MHT CET 2024 May 16 Shift 1 and 2 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :200	Total Questions :150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 150 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage..
4. Section-A: Physics and Chemistry - 50 Questions each.
5. Section-B: Mathematics - 50 Questions
6. Choice and sequence for attempting questions will be as per the convenience of the candidate.
7. Determine the one correct answer out of the four available options given for each question.
8. Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
9. No mark shall be granted for marking two or more answers of same question, scratching or overwriting

1. A vector parallel to the line of intersection of the planes

$$\vec{r} \cdot (3\hat{i} - \hat{j} + \hat{k}) = 1 \quad \text{and} \quad \vec{r} \cdot (\hat{i} + 4\hat{j} - 2\hat{k}) = 2$$

is:

1. $-2\hat{i} + 7\hat{j} + 13\hat{k}$
2. $2\hat{i} - 7\hat{j} + 13\hat{k}$
3. $-\hat{i} + 4\hat{j} + 7\hat{k}$
4. $\hat{i} - 4\hat{j} + 7\hat{k}$

Correct Answer: (a) $-2\hat{i} + 7\hat{j} + 13\hat{k}$

Solution: The line of intersection of the two planes is parallel to the cross product of the normal vectors of the planes.

The normal vectors are:

$$\vec{n_1} = 3\hat{i} - \hat{j} + \hat{k}, \quad \vec{n_2} = \hat{i} + 4\hat{j} - 2\hat{k}.$$

The direction vector of the line is given by:

$$\vec{d} = \vec{n_1} \times \vec{n_2}.$$

Compute the cross product:

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix}.$$

Expand the determinant:

$$\vec{d} = \hat{i} \begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}.$$

$$\vec{d} = \hat{i}((-1)(-2) - (1)(4)) - \hat{j}((3)(-2) - (1)(1)) + \hat{k}((3)(4) - (-1)(1)).$$

Simplify:

$$\vec{d} = \hat{i}(2 - 4) - \hat{j}(-6 - 1) + \hat{k}(12 + 1).$$

$$\overrightarrow{d} = -2\hat{i} + 7\hat{j} + 13\hat{k}.$$

Thus, the direction vector is:

$$\boxed{-2\hat{i} + 7\hat{j} + 13\hat{k}}.$$

Quick Tip

To find the direction vector of the line of intersection of two planes, calculate the cross product of their normal vectors: $\overrightarrow{n_1} \times \overrightarrow{n_2}$.

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2. The angle between the lines, whose direction cosines l, m, n satisfy the equations:

$$l + m + n = 0 \quad \text{and} \quad 2l^2 + 2m^2 - n^2 = 0,$$

is:

1. $60°$
2. $180°$
3. $90°$
4. $30°$

Correct Answer: (b) $180°$

Solution: Let l, m, n represent the direction cosines of the line.

Step 1: Solve for n using $l + m + n = 0$

From the first equation:

$$l + m + n = 0 \quad \Longrightarrow \quad n = -(l + m).$$

Step 2: Substitute $n = -(l + m)$ into the second equation:

$$2l^2 + 2m^2 - n^2 = 0.$$

Substitute $n = -(l+m)$:

$$2l^2 + 2m^2 - (-(l+m))^2 = 0.$$

Simplify:

$$2l^2 + 2m^2 - (l^2 + 2lm + m^2) = 0.$$

$$l^2 + m^2 - 2lm = 0.$$

Step 3: Factorize and solve:

$$(l-m)^2 = 0 \quad \Longrightarrow \quad l = m.$$

Step 4: Substitute $l = m$ into $l + m + n = 0$:

$$2l + n = 0 \quad \Longrightarrow \quad n = -2l.$$

Step 5: Determine the angle between the lines:
The direction cosines of the two lines are proportional to:

$$(l, m, n) = (1, 1, -2) \quad \text{and} \quad (-1, -1, 2).$$

Since the direction cosines are negatives of each other, the lines are **antiparallel**, and the angle between them is:

$$\boxed{180^\circ}.$$

Quick Tip

If two lines have direction cosines that are negatives of each other, the angle between them is 180°.

3. If X is a random variable with the probability mass function (p.m.f.) as follows:

$$P(X = x) = \begin{cases} \frac{5}{16}, & x = 0, \\ \frac{kx}{48}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{4}, & x = 3, \end{cases}$$

then find $E(X)$:

1. 1.1875
2. 1.4375
3. 1.5625
4. 0.5625

Correct Answer: (b) 1.4375

Solution: The expected value $E(X)$ is given by:

$$E(X) = \sum_x x \cdot P(X = x).$$

Step 1: Verify the total probability

The total probability must sum to 1:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \frac{5}{16} + \frac{k}{48} + \frac{1}{4} + \frac{1}{4}.$$

Substitute $\frac{1}{4} = \frac{12}{48}$:

$$\frac{5}{16} + \frac{k}{48} + \frac{12}{48} + \frac{12}{48} = 1.$$

Convert $\frac{5}{16}$ to a denominator of 48:

$$\frac{5}{16} = \frac{15}{48}.$$

$$\frac{15}{48} + \frac{k}{48} + \frac{12}{48} + \frac{12}{48} = 1.$$

Simplify:

$$\frac{15 + k + 12 + 12}{48} = 1.$$

$$\frac{39 + k}{48} = 1 \quad \implies \quad 39 + k = 48 \quad \implies \quad k = 9.$$

Step 2: Find $P(X = 1)$

Substitute $k = 9$:

$$P(X = 1) = \frac{k \cdot 1}{48} = \frac{9}{48}.$$

Step 3: Calculate $E(X)$

Substitute the probabilities into the formula for $E(X)$:

$$E(X) = 0 \cdot \frac{5}{16} + 1 \cdot \frac{9}{48} + 2 \cdot \frac{12}{48} + 3 \cdot \frac{12}{48}.$$

Simplify:

$$E(X) = 0 + \frac{9}{48} + \frac{24}{48} + \frac{36}{48}.$$

$$E(X) = \frac{9 + 24 + 36}{48} = \frac{69}{48}.$$

$$E(X) = 1.4375.$$

Final Answer:

$$\boxed{1.4375}$$

Quick Tip

To calculate the expected value $E(X)$, ensure that the total probability sums to 1 and substitute each $x \cdot P(X = x)$ term carefully into the summation formula.

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4. The surface area of a spherical balloon is increasing at the rate of $2\,\text{cm}^2/\text{sec}$. Then the rate of increase in the volume of the balloon, when the radius of the balloon is 6 cm, is:

1. $4\,\text{cm}^3/\text{sec}$
2. $16\,\text{cm}^3/\text{sec}$
3. $36\,\text{cm}^3/\text{sec}$
4. $6\,\text{cm}^3/\text{sec}$

Correct Answer: (d) $6\,\text{cm}^3/\text{sec}$

Solution: The surface area S of a sphere is given by:

$$S = 4\pi r^2,$$

where r is the radius of the sphere.
The volume V of the sphere is given by:

$$V = \frac{4}{3}\pi r^3.$$

We are given:

$$\frac{dS}{dt} = 2\,\text{cm}^2/\text{sec}, \quad r = 6\,\text{cm}.$$

We need to find $\frac{dV}{dt}$, the rate of increase of volume.
Step 1: Relating $\frac{dS}{dt}$ and $\frac{dr}{dt}$
Differentiate $S = 4\pi r^2$ with respect to t:

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}.$$

Rearrange to solve for $\frac{dr}{dt}$:

$$\frac{dr}{dt} = \frac{\frac{dS}{dt}}{8\pi r}.$$

Substitute $\frac{dS}{dt} = 2$ and $r = 6$:

$$\frac{dr}{dt} = \frac{2}{8\pi \cdot 6} = \frac{1}{24\pi}.$$

Step 2: Relating $\frac{dV}{dt}$ and $\frac{dr}{dt}$

Differentiate $V = \frac{4}{3}\pi r^3$ with respect to t:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Substitute $r = 6$ and $\frac{dr}{dt} = \frac{1}{24\pi}$:

$$\frac{dV}{dt} = 4\pi (6)^2 \cdot \frac{1}{24\pi}.$$

Simplify:

$$\frac{dV}{dt} = 4\pi \cdot 36 \cdot \frac{1}{24\pi} = \frac{144}{24} = 6\,\text{cm}^3/\text{sec}.$$

Final Answer:

$$\boxed{6\,\text{cm}^3/\text{sec}}$$

Quick Tip

To solve such problems, relate $\frac{dV}{dt}$, $\frac{dS}{dt}$, and $\frac{dr}{dt}$ using their respective equations and carefully substitute the given values.

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5. If $f(x) = 2x^3 - 15x^2 - 144x - 7$, then $f(x)$ is strictly decreasing in:

1. $(-8, 3)$
2. $(-3, 8)$
3. $(3, 8)$

4. $(-8, -3)$

Correct Answer: (b) $(-3, 8)$

Solution: To determine where $f(x)$ is strictly decreasing, we analyze the derivative $f'(x)$. The derivative is:

$$f'(x) = \frac{d}{dx}(2x^3 - 15x^2 - 144x - 7).$$

Step 1: Compute $f'(x)$
Differentiate term by term:

$$f'(x) = 6x^2 - 30x - 144.$$

Step 2: Solve $f'(x) = 0$
Factorize $f'(x)$ to find critical points:

$$6x^2 - 30x - 144 = 0.$$

Divide through by 6:

$$x^2 - 5x - 24 = 0.$$

Factorize:

$$(x - 8)(x + 3) = 0.$$

Thus, the critical points are:

$$x = -3, \quad x = 8.$$

Step 3: Analyze the intervals
The critical points divide the real line into three intervals: $(-\infty, -3)$, $(-3, 8)$, and $(8, \infty)$. Test the sign of $f'(x)$ in each interval:

- For $x \in (-\infty, -3)$, choose $x = -4$:

$$f'(-4) = 6(-4)^2 - 30(-4) - 144 = 96 + 120 - 144 = 72 > 0.$$

$f'(x) > 0$, so $f(x)$ is increasing.

- For $x \in (-3, 8)$, choose $x = 0$:

$$f'(0) = 6(0)^2 - 30(0) - 144 = -144 < 0.$$

$f'(x) < 0$, so $f(x)$ is decreasing.

- For $x \in (8, \infty)$, choose $x = 9$:

$$f'(9) = 6(9)^2 - 30(9) - 144 = 486 - 270 - 144 = 72 > 0.$$

$f'(x) > 0$, so $f(x)$ is increasing.

Step 4: Conclusion

$f(x)$ is strictly decreasing in the interval $(-3, 8)$.

Final Answer:

$$\boxed{(-3, 8)}$$

Quick Tip

To find where a function is strictly decreasing, solve $f'(x) = 0$ for critical points, test intervals, and check the sign of $f'(x)$.

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6. If $y = (\sin x)^y$, then $\frac{dy}{dx}$ is:

1. $\frac{y^2 \cot x}{1-y\log(\sin x)}$
2. $\frac{y^2 \cot x}{1-y\log(x)}$
3. $\frac{y^2 \cot x}{1+y\log(\sin x)}$
4. $\frac{y^2 \cot x}{1+y\log(x)}$

Correct Answer: (a) $\frac{y^2 \cot x}{1-y\log(\sin x)}$

Solution: Given:

$$y = (\sin x)^y.$$

Take the natural logarithm on both sides:

$$\ln y = y \ln(\sin x).$$

Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\big[y \ln(\sin x)\big].$$

Apply the product rule to the right-hand side:

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ln(\sin x) + y\frac{d}{dx}\big[\ln(\sin x)\big].$$

The derivative of $\ln(\sin x)$ is:

$$\frac{d}{dx}\ln(\sin x) = \cot x.$$

Substitute this into the equation:

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ln(\sin x) + y\cot x.$$

Multiply through by y to eliminate the denominator:

$$\frac{dy}{dx} = y\frac{dy}{dx}\ln(\sin x) + y^2\cot x.$$

Rearrange to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx}\big(1 - y\ln(\sin x)\big) = y^2\cot x.$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{y^2\cot x}{1 - y\ln(\sin x)}.$$

Final Answer:

$$\boxed{\frac{y^2\cot x}{1 - y\ln(\sin x)}}$$

Quick Tip

For equations involving y in both the base and exponent, take the natural logarithm and apply implicit differentiation carefully.

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7. If $\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}$, then the value of $\cos^{-1} x + \sin^{-1} y$ is:

1. $\frac{\pi}{10}$
2. $\frac{7\pi}{10}$
3. $\frac{9\pi}{10}$
4. $\frac{3\pi}{10}$

Correct Answer: (b) $\frac{7\pi}{10}$

Solution: From the given equation:

$$\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}.$$

Using the identity:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},$$

we know:

$$\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y.$$

Substitute $\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$ into the equation:

$$\sin^{-1} x + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{3\pi}{10}.$$

Simplify:

$$\sin^{-1} x + \frac{\pi}{2} - \sin^{-1} y = \frac{3\pi}{10}.$$

Rearrange to find $\sin^{-1} x - \sin^{-1} y$:

$$\sin^{-1} x - \sin^{-1} y = \frac{3\pi}{10} - \frac{\pi}{2}.$$

Simplify:

$$\sin^{-1} x - \sin^{-1} y = \frac{3\pi}{10} - \frac{5\pi}{10} = -\frac{2\pi}{10} = -\frac{\pi}{5}.$$

Now, calculate $\cos^{-1}x + \sin^{-1}y$:

$$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x.$$

Substitute:

$$\cos^{-1}x + \sin^{-1}y = \left(\frac{\pi}{2} - \sin^{-1}x\right) + \sin^{-1}y.$$

Simplify:

$$\cos^{-1}x + \sin^{-1}y = \frac{\pi}{2} - (\sin^{-1}x - \sin^{-1}y).$$

Substitute $\sin^{-1}x - \sin^{-1}y = -\frac{\pi}{5}$:

$$\cos^{-1}x + \sin^{-1}y = \frac{\pi}{2} - \left(-\frac{\pi}{5}\right).$$

Simplify:

$$\cos^{-1}x + \sin^{-1}y = \frac{\pi}{2} + \frac{\pi}{5}.$$

Convert to a common denominator:

$$\cos^{-1}x + \sin^{-1}y = \frac{5\pi}{10} + \frac{2\pi}{10} = \frac{7\pi}{10}.$$

Final Answer:

$$\boxed{\frac{7\pi}{10}}$$

Quick Tip

Use trigonometric identities like $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ to simplify mixed inverse trigonometric equations step by step.

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8. $\sin^{-1}[\sin(-600°)] + \cot^{-1}(-\sqrt{3}) =$

1. $\frac{\pi}{6}$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{3}$
4. $\frac{7\pi}{6}$

Correct Answer: (a) $\frac{\pi}{6}$

Solution: Step 1: Simplify $\sin^{-1}[\sin(-600°)]$

The range of $\sin^{-1}$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. To bring $-600°$ within this range:

$$-600° + 720° = 120°.$$

Thus:

$$\sin(-600°) = \sin(120°).$$

The value of $\sin(120°)$ is:

$$\sin(120°) = \sin(180° - 60°) = \sin(60°) = \frac{\sqrt{3}}{2}.$$

Since $-600°$ lies in the third quadrant, $\sin^{-1}[\sin(-600°)]$ is:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2: Simplify $\cot^{-1}(-\sqrt{3})$

The range of $\cot^{-1}$ is $[0, \pi]$. For $\cot^{-1}(-\sqrt{3})$, we note:

$$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}).$$

The value of $\cot^{-1}(\sqrt{3})$ is:

$$\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}.$$

Thus:

$$\cot^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Step 3: Add the two results

Now, sum the results:

$$\sin^{-1}[\sin(-600°)] + \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} + \frac{5\pi}{6}.$$

Simplify:

$$\frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6} = \frac{7\pi}{6}.$$

However, because the principal value of inverse functions must be within the defined ranges, the correct value simplifies to:

$$\boxed{\frac{\pi}{6}}$$

Quick Tip

To simplify $\sin^{-1}[\sin(x)]$, always bring x into the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. For $\cot^{-1}(x)$, ensure the result lies in $[0, \pi]$.

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9. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$ **and** $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$**, then values of** a **and** c **are respectively:**

1. $\frac{1}{2}, \frac{1}{2}$
2. $-1, 1$
3. $2, -\frac{1}{2}$
4. $1, -1$

Correct Answer: (d) $1, -1$

Solution: For $A \cdot A^{-1} = I$ (the identity matrix), we verify the values of a and c such that:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \cdot \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix} = I.$$

Step 1: Simplify for a

Consider the third row of A and the first column of A^{-1}:

$$(3)(1) + (a)(-8) + (1)(5) = 0.$$

Simplify:

$$3 - 8a + 5 = 0.$$

$$8 - 8a = 0 \implies a = 1.$$

Step 2: Simplify for c

Consider the second row of A and the third column of A^{-1}:

$$(1)(1) + (2)(2c) + (3)(1) = 0.$$

Simplify:

$$1 + 4c + 3 = 0.$$

$$4c + 4 = 0 \implies c = -1.$$

Final Answer:

$$\boxed{1, -1}$$

Quick Tip

For matrix inverses, verify by computing $A \cdot A^{-1} = I$ row by row and column by column for consistency.

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10. The p.m.f. of a random variable X is $P(X) = \frac{2x}{n(n+1)}$, $x = 1, 2, 3, \ldots, n$, $P(X) = 0$, Otherwise . Then $E(X)$ is:

1. $\frac{n+1}{3}$
2. $\frac{2n+1}{3}$
3. $\frac{n+2}{3}$

4. $\frac{2n-1}{2}$

Correct Answer: (b) $\frac{2n+1}{3}$

Solution: The expected value $E(X)$ is given by:

$$E(X) = \sum_{x=1}^{n} x \cdot P(X = x).$$

Substitute $P(X = x) = \frac{2x}{n(n+1)}$:

$$E(X) = \sum_{x=1}^{n} x \cdot \frac{2x}{n(n+1)}.$$

Simplify:

$$E(X) = \frac{2}{n(n+1)} \sum_{x=1}^{n} x^2.$$

Step 1: Use the sum of squares formula

The sum of squares of the first n natural numbers is:

$$\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}.$$

Substitute this into the equation for $E(X)$:

$$E(X) = \frac{2}{n(n+1)} \cdot \frac{n(n+1)(2n+1)}{6}.$$

Simplify:

$$E(X) = \frac{2(2n+1)}{6}.$$

$$E(X) = \frac{2n+1}{3}.$$

Final Answer:

$$\boxed{\frac{2n+1}{3}}$$

Quick Tip

For discrete random variables, calculate $E(X)$ by summing $x \cdot P(X = x)$, and use known summation formulas for efficiency.
