Maharashtra HSC BOARD QUESTION PAPER 2023
Mathematics

Time: 3 Hrs.   **Max. Marks: 80**

General instructions:
*The question paper is divided into **FOUR** sections.*
(1)   **Section A:**   *Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
                             *Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.*
(2)   **Section B:**   *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*
(3)   **Section C:**   *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*
(4)   **Section D:**   *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*
(5)   *Use of log table is allowed. Use of calculator is not allowed.*
(6)   *Figures to the right indicate full marks.*
(7)   *Use of graph paper is <u>not</u> necessary. Only rough sketch of graph is expected.*
(8)   *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g. (a)……. / (b)…….. /(c)…….. /(d)…….., etc. No marks shall be given, if <u>ONLY</u> the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.*
(9)   *Start answer to each section on a new page.*

SECTION – A

Q.1.   **Select and write the correct answer for the following multiple choice type of questions:**   **[16]**

i.   If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _____ respectively.
    (a)   T, T   (b)   T, F   (c)   F, T   (d)   F, F   (2)

ii.   In ΔABC, if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _____.
    (a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{6}$ (2)

iii.   The area of the triangle with vertices (1, 2, 0), (1, 0, 2) and (0, 3, 1) in sq. unit is _____.
    (a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) $\sqrt{3}$ (2)

iv.   If the corner points of the feasible solution are (0, 10), (2, 2) and (4, 0) then the point of minimum $z = 3x + 2y$ is _____.
    (a)   (2, 2)   (b)   (0, 10)   (c)   (4, 0)   (d)   (3, 4)   (2)

v.   If y is a function of x and $\log (x + y) = 2xy$, then the value of $y'(0) =$ _____.
    (a)   2   (b)   0   (c)   –1   (d)   1   (2)

vi. $\int \cos^3 x \, dx =$ _____.
    (a) $\dfrac{1}{12}\sin 3x + \dfrac{3}{4}\sin x + c$ (b) $\dfrac{1}{12}\sin 3x + \dfrac{1}{4}\sin x + c$
    (c) $\dfrac{1}{12}\sin 3x - \dfrac{3}{4}\sin x + c$ (d) $\dfrac{1}{12}\sin 3x - \dfrac{1}{4}\sin x + c$ (2)

vii.   The solution of the differential equation $\dfrac{dx}{dt} = \dfrac{x \log x}{t}$ is _____
    (a) $x = e^{ct}$ (b) $x + e^{ct} = 0$
    (c) $x = e^t + t$ (d) $xe^{ct} = 0$ (2)

viii. Let the probability mass function (p.m.f.) of a random variable X be $P(X = x) = {}^4C_x \left(\dfrac{5}{9}\right)^x \times \left(\dfrac{4}{9}\right)^{4-x}$,

for $x = 0, 1, 2, 3, 4$ then E(X) is equal to _____

   (a) $\dfrac{20}{9}$ (b) $\dfrac{9}{20}$ (c) $\dfrac{12}{9}$ (d) $\dfrac{9}{25}$ (2)

Q.2. **Answer the following questions:**   **[4]**

i.   Write the joint equation of co-ordinate axes.   (1)

ii.   Find the values of c which satisfy $\left|c\bar{u}\right| = 3$ where $\bar{u} = \hat{i} + 2\hat{j} + 3\hat{k}$.   (1)

iii. Write $\int \cot x \, dx$.   (1)

iv. Write the degree of the differential equation $e^{\frac{dy}{dx}} + \dfrac{dy}{dx} = x$ (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Write inverse and contrapositive of the following statement:

If $x < y$ then $x^2 < y^2$ (2)

Q.4. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non singular matrix, then find A^{-1} by elementary row transformations.

Hence write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (2)

Q.5. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\sqrt{2}, \dfrac{\pi}{4}\right)$.   (2)

Q.6. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$ then find the ratio of their slopes.   (2)

Q.7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB.   (2)

Q.8. Solve the following inequations graphically and write the corner points of the feasible region:

$2x + 3y \leq 6, x + y \geq 2, x \geq 0, y \geq 0$ (2)

Q.9. Show that the function $f(x) = x^3 + 10x + 7, x \in R$ is strictly increasing.   (2)

Q.10. Evaluate: $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 4x} \, dx$ (2)

Q.11. Find the area of the region bounded by the curve $y^2 = 4x$, the X-axis and the lines $x = 1$, $x = 4$ for $y \geq 0$.   (2)

Q.12. Solve the differential equation

$\cos x \cos y \, dy - \sin x \sin y \, dx = 0$ (2)

Q.13. Find the mean of number randomly selected from 1 to 15.   (2)

Q.14. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.   (2)

Attempt any EIGHT of the following questions: [24]

Q.15. Find the general solution of $\sin\theta + \sin 3\theta + \sin 5\theta = 0$ (3)

Q.16. If $-1 \le x \le 1$, the prove that $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ (3)

Q.17. If θ is the acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ then prove that

$\tan\theta = \left| \dfrac{2\sqrt{h^2 - ab}}{a + b} \right|$ (3)

Q.18. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are –2, 1, –1 and –3, –4, 1. (3)

Q.19. Find the shortest distance between lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$. (3)

Q.20. Lines $\bar{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\bar{r} = \left(4\hat{i} - 3\hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ are coplanar. Find the equation of the plane determined by them. (3)

Q.21. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + ... + \infty}}}$, then

show that $\dfrac{dy}{dx} = \dfrac{\sec^2 x}{2y - 1}$.

Find $\dfrac{dy}{dx}$ at $x = 0$. (3)

Q.22. Find the approximate value of $\sin(30°30')$.
Give that $1° = 0.0175^c$ and $\cos 30° = 0.866$ (3)

Q.23. Evaluate $\int x \tan^{-1} x \, dx$ (3)

Q.24. Find the particular solution of the differential equation $\dfrac{dy}{dx} = e^{2y} \cos x$, when $x = \dfrac{\pi}{6}$, $y = 0$ (3)

Q.25. For the following probability density function of a random variable X, find (a) $P(X < 1)$ and (b) $P(|X| < 1)$.

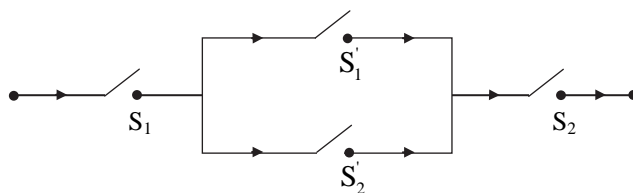$f(x) = \dfrac{x + 2}{18}$; for $-2 < x < 4$

$\quad = 0$, otherwise (3)

Q.26. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of at least 5 successes. (3)

Attempt any FIVE of the following questions: [20]

Q.27. Simplify the given circuit by writing its logical expression. Also write your conclusion.

(4)

Q.28. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$ (4)

Q.29. Prove that the volume of a tetrahedron with coterminus edges $\overline{a}$, $\overline{b}$ and $\overline{c}$ is $\dfrac{1}{6}\left[\overline{a}\ \overline{b}\ \overline{c}\right]$.

Hence, find the volume of tetrahedron whose coterminus edges are $\overline{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overline{b} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\overline{c} = 2\hat{i} + \hat{j} + 4\hat{k}$. (4)

Q.30. Find the length of the perpendicular drawn from the point P(3, 2, 1) to the line $\overline{r} = \left(7\hat{i} + 7\hat{j} + 6\hat{k}\right) + \lambda\left(-2\hat{i} + 2\hat{j} + 3\hat{k}\right)$ (4)

Q.31. If $y = \cos(m\cos^{-1} x)$ then show that $(1 - x^2)\dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} + m^2 y = 0$ (4)

Q.32. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x+4}$ on the interval [0, 5]. (4)

Q.33. Evaluate: $\displaystyle\int \dfrac{2x^2 - 3}{\left(x^2 - 5\right)\left(x^2 + 4\right)}\,dx$ (4)

Q.34. Prove that: $\displaystyle\int_{0}^{2a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(2a - x)\,dx$ (4)