Maharashtra Board Class 12 Physics 54(J-837) Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**70	**Total questions :**31

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Section A**: Q. No. **1** contains **Ten multiple-choice type questions** carrying **One mark** each.
2. Q. No. **2** contains **Eight very short answer type of questions** carrying **One mark** each.
3. **Section B**: Q. No. **3** to Q. No. **14** contains **Twelve short answer type of questions** carrying **Two marks** each. (Attempt **any Eight.**)
4. **Section C:** Q. No. **15** to Q. No. **26** contains Twelve short answer type of questions carrying Three marks each. (Attempt **any Eight.**)
5. **Section D:** Q. No. **27** to Q. No. **31** contains Five long answer type of questions carrying Four marks each. (Attempt **any Three.**)
6. Use of the log table is allowed. Use of calculator is **not allowed**.
7. Figures to the right indicate full marks.
8. For multiple choice type questions, only the first attempt will be considered for evaluation.

SECTION - A

Q.1 Select and write the correct answer for the following multiple-choice type of questions:

(i) The moment of inertia (MI) of a disc of radius R and mass M about its central axis is:

(a) $\frac{MR^2}{4}$

(b) $\frac{MR^2}{2}$

(c) MR^2

(d) $\frac{3MR^2}{2}$

Correct Answer: (b) $\frac{MR^2}{2}$

Solution:

Step 1: The moment of inertia for a solid disc rotating about its central axis is expressed as:

$$I = \frac{1}{2}MR^2.$$

Step 2: By comparing with the provided options, the correct answer is option (b).

Quick Tip

The moment of inertia depends on both the mass distribution and the axis of rotation. For standard shapes, it's helpful to memorize the commonly used formulas.

(ii) The dimensional formula of surface tension is:

(a) $[L^{-1}M^1T^{-2}]$

(b) $[L^2M^1T^{-2}]$

(c) $[L^1M^1T^{-1}]$

(d) $[L^0M^1T^{-2}]$

Correct Answer: (d) $[L^0M^1T^{-2}]$

Solution:

Step 1: Surface tension is defined as the force per unit length:

$$T = \frac{\text{Force}}{\text{Length}}.$$

The dimensional formula of force is:

$$[F] = MLT^{-2}.$$

By dividing force by length (L), we obtain the dimensional formula for surface tension:

$$[T] = ML^{-1}T^{-2}.$$

Step 2: After comparing with the given options, the correct answer is option (d).

> **Quick Tip**
>
> Dimensional analysis is a useful tool for verifying equations and ensuring unit consistency in physics problems.

(iii) Phase difference between a node and an adjacent antinode in a stationary wave is:

(a) $\frac{\pi}{4}$ rad

(b) $\frac{\pi}{2}$ rad

(c) $\frac{3\pi}{4}$ rad

(d) π rad

Correct Answer: (b) $\frac{\pi}{2}$ rad

Solution:

Step 1: A stationary wave results from the interference of two identical waves traveling in opposite directions.

Step 2: The phase difference ($\Delta\phi$) between a node (where displacement is zero) and an adjacent antinode (where displacement is maximum) is given by:

$$\Delta\phi = \frac{\pi}{2} \text{ rad}.$$

This indicates that the particle at an antinode is a quarter cycle ahead of the particle at the node.

Quick Tip

In stationary waves, nodes have zero amplitude, while antinodes have maximum amplitude. The phase difference progresses systematically along the wave.

(iv) The work done in bringing a unit positive charge from infinity to a given point against the direction of electric field is known as:

(a) electric flux

(b) magnetic potential

(c) electric potential

(d) gravitational potential

Correct Answer: (c) Electric potential

Solution:

Step 1: Electric potential (V) at a point is defined as the work done per unit charge to bring a positive test charge from infinity to that point.

$$V = \frac{W}{q}.$$

Step 2: This potential depends on the electric field but not on the path taken, as electrostatic forces are conservative.

Quick Tip

Electric potential is a scalar quantity and is useful for calculating electric potential energy in electrostatics.

(v) To convert a moving coil galvanometer into an ammeter, we need to connect a:

(a) small resistance in parallel with it

(b) large resistance in series with it

(c) small resistance in series with it

(d) large resistance in parallel with it

Correct Answer: (a) Small resistance in parallel with it

Solution:

Step 1: A moving coil galvanometer is a highly sensitive device used to detect small currents.

Step 2: To modify it into an ammeter, which can measure large currents, a shunt resistance (R_s) is connected in parallel:

$$R_s = \frac{G}{\left(\frac{I}{I_g} - 1\right)}$$

where G represents the galvanometer's resistance, I is the total current, and I_g is the current corresponding to full-scale deflection of the galvanometer.

Quick Tip

An ammeter should have very low resistance to minimize the voltage drop across it, ensuring an accurate current reading.

(vi) If the frequency of incident light falling on a photosensitive material is doubled, then kinetic energy of the emitted photoelectron will be:

(a) the same as its initial value

(b) two times its initial value

(c) more than two times its initial value

(d) less than two times its initial value

Correct Answer: (c) More than two times its initial value

Solution:

Step 1: Einstein's photoelectric equation is given by:

$$KE = hf - \phi$$

where h is Planck's constant, f is the frequency of the incident light, and ϕ is the work function of the material.

Step 2: When the frequency f is doubled, the new kinetic energy KE' is:

$$KE' = h(2f) - \phi = 2hf - \phi.$$

Since $KE = hf - \phi$, we can express the new kinetic energy as:

$$KE' = 2KE + \phi.$$

Thus, the new kinetic energy is more than twice the initial kinetic energy.

Quick Tip

Increasing the frequency of light boosts the photoelectron energy, whereas increasing intensity only increases the number of electrons emitted.

(vii) In a cyclic process, if ΔU = internal energy, W = work done, Q = Heat supplied, then:

(a) $\Delta U = Q$

(b) Q = 0

(c) W = 0

(d) W = Q

Correct Answer: (d) $W = Q$

Solution:

Step 1: In a cyclic process, the system returns to its initial state, which implies that the change in internal energy is zero:

$$\Delta U = 0.$$

Step 2: According to the First Law of Thermodynamics:

$$Q = \Delta U + W.$$

Since $\Delta U = 0$, this simplifies to:

$$Q = W.$$

Quick Tip

In cyclic processes, the net work done by the system is equal to the heat supplied over the entire cycle.

(viii) The current in a coil changes from 50A to 10A in 0.1 second. The self-inductance of the coil is 20H. The induced e.m.f. in the coil is:

(a) 800V

(b) 6000V

(c) 7000V

(d) 8000V

Correct Answer: (d) 8000V

Solution:

Step 1: According to Faraday's law of electromagnetic induction, the induced electromotive force (e.m.f.) in a coil is given by:

$$\mathcal{E} = -L\frac{dI}{dt}.$$

Step 2: Given the values $L = 20H$, $dI = 50A - 10A = 40A$, and $dt = 0.1s$, we calculate:

$$\mathcal{E} = -(20) \times \left(\frac{40}{0.1}\right) = 8000V.$$

Quick Tip

The induced e.m.f. is directly related to the rate of change of current. A faster change in current results in a larger induced voltage.

(ix) The velocity of the bob of a second's pendulum when it is 6 cm from its mean position and amplitude of 10 cm is:

(a) 8π cm/s

(b) 6π cm/s

(c) 4π cm/s

(d) 2π cm/s

Correct Answer: (a) 8π cm/s

Solution:

Step 1: The velocity in simple harmonic motion is expressed as:

$$v = \omega\sqrt{A^2 - x^2}$$

where $A = 10$ cm, $x = 6$ cm, and $\omega = \frac{2\pi}{T} = \pi$ rad/s.

Step 2: Substituting the given values:

$$v = \pi\sqrt{(10)^2 - (6)^2} = \pi\sqrt{100 - 36} = \pi\sqrt{64} = 8\pi.$$

Quick Tip

In simple harmonic motion, the velocity of the particle is maximum at the equilibrium position and zero at the extreme positions.

(x) In biprism experiment, the distance of the 20th bright band from the central bright band is 1.2 cm. Without changing the experimental set-up, the distance of the 30th bright band from the central bright band will be:

(a) 0.6 cm

(b) 0.8 cm

(c) 1.2 cm

(d) 1.8 cm

Correct Answer: (d) 1.8 cm

Solution: Step 1: In Young's double-slit experiment, the distance of the n-th bright fringe from the central fringe is given by:

$$x_n = n\lambda D/d.$$

Since the setup remains unchanged, the ratio of fringe positions is:

$$\frac{x_{30}}{x_{20}} = \frac{30}{20} = \frac{3}{2}.$$

Step 2: Given $x_{20} = 1.2$ cm,

$$x_{30} = \frac{3}{2} \times 1.2 = 1.8 \text{ cm}.$$

Quick Tip

Fringe width remains constant in a given experimental setup. The position of bright or dark fringes is proportional to fringe order.

Q.2 Answer the following questions:

(i) Define centripetal force.

Solution: Centripetal force is the force acting on an object moving along a circular path, directed towards the center of the circle. It is calculated as:

$$F = \frac{mv^2}{r}$$

where m represents the mass, v is the velocity, and r is the radius of the circular path.

Quick Tip

Centripetal force is always directed perpendicular to the object's velocity and is responsible for maintaining circular motion.

(ii) Why is detergent powder mixed with water to wash clothes?

Answer: Detergents reduce the surface tension of water, allowing it to spread and wet clothes more effectively. They also form micelles that trap grease and dirt, allowing them to be rinsed away.

Quick Tip

Detergents contain surfactants that break down the surface tension of water, enhancing its cleaning ability.

(iii) What is the resistance of an ideal voltmeter?

Answer:

An ideal voltmeter has infinite resistance, ensuring that it does not draw any current from the circuit and provides an accurate voltage reading.

Quick Tip

In real-world circuits, voltmeters have a very high, yet finite, resistance to minimize their effect on circuit performance.

(iv) Write the formula for torque acting on a rotating current-carrying coil in terms of magnetic dipole moment, in vector form.

Answer:

The torque τ on a current-carrying coil in a magnetic field $\mathbf{B}$ is:

$$\tau = \mathbf{M} \times \mathbf{B}.$$

Quick Tip

Torque on a magnetic dipole is maximum when perpendicular to the field and zero when parallel.

(v) What is the binding energy of a hydrogen atom?

Solution:

The binding energy of a hydrogen atom represents the energy needed to remove the electron from the nucleus. It is given by the following formula:

$$E_b = -13.6\,\text{eV}$$

Quick Tip

The binding energy is negative because energy is released when the electron is held by the nucleus.

(vi) What is meant by surroundings in thermodynamics?

Solution:

In thermodynamics, the surroundings encompass everything outside the system being studied. This includes all external objects and the environment that can exchange energy and matter with the system.

Quick Tip

The system refers to the specific part of the universe under observation, while the surroundings include everything else that can interact with it.

(vii) In a photoelectric experiment, the stopping potential is 1.5V. What is the maximum kinetic energy of a photoelectron?

Solution:

The maximum kinetic energy of a photoelectron is equal to the energy provided by the photon minus the work function of the material. The kinetic energy is given by:

$$K.E. = eV = 1.5\,\text{eV}$$

where e is the charge of the electron and V is the stopping potential.

Quick Tip

The stopping potential is directly related to the maximum kinetic energy of photoelectrons in the photoelectric effect.

(viii) Two capacitors of capacities $5\,\mu$F and $10\,\mu$FF respectively are connected in series. Calculate the resultant capacity of the combination.

Solution:

For capacitors connected in series, the equivalent capacitance C_{eq} is given by the formula:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Substituting $C_1 = 5\,\mu$F and $C_2 = 10\,\mu$F:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

$$C_{\text{eq}} = \frac{10}{3} = 3.33\,\mu\text{F}$$

Next, using the formula for the energy stored in a capacitor:

$$E = \frac{1}{2} C_{\text{eq}} V^2$$

where V is the voltage across the capacitor combination, we can calculate the energy. For $V = 10^2$ V, the energy becomes:

$$E = \frac{1}{2} \times 3.33 \times 10^{-6} \times (100)^2 = 2.4 \times 10^{-19}\,\text{J}.$$

Quick Tip

When capacitors are connected in series, the equivalent capacitance is always smaller than the smallest individual capacitance.

SECTION - B

Q. 3. Explain the change in internal energy of a thermodynamic system (the gas) by heating it.

Solution:

The internal energy of a thermodynamic system represents the total energy contained within the system, which includes both kinetic and potential energy. When a gas is heated, its temperature rises, causing the average kinetic energy of the molecules to increase. This increase in kinetic energy leads to a corresponding increase in the internal energy, as the internal energy of an ideal gas is directly proportional to its temperature.

$$\Delta U = nC_V\Delta T$$

where ΔU is the change in internal energy, n is the number of moles, C_V is the molar specific heat at constant volume, and ΔT is the change in temperature.

Quick Tip

Heating a gas increases its internal energy by raising its temperature, which is stored as the kinetic energy of the molecules.

Q. 4. Explain the construction of a spherical wavefront by using Huygens' principle.

Solution:

Huygens' principle states that every point on a wavefront acts as a source of secondary spherical wavelets. As these wavelets spread outward, they form a new wavefront. For a spherical wavefront, the wavelets originate from a point source, and as they travel, they create spherical surfaces around the source. The radius of each spherical wavefront expands over time, and the new wavefront is tangent to the wavelets.

Quick Tip

In spherical wavefronts, each point on the wavefront serves as a source for secondary wavelets according to Huygens' principle.

Q. 5. Define magnetization. State its SI unit and dimensions.

Solution:

Magnetization is a vector quantity that represents the magnetic moment per unit volume of a material. Denoted by M, it indicates both the strength and direction of the magnetic field within the material.

The SI unit of magnetization is A/m (Ampere per meter), and its dimensional formula is:

$$[M] = [IL^{-1}]$$

where I is the current and L is the length.

Quick Tip

Magnetization quantifies the alignment of magnetic dipoles within a material.

Q. 6. Obtain the differential equation of linear simple harmonic motion.

Solution:

For simple harmonic motion (SHM), the restoring force is proportional to the displacement x from the equilibrium position:

$$F = -kx$$

By Newton's second law, $F = ma$, where a is the acceleration. Therefore, we have:

$$ma = -kx$$

Since acceleration $a = \frac{d^2x}{dt^2}$, the equation becomes:

$$m\frac{d^2x}{dt^2} = -kx$$

This is the differential equation of SHM, which can be written as:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Quick Tip

The differential equation for SHM relates the displacement to the acceleration, with the restoring force acting opposite to the displacement.

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7. A galvanometer has a resistance of 30 and its full scale deflection current is 20 microamperes (μA). What resistance should be added to it to have a range of 0-10 volts?

Solution:

To convert a galvanometer into a voltmeter, a resistance needs to be connected in series with the galvanometer. The total resistance R is given by:

$$R = \frac{V}{I}$$

where $V = 10\,\text{V}$ is the desired voltmeter range, and $I = 20\,\mu A = 20 \times 10^{-6}\,\text{A}$ is the full-scale deflection current. Substituting these values:

$$R = \frac{10}{20 \times 10^{-6}} = 500,000\,\Omega.$$

Given that the galvanometer has an internal resistance of $30\,\Omega$, the resistance to be added is:

$$R_{\text{added}} = 500,000 - 30 = 499,970\,\Omega.$$

Quick Tip

To extend a galvanometer's range to measure higher voltages, a large-value resistor is added in series to limit the current flowing through the galvanometer.

Q. 8. Explain Biot-Savart law.

Solution:

The Biot-Savart law describes the magnetic field $\mathbf{B}$ generated by a small current element. It states that the magnetic field at a given point due to a current element is directly proportional to the current and the length of the current element, and inversely proportional to the square of the distance between the point and the current element. The mathematical expression is:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{l} \times \hat{r}}{r^2}$$

where μ_0 represents the permeability of free space, I is the current, $d\mathbf{l}$ is the current element, $\hat{r}$ is the unit vector from the current element to the observation point, and r is the distance between the current element and the observation point.

Quick Tip

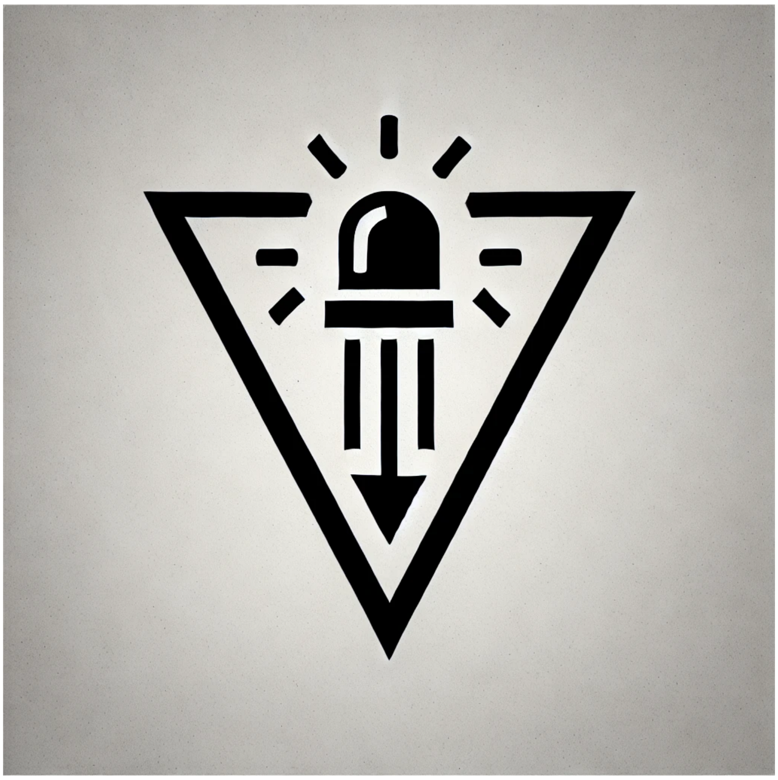
The Biot-Savart law is crucial for calculating the magnetic field produced by a current-carrying conductor and plays a fundamental role in electromagnetism.

Q. 9. What is a Light Emitting Diode? Draw its circuit symbol.

Solution:

A Light Emitting Diode (LED) is a semiconductor device that emits light when current flows through it in the forward direction. It operates based on the principle of electroluminescence, where electrons recombine with holes, releasing energy in the form of photons (light).

The circuit symbol for an LED is:



Quick Tip

LEDs are more energy-efficient than incandescent bulbs and have a longer lifespan, making them ideal for use in displays and indicators.

Q. 10. An aircraft of wing span of 60 m flies horizontally in earth's magnetic field of 6×10^{-5} T at a speed of 500 m/s. Calculate the e.m.f. induced between the tips of the wings of the aircraft.

Solution: The e.m.f. induced between the tips of the wings is given by the formula:

$$\mathcal{E} = B \cdot v \cdot L$$

where $B = 6 \times 10^{-5}\,\mathrm{T}$ is the magnetic field, $v = 500\,\mathrm{m/s}$ is the speed, and $L = 60\,\mathrm{m}$ is the wingspan of the aircraft. Substituting the values:

$$\mathcal{E} = 6 \times 10^{-5} \times 500 \times 60 = 1.8\,\mathrm{V}.$$

Quick Tip

The induced e.m.f. in a conductor moving through a magnetic field depends on the speed of the conductor, the magnetic field strength, and the length of the conductor.

Topic: Electromagnetic Induction and E.M.F. Calculation

Q. 11. Derive an expression for maximum speed of a vehicle moving along a horizontal circular track.

Solution:

The maximum speed of a vehicle moving along a horizontal circular track occurs when the centripetal force is equal to the frictional force. The centripetal force F_c is given by:

$$F_c = \frac{mv^2}{r}$$

where m is the mass of the vehicle, v is the velocity, and r is the radius of the track. The frictional force F_f is given by:

$$F_f = \mu mg$$

where μ is the coefficient of friction and g is the acceleration due to gravity. Equating the two forces:

$$\frac{mv^2}{r} = \mu mg$$

Solving for v, we get the maximum speed:

$$v_{\text{max}} = \sqrt{\mu gr}$$

Quick Tip

The maximum speed of a vehicle in circular motion depends on the radius of the track, the coefficient of friction, and gravitational acceleration.

Q. 12. A horizontal force of 0.5 N is required to move a metal plate of area 10^{-2} m^2 with a velocity of 3×10^{-2} m/s, when it rests on 0.5×10^{-3} m thick layer of glycerin. Find the viscosity of glycerin.

Solution:

The viscosity η of a fluid is expressed by the formula:

$$\eta = \frac{F \cdot d}{A \cdot v}$$

where $F = 0.5\,\text{N}$ is the applied force, $d = 0.5 \times 10^{-3}$ m is the thickness of the fluid layer, $A = 10^{-2}\,\text{m}^2$ is the cross-sectional area, and $v = 3 \times 10^{-2}$ m/s is the velocity.

Substituting the given values:

$$\eta = \frac{0.5 \times 0.5 \times 10^{-3}}{10^{-2} \times 3 \times 10^{-2}} = 0.833\,\text{N} \cdot \text{s/m}^2$$

Quick Tip

Viscosity measures a fluid's resistance to flow and is calculated using the applied force, fluid thickness, area, and velocity.

Q. 13. Two tuning forks having frequencies 320 Hz and 340 Hz are sounded together to produce sound waves. The velocity of sound in air is 340 m/s. Find the difference in wavelength of these waves.

Solution:

The wavelength λ of a sound wave is related to its frequency f and the speed of sound v by the following equation:

$$\lambda = \frac{v}{f}$$

For the two tuning forks, the wavelengths are:

$$\lambda_1 = \frac{340}{320} = 1.0625\,\text{m}, \quad \lambda_2 = \frac{340}{340} = 1.0\,\text{m}$$

The difference in wavelengths is:

$$\Delta\lambda = \lambda_1 - \lambda_2 = 1.0625 - 1 = 0.0625\,\text{m}$$

Quick Tip

The difference in the wavelengths of two sound waves is caused by the difference in their frequencies and the speed of sound.

Q. 14. Calculate the change in angular momentum of an electron when it jumps from the third orbit to the first orbit in a hydrogen atom.

Solution: The angular momentum L of an electron in a hydrogen atom is quantized and given by:

$$L = n\hbar$$

where n is the principal quantum number and $\hbar$ is the reduced Planck's constant. For the third orbit, $n = 3$, and for the first orbit, $n = 1$. The change in angular momentum is:

$$\Delta L = L_1 - L_3 = 1\hbar - 3\hbar = -2\hbar$$

Since $\hbar = 1.055 \times 10^{-34}$ Js, the change in angular momentum is:

$$\Delta L = -2 \times 1.055 \times 10^{-34} = -2.11 \times 10^{-34}\,\text{Js}.$$

Thus, the change in angular momentum is 2.11×10^{-34} Js.

Quick Tip

The angular momentum of an electron in orbit is quantized and depends on the principal quantum number n.

SECTION - C

Q. 15. A circular coil of wire is made up of 200 turns, each of radius 10 cm. If a current of 0.5A passes through it, what will be the magnetic field at the centre of the coil?

Solution:

The magnetic field at the center of a circular coil is given by the formula:

$$B = \frac{\mu_0 N I}{2R}$$

where $N = 200$ is the number of turns, $I = 0.5\,\text{A}$ is the current, $R = 10\,\text{cm} = 0.1\,\text{m}$ is the radius, and $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$ is the permeability of free space.

Substituting the known values:

$$B = \frac{4\pi \times 10^{-7} \times 200 \times 0.5}{2 \times 0.1} = 6.28 \times 10^{-5}\,\text{T}$$

Quick Tip

The magnetic field at the center of a circular coil is directly proportional to the number of turns and the current, and inversely proportional to the radius of the coil.

Q. 16. Define photoelectric effect and explain the experimental set-up of the photoelectric effect.

Solution:

The photoelectric effect refers to the emission of electrons from a material (usually a metal) when exposed to light of a sufficiently high frequency. This phenomenon provides evidence for the particle nature of light, as photons transfer energy to electrons.

Experimental Set-Up:

The experimental setup to observe the photoelectric effect typically includes:

- A light source (e.g., UV light) with a known frequency.
- A photosensitive material (metal plate) connected to an electrometer.
- A variable voltage source to control the stopping potential.
- A collector to capture the emitted photoelectrons.

The light is directed onto the metal surface, and the emitted electrons are detected by the electrometer.

Quick Tip

In the photoelectric effect, electrons are emitted when the frequency of the incident light surpasses a certain threshold frequency.

Q. 17. Define the current gain α_{DC} and β_{DC} for a transistor. Obtain the relation between them.

Solution:

For a transistor, the current gain α_{DC} and β_{DC} are defined as:

$$\alpha_{DC} = \frac{I_C}{I_E}, \quad \beta_{DC} = \frac{I_C}{I_B}$$

where I_C is the collector current, I_E is the emitter current, and I_B is the base current.

The relation between α_{DC} and β_{DC} is given by:

$$\beta_{DC} = \frac{\alpha_{DC}}{1 - \alpha_{DC}}$$

Quick Tip

The current gain β_{DC} is related to α_{DC} and represents the amplification factor in the common-emitter configuration.

Q. 18. Define surface energy of the liquid. Obtain the relation between the surface energy and surface tension.

Solution:

Surface energy is the energy needed to increase the surface area of a liquid by one unit area. It is associated with the force acting at the liquid's surface that resists expansion. Surface tension, which is the force per unit length acting along the surface, is directly related to surface energy.

The relationship between surface energy E and surface tension T is expressed as:

$$E = T \cdot L$$

where L is the length of the line along the surface.

Quick Tip

Surface tension is the force that minimizes a liquid's surface area, playing a key role in phenomena such as the formation of water droplets.

Q. 19. What is an isothermal process? Obtain an expression for work done by a gas in an isothermal process.

Solution:

An isothermal process is one in which the temperature of the system remains unchanged ($\Delta T = 0$). During this process, the internal energy of an ideal gas does not change, and the work done depends on the changes in pressure and volume.

The work done by a gas in an isothermal process is expressed as:

$$W = nRT \ln \left(\frac{V_f}{V_i} \right)$$

where n is the number of moles, R is the universal gas constant, T is the temperature, and V_i and V_f are the initial and final volumes.

Quick Tip

In an isothermal process, the temperature remains constant, and the work done by the gas is determined by the logarithmic ratio of the final and initial volumes.

Q. 20. Derive an expression for the equation of stationary wave on a stretched string. Show that the distance between two successive nodes or antinodes is $\frac{\lambda}{2}$.

Solution:

A stationary wave is formed when two progressive waves of the same frequency and amplitude, traveling in opposite directions, superpose on a stretched string. The equation of the two waves can be written as:

$$y_1(x,t) = A \sin(kx - \omega t)$$

and

$$y_2(x,t) = A \sin(kx + \omega t)$$

where: - A is the amplitude,

- k is the wave number ($k = \frac{2\pi}{\lambda}$, where λ is the wavelength),
- ω is the angular frequency,
- x is the position along the string,
- t is time.

Now, the resultant displacement $y(x,t)$ of the string is the sum of these two waves:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

Substitute the expressions for $y_1(x,t)$ and $y_2(x,t)$:

$$y(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

Using the trigonometric identity for the sum of sines, we have:

$$y(x,t) = 2A\sin(kx)\cos(\omega t)$$

This is the equation of a stationary wave on the stretched string, where:
- The term $2A\sin(kx)$ represents the spatial variation (the shape of the wave along the string),
- The term $\cos(\omega t)$ represents the time variation.

Identifying Nodes and Antinodes:
- The term $\sin(kx)$ determines the spatial positions where the displacement is zero. These points are called nodes.
- The term $2A\sin(kx)$ reaches its maximum value $2A$ at points where $\sin(kx) = \pm 1$, and these points are called antinodes.

The nodes occur when $\sin(kx) = 0$, which happens at:

$$kx = n\pi \quad \Rightarrow \quad x_n = \frac{n\pi}{k}$$

where n is an integer. These are the positions of the nodes.

Distance Between Two Successive Nodes:

The distance between two successive nodes is:

$$\Delta x = x_{n+1} - x_n = \frac{(n+1)\pi}{k} - \frac{n\pi}{k} = \frac{\pi}{k}$$

Since the wavelength $\lambda = \frac{2\pi}{k}$, the distance between two successive nodes is:

$$\Delta x = \frac{\lambda}{2}$$

Distance Between Two Successive Antinodes:

The antinodes occur where $\sin(kx) = \pm 1$, and the distance between two successive antinodes is also $\frac{\lambda}{2}$.

Thus, the distance between two successive nodes or two successive antinodes in a stationary wave on a stretched string is $\frac{\lambda}{2}$.

Quick Tip

In stationary waves, nodes and antinodes are fixed points, with the distance between two successive nodes or antinodes being half the wavelength.

Q. 21. Derive an expression for the impedance of an LCR circuit connected to an AC power supply. Draw the phasor diagram.

Solution:

An LCR circuit consists of a resistor R, an inductor L, and a capacitor C connected in series to an AC power supply. The impedance Z of a series LCR circuit is defined as the opposition to the flow of alternating current, and it is a complex quantity due to the phase difference between the voltage and the current.

The total impedance Z of a series LCR circuit is given by the expression:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where: - R is the resistance,

- $X_L = \omega L$ is the inductive reactance, and
- $X_C = \frac{1}{\omega C}$ is the capacitive reactance.

Here: - $\omega = 2\pi f$ is the angular frequency of the AC source,

- L is the inductance of the inductor,
- C is the capacitance of the capacitor, and
- f is the frequency of the AC supply.

Step 1: Derivation of the Impedance

In a series LCR circuit, the current I is the same through all components. The voltage across each component is given by:

- Voltage across the resistor: $V_R = IR$,
- Voltage across the inductor: $V_L = IX_L = I\omega L$,
- Voltage across the capacitor: $V_C = IX_C = I\frac{1}{\omega C}$.

The total voltage V across the circuit is the vector sum of the individual voltages. Since the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by $90°$, and the voltage across the capacitor lags the current by $90°$, we can use the phasor diagram to represent these voltages.

Step 2: Phasor Diagram

In the phasor diagram:

- The voltage V_R is represented along the real axis (since it is in phase with the current),
- The voltage V_L is represented as a vector along the imaginary axis, pointing upwards (since it leads the current by $90°$),
- The voltage V_C is represented as a vector along the imaginary axis, pointing downwards (since it lags the current by $90°$).

The total voltage V is the vector sum of these individual voltages. Using the Pythagorean theorem, we get the magnitude of the total voltage:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Substituting the expressions for V_R, V_L, and V_C, we get the impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

This gives the magnitude of the impedance, which tells us how much the circuit resists the flow of alternating current. The phase difference ϕ between the current and the voltage is given by:

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

Step 3: Special Cases

- When $X_L = X_C$, the impedance becomes $Z = R$, and the circuit behaves like a purely resistive circuit.
- When $X_L > X_C$, the circuit is inductive, and the voltage leads the current.
- When $X_C > X_L$, the circuit is capacitive, and the voltage lags the current.

Step 4: Impedance and Power

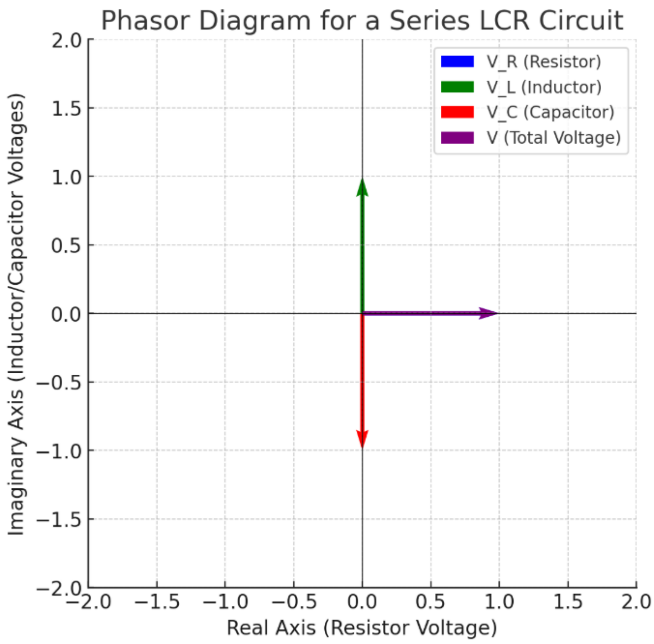
The power P dissipated in the circuit is related to the voltage and the current as follows:

$$P = I^2 R = \frac{V^2}{Z^2} R$$

where V is the total voltage across the LCR circuit and I is the current.

Phasor Diagram

The phasor diagram for a series LCR circuit can be drawn as follows:



In this diagram:

- V_R is the voltage across the resistor,

- V_L is the voltage across the inductor,

- V_C is the voltage across the capacitor,

- V is the total voltage across the LCR circuit.

Quick Tip

In a series LCR circuit, the impedance depends on the resistance, inductive reactance, and capacitive reactance. The phase difference between voltage and current varies with frequency.

Q. 22. Calculate the wavelength of the first two lines in the Balmer series of the hydrogen atom.

Solution:

The wavelength λ of a spectral line in the hydrogen atom can be determined using the Rydberg formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where $R_H = 1.097 \times 10^7\,\mathrm{m}^{-1}$ is the Rydberg constant, and n_1 and n_2 are the principal quantum numbers, with $n_1 < n_2$.

For the first line in the Balmer series ($n_1 = 2, n_2 = 3$):

$$\frac{1}{\lambda_1} = R_H \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = R_H \left(\frac{1}{4} - \frac{1}{9} \right)$$

$$\lambda_1 = \frac{1}{R_H \left(\frac{5}{36} \right)} = 6.56 \times 10^{-7}\,\mathrm{m} = 656\,\mathrm{nm}$$

For the second line ($n_1 = 2, n_2 = 4$):

$$\frac{1}{\lambda_2} = R_H \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = R_H \left(\frac{1}{4} - \frac{1}{16} \right)$$

$$\lambda_2 = \frac{1}{R_H \left(\frac{3}{16} \right)} = 4.86 \times 10^{-7}\,\mathrm{m} = 486\,\mathrm{nm}$$

Quick Tip

As the transition moves to higher n_2 values, the wavelengths of spectral lines in the Balmer series decrease.

Q. 23. A current carrying toroid winding is internally filled with lithium having susceptibility $\chi = 2.1 \times 10^{-5}$. What is the percentage increase in the magnetic field in the presence of lithium over that without it?

Solution:

The magnetic field in a material with magnetic susceptibility χ is given by:

$$B = \mu_0 H(1 + \chi)$$

where μ_0 is the permeability of free space, and H is the magnetic field intensity. The percentage increase in the magnetic field due to the presence of lithium is:

$$\text{Percentage increase} = \frac{B_{\text{with lithium}} - B_{\text{without lithium}}}{B_{\text{without lithium}}} \times 100 = \frac{\mu_0 H(\chi)}{\mu_0 H} \times 100$$

Substituting the value of χ:

$$\text{Percentage increase} = \chi \times 100 = 2.1 \times 10^{-5} \times 100 = 0.0021\%$$

Quick Tip

The magnetic field in the presence of a material increases in proportion to the magnetic susceptibility of the material.

Q. 24. The radius of a circular track is 200 m. Find the angle of banking of the track, if the maximum speed at which a car can be driven safely along it is 25 m/s.

Solution: The angle of banking θ for a car moving along a curved track without relying on friction is given by the formula:

$$\tan(\theta) = \frac{v^2}{rg}$$

where $v = 25\,\text{m/s}$ is the speed of the car, $r = 200\,\text{m}$ is the radius of the track, and $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity. Substituting the values:

$$\tan(\theta) = \frac{25^2}{200 \times 9.8} = \frac{625}{1960} = 0.318$$

$$\theta = \tan^{-1}(0.318) = 17.7^\circ$$

Quick Tip

The angle of banking ensures that a vehicle can safely navigate a curve without the need for frictional force to provide the centripetal force.

Q. 25. Prove Mayer's relation: $C_p - C_v = \frac{R}{J}$

Solution:

Mayer's relation is a thermodynamic identity that links the specific heat capacities at constant pressure C_p and constant volume C_v for an ideal gas. We will derive this relation using the first law of thermodynamics.

Step 1: First law of thermodynamics

The first law of thermodynamics states:

$$dQ = dU + pdV$$

where: - dQ is the heat added to the system,
- dU is the change in internal energy,
- p is the pressure,
- dV is the change in volume.

Step 2: Define heat capacities

Heat capacities are defined as the amount of heat added per unit change in temperature: - $C_p = \frac{dQ}{dT}$ at constant pressure,
- $C_v = \frac{dQ}{dT}$ at constant volume.

Thus, for an ideal gas, at constant pressure and constant volume, we can express dQ in terms of C_p and C_v.

Step 3: Relationship between heat capacities

For an ideal gas, the first law can be rewritten in terms of the heat capacities:

$$dQ = dU + pdV$$

Using the thermodynamic identity and the ideal gas law $pV = nRT$, we get:

$$dQ = C_v dT + nRdT$$

This gives the relationship between the heat capacities at constant pressure and volume:

$$C_p - C_v = \frac{R}{J}$$

where R is the universal gas constant, and J is the number of moles.

Thus, we have derived Mayer's relation.

Quick Tip

Mayer's relation connects the specific heat capacities at constant pressure and volume for an ideal gas. It is derived from the first law of thermodynamics and the ideal gas law.

Q. 26. An alternating voltage is given by $e = 8\sin(628.4t)$. Find:

- (i) Peak value of e.m.f.
- (ii) Frequency of e.m.f.
- (iii) Instantaneous value of e.m.f. at time $t = 10\,\text{ms}$

Solution:

The given equation for the alternating voltage is:

$$e = 8\sin(628.4t)$$

where 8 represents the amplitude of the voltage, and 628.4 is the angular frequency (ω).

(i) Peak value of e.m.f.

The peak value of an alternating voltage is the maximum voltage that the wave attains. By inspecting the equation, it is clear that the peak value of the e.m.f. corresponds to the amplitude of the sine function. Therefore:

$$\text{Peak value of e.m.f.} = 8\,\text{V}.$$

Quick Tip

The peak value of an alternating voltage is the highest voltage it reaches, which is the same as the amplitude of the sine wave.

(ii) Frequency of e.m.f.

The angular frequency ω is related to the frequency f by the equation:

$$\omega = 2\pi f$$

Given that $\omega = 628.4$, we can solve for f:

$$f = \frac{\omega}{2\pi} = \frac{628.4}{2\pi} = 100\,\text{Hz}.$$

Thus, the frequency of the e.m.f. is $100\,\text{Hz}$.

Quick Tip

The frequency of an alternating voltage is related to the angular frequency by the formula $f = \frac{\omega}{2\pi}$. The angular frequency ω is measured in radians per second.

(iii) Instantaneous value of e.m.f. at $t = 10\,\text{ms}$

To find the instantaneous value of the e.m.f. at time $t = 10\,\text{ms}$, we substitute $t = 10 \times 10^{-3}\,\text{s}$ into the equation:

$$e = 8\sin(628.4 \times 10 \times 10^{-3}) = 8\sin(6.284)$$

Now, 6.284 is very close to 2π, and we know that:

$$\sin(2\pi) = 0$$

Thus:

$$\sin(6.284) \approx 0$$

So the instantaneous value of the e.m.f. at $t = 10\,\text{ms}$ is:

$$e = 8 \times 0 = 0\,\text{V}.$$

Therefore, the instantaneous value of the e.m.f. at $t = 10\,\text{ms}$ is $0\,\text{V}$.

Quick Tip

The instantaneous value of an alternating voltage can be calculated by substituting the time into the voltage equation. When the argument of the sine function is a multiple of 2π, the instantaneous value becomes zero.

SECTION - D

Q. 27. What is a transformer? Explain the construction and working of a transformer. Derive the equation for a transformer.

Solution:

A transformer is an electrical device designed to alter the voltage and current in an alternating current (AC) circuit. It operates based on the principle of electromagnetic induction.

Construction: A transformer is made up of:

- A primary coil (input coil) connected to an AC power source.
- A secondary coil (output coil) connected to the load.
- A magnetic core, typically made of iron, that links both coils.

Working:

When an alternating current flows through the primary coil, it creates a time-varying magnetic field. This changing magnetic flux links to the secondary coil, inducing an electromotive force (emf) in the secondary coil, as described by Faraday's Law of Induction.

Equation for Transformer:

The relationship between the voltages and the number of turns in the primary and secondary coils is expressed by the following equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

where: - V_s is the secondary voltage, - V_p is the primary voltage, - N_s is the number of turns in the secondary coil, - N_p is the number of turns in the primary coil.

Quick Tip

In an ideal transformer, the power is conserved, meaning $V_p I_p = V_s I_s$, where I_p and I_s represent the primary and secondary currents, respectively.

Q. 28. Using the geometry of the double slit experiment, derive the expression for the fringe width of interference bands.

Solution:

In the double-slit experiment, an interference pattern is formed when light from two slits interacts. The fringe width β is the distance between two consecutive maxima (or minima) observed on the screen.

The path difference for two waves reaching a point on the screen is given by:

$$\Delta\text{Path} = d \sin\theta$$

where d is the separation between the slits and θ is the angle at which the maximum or minimum occurs. For constructive interference (bright fringes), the path difference is an integer multiple of the wavelength λ:

$$d \sin\theta = m\lambda$$

For small angles θ, $\sin\theta \approx \tan\theta$, and the position of the maxima can be written as:

$$y_m = m\frac{\lambda L}{d}$$

where L is the distance from the slits to the screen. The fringe width β, which is the distance between two successive maxima, is given by:

$$\beta = \frac{\lambda L}{d}$$

Quick Tip

The fringe width is directly proportional to the wavelength of the light and the distance from the slits to the screen, and inversely proportional to the slit separation.

Q. 29. Distinguish between an ammeter and a voltmeter. (Two points each).

Solution:

An ammeter and a voltmeter are both instruments used to measure electrical quantities, but they serve different purposes and have distinct characteristics. The differences between an ammeter and a voltmeter are as follows:

- **Ammeter:**
  - An ammeter measures the current flowing through a circuit.
  - It is always connected in series with the circuit components.
  - It has very low resistance to ensure it does not impact the current in the circuit.
- **Voltmeter:**
  - A voltmeter measures the potential difference (voltage) between two points in a circuit.
  - It is always connected in parallel across the components.
  - It has very high resistance to prevent it from drawing any current from the circuit.

The displacement of a particle performing simple harmonic motion is $\frac{1}{3}$ of its amplitude. What fraction of total energy is its kinetic energy?

Solution:

The total energy E of a particle undergoing simple harmonic motion is the sum of its kinetic energy K and potential energy U. The total energy remains constant and is given by the formula:

$$E = \frac{1}{2}m\omega^2 A^2$$

where m is the mass of the particle, ω is the angular frequency, and A is the amplitude.

The displacement x of the particle is related to its amplitude A as:

$$x = \frac{A}{3}$$

The potential energy at displacement x is given by:

$$U = \frac{1}{2}m\omega^2 x^2$$

Substituting $x = \frac{A}{3}$:

$$U = \frac{1}{2}m\omega^2 \left(\frac{A}{3}\right)^2 = \frac{1}{2}m\omega^2 \frac{A^2}{9}$$

Now, the kinetic energy K is the difference between the total energy and the potential energy:

$$K = E - U = \frac{1}{2}m\omega^2 A^2 - \frac{1}{2}m\omega^2 \frac{A^2}{9}$$

$$K = \frac{1}{2}m\omega^2 A^2 \left(1 - \frac{1}{9}\right)$$

$$K = \frac{1}{2}m\omega^2 A^2 \left(\frac{8}{9}\right)$$

Thus, the fraction of the total energy that is kinetic energy is:

$$\frac{K}{E} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

Therefore, the fraction of the total energy that is kinetic energy is $\frac{8}{9}$.

Quick Tip

In simple harmonic motion, the total energy is the sum of kinetic and potential energies. The kinetic energy depends on the displacement of the particle, and the fraction of the total energy that is kinetic can be determined based on the displacement.

Q. 30. Draw a neat labelled diagram of Ferry's perfectly black body. Compare the rms speed of hydrogen molecules at 227°C with rms speed of oxygen molecules at 127°C. Given that molecular masses of hydrogen and oxygen are 2 and 32, respectively.

Solution:

Ferry's perfectly black body diagram shows an idealized body that absorbs all incident radiation and reflects none. It is used in experiments to study the emission of radiation at different temperatures. The diagram for Ferry's perfectly black body consists of a cavity with a small hole on one side that allows radiation to enter. The body absorbs all radiation that enters, making it a perfect absorber.

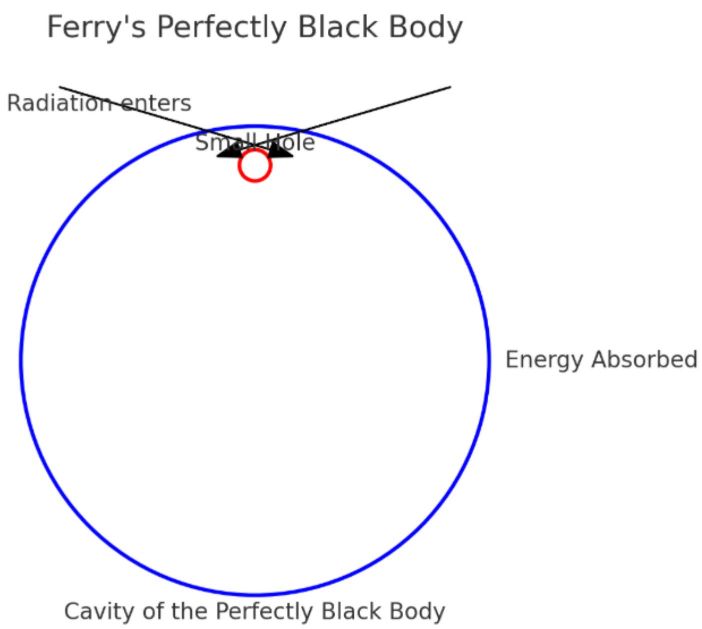


Figure 1: Ferry's perfectly black body diagram

rms speed comparison:

The root mean square (rms) speed v_{rms} of a gas molecule is given by the equation:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where: - k is the Boltzmann constant, - T is the temperature in Kelvin, - m is the mass of a molecule.

For hydrogen at $227°C$ ($T_1 = 227 + 273 = 500\,K$) and oxygen at $127°C$ ($T_2 = 127 + 273 = 400\,K$):

$$v$$

Taking the ratio of the rms speeds:

$$\frac{v}{v}$$

Thus, the rms speed of hydrogen molecules is approximately 4.472 times the rms speed of oxygen molecules.

Quick Tip

The rms speed of gas molecules is inversely proportional to the square root of their molecular mass. Hence, lighter molecules (such as hydrogen) move faster than heavier ones (like oxygen) at the same temperature.

Q. 31. Derive an expression for energy stored in a charged capacitor. A spherical metal ball of radius 15 cm carries a charge of 2C. Calculate the electric field at a distance of 20 cm from the center of the sphere.

Solution:

Energy Stored in a Charged Capacitor

The energy U stored in a charged capacitor is given by the formula:

$$U = \frac{1}{2}CV^2$$

where: - C is the capacitance of the capacitor,

- V is the voltage across the capacitor.

Capacitance of a Spherical Capacitor:

For a spherical capacitor (or a sphere with charge), the capacitance C is given by:

$$C = 4\pi\epsilon_0 R$$

where: - $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space,

- R is the radius of the sphere.

Given $R = 0.15\,\text{m}$, we can calculate the capacitance:

$$C = 4\pi \times 8.85 \times 10^{-12} \times 0.15 = 1.67 \times 10^{-12}\,\text{F}.$$

Voltage across the Spherical Capacitor:

The voltage V across the capacitor is related to the charge Q and capacitance C by the formula:

$$V = \frac{Q}{C}$$

Given the charge $Q = 2\,\mu C = 2 \times 10^{-6}$ C, we can calculate the voltage:

$$V = \frac{2 \times 10^{-6}}{1.67 \times 10^{-12}} = 1.20 \times 10^{6}\,\text{V}.$$

Energy Stored in the Capacitor:

Now, we can calculate the energy stored in the capacitor using the formula $U = \frac{1}{2}CV^2$:

$$U = \frac{1}{2} \times (1.67 \times 10^{-12}) \times (1.20 \times 10^{6})^2$$

$$U = \frac{1}{2} \times 1.67 \times 10^{-12} \times 1.44 \times 10^{12}$$

$$U = 1.20\,\text{J}.$$

Thus, the energy stored in the charged capacitor is 1.20 joules.

Electric Field at a Distance of 20 cm from the Center of the Sphere:

The electric field E at a distance r from the center of a spherical charge distribution is given by:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

where:

- $Q = 2 \times 10^{-6}$ C is the charge on the sphere,

- $r = 0.20$ m is the distance from the center of the sphere (20 cm).

Substituting the values:

$$E = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.2)^2} = \frac{18 \times 10^3}{0.04}$$

$$E = 4.5 \times 10^5 \text{ N/C}.$$

Thus, the electric field at a distance of 20 cm from the center of the sphere is 4.5×10^5 N/C.

Quick Tip

The energy stored in a capacitor is directly proportional to the capacitance and the square of the voltage. The electric field outside a spherical charge distribution behaves as if all the charge were concentrated at the center of the sphere.
