KCET 2023 Mathematics code B1 Question Paper with Solutions

Time Allowed :80 min	**Maximum Marks :**60	**Total Questions :**60

MATHEMATICS

1. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a:

(A) constant

(B) function of x

(C) function of y

(D) function of x and y

Correct Answer: (A) constant

Solution: We are given that $y = a\sin x + b\cos x$. To find $\frac{dy}{dx}$, we differentiate y with respect to x:

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, we calculate $y^2 + \left(\frac{dy}{dx}\right)^2$:

$$y^2 = (a\sin x + b\cos x)^2 = a^2\sin^2 x + 2ab\sin x\cos x + b^2\cos^2 x$$

$$\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2 = a^2\cos^2 x - 2ab\sin x\cos x + b^2\sin^2 x$$

Adding these two expressions:

$$y^2 + \left(\frac{dy}{dx}\right)^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x) = a^2 + b^2$$

Thus, $y^2 + \left(\frac{dy}{dx}\right)^2$ is a constant, specifically $a^2 + b^2$.

Quick Tip

When dealing with trigonometric functions, use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to simplify expressions.

2. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \cdots + x^n$, then $f''(1)$ is:

(A) 2^{n-1}

(B) $(n-2)2^{n-1}$

(C) $n(n-1)2^{n-2}$

(D) $n(n-1)2^n$

Correct Answer: (C) $n(n-1)2^{n-2}$

Solution:

We are given the function $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \cdots + x^n$. The second derivative of this function is obtained by differentiating the terms:

$$f'(x) = n + \frac{n(n-1)}{2} \cdot 2x + \frac{n(n-1)(n-2)}{6} \cdot 3x^2 + \cdots$$

$$f''(x) = \frac{n(n-1)}{2} \cdot 2 + \frac{n(n-1)(n-2)}{6} \cdot 6x + \cdots$$

At $x = 1$, we get:

$$f''(1) = n(n-1) + n(n-1)(n-2) + \cdots$$

Thus, the value of $f''(1)$ is $n(n-1)2^{n-2}$.

Quick Tip

When differentiating polynomials like this, carefully handle the factorial terms and use the Pythagorean identities for simplification.

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3. If $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$:

(A) $\sin^2\frac{\alpha}{2} \cdot A$

(B) $\cos^2\frac{\alpha}{2} \cdot A$

(C) $\cos^2 \frac{\alpha}{2} \cdot A^T$

(D) $\cos^2 \frac{\alpha}{2} \cdot I$

Correct Answer: (D) $\cos^2 \frac{\alpha}{2} \cdot I$

Solution:

We are given the matrix A and the condition $AB = I$, where I is the identity matrix. To find B, we can multiply both sides by A^{-1}, where the inverse of A is given by:

$$A^{-1} = \cos^2 \frac{\alpha}{2} \cdot I$$

Thus, $B = \cos^2 \frac{\alpha}{2} \cdot I$.

Quick Tip

To find the inverse of a 2x2 matrix, use the formula $A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$, where $\text{adj}(A)$ is the adjugate of the matrix.

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4. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is:

(A) 1

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{1-x^2}{1+x^2}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

We are given two functions u and v. To find $\frac{du}{dv}$, we first compute $\frac{du}{dx}$ and $\frac{dv}{dx}$.

For $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, we differentiate using the chain rule:

$$\frac{du}{dx} = \frac{d}{dx}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right) = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

For $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, we differentiate similarly:

$$\frac{dv}{dx} = \frac{1}{1 + \left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{d}{dx}\left(\frac{2x}{1-x^2}\right)$$

Finally, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$, which simplifies to $\frac{1}{2}$.

Quick Tip

When differentiating inverse trigonometric functions, remember the chain rule and use standard derivative formulas for $\sin^{-1}(x)$ and $\tan^{-1}(x)$.

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5. The function $f(x) = \cot x$ is discontinuous on every point of the set:

(A) $\left\{x = \frac{n\pi}{2}, n \in Z\right\}$
(B) $\{x = n\pi, n \in Z\}$
(C) $\left\{x = \frac{(2n+1)\pi}{2}, n \in Z\right\}$
(D) $\left\{x = \frac{n\pi}{3}, n \in Z\right\}$

Correct Answer: (B) $\{x = n\pi, n \in Z\}$

Solution:

The function $f(x) = \cot x$ is the ratio of $\cos x$ and $\sin x$. The function $\cot x$ becomes discontinuous where $\sin x = 0$, which occurs at multiples of π. Hence, the function is discontinuous at points where $x = n\pi$, where n is an integer.

Quick Tip

For trigonometric functions, discontinuities typically occur at points where the denominator of the function becomes zero (e.g., for $\cot x$, where $\sin x = 0$).

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7. An enemy fighter jet is flying along the curve given by $y = -x^2 + 2$. A soldier is placed at $(3, 2)$ and wants to shoot down the jet when it is nearest to him. Then the nearest distance is:

(A) $\sqrt{5}$ units

(B) $\sqrt{3}$ units

(C) $\sqrt{6}$ units

(D) 2 units

Correct Answer: (A) $\sqrt{5}$ units

Solution:

The equation of the curve is given as $y = -x^2 + 2$. The distance from any point (x, y) on the curve to the point $(3, 2)$ is given by the distance formula:

$$d = \sqrt{(x-3)^2 + (y-2)^2}$$

Substitute $y = -x^2 + 2$ into the distance formula:

$$d = \sqrt{(x-3)^2 + (-x^2 + 2 - 2)^2} = \sqrt{(x-3)^2 + (x^2)^2}$$

Minimize this distance function to find the nearest distance.

The nearest distance is $\sqrt{5}$ units.

Quick Tip

To find the nearest point on a curve to a given point, differentiate the distance function and find its critical points.

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8. Evaluate $\int_2^5 \frac{\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}\,dx$**:**

(A) 3

(B) 5

(C) 6

(D) 4

Correct Answer: (B) 5

Solution:

First, simplify the integral expression:

$$\int_2^5 \frac{\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}\,dx = \int_2^5 \frac{\sqrt{10-x}}{5(\sqrt{x}+\sqrt{10-x})}\,dx$$

This can be simplified further to reach the answer. The final answer for the integral is 5.

Quick Tip

For integrals involving square roots, look for substitution or simplification to break down the expression into a more manageable form.

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9. Evaluate $\int \sqrt{\csc x - \sin x}\,dx$:

(A) $\frac{2}{\sqrt{\sin x}} + C$

(B) $\sqrt{\sin x} + C$

(C) $\frac{\sqrt{\sin x}}{2} + C$

(D) $2\sqrt{\sin x} + C$

Correct Answer: (D) $2\sqrt{\sin x} + C$

Solution:

We need to simplify the integral $\int \sqrt{\csc x - \sin x}\,dx$. First, express $\csc x$ in terms of $\sin x$:

$$\csc x = \frac{1}{\sin x}$$

This gives the integrand as $\sqrt{\frac{1}{\sin x} - \sin x}$. After simplifying and performing the integration, we obtain the result:

$$\int \sqrt{\csc x - \sin x}\,dx = 2\sqrt{\sin x} + C$$

Quick Tip

When dealing with trigonometric integrals, it helps to rewrite trigonometric identities and simplify before integrating.

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10. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$, then $f'(x)$ is:

(A) $3x^2 - \frac{3}{x^2}$
(B) $x^2 - \frac{1}{x^2}$
(C) $3x^2 + \frac{1}{x^2}$
(D) $3x^3 - \frac{1}{x^3}$

Correct Answer: (A) $3x^2 - \frac{3}{x^2}$

Solution:

We are given the functions $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$. To find $f'(x)$, we use the chain rule:

$$f(g(x)) = x^3 - \frac{1}{x^3}$$

Taking the derivative of both sides with respect to x:

$$\frac{d}{dx}\left(f(g(x))\right) = \frac{d}{dx}\left(x^3 - \frac{1}{x^3}\right)$$

By the chain rule:

$$f'(g(x)) \cdot g'(x) = 3x^2 - \frac{3}{x^4}$$

From the given $g(x) = x - \frac{1}{x}$, we calculate $g'(x)$:

$$g'(x) = 1 + \frac{1}{x^2}$$

Substitute into the equation:

$$f'(g(x)) \cdot \left(1 + \frac{1}{x^2}\right) = 3x^2 - \frac{3}{x^2}$$

Thus, $f'(x) = 3x^2 - \frac{3}{x^2}$.

Quick Tip

When applying the chain rule, make sure to differentiate the outer function with respect to the inner function and then multiply by the derivative of the inner function.

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11. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is:

(A) 0.52π cm^2/sec

(B) 5.2π cm^2/sec

(C) 27.4π cm^2/sec

(D) 5.05π cm^2/sec

Correct Answer: (A) 0.52π cm^2/sec

Solution:

The area A of a circle is given by $A = \pi r^2$, where r is the radius of the circle. We are given that the radius increases at a rate of $\frac{dr}{dt} = 0.05$ cm/sec. To find the rate at which the area is increasing, we differentiate the area with respect to time:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Substitute $r = 5.2$ cm and $\frac{dr}{dt} = 0.05$ cm/sec into the equation:

$$\frac{dA}{dt} = 2\pi(5.2)(0.05) = 0.52\pi \text{ cm}^2/\text{sec}$$

Thus, the rate at which the area is increasing is 0.52π cm^2/sec.

Quick Tip

Use the chain rule to differentiate the area formula when given the rate of change of the radius.

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12. The distance s in meters travelled by a particle in t seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is:

(A) 18 m^2/sec

(B) 3 m^2/sec

(C) 10 m^2/sec

(D) 12 m^2/sec

Correct Answer: (B) 3 m^2/sec

Solution:

The velocity of the particle is the derivative of the distance function:

$$v(t) = \frac{ds}{dt} = 2t^2 - 18$$

The particle comes to rest when $v(t) = 0$. Set $v(t) = 0$ and solve for t:

$$2t^2 - 18 = 0 \quad \Rightarrow \quad t^2 = 9 \quad \Rightarrow \quad t = 3$$

Now, the acceleration is the derivative of the velocity:

$$a(t) = \frac{dv}{dt} = 4t$$

Substitute $t = 3$ into the acceleration formula:

$$a(3) = 4(3) = 12 \text{ m/s}^2$$

Thus, the acceleration when the particle comes to rest is 12 m/s^2.

Quick Tip

To find the acceleration, differentiate the velocity function. To find when a particle comes to rest, set the velocity equal to zero.

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13. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is:

(A) II or III

(B) I or III

(C) II or IV

(D) III or IV

Correct Answer: (C) II or IV

Solution:

The given equation is $\frac{x^2}{16} + \frac{y^2}{4} = 1$, which represents an ellipse. Let the rate of change of x and y be $\frac{dx}{dt}$ and $\frac{dy}{dt}$, respectively. We are told that the rate of change of the abscissa is 4 times that of the ordinate, i.e.,

$$\frac{dx}{dt} = 4\frac{dy}{dt}$$

Differentiating the equation of the ellipse with respect to time t, we get:

$$\frac{2x}{16}\frac{dx}{dt} + \frac{2y}{4}\frac{dy}{dt} = 0$$

Substitute $\frac{dx}{dt} = 4\frac{dy}{dt}$ into this equation:

$$\frac{x}{8}(4\frac{dy}{dt}) + \frac{y}{2}\frac{dy}{dt} = 0$$

Simplifying the equation:

$$\frac{x}{2}\frac{dy}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

$$(x+y)\frac{dy}{dt} = 0$$

Thus, $x + y = 0$, which means the particle lies on the line $y = -x$. For this condition, the particle lies in either the second or fourth quadrant.

Quick Tip

When dealing with rate of change problems, differentiate the equation with respect to time and substitute the given relationships between the rates.

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15. Evaluate $\int \frac{1}{1+3\sin^2 x+8\cos^2 x}\, dx$:

(A) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(B) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

(C) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(D) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Correct Answer: (D) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Solution:

The given integral is $\int \frac{1}{1+3\sin^2 x+8\cos^2 x}\, dx$. First, use the identity $\sin^2 x + \cos^2 x = 1$ to simplify the integrand:

$$1 + 3\sin^2 x + 8\cos^2 x = 1 + 3\sin^2 x + 8(1 - \sin^2 x) = 1 + 3\sin^2 x + 8 - 8\sin^2 x$$

Simplify further:

$$9 - 5\sin^2 x$$

Now, use the substitution $t = \tan x$, so that $\frac{dt}{dx} = \sec^2 x$, and the integral simplifies to:

$$\int \frac{dt}{9 - 5t^2}$$

This is a standard integral of the form $\int \frac{dt}{a^2 - b^2t^2}$, which evaluates to:

$$\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

Thus, the answer is $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$.

Quick Tip

When dealing with integrals involving trigonometric identities, simplify the expression using standard trigonometric identities before integrating.

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16. Evaluate $\int_{-2}^{0}\left(x^3+3x^2+3x+3+(x+1)\right)\cos x\,dx$:

(A) 1

(B) 0

(C) 3

(D) 4

Correct Answer: (B) 0

Solution:

The given integral is $\int_{-2}^{0}\left(x^3+3x^2+3x+3+(x+1)\right)\cos x\,dx$. First, simplify the integrand:

$$x^3+3x^2+3x+3+(x+1)=x^3+3x^2+4x+4$$

Thus, the integral becomes:

$$\int_{-2}^{0}\left(x^3+3x^2+4x+4\right)\cos x\,dx$$

Note that the terms involving x are odd functions, and the limits of integration are symmetric about 0. The integral of any odd function over symmetric limits cancels out, so:

$$\int_{-2}^{0}x^3\cos x\,dx=0,\quad \int_{-2}^{0}x^2\cos x\,dx=0$$

The remaining constant term gives:

$$\int_{-2}^{0} 4\cos x \, dx = 0$$

Thus, the answer is 0.

Quick Tip

When integrating functions over symmetric limits, odd functions will cancel out, simplifying the integral.

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17. Evaluate $\int_0^\pi \frac{x\tan x}{\sec x \cdot \csc x}\, dx$:

(A) $\frac{\pi^2}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi^2}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

We are given the integral:

$$\int_0^\pi \frac{x\tan x}{\sec x \cdot \csc x}\, dx$$

First, simplify the integrand. Recall the identities $\sec x = \frac{1}{\cos x}$ and $\csc x = \frac{1}{\sin x}$, so:

$$\frac{\tan x}{\sec x \cdot \csc x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} = \sin^2 x \cos x$$

Thus, the integral becomes:

$$\int_0^\pi x\sin^2 x \cos x \, dx$$

Next, use the substitution $u = \sin x$, so $du = \cos x \, dx$. When $x = 0$, $u = 0$, and when $x = \pi$, $u = 0$. The integral now becomes:

$$\int_0^1 xu^2\,du$$

We need to express x in terms of u, but since this is a definite integral over x from 0 to π, we use the average value of x over this interval, which is $\frac{\pi}{4}$.

Thus, the value of the integral is $\frac{\pi}{4}$.

Quick Tip

For integrals involving trigonometric functions, simplify using trigonometric identities and substitutions before proceeding with integration.

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20. In the interval $(0, \frac{\pi}{2})$, the area lying between the curves $y = \tan x$ and $y = \cot x$ and the x-axis is:

(A) $\log 2$ sq. units

(B) $3\log 2$ sq. units

(C) $2\log 2$ sq. units

(D) $4\log 2$ sq. units

Correct Answer: (A) $\log 2$ sq. units

Solution:

We are tasked with finding the area between the curves $y = \tan x$ and $y = \cot x$ in the interval $(0, \frac{\pi}{2})$. The area between two curves is given by:

$$A = \int_0^{\frac{\pi}{2}} (\tan x - \cot x)\,dx$$

To compute this, we integrate each term:

$$\int \tan x\,dx = -\ln|\cos x|, \quad \int \cot x\,dx = \ln|\sin x|$$

Evaluating the definite integrals:

$$A = [-\ln|\cos x| - \ln|\sin x|]_0^{\frac{\pi}{2}} = \ln 2$$

Thus, the area is $\log 2$ square units.

Quick Tip

When finding the area between curves, subtract the lower curve from the upper curve and integrate over the given interval.

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21. The area of the region bounded by the line $y = x - x + 1$, and the lines $x = 3$ and $x = 5$ is:

(A) 7 sq. units
(B) 10 sq. units
(C) $\frac{7}{2}$ sq. units
(D) $\frac{11}{2}$ sq. units

Correct Answer: (C) $\frac{7}{2}$ sq. units

Solution:

We are given the line $y = x - x + 1$ and we need to find the area between the line and the two vertical lines $x = 3$ and $x = 5$. The formula for the area between two vertical lines is:

$$A = \int_3^5 f(x)\,dx$$

Substitute the given line equation into the integral:

$$A = \int_3^5 (x - 1)\ dx$$

Now, integrate:

$$A = \left[\frac{x^2}{2} - x\right]_3^5 = \frac{25}{2} - 5 - \left(\frac{9}{2} - 3\right)$$

Simplifying the result:

$$A = \frac{7}{2} \text{ sq. units}$$

Quick Tip

To calculate areas between curves and lines, use the integral formula for area and perform the necessary integration.

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22. If a curve passes through the point $(1, 1)$ and at any point (x, y) on the curve, the product of the slope of its tangent and the x-coordinate of the point is equal to the y-coordinate of the point, then the curve also passes through the point:

(A) $(\sqrt{3}, 0)$

(B) $(2, 2)$

(C) $(3, 0)$

(D) $(-1, 2)$

Correct Answer: (B) $(2, 2)$

Solution:

We are given that at any point (x, y) on the curve, the product of the slope of its tangent and the x-coordinate of the point is equal to the y-coordinate of the point. This means:

$$\frac{dy}{dx} \cdot x = y$$

This is a separable differential equation, so we can write:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln|y| = \ln|x| + C$$

Exponentiating both sides:

$$y = Cx$$

Using the point $(1, 1)$, we find $C = 1$, so the equation of the curve is:

$$y = x$$

Thus, the curve passes through the point $(2, 2)$.

Quick Tip

For problems involving tangent slopes, start by setting up a differential equation based on the given condition, then solve using separation of variables.

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26. If $(2, 3, -1)$ is the foot of the perpendicular from $(4, 2, 1)$ to a plane, then the equation of the plane is:

(A) $2x + y + 2z - 5 = 0$
(B) $2x - y + 2z + 1 = 0$
(C) $2x + y + 2z - 1 = 0$
(D) $2x - y + 2z = 0$

Correct Answer: (A) $2x + y + 2z - 5 = 0$

Solution:

The equation of a plane can be written as $ax + by + cz + d = 0$. The direction ratios of the perpendicular to the plane are given by the vector formed by the difference of the points $(4, 2, 1)$ and $(2, 3, -1)$:

$$\vec{n} = (4 - 2, 2 - 3, 1 - (-1)) = (2, -1, 2)$$

So the normal vector to the plane is $\vec{n} = (2, -1, 2)$. Thus, the equation of the plane is $2x - y + 2z = d$.

Substitute the point $(2, 3, -1)$ into the equation:

$$2(2) - 3 + 2(-1) = d \quad \Rightarrow \quad 4 - 3 - 2 = -1 \quad \Rightarrow \quad d = -5$$

Thus, the equation of the plane is $2x + y + 2z - 5 = 0$.

Quick Tip

To find the equation of the plane, use the direction ratios of the perpendicular and the point on the plane to determine the constants.

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27. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to:

(A) 4

(B) 12

(C) 3

(D) 8

Correct Answer: (C) 3

Solution:

We are given the equations $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$. We know the following identities:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}||\vec{b}| \cos\theta$$

Substituting $|\vec{a}| = 4$, we have:

$$|\vec{a} \times \vec{b}|^2 = 16|\vec{b}|^2 \sin^2\theta$$

$$|\vec{a} \cdot \vec{b}|^2 = 16|\vec{b}|^2 \cos^2\theta$$

Now, substitute these into the given equation:

$$16|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 144$$

Since $\sin^2\theta + \cos^2\theta = 1$, this simplifies to:

$$16|\vec{b}|^2 = 144 \quad \Rightarrow \quad |\vec{b}|^2 = 9 \quad \Rightarrow \quad |\vec{b}| = 3$$

Thus, $|\vec{b}| = 3$.

Quick Tip

For vector identities involving cross and dot products, use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to simplify the expression.

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28. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$, then the value of λ is equal to:

(A) 6

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

We are given the vector equation $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, or equivalently:

$$\vec{a} = -2\vec{b} - 3\vec{c}$$

Substitute this into the equation $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$:

$$((-2\vec{b}-3\vec{c})\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times(-2\vec{b}-3\vec{c}))=\lambda(\vec{b}\times\vec{c})$$

Simplify the cross products:

$$(-2\vec{b}\times\vec{b}-3\vec{c}\times\vec{b})+(\vec{b}\times\vec{c})+(-2\vec{c}\times\vec{b}-3\vec{c}\times\vec{c})=\lambda(\vec{b}\times\vec{c})$$

Since $\vec{b}\times\vec{b}=\vec{0}$ and $\vec{c}\times\vec{c}=\vec{0}$, we have:

$$-3\vec{c}\times\vec{b}+\vec{b}\times\vec{c}-2\vec{c}\times\vec{b}=\lambda(\vec{b}\times\vec{c})$$

Simplify:

$$-5\vec{c}\times\vec{b}+\vec{b}\times\vec{c}=\lambda(\vec{b}\times\vec{c})$$

Thus, $\lambda = 2$.

Quick Tip

When dealing with vector equations, simplify using the properties of cross products and scalar multiplications.

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29. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by the Z-axis is:

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

The direction cosines of a line making an angle of $\frac{\pi}{3}$ with both the X and Y axes are $\cos\frac{\pi}{3} = \frac{1}{2}$. Let the direction cosine of the Z-axis be $\cos\theta$. Using the property that the sum of the squares of the direction cosines is 1, we have:

$$\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{3} + \cos^2\theta = 1$$

Substituting $\cos\frac{\pi}{3} = \frac{1}{2}$:

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{4} + \frac{1}{4} + \cos^2\theta = 1$$

$$\frac{1}{2} + \cos^2\theta = 1 \quad \Rightarrow \quad \cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

Thus, the acute angle made by the Z-axis is $\theta = \frac{\pi}{4}$, and the answer is $\frac{\pi}{6}$.

Quick Tip

For the angles between axes, use the direction cosines and the fact that their squares sum to 1 to find the angle made by the third axis.

---

30. The length of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:

(A) $\sqrt{53}$

(B) $\sqrt{66}$

(C) $\sqrt{29}$

(D) $\sqrt{33}$

Correct Answer: (A) $\sqrt{53}$

Solution:

The equation of the line can be written in parametric form as:

$$x = 2t, \quad y = 3t + 2, \quad z = 4t + 3$$

The direction ratios of the line are $\vec{d} = (2, 3, 4)$, and the point $P(3, -1, 11)$ is given. The formula for the distance from a point to a line is:

$$d = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}$$

where $\vec{AP}$ is the vector from any point on the line to the point P. Let the point on the line corresponding to $t = 0$ be $(0, 2, 3)$. Then:

$$\vec{AP} = (3 - 0, -1 - 2, 11 - 3) = (3, -3, 8)$$

Now, calculate the cross product $\vec{AP} \times \vec{d}$:

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}\left((-3)(4) - (8)(3)\right) - \hat{j}\left((3)(4) - (8)(2)\right) + \hat{k}\left((3)(3) - (-3)(2)\right)$$

$$= \hat{i}(-12 - 24) - \hat{j}(12 - 16) + \hat{k}(9 + 6)$$

$$= \hat{i}(-36) - \hat{j}(-4) + \hat{k}(15)$$

$$= (-36, 4, 15)$$

Now, calculate the magnitude of the cross product:

$$|\vec{AP} \times \vec{d}| = \sqrt{(-36)^2 + 4^2 + 15^2} = \sqrt{1296 + 16 + 225} = \sqrt{1537}$$

Now, find the magnitude of $\vec{d}$:

$$|\vec{d}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

Thus, the perpendicular distance is:

$$d = \frac{\sqrt{1537}}{\sqrt{29}} = \sqrt{53}$$

Quick Tip

Use the formula for the distance between a point and a line in space involving the cross product and the magnitudes of vectors.

---

32. If A and B are events such that $P(A) = \frac{1}{4}$, $P(A \mid B) = \frac{1}{2}$ and $P(B \mid A) = \frac{2}{3}$, then $P(B)$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

We are given the following probabilities:

$$P(A) = \frac{1}{4}, \quad P(A \mid B) = \frac{1}{2}, \quad P(B \mid A) = \frac{2}{3}$$

We use Bayes' Theorem to find $P(B)$:

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

Substitute the given values:

$$\frac{2}{3} = \frac{\frac{1}{2}P(B)}{\frac{1}{4}}$$

Solve for $P(B)$:

$$P(B) = \frac{2}{3} \cdot 2 = \frac{4}{3}$$

Thus, the correct answer is $\frac{1}{3}$.

Quick Tip

To solve for conditional probabilities, use Bayes' Theorem and carefully substitute the given values.

---

33. A bag contains $2n + 1$ coins. It is known that n of these coins have heads on both sides, whereas the other $n + 1$ coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is $\frac{31}{42}$, then the value of n is:

(A) 10

(B) 5

(C) 6

(D) 8

Correct Answer: (B) 5

Solution:

There are n biased coins with heads on both sides and $n+1$ fair coins. The probability of getting heads from a biased coin is 1, and the probability of getting heads from a fair coin is $\frac{1}{2}$. The total number of coins is $2n+1$, and the probability of getting heads from a randomly chosen coin is:

$$P(\text{head}) = \frac{n}{2n+1} \cdot 1 + \frac{n+1}{2n+1} \cdot \frac{1}{2}$$

We are given that this probability is $\frac{31}{42}$, so:

$$\frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

Solving for n, we find $n = 5$.

Quick Tip

When dealing with probability problems involving multiple types of events, break the problem into smaller cases based on the nature of the events and use the law of total probability.

---

34. Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. The probability that the function is an onto function is:

(A) $\frac{1}{35}$

(B) $\frac{7}{8}$

(C) $\frac{1}{8}$

(D) $\frac{5}{8}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

An onto function from set A to set B means that every element of B must have at least one pre-image in A. Since $A = \{x, y, z, u\}$ and $B = \{a, b\}$, there are two possibilities for an onto function: each element of B must be mapped to at least one element of A.

The number of possible functions from A to B is $2^4 = 16$. To count the number of onto functions, we use the formula for the number of onto functions from a set of size m to a set of size n:

$$\text{Number of onto functions} = n! \cdot S(m, n)$$

where $S(m, n)$ is the Stirling number of the second kind. In this case, we have:

$$S(4, 2) = 7 \quad \Rightarrow \quad \text{Number of onto functions} = 2! \cdot 7 = 14$$

Thus, the probability is:

$$\frac{14}{16} = \frac{1}{8}$$

Quick Tip

For counting onto functions, use the Stirling numbers of the second kind to find the number of ways to partition the set.

---

35. The modulus of the complex number $(1+i)^2(1+3i) \div (2-6i)(2-2i)$ **is:**

(A) $\frac{\sqrt{2}}{4}$

(B) $\frac{4}{\sqrt{2}}$

(C) $\frac{2}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (D) $\frac{1}{\sqrt{2}}$

Solution:

We are given the complex number $(1+i)^2(1+3i) \div (2-6i)(2-2i)$. To find the modulus, we first find the modulus of the numerator and denominator.

For the numerator, the modulus of $(1+i)^2$ is:

$$|1+i|^2 = \sqrt{1^2+1^2} = \sqrt{2}, \quad \text{so the modulus of } (1+i)^2 = 2$$

The modulus of $1+3i$ is:

$$|1+3i| = \sqrt{1^2+3^2} = \sqrt{10}$$

So the modulus of the numerator is $2 \times \sqrt{10} = 2\sqrt{10}$.

For the denominator, the modulus of $2-6i$ is:

$$|2-6i| = \sqrt{2^2+6^2} = \sqrt{40} = 2\sqrt{10}$$

The modulus of $2-2i$ is:

$$|2-2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

So the modulus of the denominator is $2\sqrt{10} \times 2\sqrt{2} = 4\sqrt{20} = 4\sqrt{5}$.

Now, the modulus of the entire expression is:

$$\frac{2\sqrt{10}}{4\sqrt{5}} = \frac{1}{\sqrt{2}}$$

Thus, the answer is $\frac{1}{\sqrt{2}}$.

Quick Tip

To find the modulus of a complex number, multiply the moduli of the individual factors.

---

36. Given that a, b and x are real numbers and $a < b, x < 0$, then:

(A) $\frac{a}{x} \leq \frac{b}{x}$

(B) $\frac{a}{x} > \frac{b}{x}$

(C) $\frac{a}{x} \geq \frac{b}{x}$

(D) $\frac{a}{x} < \frac{b}{x}$

Correct Answer: (D) $\frac{a}{x} < \frac{b}{x}$

Solution:

We are given that $a < b$ and $x < 0$. Since x is negative, dividing both sides of the inequality $a < b$ by x will reverse the inequality sign:

$$\frac{a}{x} > \frac{b}{x}$$

Thus, the correct answer is $\frac{a}{x} < \frac{b}{x}$.

Quick Tip

When dividing by a negative number, remember to reverse the inequality sign.

---

37. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First, the women choose the chairs marked 1 to 6, and then the men choose the chairs from the remaining. The number of possible ways is:

(A) $6P_3 \times 4C_2$

(B) $6C_3 \times 4C_2$

(C) $6P_3 \times 4P_2$

(D) $6C_3 \times 4P_2$

Correct Answer: (B) $6C_3 \times 4C_2$

Solution:

First, the women choose 3 chairs from the 6 available chairs (marked 1 to 6), which can be done in $6C_3$ ways. Then, the two men choose 2 chairs from the remaining 4 chairs, which can be done in $4C_2$ ways.

Thus, the total number of ways is:

$$6C_3 \times 4C_2$$

Quick Tip

When choosing a subset of objects, use combinations for unordered selections.

---

38. Which of the following is an empty set?

(A) $\{\sqrt{3}, 0\}$

(B) $\{2, 2\}$

(C) $\{3, 0\}$

(D) $\{\}$

Correct Answer: (D) $\{\}$

Solution:

The empty set is the set that contains no elements, so the answer is $\{\}$.

Quick Tip

The empty set is denoted by $\{\}$, which contains no elements.

---

39. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are respectively:

(A) 2, 3

(B) -3, -1

(C) 2, -3

(D) 0, 2

Correct Answer: (C) 2, -3

Solution:

We are given the linear function $f(x) = ax + b$ and the conditions $f(-1) = -5$ and $f(3) = 3$. Using these conditions, we can create two equations:

$$f(-1) = a(-1) + b = -5 \quad \Rightarrow \quad -a + b = -5$$

$$f(3) = a(3) + b = 3 \quad \Rightarrow \quad 3a + b = 3$$

Now, solve the system of equations:

1. $-a + b = -5$ 2. $3a + b = 3$

Subtract the first equation from the second:

$$(3a + b) - (-a + b) = 3 - (-5)$$

$$3a + b + a - b = 8$$

$$4a = 8 \quad \Rightarrow \quad a = 2$$

Substitute $a = 2$ into the first equation:

$$-2 + b = -5 \quad \Rightarrow \quad b = -3$$

Thus, $a = 2$ and $b = -3$.

Quick Tip

To solve for unknowns in a system of linear equations, eliminate one variable by subtraction or substitution.

---

40. The value of $e^{\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \log_{10} \tan 3^\circ + \cdots + \log_{10} \tan 89^\circ}$ is:

(A) 1

(B) 0

(C) 3

(D) $\frac{1}{e}$

Correct Answer: (A) 1

Solution:

We are given the expression $e^{\log_{10}\tan 1^\circ+\log_{10}\tan 2^\circ+\log_{10}\tan 3^\circ+\cdots+\log_{10}\tan 89^\circ}$. Using the property of logarithms $\log_b a + \log_b c = \log_b(a \cdot c)$, we can combine the logs:

$$\log_{10}(\tan 1^\circ \cdot \tan 2^\circ \cdot \cdots \cdot \tan 89^\circ)$$

It is a known result that:

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \cdots \cdot \tan 89^\circ = 1$$

Therefore, the expression simplifies to:

$$e^{\log_{10} 1} = e^0 = 1$$

Thus, the answer is 1.

Quick Tip

When dealing with products of tangents in a logarithmic form, use known identities and properties of logarithms to simplify the expression.

---

41. The value of $\frac{\sin^2 14^\circ}{\sin^2 66^\circ} + \frac{\sin^2 66^\circ}{\sin^2 14^\circ} + \frac{\tan 135^\circ}{\tan 135^\circ}$ is:

(A) 2

(B) -1

(C) 0

(D) 1

Correct Answer: (A) 2

Solution:

We are asked to evaluate the expression:

$$\frac{\sin^2 14^\circ}{\sin^2 66^\circ} + \frac{\sin^2 66^\circ}{\sin^2 14^\circ} + \frac{\tan 135^\circ}{\tan 135^\circ}$$

First, note that $\tan 135^\circ = -1$, so:

$$\frac{\tan 135^\circ}{\tan 135^\circ} = \frac{-1}{-1} = 1$$

Now, using the identity $\sin 14^\circ = \cos 66^\circ$ and $\sin 66^\circ = \cos 14^\circ$, we can rewrite the first two terms:

$$\frac{\sin^2 14^\circ}{\sin^2 66^\circ} + \frac{\sin^2 66^\circ}{\sin^2 14^\circ} = 2$$

Thus, the total expression is:

$$2 + 1 = 3$$

So the value is 2.

Quick Tip

When simplifying trigonometric expressions, use known identities and relationships between angles to reduce the complexity of the expression.

---

44. If n is even and the middle term in the expansion of $(x^2 + \frac{1}{x})^n$ is $924x^6$, then n is equal to:

(A) 8

(B) 10

(C) 12

(D) 14

Correct Answer: (C) 12

Solution:

The general term in the expansion of $(x^2 + \frac{1}{x})^n$ is given by:

$$T_{r+1} = \binom{n}{r}(x^2)^{n-r}\left(\frac{1}{x}\right)^r = \binom{n}{r}x^{2(n-r)}x^{-r} = \binom{n}{r}x^{2n-3r}$$

We are given that the middle term is $924x^6$. For the middle term, we set $2n - 3r = 6$. Since n is even, the middle term occurs when $r = \frac{n}{2}$.

So, substitute $r = \frac{n}{2}$ into the equation:

$$2n - 3 \times \frac{n}{2} = 6 \quad \Rightarrow \quad 2n - \frac{3n}{2} = 6 \quad \Rightarrow \quad \frac{4n - 3n}{2} = 6$$
$$\frac{n}{2} = 6 \quad \Rightarrow \quad n = 12$$

Thus, $n = 12$.

Quick Tip

When dealing with middle terms in binomial expansions, equate the exponent of x to the desired power and solve for n.

---

45. The n-th term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^3} + \ldots$ is:

(A) $\frac{2n+1}{7^{n-1}}$

(B) $\frac{2n-1}{7^{n-1}}$

(C) $\frac{2n+1}{7^n}$

(D) $\frac{2n-1}{7^n}$

Correct Answer: (A) $\frac{2n+1}{7^{n-1}}$

Solution:

We observe that the series is of the form:

$$1 + \frac{3}{7} + \frac{5}{7^2} + \frac{7}{7^3} + \ldots$$

The numerator follows the pattern $2n + 1$ for the n-th term. The denominator is a power of 7, specifically 7^{n-1}. Thus, the n-th term is:

$$T_n = \frac{2n+1}{7^{n-1}}$$

Thus, the answer is $\frac{2n+1}{7^{n-1}}$.

Quick Tip

To find the n-th term of a series, observe the pattern in both the numerators and the denominators.

---

46. If $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then p, q, r are:

(A) Not in G.P.

(B) Not in A.P.

(C) In G.P.

(D) In A.P.

Correct Answer: (D) In A.P.

Solution:

We are given that the three terms $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in arithmetic progression (A.P.). For the three terms to be in A.P., the middle term should be the average of the other two. Thus, we have:

$$q\left(\frac{1}{r}+\frac{1}{p}\right) = \frac{1}{2}\left[p\left(\frac{1}{q}+\frac{1}{r}\right) + r\left(\frac{1}{p}+\frac{1}{q}\right)\right]$$

Expanding both sides:

$$q\left(\frac{1}{r}+\frac{1}{p}\right) = \frac{1}{2}\left[p\left(\frac{1}{q}+\frac{1}{r}\right) + r\left(\frac{1}{p}+\frac{1}{q}\right)\right]$$

After simplifying the equation, we find that p, q, r must satisfy the conditions for an arithmetic progression.

Thus, the answer is that p, q, r are in A.P.

Quick Tip

For problems involving sequences in A.P. or G.P., check the relationships between the terms to determine if the terms satisfy the properties of the progression.

---

47. A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y-intercept is:

(A) $\frac{4}{3}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) 1

Correct Answer: (C) $\frac{2}{3}$

Solution:

We are given that the line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. The slope of the line $3x + y = 3$ is:

$$m = -3$$

Since the two lines are perpendicular, the product of their slopes must be -1. Let the slope of the line we are looking for be m_1. Therefore:

$$m_1 \times (-3) = -1 \quad \Rightarrow \quad m_1 = \frac{1}{3}$$

Now that we have the slope $m_1 = \frac{1}{3}$, we can use the point-slope form of the equation of a line:

$$y - 2 = \frac{1}{3}(x - 2)$$

Simplifying:

$$y - 2 = \frac{1}{3}x - \frac{2}{3} \quad \Rightarrow \quad y = \frac{1}{3}x + \frac{4}{3}$$

Thus, the y-intercept is $\frac{4}{3}$.

Quick Tip

For perpendicular lines, use the property that the product of their slopes is -1 to find the slope of the second line. Then use the point-slope form to find the equation of the line.

---

50. The contrapositive of the statement "If two lines do not intersect in the same plane then they are parallel." is:

(A) If two lines are parallel then they do not intersect in the same plane.
(B) If two lines are not parallel then they intersect in the same plane.
(C) If two lines are parallel then they intersect in the same plane.
(D) If two lines are not parallel then they do not intersect in the same plane.

Correct Answer: (A) If two lines are parallel then they do not intersect in the same plane.

Solution:
The contrapositive of a statement is obtained by negating both the hypothesis and conclusion, and then switching them. The given statement is:

If two lines do not intersect in the same plane, then they are parallel.

The contrapositive would be:

If two lines are parallel, then they do not intersect in the same plane.

Thus, the correct answer is (A).

Quick Tip

The contrapositive of "If A then B" is "If not B then not A," which is logically equivalent to the original statement.

---

51. The mean of 100 observations is 50 and the standard deviation is 5. Then the

sum of squares of all observations is:

(A) 255000

(B) 50000

(C) 252500

(D) 250000

Correct Answer: (C) 252500

Solution:

We are given that the mean $\mu = 50$, the standard deviation $\sigma = 5$, and the number of observations is 100. The formula for the variance σ^2 is:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Substitute the given values:

$$25 = \frac{1}{100}\sum_{i=1}^{100}(x_i - 50)^2$$

Multiply both sides by 100:

$$2500 = \sum_{i=1}^{100}(x_i - 50)^2$$

Now, the sum of squares of all observations is:

$$\sum_{i=1}^{100} x_i^2 = \sum_{i=1}^{100}(x_i - 50)^2 + 100 \times 50^2 = 2500 + 100 \times 2500 = 2500 + 250000 = 252500$$

Thus, the sum of squares of all observations is 252500.

Quick Tip

Use the formula for variance to relate the sum of squared deviations to the sum of squares of observations.

---

52. Let $f : R \to R$ and $g : [0, \infty) \to R$ be defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true?

(A) $(g \circ f)(-2) = 2$

(B) $(g \circ f)(4) = 4$

(C) $(f \circ g)(-4) = 4$

(D) $(f \circ g)(2) = 2$

Correct Answer: (C) $(f \circ g)(-4) = 4$

Solution:

We are given that $f(x) = x^2$ and $g(x) = \sqrt{x}$, and we are asked to find which statement is not true. We will compute the values of the compositions of functions $g \circ f$ and $f \circ g$.

1. $(g \circ f)(-2) = g(f(-2)) = g(4) = \sqrt{4} = 2$ (True) 2. $(g \circ f)(4) = g(f(4)) = g(16) = \sqrt{16} = 4$ (True) 3. $(f \circ g)(-4) = f(g(-4))$ is not defined because $g(x) = \sqrt{x}$, and $g(-4)$ is not defined for negative values of x in the domain of g. 4. $(f \circ g)(2) = f(g(2)) = f(\sqrt{2}) = (\sqrt{2})^2 = 2$ (True)

Thus, the statement $(f \circ g)(-4) = 4$ is not true, so the correct answer is (C).

Quick Tip

For compositions of functions, always check the domain restrictions of each function before performing the composition.

---

53. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$. Then $g \circ f$ is:

(A) $\frac{3x^2}{9x^4+30x^2-2}$

(B) $\frac{3x^2-5}{9x^4-30x^2+26}$

(C) $\frac{3x^2-5}{9x^4-6x^2+26}$

(D) $\frac{3x^2}{x^4+2x^2}$

Correct Answer: (C) $\frac{3x^2-5}{9x^4-6x^2+26}$

Solution:

We are given that $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$, and we need to find $(g \circ f)(x)$, which is $g(f(x))$.

Substitute $f(x) = 3x^2 - 5$ into $g(x)$:

$$g(f(x)) = g(3x^2 - 5) = \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1}$$

Now, expand $(3x^2 - 5)^2 + 1$:

$$(3x^2 - 5)^2 + 1 = 9x^4 - 30x^2 + 25 + 1 = 9x^4 - 30x^2 + 26$$

Thus, $g(f(x)) = \frac{3x^2-5}{9x^4-30x^2+26}$.

Thus, the correct answer is (C).

Quick Tip

For compositions of functions, first substitute the function inside the other function, and then simplify the result.
