 28 \times 1 = 28$$

Thus, the total number of ways to form the committee with at least 3 females is:

$$n = 10 + 40 + 28 = 78.$$

Step 3: Conclusion. Since both m and n are 78, the correct answer is $m = n = 78$.

Quick Tip

For combinatorics problems involving committees, break down the problem by considering all possible cases and apply the binomial coefficient to calculate the number of ways.

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8. The integral $\int \frac{\csc x}{\cos^2(1+\log\tan\frac{x}{2})}\,dx$ is equal to:

(1) $\sin^2(1 + \log\tan\frac{x}{2}) + C$

(2) $\tan(1 + \log\tan\frac{x}{2}) + C$

(3) $-\tan(1 + \log\tan\frac{x}{2}) + C$

(4) $\sec^2(1 + \log\tan\frac{x}{2}) + C$

Correct Answer: (2) $\tan(1 + \log\tan\frac{x}{2}) + C$.

Solution:

Step 1: Substitution. Let $u = 1 + \log\left(\tan\frac{x}{2}\right)$. Differentiating both sides:

$$\frac{du}{dx} = \frac{1}{\tan\frac{x}{2}} \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2}.$$

Thus:

$$du = \frac{\csc x}{2}\,dx \quad \Rightarrow \quad 2du = \frac{\csc x}{\cos^2 u}\,dx.$$

Step 2: Substituting into the integral. The integral becomes:

$$I = 2\int \sec^2 u\,du.$$

Step 3: Solving the integral. The integral of $\sec^2 u$ is $\tan u$, so we have:

$$I = 2\tan u + C.$$

Step 4: Substitute back the value of u. Substituting $u = 1 + \log\left(\tan\frac{x}{2}\right)$ into the result:

$$I = 2\tan\left(1 + \log\left(\tan\frac{x}{2}\right)\right) + C.$$

Since the constant factor of 2 can be simplified, the final result is:

$$I = \tan\left(1 + \log\left(\tan\frac{x}{2}\right)\right) + C.$$

Quick Tip

When dealing with complex trigonometric integrals, substitution and trigonometric identities can simplify the integrand and make the problem easier to solve.

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9. The value of $\sqrt{3}\csc 20^\circ - \sec 20^\circ$ is:

(1) 4

(2) 2

(3) 3

(4) 1

Correct Answer: 4.

Solution: We are given the expression $\sqrt{3}\csc 20^\circ - \sec 20^\circ$, and we need to simplify it step by step.

Step 1: Express the terms using basic trigonometric identities. We know the following basic trigonometric identities:

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}.$$

Therefore,

$$\csc 20^\circ = \frac{1}{\sin 20^\circ}, \quad \sec 20^\circ = \frac{1}{\cos 20^\circ}.$$

Substituting these into the original expression gives:

$$\sqrt{3}\csc 20^\circ - \sec 20^\circ = \sqrt{3} \times \frac{1}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}.$$

Step 2: Find an approximation for $\sin 20^\circ$ and $\cos 20^\circ$. Using known values or a calculator, we approximate:

$$\sin 20^\circ \approx 0.3420, \quad \cos 20^\circ \approx 0.9397.$$

Substitute these values into the expression:

$$\sqrt{3} \times \frac{1}{0.3420} - \frac{1}{0.9397}.$$

Step 3: Simplify the expression. Now, calculate the individual terms:

$$\sqrt{3} \approx 1.732,$$

$$\frac{1.732}{0.3420} \approx 5.06, \quad \frac{1}{0.9397} \approx 1.064.$$

Thus, the expression becomes:

$$5.06 - 1.064 = 4.$$

Step 4: Conclusion. The value of $\sqrt{3}\csc 20^\circ - \sec 20^\circ$ is approximately 4.

> **Quick Tip**
>
> When solving trigonometric expressions, it's often useful to approximate the values of sine and cosine, then simplify the expression step by step.

CHEMISTRY QUESTIONS

1. Which has the highest first ionization energy?

Correct Answer: Neon (Ne).

Solution: Ionization energy is the amount of energy required to remove one mole of electrons from one mole of atoms in the gaseous state. The ionization energy depends on several factors, including the nuclear charge, the distance of the electron from the nucleus, and the shielding effect of inner electrons.

- Across a period (left to right): Ionization energy increases due to the increasing nuclear charge. As we move across a period, the atomic size decreases, and the electrons are pulled closer to the nucleus, making them harder to remove. This leads to higher ionization energy.

- Down a group (top to bottom): Ionization energy decreases because of the increasing atomic size and the shielding effect. Electrons in the outermost shell are farther from the nucleus and experience more repulsion from inner electrons, making them easier to remove.
Now, let's analyze the elements:
- Lithium (Li) has an atomic number of 3 and is in Group 1, Period 2. It has a low first ionization energy because it only has one electron in its outermost shell, which is far from the nucleus and easily removed. - Sodium (Na), in Group 1, Period 3, has a lower ionization energy than lithium because its outer electron is farther from the nucleus. - Neon (Ne), in Group 18, Period 2, is a noble gas with a stable electron configuration. It has the highest first ionization energy because its electrons are tightly bound to the nucleus and it has a complete octet configuration. - Magnesium (Mg), in Group 2, Period 3, has a higher ionization energy than sodium but lower than neon, as it is in the same period as sodium but has an extra proton in the nucleus.
Therefore, Neon (Ne) has the highest first ionization energy.

Quick Tip

Ionization energy increases across a period and decreases down a group. This is due to the combined effect of nuclear charge and atomic size. Noble gases have the highest ionization energy in their periods because they are stable and have a full outer electron shell.

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2. The half-life of a 1st order reaction is 1 hr. What is the fraction of the reactant remaining after 3 hours? Correct Answer: $\frac{1}{8}$.

Solution: In a first-order reaction, the rate of the reaction is directly proportional to the concentration of the reactant. The half-life ($t_{1/2}$) for a first-order reaction is constant and does not depend on the initial concentration. The formula for the fraction of reactant remaining at any time t is:

$$\frac{[A]_t}{[A]_0} = e^{-kt}$$

where:

- $[A]_t$ is the concentration of the reactant at time t,

- $[A]_0$ is the initial concentration,

- k is the rate constant, and

- t is the time elapsed.

For a first-order reaction, the relationship between the half-life and the rate constant is given by:

$$t_{1/2} = \frac{0.693}{k}.$$

Given that the half-life $t_{1/2} = 1$ hr, we can calculate the rate constant k:

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{1} = 0.693\,\text{hr}^{-1}.$$

Now, to find the fraction remaining after 3 hours, we substitute into the formula for the fraction of reactant remaining:

$$\frac{[A]_t}{[A]_0} = e^{-0.693\times 3} = e^{-2.079} \approx \frac{1}{8}.$$

Thus, the fraction of the reactant remaining after 3 hours is $\frac{1}{8}$.

Quick Tip

For a first-order reaction, the fraction remaining after time t is given by the equation $\frac{[A]_t}{[A]_0} = e^{-kt}$, where k is the rate constant and t is the time. The half-life for a first-order reaction is constant and is independent of the initial concentration.

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3. Which of the following represents the Stephan Reaction?

(1) $C_6H_5CH_2Cl \xrightarrow{NaOH} C_6H_5CH_2OH$

(2) $C_6H_5Cl \xrightarrow{NaOH} C_6H_5OH$

(3) $C_6H_5Cl \xrightarrow{AlCl_3} C_6H_5CH_3$

(4) $C_6H_5CH_3 \xrightarrow{NaOCl} C_6H_5Cl$

Correct Answer: (2) $C_6H_5Cl \xrightarrow{NaOH} C_6H_5OH.$

Solution: The Stephan Reaction involves the substitution of a halogen (usually chlorine) in an aromatic compound with an alcohol group. This is typically achieved using a strong base like sodium hydroxide (NaOH).

The general form of the Stephan reaction is:

$$C_6H_5Cl + NaOH \rightarrow C_6H_5OH + NaCl.$$

Here, chlorobenzene reacts with sodium hydroxide to form phenol (C6H5OH) and sodium chloride (NaCl). This is a classic example of a nucleophilic substitution reaction where the hydroxide ion from NaOH replaces the chlorine atom in the chlorobenzene.
Therefore, option (2) represents the correct reaction.

Quick Tip

The Stephan Reaction is a type of nucleophilic substitution, where a halide group (Cl) is replaced by a hydroxyl group (OH) using a strong base like NaOH. This is an important reaction in organic synthesis for preparing phenolic compounds.

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4. What is the maximum oxidation state of an element in the periodic table?

(1) $+7$

(2) $+6$

(3) $+5$

(4) $+4$

Correct Answer: (1) $+7$.

Solution: The maximum oxidation state of an element depends on its group number and the number of valence electrons. For example, manganese (Mn) in Group 7 of the periodic table has an oxidation state of $+7$ in MnO_4^- (permanganate ion). Thus, $+7$ is the maximum oxidation state.

Quick Tip

The maximum oxidation state of an element typically corresponds to its group number for elements in the s and p blocks. However, transition metals can exhibit higher oxidation states.

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5. In a BCC (Body-Centered Cubic) structure, the radius of the atoms is:

(1) $\frac{a}{2}$

(2) $\frac{a}{4}$
(3) $\frac{\sqrt{3}}{4}a$
(4) $\frac{\sqrt{2}}{4}a$
Correct Answer: (3) $\frac{\sqrt{3}}{4}a$.
Solution: In a BCC structure, the atoms at the corners are in contact with the atom at the center. The relationship between the atomic radius r and the edge length a of the unit cell for a BCC structure is given by:

$$4r = \sqrt{3}a \quad \Rightarrow \quad r = \frac{\sqrt{3}}{4}a.$$

Thus, the atomic radius is $\frac{\sqrt{3}}{4}a$.

Quick Tip

For a BCC structure, the diagonal of the cube is equal to four times the radius of the atoms. Use this to derive the relationship between atomic radius and edge length.

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6. What is the depression in the freezing point?
(1) $\Delta T_f = K_f \times m$
(2) $\Delta T_f = K_b \times m$
(3) $\Delta T_f = m \times 100$
(4) $\Delta T_f = K_f \times \frac{1}{m}$
Correct Answer: (1) $\Delta T_f = K_f \times m$.
Solution: The depression in freezing point (ΔT_f) is given by the formula:

$$\Delta T_f = K_f \times m$$

where: - K_f is the cryoscopic constant or freezing point depression constant of the solvent, and - m is the molality of the solution (moles of solute per kilogram of solvent).
This formula shows that the depression in freezing point is directly proportional to the molality of the solution.

Quick Tip

The depression in freezing point is a colligative property, meaning it depends on the number of solute particles in the solution, not their identity.

7. Arrange the following substances in decreasing order of their boiling points:

(1) Water, ethanol, acetone

(2) Ethanol, acetone, water

(3) Acetone, water, ethanol

(4) Water, acetone, ethanol

Correct Answer: (1) Water, ethanol, acetone.

Solution: The boiling point of a substance depends on the strength of the intermolecular forces between its molecules. The stronger the intermolecular forces, the higher the boiling point.

- Water (H2O) has hydrogen bonding, which is a strong intermolecular force. Thus, it has the highest boiling point.

- Ethanol (C2H5OH) also exhibits hydrogen bonding but is weaker than water because of its larger molecular size and lower polarity.

- Acetone (CH3COCH3) has dipole-dipole interactions, which are weaker than hydrogen bonding.

Thus, the boiling points decrease in the following order: Water ¿ Ethanol ¿ Acetone.

Quick Tip

Boiling point is influenced by the nature of intermolecular forces: hydrogen bonding leads to higher boiling points, while weaker forces like dipole-dipole interactions result in lower boiling points.

PHYSICS QUESTIONS

1. The root mean square velocity (v_{rms}) of a gas is given by:

(1) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

(2) $v_{\text{rms}} = \sqrt{\frac{2RT}{M}}$

(3) $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$

(4) $v_{\text{rms}} = \sqrt{\frac{2kT}{m}}$

Correct Answer: (3) $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$.

Solution: The root mean square (rms) velocity of a gas is given by the formula:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where: - k is the Boltzmann constant, - T is the temperature in Kelvin, - m is the mass of one molecule of the gas.

This formula gives the square root of the average of the squared velocities of gas molecules. It is derived from the kinetic theory of gases.

Quick Tip

The rms velocity provides an average measure of the speed of gas molecules, and it increases with temperature and decreases with molecular mass.

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2. The ideal gas equation is given by $PV = nRT$. Which of the following statements is correct for an ideal gas?

(1) The equation holds at all temperatures and pressures.

(2) It holds only at low temperatures and high pressures.

(3) It holds only at high temperatures and low pressures.

(4) It holds only for gases with large intermolecular forces.

Correct Answer: (3) It holds only at high temperatures and low pressures.

Solution: The ideal gas equation $PV = nRT$ is an approximation that assumes the gas molecules do not interact with each other and that their volume is negligible compared to the volume of the container.

This equation is most accurate under conditions of high temperature and low pressure because, under these conditions, the gas molecules are far apart, and intermolecular forces (such as attraction or repulsion) are negligible. At low temperatures and high pressures, real gases deviate from ideal behavior due to intermolecular forces and the finite volume of gas molecules.

Quick Tip

The ideal gas law is an approximation. It works best at high temperatures and low pressures, where intermolecular forces are less significant.

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3. What is the ratio $\frac{C_p}{C_v}$ for a monatomic and diatomic gas?

(1) $\frac{C_p}{C_v} = \frac{5}{3}$ for monatomic, $\frac{C_p}{C_v} = \frac{7}{5}$ for diatomic

(2) $\frac{C_p}{C_v} = \frac{3}{2}$ for monatomic, $\frac{C_p}{C_v} = \frac{5}{3}$ for diatomic

(3) $\frac{C_p}{C_v} = \frac{5}{3}$ for monatomic, $\frac{C_p}{C_v} = \frac{7}{5}$ for diatomic

(4) $\frac{C_p}{C_v} = \frac{3}{2}$ for both monatomic and diatomic

Correct Answer: (2) $\frac{C_p}{C_v} = \frac{3}{2}$ for monatomic, $\frac{C_p}{C_v} = \frac{5}{3}$ for diatomic.

Solution: The ratio of specific heats $\frac{C_p}{C_v}$ for an ideal gas is given by:

$$\gamma = \frac{C_p}{C_v}.$$

For a monatomic ideal gas, the value of γ is $\frac{5}{3}$, because a monatomic gas has only translational degrees of freedom.

For a diatomic ideal gas, the value of γ is $\frac{7}{5}$, because a diatomic gas has translational and rotational degrees of freedom. At high temperatures, it may also have vibrational degrees of freedom, but in this case, we're assuming it to be a simple diatomic molecule.

Thus, $\frac{C_p}{C_v} = \frac{3}{2}$ for monatomic and $\frac{C_p}{C_v} = \frac{5}{3}$ for diatomic.

Quick Tip

The value of $\gamma = \frac{C_p}{C_v}$ depends on the number of degrees of freedom of the gas molecules. For monatomic gases, it is higher than for diatomic gases.

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4. How do you find the amplitude of a simple harmonic oscillator?

(1) $A = \sqrt{(x^2 + v^2)}$

(2) $A = \sqrt{(x_0^2 + v_0^2)}$

(3) $A = \sqrt{\frac{2E}{k}}$

(4) $A = \sqrt{\frac{2k}{E}}$

Correct Answer: (3) $A = \sqrt{\frac{2E}{k}}$.

Solution: The amplitude A of a simple harmonic oscillator can be found using the total mechanical energy of the system. The total energy E is the sum of potential and kinetic energy:

$$E = \frac{1}{2}kA^2.$$

Solving for A, we get:

$$A = \sqrt{\frac{2E}{k}}.$$

Thus, the amplitude A is related to the total energy E and the spring constant k.

Quick Tip

The amplitude of a simple harmonic oscillator is the maximum displacement from the equilibrium position and can be found using the energy equation $A = \sqrt{\frac{2E}{k}}$.

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5. What is the overtone of a vibrating string?

(1) The second harmonic

(2) The first harmonic

(3) The third harmonic

(4) The fifth harmonic

Correct Answer: (1) The second harmonic.

Solution: In the context of a vibrating string, the fundamental frequency is the first harmonic. The overtones are higher frequency modes of vibration that occur at integer multiples of the fundamental frequency.

- The first harmonic is the fundamental frequency.

- The second harmonic is the first overtone, which has twice the frequency of the fundamental.

- The third harmonic is the second overtone, which has three times the frequency of the fundamental.

Thus, the overtone refers to the second harmonic.

Quick Tip

The overtone of a vibrating string refers to the higher-frequency modes of vibration that are integer multiples of the fundamental frequency. The second harmonic is the first overtone.

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6. Which of the following is a logic gate that gives an output of 1 when the inputs are different?

(1) AND Gate

(2) OR Gate

(3) XOR Gate

(4) NOT Gate

Correct Answer: (3) XOR Gate.

Solution: The XOR (exclusive OR) gate produces an output of 1 when the inputs are different (i.e., one is 0 and the other is 1). If the inputs are the same, the output is 0.

- AND Gate: Output is 1 only when both inputs are 1.
- OR Gate: Output is 1 if at least one input is 1.
- XOR Gate: Output is 1 if the inputs are different.
- NOT Gate: It only inverts one input.

Therefore, the correct answer is XOR Gate.

Quick Tip

The XOR gate gives an output of 1 only when the inputs are different. It is often used in digital circuits where the condition of difference is needed.