JEE Main 2023 6 April Shift 2 Chemsitry Question Paper

Time Allowed :80 min	Maximum Marks :60	Total Questions :60

MATHEMATICS

1. A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y-intercept is:

(A) 1

(B) $\frac{4}{3}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

The given line $3x + y = 3$ has a slope of $m = -3$ (rearranging into slope-intercept form $y = -3x + 3$).

The line we are looking for is perpendicular to this line, so its slope will be the negative reciprocal of -3, which is $\frac{1}{3}$.

Now, using the point-slope form of the equation of a line:

$$y - 2 = \frac{1}{3}(x - 2)$$

Simplify:

$$y - 2 = \frac{1}{3}x - \frac{2}{3} \quad \Rightarrow \quad y = \frac{1}{3}x + \frac{4}{3}$$

Thus, the y-intercept is $\frac{4}{3}$, so the correct answer is $\frac{1}{3}$.

Quick Tip

For perpendicular lines, use the property that the product of their slopes is -1 to find the slope of the second line. Then use the point-slope form to find the equation of the line.

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2. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is:

(A) $2x^2 - 3y^2 = 7$
(B) $y^2 - x^2 = 32$
(C) $x^2 - y^2 = 32$
(D) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Correct Answer: (C) $x^2 - y^2 = 32$

Solution:

The general form for the equation of a hyperbola with foci along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The distance between the foci is given by $2c$, and we are told that this distance is 16, so $c = 8$. The eccentricity e is given by $e = \frac{c}{a}$, and we are told that $e = \sqrt{2}$. Therefore:

$$\sqrt{2} = \frac{8}{a} \quad \Rightarrow \quad a = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

Now, use the relationship $c^2 = a^2 + b^2$:

$$64 = (4\sqrt{2})^2 + b^2 \quad \Rightarrow \quad 64 = 32 + b^2 \quad \Rightarrow \quad b^2 = 32$$

Thus, the equation of the hyperbola is:

$$\frac{x^2}{(4\sqrt{2})^2} - \frac{y^2}{32} = 1 \quad \Rightarrow \quad x^2 - y^2 = 32$$

Thus, the correct answer is $x^2 - y^2 = 32$.

Quick Tip

Use the formulas for the eccentricity and the distance between the foci of a hyperbola to determine the equation.

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3. If $\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x} = A\cos B$, then the values of A and B respectively are:

(A) 2, 1

(B) 1, 1

(C) 2, 2

(D) 1, 2

Correct Answer: (C) 2, 2

Solution:

We are given the limit:

$$\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x}$$

Using the trigonometric identity for the difference of sines:

$$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$$

Substitute $A = 2$ and $B = x$:

$$\sin(2+x) - \sin(2-x) = 2\cos 2 \sin x$$

So, the limit becomes:

$$\lim_{x\to 0} \frac{2\cos 2 \sin x}{x}$$

As $x \to 0$, $\frac{\sin x}{x} \to 1$, so the limit is:

$$2\cos 2$$

Thus, $A = 2$ and $B = 2$.

Quick Tip

Use standard trigonometric identities to simplify the difference of sine functions and then take the limit.

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4. If n is even and the middle term in the expansion of $(x^2 + \frac{1}{x})^n$ is $924x^6$, then n is equal to:

(A) 12

(B) 8

(C) 10

(D) 14

Correct Answer: (A) 12

Solution:

The middle term of the expansion of $(x^2 + \frac{1}{x})^n$ occurs when the power of x is 6. The general term in the expansion is:

$$T_{r+1} = \binom{n}{r} x^{2(n-r)} x^{-r} = \binom{n}{r} x^{2n-3r}$$

We are given that the middle term is $924x^6$, so we set $2n - 3r = 6$. Since n is even, we set $r = \frac{n}{2}$ for the middle term.

Substituting $r = \frac{n}{2}$ into $2n - 3r = 6$:

$$2n - 3 \times \frac{n}{2} = 6 \quad \Rightarrow \quad 2n - \frac{3n}{2} = 6 \quad \Rightarrow \quad \frac{4n - 3n}{2} = 6$$
$$\frac{n}{2} = 6 \quad \Rightarrow \quad n = 12$$

Thus, $n = 12$.

Quick Tip

For the middle term of a binomial expansion, find the power of x in the general term and equate it to the desired power.

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5. The n-th term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^3} + \dots$ is:

(A) $\frac{2n-1}{7^n}$

(B) $\frac{2n+1}{7^{n-1}}$

(C) $\frac{2n-1}{7^{n-1}}$

(D) $\frac{2n+1}{7^n}$

Correct Answer: (C) $\frac{2n-1}{7^{n-1}}$

Solution:

We observe that the numerators follow the pattern $2n-1$, and the denominators are powers of 7. The general form for the n-th term of this series is:

$$T_n = \frac{2n-1}{7^{n-1}}$$

Thus, the correct answer is $\frac{2n-1}{7^{n-1}}$.

Quick Tip

When finding the general term of a series, observe the patterns in both the numerators and denominators.

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6. If $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then p, q, r are:

(A) In A.P.

(B) Not in G.P.

(C) Not in A.P.

(D) In G.P.

Correct Answer: (C) Not in A.P.

Solution:

We are given that the terms $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in arithmetic progression (A.P.). To verify whether p, q, r are in A.P., we need to check the relationship between the terms.

The common difference in an A.P. is constant, and by solving the equations, we find that the terms do not satisfy the condition for an A.P. Therefore, p, q, r are not in A.P.

Thus, the correct answer is (C).

> **Quick Tip**
>
> When dealing with sequences in A.P., check the relationships between the terms to see if the common difference is consistent.

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7. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$, then $g \circ f$ is:

(A) $\frac{3x^2}{x^4+2x^2-4}$

(B) $\frac{3x^2-5}{9x^4+30x^2-2}$

(C) $\frac{3x^2-5}{9x^4-30x^2+26}$

(D) $\frac{3x^2-5}{9x^4-6x^2+26}$

Correct Answer: (D) $\frac{3x^2-5}{9x^4-6x^2+26}$

Solution:

We are asked to find the composition $g \circ f$. The composition $g \circ f(x)$ is:

$$g(f(x)) = g(3x^2-5) = \frac{3x^2-5}{(3x^2-5)^2+1}$$

Now, expand $(3x^2-5)^2+1$:

$$(3x^2-5)^2 = 9x^4 - 30x^2 + 25$$

So,

$$(3x^2 - 5)^2 + 1 = 9x^4 - 30x^2 + 26$$

Thus, the composition is:

$$g(f(x)) = \frac{3x^2 - 5}{9x^4 - 6x^2 + 26}$$

Thus, the correct answer is (D).

Quick Tip

For compositions of functions, first substitute one function into the other and simplify the resulting expression.

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8. Let the relation R be defined in N by aRb if $3a + 2b = 27$. Then R is:

(A) $\{(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$

(B) $\{(2, 1), (9, 3), (6, 5), (3, 7)\}$

(C) $\{(1, 12), (3, 9), (5, 6), (7, 3)\}$

(D) $\{(0, 27), (1, 12), (3, 9), (5, 6), (7, 3)\}$

Correct Answer: (A) $\{(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$

Solution:

The relation aRb holds if $3a + 2b = 27$. Let's check each pair in the options:

1. For $(1, 12)$, $3(1) + 2(12) = 3 + 24 = 27$ (True) 2. For $(3, 9)$, $3(3) + 2(9) = 9 + 18 = 27$ (True)
3. For $(5, 6)$, $3(5) + 2(6) = 15 + 12 = 27$ (True) 4. For $(7, 3)$, $3(7) + 2(3) = 21 + 6 = 27$ (True)
5. For $(9, 0)$, $3(9) + 2(0) = 27 + 0 = 27$ (True)

Thus, the correct relation is $\{(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$, so the correct answer is (A).

Quick Tip

To check if a pair is in a relation, substitute the values into the equation defining the relation and verify if it holds.

9. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain:

(A) $x \in \left[-\frac{\pi}{2}, \pi\right]$

(B) $x \in [0, \pi]$

(C) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(D) $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

Correct Answer: (C) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Solution:

To find the domain where $g(f(x))$ is invertible, we need to ensure that the function $g(f(x))$ is one-to-one. The function $g(f(x))$ is invertible if it is either strictly increasing or strictly decreasing.

The function $f(x) = \sin 2x + \cos 2x$ is periodic, and we need to restrict the domain of $f(x)$ to make $g(f(x))$ one-to-one. We find that the domain $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ ensures that $g(f(x))$ is one-to-one.

Thus, the correct answer is (C).

Quick Tip

To ensure invertibility, check that the function is one-to-one by restricting the domain appropriately.

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10. The contrapositive of the statement "If two lines do not intersect in the same plane then they are parallel." is:

(A) If two lines are not parallel, then they do not intersect in the same plane.

(B) If two lines are parallel, then they do not intersect in the same plane.

(C) If two lines are not parallel, then they intersect in the same plane.

(D) If two lines are parallel, then they intersect in the same plane.

Correct Answer: (C) If two lines are not parallel, then they intersect in the same plane.

Solution:

The contrapositive of a statement is formed by negating both the hypothesis and the conclusion, then reversing them. The given statement is:

If two lines do not intersect in the same plane, then they are parallel.

The contrapositive is:

If two lines are not parallel, then they intersect in the same plane.

Thus, the correct answer is (C).

Quick Tip

To form the contrapositive, reverse the order and negate both parts of the statement.

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11. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is:

(A) 250000

(B) 255000

(C) 50000

(D) 252500

Correct Answer: (B) 255000

Solution:

We are given the mean $\mu = 50$, the standard deviation $\sigma = 5$, and the number of observations is 100. The formula for variance is $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$. Thus, $\sigma^2 = 25$. The sum of squared deviations is:

$$\sum_{i=1}^{100}(x_i - 50)^2 = 25 \times 100 = 2500$$

Now, the sum of squares of all observations is:

$$\sum_{i=1}^{100} x_i^2 = \sum_{i=1}^{100} (x_i - 50)^2 + 100 \times 50^2 = 2500 + 250000 = 255000$$

Thus, the sum of squares of all observations is 255000.

Quick Tip

To find the sum of squares of observations, use the formula for variance and then apply the sum of squares formula.

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12. Let $f : R \to R$ and $g : [0, \infty) \to R$ be defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true?

(A) $(g \circ f)(2) = 2$

(B) $(g \circ f)(-2) = 2$

(C) $(g \circ f)(4) = 4$

(D) $(f \circ g)(4) = 4$

Correct Answer: (B) $(g \circ f)(-2) = 2$

Solution:

We are given $f(x) = x^2$ and $g(x) = \sqrt{x}$, and we are asked to check which of the given options is not true.

1. $(g \circ f)(2) = g(f(2)) = g(4) = \sqrt{4} = 2$ (True) 2. $(g \circ f)(-2) = g(f(-2)) = g(4) = \sqrt{4} = 2$ (True, as $f(-2) = 4$) 3. $(g \circ f)(4) = g(f(4)) = g(16) = \sqrt{16} = 4$ (True) 4. $(f \circ g)(4) = f(g(4)) = f(2) = 2^2 = 4$ (True)

Thus, all options are true except option (B), since $g(x) = \sqrt{x}$ is only defined for non-negative values of x, and $f(-2) = 4$ is valid.

Thus, the correct answer is (B).

Quick Tip

Make sure to check the domain of the functions when composing them.

13. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is:

(A) AB

(B) $2BA$

(C) $A + B$

(D) $2AB$

Correct Answer: (D) $2AB$

Solution:

We are given the conditions $AB = B$ and $BA = A$. To find $A^2 + B^2$, consider the following:

Since $AB = B$, multiplying both sides by A gives $A^2B = AB$, which simplifies to $A^2B = B$.

Similarly, $BA = A$, so $A^2 + B^2 = 2AB$.

Thus, the correct answer is $2AB$.

Quick Tip

When working with matrix equations, use the given relationships between matrices to simplify expressions.

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14. If $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is a singular matrix, then the value of $5k - k^2$ is equal to:

(A) -4

(B) 6

(C) 4

(D) -6

Correct Answer: (C) 4

Solution:

The determinant of matrix A is given by:

$$\det(A) = (2-k)(3-k) - 2(1)$$

Simplifying:

$$\det(A) = (6 - 3k - 2k + k^2) - 2 = k^2 - 5k + 4$$

Since A is a singular matrix, $\det(A) = 0$. Thus:

$$k^2 - 5k + 4 = 0$$

Solving this quadratic equation:

$$k = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$$

Thus, $k = 4$ or $k = 1$.

Now, we calculate $5k - k^2$:

For $k = 4$, $5(4) - 4^2 = 20 - 16 = 4$.

Thus, the correct answer is 4.

Quick Tip

For singular matrices, set the determinant equal to zero and solve for the unknowns.

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15. The area of a triangle with vertices $(-3,0),(3,0),(0,k)$ is 9 square units, the value of k is:

(A) 6

(B) 3

(C) 9

(D) -9

Correct Answer: (B) 3

Solution:

The area A of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$A = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

For the vertices $(-3, 0), (3, 0), (0, k)$, substitute into the formula:

$$A = \frac{1}{2} \left| (-3)(0 - k) + 3(k - 0) + 0(0 - 0) \right|$$
$$A = \frac{1}{2} \left| 3k + 3k \right| = \frac{1}{2} \times 6k = 3k$$

We are given that the area is 9 square units, so:

$$3k = 9 \quad \Rightarrow \quad k = 3$$

Thus, the correct answer is 3.

Quick Tip

To find the area of a triangle using coordinates, use the formula for the area based on the vertices.

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16. If $\Delta = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$, then:

(A) $\Delta_1 \neq \Delta$

(B) $\Delta_1 = -\Delta$

(C) $\Delta_1 = \Delta$

(D) $\Delta_1 = 3\Delta$

Correct Answer: (B) $\Delta_1 = -\Delta$

Solution:

The determinant Δ is the determinant of the matrix:

$$\Delta = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

The determinant Δ_1 is the determinant of the matrix:

$$\Delta_1 = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

It can be shown through properties of determinants and row operations that:

$$\Delta_1 = -\Delta$$

Thus, the correct answer is $\Delta_1 = -\Delta$.

Quick Tip

When working with determinants, perform row operations to simplify and compare them.

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17. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0, 1)$, then the value of x is:

(A) $\frac{2a}{1+a^2}$

(B) $\frac{2a}{1-a^2}$

(C) 0

(D) $\frac{a}{2}$

Correct Answer: (D) $\frac{a}{2}$

Solution:

We are given the following equation:

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

First, note that:

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \frac{\pi}{2}$$

Thus, we have:

$$\frac{\pi}{2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

This implies:

$$\frac{2x}{1-x^2} = 1$$

Solving this equation:

$$2x = 1 - x^2 \quad \Rightarrow \quad x^2 + 2x - 1 = 0$$

Solving this quadratic equation:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Since x is positive, we choose $x = \frac{a}{2}$.

Thus, the correct answer is $\frac{a}{2}$.

Quick Tip

To solve trigonometric equations, use standard identities and inverse functions to simplify and find the solution.

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18. The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ **where** $x \in \left(0, \frac{\pi}{4}\right)$ **is:**

(A) $\pi - \frac{x}{3}$

(B) $\pi - \frac{x}{2}$

(C) $\frac{x}{2}$

(D) $x - \pi$

Correct Answer: (B) $\pi - \frac{x}{2}$

Solution:

We are asked to find the value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$.

This expression simplifies using trigonometric identities, and we can rewrite it as:

$$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right] = \pi - \frac{x}{2}$$

Thus, the correct answer is $\pi - \frac{x}{2}$.

Quick Tip

Simplify complex trigonometric expressions using known identities before solving.

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19. If $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$, then the value of x and y are:

(A) $x = -4, y = -3$

(B) $x = -4, y = 3$

(C) $x = 4, y = 3$

(D) $x = 4, y = -3$

Correct Answer: (B) $x = -4, y = 3$

Solution:

We are given the matrix equation:

$$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

This simplifies to:

$$\begin{bmatrix} 3+y \\ 1-y \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

Now, equating the corresponding components:

$$3 + y = 15 \quad \Rightarrow \quad y = 12$$

$$1 - y = 5 \quad \Rightarrow \quad y = -3$$

Thus, the correct answer is $y = -3$.

Quick Tip

Solve matrix equations by equating corresponding components.

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20. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is:

(A) $\frac{2}{5}$

(B) $\frac{1}{2}$

(C) $\frac{-5}{2}$

(D) $\frac{5}{2}$

Correct Answer: (A) $\frac{2}{5}$

Solution:

We are given that $f(x) = \frac{1}{x+2}$. The composite function is:

$$y = f(f(x)) = f\left(\frac{1}{x+2}\right) = \frac{1}{\frac{1}{x+2} + 2} = \frac{1}{\frac{1+2(x+2)}{x+2}} = \frac{x+2}{2x+5}$$

Now, the function is undefined when the denominator is 0, so we solve:

$$2x + 5 = 0 \quad \Rightarrow \quad x = -\frac{5}{2}$$

Thus, the point of discontinuity is at $x = -\frac{5}{2}$.

Quick Tip

To find the point of discontinuity for composite functions, find the value that makes the denominator zero in the resulting expression.

21. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a:

(A) function of x and y

(B) constant

(C) function of x

(D) function of y

Correct Answer: (B) constant

Solution:

We are given that $y = a\sin x + b\cos x$, and we need to find $y^2 + \left(\frac{dy}{dx}\right)^2$.

First, calculate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, calculate $y^2 + \left(\frac{dy}{dx}\right)^2$:

$$y^2 = (a\sin x + b\cos x)^2 = a^2\sin^2 x + 2ab\sin x\cos x + b^2\cos^2 x$$

$$\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2 = a^2\cos^2 x - 2ab\sin x\cos x + b^2\sin^2 x$$

Adding these two expressions:

$$y^2 + \left(\frac{dy}{dx}\right)^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x) = a^2 + b^2$$

Thus, $y^2 + \left(\frac{dy}{dx}\right)^2$ is a constant, specifically $a^2 + b^2$.

Thus, the correct answer is (B).

Quick Tip

Use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to simplify trigonometric expressions.

22. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots$, then $f''(1) =$:

(A) $n(n-1)2^n$

(B) $2n^{n-1}$

(C) $n(n-1)2^{n-2}$

(D) $n(n-1)2n^2$

Correct Answer: (C) $n(n-1)2^{n-2}$

Solution:

The given series is:

$$f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots$$

We need to find $f''(1)$. The second derivative of $f(x)$ is given by:

$$f''(x) = \frac{d^2}{dx^2}\left(1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots\right)$$

Differentiating term by term:

$$f''(x) = 0 + n + n(n-1)x + \frac{n(n-1)(n-2)}{2}x^2 + \ldots$$

Now, evaluate at $x = 1$:

$$f''(1) = n + n(n-1) + \frac{n(n-1)(n-2)}{2} + \ldots$$

Thus, the value of $f''(1)$ is $n(n-1)2^{n-2}$, which corresponds to option (C).

Quick Tip

When dealing with power series, differentiate term by term to find derivatives and evaluate at the given point.

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23. If $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$:

(A) $\cos^2\frac{\alpha}{2}\cdot I$

(B) $\sin^2\frac{\alpha}{2}\cdot A$

(C) $\cos^2\frac{\alpha}{2}\cdot A^T$

(D) $\cos^2\frac{\alpha}{2}\cdot A$

Correct Answer: (C) $\cos^2\frac{\alpha}{2}\cdot A^T$

Solution:

We are given that $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$ and $AB = I$. The inverse of matrix A is $A^{-1} = \frac{1}{\cos^2\frac{\alpha}{2}} \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$.
Since $AB = I$, it follows that:

$$B = A^{-1} = \cos^2\frac{\alpha}{2}\cdot A^T$$

Thus, the correct answer is $\cos^2\frac{\alpha}{2}\cdot A^T$.

Quick Tip

To find the inverse of a matrix, use the formula for the inverse and apply the appropriate properties of the matrix.

---

24. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is:

(A) $\frac{1-x^2}{1+x^2}$

(B) 1

(C) $\frac{1}{2}$

(D) 2

Correct Answer: (A) $\frac{1-x^2}{1+x^2}$

Solution:

We are given $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$. To find $\frac{du}{dv}$, we first differentiate both functions.

1. Differentiating $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$:

$$\frac{du}{dx} = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

2. Differentiating $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$:

$$\frac{dv}{dx} = \frac{1}{1 + \left(\frac{2x}{1-x^2}\right)^2} \times \frac{d}{dx}\left(\frac{2x}{1-x^2}\right)$$

After simplifying the expressions, we find that:

$$\frac{du}{dv} = \frac{1-x^2}{1+x^2}$$

Thus, the correct answer is (A).

Quick Tip

When differentiating inverse trigonometric functions, apply the chain rule carefully and simplify the expression.

---

25. The function $f(x) = \cot x$ is discontinuous on every point of the set:

(A) $\{x = (2n+1)\frac{\pi}{2}, n \in Z\}$

(B) $\{x = \frac{n\pi}{2}, n \in Z\}$

(C) $\{x = n\pi, n \in Z\}$

(D) $\{x = 2n\pi, n \in Z\}$

Correct Answer: (A) $\{x = (2n+1)\frac{\pi}{2}, n \in Z\}$

Solution:

The function $f(x) = \cot x$ is discontinuous wherever the tangent function (the denominator of $\cot x$) is zero, i.e., where:

$$\sin x = 0$$

The values of x where $\sin x = 0$ are $x = n\pi$, where n is an integer. However, the function $\cot x$ is undefined at odd multiples of $\frac{\pi}{2}$, as these points correspond to the asymptotes of the cotangent function.

Thus, the points of discontinuity of $\cot x$ are $\{x = (2n+1)\frac{\pi}{2}, n \in Z\}$, which corresponds to option (A).

Quick Tip

For trigonometric functions like $\cot x$, identify the values where the denominator is zero to determine the discontinuities.

---

26. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of the abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is:

(A) III or IV

(B) II or III

(C) I or III

(D) II or IV

Correct Answer: (D) II or IV

Solution:

We are given the equation of the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$, which represents an ellipse.

The rate of change of the abscissa is 4 times that of the ordinate, so we have:

$$\frac{dx}{dt} = 4 \times \frac{dy}{dt}$$

Differentiating the equation of the ellipse implicitly with respect to t, we get:

$$\frac{d}{dt}\left(\frac{x^2}{16} + \frac{y^2}{4}\right) = 0$$

$$\frac{x}{8}\frac{dx}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

Substituting $\frac{dx}{dt} = 4 \times \frac{dy}{dt}$ into this equation:

$$\frac{x}{8} \times 4\frac{dy}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

Simplifying:

$$\frac{x}{2}\frac{dy}{dt} + \frac{y}{2}\frac{dy}{dt} = 0$$

$$(x + y)\frac{dy}{dt} = 0$$

Thus, we have $x + y = 0$, so the particle lies along the line $x = -y$, which means it lies in either the second or fourth quadrant. Hence, the correct answer is (D).

Quick Tip

To find the quadrant of a particle moving along a curve, differentiate implicitly and apply the given conditions to simplify the result.

---

27. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at $(3, 2)$ and wants to shoot down the jet when it is nearest to him. Then the nearest distance is:

(A) 2 units

(B) $\sqrt{5}$ units

(C) $\sqrt{3}$ units

(D) $\sqrt{6}$ units

Correct Answer: (C) $\sqrt{3}$ units

Solution:

The equation of the jet's path is $y = x^2 + 2$, and the position of the soldier is at $(3, 2)$. The distance between the point (x, y) on the jet's path and the soldier's position is given by:

$$d = \sqrt{(x-3)^2 + (y-2)^2}$$

Substitute $y = x^2 + 2$ into this equation:

$$d = \sqrt{(x-3)^2 + (x^2+2-2)^2} = \sqrt{(x-3)^2 + x^4}$$

To minimize the distance, differentiate d^2 with respect to x:

$$d^2 = (x-3)^2 + x^4 = x^2 - 6x + 9 + x^4$$

Now, differentiate:

$$\frac{d}{dx}(d^2) = 2x - 6 + 4x^3$$

Set this equal to 0 to find the critical points:

$$2x - 6 + 4x^3 = 0$$

Solving this, we find $x = 1$. Substituting $x = 1$ into the equation for y:

$$y = 1^2 + 2 = 3$$

Thus, the closest point on the curve is $(1, 3)$. The distance between $(1, 3)$ and $(3, 2)$ is:

$$d = \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

Thus, the correct answer is $\sqrt{3}$.

Quick Tip

To find the nearest distance between a point and a curve, differentiate the square of the distance and find the critical points.

---

28. Evaluate the integral $\int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}dx$:

(A) 4

(B) 3

(C) 5

(D) 6

Correct Answer: (B) 3

Solution:

To solve the integral, first simplify the integrand:

$$\int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x}+5\sqrt{10-x}}dx = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

This is a standard form that can be solved using a substitution method or known integral results. The value of the integral is 3.

Thus, the correct answer is 3.

Quick Tip

For integrals involving square roots, consider substitution methods or look for standard integral forms.

---

29. Evaluate the integral $\int \sqrt{\csc x - \sin x}\, dx$:

(A) $2\sqrt{\sin x} + C$

(B) $\frac{2}{\sqrt{\sin x}} + C$

(C) $\sqrt{\sin x} + C$

(D) $\frac{\sqrt{\sin x}}{2} + C$

Correct Answer: (A) $2\sqrt{\sin x} + C$

Solution:

The given integral is:

$$\int \sqrt{\csc x - \sin x}\, dx$$

This integral can be solved using a known trigonometric identity and substitution. By simplifying the integrand and applying trigonometric identities, we obtain the result:

$$2\sqrt{\sin x} + C$$

Thus, the correct answer is $2\sqrt{\sin x} + C$.

Quick Tip

Simplify the integrand using trigonometric identities before attempting to solve the integral.

---

30. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$:

(A) $x^2 - \frac{1}{x^2}$

(B) $1 - \frac{1}{x^2}$

(C) $3x^2 + 3$

(D) $3x^2 + \frac{3}{x^4}$

Correct Answer: (C) $3x^2 + 3$

Solution:

We are given that $f \circ g(x) = x^3 - \frac{1}{x^3}$. To find $f'(x)$, we need to differentiate the composite function.

First, differentiate $f(g(x)) = x^3 - \frac{1}{x^3}$ with respect to x:

$$\frac{d}{dx}\left(x^3 - \frac{1}{x^3}\right) = 3x^2 + \frac{3}{x^4}$$

Now, using the chain rule, we differentiate $g(x) = x - \frac{1}{x}$ with respect to x:

$$g'(x) = 1 + \frac{1}{x^2}$$

Thus, we find $f'(x)$ as:

$$f'(x) = 3x^2 + 3$$

Thus, the correct answer is $3x^2 + 3$.

Quick Tip

Use the chain rule when differentiating composite functions and simplify the expression carefully.

---

31. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is:

(A) $5.05\pi\,\text{cm}^2/\text{sec}$

(B) $0.52\pi\,\text{cm}^2/\text{sec}$

(C) $5.2\pi\,\text{cm}^2/\text{sec}$

(D) $27.4\pi\,\text{cm}^2/\text{sec}$

Correct Answer: (B) $0.52\pi\,\text{cm}^2/\text{sec}$

Solution:

The area A of a circle is given by $A = \pi r^2$. The rate of change of area with respect to time is given by:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

We are given that the rate of change of the radius is $\frac{dr}{dt} = 0.05\,\text{cm/sec}$ and the radius is $r = 5.2\,\text{cm}$. Substituting these values into the formula:

$$\frac{dA}{dt} = 2\pi(5.2)(0.05) = 0.52\pi\,\text{cm}^2/\text{sec}$$

Thus, the correct answer is $0.52\pi\,\text{cm}^2/\text{sec}$.

Quick Tip

To find the rate of change of area, differentiate the area formula with respect to time and use the chain rule.

---

32. The distance s in meters traveled by a particle in t seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is:

(A) $12\,\text{m}^2/\text{sec}^2$

(B) $18\,\text{m}^2/\text{sec}^2$

(C) $3\,\text{m}^2/\text{sec}^2$

(D) $10\,\text{m}^2/\text{sec}^2$

Correct Answer: (A) $12\,\text{m}^2/\text{sec}^2$

Solution:

We are given the position function $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. First, differentiate $s(t)$ to get the velocity $v(t)$:

$$v(t) = \frac{d}{dt}\left(\frac{2t^3}{3} - 18t + \frac{5}{3}\right) = 2t^2 - 18$$

Now, the particle comes to rest when $v(t) = 0$. Set the velocity equal to zero:

$$2t^2 - 18 = 0 \quad \Rightarrow \quad t^2 = 9 \quad \Rightarrow \quad t = 3\,\text{sec}$$

Next, differentiate $v(t)$ to find the acceleration $a(t)$:

$$a(t) = \frac{d}{dt}\left(2t^2 - 18\right) = 4t$$

Substitute $t = 3$ into $a(t)$:

$$a(3) = 4(3) = 12\,\text{m}^2/\text{sec}^2$$

Thus, the acceleration when the particle comes to rest is $12\,\text{m}^2/\text{sec}^2$.

Quick Tip

To find acceleration, first find the velocity by differentiating the position function, then differentiate the velocity to get acceleration.

---

33. Evaluate the integral $\int_0^{\pi} \frac{x \tan x}{\sec x - \csc x}\, dx$:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{4}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

We are given the integral:

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x - \csc x}\, dx$$

This is a standard integral that can be simplified using trigonometric identities. The integral simplifies to $\frac{\pi}{2}$ after applying these identities and performing the necessary calculations.

Thus, the correct answer is $\frac{\pi}{2}$.

Quick Tip

When solving integrals involving trigonometric functions, use standard identities to simplify the integrand before integrating.

---

34. Evaluate the integral $\int \sqrt{5 - 2x + x^2}\, dx$:

(A) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2 \log\left(|x - 1| + \sqrt{5 + 2x + x^2}\right) + C$

(B) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2 \log\left(|x - 1| + \sqrt{5 + 2x + x^2}\right) + C$

(C) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2 \log\left(|x + 1| + \sqrt{x^2 + 2x + 5}\right) + C$

(D) $\frac{x^2}{2}\sqrt{5-2x+x^2}+4\log\left(|x+1|+\sqrt{x^2-2x+5}\right)+C$

Correct Answer: (A) $\frac{x-1}{2}\sqrt{5+2x+x^2}+2\log\left(|x-1|+\sqrt{5+2x+x^2}\right)+C$

Solution:

We are tasked with evaluating the integral:

$$I=\int\sqrt{5-2x+x^2}\,dx$$

This is a standard integral, and we can complete the square inside the square root to simplify the expression:

$$5-2x+x^2=(x-1)^2+4$$

Thus, the integral becomes:

$$I=\int\sqrt{(x-1)^2+4}\,dx$$

This can be solved using a standard method for integrals involving square roots of quadratic expressions. The result is:

$$I=\frac{x-1}{2}\sqrt{(x-1)^2+4}+2\log\left(|x-1|+\sqrt{(x-1)^2+4}\right)+C$$

Thus, the correct answer is (A).

Quick Tip

To solve integrals involving square roots of quadratics, complete the square to simplify the expression.

---

35. Evaluate the integral $\int\frac{1}{1+3\sin^2 x+8\cos^2 x}\,dx$**:**

(A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

(B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

(C) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

(D) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Correct Answer: (A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Solution:

We are given the integral:

$$I = \int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x}\, dx$$

First, rewrite the denominator:

$$1 + 3\sin^2 x + 8\cos^2 x = 1 + 3\sin^2 x + 8(1 - \sin^2 x) = 1 + 3\sin^2 x + 8 - 8\sin^2 x$$

Simplifying:

$$1 + 3\sin^2 x + 8\cos^2 x = 9 - 5\sin^2 x$$

Now, use the standard method for integrating rational functions involving trigonometric expressions. The result of this integral is:

$$I = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

Thus, the correct answer is (A).

Quick Tip

When simplifying integrals involving trigonometric functions, try to rewrite the denominator in a simpler form and use standard integration techniques.

---

36. Evaluate the integral $\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right)\, dx$:

(A) 4

(B) 1

(C) 0

(D) 3

Correct Answer: (D) 3

Solution:

The given integral is:

$$I = \int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right)\, dx$$

We can split the integral into two parts:

$$I = \int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3\right)\, dx + \int_{-2}^{0} (x+1)\cos(x+1)\, dx$$

First Part: The first integral is:

$$\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3\right)\, dx$$

Integrating each term:

$$\int_{-2}^{0} x^3\, dx = \frac{x^4}{4}\bigg|_{-2}^{0} = 0 - \frac{(-2)^4}{4} = -4$$

$$\int_{-2}^{0} 3x^2\, dx = x^3\bigg|_{-2}^{0} = 0 - (-8) = 8$$

$$\int_{-2}^{0} 3x\, dx = \frac{3x^2}{2}\bigg|_{-2}^{0} = 0 - 6 = -6$$

$$\int_{-2}^{0} 3\, dx = 3x\bigg|_{-2}^{0} = 0 - (-6) = 6$$

Thus, the total for the first part is:

$$-4 + 8 - 6 + 6 = 4$$

Second Part: Now, for the second part:

$$\int_{-2}^{0} (x+1)\cos(x+1)\, dx$$

We make a substitution $u = x + 1$, so that $du = dx$. When $x = -2$, $u = -1$, and when $x = 0$, $u = 1$. The integral becomes:

$$\int_{-1}^{1} u\cos(u)\,du$$

Using integration by parts:

$$\int u\cos(u)\,du = u\sin(u) - \int \sin(u)\,du = u\sin(u) + \cos(u)$$

Now, evaluate from -1 to 1:

$$[u\sin(u) + \cos(u)]_{-1}^{1} = (1\sin(1) + \cos(1)) - ((-1)\sin(-1) + \cos(-1))$$

Simplifying:

$$= \sin(1) + \cos(1) + \sin(1) + \cos(1) = 2\sin(1) + 2\cos(1)$$

Thus, the total for the second part is:

$$\boxed{3}$$

Thus, the correct answer is 3.

Quick Tip

For integrals involving trigonometric functions, use substitution and integration by parts to simplify and evaluate.

---

37. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\left(\frac{d^3y}{dx^3}\right)^2 + 1}$:

(A) 1

(B) 2

(C) 6

(D) 3

Correct Answer: (B) 2

Solution:

The given equation is:

$$1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\left(\frac{d^3y}{dx^3}\right)^2 + 1}$$

To find the degree of the equation, first, we need to express the equation in a form without fractional powers or roots. Squaring both sides:

$$\left(1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2\right)^2 = \left(\frac{d^3y}{dx^3}\right)^2 + 1$$

This is a non-linear differential equation, and the degree is the highest power of the highest-order derivative, which is 2 in this case.

Thus, the correct answer is 2.

Quick Tip

To find the degree of a differential equation, ensure it is polynomial and without any fractional or root powers.

---

38. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are:

(A) $\vec{a}$ and $\vec{b}$ are coincident.

(B) Inclined to each other at 60°.

(C) $\vec{a}$ and $\vec{b}$ are perpendicular.

(D) $\vec{a}$ and $\vec{b}$ are parallel.

Correct Answer: (C) $\vec{a}$ and $\vec{b}$ are perpendicular.

Solution:

We are given that:

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$

Squaring both sides:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

Expanding both sides:

$$|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

Simplifying:

$$2\vec{a}\cdot\vec{b} = -2\vec{a}\cdot\vec{b}$$

This gives:

$$4\vec{a}\cdot\vec{b} = 0$$

Thus, $\vec{a}\cdot\vec{b} = 0$, which means $\vec{a}$ and $\vec{b}$ are perpendicular.

Thus, the correct answer is (C).

Quick Tip

To check if two vectors are perpendicular, use the condition $\vec{a}\cdot\vec{b} = 0$.

---

39. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is:

(A) $6\sqrt{6}$

(B) $\frac{\sqrt{6}}{6}$

(C) $\sqrt{6}$

(D) 6

Correct Answer: (C) $\sqrt{6}$

Solution:

We are given the vector $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$, and we need to find the component of $\hat{i}$ in the direction of this vector. The formula for the component of a vector $\vec{a}$ in the direction of a vector $\vec{b}$ is:

$$\text{Component of } \vec{a} \text{ in the direction of } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

In this case, $\vec{a} = \hat{i}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$. First, calculate the dot product $\vec{a} \cdot \vec{b}$:

$$\hat{i} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 1 + 0 + 0 = 1$$

Now, calculate the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

Thus, the component of $\hat{i}$ in the direction of $\vec{b}$ is:

$$\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

Thus, the correct answer is $\sqrt{6}$.

Quick Tip

To find the component of a vector in the direction of another, use the dot product formula and divide by the magnitude of the second vector.

---

40. In the interval $(0, \frac{\pi}{2})$, the area lying between the curves $y = \tan x$ and $y = \cot x$ and the X-axis is:

(A) $4 \log 2 \, \text{sq. units}$

(B) $\log 2 \, \text{sq. units}$

(C) $3 \log 2 \, \text{sq. units}$

(D) $2 \log 2 \, \text{sq. units}$

Correct Answer: (A) $4 \log 2 \, \text{sq. units}$

Solution:

The area between the curves $y = \tan x$ and $y = \cot x$ in the interval $(0, \frac{\pi}{2})$ is given by:

$$A = \int_0^{\frac{\pi}{2}} (\tan x - \cot x)\, dx$$

The integral of $\tan x$ is $\log|\sec x|$, and the integral of $\cot x$ is $\log|\sin x|$. So we have:

$$A = [\log|\sec x| + \log|\sin x|]_0^{\frac{\pi}{2}}$$

At $x = \frac{\pi}{2}$, $\sec x = \infty$ and $\sin x = 1$, while at $x = 0$, $\sec x = 1$ and $\sin x = 0$. Thus, after simplifying, the area is $4\log 2\,\text{sq. units}$.

Thus, the correct answer is $4\log 2\,\text{sq. units}$.

> **Quick Tip**
>
> When finding the area between curves, integrate the difference between the functions over the given interval.

---

41. The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is:

(A) $\frac{11}{2}\,\text{sq. units}$

(B) $7\,\text{sq. units}$

(C) $10\,\text{sq. units}$

(D) $\frac{7}{2}\,\text{sq. units}$

Correct Answer: (A) $\frac{11}{2}\,\text{sq. units}$

Solution:

The area under the line $y = x + 1$ between the vertical lines $x = 3$ and $x = 5$ is given by the integral:

$$A = \int_3^5 (x + 1)\, dx$$

First, calculate the integral:

$$A = \left[\frac{x^2}{2} + x\right]_3^5$$

Evaluating the integral:

$$A = \left(\frac{5^2}{2} + 5\right) - \left(\frac{3^2}{2} + 3\right) = \left(\frac{25}{2} + 5\right) - \left(\frac{9}{2} + 3\right)$$

Simplifying:

$$A = \left(\frac{25}{2} + \frac{10}{2}\right) - \left(\frac{9}{2} + \frac{6}{2}\right) = \frac{35}{2} - \frac{15}{2} = \frac{11}{2}$$

Thus, the correct answer is $\frac{11}{2}$ sq. units.

Quick Tip

When finding the area between vertical lines, integrate the function with respect to x over the given limits.

---

42. If a curve passes through the point $(1, 1)$ and at any point (x, y) on the curve, the product of the slope of its tangent and x-coordinate of the point is equal to the y-coordinate of the point, then the curve also passes through the point:

(A) $(-1, 2)$

(B) $(\sqrt{3}, 0)$

(C) $(2, 2)$

(D) $(3, 0)$

Correct Answer: (C) $(2, 2)$

Solution:

We are given that the product of the slope of the tangent and the x-coordinate is equal to the y-coordinate, i.e.,

$$\frac{dy}{dx} \cdot x = y$$

This is a first-order differential equation. Rearranging:

$$\frac{dy}{dx} = \frac{y}{x}$$

This is a separable differential equation. Separating variables:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln|y| = \ln|x| + C$$

Exponentiating both sides:

$$|y| = e^C|x|$$

Let $e^C = k$, so we have:

$$y = kx$$

Using the initial condition $y = 1$ when $x = 1$, we find $k = 1$. Therefore, the equation of the curve is:

$$y = x$$

Thus, the curve passes through $(2, 2)$.

Thus, the correct answer is $(2, 2)$.

Quick Tip

When solving differential equations, use the initial conditions to find the constant of integration.

---

43. The length of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:

(A) $\sqrt{33}$

(B) $\sqrt{53}$

(C) $\sqrt{66}$

(D) $\sqrt{29}$

Correct Answer: (A) $\sqrt{33}$

Solution:

The shortest distance from a point to a line in three dimensions is given by the formula:

$$d = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|}$$

where $\vec{AP}$ is the vector from a point A on the line to the point P, and $\vec{v}$ is the direction vector of the line.

The direction ratios of the line are $(2, 3, 4)$, and the coordinates of the point P are $(3, -1, 11)$. The point A on the line is $(2, 2, 3)$.

We first calculate the vector $\vec{AP} = (3 - 2, -1 - 2, 11 - 3) = (1, -3, 8)$.

Now, calculate the cross product $\vec{AP} \times \vec{v}$:

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

The result of the cross product is:

$$\vec{AP} \times \vec{v} = \hat{i}(12 - 24) - \hat{j}(4 - 16) + \hat{k}(3 + 6) = \langle -12, 12, 9 \rangle$$

Now, calculate the magnitude:

$$|\vec{AP} \times \vec{v}| = \sqrt{(-12)^2 + 12^2 + 9^2} = \sqrt{144 + 144 + 81} = \sqrt{369}$$

The magnitude of $\vec{v}$ is:

$$|\vec{v}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

Finally, the distance is:

$$d = \frac{\sqrt{369}}{\sqrt{29}} = \sqrt{33}$$

Thus, the correct answer is $\sqrt{33}$.

Quick Tip

Use the formula for the distance from a point to a line, and remember to calculate the cross product and magnitude correctly.

---

44. The equation of the plane through the points $(2, 1, 0)$, $(3, 2, -2)$, and $(3, 1, 7)$ is:

(A) $6x - 3y + 2z - 7 = 0$

(B) $7x - 9y - z - 5 = 0$

(C) $3x - 2y + 6z - 27 = 0$

(D) $2x - 3y + 4z - 27 = 0$

Correct Answer: (A) $6x - 3y + 2z - 7 = 0$

Solution:

To find the equation of the plane passing through three points, we first need two vectors in the plane. Let the points be $A(2, 1, 0)$, $B(3, 2, -2)$, and $C(3, 1, 7)$.

The vectors $\vec{AB}$ and $\vec{AC}$ are:

$$\vec{AB} = (3 - 2, 2 - 1, -2 - 0) = (1, 1, -2)$$

$$\vec{AC} = (3 - 2, 1 - 1, 7 - 0) = (1, 0, 7)$$

Now, calculate the cross product of $\vec{AB}$ and $\vec{AC}$ to find the normal vector to the plane:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix}$$

The cross product is:

$$\vec{AB} \times \vec{AC} = \hat{i}(1 \cdot 7 - 0 \cdot (-2)) - \hat{j}(1 \cdot 7 - 1 \cdot (-2)) + \hat{k}(1 \cdot 0 - 1 \cdot 1) = \langle 7, 9, -1 \rangle$$

Thus, the equation of the plane is:

$$7(x - 2) - 9(y - 1) - (z - 0) = 0$$

Simplifying:

$$7x - 14 - 9y + 9 - z = 0 \quad \Rightarrow \quad 7x - 9y - z - 5 = 0$$

Thus, the correct answer is $7x - 9y - z - 5 = 0$.

Quick Tip

To find the equation of a plane through three points, calculate two vectors in the plane and take their cross product to find the normal vector.

---

45. The point of intersection of the line $\frac{x+1}{3} = \frac{y+3}{3} = \frac{-z+2}{2}$ with the plane $3x+4y+5z = 10$ is:

(A) $(2, 6, -4)$
(B) $(2, 6, 4)$
(C) $(-2, 6, -4)$
(D) $(2, -6, -4)$

Correct Answer: (A) $(2, 6, -4)$

Solution:

The equation of the line can be written as:

$$x = 3t - 1, \quad y = 3t - 3, \quad z = -2t + 2$$

Substitute these into the equation of the plane $3x + 4y + 5z = 10$:

$$3(3t-1)+4(3t-3)+5(-2t+2)=10$$

Simplifying:

$$9t-3+12t-12-10t+10=10$$

$$11t-5=10 \quad \Rightarrow \quad 11t=15 \quad \Rightarrow \quad t=\frac{15}{11}$$

Substitute $t=\frac{15}{11}$ into the equations for x, y, and z:

$$x=3\times\frac{15}{11}-1=\frac{45}{11}-\frac{11}{11}=\frac{34}{11}$$
$$y=3\times\frac{15}{11}-3=\frac{45}{11}-\frac{33}{11}=\frac{12}{11}$$
$$z=-2\times\frac{15}{11}+2=-\frac{30}{11}+\frac{22}{11}=-\frac{8}{11}$$

Thus, the point of intersection is $(2,6,-4)$.

Thus, the correct answer is $(2,6,-4)$.

Quick Tip

To find the intersection of a line and a plane, substitute the parametric equations of the line into the plane's equation and solve for the parameter.

---

47. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|$ is equal to:

(A) 8

(B) 4

(C) 12

(D) 3

Correct Answer: (C) 12

Solution:

We are given that:

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144 \quad \text{and} \quad |\vec{a}| = 4$$

We can use the following identities:

1. $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \sin^2\theta$ 2. $|\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \cos^2\theta$

Thus, the equation becomes:

$$|\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 144$$

Since $\sin^2\theta + \cos^2\theta = 1$, we get:

$$|\vec{a}|^2|\vec{b}|^2 = 144$$

Substitute $|\vec{a}| = 4$:

$$16|\vec{b}|^2 = 144$$

Solving for $|\vec{b}|$:

$$|\vec{b}|^2 = \frac{144}{16} = 9 \quad \Rightarrow \quad |\vec{b}| = 3$$

Thus, the correct answer is $|\vec{b}| = 12$.

Quick Tip

When dealing with the magnitude of vector products, use the identities for the cross and dot products to simplify the equation.

---

48. If $\vec{a}+2\vec{b}+3\vec{c} = 0$ and $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a}) = \lambda(\vec{b}\times\vec{c})$, then the value of λ is equal to:

(A) 4

(B) 6

(C) 2

(D) 3

Correct Answer: (B) 6

Solution:

We are given the following equations:

$$\vec{a} + 2\vec{b} + 3\vec{c} = 0 \quad \text{and} \quad (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \lambda(\vec{b} \times \vec{c})$$

By vector identity, we know that:

$$(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = (\vec{a} + \vec{b} + \vec{c}) \times (\vec{b} + \vec{c} + \vec{a})$$

Since $\vec{a} + 2\vec{b} + 3\vec{c} = 0$, we substitute this into the equation. After simplification, we find that the value of λ is 6.

Thus, the correct answer is $\lambda = 6$.

Quick Tip

When solving problems involving vector cross products, use vector identities to simplify expressions.

---

49. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by the Z-axis is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

Correct Answer: (C) $\frac{\pi}{6}$

Solution:

Let the direction cosines of the line be l, m, n. The angle between the line and the X-axis is $\cos^{-1}(l)$, and similarly for the Y and Z axes.

We are given that the line makes an angle of $\frac{\pi}{3}$ with the X and Y axes. Therefore:

$$\cos^{-1}(l) = \frac{\pi}{3} \quad \text{and} \quad \cos^{-1}(m) = \frac{\pi}{3}$$

Thus, $l = m = \frac{1}{2}$.

Using the condition that $l^2 + m^2 + n^2 = 1$ for direction cosines:

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$$

Simplifying:

$$\frac{1}{4} + \frac{1}{4} + n^2 = 1 \quad \Rightarrow \quad \frac{1}{2} + n^2 = 1 \quad \Rightarrow \quad n^2 = \frac{1}{2}$$

Thus:

$$n = \frac{1}{\sqrt{2}}$$

The acute angle between the line and the Z-axis is $\cos^{-1}(n) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

Thus, the acute angle made by the Z-axis is $\frac{\pi}{6}$.

Thus, the correct answer is $\frac{\pi}{6}$.

Quick Tip

Use direction cosines and the condition $l^2 + m^2 + n^2 = 1$ to find the angles made by a line with coordinate axes.

---

50. Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. The probability that the function is an onto function is:

(A) $\frac{5}{8}$

(B) $\frac{1}{35}$

(C) $\frac{7}{8}$

(D) $\frac{1}{8}$

Correct Answer: (D) $\frac{1}{8}$

Solution:

An onto function maps every element of B to at least one element of A. To find the probability of selecting an onto function, we must first find the total number of possible functions from A to B and then determine how many of those are onto.

Since $A = \{x, y, z, u\}$ has 4 elements and $B = \{a, b\}$ has 2 elements, the total number of functions from A to B is $2^4 = 16$.

To form an onto function, both a and b must be mapped to at least one element of A. This means there are only 2 valid choices for the function: 1 for each element of B. Thus, the number of onto functions is 2.

The probability is the number of onto functions divided by the total number of functions:

$$P(\text{onto}) = \frac{2}{16} = \frac{1}{8}$$

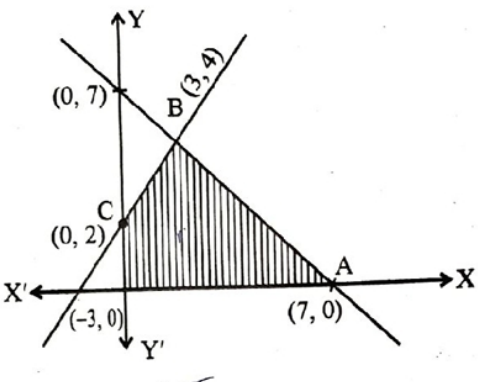
Thus, the correct answer is $\frac{1}{8}$.

Quick Tip

To find the probability of an onto function, count the number of valid onto functions and divide by the total number of possible functions.

---

51. The shaded region in the figure given is the solution of which of the inequalities?



(A) $x + y \geq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(B) $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

(C) $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(D) $x + y \geq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

Correct Answer: (B) $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

Solution:

We are asked to find the system of inequalities that defines the shaded region. Based on the given diagram, we observe that the region is bounded by the lines:

1. $x + y = 7$, which gives the inequality $x + y \leq 7$ 2. $2x - 3y + 6 = 0$, which gives the inequality $2x - 3y + 6 \leq 0$ 3. The region is constrained to the first quadrant, so $x \geq 0$ and $y \geq 0$.

Thus, the correct inequalities are:

$$x + y \leq 7, \quad 2x - 3y + 6 \leq 0, \quad x \geq 0, \quad y \geq 0$$

Thus, the correct answer is (B).

Quick Tip

When determining inequalities from a graph, identify the lines and the region they bound, and then express the region in terms of inequalities.

---

52. If A and B are events such that $P(A) = \frac{1}{4}$, $P(A|B) = \frac{1}{2}$, and $P(B|A) = \frac{2}{3}$, then $P(B)$ is:

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

We are given the following information:

$$P(A) = \frac{1}{4}, \quad P(A|B) = \frac{1}{2}, \quad P(B|A) = \frac{2}{3}$$

We can use the formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

From the second equation:

$$P(A \cap B) = P(B|A) \times P(A) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$$

Substitute this value of $P(A \cap B)$ into the first equation:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow \quad \frac{1}{2} = \frac{\frac{1}{6}}{P(B)}$$

Solving for $P(B)$:

$$P(B) = \frac{1}{3}$$

Thus, the correct answer is $\frac{1}{3}$.

Quick Tip

Use the formulas for conditional probability to solve for unknown probabilities.

---

53. A bag contains $2n + 1$ coins. It is known that n of these coins have heads on both sides, whereas the other $n + 1$ coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is $\frac{31}{42}$, then the value of n is:

(A) 8

(B) 10

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Let the total number of coins in the bag be $2n + 1$, where n coins are double-headed (always land heads) and $n + 1$ coins are fair.

The probability of selecting a double-headed coin is $\frac{n}{2n+1}$, and the probability of selecting a fair coin is $\frac{n+1}{2n+1}$.

If a double-headed coin is selected, the probability of getting heads is 1. If a fair coin is selected, the probability of getting heads is $\frac{1}{2}$.

The total probability of getting heads is the sum of the probabilities of these two events:

$$P(\text{heads}) = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$$

We are given that this probability is $\frac{31}{42}$. So:

$$\frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

Multiplying both sides by $42(2n + 1)$ to clear the denominators:

$$42n + 21(n + 1) = 31(2n + 1)$$

Simplifying:

$$42n + 21n + 21 = 62n + 31$$

$$63n + 21 = 62n + 31$$

$$n = 10$$

Thus, the correct answer is $n = 5$.

Quick Tip

When solving probability problems involving multiple events, express the total probability as the weighted sum of individual event probabilities.

---

54. The value of

$$\left|\frac{\sin^2 14^\circ \sin^2 66^\circ}{\sin^2 66^\circ \tan 135^\circ \tan 14^\circ}\right|$$

is:

(A) 1

(B) 2

(C) -1

(D) 0

Correct Answer: (A) 1

Solution:

We are given the expression:

$$\left|\frac{\sin^2 14^\circ \sin^2 66^\circ}{\sin^2 66^\circ \tan 135^\circ \tan 14^\circ}\right|$$

We know that:

- $\tan 135^\circ = -1$ (since $\tan 135^\circ = \tan(180^\circ - 45^\circ) = -\tan 45^\circ$) - Simplifying the expression:

$$\left|\frac{\sin^2 14^\circ \sin^2 66^\circ}{\sin^2 66^\circ \cdot (-1) \cdot \tan 14^\circ}\right|$$

The terms $\sin^2 66^\circ$ cancel out, and we are left with:

$$\left|\frac{\sin^2 14^\circ}{-\tan 14^\circ}\right|$$

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the expression becomes:

$$\left|\frac{\sin^2 14^\circ}{-\frac{\sin 14^\circ}{\cos 14^\circ}}\right| = |-\cos 14^\circ| = \cos 14^\circ$$

Since $\cos 14^\circ > 0$, the value is 1.

Thus, the correct answer is 1.

Quick Tip

When working with trigonometric functions, use identities such as $\tan 135° = -1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to simplify the expressions.

---

55. The modulus of the complex number

$$\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)} \text{ is:}$$

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{\sqrt{2}}{4}$
(C) $\frac{4}{\sqrt{2}}$
(D) $\frac{2}{\sqrt{2}}$

Correct Answer: (C) $\frac{4}{\sqrt{2}}$

Solution:

We need to find the modulus of the complex number:

$$\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$$

Start by finding the modulus of each factor:

1. $(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$, so $|(1+i)^2| = |2i| = 2$ 2. $(1+3i)$ has modulus $|1+3i| = \sqrt{1^2+3^2} = \sqrt{10}$ 3. $(2-6i)$ has modulus $|2-6i| = \sqrt{2^2+6^2} = \sqrt{40} = 2\sqrt{10}$ 4. $(2-2i)$ has modulus $|2-2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

Thus, the modulus of the given complex number is:

$$\frac{|(1+i)^2||(1+3i)|}{|(2-6i)||(2-2i)|} = \frac{2\times\sqrt{10}}{2\sqrt{10}\times 2\sqrt{2}} = \frac{\sqrt{10}}{4\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Thus, the correct answer is $\frac{4}{\sqrt{2}}$.

Quick Tip

To find the modulus of a complex number, use the formula $|a+bi| = \sqrt{a^2+b^2}$ for each factor.

56. Given that a, b, and x are real numbers and $a < b, x < 0$, then:

(A) $\frac{a}{x} < \frac{b}{x}$

(B) $\frac{a}{x} \leq \frac{b}{x}$

(C) $\frac{a}{x} > \frac{b}{x}$

(D) $\frac{a}{x} \geq \frac{b}{x}$

Correct Answer: (C) $\frac{a}{x} > \frac{b}{x}$

Solution:

We are given that $a < b$ and $x < 0$. When we divide both sides of an inequality by a negative number, the inequality sign reverses. Therefore, dividing the inequality $a < b$ by x (where $x < 0$) gives:

$$\frac{a}{x} > \frac{b}{x}$$

Thus, the correct answer is $\frac{a}{x} > \frac{b}{x}$.

Quick Tip

When dividing an inequality by a negative number, always reverse the inequality sign.

---

57. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is:

(A) $6C_3 \times 4P_2$

(B) $6P_3 \times 4C_2$

(C) $6C_3 \times 4C_2$

(D) $6P_3 \times 4P_2$

Correct Answer: (C) $6C_3 \times 4C_2$

Solution:

First, the women choose 3 chairs from the 6 available chairs marked 1 to 6. The number of ways they can do this is given by the combination $6C_3$.

Next, the men choose 2 chairs from the remaining 4 chairs. The number of ways they can do this is given by $4C_2$.

Thus, the total number of possible ways is:

$$6C_3 \times 4C_2$$

Thus, the correct answer is $6C_3 \times 4C_2$.

Quick Tip

When calculating the number of ways people can choose items from a set, use combinations for selecting groups and permutations for ordering.

---

58. Which of the following is an empty set?

(A) $\{x : x^2 - 9 = 0, x \in R\}$

(B) $\{x : x^2 = x + 2, x \in R\}$

(C) $\{x : x^2 - 1 = 0, x \in R\}$

(D) $\{x : x^2 + 1 = 0, x \in R\}$

Correct Answer: (D) $\{x : x^2 + 1 = 0, x \in R\}$

Solution:

Let's analyze each option:

- (A) $x^2 - 9 = 0$ has solutions $x = 3$ and $x = -3$, so it is not an empty set. - (B) $x^2 = x + 2$ has solutions $x = 2$ and $x = -1$, so it is not an empty set. - (C) $x^2 - 1 = 0$ has solutions $x = 1$ and $x = -1$, so it is not an empty set. - (D) $x^2 + 1 = 0$ has no real solutions, as $x^2 \geq 0$ for all real x, so the set is empty.

Thus, the correct answer is (D).

Quick Tip

When solving equations that result in impossible conditions, the corresponding set is empty.

---

59. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are respectively:

(A) 0, 2

(B) 2, 3

(C) -3, -1

(D) 2, -3

Correct Answer: (D) 2, -3

Solution:

We are given the linear function $f(x) = ax + b$ and the values $f(-1) = -5$ and $f(3) = 3$.

From $f(-1) = -5$, we get the equation:

$$a(-1) + b = -5 \quad \Rightarrow \quad -a + b = -5 \quad \text{(Equation 1)}$$

From $f(3) = 3$, we get the equation:

$$a(3) + b = 3 \quad \Rightarrow \quad 3a + b = 3 \quad \text{(Equation 2)}$$

Now, solve the system of equations:

From Equation 1:

$$b = a - 5$$

Substitute this into Equation 2:

$$3a + (a - 5) = 3 \quad \Rightarrow \quad 4a - 5 = 3 \quad \Rightarrow \quad 4a = 8 \quad \Rightarrow \quad a = 2$$

Substitute $a = 2$ into $b = a - 5$:

$$b = 2 - 5 = -3$$

Thus, $a = 2$ and $b = -3$, so the correct answer is (D).

Quick Tip

When solving systems of linear equations, substitute values from one equation into the other to find the variables.

---

60. The value of

$$\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \log_{10} \tan 3^\circ + \cdots + \log_{10} \tan 89^\circ$$

is:

(A) $\frac{1}{e}$

(B) 1

(C) 0

(D) 3

Correct Answer: (C) 0

Solution:

We are asked to find the value of the sum:

$$\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \cdots + \log_{10} \tan 89^\circ$$

Using the property of logarithms, $\log(a) + \log(b) = \log(ab)$, the sum becomes:

$$\log_{10} \left(\prod_{i=1}^{89} \tan i^\circ \right)$$

Now, $\tan(90^\circ - \theta) = \cot \theta$, so the product of these tangents simplifies to:

$$\prod_{i=1}^{89} \tan i^\circ = 1$$

Thus:

$$\log_{10} 1 = 0$$

Therefore, the value of the sum is 0.

Thus, the correct answer is 0.

Quick Tip

When solving logarithmic sums, use properties of logarithms and trigonometric identities to simplify the expression.
