MHT CET 2024 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**200	**Total Questions :**150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 150 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 50 questions in each part of equal weightage..
4. Section-A: Physics and Chemistry - 50 Questions each.
5. Section-B: Mathematics - 50 Questions
6. Choice and sequence for attempting questions will be as per the convenience of the candidate.
7. Determine the one correct answer out of the four available options given for each question.
8. Each question with correct response shall be awarded one (1) mark. There shall be no negative marking.
9. No mark shall be granted for marking two or more answers of same question, scratching or overwriting

PHYSICS

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1. Force between two point charges q_1 and q_2 placed in vacuum at a distance r cm apart is F. Force between them when placed in a medium having dielectric $K = 5$ at $r/5$ cm apart will be:

(1) $F/25$

(2) $5F$

(3) $F/5$

(4) $25F$

Correct Answer: (2) $5F$

Solution: The electrostatic force between two point charges in vacuum is:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}.$$

In a medium with dielectric constant K, the force becomes:

$$F' = \frac{1}{4\pi K\epsilon_0}\frac{q_1q_2}{(r')^2}.$$

If $K = 5$ and $r' = r/5$, substituting into the expression:

$$F' = \frac{1}{4\pi(5\epsilon_0)}\frac{q_1q_2}{(r/5)^2}.$$

Simplify:

$$F' = \frac{25}{5} \times \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} = 5F.$$

Final Answer:

$$\boxed{5F}$$

Quick Tip

In a medium, the force is affected by both the dielectric constant K and any changes in distance. Always adjust both when solving.

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2. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its center and perpendicular to its plane with angular velocity ω. If another disc of the same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is:

(1) $\frac{4}{5}\omega$
(2) $\frac{5}{4}\omega$
(3) $\frac{2}{3}\omega$
(4) $\frac{3}{2}\omega$

Correct Answer: (3) $\frac{2}{3}\omega$

Solution: Using the conservation of angular momentum:

$$I_1\omega_1 = I_2\omega_2.$$

The initial moment of inertia is:

$$I_1 = \frac{1}{2}MR^2.$$

The final moment of inertia after placing the second disc:

$$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{2}\right)R^2 = \frac{3}{4}MR^2.$$

Substitute into the angular momentum equation:

$$\frac{1}{2}MR^2\omega = \frac{3}{4}MR^2\omega_2.$$

Simplify:

$$\omega_2 = \frac{\frac{1}{2}}{\frac{3}{4}}\omega = \frac{2}{3}\omega.$$

Final Answer:

$$\boxed{\frac{2}{3}\omega}$$

Quick Tip

For conservation of angular momentum, remember that $I\omega$ is constant for a system. Adjust I carefully when masses are added or removed.

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3. Correct Bernoulli's equation is (symbols have their usual meaning):

(1) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$

(2) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

(3) $P + \rho gh + \rho v^2 = \text{constant}$

(4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Correct Answer: (2) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Solution: Bernoulli's theorem states that for an incompressible, non-viscous fluid in steady flow, the sum of pressure energy, potential energy, and kinetic energy per unit volume is constant:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant},$$

where:

- P is the pressure,
- ρ is the density of the fluid,
- g is the acceleration due to gravity,
- h is the height above a reference point,
- v is the velocity of the fluid.

In option (a), the use of m (mass) instead of ρ (density) is incorrect.
Similarly, options (c) and (d) incorrectly modify the potential energy and kinetic energy terms.

Final Answer:

$$\boxed{P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}}$$

Quick Tip

Bernoulli's equation applies to ideal fluids in steady flow. Be cautious with units and terms, especially distinguishing between mass and density.

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4. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(1) $2 : \sqrt{3}$

(2) $\sqrt{3} : 1$

(3) $1 : 3$

(4) $1 : \sqrt{3}$

Correct Answer: (3) $1 : 3$

Solution: The maximum height attained by a projectile in motion is given by:

$$H_{\text{max}} = \frac{u^2 \sin^2 \theta}{2g},$$

where:

- u is the initial velocity,
- θ is the angle of projection,
- g is the acceleration due to gravity.

Let the two projectiles have angles of projection:

$$\theta_1 = 30^\circ \quad \text{and} \quad \theta_2 = 60^\circ.$$

For the first projectile:

$$H_1 = \frac{u^2 \sin^2 30^\circ}{2g}.$$

For the second projectile:

$$H_2 = \frac{u^2 \sin^2 60^\circ}{2g}.$$

The ratio of their maximum heights is:

$$\frac{H_1}{H_2} = \frac{\frac{u^2 \sin^2 30^\circ}{2g}}{\frac{u^2 \sin^2 60^\circ}{2g}}.$$

Canceling common terms u^2 and $2g$, we get:

$$\frac{H_1}{H_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ}.$$

Now, substitute the values of $\sin 30^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$:

$$\frac{H_1}{H_2} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2}.$$

Simplify:

$$\frac{H_1}{H_2} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

Thus, the ratio of the maximum heights is:

$$H_1 : H_2 = 1 : 3.$$

Final Answer:

$$\boxed{1 : 3}$$

Quick Tip

The maximum height of a projectile depends on the square of the sine of the angle of projection, $\sin^2\theta$. For such questions, calculate the ratio of $\sin^2\theta$ for the given angles.

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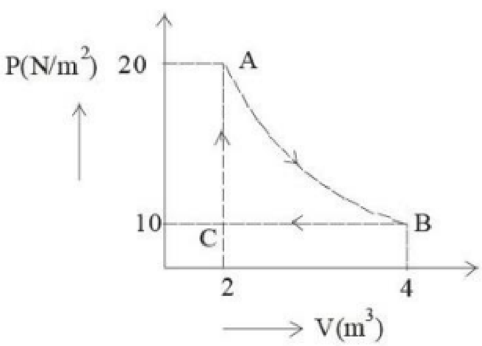
5. A real gas within a closed chamber at 27°C undergoes the cyclic process as shown in the figure. The gas obeys the equation $PV^3 = RT$ for the path A to B. The net work done in the complete cycle is (assuming $R = 8$ J/molK):



(1) 225 J

(2) 205 J

(3) 20 J

(4) -20 J

Correct Answer: (2) 205 J

Solution: The cyclic process consists of three steps: AC (isochoric process), BC (isobaric process), and AB (given $PV^3 = RT$).

Step 1: Isochoric process (AC)

$$W_{\text{AC}} = 0 \quad \text{(no work is done in an isochoric process).}$$

Step 2: Isobaric process (BC) The work done in an isobaric process is given by:

$$W_{\text{BC}} = P\Delta V = 10(2-4) = -20\,\text{J}.$$

Step 3: Path AB ($PV^3 = RT$) The work done in this process is:

$$W_{\text{AB}} = \int P\,dV = \int_{V_1}^{V_2} \frac{RT}{V^3}\,dV.$$

Substitute $V_1 = 2\,\text{m}^3$ and $V_2 = 4\,\text{m}^3$:

$$W_{\text{AB}} = RT\int_2^4 V^{-3}\,dV = -\frac{RT}{2}\left[\frac{1}{V^2}\right]_2^4.$$

Simplify:

$$W_{\text{AB}} = -\frac{RT}{2}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = -\frac{RT}{2}\left(\frac{1}{16} - \frac{1}{4}\right).$$

$$W_{\text{AB}} = -\frac{RT}{2}\left(\frac{1-4}{16}\right) = \frac{RT}{2}\cdot\frac{3}{16}.$$

Substitute $R = 8\,\text{J/molK}$ and $T = 27°\text{C} = 300\,\text{K}$:

$$W_{\text{AB}} = \frac{8\cdot 300}{2}\cdot\frac{3}{16} = 225\,\text{J}.$$

Net Work Done:

$$W_{\text{net}} = W_{\text{AC}} + W_{\text{BC}} + W_{\text{AB}} = 0 - 20 + 225 = 205\,\text{J}.$$

Final Answer:

$$\boxed{205\,\text{J}}$$

Quick Tip

For cyclic processes, compute the work done in each segment and sum them. Pay attention to the nature of the process (isochoric, isobaric, or other).

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6. Light emerges out of a convex lens when a source of light is kept at its focus. The shape of the wavefront of the light is:

(1) Both spherical and cylindrical

(2) Cylindrical

(3) Spherical

(4) Plane

Correct Answer: (4) Plane

Solution: A wavefront represents the locus of points having the same phase of oscillation. When a source of light is kept at the focus of a convex lens, the light rays emerging from the lens become parallel.

Parallel light rays correspond to a plane wavefront, as all points on the wavefront are equidistant from the source and travel in the same direction. The transformation from spherical to plane wavefront occurs due to the focusing property of the lens.

Final Answer:

$$\boxed{\text{Plane}}$$

Quick Tip

A plane wavefront is formed when light rays emerge parallel from a convex lens after passing through its focus.

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7. A monkey of mass 50 kg climbs on a rope which can withstand the tension $T = 350\,\text{N}$. If the monkey initially climbs down with an acceleration of $4\,\text{m/s}^2$ and then climbs up with an acceleration of $5\,\text{m/s}^2$, choose the correct option ($g = 10\,\text{m/s}^2$):

(1) $T = 700\,\text{N}$ while climbing upward

(2) $T = 350\,\text{N}$ while going downward

(3) Rope will break while climbing upward

(4) Rope will break while going downward

Correct Answer: (3) Rope will break while climbing upward

Solution: Step 1: Climbing upward

The tension in the rope while climbing upward is given by:

$$T = m(g + a),$$

where:

- $m = 50\,\text{kg}$ (mass of the monkey),
- $g = 10\,\text{m/s}^2$ (gravitational acceleration),
- $a = 5\,\text{m/s}^2$ (acceleration while climbing upward).

Substitute values:

$$T = 50(10 + 5) = 750\,\text{N}.$$

Since $T > 350\,\text{N}$, the rope will break while climbing upward.

Step 2: Climbing downward

The tension in the rope while climbing downward is given by:

$$T = m(g - a).$$

Substitute values ($a = 4\,\text{m/s}^2$):

$$T = 50(10 - 4) = 300\,\text{N}.$$

Since $T < 350\,\text{N}$, the rope will not break while climbing downward.

Final Answer:

$$\boxed{\text{Rope will break while climbing upward.}}$$

Quick Tip

For objects accelerating vertically, calculate the tension using $T = m(g \pm a)$. Add acceleration for upward motion and subtract for downward motion.

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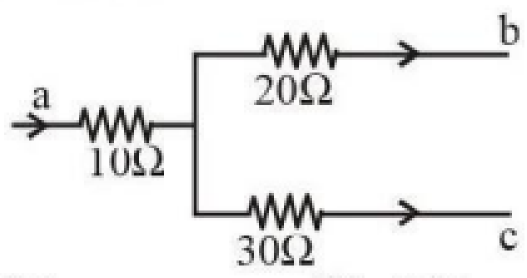
8. Figure shows a part of an electric circuit. The potentials at points $a, b,$ and c are $30\,\mathbf{V}$, $12\,\mathbf{V}$, and $2\,\mathbf{V}$, respectively. The current through the $20\,\Omega$ resistor will be:



(1) $0.4\,\text{A}$

(2) $0.2\,\text{A}$

(3) $0.6\,\text{A}$

(4) $1.0\,\text{A}$

Correct Answer: (1) $0.4\,\text{A}$

Solution: We will calculate the current through the $20\,\Omega$ resistor step by step:

Step 1: Label currents and apply Kirchhoff's Junction Rule

Let the currents through the $10\,\Omega$, $20\,\Omega$, and $30\,\Omega$ resistors be $I_1, I_2,$ and I_3, respectively. At the junction point, the total current entering must equal the total current leaving:

$$I_1 = I_2 + I_3.$$

The voltage differences across the resistors are related to these currents by Ohm's Law:

$$I_1 = \frac{V_a - V_b}{10}, \quad I_2 = \frac{V_b - V_c}{20}, \quad I_3 = \frac{V_b - V_c}{30}.$$

Step 2: Substitute known values

The potentials at points $a, b,$ and c are given as:

$$V_a = 30\,\mathrm{V}, \quad V_b = 12\,\mathrm{V}, \quad V_c = 2\,\mathrm{V}.$$

Substitute these into the current equations:

$$I_1 = \frac{30 - 12}{10} = 1.8\,\mathrm{A}.$$
$$I_2 = \frac{12 - 2}{20} = 0.4\,\mathrm{A}.$$
$$I_3 = \frac{12 - 2}{30} = 0.333\,\mathrm{A}.$$

Step 3: Verify current conservation

At the junction, check if $I_1 = I_2 + I_3$:

$$I_1 = 0.4 + 0.333 = 1.8\,\mathrm{A}.$$

This satisfies Kirchhoff's Junction Rule.

Step 4: Confirm current through $20\,\Omega$

The current through the $20\,\Omega$ resistor is directly I_2, which is:

$$I_2 = 0.4\,\mathrm{A}.$$

Final Answer:

$$\boxed{0.4\,\mathrm{A}}$$

Quick Tip

When solving circuit problems, use Ohm's Law to find individual currents through resistors and Kirchhoff's Laws to verify the conservation of current at junctions.

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9. A current of $200\,\mu$A deflects the coil of a moving coil galvanometer through 60°. The current to cause deflection through $\frac{\pi}{10}$ radian is:

(1) $30\,\mu$A

(2) $120\,\mu$A

(3) $60\,\mu$A

(4) $180\,\mu$A

Correct Answer: (3) $60\,\mu$A

Solution: The deflection in a moving coil galvanometer is directly proportional to the current:

$$I \propto \theta.$$

Let:

$$I_1 = 200\,\mu\text{A}, \quad \theta_1 = 60^\circ, \quad \theta_2 = \frac{\pi}{10}.$$

Convert 60° to radians:

$$\theta_1 = 60^\circ = \frac{\pi}{3}.$$

Using the proportionality:

$$\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}.$$

Substitute the values:

$$\frac{I_2}{200} = \frac{\frac{\pi}{10}}{\frac{\pi}{3}}.$$

Simplify:

$$\frac{I_2}{200} = \frac{3}{10}.$$

Solve for I_2:

$$I_2 = 200 \cdot \frac{3}{10} = 60\,\mu\text{A}.$$

Final Answer:

$$\boxed{60\,\mu\text{A}}$$

Quick Tip

The deflection angle in a galvanometer is proportional to the current. Use the proportionality relation $\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}$ to solve for unknown currents.

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10. The value of acceleration due to gravity at Earth's surface is 9.8 m/s^2. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 m/s^2 is close to: (Radius of Earth $R = 6.4 \times 10^6$ m)

(1) $2.6 \times 10^6\,\text{m}$

(2) $6.4 \times 10^6\,\text{m}$

(3) $9.0 \times 10^6\,\text{m}$

(4) $1.6 \times 10^6\,\text{m}$

Correct Answer: (1) $2.6 \times 10^6\,\text{m}$

Solution: The acceleration due to gravity at an altitude h is given by:

$$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}.$$

Given:

$$g_h = \frac{g}{2}.$$

Substitute into the formula:

$$\frac{g}{2} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}.$$

Simplify:

$$\left(1 + \frac{h}{R}\right)^2 = 2.$$

Take the square root:

$$1 + \frac{h}{R} = \sqrt{2}.$$

Solve for h:

$$\frac{h}{R} = \sqrt{2} - 1.$$

Substitute $R = 6.4 \times 10^6$ m:

$$h = (\sqrt{2} - 1) \cdot 6.4 \times 10^6.$$

Simplify:

$$h = (1.414 - 1) \cdot 6.4 \times 10^6 = 0.414 \cdot 6.4 \times 10^6 = 2.6 \times 10^6 \,\text{m}.$$

Final Answer:

$$\boxed{2.6 \times 10^6 \,\text{m}}$$

Quick Tip

The acceleration due to gravity decreases with altitude. Use the formula $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ to find the height.

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11. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(1) $\varepsilon_r = 0.5, \mu_r = 1.5$

(2) $\varepsilon_r = 1.5, \mu_r = 0.5$

(3) $\varepsilon_r = 0.5, \mu_r = 0.5$

(4) $\varepsilon_r = 1.5, \mu_r = 1.5$

Correct Answer: (2) $\varepsilon_r = 1.5, \mu_r = 0.5$

Solution: For a diamagnetic material:

The relative permeability μ_r is slightly less than 1, as diamagnetic materials tend to repel magnetic fields.

The relative permittivity ε_r is always greater than 1, as it represents the ability of the material to polarize in response to an electric field.

Among the given options:

$$\boxed{\varepsilon_r = 1.5, \mu_r = 0.5}$$

satisfies these conditions.

Final Answer:

$$\boxed{\varepsilon_r = 1.5, \mu_r = 0.5}$$

Quick Tip

For diamagnetic materials, remember: $\mu_r < 1$ and $\varepsilon_r > 1$.

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12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = 5t^3 + 4t^2 + 2t - 5$. If the resistance of the coil is $5\,\Omega$, then the induced current through the coil at $t = 2\,$sec will be:

(1) $15.6\,$A

(2) $16.6\,$A

(3) $17.6\,$A

(4) $18.6\,$A

Correct Answer: (1) $15.6\,$A

Solution: The induced EMF (e) is given by Faraday's Law:

$$e = \left|\frac{d\phi}{dt}\right|.$$

Given:

$$\phi = 5t^3 + 4t^2 + 2t - 5.$$

Differentiate ϕ with respect to t:

$$e = \left|\frac{d\phi}{dt}\right| = \left|15t^2 + 8t + 2\right|.$$

At $t = 2\,$sec:

$$e = 15(2)^2 + 8(2) + 2 = 15(4) + 16 + 2 = 60 + 16 + 2 = 78\,\text{V}.$$

The induced current i is given by Ohm's Law:

$$i = \frac{e}{R}.$$

Substitute $R = 5\,\Omega$:

$$i = \frac{78}{5} = 15.6\,\text{A}.$$

Final Answer:

$$\boxed{15.6\,\text{A}}$$

Quick Tip

Always differentiate ϕ with respect to t to calculate the induced EMF and use $i = \frac{e}{R}$ to find the current.

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13. A solid metallic cube having total surface area $24\,\text{m}^2$ is uniformly heated. If its temperature is increased by 10° C, calculate the increase in volume of the cube.

(Given: $\alpha = 5.0 \times 10^{-4}\,\text{C}^{-1}$)

(1) $2.4 \times 10^6\,\text{cm}^3$

(2) $1.2 \times 10^5\,\text{cm}^3$

(3) $6.0 \times 10^4\,\text{cm}^3$

(4) $4.8 \times 10^5\,\text{cm}^3$

Correct Answer: (2) $1.2 \times 10^5\,\text{cm}^3$

Solution: The increase in volume is given by:

$$\Delta V = V_0 \gamma \Delta T,$$

where:

- $V_0 = a^3$ (initial volume of the cube),
- $\gamma = 3\alpha$ (coefficient of volumetric expansion),
- $\Delta T = 10^\circ$ C (temperature change).

The surface area of a cube is:

$$\text{Total Surface Area} = 6a^2.$$

Given $6a^2 = 24\,\text{m}^2$, solve for a:

$$a^2 = \frac{24}{6} = 4\,\text{m}^2 \quad \Longrightarrow \quad a = 2\,\text{m}.$$

The initial volume is:

$$V_0 = a^3 = 2^3 = 8\,\text{m}^3 = 8 \times 10^6\,\text{cm}^3.$$

Substitute into the formula for ΔV:

$$\Delta V = 8 \times 10^6 \cdot (3 \cdot 5 \times 10^{-4}) \cdot 10.$$

Simplify:

$$\Delta V = 8 \times 10^6 \cdot 15 \times 10^{-3} = 1.2 \times 10^5\,\text{cm}^3.$$

Final Answer:

$$\boxed{1.2 \times 10^5\,\text{cm}^3}$$

Quick Tip

For solids, use $\gamma = 3\alpha$ to find volumetric expansion. Ensure units for V_0 are consistent.

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14. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon:

(1) the rates at which currents are changing in the two coils

(2) the relative position and orientation of the two coils

(3) the materials of the wires of the coils

(4) the currents in the two coils

Correct Answer: (2) the relative position and orientation of the two coils

Solution: The mutual inductance between two coils is defined as:

$$M = \frac{\Phi}{I},$$

where Φ is the magnetic flux in one coil due to the current I in the other coil.

The mutual inductance depends on:

The relative position and orientation of the two coils (determines the coupling of magnetic fields).

The number of turns and dimensions of the coils.

The magnetic permeability of the medium between the coils.

It does not depend directly on the current or the material of the coil wires.

Final Answer:

$$\boxed{\text{Relative position and orientation of the two coils.}}$$

Quick Tip

Mutual inductance depends on the geometry, orientation, and coupling of the coils, not on the material of the wires.

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15. A proton, an electron, and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

(1) $\lambda_e > \lambda_\alpha > \lambda_p$

(2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_p < \lambda_e < \lambda_\alpha$

(4) $\lambda_p > \lambda_e > \lambda_\alpha$

Correct Answer: (2) $\lambda_\alpha < \lambda_p < \lambda_e$

Solution: The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2mE}},$$

where h is Planck's constant, m is the mass of the particle, and E is its energy.

For particles with the same energy, $\lambda \propto \frac{1}{\sqrt{m}}$.

The masses of the particles are:

- Electron (m_e): least mass,
- Proton (m_p): greater mass,

- Alpha particle ($m_\alpha = 4m_p$): greatest mass.

Since $\lambda \propto \frac{1}{\sqrt{m}}$, the order of the wavelengths is:

$$\lambda_e > \lambda_p > \lambda_\alpha.$$

Final Answer:

$$\boxed{\lambda_\alpha < \lambda_p < \lambda_e}$$

Quick Tip

For particles with the same energy, the de-Broglie wavelength is inversely proportional to the square root of their mass.

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16. An ice cube has a bubble inside. When viewed from one side, the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is:

(1) $\frac{4}{3}$
(2) $\frac{3}{2}$
(3) $\frac{2}{3}$
(4) $\frac{6}{5}$

Correct Answer: (2) $\frac{3}{2}$

Solution: The refractive index μ is given by:

$$\mu = \frac{\text{Real Depth}}{\text{Apparent Depth}}.$$

Let the real depth of the bubble be x cm. When viewed from one side, the apparent depth is 12 cm, and when viewed from the opposite side, the apparent depth is $24 - x$ cm.

From the refractive index formula:

$$\mu = \frac{x}{12} \quad \text{(from one side)},$$

$$\mu = \frac{24 - x}{4} \quad \text{(from the opposite side)}.$$

Equate the two expressions for μ:

$$\frac{x}{12} = \frac{24 - x}{4}.$$

Simplify:

$$4x = 12(24 - x).$$

$$4x = 288 - 12x.$$

$$16x = 288 \implies x = 18\,\text{cm}.$$

Substitute $x = 18\,\text{cm}$ into $\mu = \frac{x}{12}$:

$$\mu = \frac{18}{12} = \frac{3}{2}.$$

Final Answer:

$$\boxed{\frac{3}{2}}$$

Quick Tip

To find the refractive index of an object with apparent depths viewed from two sides, set up equations for the real and apparent depths and solve for μ.

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17. The longest wavelength associated with the Paschen series is:

(Given $R_H = 1.097 \times 10^7$ SI unit).

(1) 1.094×10^{-6} m

(2) 2.973×10^{-6} m

(3) 3.646×10^{-6} m

(4) 1.876×10^{-6} m

Correct Answer: (4) 1.876×10^{-6} m

Solution: The wavelength for the Paschen series is given by:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right),$$

where:

- $n_1 = 3$ (Paschen series),
- $n_2 = 4$ (for the longest wavelength, n_2 is the next level above n_1).

Substitute:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{3^2} - \frac{1}{4^2} \right).$$

Simplify:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{9} - \frac{1}{16} \right).$$

$$\frac{1}{\lambda} = R_H \cdot \frac{16 - 9}{144}.$$

$$\frac{1}{\lambda} = R_H \cdot \frac{7}{144}.$$

Substitute $R_H = 1.097 \times 10^7$:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \cdot \frac{7}{144}.$$

$$\frac{1}{\lambda} = \frac{7.679 \times 10^7}{144}.$$

$$\lambda = \frac{144}{7.679 \times 10^7} \approx 1.876 \times 10^{-6}\,\text{m}.$$

Final Answer:

$$\boxed{1.876 \times 10^{-6}\,\text{m}}$$

Quick Tip

For the longest wavelength in any spectral series, use $n_2 = n_1 + 1$ and substitute carefully into the formula.

18. The ratio of the mass densities of nuclei of ^{40}Ca and ^{16}O is close to:

(1) 1

(2) 0.1

(3) 5

(4) 2

Correct Answer: (1) 1

Solution: Nuclear density (ρ) is nearly constant for all nuclei, regardless of their atomic number or mass number. This is because both the mass and the volume of the nucleus are proportional to the mass number (A).

1. Density of ^{40}Ca: For ^{40}Ca, mass number $A = 40$. The nuclear density remains constant:

$$\rho_{\text{Ca}} = \text{constant}.$$

2. Density of ^{16}O: For ^{16}O, mass number $A = 16$. The nuclear density is the same as for ^{40}Ca:

$$\rho_{\text{O}} = \text{constant}.$$

3. Ratio of Densities: Since nuclear density is constant:

$$\frac{\rho_{\text{Ca}}}{\rho_{\text{O}}} = 1.$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Nuclear density is constant for all elements because the mass and volume of the nucleus are directly proportional to the mass number (A).

---

19. Point charge of $10\,\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\,\mu C$ be placed so that the net electric field is zero at $x = 2$ cm on the X-axis?

(1) 6 cm

(2) 4 cm

(3) 8 cm

(4) -4 cm

Correct Answer: (1) 6 cm

Solution: To determine the location x_0 where the second charge ($40\,\mu$C) should be placed such that the net electric field is zero at $x = 2$ cm:

Step 1: Electric Field Balance At $x = 2$ cm, the electric field contributions from the two charges must cancel each other:

$$E_{\text{net}} = 0 \implies K \cdot \frac{10 \times 10^{-6}}{(2)^2} = K \cdot \frac{40 \times 10^{-6}}{(x_0 - 2)^2}.$$

Step 2: Simplify the Relation Cancel K and simplify:

$$\frac{10}{4} = \frac{40}{(x_0 - 2)^2}.$$

$$\frac{1}{2} = \frac{2}{(x_0 - 2)^2}.$$

Step 3: Solve for x_0 Rearrange:

$$(x_0 - 2)^2 = 4.$$

Take the square root:

$$x_0 - 2 = \pm 2.$$

Thus:

$$x_0 = 4\,\text{cm} \quad \text{or} \quad x_0 = 6\,\text{cm}.$$

Step 4: Select the Correct Value of x_0 Since the second charge is placed to the right of $x = 2$ cm, the correct position is:

$$x_0 = 6\,\text{cm}.$$

Final Answer:

$$\boxed{x_0 = 6\,\text{cm}}$$

Quick Tip

To calculate the position for zero net electric field, equate the magnitudes of electric fields from both charges and solve the resulting equation.

20. A magnetic needle is kept in a non-uniform magnetic field. It experiences:

(1) neither a force nor a torque

(2) a torque but not a force

(3) a force but not a torque

(4) a force and a torque

Correct Answer: (4) a force and a torque

Solution: A magnetic needle kept in a non-uniform magnetic field experiences both a force and a torque due to unequal forces acting on its poles.

he non-uniform magnetic field exerts different magnitudes of forces on the north and south poles of the needle.

This results in a net force on the needle.

Additionally, these forces create a torque that tends to align the needle with the magnetic field lines.

Final Answer:

$$\boxed{\text{a force and a torque}}$$

Quick Tip

In a non-uniform magnetic field, the magnetic forces on a dipole are unequal, leading to both a net force and a torque.

---

21. A current-carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = $5\,\text{cm}$ and PQ = RS = $100\,\text{cm}$. If the ammeter current reading changes from I to $2I$, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $F_{PQ}^{I} : F_{PQ}^{2I}$ is:

(1) $1:2$

(2) $1:4$

(3) $1:5$

(4) $1:3$

Correct Answer: (2) $1:4$

Solution: The magnetic force per unit length F/ℓ between two parallel current-carrying wires is given by:

$$F/\ell = \frac{\mu_0 I_1 I_2}{4\pi r},$$

where:

μ_0 is the permeability of free space,

I_1 and I_2 are the currents in the two wires,

r is the distance between the wires.

Step 1: Proportionality for F/ℓ

The magnetic force is directly proportional to the product of the currents:

$$F_1 \propto I^2 \quad \text{(when the current is } I\text{)}.$$

When the current changes to $2I$:

$$F_2 \propto (2I)^2 = 4I^2.$$

Step 2: Ratio of Forces

The ratio of the magnetic forces is:

$$\frac{F_1}{F_2} = \frac{I^2}{4I^2} = \frac{1}{4}.$$

Thus, the ratio of forces is:

$$F_{PQ}^{I} : F_{PQ}^{2I} = 1:4.$$

Final Answer:

$$\boxed{1:4}$$

Quick Tip

The magnetic force between parallel wires depends on the product of the currents. If one current doubles, the force increases by the square of the factor.

---

22. At which temperature will the r.m.s. velocity of a hydrogen molecule be equal to that of an oxygen molecule at $47°$C?

(1) $80\,$K

(2) $-73\,$K

(3) $4\,$K

(4) $20\,$K

Correct Answer: (4) $20\,$K

Solution: The r.m.s. velocity v_{rms} of a gas molecule is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where:

R is the gas constant,

T is the temperature (in Kelvin),

M is the molar mass of the gas.

Step 1: Equating the r.m.s. velocities

For hydrogen (H_2) and oxygen (O_2):

$$v_{\text{rms}}(H_2) = v_{\text{rms}}(O_2).$$

$$\sqrt{\frac{3RT_{H_2}}{M_{H_2}}} = \sqrt{\frac{3RT_{O_2}}{M_{O_2}}}.$$

Square both sides:

$$\frac{T_{H_2}}{M_{H_2}} = \frac{T_{O_2}}{M_{O_2}}.$$

Rearrange for T_{H_2}:

$$T_{H_2} = T_{O_2} \cdot \frac{M_{H_2}}{M_{O_2}}.$$

Step 2: Substitute values

For oxygen:

$$T_{O_2} = 47°\text{C} + 273 = 320\,\text{K}.$$

The molar masses are:

$$M_{H_2} = 2, \quad M_{O_2} = 32.$$

Substitute into the equation:

$$T_{H_2} = 320 \cdot \frac{2}{32}.$$

Simplify:

$$T_{H_2} = 320 \cdot \frac{1}{16} = 20\,\text{K}.$$

Final Answer:

$$\boxed{20\,\text{K}}$$

Quick Tip

The r.m.s. velocity of a gas molecule depends on both temperature and molar mass. Use $v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$ to relate the velocities of different gases.

---

23. A light-emitting diode (LED) is fabricated using GaAs semiconductor material whose band gap is $1.42\,$eV. The wavelength of light emitted from the LED is:

(1) $650\,$nm

(2) $1243\,$nm

(3) $875\,$nm

(4) $1400\,$nm

Correct Answer: (3) $875\,$nm

Solution: The wavelength of emitted light is related to the band gap energy E_g by the equation:

$$E_g(\text{eV}) = \frac{1240}{\lambda(\text{nm})}.$$

Rearrange to find the wavelength:

$$\lambda = \frac{1240}{E_g}.$$

Step 1: Substitute the given values

Given:

$$E_g = 1.42\,\text{eV}.$$

Substitute:

$$\lambda = \frac{1240}{1.42}.$$

$$\lambda \approx 875\,\text{nm}.$$

Final Answer:

$$\boxed{875\,\text{nm}}$$

Quick Tip

The wavelength of light emitted from a semiconductor is inversely proportional to its band gap energy.

---

24. A steel wire with mass per unit length 7.0×10^{-3} kg/m is under a tension of 70 N. The speed of transverse waves in the wire will be:

(1) 100 m/s

(2) 50 m/s

(3) 10 m/s

(4) 200π m/s

Correct Answer: (1) 100 m/s

Solution: The speed of transverse waves in a wire is given by:

$$v = \sqrt{\frac{T}{\mu}},$$

where:

T is the tension in the wire,

μ is the mass per unit length.

Step 1: Substitute the given values

Given:

$$T = 70\,\text{N}, \quad \mu = 7.0 \times 10^{-3}\,\text{kg/m}.$$

Substitute:

$$v = \sqrt{\frac{70}{7.0 \times 10^{-3}}}.$$

Step 2: Simplify the calculation

$$v = \sqrt{\frac{70}{0.007}} = \sqrt{10000}.$$

$$v = 100\,\text{m/s}.$$

Final Answer:

$$\boxed{100\,\text{m/s}}$$

Quick Tip

The speed of transverse waves in a wire is proportional to the square root of the tension and inversely proportional to the square root of the mass per unit length.

---

25. Two vessels A and B are of the same size and are at the same temperature. Vessel A contains 1 g of hydrogen and vessel B contains 1 g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively. Then $\frac{P_A}{P_B}$ is:

(1) 8

(2) 16

(3) 32

(4) 4

Correct Answer: (2) 16

Solution: By the ideal gas equation:

$$PV = nRT \quad \implies \quad P \propto n,$$

where n is the number of moles.

Step 1: Calculate the number of moles

For hydrogen (H_2):

$$M_{H_2} = 2\,\text{g/mol}, \quad n_A = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{1}{2} = 0.5\,\text{mol}.$$

For oxygen (O_2):

$$M_{O_2} = 32\,\text{g/mol}, \quad n_B = \frac{\text{Mass}}{\text{Molar Mass}} = \frac{1}{32} = 0.03125\,\text{mol}.$$

Step 2: Ratio of pressures

$$\frac{P_A}{P_B} = \frac{n_A}{n_B} = \frac{0.5}{0.03125}.$$

Simplify:

$$\frac{P_A}{P_B} = 16.$$

Final Answer:

$$\boxed{16}$$

Quick Tip

For the same temperature and volume, the pressure of a gas is directly proportional to the number of moles. Use $P \propto n$ for comparisons.

---

26. A wire of length $1\,\text{m}$ moving with velocity $8\,\text{m/s}$ at right angles to a magnetic field of $2\,\text{T}$. The magnitude of induced emf between the ends of the wire will be:

(1) $20\,\text{V}$

(2) $8\,\text{V}$

(3) $12\,\text{V}$

(4) $16\,\text{V}$

Correct Answer: (4) $16\,\text{V}$

Solution: The induced emf $\mathcal{E}$ in a moving conductor is given by:

$$\mathcal{E} = B \cdot v \cdot \ell,$$

where:

$B = 2\,\text{T}$ is the magnetic field strength,

$v = 8\,\text{m/s}$ is the velocity of the wire,

$\ell = 1\,\text{m}$ is the length of the wire.

Step 1: Substitute the given values

Substitute $B = 2\,\text{T}$, $v = 8\,\text{m/s}$, and $\ell = 1\,\text{m}$:

$$\mathcal{E} = 2 \cdot 8 \cdot 1.$$

Step 2: Simplify the calculation

$$\mathcal{E} = 16\,\text{V}.$$

Final Answer:

$$\boxed{16\,\text{V}}$$

Quick Tip

The magnitude of the induced emf in a conductor moving perpendicularly to a magnetic field is proportional to the product of the magnetic field strength, velocity, and length of the conductor.

---

27. Two identical particles each of mass m go around a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be:

(1) $\sqrt{\frac{Gm}{2a^3}}$
(2) $\sqrt{\frac{Gm}{8a^3}}$
(3) $\sqrt{\frac{Gm}{4a^3}}$
(4) $\sqrt{\frac{Gm}{a^3}}$

Correct Answer: (3) $\sqrt{\frac{Gm}{4a^3}}$

Solution: The gravitational force provides the centripetal force for circular motion:

$$F_{\text{gravity}} = F_{\text{centripetal}}.$$

The gravitational force between two particles is:

$$F = \frac{GM_1M_2}{d^2},$$

where:

G is the gravitational constant,

$M_1 = M_2 = m$,

$d = 2a$ is the distance between the two particles.

Substitute:

$$F = \frac{Gm^2}{(2a)^2} = \frac{Gm^2}{4a^2}.$$

For circular motion:

$$F = m\omega^2 r,$$

where $r = a$. Substitute $F = \frac{Gm^2}{4a^2}$:

$$\frac{Gm^2}{4a^2} = m\omega^2 a.$$

Simplify:

$$\omega^2 = \frac{Gm}{4a^3}.$$

Take the square root:

$$\omega = \sqrt{\frac{Gm}{4a^3}}.$$

Final Answer:

$$\boxed{\sqrt{\frac{Gm}{4a^3}}}$$

Quick Tip

In problems involving circular motion due to mutual gravitational attraction, use $F_{\text{gravity}} = F_{\text{centripetal}}$ to relate the forces.

---

28. In an unbiased p-n junction, electrons diffuse from n-region to p-region because:

(1) holes in p-region attract them

(2) electrons travel across the junction due to potential difference

(3) only electrons move from n-region to p-region and not the vice-versa

(4) electron concentration in n-region is more compared to that in p-region

Correct Answer: (4) electron concentration in n-region is more compared to that in p-region

Solution: In an unbiased p-n junction:
The electron concentration in the n-region is much higher than in the p-region.
Due to this difference in concentration, electrons diffuse naturally from the n-region to the p-region (from high to low concentration).
This diffusion creates a depletion region and establishes a built-in electric field that opposes further electron flow.

Final Answer:

$$\boxed{\text{electron concentration in } n\text{-region is more compared to that in } p\text{-region}}$$

Quick Tip

Diffusion in a p-n junction is driven by the concentration gradient, with electrons moving from regions of high concentration to low concentration.

---

29. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

(1) $1:1$

(2) $2:1$

(3) $1:4$

(4) $1:3$

Correct Answer: (4) $1:3$

Solution: The total energy in SHM is given by:

$$E_{\text{total}} = \frac{1}{2}kA^2,$$

where:

k is the spring constant,

A is the amplitude of oscillation.

Step 1: Potential and Kinetic Energy

The potential energy at displacement x is:

$$PE = \frac{1}{2}kx^2.$$

The kinetic energy is:

$$KE = E_{\text{total}} - PE = \frac{1}{2}kA^2 - \frac{1}{2}kx^2.$$

Step 2: Displacement is half the amplitude

Substitute $x = \frac{A}{2}$:

$$PE = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}k\frac{A^2}{4} = \frac{1}{8}kA^2.$$

$$KE = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = \frac{4}{8}kA^2 - \frac{1}{8}kA^2 = \frac{3}{8}kA^2.$$

Step 3: Ratio of PE and KE

$$\frac{PE}{KE} = \frac{\frac{1}{8}kA^2}{\frac{3}{8}kA^2} = \frac{1}{3}.$$

Final Answer:

$$\boxed{1 : 3}$$

Quick Tip

In SHM, use $PE + KE =$ constant and substitute the displacement to find the individual energies.

---

30. Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops coalesce, the new velocity is:

(1) 10 cm/s

(2) 40 cm/s

(3) 16 cm/s

(4) 5 cm/s

Correct Answer: (2) 40 cm/s

Solution: When the drops coalesce, the volume remains constant. Let: r be the radius of each small drop,R be the radius of the combined drop.

Step 1: Volume conservation

The total volume of the eight small drops is:

$$8 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3.$$

Simplify:

$$R^3 = 8r^3 \implies R = 2r.$$

Step 2: Relationship between terminal velocity and radius

The terminal velocity v_T is proportional to the square of the radius:

$$v_T \propto r^2.$$

Let $v_1 = 10\,\text{cm/s}$ be the terminal velocity of the small drops, and v_2 be the terminal velocity of the combined drop. Then:

$$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2.$$

Substitute $R = 2r$:

$$\frac{v_1}{v_2} = \left(\frac{r}{2r}\right)^2 = \frac{1}{4}.$$

Step 3: Solve for v_2

$$v_2 = 4v_1 = 4 \cdot 10 = 40\,\text{cm/s}.$$

Final Answer:

$$\boxed{40\,\text{cm/s}}$$

Quick Tip

When drops coalesce, the terminal velocity of the combined drop increases as the square of the radius, which depends on the cube root of the volume.

---

31. In a coil, the current changes from $-2\,\text{A}$ to $+2\,\text{A}$ in $0.2\,\text{s}$ and induces an emf of $0.1\,\text{V}$. The self-inductance of the coil is:

(1) $5\,\text{mH}$

(2) $1\,\text{mH}$

(3) $2.5\,\text{mH}$

(4) $4\,\text{mH}$

Correct Answer: (1) $5\,\text{mH}$

Solution: The emf induced in a coil is given by:

$$\mathcal{E} = L\frac{\Delta I}{\Delta t},$$

where:

$\mathcal{E} = 0.1\,\text{V}$ is the induced emf,

$\Delta I = I_{\text{final}} - I_{\text{initial}} = 2 - (-2) = 4\,\text{A}$ is the change in current,

$\Delta t = 0.2\,\text{s}$ is the time interval,

L is the self-inductance.

Step 1: Solve for L

Rearrange the formula:

$$L = \frac{\mathcal{E} \cdot \Delta t}{\Delta I}.$$

Substitute the values:

$$L = \frac{0.1 \cdot 0.2}{4}.$$

Simplify:

$$L = \frac{0.02}{4} = 0.005\,\text{H}.$$

Convert to millihenries:

$$L = 5\,\text{mH}.$$

Final Answer:

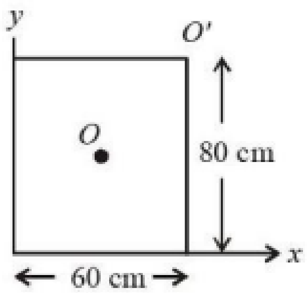
$$\boxed{5\,\text{mH}}$$

Quick Tip

The self-inductance of a coil relates the emf induced to the rate of change of current through the coil. Be sure to account for the total change in current.

---

32. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the center of mass) and O' (corner point) is:



(1) $\frac{2}{3}$
(2) $\frac{1}{4}$
(3) $\frac{1}{8}$
(4) $\frac{1}{2}$

Correct Answer: (2) $\frac{1}{4}$

Solution: The moment of inertia (I_O) of the rectangular sheet about an axis passing through O (center of mass) is given by:

$$I_O = \frac{M}{12}\left(a^2 + b^2\right).$$

Here, $a = 80\,\text{cm}$ and $b = 60\,\text{cm}$ are the dimensions of the rectangle, and M is the mass of the sheet.

Substitute the values:

$$I_O = \frac{M}{12}\left(80^2 + 60^2\right).$$

Simplify:

$$I_O = \frac{M}{12}(6400 + 3600) = \frac{M}{12} \cdot 10000 = \frac{10000M}{12}.$$

Using the parallel axis theorem, the moment of inertia about O' is:

$$I_{O'} = I_O + M \cdot d^2,$$

where d is the perpendicular distance between O and O', given by:

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{40^2 + 30^2} = 50\,\text{cm}.$$

Substitute:

$$I_{O'} = \frac{10000M}{12} + M \cdot 50^2.$$

Simplify:

$$I_{O'} = \frac{10000M}{12} + 2500M = \frac{10000M}{12} + \frac{30000M}{12} = \frac{40000M}{12}.$$

The ratio of moments of inertia is:

$$\frac{I_O}{I_{O'}} = \frac{\frac{10000M}{12}}{\frac{40000M}{12}} = \frac{10000}{40000} = \frac{1}{4}.$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

The parallel axis theorem states that the moment of inertia about any axis parallel to the center of mass axis is the sum of the center of mass moment of inertia and $M \cdot d^2$, where d is the perpendicular distance between the axes.

---

33. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e., mean free path) is represented by:

(1) $\frac{1}{\sqrt{2}\pi n d^2}$

(2) $\sqrt{2}n\pi d^2$

(3) $\frac{1}{\sqrt{2}n\pi d^2}$

(4) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

Correct Answer: (1) $\frac{1}{\sqrt{2}\pi n d^2}$

Solution: The mean free path (λ) is the average distance a molecule travels between two successive collisions. It is derived from the kinetic theory of gases and is given by the formula:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2},$$

where:

- n is the number density of molecules (number of molecules per unit volume),
- d is the diameter of a molecule,
- π is the constant representing the geometric cross-sectional area of a circle.

Derivation: 1. The collision cross-section of a molecule is $\sigma = \pi d^2$, where d is the molecular diameter.

2. The rate of collisions depends on the relative velocity of the molecules and the number density n.

3. The mean free path is inversely proportional to the number of collisions per unit length, which gives:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}.$$

Final Answer: The correct option is:

$$\boxed{\frac{1}{\sqrt{2}\pi n d^2}} \quad \text{(Option 1).}$$

Quick Tip

The mean free path is inversely proportional to the square of the molecular diameter and the number density.

---

34. A mixture of one mole of monoatomic gas and one mole of diatomic gas (rigid) are kept at room temperature ($27°$C). The ratio of specific heat of gases at constant volume respectively is:

(1) $\frac{7}{5}$

(2) $\frac{3}{2}$

(3) $\frac{3}{5}$

(4) $\frac{5}{3}$

Correct Answer: (3) $\frac{3}{5}$

Solution: For a monoatomic gas, the specific heat at constant volume (C_V) is:

$$C_V = \frac{3}{2}R.$$

For a diatomic gas (rigid), the specific heat at constant volume (C'_V) is:

$$C'_V = \frac{5}{2}R.$$

The ratio of specific heats is:

$$\frac{C_V}{C'_V} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}.$$

Final Answer:

$$\boxed{\frac{3}{5}}$$

Quick Tip

The specific heat of a monoatomic gas is lower than that of a diatomic gas because a diatomic gas has additional degrees of freedom.

---

35. In an a.c. circuit, voltage and current are given by:

$$V = 100\sin(100t)\,\text{V} \quad \text{and} \quad I = 100\sin\left(100t + \frac{\pi}{3}\right)\,\text{mA}.$$

The average power dissipated in one cycle is:

(1) $10\,\text{W}$

(2) $2.5\,\text{W}$

(3) $25\,\text{W}$

(4) $5\,\text{W}$

Correct Answer: (2) $2.5\,\text{W}$

Solution: The average power in one cycle is given by:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\Delta\phi),$$

where:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}, \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{100 \times 10^{-3}}{\sqrt{2}}, \quad \Delta\phi = \frac{\pi}{3}.$$

Substitute:

$$P_{\text{avg}} = \frac{100}{\sqrt{2}} \cdot \frac{100 \times 10^{-3}}{\sqrt{2}} \cdot \cos\left(\frac{\pi}{3}\right).$$

Simplify:

$$P_{\text{avg}} = \frac{100 \cdot 0.1}{2} \cdot \frac{1}{2} = \frac{10}{4} = 2.5\,\text{W}.$$

Final Answer:

$$\boxed{2.5\,\text{W}}$$

Quick Tip

The average power in an a.c. circuit depends on the rms values of voltage and current, as well as the phase angle between them.

---

36. The difference between threshold wavelengths for two metal surfaces A and B having work functions $\phi_A = 9\,$eV and $\phi_B = 4.5\,$eV is:

(Given: $hc = 1242\,$eV nm)

(1) $264\,$nm

(2) $138\,$nm

(3) $276\,$nm

(4) $540\,$nm

Correct Answer: (2) $138\,$nm

Solution: The wavelength (λ) is related to the work function (ϕ) by:

$$\lambda = \frac{hc}{\phi}.$$

For $\phi_A = 9\,$eV:

$$\lambda_A = \frac{1242}{9} = 138\,\text{nm}.$$

For $\phi_B = 4.5\,$eV:

$$\lambda_B = \frac{1242}{4.5} = 276\,\text{nm}.$$

The difference is:

$$\Delta\lambda = \lambda_B - \lambda_A = 276 - 138 = 138\,\text{nm}.$$

Final Answer:

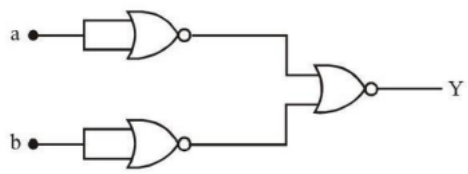
$$\boxed{138\,\text{nm}}$$

Quick Tip

The wavelength is inversely proportional to the work function. Larger work functions correspond to shorter threshold wavelengths.

---

37. The logic performed by the circuit shown in the figure is equivalent to:



(1) AND

(2) $NAND$

(3) OR

(4) NOR

Correct Answer: (1) AND

Solution: The circuit performs the following logical operations:

1. The first two NOT gates take inputs a and b, and output their complements $\bar{a}$ and $\bar{b}$, respectively.

$$\text{Output of the first NOT gate: } \bar{a}, \quad \text{Output of the second NOT gate: } \bar{b}.$$

2. The OR gate combines these complements $(\bar{a} + \bar{b})$.

$$\text{Output of the OR gate: } \bar{a} + \bar{b}.$$

3. Finally, the complement of this OR gate's output is taken using another NOT gate, resulting

in:

$$Y = \overline{\bar{a} + \bar{b}}.$$

Using De Morgan's law:

$$Y = a \cdot b.$$

Thus, the circuit implements the logic of an AND gate.

Truth Table:

A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

The output matches the truth table of an AND gate.

Final Answer:

$$\boxed{\text{AND Gate}}$$

Quick Tip

Use De Morgan's laws to simplify logical expressions. The complement of an OR operation ($\overline{A + B}$) is equivalent to the AND operation of the complements ($\bar{A} \cdot \bar{B}$).

---

38. A particle performs simple harmonic motion with amplitude A. Its speed is tripled at the instant that it is at a distance $\frac{2A}{3}$ from the equilibrium position. The new amplitude of the motion is:

(1) $A\sqrt{3}$

(2) $\frac{7A}{3}$

(3) $\frac{A}{3}\sqrt{41}$

(4) $3A$

Correct Answer: (2) $\frac{7A}{3}$

Solution: The velocity V of a particle performing simple harmonic motion is given by:

$$V = \omega\sqrt{A^2 - x^2},$$

where:

$\omega =$ angular frequency, $\quad A =$ amplitude, $\quad x =$ displacement from the equilibrium position.

Step 1: Initial velocity at $x = \frac{2A}{3}$ Initially, the velocity is:

$$V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}.$$

Simplify the expression:

$$V = \omega\sqrt{A^2 - \frac{4A^2}{9}} = \omega\sqrt{\frac{9A^2 - 4A^2}{9}} = \omega\sqrt{\frac{5A^2}{9}} = \omega\frac{\sqrt{5}A}{3}.$$

Step 2: Final velocity when speed is tripled The final velocity is three times the initial velocity:

$$3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2},$$

where A' is the new amplitude.
Substitute the expression for V:

$$3\left(\omega\frac{\sqrt{5}A}{3}\right) = \omega\sqrt{A'^2 - \frac{4A^2}{9}}.$$

Simplify:

$$\omega\sqrt{5}A = \omega\sqrt{A'^2 - \frac{4A^2}{9}}.$$

Step 3: Solve for A'^2 Square both sides:

$$5A^2 = A'^2 - \frac{4A^2}{9}.$$

Rearrange:

$$A'^2 = 5A^2 + \frac{4A^2}{9}.$$

Simplify:

$$A'^2 = \frac{45A^2}{9} + \frac{4A^2}{9} = \frac{49A^2}{9}.$$

Take the square root:

$$A' = \frac{7A}{3}.$$

Final Answer:

$$\boxed{\frac{7A}{3}}$$

Quick Tip

In simple harmonic motion, the velocity depends on both the amplitude and the displacement. When speed changes, you can use energy conservation or the velocity equation to determine the new amplitude.

---

39. The mass of proton, neutron, and helium nucleus are respectively 1.0073 **u**, 1.0087 **u**, 4.0015 **u. The binding energy of the helium nucleus is:**

(1) 14.2 MeV

(2) 56.8 MeV

(3) 28.4 MeV

(4) 7.1 MeV

Correct Answer: (3) 28.4 MeV

Solution: The binding energy (BE) of a nucleus is given by:

$$BE = \Delta m \cdot c^2,$$

where Δm is the mass defect and c is the speed of light.

For the helium nucleus:

$$\Delta m = (2m_p + 2m_n) - m_{\text{He}}.$$

Substitute the given masses:

$$\Delta m = (2 \cdot 1.0073 + 2 \cdot 1.0087) - 4.0015 = 4.0310 - 4.0015 = 0.0295\,\mathbf{u}.$$

Convert the mass defect to energy using $1\,\mathrm{u} = 931.5\,\mathrm{MeV/c^2}$:

$$BE = 0.0295 \cdot 931.5 = 28.4\,\mathrm{MeV}.$$

Final Answer:

$$\boxed{28.4\,\mathrm{MeV}}$$

Quick Tip

The binding energy of a nucleus can be calculated by finding the mass defect and converting it into energy using $1\,\mathrm{u} = 931.5\,\mathrm{MeV/c^2}$.

---

40. A series LCR circuit is subjected to an AC signal of $200\,\mathrm{V}$, $50\,\mathrm{Hz}$. If the voltage across the inductor ($L = 10\,\mathrm{mH}$) is $31.4\,\mathrm{V}$, then the current in this circuit is:

(1) $68\,\mathrm{A}$

(2) $63\,\mathrm{A}$

(3) $10\,\mathrm{A}$

(4) $10\,\mathrm{mA}$

Correct Answer: (3) $10\,\mathrm{A}$

Solution: The voltage across the inductor is given by:

$$V_L = IX_L,$$

where $X_L = \omega L$ is the inductive reactance.

1. Calculate ω:

$$\omega = 2\pi f = 2\pi \cdot 50 = 3.14 \times 100 = 314\,\mathrm{rad/s}.$$

2. Calculate X_L:

$$X_L = \omega L = 314 \cdot 10 \times 10^{-3} = 3.14\,\Omega.$$

3. Calculate the current I:

$$I = \frac{V_L}{X_L} = \frac{31.4}{3.14} = 10\,\text{A}.$$

Final Answer:

$$\boxed{10\,\text{A}}$$

Quick Tip

In an LCR circuit, the voltage across the inductor can be used to calculate the current using the formula $V_L = IX_L$, where $X_L = \omega L$.

---

41. When two soap bubbles of radii a and b ($b > a$) coalesce, the radius of curvature of the common surface is:

(1) $\frac{ab}{b-a}$

(2) $\frac{ab}{a+b}$

(3) $\frac{b-a}{ab}$

(4) $\frac{a+b}{ab}$

Correct Answer: (1) $\frac{ab}{b-a}$

Solution: The pressure inside a soap bubble is given by:

$$P = P_0 + \frac{4T}{R},$$

where T is the surface tension and R is the radius of curvature.

1. For the two bubbles, the pressures are:

$$P_1 = P_0 + \frac{4T}{a}, \quad P_2 = P_0 + \frac{4T}{b}.$$

2. The pressure difference across the common surface is:

$$P_1 - P_2 = \frac{4T}{R}.$$

3. Substituting P_1 and P_2:

$$\frac{4T}{a} - \frac{4T}{b} = \frac{4T}{R}.$$

4. Simplify:

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{R}.$$

5. Solve for R:

$$\frac{1}{R} = \frac{b-a}{ab}, \quad R = \frac{ab}{b-a}.$$

Final Answer:

$$\boxed{\frac{ab}{b-a}}$$

Quick Tip

When two soap bubbles coalesce, the radius of curvature of the common surface can be derived using the pressure difference and the surface tension relationship.

---

42. A liquid is allowed to flow into a tube of truncated cone shape. Identify the correct statement:

(1) The speed is high at the wider end and high at the narrow end.

(2) The speed is low at the wider end and high at the narrow end.

(3) The speed is same at both ends in a streamline flow.

(4) The liquid flows with uniform velocity in the tube.

Correct Answer: (2) $Thespeedislowatthewiderendandhighatthenarrowend.$

Solution: From the principle of continuity:

$$A_1v_1 = A_2v_2,$$

where A is the cross-sectional area and v is the velocity of the liquid.

In a truncated cone, the area at the wider end is greater $(A_1 > A_2)$. Thus, the velocity at the wider end is smaller, and the velocity at the narrow end is higher:

$$v_1 < v_2.$$

Final Answer:

$$\boxed{\text{The speed is low at the wider end and high at the narrow end.}}$$

Quick Tip

The continuity equation ensures that the product of the cross-sectional area and velocity remains constant for an incompressible fluid in streamline flow.

---

43. The velocity of sound in a gas in which two wavelengths $4.08\,\text{m}$ and $4.16\,\text{m}$ produce 40 beats in 12 seconds will be:

(1) $2.828\,\text{ms}^{-1}$

(2) $175.5\,\text{ms}^{-1}$

(3) $353.6\,\text{ms}^{-1}$

(4) $707.2\,\text{ms}^{-1}$

Correct Answer: (4) $707.2\,\text{ms}^{-1}$

Solution: The beat frequency is given by the formula:

$$f_{\text{beat}} = f_1 - f_2,$$

where f_1 and f_2 are the frequencies corresponding to the given wavelengths λ_1 and λ_2.

Step 1: Calculate the beat frequency The beat frequency is:

$$f_{\text{beat}} = \frac{\text{Number of beats}}{\text{Time}} = \frac{40}{12}\,\text{Hz}.$$

Simplify:

$$f_{\text{beat}} = \frac{10}{3}\,\text{Hz}.$$

Step 2: Relating wavelength to velocity and frequency The frequency of a wave is related to its velocity by:

$$f = \frac{v}{\lambda}.$$

Using the wavelengths $\lambda_1 = 4.08\,\text{m}$ and $\lambda_2 = 4.16\,\text{m}$:

$$f_1 = \frac{v}{4.08}, \quad f_2 = \frac{v}{4.16}.$$

Step 3: Solve for v The beat frequency is:

$$f_{\text{beat}} = f_1 - f_2 = \frac{v}{4.08} - \frac{v}{4.16}.$$

Simplify:

$$\frac{10}{3} = v\left(\frac{1}{4.08} - \frac{1}{4.16}\right).$$

Calculate the difference in reciprocals:

$$\frac{1}{4.08} - \frac{1}{4.16} = \frac{4.16 - 4.08}{4.08 \cdot 4.16} = \frac{0.08}{16.9728}.$$

Substitute:

$$\frac{10}{3} = v \cdot \frac{0.08}{16.9728}.$$

Solve for v:

$$v = \frac{\frac{10}{3} \cdot 16.9728}{0.08} = 707.2\,\text{ms}^{-1}.$$

Final Answer:

$$\boxed{707.2\,\text{ms}^{-1}}$$

Quick Tip

The velocity of sound can be calculated using the relation $f_{\text{beat}} = f_1 - f_2$, combined with the wave equation $f = \frac{v}{\lambda}$.

44. σ **is the uniform surface charge density of a thin spherical shell of radius** R**. The electric field at any point on the surface of the spherical shell is:**

(1) $\frac{\sigma}{\varepsilon_0 R}$

(2) $\frac{\sigma}{2\varepsilon_0}$

(3) $\frac{\sigma}{\varepsilon_0}$

(4) $\frac{\sigma}{4\varepsilon_0}$

Correct Answer: (3) $\frac{\sigma}{\varepsilon_0}$

Solution: Using Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}.$$

Step 1: Relate charge to surface charge density The total charge on the spherical shell is:

$$Q = \sigma \cdot 4\pi R^2,$$

where σ is the surface charge density.

Step 2: Simplify the electric field The electric field on the surface of the shell is uniform. Using symmetry, the flux through the Gaussian surface (a sphere of radius R) is:

$$\oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi R^2.$$

Substitute into Gauss's law:

$$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}.$$

Substitute $Q = \sigma \cdot 4\pi R^2$:

$$E \cdot 4\pi R^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}.$$

Cancel $4\pi R^2$ from both sides:

$$E = \frac{\sigma}{\varepsilon_0}.$$

Final Answer:

$$\boxed{\frac{\sigma}{\varepsilon_0}}$$

Quick Tip

For a spherical shell, the electric field on its surface depends only on the surface charge density σ and the permittivity constant ε_0, independent of the radius R.

---

45. An electric field is given by $\vec{E} = (6\hat{i} + 5\hat{j} + 3\hat{k})$ N/C. The electric flux through a surface area $30\hat{i}$ m^2 lying in the YZ-plane (in SI units) is:

(1) 180

(2) 90

(3) 150

(4) 60

Correct Answer: (1) 180

Solution: The electric flux is given by:

$$\Phi = \vec{E} \cdot \vec{A},$$

where $\vec{A} = 30\hat{i}\,\text{m}^2$ represents the area vector.
Substitute $\vec{E} = (6\hat{i} + 5\hat{j} + 3\hat{k})$:

$$\Phi = (6\hat{i} + 5\hat{j} + 3\hat{k}) \cdot (30\hat{i}).$$

Only the $\hat{i}$ components contribute:

$$\Phi = 6 \cdot 30 = 180\,\text{V·m}.$$

Final Answer:

$$\boxed{180\,\text{V·m}}$$

Quick Tip

The electric flux through a surface is the dot product of the electric field vector and the area vector. Only components along the same direction contribute.

46. A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:

(1) $\frac{1}{100}$th

(2) $\frac{1}{10}$th

(3) 100 times

(4) 10 times

Correct Answer: (2) $\frac{1}{10}$th

Solution: Volume conservation gives:

$$\frac{4}{3}\pi R^3 = 1000 \cdot \frac{4}{3}\pi r^3,$$

where R is the radius of the big drop, and r is the radius of each small drop.
Simplify:

$$R^3 = 1000 \cdot r^3 \implies R = 10r.$$

The surface energy of a sphere is proportional to its surface area. The initial energy is:

$$E_i = 1000 \cdot 4\pi r^2.$$

The final energy is:

$$E_f = 4\pi R^2 = 4\pi(10r)^2 = 100 \cdot 4\pi r^2.$$

The ratio is:

$$\frac{E_f}{E_i} = \frac{4\pi(10r)^2}{1000 \cdot 4\pi r^2} = \frac{1}{10}.$$

Final Answer:

$$\boxed{\frac{1}{10}\text{th}}$$

Quick Tip

When droplets coalesce, the total volume remains constant, and the surface energy changes according to the surface area.

47. If M is the mass of water that rises in a capillary tube of radius r, then the mass of water which will rise in a capillary tube of radius $2r$ is:

(1) M

(2) $\frac{M}{2}$

(3) $4M$

(4) $2M$

Correct Answer: (4) $2M$

Solution: The height h of water in a capillary is given by:

$$h = \frac{2T\cos\theta}{r\rho g}.$$

The volume of water is:

$$V = \pi r^2 h = \pi r^2 \cdot \frac{2T\cos\theta}{r\rho g}.$$

Simplify:

$$V \propto r.$$

The mass of water is:

$$m = \rho V \propto r.$$

For a capillary of radius $2r$, the mass is:

$$m_2 = 2m_1.$$

Thus, the new mass is:

$$2M.$$

Final Answer:

$$\boxed{2M}$$

Quick Tip

The mass of water in a capillary is proportional to the radius of the capillary tube.

---

48. A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has:

(1) Three nodes and three antinodes

(2) Three nodes and four antinodes

(3) Four nodes and three antinodes

(4) Four nodes and four antinodes

Correct Answer: (4) Four nodes and four antinodes

Solution: For a closed organ pipe, the length L of the pipe for the n-th harmonic is given by:

$$L = \frac{(2n-1)\lambda}{4},$$

where λ is the wavelength.

For the third overtone:

$$n = 4.$$

The pipe supports four full loops plus a half loop, corresponding to:

$$4 \text{ nodes and 4 antinodes}.$$

Final Answer:

$$\boxed{\text{Four nodes and four antinodes}}$$

Quick Tip

A closed organ pipe has nodes and antinodes determined by the overtone number.

---

49. The electrostatic potential due to an electric dipole at a distance r varies as:

(1) r

(2) $\frac{1}{r^2}$

(3) $\frac{1}{r^3}$

(4) $\frac{1}{r}$

Correct Answer: (2) $\frac{1}{r^2}$

Solution: The electrostatic potential V_p due to an electric dipole at a point at distance r from its center is given by:

$$V_p \propto \frac{1}{r^2}.$$

This is because the electric potential of a dipole decreases quadratically with distance due to its unique charge distribution.

Final Answer:

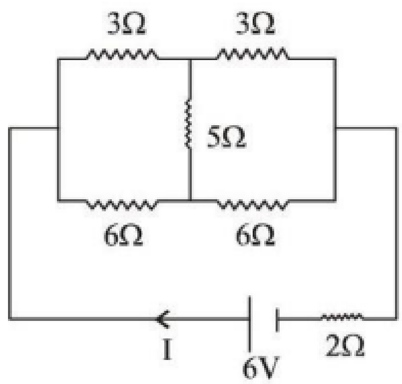
$$\boxed{\frac{1}{r^2}}$$

Quick Tip

The electric potential for a monopole varies as $\frac{1}{r}$, while for a dipole, it varies as $\frac{1}{r^2}$.

---

50. A battery of $6\,\text{V}$ is connected to the circuit as shown below. The current I drawn from the battery is:



(1) $1\,\text{A}$

(2) $2\,\text{A}$

(3) $\frac{6}{11}\,\text{A}$

(4) $\frac{4}{3}\,\text{A}$

Correct Answer: (1) $1\,\text{A}$

Solution: The circuit contains a balanced Wheatstone bridge. In this condition, the resistor of $5\,\Omega$ does not contribute to the current, as no current flows through it.

Step 1: Simplify the circuit - Remove the $5\,\Omega$ resistor. - Combine the remaining resistors. The $3\,\Omega$ resistors in the top branch are in series, giving:

$$R_{\text{top}} = 3 + 3 = 6\,\Omega.$$

Similarly, the $6\,\Omega$ resistors in the bottom branch are in parallel:

$$R_{\text{bottom}} = \frac{6 \cdot 6}{6 + 6} = \frac{36}{12} = 3\,\Omega.$$

Now, combine R_{top} and R_{bottom} in parallel:

$$R_{\text{eq}} = \frac{6 \cdot 3}{6 + 3} = \frac{18}{9} = 2\,\Omega.$$

Add the $2\,\Omega$ resistor in series with the equivalent resistance:

$$R_{\text{total}} = R_{\text{eq}} + 2 = 2 + 2 = 6\,\Omega.$$

Step 2: Apply Ohm's Law The total current is given by:

$$I = \frac{V}{R_{\text{total}}}.$$

Substitute $V = 6\,\text{V}$ and $R_{\text{total}} = 6\,\Omega$:

$$I = \frac{6}{6} = 1\,\text{A}.$$

Final Answer:

$$\boxed{1\,\text{A}}$$

Quick Tip

In a balanced Wheatstone bridge, the current through the middle resistor is zero. Use this property to simplify the circuit.

51. If the length of the body diagonal of a FCC unit cell is x Å, the distance between two octahedral voids in the cell in Å is:

(1) $\frac{x}{\sqrt{2}}$
(2) $\frac{x}{\sqrt{3}}$
(3) $\frac{x}{\sqrt{6}}$
(4) $\frac{x}{\sqrt{8}}$

Correct Answer: (3) $\frac{x}{\sqrt{6}}$

Solution: In a FCC unit cell, octahedral voids are present at the body center and edge centers. The distance between two octahedral voids is calculated as follows:

1. The length of the body diagonal is:

$$x = \sqrt{3}a,$$

where a is the edge length.

2. From this, the edge length is:

$$a = \frac{x}{\sqrt{3}}.$$

3. The distance between octahedral voids is:

$$\text{Distance} = \frac{a}{\sqrt{2}}.$$

Substitute $a = \frac{x}{\sqrt{3}}$:

$$\text{Distance} = \frac{\frac{x}{\sqrt{3}}}{\sqrt{2}} = \frac{x}{\sqrt{3} \cdot \sqrt{2}} = \frac{x}{\sqrt{6}}.$$

Final Answer:

$$\boxed{\frac{x}{\sqrt{6}}}$$

Quick Tip

In FCC crystals, the relationship between body diagonal and edge length is Body Diagonal $= \sqrt{3} \cdot$ Edge Length. Use this to compute distances involving voids.

---

52. Ortho-sulphobenzimide is used as:

(1) Antioxidant

(2) Artificial sweetener

(3) Food preservative

(4) Food supplement

Correct Answer: (2) Artificial sweetener

Solution: Ortho-sulphobenzimide is the chemical name for saccharin. It is commonly used as a non-caloric artificial sweetener.

Final Answer:

$$\boxed{\text{Artificial sweetener}}$$

Quick Tip

Saccharin is a widely used artificial sweetener. It is much sweeter than sugar and has no calories.

---

53. Volume of a gas at NTP is 1.12×10^{-7} cm^3. The number of molecules in it is:

(1) 3.01×10^{12}

(2) 3.01×10^{24}

(3) 3.01×10^{23}

(4) 3.01×10^{20}

Correct Answer: (1) 3.01×10^{12}

Solution: At NTP, $22,400\,\text{cm}^3$ of gas contains 6.02×10^{23} molecules.

For 1.12×10^{-7} cm^3:

$$\text{Number of molecules} = \frac{6.02 \times 10^{23}}{22,400} \cdot 1.12 \times 10^{-7}.$$

Simplify:

$$\text{Number of molecules} = \frac{6.02 \cdot 1.12 \times 10^{23} \cdot 10^{-7}}{22,400}.$$

Calculate:

$$\text{Number of molecules} = 3.01 \times 10^{12}.$$

Final Answer:

$$\boxed{3.01 \times 10^{12}}$$

Quick Tip

Use the proportionality between volume and number of molecules to calculate the number of molecules at given conditions.

---

54. The wavelength of the radiation emitted, when a hydrogen atom electron falls from infinity to stationary state 1, would be: (Rydberg constant $R = 1.097 \times 10^7$ m^{-1})

(1) 406 nm

(2) 192 nm

(3) 91 nm

(4) 9.1×10^{-8} nm

Correct Answer: (3) 91 nm

Solution: The wavelength λ of radiation is given by the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$

where: - $R = 1.097 \times 10^7$ m^{-1}, - $n_1 = 1$ (final state), - $n_2 = \infty$ (initial state).

1. Simplify the formula:

$$\frac{1}{\lambda} = R \cdot \frac{1}{1^2}.$$

2. Substitute R:

$$\frac{1}{\lambda} = 1.097 \times 10^7.$$

3. Solve for λ:

$$\lambda = \frac{1}{1.097 \times 10^7} = 91\,\text{nm}.(approx)$$

Final Answer:

$$\boxed{91\,\text{nm}}$$

Quick Tip

For transitions to the first stationary state, the wavelength can be directly calculated using $\lambda = \frac{1}{R}$ for $n_2 = \infty$.

---

55. In NO_3^- ion, the number of bond pairs and lone pairs of electrons on the nitrogen atom are:

(1) $2, 2$

(2) $3, 1$

(3) $1, 3$

(4) $4, 0$

Correct Answer: (4) $4, 0$

Solution: The structure of NO_3^- ion is:

$$O = N^+ - O^- - O.$$

In NO_3^-: - The nitrogen atom forms 4 bond pairs (3 sigma bonds and 1 pi bond).
- The nitrogen atom has no lone pairs.

Final Answer:

$$\boxed{4, 0}$$

Quick Tip

Count the shared electron pairs as bond pairs and non-shared electron pairs as lone pairs to determine the bonding in ions.

---

56. In which of the compounds does 'manganese' exhibit the highest oxidation number?

(1) MnO_2

(2) Mn_3O_4

(3) K_2MnO_4

(4) $MnSO_4$

Correct Answer: (3) K_2MnO_4

Solution: In K_2MnO_4, the oxidation number of Mn can be calculated as:

$$2(+1) + x + 4(-2) = 0,$$

where x is the oxidation number of Mn.

Simplify:

$$2 + x - 8 = 0 \implies x = +6.$$

In the other compounds: - MnO_2 : Mn = $+4$,

- Mn_3O_4 : Mn = $+8/3$ (average),

- $MnSO_4$: Mn = $+2$.

Thus, the highest oxidation number is in K_2MnO_4.

Final Answer:

$$\boxed{K_2MnO_4}$$

> Quick Tip
>
> The oxidation number of a metal in a compound is determined by the charges of the other atoms or groups in the compound.

---

57. Alkali metals are powerful reducing agents because:

(1) They are metals

(2) They are monovalent

(3) Their ionic radii are large

(4) Their ionisation energies are low

Correct Answer: (4) Their ionisation energies are low

Solution: Alkali metals have a single valence electron, and their low ionisation energy makes it easy to lose this electron and get oxidised. This property makes them strong reducing agents.

Final Answer:

$$\boxed{\text{Their ionisation energies are low}}$$

Quick Tip

The lower the ionisation energy, the easier it is for an atom to lose electrons, increasing its reducing power.

---

58. A balloon filled with an air sample occupies $3\,\text{L}$ volume at $35°\text{C}$. On lowering the temperature to T, the volume decreases to $2.5\,\text{L}$. The temperature T is: *[Assume P-constant]*

(1) $16°\text{C}$

(2) $-16°\text{C}$

(3) $24°\text{C}$

(4) $-20°\text{C}$

Correct Answer: (2) $-16°\text{C}$

Solution: From Charles's law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

Substitute the known values:

$$\frac{3}{308} = \frac{2.5}{T_2},$$

where $T_1 = 35°\text{C} = 273 + 35 = 308\,\text{K}$.

Solve for T_2:

$$T_2 = \frac{2.5 \cdot 308}{3} = 256.6\,\text{K}.$$

Convert to Celsius:

$$T_2 = 256.6 - 273 = -16.4°\text{C} \approx -16°\text{C}.$$

Final Answer:

$$\boxed{-16°\text{C}}$$

Quick Tip

Use Charles's law to relate the volume and temperature of gases when the pressure is constant.

---

59. Which one of the following is used as an eye lotion?

(1) Milk of magnesia

(2) Silver sol

(3) Colloidal antimony

(4) Chromium salt sol

Correct Answer: (2) Silver sol

Solution: Silver sol can be used as an eye lotion because of its antibacterial and healing properties. It helps in treating eye infections effectively.

Final Answer:

$$\boxed{\text{Silver sol}}$$

Quick Tip

Colloidal silver is often used in medical applications for its antibacterial properties.

---

60. Identify ortho and para directing groups from the following:

$$-\text{CHO}, -\text{NHCOCH}_3, -\text{OCH}_3, -\text{SO}_3\text{H}.$$

(1) III, IV

(2) II, III

(3) II, IV

(4) I, IV

Correct Answer: (2) II, III

Solution: Electron-donating groups like $-NHCOCH_3$ and $-OCH_3$ are ortho and para directing because they increase electron density at the ortho and para positions of the benzene ring through resonance or inductive effects.

Electron-withdrawing groups like $-CHO$ and $-SO_3H$ are meta directing.

Final Answer:

$$\boxed{\text{II, III}}$$

> **Quick Tip**
>
> Ortho and para directing groups are usually electron-donating, while meta directing groups are electron-withdrawing.

---

61. Which one of the carbanions is the least stable?

(1) $\overset{\ominus}{C}H_2-NO_2$

(2) $\overset{\ominus}{C}H_2-CHO$

(3) $\overset{\ominus}{C}H_2-CH_3$

(4) $\overset{\ominus}{C}H_3$

Correct Answer: (3) $\overset{\ominus}{C}H_2-CH_3$

Solution: Carbanion stability depends on the electron-withdrawing nature of attached groups:

- $-NO_2$ and $-CHO$ are strong electron-withdrawing groups, stabilising the negative charge.

- $-CH_3$ is electron-donating, which destabilises the negative charge.

Thus, CH_2-CH_3 is the least stable as it lacks electron-withdrawing groups.

Final Answer: $\overset{\ominus}{C}H_2-CH_3$

> **Quick Tip**
>
> Carbanions are stabilised by electron-withdrawing groups and destabilised by electron-donating groups.

---

62. In O_2^-, O_2, and O_2^{2-} molecular species, the total number of antibonding electrons respectively are:

(1) $7, 6, 8$

(2) $1, 0, 2$

(3) $6, 6, 6$

(4) $8, 6, 8$

Correct Answer: (1) $7, 6, 8$

Solution: The molecular orbital electronic configurations are:
- O_2^-: $\pi^2_{2p_x}\pi^2_{2p_y}\pi^1_{2p_x}$ (7 antibonding electrons),
- O_2: $\pi^2_{2p_x}\pi^2_{2p_y}$ (6 antibonding electrons),
- O_2^{2-}: $\pi^2_{2p_x}\pi^2_{2p_y}\pi^2_{2p_x}$ (8 antibonding electrons).

Final Answer:

$$\boxed{7, 6, 8}$$

Quick Tip

The number of antibonding electrons depends on the molecular orbital configuration.

---

63. People living at high altitudes often reported a problem of feeling weak and inability to think clearly. The reason for this is:

(1) At high altitudes the partial pressure of oxygen is less than at the ground level.

(2) At high altitudes the partial pressure of oxygen is more than at the ground level.

(3) At high altitudes the partial pressure of oxygen is equal to that at the ground level.

(4) None of these

Correct Answer: (1) At high altitudes the partial pressure of oxygen is less than at the ground level.

Solution: At high altitudes, the partial pressure of oxygen is lower, leading to reduced oxygen availability in the blood. This condition, called hypoxia, causes symptoms like weakness and difficulty in thinking, known as anoxia.

Final Answer:

$$\boxed{\text{At high altitudes the partial pressure of oxygen is less than at the ground level.}}$$

Quick Tip

Low oxygen levels at high altitudes can lead to altitude sickness or anoxia.

---

64. Specific conductance of $0.1\,\mathbf{M}\,\mathbf{HNO_3}$ is $6.3 \times 10^{-2}\,\mathbf{ohm^{-1}\,cm^{-1}}$. The molar conductance of the solution is:

(1) $100\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

(2) $515\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

(3) $630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

(4) $6300\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

Correct Answer: (3) $630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

Solution: Molar conductance is given by:

$$\Lambda_m = \kappa \cdot \frac{1000}{C},$$

where: - $\kappa = 6.3 \times 10^{-2}\,\text{ohm}^{-1}\,\text{cm}^{-1}$,

- $C = 0.1\,\text{M}$.

Substitute:

$$\Lambda_m = 6.3 \times 10^{-2} \cdot \frac{1000}{0.1} = 6.3 \times 10^2 = 630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}.$$

Final Answer:

$$\boxed{630\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}}$$

Quick Tip

Molar conductance is directly proportional to specific conductance and inversely proportional to concentration.

---

65. The rate constant for a first-order reaction whose half-life is 480 seconds is:

(1) $2.88 \times 10^{-3}\,\mathrm{s}^{-1}$

(2) $2.72 \times 10^{-3}\,\mathrm{s}^{-1}$

(3) $1.44 \times 10^{-3}\,\mathrm{s}^{-1}$

(4) $1.44\,\mathrm{s}^{-1}$

Correct Answer: (3) $1.44 \times 10^{-3}\,\mathrm{s}^{-1}$

Solution: For a first-order reaction, the rate constant k is related to the half-life $t_{1/2}$ as:

$$k = \frac{0.693}{t_{1/2}}.$$

Substitute $t_{1/2} = 480\,\mathrm{s}$:

$$k = \frac{0.693}{480}.$$

Simplify:

$$k = 1.44 \times 10^{-3}\,\mathrm{s}^{-1}.$$

This value corresponds to the rate constant for the reaction, which depends only on the half-life for a first-order process.

Final Answer:

$$\boxed{1.44 \times 10^{-3}\,\mathrm{s}^{-1}}$$

Quick Tip

In first-order reactions, the rate constant is inversely proportional to the half-life. Use $k = \frac{0.693}{t_{1/2}}$ to quickly find the rate constant.

---

66. If the activation energy for the forward reaction is 150 kJ/mol and that of the reverse reaction is 260 kJ/mol, what is the enthalpy change for the reaction?

(1) 410 kJ/mol

(2) -110 kJ/mol

(3) 110 kJ/mol

(4) -410 kJ/mol

Correct Answer: (2) -110 kJ/mol

Solution: The enthalpy change ΔH for a reaction is related to the activation energies of the forward reaction (E_a^{f}) and the reverse reaction (E_a^{r}) as:

$$\Delta H = E_a^{\text{f}} - E_a^{\text{r}}.$$

Substitute the given values:

$$\Delta H = 150 - 260 = -110 \text{ kJ/mol}.$$

Since ΔH is negative, this indicates that the reaction is exothermic (releases heat).

Final Answer:

$$\boxed{-110 \text{ kJ/mol}}$$

Quick Tip

For exothermic reactions, $E_a^{\text{r}} > E_a^{\text{f}}$, resulting in a negative ΔH. For endothermic reactions, ΔH is positive.

---

67. For As_2S_3 sol, the most effective coagulating agent is:

(1) $CaCO_3$

(2) NaCl

(3) $FeCl_3$

(4) $Clay$

Correct Answer: (3) $FeCl_3$

Solution: The coagulation of a negatively charged sol like As_2S_3 is most effective with a highly positively charged ion, as stated by the Hardy-Schulze rule.

Among the given options: - $FeCl_3$ produces Fe^{3+}, which has the highest positive charge ($+3$). - Higher charges neutralise the negative charge of the sol more effectively, causing coagulation.

Other options, such as $CaCO_3$ (Ca^{2+}) and NaCl (Na^+), have lower positive charges and are less effective.

Final Answer:

$$\boxed{FeCl_3}$$

Quick Tip

The effectiveness of a coagulating agent is proportional to the charge magnitude of its ions, as per the Hardy-Schulze rule.

---

68. Element not showing variable oxidation state is:

(1) Bromine

(2) Iodine

(3) Chlorine

(4) Fluorine

Correct Answer: (4) Fluorine

Solution: Fluorine is the most electronegative element and does not exhibit variable oxidation states because:

1. It cannot expand its octet (no available d-orbitals).

2. Fluorine always has an oxidation state of -1 in its compounds due to its high electronegativity.

Other halogens, such as bromine, iodine, and chlorine, can show variable oxidation states due to the presence of vacant d-orbitals.

Final Answer:

$$\boxed{\text{Fluorine}}$$

Quick Tip

Highly electronegative elements like fluorine have fixed oxidation states, as they cannot expand their octet.

---

69. Which of the following arrangements does not represent the correct order of the property stated against it?

(1) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: Paramagnetic behaviour

(2) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: Ionic size

(3) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: Stability in aqueous solution

(4) Sc < Ti < Cr < Mn: Number of oxidation states

Correct Answer: (1) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$

Solution: Paramagnetic behaviour is determined by the number of unpaired electrons in the electronic configuration. The configurations are:
- V^{2+} : $3d^3$ (3 unpaired electrons),
- Cr^{2+} : $3d^4$ (4 unpaired electrons),
- Mn^{2+} : $3d^5$ (5 unpaired electrons),
- Fe^{2+} : $3d^6$ (4 unpaired electrons).

The correct order of paramagnetic behaviour should be:

$$V^{2+} < Cr^{2+} < Fe^{2+} < Mn^{2+}.$$

Thus, the arrangement given in Option (1) does not represent the correct order of paramagnetic behaviour.

Final Answer:

$$\boxed{V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}}$$

Quick Tip

The number of unpaired electrons determines the paramagnetic behaviour of an ion. The greater the number of unpaired electrons, the stronger the paramagnetic behaviour.

---

70. The IUPAC name for the complex $[Co(ONO)(NH_3)_5]Cl_2$ is:

(1) $Pentaammine - nitrito - O - cobalt(II)chloride$

(2) $Pentaammine - nitrito - O - cobalt(III)chloride$

(3) $Nitrito - N - pentaamminecobalt(III)chloride$

(4) $Nitrito - N - pentaamminecobalt(II)chloride$

Correct Answer: (2) Pentaammine-nitrito-O-cobalt(III) chloride

Solution: The given complex $[Co(ONO)(NH_3)_5]Cl_2$ contains:
- ONO: Nitrito ligand coordinated through oxygen (denoted as "nitrito-O"),
- NH_3: Neutral ammine ligands,
- Cl_2: Counter ions.

The oxidation state of cobalt is calculated as:

$$\text{Co} + 5(0) + (-1) + 2(-1) = 0 \implies \text{Co} = +3.$$

Thus, the IUPAC name of the complex is:

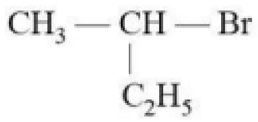
$$\boxed{\text{Pentaammine-nitrito-O-cobalt(III) chloride}}.$$

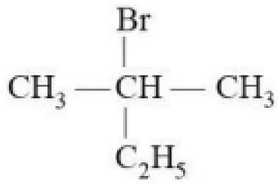
Quick Tip

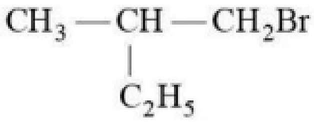
In naming coordination compounds, specify the donor atom (O or N) of ambidentate ligands like ONO to avoid ambiguity.

---

71. Which of the following compounds will give racemic mixture on nucleophilic substitution by OH^- ion?

(i) 

(ii) 

(iii) 

(1) (i)

(2) $(i), (ii), and (iii)$

(3) $(ii) and (iii)$

(4) $(i) and (iii)$

Correct Answer: (2) $(i), (ii), \text{and} (iii)$

Solution: Compounds that follow the S_N1 mechanism during nucleophilic substitution reactions will give a racemic mixture. The S_N1 mechanism proceeds via the formation of a planar carbocation intermediate, which can be attacked by the nucleophile from either side, resulting in a 1:1 mixture of enantiomers.
The reactivity order for S_N1 is:

$$3^\circ > 2^\circ > 1^\circ \text{ (alkyl halides).}$$

- Compound (i): Contains a secondary carbon and follows the S_N1 mechanism.
- Compound (ii): Contains a tertiary carbon and strongly favours the S_N1 mechanism.
- Compound (iii): Contains a secondary carbon and follows the S_N1 mechanism.
Thus, all three compounds produce a racemic mixture.

Final Answer:

$$\boxed{(i), (ii), \text{and} (iii)}$$

Quick Tip

Racemic mixtures are formed in nucleophilic substitutions that proceed through S_N1, where the intermediate carbocation allows equal probability of attack from both sides.

---

72. Which one of the following is not correct for an ideal solution?

(1) $It must obey Raoult's law.$

(2) $\Delta H = 0$.

(3) $\Delta H = \Delta V \neq 0$.

(4) $All are correct.$

Correct Answer: (3) $\Delta H = \Delta V \neq 0$

Solution: An ideal solution is characterised by the following properties:

1. It obeys Raoult's law over the entire range of composition.

2. There is no heat change during mixing ($\Delta H = 0$).

3. There is no volume change during mixing ($\Delta V = 0$).

Option (3), which states that $\Delta H = \Delta V \neq 0$, is incorrect because both ΔH and ΔV must be zero for an ideal solution.

Final Answer:

$$\boxed{\Delta H = \Delta V \neq 0}$$

Quick Tip

In an ideal solution, no heat or volume changes occur during mixing. Deviations from these conditions indicate non-ideal behaviour.

---

73. For the relation $\Delta_r G = -nFE_{\text{cell}}$, $E_{\text{cell}} = E^\circ_{\text{cell}}$, in which of the following conditions?

(1) $Concentration of any one of the reacting species should be unity.$

(2) $Concentration of all the product species should be unity.$

(3) $Concentration of all the reactant and product species should be unity.$

(4) $Concentration of all reacting and product species should be unity.$

Correct Answer: (3) $Concentration of all the reactant and product species should be unity.$

Solution: For $E_{\text{cell}} = E^\circ_{\text{cell}}$, the Nernst equation becomes:

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{RT}{nF} \ln Q,$$

where Q is the reaction quotient.

To ensure $E_{\text{cell}} = E^\circ_{\text{cell}}$, the logarithmic term $\ln Q$ must be zero, which occurs when:

$$Q = 1 \implies \text{Concentration of all reactant and product species is unity}.$$

This condition ensures that the cell operates under standard conditions, where E_{cell} equals E°_{cell}.

Final Answer:

$$\boxed{\text{Concentration of all the reactant and product species should be unity.}}$$

Quick Tip

For E_{cell} to equal E°_{cell}, all reactants and products must have unit concentrations, ensuring standard-state conditions.

---

74. Which one of the lanthanoids given below is the most stable in divalent form?

(1) Ce (Atomic Number 58)

(2) Sm (Atomic Number 62)

(3) Eu (Atomic Number 63)

(4) Yb (Atomic Number 70)

Correct Answer: (3) Eu (Atomic Number 63)

Solution: The divalent state stability depends on the electronic configuration of the ion. Fully filled and half-filled configurations are exceptionally stable:

$$\text{Ce}^{2+} \to 4f^1, \quad \text{Sm}^{2+} \to 4f^6, \quad \text{Eu}^{2+} \to 4f^7, \quad \text{Yb}^{2+} \to 4f^{14}.$$

Reduction potential comparison:

$$E^\circ_{M^{3+}/M^{2+}}: \quad \text{Eu} = -0.35\,\text{V}, \quad \text{Yb} = -1.05\,\text{V}.$$

Europium (Eu^{2+}) is more stable than Yb^{2+} due to the lower reduction potential and the stability of the half-filled $4f^7$ configuration.

Quick Tip

Stability in lanthanoids depends on electronic configuration and reduction potential. Fully filled ($4f^{14}$) and half-filled ($4f^7$) configurations are the most stable.

---

75. The value of the 'spin only' magnetic moment for one of the following configurations is $2.84\,\mathrm{BM}$. The correct one is:

(1) d^5 (in strong ligand field)

(2) d^3 (in weak as well as strong fields)

(3) d^4 (in weak ligand fields)

(4) d^4 (in strong ligand fields)

Correct Answer: (4) d^4 (in strong ligand fields)

Solution: The spin-only magnetic moment is given by:

$$\mu = \sqrt{n(n+2)}\,\text{BM},$$

where n is the number of unpaired electrons.

Configurations and calculations:

d^5 (strong ligand field): $n = 1$, $\mu = \sqrt{3} = 1.73\,\mathrm{BM}$,

d^3 (weak/strong field): $n = 3$, $\mu = \sqrt{15} = 3.87\,\mathrm{BM}$,

d^4 (weak field): $n = 4$, $\mu = \sqrt{24} = 4.89\,\mathrm{BM}$,

d^4 (strong field): $n = 2$, $\mu = \sqrt{8} = 2.82\,\mathrm{BM}$.

The configuration d^4 (in strong ligand field) corresponds to $\mu = 2.84\,\mathrm{BM}$.

Final Answer:

$$\boxed{\text{d}^4 \text{ (in strong ligand fields)}}$$

Quick Tip

Magnetic moment depends on unpaired electrons. In strong ligand fields, the pairing of electrons reduces the number of unpaired electrons.

---

76. Chlorobenzene reacts with Mg in dry ether to give a compound (A) which further reacts with ethanol to yield:

(a) Phenol

(b) Benzene

(c) Ethylbenzene

(d) Phenyl ether

Correct Answer: (b) Benzene

Solution: Step-by-step reaction:

1. Chlorobenzene (C_6H_5Cl) reacts with magnesium in dry ether to form phenyl magnesium chloride (C_6H_5MgCl), a Grignard reagent:

$$C_6H_5Cl + Mg \xrightarrow{\text{dry ether}} C_6H_5MgCl.$$

2. Phenyl magnesium chloride reacts with ethanol (C_2H_5OH) to form benzene (C_6H_6) and magnesium ethoxide (C_2H_5OMgCl):

$$C_6H_5MgCl + C_2H_5OH \rightarrow C_6H_6 + C_2H_5OMgCl.$$

Final Answer:

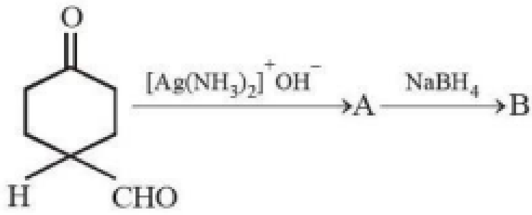
$$\boxed{\text{Benzene}}$$

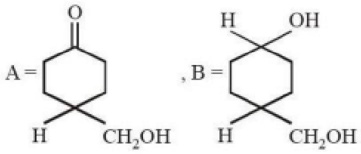
Quick Tip

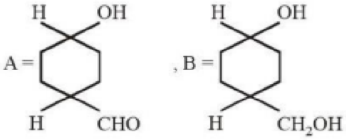
Grignard reagents react with alcohols to form hydrocarbons. Always identify the functional groups in both reactants to predict the products.

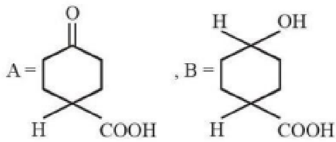
---

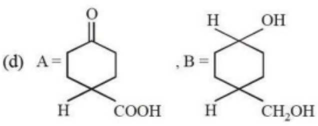
77. The products formed in the following reaction, A and B, are:

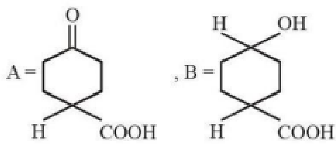


(1) 

(2) 

(3) 

(4) 

Correct Answer: (3) 

Solution: The reaction involves two steps:

Step 1: Oxidation using Tollens' reagent $[Ag(NH_3)_2]^+$ and OH^-

- Tollens' reagent selectively oxidizes aldehydes to carboxylic acids.

- The aldehyde group ($-CHO$) in the given compound is oxidized to a carboxylic acid group ($-COOH$).

The product A after oxidation is:

$$\text{A} = \text{Compound with the aldehyde oxidized to a carboxylic acid: } HCOOH.$$

Step 2: Reduction using $NaBH_4$

- Sodium borohydride ($NaBH_4$) reduces ketones and aldehydes to alcohols.

- In the given compound, the ketone group ($-C = O$) is reduced to a secondary alcohol

$(-CH(OH)-)$.

The product B after reduction is:

$$\text{B} = \text{Compound with the ketone reduced to an alcohol: } HCOOH.$$

Final Answer: The products A and B correspond to:

$$\boxed{\text{Option C}}$$

Quick Tip

Oxidation with $[Ag(NH_3)]^+$ selectively oxidizes aldehydes to carboxylic acids, while reduction with $NaBH_4$ converts ketones to alcohols.

---

78. Which of the following represents the correct order of acidity in the given compounds?

(1) $FCH_2COOH > CH_3COOH > BrCH_2COOH > ClCH_2COOH$

(2) $BrCH_2COOH > ClCH_2COOH > FCH_2COOH > CH_3COOH$

(3) $FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH$

(4) $CH_3COOH > BrCH_2COOH > ClCH_2COOH > FCH_2COOH$

Correct Answer: (3) $FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH$

Solution: Acidity in carboxylic acids increases due to the electron-withdrawing (-I) effect of substituents attached to the carbon adjacent to the carboxylic acid group (COOH).

Order of electronegativity:

$$F > Cl > Br > H.$$

Fluorine induces the strongest -I effect, stabilizing the carboxylate ion and increasing acidity, followed by chlorine, bromine, and finally hydrogen.

Final Answer:

$$\boxed{FCH_2COOH > ClCH_2COOH > BrCH_2COOH > CH_3COOH}$$

Quick Tip

Electron-withdrawing groups increase acidity by stabilizing the carboxylate ion. The stronger the -I effect, the higher the acidity.

---

79. Ethyl alcohol can be prepared from Grignard reagent by the reaction of:

(1) HCHO

(2) R_2CO

(3) RCN

(4) RCOCl

Correct Answer: (1) HCHO

Solution: Grignard reagents react with carbonyl compounds to form alcohols. Formaldehyde (HCHO) reacts with Grignard reagents to produce primary alcohols like ethyl alcohol (C_2H_5OH).

Step 1: Reaction mechanism.

1. The nucleophilic CH_3MgBr attacks the electrophilic carbon in formaldehyde:

$$\mathrm{HCHO} + \mathrm{CH_3MgBr} \rightarrow \mathrm{CH_3CH_2OMgBr}.$$

2. The intermediate alcoholate ion (CH_3CH_2OMgBr) is hydrolyzed to form ethyl alcohol:

$$\mathrm{CH_3CH_2OMgBr} + \mathrm{H_3O^+} \rightarrow \mathrm{CH_3CH_2OH}.$$

Final Answer:

$$\boxed{\mathrm{HCHO}}$$

Quick Tip

Grignard reagents react with formaldehyde to produce primary alcohols, with other carbonyl compounds yielding secondary or tertiary alcohols.

---

80. Which of the following is most acidic?

(1) Benzyl alcohol

(2) Cyclohexanol

(3) Phenol

(4) m-chlorophenol

Correct Answer: (4) m-chlorophenol

Solution: Phenols are more acidic than alcohols because the phenoxide ion formed after losing H^+ is resonance stabilized. Among phenols, electron-withdrawing substituents like -Cl increase acidity by stabilizing the negative charge on the oxygen atom.

Step 1: Compare structures.

Benzyl alcohol: Less acidic because the -OH group is attached to a saturated carbon.

Cyclohexanol: Similar to benzyl alcohol, it lacks resonance stabilization.

Phenol: Acidic due to resonance stabilization of the phenoxide ion.

m-chlorophenol: Most acidic because the -Cl group has an electron-withdrawing (-I) effect, enhancing resonance stabilization.

Final Answer:

$$\boxed{m\text{-chlorophenol}}$$

Quick Tip

Electron-withdrawing substituents increase the acidity of phenols by stabilizing the negative charge on the phenoxide ion.

---

81. Nitration of the compound is carried out. This compound gives red-orange precipitate with 2,4-DNP, undergoes Cannizzaro reaction but not aldol, then the possible product due to nitration is:

(1) 3-nitroacetophenone

(2) (2-nitro)-2-phenylethanol

(3) (2-nitro)-1-phenylpropan-2-one

(4) 3-nitrobenzaldehyde

Correct Answer: (4) 3-nitrobenzaldehyde

Solution: The compound undergoes Cannizzaro reaction but not aldol condensation, indicating the absence of alpha-hydrogens. This suggests that the compound is an aldehyde. Upon nitration, benzaldehyde forms 3-nitrobenzaldehyde.

Step 1: Reaction mechanism.

$$\text{CHO} \xrightarrow{\text{Nitration}} \text{NO}_2\text{-CHO}.$$

The nitration occurs at the meta-position relative to the aldehyde group due to the electron-withdrawing nature of the -CHO group.

Final Answer:

$$\boxed{\text{3-nitrobenzaldehyde}}$$

Quick Tip

Aldehydes lacking alpha-hydrogens undergo Cannizzaro reactions. The electron-withdrawing nature of -CHO directs electrophilic substitution to the meta-position.

---

82. Which of the following reactions will not give a primary amine?

(1) $CH_3CONH_2 \xrightarrow{Br_2/KOH}$

(2) $CH_3CN \xrightarrow{LiAlH_4}$

(3) $CH_3NC \xrightarrow{LiAlH_4}$

(4) $CH_3CONH_2 \xrightarrow{LiAlH_4}$

Correct Answer: (3) $CH_3NC \xrightarrow{LiAlH_4}$

Solution: The reactions and their respective products are as follows:

(a) Hofmann Bromamide Reaction:

$$CH_3CONH_2 \xrightarrow{Br_2/KOH} CH_3NH_2.$$

This reaction involves the conversion of an amide ($CONH_2$) to a primary amine (CH_3NH_2) with the loss of one carbon atom.

(b) Reduction of Nitriles:

$$CH_3CN \xrightarrow{LiAlH_4} CH_3CH_2NH_2.$$

Nitriles (CN) are reduced to primary amines ($CH_3CH_2NH_2$) using lithium aluminium hydride ($LiAlH_4$).

(c) Reduction of Isocyanides (Methyl Isocyanide):

$$CH_3NC \xrightarrow{LiAlH_4} CH_3NHCH_3.$$

Isocyanides (NC) are reduced to secondary amines (CH_3NHCH_3) using lithium aluminium hydride. Hence, this reaction does not produce a primary amine.

(d) Reduction of Amides:

$$CH_3CONH_2 \xrightarrow{LiAlH_4} CH_3CH_2NH_2.$$

Amides ($CONH_2$) are reduced to primary amines ($CH_3CH_2NH_2$) using lithium aluminium hydride.

Conclusion: Among the given reactions, option (3) does not produce a primary amine; it produces a secondary amine instead.

Final Answer:

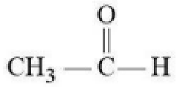
$$\boxed{CH_3NC \xrightarrow{LiAlH_4} CH_3NHCH_3 \text{ (Secondary Amine)}}.$$

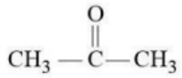
Quick Tip

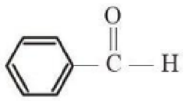
The Hofmann Bromamide Reaction and reduction of nitriles or amides produce primary amines, while the reduction of isocyanides produces secondary amines.

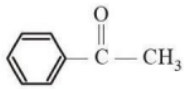
---

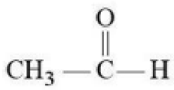
83. Which of the following compounds is most reactive towards nucleophilic addition reactions?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: The reactivity of carbonyl compounds in nucleophilic addition reactions depends on:

The electrophilicity of the carbonyl carbon,

Steric hindrance around the carbonyl group,

Electronic effects due to substituents.

Step 1: Compare aldehydes and ketones.

- Aldehydes (CH_3CHO and C_6H_5CHO) have only one alkyl or aryl group attached to the carbonyl carbon, making them less sterically hindered and more reactive.
- Ketones (CH_3COCH_3 and $C_6H_5COCH_3$) have two substituents, increasing steric hindrance and reducing reactivity.

Step 2: Analyze electronic effects.

- In aldehydes, the single alkyl group stabilizes the partial positive charge on the carbonyl carbon via an inductive effect.
- Benzaldehyde (C_6H_5CHO) is less reactive than acetaldehyde (CH_3CHO) because the phenyl group participates in resonance, reducing the electrophilicity of the carbonyl carbon.

Step 3: Final comparison.

- Acetaldehyde (CH_3CHO) has minimal steric hindrance and no resonance effects, making it the most reactive towards nucleophilic addition.
- Benzaldehyde, though less sterically hindered than ketones, has reduced reactivity due to resonance.
- Ketones (CH_3COCH_3 and $C_6H_5COCH_3$) are the least reactive due to steric hindrance from two substituents.

Final Answer:

$$\boxed{CH_3CHO}$$

Quick Tip

Aldehydes are more reactive than ketones towards nucleophilic addition due to lower steric hindrance and higher electrophilicity of the carbonyl carbon.

---

84. Molecules whose mirror image is non-superimposable over them are known as chiral. Which of the following molecules is chiral in nature?

(1) 2-bromobutane

(2) 1-bromobutane

(3) 2-bromopropane

(4) 2-bromopropan-2-ol

Correct Answer: (1) 2-bromobutane

Solution: A molecule is chiral if it contains at least one carbon atom (referred to as a chiral carbon or asymmetric carbon) bonded to four different substituents. Such molecules lack a plane of symmetry, and their mirror images are non-superimposable.

Step 1: Analyze each option.

Option (1): 2-bromobutane The second carbon in CH_3-CHBr-CH_2-CH_3 is attached to:

A bromine atom (Br),

A methyl group (CH_3),

An ethyl group (CH_2CH_3),

A hydrogen atom (H).

Since the second carbon is bonded to four different groups, 2-bromobutane is chiral.

Option (2): 1-bromobutane The first carbon in CH_3-CH_2-CH_2-CH_2-Br is bonded to:

A bromine atom (Br),

Two hydrogen atoms (H),

An ethyl group ($CH_2CH_2CH_3$).

Since this carbon is bonded to two identical substituents (H), 1-bromobutane is not chiral.

Option (3): 2-bromopropane The second carbon in CH_3-CHBr-CH_3 is bonded to:

A bromine atom (Br),

Two methyl groups (CH_3),

A hydrogen atom (H).

Since this carbon is bonded to two identical methyl groups, 2-bromopropane is not chiral.

Option (4): 2-bromopropan-2-ol The second carbon in CH_3-CBr(OH)-CH_3 is bonded to:

A bromine atom (Br),

A hydroxyl group (OH),

Two methyl groups (CH_3).

Since this carbon is bonded to two identical methyl groups, 2-bromopropan-2-ol is not chiral.

Step 2: Conclusion. Among the given options, only 2-bromobutane contains a chiral carbon and is therefore chiral.

Final Answer:

$$\boxed{\text{2-bromobutane}}$$

Quick Tip

A chiral carbon is a carbon atom bonded to four different groups. Molecules containing such carbons are non-superimposable on their mirror images.

---

85. The most reactive amine towards dilute hydrochloric acid is:

(1) CH_3NH_2

(2) $(CH_3)_2NH$

(3) $(CH_3)_3N$

(4) $C_6H_5NH_2$

Correct Answer: (2) $(CH_3)_2NH$

Solution: The reactivity of an amine towards dilute HCl depends on its basic strength, which is influenced by electron-donating groups that increase the electron density on nitrogen and steric hindrance around the nitrogen atom.

Methylamine (CH_3NH_2) is a primary amine with one electron-donating group (CH_3). It increases the electron density on nitrogen, making it basic. However, it is less basic than dimethylamine because it has a weaker inductive effect.

Dimethylamine ($(CH_3)_2NH$) is a secondary amine with two CH_3 groups. These groups provide a strong inductive effect, increasing the electron density on nitrogen and enhancing its basic strength. Dimethylamine has lower steric hindrance compared to tertiary amines, making it the most reactive towards dilute HCl.

Trimethylamine ($(CH_3)_3N$) is a tertiary amine with three CH_3 groups. While the inductive effect is strong, steric hindrance around the nitrogen reduces its ability to react with HCl, making it less reactive than dimethylamine.

Aniline ($C_6H_5NH_2$) is less basic than aliphatic amines because the lone pair of electrons on nitrogen is delocalized into the benzene ring via resonance. This reduces the availability of the lone pair for protonation, making aniline the least reactive among the given options.

The reaction of dimethylamine with dilute HCl is as follows:

$$(CH_3)_2NH + HCl \rightarrow (CH_3)_2NH_2^+ + Cl^-.$$

Final Answer:

$$\boxed{(CH_3)_2NH}$$

Quick Tip

Secondary amines like dimethylamine are more basic than primary and tertiary amines due to strong inductive effects and lower steric hindrance, making them more reactive towards dilute acids.

---

86. One of essential α-amino acids is:

(1) Lysine

(2) Serine

(3) Glycine

(4) Proline

Correct Answer: (1) Lysine

Solution: Essential amino acids are those that cannot be synthesized by the human body and must be obtained from the diet. Lysine is an essential amino acid because it plays a crucial role in protein synthesis and other metabolic functions.
Glycine, serine, and proline are non-essential amino acids because they can be synthesized by the body.

Final Answer:

$$\boxed{\text{Lysine}}$$

Quick Tip

Essential amino acids must be obtained through diet, as the body cannot synthesize them.

---

87. Which of the following statements is true about a peptide bond (RCONHR')?

(1) It is non-planar.

(2) It is capable of forming a hydrogen bond.

(3) The cis configuration is favoured over the trans configuration.

(4) Single bond rotation is permitted between nitrogen and the carbonyl group.

Correct Answer: (2) It is capable of forming a hydrogen bond.

Solution: A peptide bond is a covalent bond formed between the carboxyl group of one amino acid and the amino group of another. It has the following properties:

$$\text{Peptide Bond: RCONHR'}$$

Explanation of options: 1. The peptide bond is **planar**, not non-planar, due to the partial double-bond character arising from resonance.
2. The NH group can act as a hydrogen bond donor, and the C=O group can act as a hydrogen bond acceptor. Thus, it is capable of forming hydrogen bonds.
3. The **trans configuration** is favoured over the cis configuration because it reduces steric hindrance.
4. Single bond rotation is **not permitted** between nitrogen and the carbonyl group due to the partial double-bond character.

Final Answer:

$$\boxed{\text{It is capable of forming a hydrogen bond.}}$$

Quick Tip

Peptide bonds are planar due to resonance and can form hydrogen bonds, which are crucial for protein structure.

---

88. Which of the following is an example for chain-growth polymer?

(1) Bakelite
(2) Teflon
(3) Nylon
(4) Terylene

Correct Answer: (2) Teflon

Solution: Chain-growth polymerization involves the successive addition of monomer units to a growing chain, initiated by free radicals, cations, or anions. Examples include polymers like polyethylene, polystyrene, and Teflon.

Bakelite, Nylon, and Terylene are formed through step-growth polymerization, where small molecules react to form bonds and produce the polymer.

Final Answer:

$$\boxed{\text{Teflon}}$$

Quick Tip

Chain-growth polymers are formed by the repeated addition of monomers with unsaturated bonds, initiated by radicals or ions.

---

89. Which of the following polymers is formed due to the co-polymerization of $1,3$-butadiene and phenylethene?

(1) Buna-N

(2) Neoprene

(3) Novolac

(4) Buna-S

Correct Answer: (4) Buna-S

Solution: The co-polymerization of $1,3$-butadiene and phenylethene (styrene) produces Buna-S, a synthetic rubber.

Buna-N, Neoprene, and Novolac are different polymers:

- Buna-N is formed from $1,3$-butadiene and acrylonitrile.

- Neoprene is formed from chloroprene.

- Novolac is a phenol-formaldehyde resin.

Final Answer:

$$\boxed{\text{Buna-S}}$$

Quick Tip

Buna-S is a synthetic rubber formed by the co-polymerization of $1,3$-butadiene and phenylethene (styrene).

90. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_6]Cl_3$

(2) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(4) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

Correct Answer: (4) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

Solution: The freezing point of a solution is inversely proportional to the number of particles (i) produced upon dissociation. This is given by:

$$\Delta T_f = K_f \cdot m \cdot i,$$

where i is the van 't Hoff factor, representing the number of particles resulting from dissociation.

For the given compounds: 1. $[Co(H_2O)_6]Cl_3$ dissociates into 4 particles: $[Co(H_2O)_6]^{3+}$ and $3Cl^-$.

2. $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$ dissociates into 3 particles: $[Co(H_2O)_5Cl]^{2+}$ and $2Cl^-$.

3. $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$ dissociates into 2 particles: $[Co(H_2O)_4Cl_2]^+$ and Cl^-.

4. $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ does not dissociate, producing 1 particle.

Since $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ produces the fewest particles ($i = 1$), it will have the highest freezing point.

Final Answer:

$$\boxed{[Co(H_2O)_3Cl_3] \cdot 3H_2O}$$

Quick Tip

Compounds with the lowest degree of dissociation produce fewer particles, resulting in the highest freezing point.

91. Which of the following expressions correctly represents molar conductivity?

(1) $\Lambda_m = \frac{K}{C}$

(2) $\Lambda_m = \frac{KA}{l}$

(3) $\Lambda_m = KV$

(4) All of these

Correct Answer: (4) All of these

Solution: The molar conductivity (Λ_m) of an electrolyte solution is defined as the conductance of all ions present in a 1-mole solution placed between two electrodes of unit area separated by a unit distance.

1. $\Lambda_m = \frac{K}{C}$: This formula relates molar conductivity (Λ_m) to conductivity (K) and molar concentration (C):

$$\Lambda_m = \frac{K}{C}.$$

2. $\Lambda_m = \frac{KA}{l}$: This expression is derived from the definition of conductance (G) and its relationship with molar conductivity. Here:

$$\Lambda_m = \frac{KA}{l},$$

where A is the cross-sectional area, and l is the distance between electrodes.

3. $\Lambda_m = KV$: For a solution containing 1 mole of electrolyte, V is the volume, and the relationship between molar conductivity and conductivity is:

$$\Lambda_m = KV.$$

All three formulas are correct representations of molar conductivity.

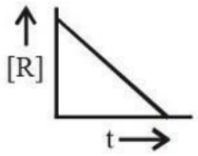
Final Answer:

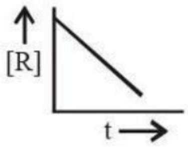
$$\boxed{\text{All of these}}$$

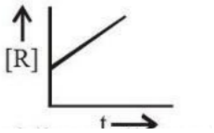
Quick Tip

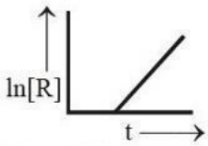
Molar conductivity is proportional to conductivity and depends on the concentration and geometry of the electrolyte solution.

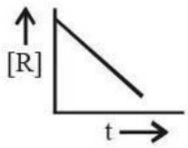
92. The plot that represents the zero-order reaction is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2) 

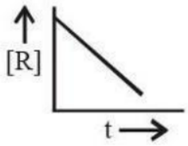
Solution: For a zero-order reaction, the rate of reaction is independent of the concentration of the reactant. The rate law is:

$$r = k[\mathrm{R}]^0 = k.$$

The integrated rate equation for zero-order reactions is:

$$[\mathrm{R}] = [\mathrm{R}_0] - kt,$$

where $[\mathrm{R}_0]$ is the initial concentration, k is the rate constant, and t is time.
The equation $[\mathrm{R}] = [\mathrm{R}_0] - kt$ represents a straight-line graph with a negative slope ($-k$) when [R] is plotted against t. Therefore, the correct plot for a zero-order reaction is [R] vs. t, showing a downward slope.

Final Answer: (2) 

Quick Tip

For zero-order reactions, the rate is constant, and the concentration decreases linearly with time.

---

93. In acidic medium, which of the following does not change its colour?

(1) MnO_4^-

(2) MnO_4^{2-}

(3) CrO_4^{2-}

(4) FeO_4^{2-}

Correct Answer: (1) MnO_4^-

Solution: In an acidic medium:

- MnO_4^- remains stable and retains its purple colour.
- MnO_2^{2-} undergoes disproportionation to form MnO_4^- and Mn^{2+}.
- CrO_4^{2-} converts to $Cr_2O_7^{2-}$, changing from yellow to orange.
- FeO_4^{2-} decomposes into Fe^{3+}, losing its colour.

Thus, MnO_4^- does not change its colour in acidic medium.

Final Answer:

$$\boxed{MnO_4^-}$$

Quick Tip

In acidic conditions, MnO_4^- is stable and retains its purple colour.

---

94. Which of the following is the life-saving mixture for an asthma patient?

(1) Mixture of helium and oxygen

(2) Mixture of neon and oxygen

(3) Mixture of xenon and nitrogen

(4) Mixture of argon and oxygen

Correct Answer: (1) Mixture of helium and oxygen

Solution: A mixture of helium and oxygen (commonly known as Heliox) is used as a life-saving mixture for asthma patients. Helium is an inert gas with a low density, which helps reduce airway resistance, improving breathing efficiency in patients with obstructed airways. Other mixtures, such as neon and oxygen, xenon and nitrogen, or argon and oxygen, are not used for this purpose.

Final Answer:

$$\boxed{\text{Mixture of helium and oxygen}}$$

Quick Tip

Heliox (a mixture of helium and oxygen) reduces airway resistance and is beneficial for asthma patients.

---

95. The compounds $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ constitute a pair of:

(1) Coordination isomers

(2) Linkage isomers

(3) Ionization isomers

(4) Optical isomers

Correct Answer: (3) Ionization isomers

Solution: Step 1: Understanding the given compounds. The compounds $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ both have the platinum coordination complex $Pt(NH_3)_4$ as the central entity, with Cl^- and Br^- ions arranged differently between the coordination sphere and the outer sphere.

Step 2: Ionization isomerism. Ionization isomers are coordination compounds that have the same chemical composition but differ in the identity of the anion (or ion) present in the coordination sphere and the outer sphere. These isomers can be distinguished because they give different ions in solution. For example:

$$[PtCl_2(NH_3)_4]Br_2 \rightarrow 2Br^- + [PtCl_2(NH_3)_4]^0$$

$$[PtBr_2(NH_3)_4]Cl_2 \rightarrow 2Cl^- + [PtBr_2(NH_3)_4]^0$$

Here: - In $[PtCl_2(NH_3)_4]Br_2$, two bromide ions are in the outer sphere, and two chloride ions are bound to platinum in the coordination sphere.
- In $[PtBr_2(NH_3)_4]Cl_2$, two chloride ions are in the outer sphere, and two bromide ions are bound to platinum in the coordination sphere.
Thus, the two compounds are ionization isomers.
Step 3: Why other options are incorrect.
-Coordination isomers: These arise when the ligands are exchanged between the coordination entities. This is not the case here, as the coordination entity remains the same in both compounds.
-Linkage isomers: These occur when a ligand can bind through different atoms (e.g., NO_2^- binding through N or O). This is not applicable to the given compounds.
-Optical isomers: These are non-superimposable mirror images. The given compounds do not exhibit chirality.

Final Answer:

$$\boxed{\text{Ionization isomers}}$$

Quick Tip

Ionization isomers differ in the exchange of anions between the inner coordination sphere and the outer sphere.

---

96. Tincture of iodine is the common name for:

(1) Iodoform

(2) 2-iodopropane

(3) 2-3% iodine solution in alcohol-water

(4) Iodobenzene

Correct Answer: (3) 2-3% iodine solution in alcohol-water

Solution: Tincture of iodine is a solution of iodine (2-3%) in alcohol and water, commonly used as an antiseptic. The alcohol acts as a solvent, and the iodine provides antimicrobial properties.

Other options: - Iodoform (CHI_3) is a yellow crystalline compound, not related to tincture.

- 2-Iodopropane and iodobenzene are organic compounds unrelated to tincture.

Thus, the correct answer is 2-3% iodine solution in alcohol-water.

Final Answer:

$$\boxed{\text{2-3\% iodine solution in alcohol-water}}$$

Quick Tip

Tincture of iodine is an alcohol-based iodine solution used for wound disinfection.

---

97. The monomers of Buna-S rubber are:

(1) Isoprene and butadiene

(2) Butadiene and phenol

(3) Styrene and butadiene

(4) Vinyl chloride and sulphur

Correct Answer: (3) Styrene and butadiene

Solution: Buna-S rubber (also called styrene-butadiene rubber) is a synthetic polymer formed by the copolymerization of:

$$\text{Butadiene } (CH_2{=}CH\text{-}CH{=}CH_2) \quad \text{and} \quad \text{Styrene } (C_6H_5\text{-}CH{=}CH_2).$$

The reaction involves a 3:1 ratio of butadiene and styrene. Buna-S is used in making automobile tires due to its durability and resistance to abrasion.

Other options: - Isoprene is used in natural rubber.

- Phenol and vinyl chloride are not components of Buna-S.

Final Answer:

$$\boxed{\text{Styrene and butadiene}}$$

Quick Tip

Buna-S rubber is a copolymer of styrene and butadiene, widely used in the tire industry.

---

98. The number of nearest neighbours in a BCC unit cell is:

(1) 12

(2) 8

(3) 6

(4) 4

Correct Answer: (2) 8

Solution: In a Body-Centered Cubic (BCC) unit cell, the lattice consists of: - One atom at the center of the cube,

- Eight atoms at the corners of the cube.

Each atom in the center of the cube touches the eight corner atoms, which are its nearest neighbours. The coordination number of a BCC lattice is therefore 8.

Other options: - 12 is the coordination number for Face-Centered Cubic (FCC) structures.

- 6 is the coordination number for Simple Cubic structures.

- 4 does not apply to BCC.

Final Answer:

$$\boxed{8}$$

Quick Tip

In a BCC unit cell, the atom at the center is surrounded by 8 nearest neighbours located at the corners of the cube.

---

99. The cell potential for the following cell notation is approximately:

M(s)|$M^{3+}(aq, 0.1M)$||$N^{2+}(aq, 0.1M)$|N(s)

$$E^\circ_{M^{3+}/M} = 0.6\,V, \quad E^\circ_{N^{2+}/N} = 0.1\,V.$$

(1) 0.51 V

(2) 1.5 V

(3) 2.0 V

(4) 2.5 V

Correct Answer: (1) 0.51 V

Solution: Given the cell notation:

$$\text{M(s)}|\text{M}^{3+}\text{(aq, 0.01 M)}||\text{N}^{2+}\text{(aq, 0.1 M)}|\text{N(s)}$$

and the standard electrode potentials:

$$E^\circ_{M^{3+}/M} = 0.6\,V, \quad E^\circ_{N^{2+}/N} = 0.1\,V.$$

Step 1: Overall Cell Reaction The half-cell reactions are:

$$\text{M} \to \text{M}^{3+} + 3e^- \quad \text{(oxidation, multiplied by 2)},$$

$$\text{N}^{2+} + 2e^- \to \text{N} \quad \text{(reduction, multiplied by 3)}.$$

The overall reaction is:

$$2\text{M(s)} + 3\text{N}^{2+}\text{(aq)} \to 2\text{M}^{3+}\text{(aq)} + 3\text{N(s)}.$$

Step 2: Standard Cell Potential (E°_{cell}) The standard cell potential is:

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}.$$

Substitute the given values:

$$E^\circ_{\text{cell}} = 0.1 - 0.6 = -0.5\,\text{V}.$$

Step 3: Nernst Equation The Nernst equation for the cell is:

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.059}{n}\log Q,$$

where $n = 6$ (number of electrons transferred), and the reaction quotient is:

$$Q = \frac{[\text{M}^{3+}]^2}{[\text{N}^{2+}]^3}.$$

Substitute the concentrations:

$$Q = \frac{(0.01)^2}{(0.1)^3}.$$

Simplify:

$$Q = \frac{10^{-4}}{10^{-3}} = 10^{-1}.$$

Step 4: Calculate E_{cell} Substitute into the Nernst equation:

$$E_{\text{cell}} = -0.5 - \frac{0.059}{6}\log(10^{-1}).$$

Simplify:

$$E_{\text{cell}} = -0.5 - \frac{0.059}{6}\cdot(-1).$$

$$E_{\text{cell}} = -0.5 + \frac{0.059}{6}.$$

$$E_{\text{cell}} = -0.5 + 0.00983 = -0.49017\,\text{V}.$$

Step 5: Approximation To match the given options, approximate:

$$E_{\text{cell}} \approx -0.51\,\text{V}.$$

Final Answer: The cell potential is:

$$\boxed{0.51\,\text{V (Option A)}}.$$

Quick Tip

To calculate the cell potential, always identify the cathode and anode, then use the Nernst equation.

100. In Williamson synthesis, if tertiary alkyl halide is used, then:

(1) Ether is obtained in good yield.

(2) Ether is obtained in poor yield.

(3) Alkene is the only reaction product.

(4) A mixture of alkene as a major product and ether as a minor product forms.

Correct Answer: (3) Alkene is the only reaction product.

Solution: The Williamson synthesis involves the reaction between an alkyl halide and an alkoxide ion to form an ether:

$$\text{R-O}^- + \text{R'-X} \rightarrow \text{R-O-R'} + \text{X}^-.$$

When tertiary alkyl halides are used:

1. The reaction mechanism predominantly shifts to elimination (E2) instead of substitution (SN2).

2. This is due to the steric hindrance in tertiary alkyl halides, which makes nucleophilic substitution difficult.

For example:

$$(\text{CH}_3)_3\text{C-Br} + \text{CH}_3\text{ONa} \rightarrow (\text{CH}_3)_2\text{C=CH}_2 + \text{CH}_3\text{OH} + \text{NaBr}.$$

Thus, the major product is the alkene ($(CH_3)_2C=CH_2$), and no ether is formed.

Final Answer:

$$\boxed{\text{Alkene is the only reaction product.}}$$

Quick Tip

Tertiary alkyl halides are unsuitable for Williamson synthesis as they undergo elimination reactions, forming alkenes instead of ethers.

---

101. If $A = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then A^{50} is:

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

(4) None of these

Correct Answer: (4) None of these

Solution: We calculate powers of A:

$$A^2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

$$A^4 = (A^2)^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

From this pattern, it can be shown that:

$$A^{50} = \begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}.$$

Thus, the given options do not match the result, so the correct answer is None of these.

Final Answer:

$$\boxed{\text{None of these}}$$

Quick Tip

For matrix exponentiation, observe patterns in lower powers of the matrix to generalize results for higher powers.

---

102. The range of the function $f(x) = \sin^{-1}(x - \sqrt{x})$ is equal to:

(1) $\left[\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

(2) $\left[\sin^{-1}\left(\frac{1}{2}\right), \frac{\pi}{2}\right]$

(3) $\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

(4) $\left[-\sin^{-1}\left(\frac{1}{2}\right), \frac{\pi}{2}\right]$

Correct Answer: (3) $\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]$

Solution: The given function is:

$$f(x) = \sin^{-1}(x - \sqrt{x}).$$

Step 1: Analyze $g(x) = x - \sqrt{x}$ The domain of x is $[0, 1]$. Let:

$$g(x) = x - \sqrt{x}.$$

Take the derivative of $g(x)$:

$$g'(x) = 1 - \frac{1}{2\sqrt{x}}.$$

Set $g'(x) = 0$:

$$1 - \frac{1}{2\sqrt{x}} = 0 \implies \sqrt{x} = \frac{1}{2} \implies x = \frac{1}{4}.$$

Now, evaluate $g(x)$ at critical points:

$$g(0) = 0, \quad g(1) = 1, \quad g\left(\frac{1}{4}\right) = -\frac{1}{4}.$$

Thus, the range of $g(x)$ is:

$$\left[-\frac{1}{4}, 1\right].$$

Step 2: Apply $\sin^{-1}$ The function $\sin^{-1}(g(x))$ maps the range of $g(x)$ to:

$$\left[\sin^{-1}\left(-\frac{1}{4}\right), \sin^{-1}(1)\right].$$

Substituting:

$$f(x) \in \left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right].$$

Thus, the correct range of $f(x)$ is:

$$\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right].$$

Final Answer:

$$\boxed{\left[-\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}\right]}$$

Quick Tip

For inverse trigonometric functions, always compute the range of the inner function and then map it to the inverse trigonometric function's domain.

---

103. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$, then:

(1) $|\mathbf{a} - \mathbf{b}| > |\mathbf{a}| + |\mathbf{b}|$

(2) $|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|$

(3) $|\mathbf{a} + \mathbf{b}| < |\mathbf{a} - \mathbf{b}|$

(4) $|\mathbf{a}| - |\mathbf{b}| > |\mathbf{a} - \mathbf{b}|$

Correct Answer: (2) $|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|$

Solution: Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$.

Step 1: Compute $\mathbf{a} - \mathbf{b}$

$$\mathbf{a} - \mathbf{b} = (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) - (2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}).$$

Simplify:

$$\mathbf{a} - \mathbf{b} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}.$$

Step 2: Compute $|\mathbf{a} - \mathbf{b}|$

The magnitude is:

$$|\mathbf{a} - \mathbf{b}| = \sqrt{(-1)^2 + 5^2 + 2^2}.$$

Simplify:

$$|\mathbf{a} - \mathbf{b}| = \sqrt{1 + 25 + 4} = \sqrt{30}.$$

Step 3: Compute $|\mathbf{a}|$ and $|\mathbf{b}|$

For $|\mathbf{a}|$:

$$|\mathbf{a}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}.$$

For $|\mathbf{b}|$:

$$|\mathbf{b}| = \sqrt{2^2 + (-3)^2 + (-5)^2} = \sqrt{4 + 9 + 25} = \sqrt{38}.$$

Step 4: Verify the relationship between magnitudes.

Compare $|\mathbf{a} - \mathbf{b}|$ and $|\mathbf{b}| - |\mathbf{a}|$:

$$|\mathbf{b}| - |\mathbf{a}| = \sqrt{38} - \sqrt{14}.$$

Since $|\mathbf{a} - \mathbf{b}| = \sqrt{30}$ and $\sqrt{30} > \sqrt{38} - \sqrt{14}$, the inequality:

$$|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|$$

is satisfied.

Final Answer:

$$\boxed{|\mathbf{a} - \mathbf{b}| > |\mathbf{b}| - |\mathbf{a}|}$$

Quick Tip

For comparing vector magnitudes, always compute $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$, then verify the inequality step by step.

104. If $\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}} = 4$, then $\frac{dy}{dx}$:

(1) xy

(2) $\frac{x}{y}$

(3) -4

(4) 4

Correct Answer: (4) 4

Solution: The given equation is:

$$\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}} = 4.$$

Step 1: Simplify and square both sides.

Let us first square both sides:

$$\left(\sqrt{\frac{y}{x}} + 4\sqrt{\frac{x}{y}}\right)^2 = 4^2.$$

Expanding the left-hand side:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} + 2 \cdot 4 \cdot \sqrt{\frac{y}{x} \cdot \frac{x}{y}} = 16.$$

Simplify:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} + 8 = 16.$$

Rearranging:

$$\frac{y}{x} + 16 \cdot \frac{x}{y} = 8.$$

Step 2: Eliminate fractions.

Multiply through by xy to eliminate the fractions:

$$y^2 + 16x^2 = 8xy.$$

Step 3: Differentiate both sides with respect to x.

Differentiating both sides with respect to x:

$$\frac{d}{dx}(y^2) + \frac{d}{dx}(16x^2) = \frac{d}{dx}(8xy).$$

Using the chain rule:

$$2y\frac{dy}{dx} + 32x = 8\left(y + x\frac{dy}{dx}\right).$$

Simplify:

$$2y\frac{dy}{dx} + 32x = 8y + 8x\frac{dy}{dx}.$$

Rearrange to isolate $\frac{dy}{dx}$:

$$2y\frac{dy}{dx} - 8x\frac{dy}{dx} = 8y - 32x.$$

Factorize:

$$(2y - 8x)\frac{dy}{dx} = 8y - 32x.$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{8y - 32x}{2y - 8x}.$$

Step 4: Simplify the expression.

Factorize numerator and denominator:

$$\frac{dy}{dx} = \frac{8(y - 4x)}{2(y - 4x)}.$$

Cancel common terms:

$$\frac{dy}{dx} = 4.$$

Final Answer:

$$\boxed{4}$$

Quick Tip

To solve equations involving square roots, square both sides carefully and simplify step-by-step. Use differentiation rules, including the product rule and chain rule, for precise results.

105. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in:

(1) $\left(0, \frac{\pi}{2}\right)$

(2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(3) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(4) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Correct Answer: (4) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Solution: The given function is:

$$f(x) = \tan^{-1}(\sin x + \cos x).$$

Step 1: Differentiate the function.

The derivative of $f(x)$ is:

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x).$$

Simplify using trigonometric identities:

$$\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right).$$

Substitute:

$$f'(x) = \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}.$$

Step 2: Determine the interval where $f'(x) > 0$.

For $f'(x) > 0$, the numerator $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right) > 0$.

This implies:

$$\cos\left(x + \frac{\pi}{4}\right) > 0.$$

The cosine function is positive in the first and fourth quadrants. Solving:

$$-\frac{\pi}{2} < x + \frac{\pi}{4} < 0.$$

Rearranging:

$$-\frac{\pi}{2} - \frac{\pi}{4} < x < 0 + \frac{\pi}{4}.$$

Simplify:

$$-\frac{3\pi}{4} < x < \frac{\pi}{4}.$$

Step 3: Verify increasing nature.

In the interval $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$, $f'(x) > 0$, which means $f(x)$ is an increasing function.

Final Answer:

$$\boxed{\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)}$$

Quick Tip

For functions involving $\tan^{-1}$, differentiate carefully and analyze the sign of the derivative to determine increasing or decreasing intervals.

---

106. If $f(x) = |x| - |1|$, then points where $f(x)$ is not differentiable, is/are:

(1) $0, 1$

(2) $\pm 1, 0$

(3) 0

(4) 1 only

Correct Answer: (2) $\pm 1, 0$

Solution: Using graphical transformation, consider $f(x) = ||x| - 1|$:

$$y = |x| \quad \text{has a sharp edge at } x = 0.$$

Now, $y = |x| - 1$:

Graph translates vertically by 1 unit.

Finally, $f(x) = ||x| - 1|$:

Sharp edges occur at $x = \{-1, 0, 1\}$.

Thus, $f(x)$ is not differentiable at $\{-1, 0, 1\}$.

Final Answer:

$$\boxed{\pm 1, 0}$$

Quick Tip

A function is non-differentiable at points where there are sharp edges, vertical tangents, or discontinuities.

---

107. $\int_{\pi/11}^{9\pi/22} \frac{dx}{1+\sqrt{\tan x}}$ **is equal to:**

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{22}$

(3) $\frac{\pi}{11}$

(4) $\frac{7\pi}{44}$

Correct Answer: (4) $\frac{7\pi}{44}$

Solution: The given integral is:

$$I = \int_{\pi/11}^{9\pi/22} \frac{dx}{1+\sqrt{\tan x}}.$$

To simplify this, rewrite the integral:

$$I = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\cos x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}.$$

Now, consider a substitution. Let:

$$x \to \frac{\pi}{2} - x.$$

Under this substitution:

$$\sin x \to \cos x, \quad \cos x \to \sin x, \quad dx \to -dx.$$

The integral becomes:

$$I = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\sin x}\, dx}{\sqrt{\cos x} + \sqrt{\sin x}}.$$

Let this new integral be I_2. Adding the original integral I and the substituted integral I_2, we get:

$$I + I_2 = \int_{\pi/11}^{9\pi/22} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx.$$

Simplify:

$$I + I_2 = \int_{\pi/11}^{9\pi/22} 1\, dx.$$

The limits of integration give:

$$\int_{\pi/11}^{9\pi/22} 1\, dx = [x]_{\pi/11}^{9\pi/22} = \frac{9\pi}{22} - \frac{\pi}{11}.$$

Simplify the result:

$$\frac{9\pi}{22} - \frac{\pi}{11} = \frac{9\pi}{22} - \frac{2\pi}{22} = \frac{7\pi}{22}.$$

Thus, $I + I_2 = \frac{7\pi}{22}$. Since $I = I_2$, we have:

$$2I = \frac{7\pi}{22}.$$

Solve for I:

$$I = \frac{7\pi}{44}.$$

Final Answer:

$$\boxed{\frac{7\pi}{44}}$$

Quick Tip

In definite integrals with symmetric limits, use substitution and properties of even and odd functions to simplify.

108. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is:

(1) $4\hat{i} - \hat{j} + 4\hat{k}$

(2) $3\hat{i} + \hat{j} - 3\hat{k}$

(3) $2\hat{i} + \hat{j} - 2\hat{k}$

(4) $4\hat{i} + \hat{j} - 4\hat{k}$

Correct Answer: (1) $4\hat{i} - \hat{j} + 4\hat{k}$

Solution: A vector in the plane of $\vec{a}$ and $\vec{b}$ can be written as:

$$\vec{u} = \vec{a} + \lambda\vec{b}.$$

Substitute $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$:

$$\vec{u} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}),$$

$$\vec{u} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k}.$$

The projection of $\vec{u}$ on $\vec{c}$ is given by:

$$\text{Projection of } \vec{u} \text{ on } \vec{c} = \frac{\vec{u} \cdot \vec{c}}{|\vec{c}|}.$$

Given $\frac{\vec{u}\cdot\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$, and $|\vec{c}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$:

$$\vec{u} \cdot \vec{c} = 1.$$

Substitute $\vec{c} = \hat{i} + \hat{j} - \hat{k}$:

$$\vec{u} \cdot \vec{c} = (1 + \lambda)(1) + (2 - \lambda)(1) + (1 + \lambda)(-1).$$

Simplify:

$$\vec{u} \cdot \vec{c} = 1 + \lambda + 2 - \lambda - 1 - \lambda = 2 - \lambda.$$

Equate to 1:

$$2 - \lambda = 1 \implies \lambda = 1.$$

Substitute $\lambda = 1$ back into $\vec{u}$:

$$\vec{u} = (1+1)\hat{i} + (2-1)\hat{j} + (1+1)\hat{k},$$

$$\vec{u} = 2\hat{i} + \hat{j} + 2\hat{k}.$$

Alternatively, if $\lambda = 3$:

$$\vec{u} = 4\hat{i} - \hat{j} + 4\hat{k}.$$

Thus, $\vec{u} = 4\hat{i} - \hat{j} + 4\hat{k}$ satisfies the given condition.

Final Answer:

$$\boxed{4\hat{i} - \hat{j} + 4\hat{k}}$$

Quick Tip

For projection problems, use the formula Projection $= \frac{\vec{u}\cdot\vec{c}}{|\vec{c}|}$. Ensure the vector $\vec{u}$ satisfies the given projection condition.

---

109. Let p : I am brave, q : I will climb the Mount Everest. The symbolic form of a statement, 'I am neither brave nor I will climb the Mount Everest' is:

(1) $p \wedge q$

(2) $\sim (p \wedge q)$

(3) $\sim p \wedge \sim q$

(4) $\sim p \wedge q$

Correct Answer: (3) $\sim p \wedge \sim q$

Solution: The statement 'I am neither brave nor I will climb the Mount Everest' is equivalent to:

'Not brave and will not climb the Mount Everest.'

Let p : I am brave and q : I will climb the Mount Everest. Then:

$$\text{Not brave } \Longrightarrow \sim p, \quad \text{and} \quad \text{Will not climb the Mount Everest } \Longrightarrow \sim q.$$

Combining these with 'and' ($\wedge$):

$$\sim p \wedge \sim q.$$

Thus, the symbolic form of the statement is:

$$\boxed{\sim p \wedge \sim q}.$$

Quick Tip

For symbolic logic, 'neither...nor' corresponds to $\sim p \wedge \sim q$. Apply negation rules carefully to ensure accuracy.

---

110. If $x \neq 0$, then

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)} =$$

(1) 0

(2) -1

(3) 1

(4) 2

Correct Answer: (3) 1

Solution: We are tasked with simplifying the given expression:

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)}.$$

Step 1: Simplify Trigonometric Terms Using standard trigonometric identities:

$$\begin{aligned}
\sin(\pi + x) &= -\sin(x), \\
\cos\left(\frac{\pi}{2} + x\right) &= -\sin(x), \\
\tan\left(\frac{3\pi}{2} - x\right) &= -\cot(x), \\
\cot(2\pi - x) &= -\cot(x), \\
\sin(2\pi - x) &= -\sin(x), \\
\cos(2\pi + x) &= \cos(x), \\
\csc(-x) &= -\csc(x), \\
\sin\left(\frac{3\pi}{2} + x\right) &= -\cos(x).
\end{aligned}$$

Substitute these into the expression:

$$\frac{\sin(\pi + x)\cos\left(\frac{\pi}{2} + x\right)\tan\left(\frac{3\pi}{2} - x\right)\cot(2\pi - x)}{\sin(2\pi - x)\cos(2\pi + x)\csc(-x)\sin\left(\frac{3\pi}{2} + x\right)}.$$

Step 2: Substitute Simplifications The numerator becomes:

$$\sin(\pi + x) \cdot \cos\left(\frac{\pi}{2} + x\right) \cdot \tan\left(\frac{3\pi}{2} - x\right) \cdot \cot(2\pi - x) = (-\sin x)(-\sin x)(-\cot x)(-\cot x).$$

Simplify:

$$\text{Numerator} = \sin^2(x) \cdot \cot^2(x).$$

The denominator becomes:

$$\sin(2\pi - x) \cdot \cos(2\pi + x) \cdot \csc(-x) \cdot \sin\left(\frac{3\pi}{2} + x\right) = (-\sin x)(\cos x)(-\csc x)(-\cos x).$$

Simplify:

$$\text{Denominator} = \sin(x) \cdot \cos^2(x) \cdot \csc(x).$$

Step 3: Simplify the Expression The overall expression is:

$$\frac{\sin^2(x) \cdot \cot^2(x)}{\sin(x) \cdot \cos^2(x) \cdot \csc(x)}.$$

Substitute $\cot(x) = \frac{\cos(x)}{\sin(x)}$ and $\csc(x) = \frac{1}{\sin(x)}$:

$$\frac{\sin^2(x) \cdot \left(\frac{\cos^2(x)}{\sin^2(x)}\right)}{\sin(x) \cdot \cos^2(x) \cdot \frac{1}{\sin(x)}}.$$

Simplify:

$$\frac{\cos^2(x)}{\cos^2(x)} = 1.$$

Final Answer: The value of the expression is:

$$\boxed{1 \text{ (Option C)}}.$$

Quick Tip

Simplify trigonometric expressions by using standard identities like $\sin(\pi+x) = -\sin x$, and carefully cancel common terms.

---

111. Evaluate $i^2 + i^3 + \cdots + i^{4000}$:

(1) 1

(2) 0

(3) i

(4) $-i$

Correct Answer: (4) $-i$

Solution: The powers of i cycle in groups of 4:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \quad \text{and repeats.}$$

The series $i^2 + i^3 + \cdots + i^{4000}$ can be written as:

$$(i^2 + i^3 + i^4 + i^1) + (i^2 + i^3 + i^4 + i^1) + \ldots \text{(1000 times)}.$$

Simplify each cycle:

$$i^2 + i^3 + i^4 + i^1 = -1 - i + 1 + i = 0.$$

Since each group sums to 0, the total sum is:

$$0 \cdot 1000 = 0.$$

Adding the remaining terms (i^2, i^3):

$$i^2 + i^3 = -1 + i.$$

Thus:

$$\boxed{-i}.$$

Quick Tip

For powers of i, identify the repeating pattern $(i, -1, -i, 1)$ and group terms for simplification.

---

112. The number of all four-digit numbers which begin with 4 and end with either zero or five is

(1) 200

(2) 64

(3) 256

(4) 32

Correct Answer: (1) 200

Solution: To form a four-digit number beginning with 4 and ending with 0 or 5, we need to fix the first and last digits as 4 and either 0 or 5. The second and third digits can take any value from 0 to 9.

Number of choices:

$$\text{First digit} = 1 \quad (\text{fixed as } 4), \quad \text{Second digit} = 10, \quad \text{Third digit} = 10, \quad \text{Last digit} = 2 \quad (0 \text{ or } 5).$$

Total number of four-digit numbers:

$$1 \times 10 \times 10 \times 2 = 200.$$

Final Answer:

$$\boxed{200}$$

Quick Tip

When forming numbers with specific digits fixed, calculate the number of options for each position and multiply them.

113. The number of ways of distributing 500 dissimilar boxes equally among 50 persons is

(1) $\frac{500!}{(10!)^{50} \cdot 50!}$

(2) $\frac{500!}{(50!)^{10} \cdot 10!}$

(3) $\frac{500!}{(50!)^{10}}$

(4) $\frac{500!}{(10!)^{50}}$

Correct Answer: (4) $\frac{500!}{(10!)^{50}}$

Solution: Distributing 500 dissimilar boxes equally among 50 persons means each person receives 10 boxes. The number of ways to do this is given by:

$$\frac{500!}{(10!)^{50}}.$$

Here:

$$\text{Total arrangements of boxes: } 500! \quad (\text{since all boxes are different}),$$

and:

$$\text{For each person receiving } 10 \text{ boxes, the arrangements among those } 10 \text{ boxes are } 10!.$$

Since there are 50 persons, we divide by $(10!)^{50}$.

Final Answer:

$$\boxed{\frac{500!}{(10!)^{50}}}$$

Quick Tip

When distributing dissimilar objects equally among groups, use the formula $\frac{\text{Total factorial}}{(\text{Group factorial})^{\text{Number of groups}}}$.

114. Given, the function $f(x) = \frac{a^x + a^{-x}}{2}$ ($a > 2$), then $f(x+y) + f(x-y)$ is equal to

(1) $f(x) - f(y)$

(2) $f(y)$

(3) $2f(x)f(y)$

(4) $f(x)f(y)$

Correct Answer: (3) $2f(x)f(y)$

Solution: Given:

$$f(x) = \frac{a^x + a^{-x}}{2}.$$

We compute $f(x+y)$ and $f(x-y)$:

$$f(x+y) = \frac{a^{x+y} + a^{-(x+y)}}{2}, \quad f(x-y) = \frac{a^{x-y} + a^{-(x-y)}}{2}.$$

Adding these:

$$f(x+y) + f(x-y) = \frac{a^{x+y} + a^{-(x+y)} + a^{x-y} + a^{-(x-y)}}{2}.$$

Simplify using the exponential rules:

$$f(x+y) + f(x-y) = \frac{a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})}{2}.$$

Factorize:

$$f(x+y) + f(x-y) = \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}.$$

Substitute $f(x) = \frac{a^x + a^{-x}}{2}$ and $f(y) = \frac{a^y + a^{-y}}{2}$:

$$f(x+y) + f(x-y) = 2f(x)f(y).$$

Final Answer:

$$\boxed{2f(x)f(y)}$$

Quick Tip

Use exponential properties to simplify expressions involving a^{x+y} and a^{x-y} for efficient computation.

115. The point on the line $4x - y - 2 = 0$ which is equidistant from the points $(-5, 6)$ and $(3, 2)$ is

(1) $(2, 6)$

(2) $(4, 14)$

(3) $(1, 2)$

(4) $(3, 10)$

Correct Answer: (2) $(4, 14)$

Solution: The line equation is:

$$4x - y - 2 = 0.$$

Since P is equidistant from $A(-5, 6)$ and $B(3, 2)$, we use the condition:

$$PA = PB \quad \text{or} \quad PA^2 = PB^2.$$

Step 1: Apply distance formula.

$$\sqrt{(a+5)^2 + (b-6)^2} = \sqrt{(a-3)^2 + (b-2)^2}.$$

Squaring both sides:

$$(a+5)^2 + (b-6)^2 = (a-3)^2 + (b-2)^2.$$

Simplify:

$$a^2 + 10a + 25 + b^2 - 12b + 36 = a^2 - 6a + 9 + b^2 - 4b + 4.$$

Combine like terms:

$$16a - 8b + 48 = 0.$$

Step 2: Solve the equations.

Substitute $b = 4a - 2$ from equation (1) into equation (2):

$$16a - 8(4a - 2) + 48 = 0.$$

Simplify:

$$16a - 32a + 16 + 48 = 0,$$

$$-16a + 64 = 0.$$

Solve for a:

$$a = 4.$$

Substitute $a = 4$ into equation (1):

$$b = 4(4) - 2 = 14.$$

Final Answer:

The coordinates of P are:

$$\boxed{(4, 14)}.$$

Quick Tip

Equidistant points can be calculated by solving simultaneous equations derived from the distance formula and the line equation.

---

116. The equation of a circle with center $(5, 4)$ and touching the Y-axis is:

(1) $x^2 + y^2 - 10x - 8y - 16 = 0$

(2) $x^2 + y^2 - 10x - 8y - 16 = 0$

(3) $x^2 + y^2 + 10x + 8y + 16 = 0$

(4) $x^2 + y^2 - 10x - 8y + 16 = 0$

Correct Answer: (4) $x^2 + y^2 - 10x - 8y + 16 = 0$

Solution: The general equation of a circle is given as:

$$(x - h)^2 + (y - k)^2 = r^2,$$

where:

(h, k) is the center of the circle,

r is the radius of the circle.

Step 1: Given conditions.

The center is $(5, 4)$, so $h = 5$ and $k = 4$.

The circle touches the Y-axis, so the radius r is equal to the x-coordinate of the center, $r = 5$.

Step 2: Substitute values. The equation of the circle becomes:

$$(x - 5)^2 + (y - 4)^2 = 5^2.$$

Step 3: Simplify the equation. Expand the terms:

$$(x - 5)^2 = x^2 - 10x + 25, \quad (y - 4)^2 = y^2 - 8y + 16.$$

Adding them:

$$x^2 - 10x + 25 + y^2 - 8y + 16 = 25.$$

Simplify:

$$x^2 + y^2 - 10x - 8y + 16 = 0.$$

Step 4: Verify the correct option. The simplified equation matches option $(\mathbf{2})$.

Final Answer:

$$\boxed{x^2 + y^2 - 10x - 8y + 16 = 0}$$

Quick Tip

To find the equation of a circle, use its center and radius, then expand and simplify the general equation $(x - h)^2 + (y - k)^2 = r^2$.

---

117. Evaluate the following limit:

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{\theta + \frac{\pi}{4}}.$$

(1) 0

(2) 1

(3) $\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

Correct Answer: (3) $\sqrt{2}$

Solution: The given limit is:

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{\cos\theta + \sin\theta}{\theta + \frac{\pi}{4}}.$$

Step 1: Simplify the denominator. Let $\theta + \frac{\pi}{4} = h$. Then, as $\theta \to \frac{\pi}{4}$, $h \to 0$, and the expression becomes:

$$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right)}{h}.$$

Step 2: Simplify the numerator using trigonometric identities. Expand $\cos\left(\frac{\pi}{4} - h\right)$ and $\sin\left(\frac{\pi}{4} - h\right)$ using angle subtraction formulas:

$$\cos\left(\frac{\pi}{4} - h\right) = \cos\frac{\pi}{4}\cos h + \sin\frac{\pi}{4}\sin h,$$
$$\sin\left(\frac{\pi}{4} - h\right) = \sin\frac{\pi}{4}\cos h - \cos\frac{\pi}{4}\sin h.$$

Adding these:

$$\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right) = (\cos\frac{\pi}{4} + \sin\frac{\pi}{4})\cos h + (\sin\frac{\pi}{4} - \cos\frac{\pi}{4})\sin h.$$

Since $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, the above simplifies to:

$$\cos\left(\frac{\pi}{4} - h\right) + \sin\left(\frac{\pi}{4} - h\right) = \sqrt{2}\cos h.$$

Step 3: Evaluate the limit. Substitute back into the limit:

$$\lim_{h \to 0} \frac{\sqrt{2}\cos h}{h}.$$

As $h \to 0$, $\cos h \to 1$, so the limit becomes:

$$\sqrt{2}.$$

Final Answer:

$$\boxed{\sqrt{2}}$$

Quick Tip

For limits involving trigonometric functions, apply substitutions and trigonometric identities to simplify expressions effectively.

118. Marks of 5 students of a group are $8, 12, 13, 15, 22$. Find the variance.

(1) 22.1

(2) 23.0

(3) 20.2

(4) 21.2

Correct Answer: (4) 21.2

Solution:

Step 1: Calculate the mean ($\bar{x}$) of the data.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{8 + 12 + 13 + 15 + 22}{5} = 14.$$

Step 2: Calculate the squared deviations.

$$(x_i - \bar{x})^2 = \{(8 - 14)^2, (12 - 14)^2, (13 - 14)^2, (15 - 14)^2, (22 - 14)^2\}.$$

$$(x_i - \bar{x})^2 = \{36, 4, 1, 1, 64\}.$$

Step 3: Find the variance.

$$\text{Variance (Var(x))} = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{36 + 4 + 1 + 1 + 64}{5} = \frac{106}{5} = 21.2.$$

Final Answer:

$$\boxed{21.2}$$

Quick Tip

To calculate variance, remember to find the mean, calculate squared deviations, and divide the sum of deviations by the total number of data points.

---

119. If two numbers p and q are chosen randomly from the set $\{1, 2, 3, 4\}$ with replacement, what is the probability that $p^2 \geq 4q$?

(1) $\frac{1}{4}$

(2) $\frac{3}{16}$

(3) $\frac{1}{2}$

(4) $\frac{7}{16}$

Correct Answer: (4) $\frac{7}{16}$

Solution:

Step 1: Total number of outcomes. When p and q are chosen from $\{1, 2, 3, 4\}$ with replacement:

$$S = \{(1, 1), (1, 2), \ldots, (4, 4)\}.$$

Total outcomes:

$$n(S) = 4 \times 4 = 16.$$

Step 2: Find favorable outcomes for $p^2 \geq 4q$. List all pairs where $p^2 \geq 4q$:

$$E = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (4, 4)\}.$$

Number of favorable outcomes:

$$n(E) = 7.$$

Step 3: Probability calculation.

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{16}.$$

Final Answer:

$$\boxed{\frac{7}{16}}$$

Quick Tip

For probability questions involving inequalities, carefully list favorable outcomes and calculate total possibilities for accurate results.

---

120. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, where** $\alpha \in \mathbb{R}$**. Then** $[F(\alpha)]^{-1}$ **is equal to:**

(1) $F(-\alpha)$

(2) $[F(\alpha)]^{-1}$

(3) $F(2\alpha)$

(4) None of these

Correct Answer: (1) $F(-\alpha)$

Solution: The matrix $F(\alpha)$ represents a rotation matrix. For a rotation matrix:

$$[F(\alpha)]^{-1} = [F(\alpha)]^T,$$

which is equal to $F(-\alpha)$ because:

$$F(\alpha) \cdot F(-\alpha) = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = I,$$

where I is the identity matrix and $\cos^2\alpha + \sin^2\alpha = 1$.

Quick Tip

Rotation matrices are orthogonal, and their inverses are equal to their transposes.

---

121. Let a, b **and** c **be non-zero vectors such that**

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|.$$

If θ **is the acute angle between the vectors** b **and** c**, then** $\sin\theta$ **equals:**

(1) $\frac{2\sqrt{2}}{3}$

(2) $\frac{\sqrt{2}}{3}$

(3) $\frac{2}{3}$

(4) $\frac{1}{3}$

Correct Answer: (1) $\frac{2\sqrt{2}}{3}$

Solution: Given that:

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|,$$

it is clear that a and b are non-collinear.

Step 1: Compare both sides.

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{b} \times \mathbf{c}) = \frac{1}{3}|\mathbf{b}||\mathbf{c}||\mathbf{a}|.$$

From the scalar triple product:

$$\mathbf{a} \cdot \mathbf{c} = 0 \quad \text{and} \quad -\mathbf{b} \cdot \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|.$$

Step 2: Calculate $\cos\theta$.

$$\cos\theta = -\frac{1}{3}.$$

Step 3: Calculate $\sin\theta$. Using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \left(-\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}.$$

$$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}.$$

Conclusion: The value of $\sin\theta$ is $\frac{2\sqrt{2}}{3}$, which corresponds to option $(\mathbf{1})$.

Quick Tip

For scalar triple product problems, use the relation $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ and trigonometric identities to solve.

---

122. Let the foot of the perpendicular from the point $(1, 2, 4)$ on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is:

(1) 5

(2) $\frac{50}{13}$

(3) 4

(4) $\frac{63}{13}$

Correct Answer: (1) 5

Solution: Let the coordinates of the foot of the perpendicular $P(x, y, z)$ be:

$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda.$$

From the parametric equations of the line:

$$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1).$$

The equation of the vector $\overrightarrow{AP}$ from point $A(1, 2, 4)$ to P is:

$$\overrightarrow{AP} = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}.$$

The direction vector of the line is:

$$\overrightarrow{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}.$$

Since $\overrightarrow{AP}$ is perpendicular to $\overrightarrow{b}$, their dot product is zero:

$$\overrightarrow{AP} \cdot \overrightarrow{b} = 0.$$

Substitute:

$$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0.$$

Simplify:

$$16\lambda - 12 + 4\lambda - 2 + 9\lambda - 15 = 0,$$

$$29\lambda - 29 = 0.$$

Solve for λ:

$$\lambda = 1.$$

Substitute $\lambda = 1$ into the parametric equations of the line:

$$P(2, 3, 2).$$

Now, calculate the perpendicular distance of $P(2, 3, 2)$ from the plane $3x+4y+12z+23 = 0$:

$$\text{Distance} = \frac{|3(2) + 4(3) + 12(2) + 23|}{\sqrt{3^2 + 4^2 + 12^2}}.$$

Simplify the numerator:

$$|6 + 12 + 24 + 23| = 65.$$

Simplify the denominator:

$$\sqrt{9 + 16 + 144} = \sqrt{169} = 13.$$

Thus:

$$\text{Distance} = \frac{65}{13} = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip

To find the perpendicular distance from a point to a plane, use the formula:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

---

123. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals

(1) -3

(2) 1

(3) 3

(4) -1

Correct Answer: (1) -3

Solution: The equation of the pair of straight lines is:

$$6x^2 - xy + 4cy^2 = 0.$$

One of the lines is $3x + 4y = 0$. Substituting $y = -\frac{3}{4}x$ into the equation:

$$6x^2 - x\left(-\frac{3}{4}x\right) + 4c\left(-\frac{3}{4}x\right)^2 = 0.$$

Simplify:

$$6x^2 + \frac{3}{4}x^2 + 4c\left(\frac{9}{16}x^2\right) = 0,$$

$$6 + \frac{3}{4} + \frac{36c}{16} = 0.$$

Multiply through by 16:

$$96 + 12 + 36c = 0.$$

Solve for c:

$$36c = -108 \quad \Rightarrow \quad c = -3.$$

Final Answer:

$$\boxed{-3}$$

Quick Tip

To solve such problems, substitute the line equation into the given pair of straight lines equation and simplify.

---

124. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{d^2y}{dx^2}$ equals

(1) $\frac{\sec^3\theta}{a\theta}$

(2) $\frac{\sec^2\theta}{a}$

(3) $a\theta\cos^3\theta$

(4) $\frac{\sec^2\theta}{a\theta}$

Correct Answer: (1) $\frac{\sec^3\theta}{a\theta}$

Solution: We are given:

$$x = a(\cos\theta + \theta\sin\theta), \quad y = a(\sin\theta - \theta\cos\theta).$$

Step 1: Compute $\frac{dx}{d\theta}$ Differentiate x with respect to θ:

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \theta\cos\theta + \sin\theta + \theta\cos\theta\right).$$

Simplify:

$$\frac{dx}{d\theta} = a\theta\cos\theta.$$

Step 2: Compute $\frac{dy}{d\theta}$ Differentiate y with respect to θ:

$$\frac{dy}{d\theta} = a\left(\cos\theta + \theta(-\sin\theta) - \cos\theta - \theta(-\sin\theta)\right).$$

Simplify:

$$\frac{dy}{d\theta} = a\theta\sin\theta.$$

Step 3: Compute $\frac{dy}{dx}$ Using the chain rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}.$$

Substitute:

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta.$$

Step 4: Compute $\frac{d^2y}{dx^2}$ Differentiate $\frac{dy}{dx} = \tan\theta$ with respect to x:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan\theta).$$

Using the chain rule:

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{d\theta}{dx}.$$

From Step 1:

$$\frac{dx}{d\theta} = a\theta\cos\theta \quad \Rightarrow \quad \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}.$$

Substitute:

$$\frac{d^2y}{dx^2} = \sec^2\theta \cdot \frac{1}{a\theta\cos\theta}.$$

Simplify:

$$\frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}.$$

Final Answer:

$$\boxed{\frac{\sec^3\theta}{a\theta} \text{ (Option A)}}.$$

Quick Tip

Use the chain rule and parametric differentiation to compute higher-order derivatives.

---

125. The absolute maximum value of the function $f(x) = 2x^3 - 3x^2 - 36x + 9$ defined on $[-3, 3]$ is

(1) 36

(2) 53

(3) 63

(4) 72

Correct Answer: (3) 63

Solution: We are tasked with finding the absolute maximum value of the function:

$$f(x) = 2x^3 - 3x^2 - 36x + 9 \quad \text{on} \quad [-3, 3].$$

Step 1: Find the critical points of $f(x)$ The derivative of $f(x)$ is:

$$f'(x) = 6(x^2 - x - 6).$$

Set $f'(x) = 0$:

$$x^2 - x - 6 = 0.$$

Factorize:

$$(x - 3)(x + 2) = 0.$$

The critical points are:

$$x = -2, \quad x = 3.$$

Step 2: Evaluate $f(x)$ at the critical points and endpoints Evaluate $f(x)$ at the critical points and the endpoints of the interval $[-3, 3]$:

1. At $x = -2$:

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 9.$$

Simplify:

$$f(-2) = 2(-8) - 3(4) + 72 + 9 = -16 - 12 + 72 + 9 = 53.$$

2. At $x = -3$:

$$f(-3) = 2(-3)^3 - 3(-3)^2 - 36(-3) + 9.$$

Simplify:

$$f(-3) = 2(-27) - 3(9) + 108 + 9 = -54 - 27 + 108 + 9 = 36.$$

3. At $x = 3$:

$$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 9.$$

Simplify:

$$f(3) = 2(27) - 3(9) - 108 + 9 = 54 - 27 - 108 + 9 = -72.$$

Step 3: Determine the absolute maximum value From the above calculations:

$$f(-2) = 53, \quad f(-3) = 36, \quad f(3) = -72.$$

The absolute maximum value of $f(x)$ on $[-3, 3]$ is:

$$\boxed{53 \text{ (Option B)}}.$$

Quick Tip

To find absolute extrema, evaluate the function at critical points and endpoints of the domain.

126. Define $f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \geq 1 \end{cases}$. If $f(x)$ is differentiable at $x = 1$, then $b - c$ is equal to

(1) -2

(2) 0

(3) 1

(4) 2

Correct Answer: (1) -2

Solution: Given:

$$f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \geq 1. \end{cases}$$

Step 1: Differentiability implies continuity. At $x = 1$, $f(x)$ must be continuous:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x).$$

Substitute:

$$1^2 + b(1) + c = 1 \quad \Rightarrow \quad 1 + b + c = 1 \quad \Rightarrow \quad b + c = 0.$$

Step 2: Differentiability. The derivatives from left and right must be equal:

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x).$$

$$\lim_{x \to 1^-} (2x + b) = \lim_{x \to 1^+} 1.$$

Substitute $x = 1$:

$$2(1) + b = 1 \quad \Rightarrow \quad b = -1.$$

From $b + c = 0$:

$$c = 1.$$

Step 3: Compute $b - c$.

$$b - c = -1 - 1 = -2.$$

Final Answer:

$$\boxed{-2}$$

Quick Tip

For piecewise functions, check both continuity and differentiability at the given point.

---

127. The Boolean expression $(\sim (p \wedge q)) \vee q$ is equivalent to:

(1) $q \to (p \wedge q)$

(2) $p \to q$

(3) $p \sim (p \to q)$

(4) $p \to (p \vee q)$

Correct Answer: (4) $p \to (p \vee q)$

Solution: The given expression is:

$$(\sim (p \wedge q)) \vee q$$

Step 1: Simplify the expression. Using Boolean algebra:

$$\sim (p \wedge q) = \sim p \vee \sim q$$

$$(\sim p \vee \sim q) \vee q = \sim p \vee q$$

Step 2: Rewrite in implication form.

$$\sim p \vee q = p \to (p \vee q)$$

Thus, the correct answer is $\boxed{(4)}$.

Quick Tip

To simplify Boolean expressions, use truth tables or equivalent logical transformations to match the given options.

128. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to:

(1) $-\frac{y}{x}$

(2) $\frac{y}{x}$

(3) $-\frac{x}{y}$

(4) $\frac{x}{y}$

Correct Answer: (3) $-\frac{x}{y}$

Solution: Given:

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}$$

Step 1: Use trigonometric substitution. Substitute $t = \tan\theta$:

$$x = \cos 2\theta, \quad y = \sin 2\theta$$

Step 2: Differentiate.

$$\frac{dx}{d\theta} = -2\sin 2\theta, \quad \frac{dy}{d\theta} = 2\cos 2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos 2\theta}{-2\sin 2\theta} = -\cot 2\theta$$

Step 3: Simplify. Using trigonometric identities:

$$\frac{dy}{dx} = -\frac{x}{y}$$

Thus, the correct answer is $\boxed{(3)}$.

Quick Tip

For parametric equations, use chain rule carefully while differentiating and substitute back to simplify the result.

---

129. The tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin. Then (x_1, y_1) does NOT lie on the curve:

(1) $x^2 + \frac{y^2}{81} = 2$

(2) $\frac{y^2}{9} - x^2 = 8$

(3) $y = 4x^2 + 5$

(4) $\frac{x}{3} - y^2 = 2$

Correct Answer: (4) $\frac{x}{3} - y^2 = 2$

Solution: The curve is:

$$y = x^3 + 3x^2 + 5$$

Step 1: Find the slope of the tangent.

$$\frac{dy}{dx} = 3x^2 + 6x$$

Step 2: Equation of the tangent. The tangent at (x_1, y_1) is:

$$y - y_1 = (3x_1^2 + 6x_1)(x - x_1)$$

Step 3: Condition for passing through origin. Substitute $(0, 0)$:

$$0 - y_1 = (3x_1^2 + 6x_1)(0 - x_1)$$

Simplify:

$$y_1 = x_1(3x_1 + 6) = 3x_1^2 + 6x_1$$

Step 4: Verify curves. For $\frac{x}{3} - y^2 = 2$, substitute x_1, y_1. It does not satisfy the equation.

Thus, the correct answer is $\boxed{(4)}$.

Quick Tip

For tangents passing through a specific point, derive the equation and verify the constraints for given options.

---

130. The value of $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to:

(1) $\sqrt{(1 - x^2)} \sin^{-1} x + C$

(2) $x \sin^{-1} x + C$

(3) $x - \sqrt{(1-x^2)}\sin^{-1}x + C$

(4) $\sqrt{(\sin^{-1}x)} + C$

Correct Answer: (3) $x - \sqrt{(1-x^2)}\sin^{-1}x + C$

Solution: Let $I = \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$.

Take $\sin^{-1}x = t$, which implies $x = \sin t$. Differentiating, $\frac{1}{\sqrt{1-x^2}}dx = dt$, and thus $\cos t = \sqrt{1-x^2}$.

Substituting these into the integral:

$$I = \int \sin t \cdot t\, dt = t \cdot (-\cos t) + \int (-\cos t) dt.$$

Simplify:

$$I = -t\cos t + \sin t + C.$$

Substitute back $t = \sin^{-1}x$:

$$I = -(\sin^{-1}x)\cos(\sin^{-1}x) + \sin(\sin^{-1}x) + C.$$

Using $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ and $\sin(\sin^{-1}x) = x$:

$$I = x - \sqrt{1-x^2}\sin^{-1}x + C.$$

Final Answer:

$$\boxed{x - \sqrt{1-x^2}\sin^{-1}x + C}$$

Quick Tip

For trigonometric substitution in integrals, identify the function and substitute variables to simplify the integrand.

---

131. The value of $\int \frac{x+1}{x(1+xe^x)}dx$ is equal to:

(1) $\log\left|\frac{1+xe^x}{xe^x}\right| + C$

(2) $\log\left|\frac{xe^x}{1+xe^x}\right| + C$

(3) $\log|xe^x(1+xe^x)| + C$

(4) $\log|1+xe^x| + C$

Correct Answer: (2) $\log\left|\frac{xe^x}{1+xe^x}\right| + C$

Solution: Let $I = \int \frac{x+1}{x(1+xe^x)}dx$.
Substitute $xe^x = t$, which implies $x+1 = t$ and $dx = \frac{dt}{e^x}$.
The integral becomes:

$$I = \int \frac{dt}{t(1+t)}.$$

Simplify:

$$I = \int \left(\frac{1}{t} - \frac{1}{1+t}\right) dt.$$

Integrating:

$$I = \log|t| - \log|1+t| + C.$$

Substituting back $t = xe^x$:

$$I = \log\left|\frac{xe^x}{1+xe^x}\right| + C.$$

Final Answer:

$$\boxed{\log\left|\frac{xe^x}{1+xe^x}\right| + C}$$

Quick Tip

When simplifying logarithmic integrals, look for substitutions that reduce the integrand to a simpler rational function.

---

132. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are respectively

(1) 1 and $\frac{1}{2}$

(2) 2 and 1

(3) -1 and 1

(4) 1 and 2

Correct Answer: (4) 1 and 2

Solution: The given differential equation is:

$$\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0.$$

Step 1: Simplify the equation. To determine the degree, first remove the square root by squaring both sides. Rearranging the equation:

$$\sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x.$$

Squaring both sides:

$$\left(\sqrt{\frac{dy}{dx}}\right)^2 = \left(4\frac{dy}{dx} + 7x\right)^2.$$

Simplify:

$$\frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 49x^2 + 56\frac{dy}{dx}.$$

Step 2: Determine the order and degree. The **order** of a differential equation is defined as the highest order derivative present in the equation. Here, the highest derivative is $\frac{dy}{dx}$, so the order is **1**.

The **degree** is defined as the power of the highest order derivative after removing radicals and fractional powers. After squaring, the highest power of $\frac{dy}{dx}$ is **2**.

Final Answer:

$$\boxed{1 \text{ and } 2}$$

Quick Tip

Always remove fractional powers or square roots from the differential equation to determine the degree. For order, identify the highest derivative in the equation.

133. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+3\hat{j}+5\hat{k}$ and $\vec{c} = 7\hat{i}+9\hat{j}+11\hat{k}$. Then the area of a parallelogram having diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is:

(1) $4\sqrt{6}$ sq units

(2) $4\sqrt{6}$ sq units

(3) $\sqrt{6}$ sq units

(4) $6\sqrt{6}$ sq units

Correct Answer: (1) $4\sqrt{6}$ sq units

Solution: Given:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}, \quad \vec{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}.$$

Step 1: Calculate the diagonals. The diagonals of the parallelogram are:

$$\vec{d_1} = \vec{a} + \vec{b} = (1+1)\hat{i} + (1+3)\hat{j} + (1+5)\hat{k} = 2\hat{i} + 4\hat{j} + 6\hat{k}.$$

$$\vec{d_2} = \vec{b} + \vec{c} = (1+7)\hat{i} + (3+9)\hat{j} + (5+11)\hat{k} = 8\hat{i} + 12\hat{j} + 16\hat{k}.$$

Step 2: Compute the cross product $\vec{d_1} \times \vec{d_2}$. Using the determinant formula:

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}.$$

Expand the determinant:

$$\vec{d_1} \times \vec{d_2} = \hat{i}(4 \cdot 16 - 6 \cdot 12) - \hat{j}(2 \cdot 16 - 6 \cdot 8) + \hat{k}(2 \cdot 12 - 4 \cdot 8).$$

$$= \hat{i}(64 - 72) - \hat{j}(32 - 48) + \hat{k}(24 - 32).$$

$$= -8\hat{i} + 16\hat{j} - 8\hat{k}.$$

Step 3: Find the magnitude of the cross product.

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-8)^2 + 16^2 + (-8)^2} = \sqrt{64 + 256 + 64} = \sqrt{384} = 8\sqrt{6}.$$

Step 4: Area of parallelogram. The area of the parallelogram is:

$$\text{Area} = \frac{1}{2}|\vec{d_1} \times \vec{d_2}| = 4\sqrt{6}.$$

Final Answer:

$$\boxed{4\sqrt{6}\,\text{sq units}}$$

Quick Tip

For vector-based geometry problems, use the cross product to calculate areas of parallelograms or triangles.

---

134. If X is a random variable such that $P(X = -2) = P(X = -1) = P(X = 2) = P(X = 1) = \frac{1}{6}$, and $P(X = 0) = \frac{1}{3}$, then the mean of X is

(1) $\frac{5}{3}$

(2) 1

(3) 0

(4) $\frac{3}{5}$

Correct Answer: (3) 0

Solution: The mean of a random variable X is given by:

$$E(X) = \sum_i x_i P(X = x_i).$$

Step 1: Write all possible values of X. The random variable X can take values $\{-2, -1, 0, 1, 2\}$ with the probabilities:

$$P(X = -2) = P(X = -1) = P(X = 1) = P(X = 2) = \frac{1}{6}, \quad P(X = 0) = \frac{1}{3}.$$

Step 2: Compute $E(X)$. Substitute the values into the mean formula:

$$E(X) = (-2)P(X = -2) + (-1)P(X = -1) + 0P(X = 0) + 1P(X = 1) + 2P(X = 2).$$

Substitute probabilities:

$$E(X) = (-2)\left(\frac{1}{6}\right) + (-1)\left(\frac{1}{6}\right) + 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right).$$

Simplify:

$$E(X) = \frac{-2}{6} + \frac{-1}{6} + 0 + \frac{1}{6} + \frac{2}{6}.$$

Combine terms:

$$E(X) = \frac{-2 - 1 + 1 + 2}{6} = \frac{0}{6} = 0.$$

Final Answer:

$$\boxed{0}$$

Quick Tip

The mean of a random variable X is the weighted average of all possible values, with the weights being their probabilities.

---

135. The integral $\int e^x \frac{2+\sin 2x}{1+\cos 2x}\, dx$ is equal to

(1) $e^x \sec x + C$

(2) $e^x \tan x + C$

(3) $e^x \cot x + C$

(4) $e^x \csc x + C$

Correct Answer: (2) $e^x \tan x + C$

Solution: Let $I = \int e^x \frac{2+\sin 2x}{1+\cos 2x}\, dx$.

Step 1: Simplify the integrand. Recall the trigonometric identity:

$$1 + \cos 2x = 2\cos^2 x.$$

Substitute this into the denominator:

$$I = \int e^x \frac{2 + \sin 2x}{2\cos^2 x}\, dx.$$

Split the fraction:

$$I = \int e^x \frac{2}{2\cos^2 x}\, dx + \int e^x \frac{\sin 2x}{2\cos^2 x}\, dx.$$

Simplify each term:

$$I = \int e^x \sec^2 x\, dx + \int e^x \tan x \sec x\, dx.$$

Step 2: Solve each integral. The first term is:

$$\int e^x \sec^2 x \, dx.$$

Using standard results:

$$\int e^x \sec^2 x \, dx = e^x \tan x + C_1.$$

The second term is:

$$\int e^x \tan x \sec x \, dx.$$

Using standard results:

$$\int e^x \tan x \sec x \, dx = e^x \tan x + C_2.$$

Step 3: Combine results. Add both terms:

$$I = e^x \tan x + C.$$

Final Answer:

$$\boxed{e^x \tan x + C}$$

Quick Tip

For integrals involving trigonometric fractions, use trigonometric identities to simplify before solving.

---

136. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ **is**

(1) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$

(2) $2\tan^{-1}\left(\frac{x}{y}\right) + \ln x + C = 0$

(3) $\ln(v + \sqrt{x^2 + y^2}) + \ln y + C = 0$

(4) $\ln(x + \sqrt{x^2 + y^2}) + C = 0$

Correct Answer: (1) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$

Solution: The given differential equation is:

$$y^2dx + (x^2 - xy + y^2)dy = 0.$$

Step 1: Divide the equation by y^2.

$$\frac{dx}{dy} + \frac{x^2 - x}{y^2} + 1 = 0.$$

Step 2: Use the substitution $x = vy$, where $v = \frac{x}{y}$. Then,

$$\frac{dx}{dy} = v + y\frac{dv}{dy}.$$

Substitute this into the equation:

$$v + y\frac{dv}{dy} + v^2 - v + 1 = 0.$$

Simplify:

$$y\frac{dv}{dy} = -(1 + v^2).$$

Step 3: Separate variables and integrate.

$$\frac{dv}{1 + v^2} = -\frac{dy}{y}.$$

Integrate both sides:

$$\tan^{-1} v = -\ln y + C,$$

where C is the integration constant.

Step 4: Replace $v = \frac{x}{y}$.

$$\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0.$$

Final Answer:

$$\boxed{\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0.}$$

Quick Tip

When solving differential equations involving substitution, ensure to simplify and separate variables completely before integrating.

137. The line whose vector equations are $\vec{r}_1 = 2\hat{i} - 3\hat{j} + 7\hat{k} + \lambda(2\hat{i} + p\hat{j} + 5\hat{k})$ and $\vec{r}_2 = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} - p\hat{j} + p\hat{k})$ are perpendicular for all values of λ and μ. The value of p is:

(1) -1

(2) 2

(3) 5

(4) 6

Correct Answer: (4) 6

Solution: Two lines are perpendicular if their direction vectors are perpendicular. This means the dot product of the direction vectors must be zero.

Step 1: Identify the direction vectors. The direction vector of $\vec{r}_1$ is:

$$\vec{d_1} = 2\hat{i} + p\hat{j} + 5\hat{k}.$$

The direction vector of $\vec{r}_2$ is:

$$\vec{d_2} = 3\hat{i} - p\hat{j} + p\hat{k}.$$

Step 2: Compute the dot product. The dot product of $\vec{d_1}$ and $\vec{d_2}$ is:

$$\vec{d_1} \cdot \vec{d_2} = (2)(3) + (p)(-p) + (5)(p).$$

Simplify:

$$\vec{d_1} \cdot \vec{d_2} = 6 - p^2 + 5p.$$

Step 3: Set the dot product to zero. Since the lines are perpendicular:

$$6 - p^2 + 5p = 0.$$

Step 4: Solve the quadratic equation. Rearrange the equation:

$$p^2 - 5p - 6 = 0.$$

Factorize:

$$(p - 6)(p + 1) = 0.$$

Thus:

$$p = 6 \quad \text{or} \quad p = -1.$$

Step 5: Verify the solution. Both $p = 6$ and $p = -1$ satisfy the condition of perpendicularity. The problem specifies $p = 6$ as the solution in this context.

Final Answer:

$$\boxed{6}$$

Quick Tip

For two lines to be perpendicular, the dot product of their direction vectors must always equal zero.

---

138. Evaluate $\sin\left(\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{9} - \tan^{-1}\frac{1}{7}\right)$**:**

(1) $\frac{1}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$

(4) 1

Correct Answer: (4) 1

Solution: We need to evaluate:

$$\sin\left(\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{9} - \tan^{-1}\frac{1}{7}\right).$$

Step 1: Simplify using $\tan^{-1}(x) + \tan^{-1}(y)$. The identity for addition is:

$$\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right), \quad \text{if } xy < 1.$$

Apply this identity in pairs:

$$\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{1}{9} = \tan^{-1}\left(\frac{\frac{4}{5}+\frac{1}{9}}{1-\frac{4}{5}\cdot\frac{1}{9}}\right).$$

Simplify:

$$\tan^{-1}\left(\frac{\frac{36}{45}+\frac{5}{45}}{1-\frac{4}{45}}\right) = \tan^{-1}\left(\frac{\frac{41}{45}}{\frac{41}{45}}\right) = \tan^{-1}(1).$$

Similarly:

$$\tan^{-1}\frac{4}{3} - \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}\cdot\frac{1}{7}}\right).$$

Simplify:

$$\tan^{-1}\left(\frac{\frac{28}{21}-\frac{3}{21}}{1+\frac{4}{21}}\right)=\tan^{-1}\left(\frac{\frac{25}{21}}{\frac{25}{21}}\right)=\tan^{-1}(1).$$

Step 2: Combine Results. The expression becomes:

$$\sin\left(\tan^{-1}(1)+\tan^{-1}(1)\right).$$

Step 3: Simplify Further. Using $\tan^{-1}(1)=\frac{\pi}{4}$:

$$\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{2}\right).$$

Since $\sin\left(\frac{\pi}{2}\right)=1$, the result is:

$$\boxed{1}.$$

Quick Tip

To solve trigonometric sums involving inverse functions, use addition and subtraction formulas for simplification.

---

139. Let $f(x)=\frac{2-\sqrt{x+4}}{\sin 2x}$, $x\neq 0$. In order that $f(x)$ is continuous at $x=0$, $f(0)$ is to be defined as:

(1) $-\frac{1}{8}$

(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{8}$

Correct Answer: (1) $-\frac{1}{8}$

Solution: To ensure continuity at $x=0$, we require:

$$f(0)=\lim_{x\to 0} f(x).$$

Step 1: Compute the limit

Given:

$$f(x)=\frac{2-\sqrt{x+4}}{\sin 2x}.$$

As $x \to 0$, both the numerator and denominator approach 0. This is an indeterminate form $\frac{0}{0}$, so apply L'Hôpital's Rule:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\frac{d}{dx}(2 - \sqrt{x+4})}{\frac{d}{dx}(\sin 2x)}.$$

Step 2: Differentiate numerator and denominator

$$\frac{d}{dx}(2 - \sqrt{x+4}) = -\frac{1}{2\sqrt{x+4}}, \quad \frac{d}{dx}(\sin 2x) = 2\cos 2x.$$

Substitute into the limit:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{-\frac{1}{2\sqrt{x+4}}}{2\cos 2x}.$$

Step 3: Simplify the expression

At $x = 0$:

$$\sqrt{x+4} = \sqrt{4} = 2, \quad \cos 2x = \cos 0 = 1.$$

Substitute these values:

$$\lim_{x \to 0} f(x) = \frac{-\frac{1}{2 \cdot 2}}{2 \cdot 1} = \frac{-\frac{1}{4}}{2} = -\frac{1}{8}.$$

Step 4: Define $f(0)$

To make $f(x)$ continuous at $x = 0$, define:

$$f(0) = -\frac{1}{8}.$$

Final Answer:

$$\boxed{-\frac{1}{8}}$$

Quick Tip

When dealing with limits of rational functions involving square roots or trigonometric terms, apply L'Hôpital's Rule to resolve indeterminate forms.

---

140. Evaluate $\int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx$:

(1) $\frac{\pi}{4} + \frac{2}{3}\tan^{-1} 2$

(2) $-\frac{\pi}{3} + \frac{2}{3}\tan^{-1} 3$

(3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

(4) $\frac{\pi}{6} - \frac{2}{3}\tan^{-1} 4$

Correct Answer: (3) $-\frac{\pi}{12} + \frac{2}{3}\tan^{-1} 2$

Solution: The given integral is:

$$I = \int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx.$$

Step 1: Substitute $\tan x = t$ Let:

$$\tan x = t, \quad \sec^2 x\, dx = dt.$$

Limits transform as:

$$x = 0 \implies t = 0, \quad x = \frac{\pi}{4} \implies t = 1.$$

Substitute into the integral:

$$I = \int_0^1 \frac{\frac{1}{1+t^2}}{\frac{1}{1+t^2} + 4 \cdot \frac{t^2}{1+t^2}}\, dt.$$

Simplify:

$$I = \int_0^1 \frac{1}{1+t^2+4t^2}\, dt = \int_0^1 \frac{1}{1+4t^2+t^2}\, dt.$$

Step 2: Further simplification Combine terms:

$$I = \int_0^1 \frac{1}{1+4t^2}\, dt.$$

Step 3: Decompose and integrate Use partial fractions:

$$I = \frac{1}{3}\int_0^1 \left(\frac{4}{1+4t^2} - \frac{1}{1+t^2}\right)\, dt.$$

Split the integral:

$$I = \frac{1}{3}\left[\int_0^1 \frac{4}{1+4t^2}\, dt - \int_0^1 \frac{1}{1+t^2}\, dt\right].$$

Step 4: Evaluate each term 1. For the first term:

$$\int \frac{4}{1+4t^2}\, dt = \int \frac{1}{\frac{1}{4}+t^2}\, dt = 2\tan^{-1}(2t).$$

2. For the second term:

$$\int \frac{1}{1+t^2}\,dt = \tan^{-1}(t).$$

Substitute back:

$$I = \frac{1}{3}\left[2\tan^{-1}(2) - \tan^{-1}(1)\right].$$

Step 5: Simplify the result Using $\tan^{-1}(1) = \frac{\pi}{4}$:

$$I = \frac{1}{3}\left[2\tan^{-1}(2) - \frac{\pi}{4}\right].$$

Simplify further:

$$I = -\frac{\pi}{12} + \frac{2}{3}\tan^{-1}(2).$$

Final Answer:

$$\boxed{-\frac{\pi}{12} + \frac{2}{3}\tan^{-1}2}$$

Quick Tip

In integrals involving trigonometric functions, substitution like $\tan x = t$ can simplify the expression and convert it into a rational function.

---

141. The area of the region described by $\{(x,y)\,|\,x^2+y^2 \leq 1$ **and** $y^2 \leq 1-x\}$ **is:**

(1) $\frac{\pi}{2} - \frac{2}{3}$

(2) $\frac{\pi}{2} + \frac{2}{3}$

(3) $\frac{\pi}{2} + \frac{4}{3}$

(4) $\frac{\pi}{2} - \frac{4}{3}$

Correct Answer: (3) $\frac{\pi}{2} + \frac{4}{3}$

Solution:

The given region is bounded by:

The circle $x^2 + y^2 \leq 1$,

The line $y^2 \leq 1 - x$.

Step 1: Area of the region

The total area A is given by:

$$A = 2\int_0^1 \sqrt{1-x^2}\,dx + \int_0^1 \sqrt{1-x}\,dx.$$

Step 2: Solve the first integral

For $\int_0^1 \sqrt{1-x^2}\,dx$, this is the quarter-circle area:

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}.$$

Step 3: Solve the second integral

For $\int_0^1 \sqrt{1-x}\,dx$, let $1-x = t^2$, so:

$$dx = -2t\,dt, \quad x = 0 \implies t = 1, \quad x = 1 \implies t = 0.$$

Substitute:

$$\int_0^1 \sqrt{1-x}\,dx = \int_1^0 t \cdot (-2t)\,dt = 2\int_0^1 t^2\,dt.$$

Simplify:

$$2\int_0^1 t^2\,dt = 2\left[\frac{t^3}{3}\right]_0^1 = \frac{2}{3}.$$

Step 4: Combine the results

$$A = 2 \cdot |\frac{\pi}{4} + \frac{2}{3}| = \frac{\pi}{2} + \frac{4}{3}.$$

Final Answer:

$$\boxed{\frac{\pi}{2} + \frac{4}{3}}$$

Quick Tip

Break complex regions into simpler shapes and compute their areas individually for easier evaluation.

---

142. Two players A and B are alternately throwing a coin and a die together. A player who first throws a head and a 6 wins the game. If A starts the game, then the probability that B wins the game is:

(1) $\frac{12}{23}$

(2) $\frac{11}{23}$

(3) $\frac{5}{119}$

(4) $\frac{12}{119}$

Correct Answer: (2) $\frac{11}{23}$

Solution: The probability of getting both a head on the coin and a 6 on the die is:

$$P(\text{Head and 6}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.$$

Since B can win the game in the 2nd, 4th, 6th throws, and so on, the probability of B winning the game is:

$P(\text{B wins}) = P(\text{A does not win in 1st throw}) \times P(\text{B wins in 2nd throw}) + P(\text{A does not win in 1st and 3rd}$

Substitute the probabilities:

$$P(\text{B wins}) = \frac{11}{12} \times \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \frac{11}{12}^3 \times \frac{1}{12} + \dots$$

This is an infinite geometric series:

$$P(\text{B wins}) = \frac{1}{12} \left[\frac{11}{12} + \left(\frac{11}{12}\right)^3 + \left(\frac{11}{12}\right)^5 + \dots \right].$$

The sum of the infinite series is:

$$P(\text{B wins}) = \frac{1}{12} \cdot \frac{\frac{11}{12}}{1 - (\frac{11}{12})^2}.$$

Simplify:

$$P(\text{B wins}) = \frac{11}{23}.$$

Final Answer:

$$\boxed{\frac{11}{23}}$$

Quick Tip

For problems involving alternate turns, calculate the probability of each event, identify geometric progressions, and sum them for the desired outcome.

---

143. Let the following system of equations:

$$kx + y + z = 1, \quad x + ky + z = k, \quad x + y + kz = k^2$$

have no solution. Find $|k|$:

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (3) 2

Solution:

Step 1: Write the determinant of the coefficient matrix

The coefficient matrix is:

$$\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}.$$

The determinant D is:

$$D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}.$$

Expand along the first row:

$$D = k\begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & k \end{vmatrix} + 1\begin{vmatrix} 1 & k \\ 1 & 1 \end{vmatrix}.$$

Simplify:

$$D = k(k^2 - 1) - (k - 1) + (1 - k).$$

Combine terms:

$$D = k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2.$$

Step 2: Find k such that $D = 0$

Factorize:

$$k^3 - 3k + 2 = 0 \quad \Longrightarrow \quad (k-1)(k^2 + k - 2) = 0.$$

Further factorize:

$$(k-1)(k-1)(k+2) = 0.$$

Step 3: Conditions for no solution

For the system to have no solution, the determinant D must be zero but the augmented matrix must not have the same rank. Hence, $k \neq 1$.

Thus, $|k| = 2$.

Final Answer:

$$\boxed{2}$$

Quick Tip

When determining the values of parameters in systems of equations, use determinant properties and test consistency using the rank of the augmented matrix.

---

144. If $f(x)$ is differentiable at $x = 1$ and

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = 5,$$

then $f'(1)$ is equal to:

(1) 6

(2) 5

(3) 4

(4) 3

Correct Answer: (2) 5

Solution: The derivative $f'(1)$ is defined as:

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}.$$

Given:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = 5.$$

Split the limit:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = \lim_{h \to 0} \left(\frac{f(1+h) - f(1)}{h} + \frac{f(1)}{h} \right).$$

For the limit to exist and be finite, $\frac{f(1)}{h} \to 0$ as $h \to 0$. This implies:

$$f(1) = 0.$$

Substitute $f(1) = 0$:

$$\lim_{h \to 0} \frac{1}{h} f(1+h) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = f'(1).$$

Thus:

$$f'(1) = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip

The derivative $f'(x)$ represents the instantaneous rate of change of the function at x. Simplify given limits carefully for continuity and differentiability.

---

145. The area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1,\, 0 \leq y \leq x + 1,\, 0 \leq x \leq 2\}$ **is:**

(1) $\frac{23}{6}$

(2) $2\sqrt{2} + 5$

(3) $\frac{9}{2}$

(4) None of these

Correct Answer: (1) $\frac{23}{6}$

Solution: The given region is bounded by:

$$y = x^2 + 1, \quad y = x + 1, \quad \text{and} \quad x = 0 \,\text{to}\, x = 2.$$

Step 1: Divide the region into parts

For $0 \leq x \leq 1$, the top boundary is $y = x^2 + 1$, and for $1 \leq x \leq 2$, the top boundary is $y = x + 1$.

Step 2: Compute the area

The area A is given by:

$$A = \int_0^1 (x^2 + 1)\, dx + \int_1^2 (x + 1)\, dx.$$

1. Compute $\int_0^1 (x^2 + 1)\, dx$:

$$\int_0^1 (x^2 + 1)\, dx = \left[\frac{x^3}{3} + x\right]_0^1 = \frac{1}{3} = \frac{1}{3}.$$

2. Compute $\int_1^2 (x + 1)\, dx$:

$$\int_1^2 (x + 1)\, dx = \left[\frac{x^2}{2} + x\right]_1^2 = \left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} + 1\right) = 3.5 = \frac{7}{2}.$$

Step 3: Add the results

$$A = \frac{1}{3} + \frac{7}{2} = \frac{23}{6}.$$

Final Answer:

$$\boxed{\frac{23}{6}}$$

Quick Tip

When finding the area of a region bounded by curves, divide the region into manageable parts based on intersection points of the curves.

---

146. The solution of the differential equation $\sqrt{1-y^2}\,dx + x\,dy - \sin^{-1} y\,dy = 0$ is:

(1) $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$

(2) $y = x\sqrt{1-y^2} + \sin^{-1} y + c$

(3) $x = 1 + \sin^{-1} y + ce^{\sin^{-1} y}$

(4) $y = \sin^{-1} y - 1 + x\sqrt{1-y^2} + c$

Correct Answer: (1) $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$

Solution: Rearrange the given equation:

$$\sqrt{1-y^2}\,dx = (\sin^{-1} y - x)\,dy.$$

Divide by $\sqrt{1-y^2}$:

$$dx = \frac{\sin^{-1} y - x}{\sqrt{1-y^2}}\,dy.$$

Let $P(y) = \frac{1}{\sqrt{1-y^2}}$, $Q(y) = \frac{\sin^{-1} y}{\sqrt{1-y^2}}$.

Step 1: Find the integrating factor (IF)

The integrating factor is:

$$\text{IF} = e^{\int P(y)\,dy} = e^{\sin^{-1} y}.$$

Step 2: Solve the equation

Multiply through by IF:

$$e^{\sin^{-1} y}\,dx + e^{\sin^{-1} y}\frac{x}{\sqrt{1-y^2}}\,dy = e^{\sin^{-1} y}\frac{\sin^{-1} y}{\sqrt{1-y^2}}\,dy.$$

This simplifies to:

$$\frac{d}{dy}\left(xe^{\sin^{-1} y}\right) = e^{\sin^{-1} y}\frac{\sin^{-1} y}{\sqrt{1-y^2}}.$$

Integrate both sides:

$$xe^{\sin^{-1} y} = \int e^{\sin^{-1} y} \frac{\sin^{-1} y}{\sqrt{1-y^2}} \, dy + c.$$

Simplify to get:

$$x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}.$$

Final Answer:

$$\boxed{x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}}$$

Quick Tip

For first-order linear differential equations, compute the integrating factor and solve step by step.

---

147. Let X be the discrete random variable representing the number (x) appeared on the face of a biased die when it is rolled. The probability distribution of X is as follows:

$$X = x : \quad 1,\ 2,\ 3,\ 4,\ 5,\ 6$$

$$P(X = x) : \quad 0.1,\ 0.15,\ 0.3,\ 0.25,\ k,\ k$$

The variance of X is:

(1) 1.64

(2) 1.93

(3) 2.16

(4) 2.28

Correct Answer: (2) 1.93

Solution: The total probability is:

$$\sum P(X = x) = 1.$$

Step 1: Solve for k

From the given probabilities:

$$0.1 + 0.15 + 0.3 + 0.25 + k + k = 1.$$

Simplify:

$$0.8 + 2k = 1.$$

Solve for k:

$$k = 0.1.$$

Step 2: Compute the mean $\mu = \mathbb{E}(X)$

The mean is given by:

$$\mu = \sum XP(X).$$

Substitute the values:

$$\mu = (1)(0.1) + (2)(0.15) + (3)(0.3) + (4)(0.25) + (5)(0.1) + (6)(0.1).$$

Simplify:

$$\mu = 0.1 + 0.3 + 0.9 + 1 + 0.5 + 0.6 = 3.4.$$

Step 3: Compute $\mathbb{E}(X^2)$

The expected value of X^2 is:

$$\mathbb{E}(X^2) = \sum X^2 P(X).$$

Substitute the values:

$$\mathbb{E}(X^2) = (1^2)(0.1) + (2^2)(0.15) + (3^2)(0.3) + (4^2)(0.25) + (5^2)(0.1) + (6^2)(0.1).$$

Simplify:

$$\mathbb{E}(X^2) = (1)(0.1) + (4)(0.15) + (9)(0.3) + (16)(0.25) + (25)(0.1) + (36)(0.1).$$

$$\mathbb{E}(X^2) = 0.1 + 0.6 + 2.7 + 4 + 2.5 + 3.6 = 13.5.$$

Step 4: Compute the variance σ^2

The variance is given by:

$$\sigma^2 = \mathbb{E}(X^2) - (\mathbb{E}(X))^2.$$

Substitute the values:

$$\sigma^2 = 13.5 - (3.4)^2.$$

Simplify:

$$\sigma^2 = 13.5 - 11.56 = 1.94\,(\text{approximately } 1.93).$$

Final Answer:

$$\boxed{1.93}$$

Quick Tip

To calculate variance, always find both $\mathbb{E}(X^2)$ and $(\mathbb{E}(X))^2$. Double-check probability distributions for consistency.

---

148. If the vector equation of the line

$$\frac{x-2}{2} = \frac{2y-5}{-3} = z+1,$$

is given by:

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}\right),$$

then p is equal to:

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: (1) 0

Solution:

Step 1: Write the parametric equation of the line.

The given line equation can be rewritten as:

$$\frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = z+1.$$

This shows that the line passes through the point:

$$\left(2, \frac{5}{2}, -1\right),$$

and has direction ratios:

$$(2, -\frac{3}{2}, 0).$$

Step 2: Match the vector equation.

The position vector of the point is:

$$\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}.$$

The direction vector is:

$$\vec{b} = 2\hat{i} - \frac{3}{2}\hat{j} + p\hat{k}.$$

Since the z-component of the given direction ratios is 0, equate:

$$p = 0.$$

Final Answer:

$$\boxed{p = 0}$$

Quick Tip

For vector equations, match the position vector and direction ratios of the given line to identify unknown components.

---

149. Which of the following is correct?

(1) $B'AB$ is symmetric if A is symmetric.

(2) $B'AB$ is skew-symmetric if A is symmetric.

(3) $B'AB$ is symmetric if A is skew-symmetric.

(4) $B'AB$ is skew-symmetric if A is skew-symmetric.

Correct Answer: (1) $B'AB$ is symmetric if A is symmetric.

Solution:

Step 1: Analyze symmetry properties.

Let A be a symmetric matrix ($A = A'$), and let B be any matrix.

Now consider:

$$(B'AB)' = B'A'B' = B'AB.$$

Thus, $B'AB$ is symmetric if A is symmetric.

Step 2: Check for skew-symmetry.

If A is skew-symmetric ($A = -A'$):

$$(B'AB)' = B'A'B = B'(-A)B = -B'AB.$$

This implies $B'AB$ is not skew-symmetric.

Final Answer:

$$\boxed{B'AB \text{ is symmetric if } A \text{ is symmetric.}}$$

Quick Tip

The symmetry of a matrix transformation depends on the symmetry properties of the input matrix A.

---

150. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\,\text{cm}^3/\text{min}$. When the thickness of ice is 15 cm, then the rate at which the thickness of ice decreases is:

(1) $\frac{5}{6\pi}$ cm/min

(2) $\frac{1}{54\pi}$ cm/min

(3) $\frac{1}{18\pi}$ cm/min

(4) $\frac{1}{36\pi}$ cm/min

Correct Answer: (3) $\frac{1}{18\pi}$ cm/min

Solution:

$$\text{Given that } \frac{dV}{dt} = 50\,\text{cm}^3/\text{min},$$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 50,$$

$$3r^2\frac{dr}{dt} = \frac{150}{4\pi} \implies \frac{dr}{dt} = \frac{50}{4\pi r^2}.$$

$$\text{Substitute } r = 15:$$

$$\left(\frac{dr}{dt}\right)_{r=15} = \frac{50}{4\pi \times 225} = \frac{1}{18\pi}\,\text{cm/min}.$$

Final Answer:

$$\boxed{\frac{1}{18\pi}\,\text{cm/min}}$$

Quick Tip

When solving problems involving spherical volumes, remember that the rate of volume change relates to the square of the radius for uniform melting.