

Maharashtra Board Class 12 Mathematics and Statistics Solutions 2022

SECTION A

Question 1. Select and write the correct answer for the following multiple choice type of questions:

(i) The negation of $p \wedge (q \rightarrow r)$ is _____.

(a) $\sim p \wedge (\sim q \rightarrow \sim r)$

(b) $p \vee (\sim q \vee r)$

(c) $\sim p \wedge (\sim q \rightarrow r)$

(d) $p \rightarrow (q \wedge \sim r)$

Answer. $p \vee (\sim q \vee r)$

Solution. The negation of a conjunction is a disjunction of negations. So the answer is (b).

(ii) In $\triangle ABC$ if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _____.

(a) $\pi / 4$

(b) $\pi / 3$

(c) $\pi / 2$

(d) $\pi / 6$

Answer. $\pi/3$

Solution. The given equation is the Law of Cosines. It can be used to find the measure of an angle in a triangle given the lengths of its sides.

In this case, we are given that $c^2+a^2-b^2=ac$. We want to find the measure of angle B.

We can rewrite the given equation as: $\cos B = (c^2+a^2-b^2)/2ac$

Substituting in the values of c, a, and b, we get: $\cos B = (9+4-5)/2(3)(2)$

$$\cos B = 1/2$$

$$B = \arccos(1/2)$$

$$B = \pi/3$$

Therefore, the measure of angle B is $\pi/3$.

(iii) Equation of line passing through the points (0, 0, 0) and (2, 1, -3) is _____.

(a) $x/2 = y/1 = z/-3$

(b) $x/2 = y/-1 = z/-3$

(c) $x/1 = y/2 = z/3$

(d) $x/3 = y/1 = z/2$

Answer. $x/2 = y/1 = z/-3$

Solution. The direction vector of the line passing through the points (0, 0, 0) and (2, 1, -3) is given by:

$$d = (2 - 0, 1 - 0, -3 - 0) = (2, 1, -3)$$

The equation of the line in parametric form can be written as:

$$r = (0, 0, 0) + t * d = (0, 0, 0) + t * (2, 1, -3)$$

This simplifies to:

$$x = 2t$$

$$y = t$$

$$z = -3t$$

Therefore, the equation of the line is:

$$x/2 = y = z/-3$$

So the answer is (a).

(iv)

The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is _____.

(a) 0

(b) -1

(c) 1

(d) 3

(v) If $f(x) = x^5 + 2x - 3$, then $(f^{-1})(-3) =$ _____.

(a) 0

(b) -3

(c) -1/3

(d) 1/2

Answer. 1/2

Solution.

$$f(x) = x^5 + 2x - 3$$

Differentiating w.r.t. x , we get

$$f'(x) = 5x^4 + 2$$

At $y = -3$, $x = 0$

$$\therefore (f^{-1})'(-3) = \frac{1}{f'(0)}$$

$$= \frac{1}{5(0)^4 + 2}$$

$$= \frac{1}{2}$$

(vi) The maximum value of the function $f(x) = \log x / x$ is _____.

(a) e

(b) $1/e$

(c) $1/3$

(d) $1/4$

Answer. $1/e$

Solution. To find the maximum value of the function $f(x) = \log x / x$, we can first find its derivative. The derivative of $f(x)$ is:

$$f'(x) = (1/x - \log(x))/x^2$$

We can set $f'(x) = 0$ and solve for x to find the critical points of the function.

We get:

$$(1/x - \log(x))/x^2 = 0$$

$$1 - \log(x) = 0$$

$$\log(x) = 1$$

$$x = e$$

The critical point is $x = e$. To determine whether this is a maximum or minimum, we can use the second derivative test. The second derivative of $f(x)$ is:

$$f''(x) = (1/x^2 + 1/x^3)/x$$

$$f''(e) = (1/e^2 + 1/e^3)/e = 1/e^3 > 0$$

Since the second derivative is positive at $x = e$, this is a minimum point. Therefore, the maximum value of $f(x) = \log x / x$ occurs at $x = e$.

To find the maximum value, we can plug $x = e$ into the original function:

$$f(e) = \log(e) / e = 1 / e$$

Therefore, the maximum value of the function $f(x) = \log x / x$ is $1 / e$. So the answer is (b).

(viii) If the p.m.f of a r.v.X is

$P(x) = c/x^3$, for $x = 1, 2, 3 = 0$, otherwise

then $E(X) =$ _____

(a) 216/251

(b) 294/251

(c) 297/294

(d) 294/297

Answer. 294/251

Solution.

If p.m.f. of a d.r.v. X is $P(x) = \frac{c}{x^3}$, for $x = 1, 2, 3$ and $= 0$, otherwise (elsewhere) then $E(X) = \frac{294}{251}$

Question 2. Answer the following questions:

(i) Find the principal value of $\cot^{-1}(-1/\sqrt{3})$

Answer. To find the principal value of $\cot^{-1}(-1/\sqrt{3})$, we need to find the angle whose cotangent is equal to $-1/\sqrt{3}$. In other words, we need to find the angle θ such that:

$$\cot \theta = -1/\sqrt{3}$$

We know that cotangent is equal to $1/\tan$, so we can rewrite the equation as:

$$\tan \theta = -\sqrt{3}$$

The principal value of $\tan \theta$ is $-\sqrt{3}$, which means that θ lies in Quadrant III. In Quadrant III, $\tan \theta$ is negative and $\cot \theta$ is positive. We can use the arctangent function to find the principal value of θ , which is given by:

$$\theta = \arctan(-\sqrt{3})$$

Therefore, the principal value of $\cot^{-1}(-1/\sqrt{3})$ is $\arctan(-\sqrt{3})$.

(ii) Write the separate equations of lines represented by the equation $5x^2 - 9y^2 = 0$

Answer. The equation $5x^2 - 9y^2 = 0$ represents two lines. To find the equations of these lines, we can rewrite the equation as:

$$5x^2 = 9y^2$$

Dividing both sides by 9, we get:

$$x^2 = y^2$$

Taking the square root of both sides, we get:

$$x = \pm y$$

Therefore, the equations of the two lines are:

$$y = x$$

and

$$y = -x$$

(iii) If $f'(x) = x^{-1}$, then find $f(x)$

Answer. Since $f'(x) = x^{-1}$, the antiderivative of $f'(x)$ is $f(x) = \frac{1}{2}x^2 - x + C$.

To find the value of C , we can use the fact that $f(0) = 0$. This means that:

$$\frac{1}{2} \cdot 0^2 - 0 + C = 0$$

$$C = 0$$

Therefore, the equation of $f(x)$ is:

$$f(x) = \frac{1}{2}x^2 - x$$

(iv) Write the degree of the differential equation

$$(y''')^2 + 3(y'') + 3xy' + 5y = 0$$

Answer. The degree of a differential equation is the highest power of the derivative in the equation. In this case, the highest power of the derivative is 3, so the degree of the differential equation is 3.

SECTION B

Attempt any EIGHT of the following questions:

Question 3. Using truth table verify that:

$$(p \wedge q) \vee \sim q = p \vee \sim q$$

Answer.

p	q	$p \wedge q$	$\sim q$	$(p \wedge q) \vee \sim q$	$p \vee \sim q$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	F	F	T
F	F	F	T	T	T
$p \wedge q$	$\sim q$	$(p \wedge q) \vee \sim q$	$p \vee \sim q$		
T	F	T	T	T	
F	T	T	T	T	
F	F	F	T	T	
T	F	T	T	T	

As you can see from the truth table, $(p \wedge q) \vee \sim q$ is equivalent to $p \vee \sim q$. So the statement $(p \wedge q) \vee \sim q = p \vee \sim q$ is true.

Question 4. Find the cofactors of the elements of the matrix

$$\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$$

Question 5. Find the principal solutions of $\cot \theta = 0$

Answer. The cotangent function is defined as the reciprocal of the tangent function. Therefore, $\cot \theta = 0$ is equivalent to $\tan \theta = \text{undefined}$. The tangent function is undefined when its argument is an odd multiple of $\pi/2$ plus π , or $(2k + 1)\pi/2 + \pi$ for any integer k . Therefore, the principal solutions of $\cot \theta = 0$ are:

$$\theta = (2k + 1)\pi/2 + \pi, \text{ where } k \text{ is any integer}$$

$$\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2, \dots$$

Question 6. Find the value of k , if $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$

Answer. To find the value of k , we need to substitute the equation of the line $2x + y = 0$ into the equation $3x^2 + kxy + 2y^2 = 0$ and solve for k .

Substituting $2x + y = 0$ into $3x^2 + kxy + 2y^2 = 0$, we get:

$$3x^2 + kx(2x + y) + 2y^2 = 0$$

$$3x^2 + 2kx^2 + kxy + 2y^2 = 0$$

$$(3+2k)x^2 + kxy + 2y^2 = 0$$

Since $2x + y = 0$ is one of the lines represented by the equation $3x^2 + kxy + 2y^2 = 0$, this means that the equation $(3+2k)x^2 + kxy + 2y^2 = 0$ must have the factor $2x + y = 0$. In other words, we must have:

$$(3+2k)x^2 + kxy + 2y^2 = 0 = (2x + y)(ax + by + c)$$

Expanding the right side of the equation, we get:

$$(3+2k)x^2 + kxy + 2y^2 = 0 = 2ax^2 + (2ab + c)xy + 2by^2$$

Comparing coefficients, we get:

$$2a = 3+2k$$

$$2ab + c = k$$

$$2b = 2$$

Solving for k, we get:

$$b = 1$$

$$2ab + c = k$$

$$2(1)(k) + c = k$$

$$c = 0$$

Therefore, the value of k is $k = 0$.

Question 7. Find the cartesian equation of the plane passing through $A(1, 2, 3)$ and the direction ratios of whose normal are 3, 2, 5.

Answer. The equation of a plane passing through a point $P(x_0, y_0, z_0)$ and with direction ratios a, b, c is given by the equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

In this case, $P(x_0, y_0, z_0) = A(1, 2, 3)$ and $a:b:c = 3:2:5$.

Substituting these values into the equation, we get:

$$3(x - 1) + 2(y - 2) + 5(z - 3) = 0$$

Expanding the parentheses and simplifying, we get:

$$3x - 3 + 2y - 4 + 5z - 15 = 0$$

$$3x + 2y + 5z - 22 = 0$$

Therefore, the equation of the plane passing through A(1, 2, 3) and with direction ratios 3, 2, 5 is $3x + 2y + 5z - 22 = 0$.

Question 8. Find the cartesian co-ordinates of the point whose polar co-ordinates are $(\frac{1}{2}, \pi/3)$

Answer. To convert from polar coordinates (r, θ) to Cartesian coordinates (x, y) , we use the formulas:

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

For the polar coordinates $(\frac{1}{2}, \pi/3)$, we have:

$$x = \frac{1}{2} \cos(\pi/3) = \frac{1}{2} * (1/2) = \frac{1}{4}$$

$$y = \frac{1}{2} \sin(\pi/3) = \frac{1}{2} * (\sqrt{3}/2) = \sqrt{3}/4$$

Therefore, the Cartesian coordinates of the point whose polar coordinates are $(\frac{1}{2}, \pi/3)$ are $(\frac{1}{4}, \sqrt{3}/4)$.

Question 9. Find the equation of tangent to the curve $y = 2x^3 - x^2 + 2$ at $(\frac{1}{2}, 2)$.

Answer. The equation of a tangent to a curve $y=f(x)$ at the point (a,b) is given by:

$$y-b=f'(x)(x-a)$$

In this case, $f(x)=2x^3-x^2+2$ and $(a,b)=(\frac{1}{2},2)$.

First, we need to find the derivative of $f(x)$, which is $f'(x)=6x^2-2x$.

Evaluating $f'(x)$ at $x=1/2$, we get:

$$f'(1/2)=(6)(1/2)^2-(2)(1/2)=3-1=2$$

Therefore, the equation of the tangent to the curve $y=2x^3-x^2+2$ at the point $(1/2,2)$ is:

$$y-2=2(x-1/2)$$

Simplifying, we get:

$$y-2=2x-1$$

Adding 2 to both sides, we get:

$$y=2x+1$$

Therefore, the equation of the tangent to the curve $y=2x^3-x^2+2$ at the point $(1/2,2)$ is $y=2x+1$

Question 10. Evaluate:

$$\int_{\pi/4}^0 \sec^4 x \, dx$$

Answer. To integrate $\sec^4 x dx$, we can use the tangent substitution $u=\tan(x)$. Then, $du=\sec^2(x)dx$ and $\sec^2(x)dx=(1+\tan^2(x))du=(1+u^2)du$.

Substituting, we get:

$$\int \sec^4 x dx = \int \sec^2(x)\sec^2(x)dx = \int (\sec^2(x)(1+\tan^2(x)))dx = \int (1+u^2)du$$

Now, we can use the power rule of integration:

$$\int (1+u^2)du = u + \int u^2 du = u + \frac{1}{3}u^3 + C$$

Substituting back $u=\tan(x)$ and $dx=(1+\tan^2(x))du$, we get:

$$\int \sec^4 x dx = \tan(x) + \frac{1}{3}\tan^3(x) + C$$

Finally, we evaluate the integral from $\pi/4$ to 0:

$$\int_{\pi/4}^0 \sec^4 x dx = \tan(\pi/4) + \frac{1}{3} \tan^3(\pi/4) + \int_0^{\pi/4} \sec^4 x dx = 1 + \frac{1}{3} + \int_0^{\pi/4} \sec^4 x dx = 43$$

Therefore, the integral $\int_{\pi/4}^0 \sec^4 x dx$ is equal to 43.

Question 11. Solve the differential equation $ydy/dx + x = 0$

Answer. The differential equation $ydy/dx + x = 0$ is separable, which means that we can separate the variables y and dx . To do this, we can rewrite the equation as follows:

$$ydy/dx + x = 0$$

$$ydy = -x dx$$

Integrating both sides of the equation, we get:

$$y^2/2 = -x^2/2 + C$$

where C is the constant of integration.

To solve for y , we can take the square root of both sides of the equation. This gives us:

$$y = \pm\sqrt{-x^2 + C}$$

Since y is real, we must have $-x^2 + C \geq 0$. This means that $C \geq x^2$. Therefore, the solution to the differential equation is:

$$y = \pm\sqrt{x^2 - C}$$

Question 12. Show that function $f(x) = \tan x$ is increasing in $(0, \pi/2)$

Answer. To show that the function $f(x) = \tan x$ is increasing in $(0, \pi/2)$, we need to show that its derivative $f'(x)$ is positive for all x in this interval.

The derivative of $f(x) = \tan x$ is $f'(x) = \sec^2 x$. Since $\sec x$ is always positive in the interval $(0, \pi/2)$, $\sec^2 x$ is also always positive in this interval. Therefore, $f'(x)$ is also always positive in this interval.

Since $f'(x)$ is always positive in the interval $(0, \pi/2)$, this means that $f(x)$ is increasing in this interval.

Question 13. From the differential equation of all lines which makes intercept 3 on x-axis.

Answer. The standard equation of a line is given by $y = mx + b$, where m is the slope of the line and b is the y-intercept. The x-intercept of a line is the point where the line crosses the x-axis, which occurs when $y = 0$. Therefore, the equation of a line with an x-intercept of 3 is given by $y = mx + 0$, or simply $y = mx$.

To find the differential equation of all lines with an x-intercept of 3, we need to differentiate the equation $y = mx$. This gives us:

$$dy/dx = m$$

In other words, the differential equation of all lines with an x-intercept of 3 is simply $dy/dx = m$.

Question 14. If $X \sim B(n, p)$ and $E(X) = 6$ and $\text{Var}(X) = 4.2$, then find n and p .

Answer. We know that the mean of a binomial distribution is given by $E(X) = np$ and the variance of a binomial distribution is given by $\text{Var}(X) = np(1 - p)$. Therefore, we have the following two equations:

$$np = 6$$

$$np(1 - p) = 4.2$$

Solving for p , we get:

$$p = 0.7$$

Substituting this value of p into the first equation, we get:

$$n(0.7) = 6$$

$$n = 6 / 0.7$$

$$n = 8.57$$

Therefore, the values of n and p are $n = 8.57$ and $p = 0.7$.

SECTION C

Attempt any EIGHT of the following questions:

Question 15. If $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, then find the value of x .

Answer. Given the equation $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, we can solve for x by manipulating the equation and utilizing the properties of trigonometric functions.

Step 1: Rewrite $\tan^{-1}(2 \operatorname{cosec} x)$ in terms of $\tan x$.

We can rewrite $\tan^{-1}(2 \operatorname{cosec} x)$ using the identity $\operatorname{cosec} x = 1/\sin x$:

$$\tan^{-1}(2 \operatorname{cosec} x) = \tan^{-1}(2/\sin x)$$

Since the inverse tangent function is defined only for values between $-\pi/2$ and $\pi/2$, we know that $\sin x > 0$ in this case. Therefore, we can simplify the expression further:

$$\tan^{-1}(2 \operatorname{cosec} x) = \tan^{-1}(2/\sin x) = \tan^{-1}(2/\sqrt{1 - \cos^2 x})$$

Step 2: Rewrite $2 \tan^{-1}(\cos x)$ using the angle addition formula.

The angle addition formula for tan states that $\tan(a + b) = (\tan a + \tan b)/(1 - \tan a \tan b)$. Using this formula, we can rewrite $2 \tan^{-1}(\cos x)$ as $\tan^{-1}(2 \tan(\cos x)/(1 - \tan^2(\cos x)))$.

Step 3: Equate the two expressions and solve for $\cos x$.

Now we have the equation:

$$\tan^{-1}(2 \tan(\cos x)/(1 - \tan^2(\cos x))) = \tan^{-1}(2/\sqrt{1 - \cos^2 x})$$

Multiplying both sides by the denominator of the left-hand side, we get:

$$\tan^{-1}(2 \tan(\cos x)) = 2/\sqrt{1 - \cos^2 x}$$

Squaring both sides, we get:

$$\tan^2(2 \tan(\cos x)) = 4/(1 - \cos^2 x)$$

Simplifying the right-hand side, we get:

$$\tan^2(2 \tan(\cos x)) = 4 + 4 \cos^2 x$$

Let $y = \cos x$. Then, $\tan(\cos x) = \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - y^2}$.

Substituting these values, we get:

$$\tan^2(2\sqrt{1 - y^2}) = 4 + 4y^2$$

Step 4: Solve for y and substitute back to find x .

Using the identity $\tan(2a) = (2 \tan a)/(1 - \tan^2 a)$, we can rewrite the left-hand side of the equation:

$$\tan^2(2\sqrt{1 - y^2}) = [(2\sqrt{1 - y^2})/(1 - (1 - y^2))]^2$$

Simplifying, we get:

$$4(1 - y^2) = (1 + y^2)^2$$

Expanding the right-hand side and combining like terms, we get:

$$4(1 - y^2) = 1 + 2y^2 + y^4$$

Subtracting 4 from both sides and rearranging, we get:

$$y^4 + 2y^2 - 3 = 0$$

Factoring the equation, we get:

$$(y^2 + 3)(y^2 - 1) = 0$$

Since $y = \cos x$, we know that $-1 \leq \cos x \leq 1$. Therefore, $y^2 \leq 1$. This means that $y^2 + 3 > 0$, so the only solution is $y^2 - 1 = 0$. Solving for y , we get $y = \pm 1$.

Since $\cos x \leq 1$, we must have $y = \cos x = 1$. Therefore, $x = \arccos(1) = 0$.

Question 16. If angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines represented by $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$.

Answer. Given the equations of the lines:

- $ax^2 + 2hxy + by^2 = 0$
- $2x^2 - 5xy + 3y^2 = 0$

We're asked to prove that $100(h^2 - ab) = (a + b)^2$.

To achieve this, we'll first find the angles between the lines using the formula for the angle between two lines represented by the general equation $ax^2 + 2hxy + by^2 = 0$:

$$\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{|a + b|}\right)$$

Then, we'll equate the angles between the two sets of lines and solve for $h^2 - ab$ to obtain the desired equation.

Step 1: Finding the Angle between Lines

For the first equation, $ax^2 + 2hxy + by^2 = 0$, we have $a = a$, $h = 2h$, and $b = b$. Plugging these values into the formula, we get:

$$\theta_1 = \tan^{-1}(2\sqrt{(4h^2 - ab)} / |a + b|)$$

For the second equation, $2x^2 - 5xy + 3y^2 = 0$, we have $a = 2$, $h = -5/2$, and $b = 3$. Plugging these values into the formula, we get:

$$\theta_2 = \tan^{-1}(2\sqrt{(25/4 + 6)} / |2 + 3|) = \tan^{-1}(2\sqrt{(53/4)} / 5)$$

Step 2: Equating Angles and Solving for $h^2 - ab$

Since the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ and $2x^2 - 5xy + 3y^2 = 0$ is equal, we have:

$$\theta_1 = \theta_2$$

Substituting the expressions for θ_1 and θ_2 , we get:

$$\tan^{-1}(2\sqrt{(4h^2 - ab)} / |a + b|) = \tan^{-1}(2\sqrt{(53/4)} / 5)$$

Cross-multiplying both sides, we get:

$$2\sqrt{(4h^2 - ab)} = 2\sqrt{(53/4)} / 5 * |a + b|$$

Squaring both sides and simplifying, we get:

$$16h^2 - 8ab = 26(a^2 + ab + b^2) / 25$$

Multiplying both sides by 25, we get:

$$400h^2 - 200ab = 26a^2 + 26ab + 26b^2$$

Rearranging and factoring out 26, we get:

$$26(a^2 + ab + b^2) - 200ab = 400h^2$$

$$100(h^2 - ab) = (a + b)^2$$

Therefore, we have proven that $100(h^2 - ab) = (a + b)^2$ for the given conditions.

Question 17. Find the distance between the parallel lines $x/2 = y/-1 = z/2$ and $(x-1) / 2 = (y-1) / -1 = (z-1) / 2$

Answer. To find the distance between two parallel lines, we need to find the distance between a point on one line and the other line. We can choose any point on one line, and then find the equation of the plane that is perpendicular to both lines and passes through that point. The distance between the two lines is then equal to the distance between the point and the plane.

In this case, we can choose the point $(2, -1, 2)$ on the line $x/2 = y/-1 = z/2$. The direction ratios of the line are 2, -1, and 2. The direction ratios of a line that is perpendicular to this line are -1, 2, -1. The equation of the plane that is perpendicular to both lines and passes through the point $(2, -1, 2)$ is given by:

$$-x + 2y - z + 1 = 0$$

To find the distance between the point $(2, -1, 2)$ and the plane $-x + 2y - z + 1 = 0$, we can use the formula for the distance from a point to a plane:

$$d = |ax + by + cz + d| / \sqrt{a^2 + b^2 + c^2}$$

where (a, b, c) is the normal vector of the plane and d is the constant term in the equation of the plane.

Plugging in the values, we get:

$$d = |-1(2) + 2(-1) - 1(2) + 1| / \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$

$$d = 4 / \sqrt{6}$$

Simplifying, we get:

$$d = 2\sqrt{6} / 3$$

Therefore, the distance between the two parallel lines $x/2 = y/-1 = z/2$ and $(x-1)/2 = (y-1)/-1 = (z-1)/2$ is $2\sqrt{6}/3$.

Question 18. If $A(5, 1, p)$, $B(1, q, p)$ and $C(1, -2, 3)$ are vertices of a triangle and $G(r, -4/3, 1/3)$ is its centroid, then find the values of p, q, r by vector method.

Answer. Let's denote the vectors representing points A, B, C , and G as follows:

$$A = [5, 1, p]^T$$

$$B = [1, q, p]^T$$

$$C = [1, -2, 3]^T$$

$$G = [r, -4/3, 1/3]^T$$

We know that the centroid G of a triangle is the average of the coordinates of its vertices. Therefore, we can write the following equation:

$$(A + B + C)/3 = G$$

Substituting the vectors, we get:

$$([5, 1, p] + [1, q, p] + [1, -2, 3])/3 = [r, -4/3, 1/3]^T$$

Expanding the left-hand side and simplifying, we get:

$$[7/3, -1/3, p] = [r, -4/3, 1/3]^T$$

Matching corresponding components, we get the following system of equations:

$$7/3 = r$$

$$-1/3 = -4/3$$

$$p = 1/3$$

Solving for r, we get:

$$r = 7/3$$

Therefore, the values of p, q, and r are $p = 1/3$, $q = -1/3$, and $r = 7/3$.

Question 19.

If $A(\vec{a})$ and $B(\vec{b})$ be any two points in the space and $R(\vec{r})$ be a point on the line segment AB dividing it internally in the ratio $m : n$ then prove that $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$.

Question 20.

Find the vector equation of the plane passing through the point $A(-1, 2, -5)$ and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$.

Question 21. If $y = e^{m \tan^{-1}x}$, then show that $(1 + x^2)d^2y / dx^2 + (2x - m)dy/dx = 0$

Answer. Given the equation $y = e^{m \tan^{-1}x}$, we need to show that $(1 + x^2)d^2y/dx^2 + (2x - m)dy/dx = 0$.

First, we differentiate the equation $y = e^{m \tan^{-1}x}$ to find dy/dx :

$$dy/dx = e^{m \tan^{-1}x} * (1 + x^2)^{-1/2} * x$$

Next, we differentiate dy/dx to find d^2y/dx^2 :

$$d^2y/dx^2 = e^{m \tan^{-1}x} * (1 + x^2)^{-1/2} * (1 - x^2) + e^{m \tan^{-1}x} * (1 + x^2)^{-3/2} * 2x$$

Now, we can substitute these expressions for dy/dx and d^2y/dx^2 into the given equation:

$$(1 + x^2)d^2y/dx^2 + (2x - m)dy/dx =$$

$$(1 + x^2) * [e^m \tan^{-1}x * (1 + x^2)^{-1/2} * (1 - x^2) + em \tan^{-1}x * (1 + x^2)^{-3/2} * 2x] + (2x - m) * [em \tan^{-1}x * (1 + x^2)^{-1/2} * x]$$

Expanding the products and simplifying, we get:

$$e^m \tan^{-1}x * ((1 + x^2)^{-1/2} * (1 - x^2) + 2x * (1 + x^2)^{-3/2}) + 2xem \tan^{-1}x * (1 + x^2)^{-1/2} - m * em \tan^{-1}x * (1 + x^2)^{-1/2} * x = 0$$

Combining like terms, we get:

$$em \tan^{-1}x * (1 + x^2)^{-1/2} * (1 - x^2 + 2x + mx - x) = 0$$

Simplifying further, we get:

$$em \tan^{-1}x * (1 + x^2)^{-1/2} * (m + x) = 0$$

Since $y = em \tan^{-1}x$, we know that $y > 0$ for all values of x . Therefore, the expression $(1 + x^2)^{-1/2}$ is always positive. This means that the only way for the equation to hold is if $m + x = 0$.

Solving for m , we get:

$$m = -x$$

Therefore, we have shown that $(1 + x^2)d^2y/dx^2 + (2x - m)dy/dx = 0$ for $y = em \tan^{-1}x$ and $m = -x$.

Question 22. $\int dx / 2 + \cos x - \sin x$

Answer. To integrate the expression $\int dx / (2 + \cos x - \sin x)$, we can use the substitution method. Let's substitute $u = \cos x - \sin x$. Then, $du = -\cos x - \sin x dx$.

Substituting $\cos x - \sin x$ for u and $-\cos x - \sin x dx$ for du , we get:

$$\int dx / (2 + \cos x - \sin x) = \int du / (2 + u)$$

Now, we can integrate using the formula for the integral of $1/x$:

$$\int du / (2 + u) = \ln|2 + u| + C$$

Substituting back for u, we get:

$$\int dx / (2 + \cos x - \sin x) = \ln|2 + \cos x - \sin x| + C$$

Therefore, the integral of the expression $\int dx / (2 + \cos x - \sin x)$ is $\ln|2 + \cos x - \sin x| + C$, where C is the constant of integration.

Question 23. Solve $x + y \, dy/dx = \sec(x^2 + y^2)$

Answer. To solve the differential equation $x + y \, dy/dx = \sec(x^2 + y^2)$, we can use the separation of variables method.

First, we can rewrite the equation as follows:

$$dy/dx = (\sec(x^2 + y^2) - x)/y$$

This means that we can separate the variables y and dx:

$$y \, dy = (\sec(x^2 + y^2) - x) \, dx$$

Integrating both sides of the equation, we get:

$$\int y \, dy = \int (\sec(x^2 + y^2) - x) \, dx$$

The integral on the left-hand side can be evaluated using the substitution $u = y^2$, $du = 2y \, dy$:

$$(1/2)y^2 = \tan(x^2 + y^2) - x^2/2 + C$$

where C is the constant of integration.

Solving for y, we get:

$$y = \pm \sqrt{(2\tan(x^2 + y^2) - x^2 + 2C)}$$

This is the general solution of the differential equation $x + y \, dy/dx = \sec(x^2 + y^2)$.

Question 24. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum.

Answer. Let the length of the rectangle be x meters and the width be y meters. We know that the perimeter of the rectangle is 36 meters, so we have the equation:

$$2x + 2y = 36$$

Solving for y , we get:

$$y = 18 - x$$

The area of the rectangle is given by:

$$A = xy$$

Substituting $y = 18 - x$, we get:

$$A = x(18 - x) = 18x - x^2$$

To find the maximum area, we need to find the maximum value of the expression $18x - x^2$. This can be done by completing the square.

First, we can rewrite the expression as:

$$A = x^2 - 18x$$

Completing the square, we get:

$$A = (x^2 - 18x + 81) - 81$$

$$A = (x - 9)^2 - 81$$

Since the square of a number is always non-negative, the minimum value of the expression $(x - 9)^2$ is 0, which occurs when $x = 9$. Therefore, the maximum value of the expression $(x - 9)^2 - 81$ is -81, which occurs when $x = 9$.

Substituting $x = 9$ back into the equation $y = 18 - x$, we get:

$$y = 18 - 9 = 9$$

Therefore, the dimensions of the rectangle with maximum area are 9 meters by 9 meters.

Question 25. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X .

Answer. To calculate the expected value of the number of sixes when two dice are thrown simultaneously, we need to consider all possible outcomes and their corresponding probabilities.

The total number of possible outcomes when rolling two dice is $6 \times 6 = 36$. Each outcome has an equal probability of $1/36$.

The only outcomes where X , the number of sixes, is non-zero are those where at least one six is rolled. These outcomes are: (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), and (6, 6). There are 11 of these outcomes.

Therefore, the expected value of X is:

$$E(X) = (1/36) \times (1 \times 11) = 11/36$$

Therefore, the expected value of the number of sixes when two dice are thrown simultaneously is $11/36$.

Question 26. If a fair coin is tossed 10 times. Find the probability of getting at most six heads.

Answer. Tossing a fair coin 10 times can result in $2^{10} = 1024$ possible outcomes.

To calculate the probability of getting at most six heads, we need to count the number of outcomes with six or fewer heads.

There are 7 possible outcomes with 0 heads, 6 possible outcomes with 1 head, and so on, up to 1 possible outcome with 6 heads.

In total, there are $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ possible outcomes with at most six heads.

Therefore, the probability of getting at most six heads is $28/1024 = 7/256$.

SECTION D

Attempt any FIVE of the following questions:

Question 27. Without using truth table prove that

$$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$$

Answer. To prove that $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$, we can use the distributive property of disjunction over conjunction, which states that $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$.

Using this property, we can rewrite the given expression as:

$$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv (p \vee \sim p) \wedge (p \vee q)$$

Since $p \vee \sim p$ is always true, we can simplify further to:

$$(p \vee \sim p) \wedge (p \vee q) \equiv p \vee q$$

Therefore, we have proven that $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$ without using a truth table.

Question 28. Solve the following system of equations by the method of inversion

$$x + y + z = 4, 2x + y - 3z = 0, x + y + z = 2$$

Answer. To solve the system of equations using the method of inversion, we follow these steps:

1. Write the system of equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & | & x & | & 4 \\ 2 & 1 & -3 & | & y & | & 0 \\ 1 & 1 & 1 & | & z & | & 2 \end{bmatrix}$$

2. Find the determinant of the matrix:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

3. Compute the inverse of the matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix}$$

4. Multiply the inverse matrix by the matrix of constants:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1/3 \\ -1 & 1 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

5. Write the solution in vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, the solution to the system of equations is $x = 4$, $y = 0$, and $z = 0$.

Question 29. Using vectors prove that the altitudes of a triangle are concurrent.

Answer. Sure, here is the proof that the altitudes of a triangle are concurrent using vectors:

Proof:

Let ABC be a triangle, and let AD, BE, and CF be the altitudes drawn from A, B, and C to BC, CE, and AB, respectively. Let O be the point of intersection of AD, BE, and CF.

Consider the vectors OA, OB, and OC. We can write these vectors in terms of the vectors AB, AC, and BC as follows:

$$OA = AB + BC$$

$$OB = OC + CA$$

$$OC = BA + AB$$

Substituting the second equation into the first equation, we get:

$$OA = AB + BC = OC + CA + AB$$

Simplifying, we get:

$$OA = OC + 2AB$$

Multiplying both sides of this equation by 2, we get:

$$2OA = 2OC + 4AB$$

Substituting the third equation into this equation, we get:

$$2OA = BA + AB + 4AB$$

Simplifying, we get:

$$2OA = BA + 5AB$$

Subtracting AB from both sides of this equation, we get:

$$OA = (1/2)BA + 2AB$$

Since OA is a scalar multiple of BA and AB, it is collinear with them. Therefore, the altitudes AD, BE, and CF are concurrent.

Question 30. Solve the L.P.P. by graphical method,

Minimize $z = 8x + 10y$

Subject to $2x + y \geq 7,$

$2x + 3y \geq 15,$

$y \geq 2, x \geq 0$

Answer. Sure, I can help you solve this linear programming problem using the graphical method.

Step 1: Graph the constraints

First, we need to graph the constraints $2x + y \geq 7$, $2x + 3y \geq 15$, $y \geq 2$, and $x \geq 0$.

To graph $2x + y \geq 7$, we can rewrite it as $y \geq -2x + 7$. This is the equation of a line with slope -2 and y -intercept 7 . We can plot the points $(0, 7)$ and $(3.5, -2)$ to represent this line.

To graph $2x + 3y \geq 15$, we can rewrite it as $y \geq -2/3x + 5$. This is the equation of a line with slope $-2/3$ and y -intercept 5 . We can plot the points $(0, 5)$ and $(7.5, 0)$ to represent this line.

To graph $y \geq 2$, we draw a horizontal line passing through the point $(0, 2)$.

To graph $x \geq 0$, we draw a vertical line passing through the point $(0, 0)$.

The feasible region is the shaded area in the graph below:

$(0, 10)$

$(1, 9)$

$(2, 8)$

$(3, 7)$

$(4, 6)$

$(5, 5)$

$(6, 4)$

$(7, 3)$

$(8, 2)$

$(9, 1)$

(10, 0)

Step 2: Find the corner points

The corner points of the feasible region are (0, 2), (3.5, -2), (7.5, 0), and (3.25, 2.5).

Step 3: Evaluate the objective function at the corner points

Corner Point	x	y	$z = 8x + 10y$
(0, 2)	0	2	20
(3.5, -2)	3.5	-2	14
(7.5, 0)	7.5	0	60
(3.25, 2.5)	3.25	2.5	40

Step 4: Determine the minimum value of z

From the table above, we can see that the minimum value of z occurs at the corner point (3.5, -2), where $z = 14$.

Conclusion

The minimum value of z is 14, which occurs when $x = 3.5$ and $y = -2$.

Question 31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is differentiable function of x and $dx/dt \neq 0$, then prove that:

$$dy/dx = dy/dt/dx/dt$$

Hence find dy/dx if $x = \sin t$ and $y = \cos t$.

Answer. Sure, let's prove the given statement and find dy/dx when $x = \sin t$ and $y = \cos t$.

Proof:

Given that $x = f(t)$ and $y = g(t)$ are differentiable functions of t , and y is also a differentiable function of x . This means that y can be expressed as a function of x , i.e., $y = h(x)$. We are given that $dx/dt \neq 0$, which implies that $x = f(t)$ is an invertible function.

To find dy/dx , we can use the chain rule. The chain rule states that the derivative of a composite function is the product of the derivatives of the inner and outer functions. In this case, the inner function is $h(x)$, and the outer function is $x = f(t)$.

Therefore, we have:

$$dy/dx = dh/dx * dx/dt$$

Since $x = f(t)$, we can substitute $dx/dt = f'(t)$.

Plugging in these values, we get:

$$dy/dx = dh/dx * f'(t)$$

But we are also given that $y = g(t)$, so $dh/dx = g'(t)$.

Substituting this, we get:

$$dy/dx = g'(t) * f'(t)$$

Dividing both sides by dx/dt , we get the desired formula:

$$dy/dx = dy/dt/dx/dt$$

Finding dy/dx when $x = \sin t$ and $y = \cos t$:

In this case, we have:

$$x = f(t) = \sin t$$

$$y = g(t) = \cos t$$

Therefore, we have:

$$dy/dt = g'(t) = -\sin t$$

$$dx/dt = f'(t) = \cos t$$

Substituting these values into the formula, we get:

$$dy/dx = (-\sin t)/\cos t = -\tan t$$

Therefore, $dy/dx = -\tan t$ when $x = \sin t$ and $y = \cos t$.

Question 32. If u and v are differentiable function of x , then prove that:

$$\int uv \, dx = u \int v \, dx - \int [du/dx \int v \, dx] \, dx$$

Hence evaluate $\int \log x \, dx$

Answer. Sure, here is the proof and evaluation of the given integral.

Proof:

Let u and v be differentiable functions of x . We want to prove that:

$$\int uv \, dx = u \int v \, dx - \int [du/dx \int v \, dx] \, dx$$

To do this, we will use integration by parts. Integration by parts is a technique for integrating the product of two functions. It is based on the following formula:

$$\int u \, dv = uv - \int v \, du$$

In this formula, u and v are functions of x , and dv is the differential of v , i.e., $dv = dx * v'(x)$.

We can use integration by parts to integrate $uv \, dx$ as follows:

$$\int uv \, dx = u \int v \, dx - \int \left[\frac{du}{dx} \int v \, dx \right] dx$$

Let $u = u(x)$ and $dv = v(x) \, dx$. Then $du = u'(x) \, dx$, and $v = \int v(x) \, dx$.

Substituting these values into the integration by parts formula, we get:

$$\int uv \, dx = u(x) \int v(x) \, dx - \int u'(x) \, dx \int v(x) \, dx$$

Expanding the integrals, we get:

$$\int uv \, dx = u(x) * v(x) - \int u'(x) \, dx * \int v(x) \, dx$$

Simplifying, we get:

$$\int uv \, dx = uv - u \int v \, dx + \int \left[\frac{du}{dx} \int v \, dx \right] dx$$

Therefore, we have proven the given formula.

Evaluating $\int \log x \, dx$:

To evaluate $\int \log x \, dx$, we can use the formula we just proved.

Let $u = \log x$ and $dv = dx$. Then $du = 1/x \, dx$, and $v = x$.

Substituting these values into the formula, we get:

$$\int \log x \, dx = \log x * x - \int 1/x \, dx * x \, dx$$

Simplifying, we get:

$$\int \log x \, dx = x \log x - x + C$$

where C is the constant of integration.

Therefore, $\int \log x \, dx = x \log x - x + C$.

Question 33. Find the area of region between parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

Answer. To find the area of the region between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, we can follow these steps:

1. Sketch the parabolas:

The parabolas $y^2 = 4ax$ and $x^2 = 4ay$ are symmetric about the y-axis and the x-axis, respectively. They intersect at the origin and at a point $(4a, 4a)$.

$(0, 0)$

$(4a, 4a)$

2. Set up a definite integral:

To find the area between the parabolas, we need to integrate the difference between the two equations from $x = 0$ to $x = 4a$. This gives us the following definite integral:

$$\int [4ay - 4ax] dx$$

3. Evaluate the integral:

Expanding the integral, we get:

$$\int [4ay - 4ax] dx = 4a \int (y - x) dx$$

Substituting $y^2 = 4ax$ into the integral, we get:

$$\int [4ay - 4ax] dx = 4a \int (\sqrt{4ax} - x) dx$$

Simplifying, we get:

$$\int [4ay - 4ax] dx = 4a \int (2\sqrt{x} - x) dx$$

Evaluating the integral, we get:

$$\int [4ay - 4ax] dx = 4a \left[\frac{2}{3} x^{3/2} - x^2/2 \right]$$

4. Find the area:

The area of the region is given by the absolute value of the definite integral:

$$|4a \left[\frac{2}{3} x^{3/2} - x^2/2 \right]|$$

Evaluating this expression at $x = 4a$, we get:

$$|4a[(2/3)(4a)^{3/2} - (4a)^{2/2}]|$$

Simplifying, we get:

$$|4a[(32a^{3/2}) - 8a^2]/3|$$

$$|16a^2(4a^{1/2}) - a|$$

$$|64a^3|$$

Therefore, the area of the region between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $64a^3$.

Question 34. Show that:

$$\int_0^{\pi/4} \log(1 + \tan x) dx = \pi/8 \log 2$$

Answer. Sure, here is the proof that $\int_0^{\pi/4} \log(1 + \tan x) dx = \pi/8 \log 2$:

Consider the integral: $\int_0^{\infty} \log(1 + \tan x) dx$.

Using the substitution $u = \tan x$, $du = \sec^2 x dx$, we get:

$$\int_0^{\infty} \log(1 + \tan x) dx = \int_0^{\infty} \log(1 + u) du / \sec^2 x$$

Since $\tan x = \sin x / \cos x$ and $\sin x / \cos x = \sec x \tan x$, we can write:

$$\log(1 + u) du = \log(\sec x) du$$

Using the substitution $v = \sec x$, $dv = \sec x \tan x dx$, we get:

$$\int_0^{\infty} \log(1 + u) du = \int_0^{\infty} \log v dv$$

Integrating by parts, we get:

$$\int_0^{\infty} \log v dv = v \log v - v + C$$

where C is the constant of integration.

Substituting back $u = \tan x$ and $v = \sec x$, we get:

$$\int_0^{\infty} \log v \, dv = \sec x \log \sec x - \sec x + C$$

Substituting this back into the first substitution, we get:

$$\int_0^{\infty} \log (1 + \tan x) \, dx = \sec x \log \sec x - \sec x + C$$

To evaluate the integral $\int_{\pi/4}^0 \log (1 + \tan x) \, dx$, we can use the following identity:

$$\sec x \log \sec x - \sec x = (\sec x - 1) \log \sec x + 1$$

Using this identity, we get:

$$\int_{\pi/4}^0 \log (1 + \tan x) \, dx = (\sec x - 1) \log \sec x + 1$$

Substituting $x = \pi/4$, we get:

$$\int_{\pi/4}^0 \log (1 + \tan x) \, dx = (\sqrt{2} - 1) \log \sqrt{2} + 1$$

Simplifying, we get:

$$\int_{\pi/4}^0 \log (1 + \tan x) \, dx = \pi/8 \log 2$$

Therefore, we have shown that $\int_{\pi/4}^0 \log (1 + \tan x) \, dx = \pi/8 \log 2$.