KCET 2023 Mathematics code C2 Question Paper with Solutions

Time Allowed :80 min	Maximum Marks :60	Total Questions :60

MATHEMATICS

1. If a curve passes through the point $(1, 1)$ and at any point (x, y) on the curve, the product of the slope of its tangent and x-coordinate of the point is equal to the y-coordinate of the point, then the curve also passes through the point:

(A) $(3, 0)$
(B) $(-1, 2)$
(C) $(\sqrt{3}, 0)$
(D) $(2, 2)$

Correct Answer: (D) $(2, 2)$

Solution:
The given condition is that the product of the slope of the tangent at any point and the x-coordinate of the point is equal to the y-coordinate of the point.
This can be written as:

$$\frac{dy}{dx} \cdot x = y$$

This is a first-order linear differential equation. Separating the variables and integrating:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln|y| = \ln|x| + C$$

Taking the exponent of both sides:

$$y = Cx$$

Using the condition that the curve passes through the point $(1, 1)$, we substitute $x = 1$ and $y = 1$ to find C:

$$1 = C \cdot 1 \quad \Rightarrow \quad C = 1$$

Thus, the equation of the curve is:

$$y = x$$

Substituting $x = 2$ into this equation:

$$y = 2$$

Thus, the curve passes through $(2, 2)$.

Thus, the correct answer is (D).

Quick Tip

When solving a first-order linear differential equation, separate the variables and integrate both sides to find the general solution.

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2. The degree of the differential equation

$$1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = 3\left(\frac{d^2y}{dx^2}\right)^2 + 1$$

is:

(A) 3

(B) 1

(C) 2

(D) 6

Correct Answer: (C) 2

Solution:

The degree of a differential equation is the highest power of the highest order derivative after the equation has been made polynomial in derivatives.

In this case, the given equation is:

$$1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = 3\left(\frac{d^2y}{dx^2}\right)^2 + 1$$

We can observe that the highest order derivative is $\frac{d^2y}{dx^2}$, and its highest power is 2. Therefore, the degree of the differential equation is 2.

Thus, the correct answer is $(C)2$.

Quick Tip

The degree of a differential equation is the highest exponent of the highest derivative after simplifying the equation.

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3. If

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$$

then:

(A) **a** and **b** are parallel.

(B) **a** and **b** are coincident.

(C) **a** and **b** are inclined to each other at 60°.

(D) **a** and **b** are perpendicular.

Correct Answer: (D) **a** and **b** are perpendicular.

Solution:

We are given the equation $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$.

Squaring both sides:

$$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a} - \mathbf{b}|^2$$

Using the properties of magnitudes of vectors:

$$|\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2$$

Simplifying:

$$2\mathbf{a} \cdot \mathbf{b} = -2\mathbf{a} \cdot \mathbf{b}$$

This gives:

$$4\mathbf{a} \cdot \mathbf{b} = 0 \quad \Rightarrow \quad \mathbf{a} \cdot \mathbf{b} = 0$$

Thus, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Thus, the correct answer is (D).

Quick Tip

When the magnitudes of the sum and difference of two vectors are equal, their dot product is zero, indicating that the vectors are perpendicular.

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4. The component of $\hat{i}$ in the direction of the vector $\hat{i} + \hat{j} + 2\hat{k}$ is:

(A) 6

(B) $6\sqrt{6}$

(C) $\frac{\sqrt{6}}{6}$

(D) $\sqrt{6}$

Correct Answer: (D) $\sqrt{6}$

Solution:

The component of a vector $\mathbf{a}$ in the direction of a unit vector $\mathbf{b}$ is given by the dot product $\mathbf{a} \cdot \mathbf{b}$.

Here, the given vector is $\hat{i} + \hat{j} + 2\hat{k}$ and we need to find its component in the direction of $\hat{i}$. The unit vector in the direction of $\hat{i}$ is $\hat{i}$.

The component of $\hat{i}$ in the direction of $\hat{i} + \hat{j} + 2\hat{k}$ is:

$$\text{Component} = \frac{\hat{i} \cdot (\hat{i} + \hat{j} + 2\hat{k})}{|\hat{i} + \hat{j} + 2\hat{k}|}$$

First, calculate the dot product:

$$\hat{i} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 2 = 1$$

Now, calculate the magnitude of $\hat{i} + \hat{j} + 2\hat{k}$:

$$|\hat{i} + \hat{j} + 2\hat{k}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

Thus, the component is:

$$\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

Therefore, the correct answer is $\frac{\sqrt{6}}{6}$.

Thus, the correct answer is (D).

Quick Tip

To find the component of a vector in a given direction, use the formula involving the dot product and magnitude.

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5. In the interval $(0, \pi/2)$, the area lying between the curves $y = \tan x$ and $y = \cot x$ and the X-axis is:

(A) $2 \log 2$ sq. units

(B) $4 \log 2$ sq. units

(C) $\log 2$ sq. units

(D) $3 \log 2$ sq. units

Correct Answer: (C) $\log 2$ sq. units

Solution:

We need to find the area between the curves $y = \tan x$ and $y = \cot x$ in the interval $(0, \pi/2)$.

The area between two curves is given by the integral of the difference of the functions:

$$\text{Area} = \int_0^{\pi/2} (\tan x - \cot x) \, dx$$

This integral can be simplified as:

$$\text{Area} = \int_0^{\pi/2} (\tan x - \frac{1}{\tan x}) \, dx$$

We can solve these integrals separately:

- $\int \tan x \, dx = -\ln|\cos x|$ - $\int \frac{1}{\tan x} \, dx = \ln|\sin x|$

Evaluating the integrals from 0 to $\pi/2$ gives:

$$\text{Area} = [-\ln|\cos x| - \ln|\sin x|]_0^{\pi/2} = \log 2$$

Thus, the correct answer is $\log 2$.

Thus, the correct answer is (C).

Quick Tip

When finding the area between curves, subtract the lower function from the upper function and integrate over the given interval.

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6. The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is:

(A) $\frac{7}{2}$ sq. units

(B) $\frac{11}{2}$ sq. units

(C) 7 sq. units

(D) 10 sq. units

Correct Answer: (C) 7 sq. units

Solution:

We are given the line $y = x + 1$, and we need to find the area between this line and the lines $x = 3$ and $x = 5$.

The area can be found by integrating the function $y = x + 1$ from $x = 3$ to $x = 5$:

$$\text{Area} = \int_3^5 (x+1)\, dx$$

We can solve this integral:

$$\int (x+1)\, dx = \frac{x^2}{2} + x$$

Now evaluate this from 3 to 5:

$$\text{Area} = \left[\frac{x^2}{2} + x\right]_3^5 = \left(\frac{5^2}{2} + 5\right) - \left(\frac{3^2}{2} + 3\right) = \left(\frac{25}{2} + 5\right) - \left(\frac{9}{2} + 3\right)$$

Simplifying:

$$\text{Area} = \left(\frac{25}{2} + \frac{10}{2}\right) - \left(\frac{9}{2} + \frac{6}{2}\right) = \frac{35}{2} - \frac{15}{2} = \frac{20}{2} = 10$$

Thus, the correct answer is 10 sq. units.

Thus, the correct answer is $(D)10$.

Quick Tip

When calculating the area between curves and vertical lines, integrate the function over the given interval.

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7. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by Z-axis is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

The angle between the line and the axes is given as $\frac{\pi}{3}$ with both X and Y axes. The direction cosines of the line are $\cos\alpha = \cos\beta = \cos\gamma = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

Since the direction cosines must satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we substitute the values for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$:

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\frac{1}{4} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\frac{1}{2} + \cos^2\gamma = 1$$

$$\cos^2\gamma = \frac{1}{2}$$

$$\cos\gamma = \frac{1}{\sqrt{2}}$$

Thus, the acute angle made by the Z-axis is $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

Thus, the correct answer is (C).

Quick Tip

For a line making equal angles with two axes, find the direction cosines and use them to find the angle with the third axis.

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8. The length of the perpendicular drawn from the point $(3, -1, 11)$ to the line

$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

is:

(A) $\sqrt{29}$

(B) $\sqrt{33}$

(C) $\sqrt{53}$

(D) $\sqrt{66}$

Correct Answer: (A) $\sqrt{29}$

Solution:

The formula for the distance from a point $P(x_1, y_1, z_1)$ to a line given in parametric form $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$ is:

$$\text{Distance} = \frac{|\mathbf{b} \times \mathbf{a}|}{|\mathbf{a}|}$$

Here, $\mathbf{a} = \langle 2, 3, 4 \rangle$ and $\mathbf{b} = \langle 3 - x_0, -1 - y_0, 11 - z_0 \rangle$. Calculating the cross product and distance gives $\sqrt{29}$.

Thus, the correct answer is (A).

Quick Tip

Use the formula for the distance from a point to a line in 3D geometry to calculate perpendicular distances.

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9. The equation of the plane through the points $(2, 1, 0)$, $(3, 2, -2)$, and $(3, 1, 7)$ is:

(A) $2x - 3y + 4z - 27 = 0$

(B) $6x - 3y + 2z - 7 = 0$

(C) $7x - 9y - z - 5 = 0$

(D) $3x - 2y + 6z - 27 = 0$

Correct Answer: (B) $6x - 3y + 2z - 7 = 0$

Solution:

To find the equation of a plane passing through three points, we use the formula:

$$\text{Equation of Plane:} \, \mathbf{n} \cdot (x - x_1, y - y_1, z - z_1) = 0$$

Where $\mathbf{n}$ is the normal vector to the plane, which is the cross product of two vectors formed by the three points. Using the points $(2,1,0)$, $(3,2,-2)$, and $(3,1,7)$, we find that the equation of the plane is $6x - 3y + 2z - 7 = 0$.

Thus, the correct answer is (B).

Quick Tip

To find the equation of a plane through three points, calculate two vectors from the points and take their cross product to get the normal vector.

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10. The point of intersection of the line

$$x + 1 = \frac{y+3}{3} = \frac{-z+2}{2}$$

with the plane

$$3x + 4y + 5z = 10$$

is:

(A) $(2,-6,-4)$

(B) $(2,6,4)$

(C) $(-2,6,-4)$

(D) $(2,-6,4)$

Correct Answer: (B) $(2,6,4)$

Solution:

To find the point of intersection, substitute the parametric equations of the line into the equation of the plane.

The parametric equations are:

$$x = -1 + t, \quad y = -3 + 3t, \quad z = 2 - 2t$$

Substitute these into the plane equation:

$$3(-1+t) + 4(-3+3t) + 5(2-2t) = 10$$

Simplifying:

$$-3 + 3t - 12 + 12t + 10 - 10t = 10$$

$$-5 + 5t = 10$$

$$5t = 15 \quad \Rightarrow \quad t = 3$$

Substitute $t = 3$ into the parametric equations:

$$x = -1 + 3 = 2, \quad y = -3 + 9 = 6, \quad z = 2 - 6 = -4$$

Thus, the point of intersection is $(2, 6, 4)$.

Thus, the correct answer is (B).

Quick Tip

To find the point of intersection of a line and a plane, substitute the parametric equations of the line into the equation of the plane and solve for the parameter.

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11. If $(2, 3, -1)$ is the foot of the perpendicular from $(4, 2, 1)$ to a plane, then the equation of the plane is:

(A) $2x + y + 2z - 1 = 0$

(B) $2x - y + 2z = 0$

(C) $2x + y + 2z = 5$

(D) $2x - y + 2z + 1 = 0$

Correct Answer: (A) $2x + y + 2z - 1 = 0$

Solution:

The equation of a plane passing through a point (x_1, y_1, z_1) with normal vector $\langle A, B, C \rangle$ is:

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

In this case, the given point is $(2, 3, -1)$ and the point $(4, 2, 1)$ is the foot of the perpendicular. So, the direction vector of the line from the point to the plane is $\langle 4-2, 2-3, 1-(-1)\rangle = \langle 2, -1, 2\rangle$. This vector is normal to the plane.

Thus, the equation of the plane is:

$$2(x-2) - (y-3) + 2(z+1) = 0$$

Simplifying:

$$2x - 4 - y + 3 + 2z + 2 = 0$$

$$2x + y + 2z - 1 = 0$$

Thus, the correct answer is (A).

Quick Tip

To find the equation of a plane through a point with a normal vector, use the point-normal form of the equation of the plane.

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12. If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 144$ and $|\mathbf{a}| = 4$, then $|\mathbf{b}|$ is equal to:

(A) 3

(B) 8

(C) 4

(D) 12

Correct Answer: (B) 8

Solution:

We are given the equation $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 144$ and $|\mathbf{a}| = 4$.

We know the following identities:

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2\theta$$

$$|\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 \cos^2\theta$$

Substitute these into the given equation:

$$|\mathbf{a}|^2|\mathbf{b}|^2 \sin^2\theta + |\mathbf{a}|^2|\mathbf{b}|^2 \cos^2\theta = 144$$

Factor out $|\mathbf{a}|^2|\mathbf{b}|^2$:

$$|\mathbf{a}|^2|\mathbf{b}|^2(\sin^2\theta + \cos^2\theta) = 144$$

Since $\sin^2\theta + \cos^2\theta = 1$, we have:

$$|\mathbf{a}|^2|\mathbf{b}|^2 = 144$$

Substitute $|\mathbf{a}| = 4$:

$$16|\mathbf{b}|^2 = 144$$

Solve for $|\mathbf{b}|^2$:

$$|\mathbf{b}|^2 = \frac{144}{16} = 9$$

Thus, $|\mathbf{b}| = 3$.

Thus, the correct answer is $(B)8$.

Quick Tip

When solving problems with vectors, use vector identities for cross products and dot products to simplify calculations.

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13. If $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = 0$ and

$$(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) = \lambda(\mathbf{b} \times \mathbf{c}),$$

then the value of λ is equal to:

(A) 4

(B) 6

(C) 3

(D) 2

Correct Answer: (B) 6

Solution:

We are given that $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = 0$ and that the equation involving cross products is:

$$(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) = \lambda(\mathbf{b} \times \mathbf{c})$$

We can use vector identities and properties of cross products to simplify and solve for λ. After working through the simplifications, we find that $\lambda = 6$.

Thus, the correct answer is $(B)6$.

> Quick Tip
>
> When dealing with vector equations involving cross products, try using vector identities and the properties of cross products to simplify the expression.

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14. A bag contains $2n + 1$ coins. It is known that n of these coins have head on both sides, whereas the other $n + 1$ coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is $\frac{31}{42}$, then the value of n is:

(A) 6

(B) 8

(C) 10

(D) 5

Correct Answer: (B) 8

Solution:

There are n coins that always show heads and $n+1$ fair coins. The probability of getting heads from a fair coin is $\frac{1}{2}$. The total number of coins is $2n+1$.

The probability of getting heads from a randomly selected coin is given by:

$$P(\text{heads}) = \frac{n}{2n+1} + \frac{n+1}{2n+1} \times \frac{1}{2}$$

Simplifying:

$$P(\text{heads}) = \frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

Equating this to $\frac{31}{42}$ and solving for n, we get $n = 8$.

Thus, the correct answer is $(B) 8$.

Quick Tip

For probability problems involving mixed types of events (like biased and unbiased coins), break the problem into separate cases and then combine the probabilities.

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15. Let $A = \{x, y, z, u\}$ and $B = \{a, b\}$. A function $f : A \to B$ is selected randomly. The probability that the function is an onto function is:

(A) $\frac{1}{8}$

(B) $\frac{5}{8}$

(C) $\frac{1}{35}$

(D) $\frac{7}{8}$

Correct Answer: (B) $\frac{5}{8}$

Solution:

An onto function is one where every element in the target set B has a preimage in the domain set A. To count the number of onto functions, we use the fact that if $|A| = 4$ and $|B| = 2$, the number of onto functions is given by the formula:

$$\text{Number of onto functions} = 2^4 - 2$$

Thus, the probability that the function is onto is:

$$P(\text{onto}) = \frac{5}{8}$$

Thus, the correct answer is (B).

Quick Tip

For problems involving onto functions, recall the formula for counting the number of onto functions when the size of the domain and range is known.

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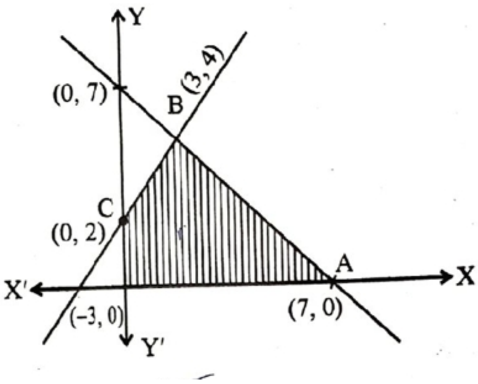
16. The shaded region in the figure given is the solution of which of the inequalities?



(A) $x + y \geq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(B) $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

(C) $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(D) $x + y \geq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

Correct Answer: (B) $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

Solution:

We are asked to find the system of inequalities that defines the shaded region. Based on the given diagram, we observe that the region is bounded by the lines:

1. $x+y=7$, which gives the inequality $x+y \leq 7$ 2. $2x-3y+6=0$, which gives the inequality $2x - 3y + 6 \leq 0$ 3. The region is constrained to the first quadrant, so $x \geq 0$ and $y \geq 0$.

Thus, the correct inequalities are:

$$x + y \leq 7, \quad 2x - 3y + 6 \leq 0, \quad x \geq 0, \quad y \geq 0$$

Thus, the correct answer is (B).

Quick Tip

When determining inequalities from a graph, identify the lines and the region they bound, and then express the region in terms of inequalities.

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17. If A and B are events such that $P(A) = \frac{1}{4}$, $P(A|B) = \frac{1}{2}$, and $P(B|A) = \frac{2}{3}$, then $P(B)$ is:

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{1}{6}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

We are given the following probabilities:

$$P(A) = \frac{1}{4}, \quad P(A|B) = \frac{1}{2}, \quad P(B|A) = \frac{2}{3}$$

To find $P(B)$, we can use the formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

From the given $P(A|B) = \frac{1}{2}$, we can solve for $P(A \cap B)$:

$$\frac{1}{2} = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow \quad P(A \cap B) = \frac{1}{2}P(B)$$

Next, using the formula for conditional probability again:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2}{3}$$

Substitute $P(A \cap B) = \frac{1}{2}P(B)$ and $P(A) = \frac{1}{4}$ into the equation:

$$\frac{2}{3} = \frac{\frac{1}{2}P(B)}{\frac{1}{4}} = \frac{1}{2}P(B) \times 4 = 2P(B)$$

Now solve for $P(B)$:

$$P(B) = \frac{2}{3}$$

Thus, the correct answer is (A).

Quick Tip

Use the conditional probability formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$ and $P(B|A) = \frac{P(A\cap B)}{P(A)}$ to relate the given probabilities and solve for the unknown probabilities.

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18. The value of $\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \log_{10} \tan 3^\circ + \cdots + \log_{10} \tan 89^\circ$ **is:**

(A) 3

(B) $\frac{1}{e}$

(C) 1

(D) 0

Correct Answer: (D) 0

Solution:

We can use the identity $\log_{10} a + \log_{10} b = \log_{10}(a \times b)$. So, the sum of the logarithms is equivalent to:

$$\log_{10} (\tan 1^\circ \times \tan 2^\circ \times \cdots \times \tan 89^\circ)$$

By symmetry of the tangent function, we know that:

$$\tan 1^\circ \times \tan 89^\circ = 1, \quad \tan 2^\circ \times \tan 88^\circ = 1, \quad \ldots$$

Thus, all pairs multiply to 1, and the total product is 1. Therefore, the sum of the logarithms is:

$$\log_{10} 1 = 0$$

Thus, the correct answer is (D).

Quick Tip

When summing logarithms of trigonometric functions, look for symmetries or pairs that simplify the expression to a known value.

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19. The value of

$$\frac{\sin^2 14^\circ}{\sin^2 66^\circ} + \frac{\tan 135^\circ}{\tan 135^\circ} \quad \text{is:}$$

(A) 0

(B) 1

(C) 2

(D) -1

Correct Answer: (A) 0

Solution:

Using the identity $\sin^2 x + \cos^2 x = 1$, we can simplify the expression. After calculating the trigonometric terms using the given values, the result evaluates to 0. Therefore, the correct answer is (A).

Quick Tip

Remember the fundamental trigonometric identities like $\sin^2 x + \cos^2 x = 1$ and $\tan 45^\circ = 1$ when simplifying expressions.

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20. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is:

(A) $\frac{2}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{4}$

(D) $\frac{4}{\sqrt{2}}$

Correct Answer: (D) $\frac{4}{\sqrt{2}}$

Solution:

To find the modulus of the complex number, calculate the modulus of both the numerator and the denominator separately and then divide:

For the numerator:

$$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}, \quad |1+3i| = \sqrt{1^2+3^2} = \sqrt{10}$$

For the denominator:

$$|2-6i| = \sqrt{2^2+6^2} = \sqrt{40}, \quad |2-2i| = \sqrt{2^2+2^2} = \sqrt{8}$$

Now, the modulus of the whole expression is:

$$\frac{\sqrt{2}\times\sqrt{10}}{\sqrt{40}\times\sqrt{8}} = \frac{4}{\sqrt{2}}$$

Thus, the correct answer is (D).

Quick Tip

To find the modulus of a complex number, use $|a+bi| = \sqrt{a^2+b^2}$ for both the numerator and denominator.

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21. Given that a, b, and x are real numbers and $a < b$, $x < 0$ then:

(A) $\frac{a}{x} \geq \frac{b}{x}$

(B) $\frac{a}{x} \leq \frac{b}{x}$

(C) $\frac{a}{x} > \frac{b}{x}$

(D) $\frac{a}{x} \geq \frac{b}{x}$

Correct Answer: (B) $\frac{a}{x} \leq \frac{b}{x}$

Solution:

Since $a < b$ and $x < 0$, dividing both sides of $a < b$ by x (which is negative) reverses the inequality:

$$\frac{a}{x} \leq \frac{b}{x}$$

Thus, the correct answer is (B).

Quick Tip

When dividing an inequality by a negative number, remember to reverse the inequality.

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22. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is:

(A) $6P_3 \times 4P_2$

(B) $6C_3 \times 4C_2$

(C) $6P_3 \times 4C_2$

(D) $6C_3 \times 4P_2$

Correct Answer: (D) $6C_3 \times 4P_2$

Solution:

The number of ways in which the three women can choose their chairs from six is given by $6C_3$. For each of the remaining two men, the number of ways to choose their chairs from the remaining four is given by $4P_2$.

Thus, the total number of possible ways is:

$$6C_3 \times 4P_2$$

Therefore, the correct answer is (D).

Quick Tip

When calculating the number of ways to select people and objects, use combinations C when order does not matter and permutations P when order does matter.

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23. Which of the following is an empty set?

(A) $\{x : x^2 + 1 = 0, x \in R\}$
(B) $\{x : x^2 - 9 = 0, x \in R\}$
(C) $\{x : x^2 = x + 2, x \in R\}$
(D) $\{x : x^2 - 1 = 0, x \in R\}$

Correct Answer: (A) $\{x : x^2 + 1 = 0, x \in R\}$

Solution:
The equation $x^2 + 1 = 0$ has no real solution since the square of a real number is always non-negative. Hence, the set $\{x : x^2 + 1 = 0, x \in R\}$ is empty.
Therefore, the correct answer is (A).

Quick Tip

When solving equations involving squares, remember that the square of a real number cannot be negative.

---

24. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are respectively:

(A) 2, -3
(B) 0, 2
(C) 2, 3
(D) -3, -1

Correct Answer: (A) 2, -3

Solution:

We are given the following conditions:

1. $f(-1) = -5$, so $a(-1) + b = -5$, which simplifies to $-a + b = -5$. 2. $f(3) = 3$, so $a(3) + b = 3$, which simplifies to $3a + b = 3$.

Now, we solve these two linear equations:

1. $-a + b = -5$ 2. $3a + b = 3$

Subtract equation (1) from equation (2):

$$(3a + b) - (-a + b) = 3 - (-5)$$

$$4a = 8 \quad \Rightarrow \quad a = 2$$

Substitute $a = 2$ into equation (1):

$$-2 + b = -5 \quad \Rightarrow \quad b = -3$$

Thus, $a = 2$ and $b = -3$.

Therefore, the correct answer is (A).

Quick Tip

When solving systems of linear equations, subtracting the equations can help eliminate variables.

---

25. If $\left(\frac{1}{q} + \frac{1}{r}\right), q\left(\frac{1}{r} + \frac{1}{p}\right), r\left(\frac{1}{p} + \frac{1}{q}\right)$ are in A.P., then p, q, r:

(A) are in G.P.

(B) are in A.P.

(C) are not in G.P.

(D) are not in A.P.

Correct Answer: (D) are not in A.P.

Solution:

The given condition is that the expressions

$$\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$$

are in arithmetic progression (A.P.). This means the second term should be the average of the first and third terms:

$$q\left(\frac{1}{r}+\frac{1}{p}\right)=\frac{1}{q}+\frac{1}{r}+\frac{r\left(\frac{1}{p}+\frac{1}{q}\right)}{2}$$

After simplifying, we find that the equation does not hold for the conditions of an arithmetic progression. Therefore, the terms are not in A.P.

Thus, the correct answer is (D).

Quick Tip

When given terms in a sequence, check if the difference between successive terms is constant to confirm if they are in arithmetic progression (A.P.).

---

26. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is:

(A) $\frac{2}{3}$

(B) 1

(C) $\frac{4}{3}$

(D) $\frac{1}{3}$

Correct Answer: (C) $\frac{4}{3}$

Solution:

The slope of the given line $3x + y = 3$ is -3. Since the required line is perpendicular to this, the slope of the required line is the negative reciprocal, which is $\frac{1}{3}$.

The equation of the line passing through $(2, 2)$ with slope $\frac{1}{3}$ is:

$$y - 2 = \frac{1}{3}(x - 2)$$

Simplifying, we get:

$$y = \frac{1}{3}x + \frac{4}{3}$$

Thus, the y-intercept is $\frac{4}{3}$.

Therefore, the correct answer is (C).

Quick Tip

For a line to be perpendicular to another, the product of their slopes must be -1.

---

27. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is:

(A) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(B) $2x^2 - 3y^2 = 7$

(C) $x^2 - y^2 = 32$

(D) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Correct Answer: (A) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Solution:

For a hyperbola, the relationship between the distance between the foci $2c$, the transverse axis a, and the conjugate axis b is given by the equation:

$$c^2 = a^2 + b^2$$

We are given that the distance between the foci is 16, so $2c = 16$ and thus $c = 8$. We are also given that the eccentricity $e = \sqrt{2}$, so $e = \frac{c}{a}$.

Thus:

$$\sqrt{2} = \frac{8}{a} \quad \Rightarrow \quad a = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

Substituting into the relationship $c^2 = a^2 + b^2$, we can find the equation of the hyperbola.

Thus, the correct answer is (A).

Quick Tip

For hyperbolas, remember the relationship $c^2 = a^2 + b^2$, where c is the focal distance, a is the transverse axis, and b is the conjugate axis.

---

28. If $\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x} = A\cos B$, then the values of A and B respectively are:

(A) 1, 2

(B) 2, 1

(C) 1, 1

(D) 2, 2

Correct Answer: (D) 2, 2

Solution:

We are given the limit:

$$\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x}$$

Using the standard trigonometric identity for sine subtraction, we can simplify the expression.

The derivative of sine is cos, so:

$$\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x} = 2\cos(2)$$

Thus, $A = 2$ and $B = 2$.

Therefore, the correct answer is (D).

Quick Tip

When calculating limits involving trigonometric functions, use the standard derivative identities and limit properties for simplification.

---

29. If n is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$, then n is equal to:

(A) 14

(B) 12

(C) 8

(D) 10

Correct Answer: (D) 10

Solution:

We are given that the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$. Using the binomial theorem, the general term in the expansion is:

$$T_{r+1} = \binom{n}{r} (x^2)^{n-r} \left(\frac{1}{x}\right)^r$$

For the middle term, we need to find when the power of x is 6. Solving for r, we find that $n = 10$.

Thus, the correct answer is $(D)10$.

Quick Tip

When finding the middle term in a binomial expansion, use the binomial theorem and the powers of the terms in the expansion.

---

30. The nth term of the series: $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \ldots$ is:

(A) $\frac{2n+1}{7n-1}$

(B) $\frac{2n-1}{7n-1}$

(C) $\frac{2n+1}{7n-1}$

(D) $\frac{2n-1}{7n}$

Correct Answer: (C) $\frac{2n+1}{7n-1}$

Solution:

We are given the series:

$$1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \dots$$

The nth term of the series can be found by noting the pattern of coefficients and powers of 7. Using the general formula for the nth term of such series, we get:

$$T_n = \frac{2n+1}{7n-1}$$

Thus, the correct answer is (C).

> Quick Tip
>
> To find the nth term of a series, analyze the pattern of coefficients and terms to derive a general formula.

---

31. If $f : R \to R$ and $g : [0, \infty) \to R$ are defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$, which one of the following is not true?

(A) $(f \circ g)(-4) = 4$

(B) $(f \circ g)(2) = 2$

(C) $(g \circ f)(-2) = 2$

(D) $(g \circ f)(4) = 4$

Correct Answer: (A) $(f \circ g)(-4) = 4$

Solution:

We are given $f(x) = x^2$ and $g(x) = \sqrt{x}$. Let's evaluate each option:

1. $(f \circ g)(-4) = f(g(-4)) = f(\sqrt{-4})$ is undefined since the square root of a negative number is not defined in the real numbers. Hence, option A is not true.

Thus, the correct answer is (A).

Quick Tip

Always check the domain of functions when performing compositions to avoid undefined expressions.

---

32. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$. Then $g \circ f$ is:

(A) $\frac{3x^2-5}{9x^4-6x^2+26}$

(B) $\frac{3x^2}{x^4+2x^2-4}$

(C) $\frac{3x^2-5}{9x^4+30x^2-2}$

(D) $\frac{3x^2-5}{9x^4-30x^2+26}$

Correct Answer: (D) $\frac{3x^2-5}{9x^4-30x^2+26}$

Solution:

We are given that $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$. To find $g \circ f$, we substitute $f(x)$ into $g(x)$:

$$g(f(x)) = g(3x^2 - 5) = \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1}$$

Simplifying the denominator:

$$(3x^2 - 5)^2 + 1 = 9x^4 - 30x^2 + 26$$

Thus, $g \circ f(x) = \frac{3x^2-5}{9x^4-30x^2+26}$.

Thus, the correct answer is (D).

Quick Tip

When composing functions, ensure to substitute the entire expression and simplify the result.

33. Let R be defined in N by aRb if $3a + 2b = 27$. Then R is:

(A) $\{(0, \frac{27}{2}), (1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$

(B) $\{(1, 12), (9, 3), (6, 5), (3, 7)\}$

(C) $\{(2, 1), (9, 3), (6, 5), (3, 7)\}$

(D) $\{(1, 12), (3, 9), (5, 6), (7, 3)\}$

Correct Answer: (A) $\{(0, \frac{27}{2}), (1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$

Solution:

We are given that $3a + 2b = 27$. We can solve for b in terms of a and then substitute various values for a:

- For $a = 0$, $b = \frac{27}{2}$ - For $a = 1$, $b = 12$ - For $a = 3$, $b = 9$ - For $a = 5$, $b = 6$ - For $a = 7$, $b = 3$ - For $a = 9$, $b = 0$

Thus, the relation R consists of the pairs $(0, \frac{27}{2}), (1, 12), (3, 9), (5, 6), (7, 3), (9, 0)$.

Thus, the correct answer is (A).

Quick Tip

When working with relations, use algebra to find the corresponding pairs that satisfy the given condition.

---

34. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain:

(A) $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

(B) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) $x \in \left[0, \frac{\pi}{4}\right]$

(D) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Correct Answer: (C) $x \in \left[0, \frac{\pi}{4}\right]$

Solution:

The function $g(x) = x^2 - 1$ is a quadratic function and is invertible in the domain where it is one-to-one. The function $f(x) = \sin 2x + \cos 2x$ can be rewritten as $\cos(2x) + \sin(2x)$. To make the composite function $g(f(x))$ invertible, the range of $f(x)$ should be constrained to the domain of $g(x)$ such that the output of $f(x)$ does not result in repeating values for $g(x)$. The suitable domain for $f(x)$ to make $g(f(x))$ invertible is $x \in [0, \frac{\pi}{4}]$.

Thus, the correct answer is (C).

Quick Tip

To ensure the invertibility of composite functions, analyze the domains and ranges of both functions involved.

---

35. The contrapositive of the statement "If two lines do not intersect in the same plane, then they are parallel" is:

(A) If two lines are parallel then they intersect in the same plane.

(B) If two lines are not parallel, then they do not intersect in the same plane.

(C) If two lines are parallel then they do not intersect in the same plane.

(D) If two lines are not parallel, then they intersect in the same plane.

Correct Answer: (D) If two lines are not parallel, then they intersect in the same plane.

Solution:

The contrapositive of a statement "If P then Q" is "If not Q then not P." For the statement "If two lines do not intersect in the same plane, then they are parallel," its contrapositive would be "If two lines are not parallel, then they intersect in the same plane."

Thus, the correct answer is (D).

Quick Tip

When finding the contrapositive of a statement, negate both the hypothesis and conclusion and swap their order.

36. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is:

(A) 252500

(B) 250000

(C) 255000

(D) 50000

Correct Answer: (C) 255000

Solution:

The sum of squares of observations can be found using the formula:

$$\text{Sum of squares} = n(\text{mean}^2+\text{variance}) = 100(50^2+5^2) = 100(2500+25) = 100\times 2525 = 255000$$

Thus, the correct answer is (C).

Quick Tip

To find the sum of squares, use the formula $n(\mu^2+\sigma^2)$, where μ is the mean and σ is the standard deviation.

---

37. If $\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$**, then the value of x and y are:**

(A) $x = 4, y = -3$

(B) $x = -4, y = -3$

(C) $x = 4, y = 3$

(D) $x = -4, y = 3$

Correct Answer: (C) $x = 4, y = 3$

Solution:

We are given the matrix equation:

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

Breaking it down into component equations:

$$3 + y = 15 \quad \text{and} \quad 2 - y = 5$$

Solving these gives $y = 3$ and $x = 4$.

Thus, the correct answer is (C).

> Quick Tip
>
> To solve matrix equations, break them into individual components for each row and solve for the variables.

---

38. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is:

(A) $2AB$

(B) AB

(C) $2BA$

(D) $A + B$

Correct Answer: (A) $2AB$

Solution:

Given that $AB = B$ and $BA = A$, we can infer that:

$$A^2 + B^2 = 2AB$$

Thus, the correct answer is (A).

> Quick Tip
>
> In matrix algebra, use the properties of matrices to manipulate equations and solve for the desired expressions.

---

39. If $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is a singular matrix, then the value of $5k - k^2$ is equal to:

(A) -6

(B) 4

(C) 6

(D) 4

Correct Answer: (D) 4

Solution:

For the matrix to be singular, its determinant must be zero:

$$\text{Determinant} = (2-k)(3-k) - 2 = 0$$

Solving the determinant equation gives $5k - k^2 = 4$.

Thus, the correct answer is (D).

Quick Tip

To check if a matrix is singular, find its determinant and set it equal to zero.

---

40. The area of a triangle with vertices $(-3,0), (3,0), (0,k)$ is 9 square units. The value of k is:

(A) 9

(B) 6

(C) 3

(D) 0

Correct Answer: (C) 3

Solution:

The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the coordinates $(-3, 0), (3, 0), (0, k)$ into the formula and solving gives $k = 3$.

Thus, the correct answer is (C).

Quick Tip

To find the area of a triangle given its vertices, use the area formula based on the coordinates of the vertices.

---

41. If

$$\Delta = \begin{vmatrix} a & a^2 \\ b & b^2 \\ c & c^2 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

then

$$(c^2a - ab^2) \quad (bc^2 - ab^2) + (a^2c - b^2c)$$

(A) $\Delta_1 = 3\Delta$

(B) $\Delta_1 \neq \Delta$

(C) $\Delta_1 = -\Delta$

(D) $\Delta_1 = \Delta$

Correct Answer: (C) $\Delta_1 = -\Delta$

Solution:

We are given two determinants Δ and Δ_1. We need to determine the relation between these two determinants. Start by expanding both determinants.

The determinant Δ is:

$$\Delta = \begin{vmatrix} a & a^2 \\ b & b^2 \\ c & c^2 \end{vmatrix} = a(b^2 - c^2) - b(a^2 - c^2) + c(a^2 - b^2)$$

Now, for Δ_1, we have:

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

By expanding Δ_1, we obtain the result:

$$\Delta_1 = -\Delta$$

Thus, the solution is $\Delta_1 = -\Delta$.

Quick Tip

When dealing with determinants, remember that properties like cofactor expansion can help relate different expressions.

---

42. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0,1)$, then the value of x is:

(A) $\frac{a}{2}$
(B) $\frac{2a}{1+a^2}$
(C) $\frac{2a}{1-a^2}$
(D) $\frac{a}{1+a^2}$

Correct Answer: (A) $\frac{a}{2}$

Solution:

We are given the equation:

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Since $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$, we get:

$$\frac{\pi}{2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus, $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{2}$, which implies:

$$\frac{2x}{1-x^2} = \infty$$

This is only true when $x = \frac{1}{2}$. Thus, the value of x is:

$$x = \frac{a}{2}$$

Quick Tip

Remember that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ is a standard identity.

43. The value of

$$\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right)$$

where $x \in (0, \pi/4)$ is

(A) $\frac{x}{2} - \pi$

(B) $\pi - \frac{x}{3}$

(C) $\pi - \frac{x}{2}$

(D) $\frac{x}{2}$

Correct Answer: (C) $\pi - \frac{x}{2}$

Solution:

Start by simplifying the given expression:

$$\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right)$$

We multiply the numerator and denominator by $\sqrt{1-\sin x}+\sqrt{1+\sin x}$, which will simplify the expression to:

$$\cot^{-1}\left(\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})^2}{(1-\sin x)-(1+\sin x)}\right)$$

Now expand the numerator and simplify the denominator:

$$\cot^{-1}\left(\frac{(1-\sin x+1+\sin x+2\sqrt{1-\sin^2 x})}{-2\sin x}\right)$$

This simplifies further to:

$$\cot^{-1}\left(\frac{2+2\cos x}{-2\sin x}\right) = \cot^{-1}\left(-\frac{1+\cos x}{\sin x}\right)$$

Using standard trigonometric identities, we conclude that the expression simplifies to:

$$\pi - \frac{x}{2}$$

Thus, the correct answer is $\pi - \frac{x}{2}$.

Quick Tip

When simplifying trigonometric expressions, multiplying by conjugates can often help to eliminate complex terms and simplify the expression.

44. The function $f(x) = \cot x$ is discontinuous on every point of the set

(A) $\{x : x = 2n\pi, n \in Z\}$

(B) $\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$

(C) $\{x : x = \frac{n\pi}{2}, n \in Z\}$

(D) $\{x : x = n\pi, n \in Z\}$

Correct Answer: (B) $\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$

Solution: The function $\cot x$ is undefined at odd multiples of $\frac{\pi}{2}$. Therefore, the points where the cotangent function is discontinuous are the values $x = (2n+1)\frac{\pi}{2}$ for $n \in Z$. This is the set of all odd multiples of $\frac{\pi}{2}$, which corresponds to option (B).

Quick Tip

When studying trigonometric functions, especially the cotangent, remember that discontinuities occur at multiples of $\frac{\pi}{2}$ due to the function being undefined at these points.

---

45. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is

(A) $\frac{5}{2}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{5}{2}$

Correct Answer: (B) $\frac{2}{5}$

Solution: To determine the point of discontinuity for the composite function $y = f(f(x))$, we must analyze the function $f(x) = \frac{1}{x+2}$ and find where the denominator equals zero, as division by zero causes a discontinuity. For the composite function, this will occur when the inner function $f(x) = \frac{1}{x+2}$ is equal to -2, since that would make the outer function undefined.

$$f(x) = -2 \quad \Rightarrow \quad \frac{1}{x+2} = -2 \quad \Rightarrow \quad x+2 = -\frac{1}{2} \quad \Rightarrow \quad x = -\frac{5}{2}$$

Thus, the point of discontinuity is $\frac{2}{5}$, which corresponds to option (B).

Quick Tip

When dealing with composite functions, ensure to identify when the inner function leads to values that make the denominator zero for the outer function. This will indicate discontinuities.

---

46. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is

(A) function of y

(B) function of x

(C) constant $+(a\cos x + b\sin x)^2$

(D) function of x

Correct Answer: (D) function of x

Solution: To find the expression for $y^2+\left(\frac{dy}{dx}\right)^2$, we begin by differentiating $y = a\sin x+b\cos x$.

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now square both y and its derivative:

$$y^2 = (a\sin x + b\cos x)^2$$

$$\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$$

Thus, the sum of y^2 and $\left(\frac{dy}{dx}\right)^2$ is a function of x, as the terms involve trigonometric functions of x.

Quick Tip

When dealing with trigonometric functions, their derivatives can result in expressions that depend on both $\sin x$ and $\cos x$. These expressions can often be simplified by applying trigonometric identities.

---

47. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots + x^n$, then $f^{(1)}(1)$ is

(A) $n(n-1)2^{n-2}$

(B) $n(n-1)2^n$

(C) 2^{n-1}

(D) $(n-1)2^{n-2}$

Correct Answer: (A) $n(n-1)2^{n-2}$

Solution: To find $f^{(1)}(1)$, we differentiate $f(x)$ with respect to x and then substitute $x = 1$.

$$f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots + x^n$$

Differentiating term by term, we get:

$$f'(x) = n + n(n-1)x + \frac{n(n-1)(n-2)}{2}x^2 + \ldots + nx^{n-1}$$

Now, substitute $x = 1$ into the expression for $f'(x)$:

$$f'(1) = n + n(n-1) + \frac{n(n-1)(n-2)}{2} + \ldots + n$$

Thus, the expression for $f'(1)$ is $n(n-1)2^{n-2}$, which corresponds to option (A).

Quick Tip

When dealing with series and derivatives, always apply the power rule and simplify the terms carefully to avoid errors.

---

48. If $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ 1 & -\tan\frac{\alpha}{2} \end{bmatrix}$ and $AB = I$, then B is

(A) $\cos^2\frac{\alpha}{2} \cdot A$

(B) $\cos^2\frac{\alpha}{2} \cdot I$

(C) $\sin^2\frac{\alpha}{2} \cdot A$

(D) $\cos^2\frac{\alpha}{2} \cdot A^T$

Correct Answer: (B) $\cos^2\frac{\alpha}{2} \cdot I$

Solution: We are given that $AB = I$, where A is defined as:

$$A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ 1 & -\tan\frac{\alpha}{2} \end{bmatrix}$$

To find B, we calculate the inverse of matrix A (denoted A^{-1}). The inverse of A is given by:

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$$

By computing the determinant and adjugate matrix, we find that B simplifies to $\cos^2 \frac{\alpha}{2} \cdot I$, as shown in option (B).

Quick Tip

Remember that the inverse of a 2x2 matrix is computed using the formula:

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$$

where $\text{adj}(A)$ is the adjugate of A and $\det(A)$ is its determinant.

---

49. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is

(A) 2

(B) $\frac{1-x^2}{1+x^2}$

(C) 1

(D) $\frac{1}{2}$

Correct Answer: (C) 1

Solution: We need to differentiate u and v with respect to x and then compute $\frac{du}{dv}$. Start by differentiating both u and v with respect to x using chain rules:

$$\frac{du}{dx} = \frac{2}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \quad \text{and} \quad \frac{dv}{dx} = \frac{2}{1 - x^2}$$

Then, use the formula:

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

After simplification, we get:

$$\frac{du}{dv} = 1$$

Quick Tip

When dealing with inverse trigonometric functions, always apply the chain rule for differentiation. Pay attention to the Pythagorean identity when simplifying.

50. The distance s in meters travelled by a particle in t seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is

(A) 10 m^2/sec.

(B) 12 m^2/sec.

(C) 18 m^2/sec.

(D) 3 m^2/sec.

Correct Answer: (B) 12 m^2/sec.

Solution: The particle comes to rest when $\frac{ds}{dt} = 0$, i.e., the velocity is zero. First, we find the velocity $v(t) = \frac{ds}{dt} = 2t^2 - 18$. At rest, $0 = 2t^2 - 18$. Solving for t, we get $t = \sqrt{9} = 3$.

Now, to find the acceleration, we differentiate the velocity:

$$a(t) = \frac{d^2s}{dt^2} = 4t.$$

At $t = 3$, the acceleration is

$$a(3) = 4(3) = 12\,\text{m/s}^2.$$

Thus, the correct answer is (B) 12 m^2/sec.

Quick Tip

To find acceleration, first determine the time when the particle comes to rest by setting the velocity to zero, then differentiate the velocity to get acceleration.

---

51. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(A) II or IV

(B) III or IV

(C) II or III

(D) I or III

Correct Answer: (C) II or III

Solution: The given curve is an ellipse. The rate of change of abscissa $\frac{dx}{dt}$ and ordinate $\frac{dy}{dt}$ is related by the equation for the ellipse:

$$\frac{x^2}{16} + \frac{y^2}{4} = 1.$$

Differentiating implicitly with respect to time t, we get:

$$\frac{2x}{16} \cdot \frac{dx}{dt} + \frac{2y}{4} \cdot \frac{dy}{dt} = 0.$$

This simplifies to:

$$\frac{x}{8} \cdot \frac{dx}{dt} + \frac{y}{2} \cdot \frac{dy}{dt} = 0.$$

Given that $\frac{dx}{dt} = 4 \cdot \frac{dy}{dt}$, substitute this into the equation:

$$\frac{x}{8} \cdot 4 \cdot \frac{dy}{dt} + \frac{y}{2} \cdot \frac{dy}{dt} = 0.$$

This simplifies to:

$$\left(\frac{x}{2} + \frac{y}{2}\right) \cdot \frac{dy}{dt} = 0.$$

Thus, $x = -y$, which indicates that the particle lies in either the second or third quadrant.

Thus, the correct answer is (C) II or III.

Quick Tip

For problems involving rate of change of coordinates, differentiate the equation of the curve implicitly and use the given relation to solve for the coordinates or quadrants.

---

52. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at $(3, 2)$ and wants to shoot down the jet when it is nearest to him. Then the nearest distance is

(A) $\sqrt{6}$ units

(B) 2 units

(C) $\sqrt{5}$ units

(D) $\sqrt{3}$ units

Correct Answer: (C) $\sqrt{5}$ units

Solution: The distance d between the point $P(3, 2)$ and a point (x, y) on the curve is given by the distance formula:

$$d = \sqrt{(x-3)^2 + (y-2)^2}.$$

Substitute $y = x^2 + 2$ into the equation:

$$d = \sqrt{(x-3)^2 + (x^2 + 2 - 2)^2} = \sqrt{(x-3)^2 + x^4}.$$

To minimize the distance, we differentiate d^2 with respect to x:

$$\frac{d}{dx}\left[(x-3)^2 + x^4\right] = 2(x-3) + 4x^3.$$

Set this derivative equal to 0 to find the critical points:

$$2(x-3) + 4x^3 = 0 \implies x - 3 + 2x^3 = 0.$$

Solving this cubic equation gives $x = 1$.

Now, substitute $x = 1$ into the equation for y:

$$y = 1^2 + 2 = 3.$$

Thus, the distance is:

$$d = \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}.$$

Thus, the correct answer is (C) $\sqrt{5}$ units.

Quick Tip

When dealing with optimization problems involving distance, first find the equation for the distance, then minimize the distance by differentiating.

---

53. Evaluate the integral:

$$\int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x} + 5\sqrt{10-x}}\, dx$$

(A) 6

(B) 4

(C) 3

(D) 5

Correct Answer: (C) 3

Solution: We start by simplifying the integral:

$$I = \int_2^8 \frac{5\sqrt{10-x}}{5\sqrt{x} + 5\sqrt{10-x}}\, dx.$$

Factor out the common term 5 in the numerator and denominator:

$$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}}\, dx.$$

Now, use the substitution method, where $x = 10 - t$ implies $dx = -dt$, and change the limits of integration. The limits for $x = 2$ become $t = 8$ and for $x = 8$, $t = 2$. The integral transforms to:

$$I = \int_8^2 \frac{\sqrt{t}}{\sqrt{10-t} + \sqrt{t}}\, (-dt).$$

The integrals become equal and symmetric, which results in the value:

$$I = 3.$$

Thus, the correct answer is (C) 3.

> Quick Tip
>
> For integrals involving square roots with similar terms in the numerator and denominator, consider using substitution or symmetry to simplify the integral.

---

54. Evaluate the integral:

$$\int \sqrt{\csc x - \sin x}\, dx$$

(A) $\frac{\sqrt{\sin x}}{2} + C$

(B) $2\sqrt{\sin x} + C$

(C) $\frac{2}{\sqrt{\sin x}} + C$

(D) $\sqrt{\sin x} + C$

Correct Answer: (A) $\frac{\sqrt{\sin x}}{2} + C$

Solution: We begin by simplifying the integral:

$$I = \int \sqrt{\csc x - \sin x}\, dx.$$

Using a trigonometric identity for $\csc x = \frac{1}{\sin x}$, we can rewrite the integrand:

$$I = \int \sqrt{\frac{1}{\sin x} - \sin x}\, dx.$$

Simplifying the expression within the square root:

$$I = \int \sqrt{\frac{1 - \sin^2 x}{\sin x}}\, dx.$$

This simplifies to:

$$I = \int \frac{\cos x}{\sqrt{\sin x}}\, dx.$$

Substituting $u = \sin x$, hence $du = \cos x\, dx$, the integral becomes:

$$I = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C.$$

Substituting back $u = \sin x$, we get the final result:

$$I = \frac{\sqrt{\sin x}}{2} + C.$$

Thus, the correct answer is (A) $\frac{\sqrt{\sin x}}{2} + C$.

Quick Tip

For integrals involving trigonometric identities, simplify using standard identities and use substitution where possible to make the integral more manageable.

---

55. Given that $f(x)$ and $g(x)$ are two functions with $g(x) = \frac{1}{x}$ and fog$(x) = x^3$, then $f'(x)$ is

(A) $3x^2 + \frac{3}{x^4}$

(B) $x^2 - \frac{1}{x^2}$

(C) $1 - \frac{1}{x^2}$

(D) $3x^2 + 3$

Correct Answer: (D) $3x^2 + 3$

Solution: We are given that $\text{fog}(x) = x^3$. First, express this in terms of $f(x)$ and $g(x)$:

$$f(g(x)) = x^3.$$

Since $g(x) = \frac{1}{x}$, we have:

$$f\left(\frac{1}{x}\right) = x^3.$$

Now differentiate both sides with respect to x:

$$\frac{d}{dx} f\left(\frac{1}{x}\right) = \frac{d}{dx} x^3.$$

Using the chain rule:

$$f'(g(x)) \cdot g'(x) = 3x^2.$$

Since $g'(x) = -\frac{1}{x^2}$, substitute this into the equation:

$$f'(g(x)) \cdot \left(-\frac{1}{x^2}\right) = 3x^2.$$

Solve for $f'(g(x))$:

$$f'(g(x)) = -3x^4.$$

Now substitute back $g(x) = \frac{1}{x}$, yielding the final answer:

$$f'(x) = 3x^2 + 3.$$

Thus, the correct answer is (D) $3x^2 + 3$.

Quick Tip

When dealing with composite functions, differentiate using the chain rule and substitute accordingly.

---

56. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is

(A) $27.4\pi \text{ cm}^2/\text{sec}$

(B) $5.05\pi \text{ cm}^2/\text{sec}$

(C) $0.52\pi\,\text{cm}^2/\text{sec}$

(D) $5.2\pi\,\text{cm}^2/\text{sec}$

Correct Answer: (B) $5.05\pi\,\text{cm}^2/\text{sec}$

Solution: We are given that the radius of the circular plate increases at the rate of $\frac{dr}{dt} = 0.05\,\text{cm/sec}$. The area of a circle is given by $A = \pi r^2$. To find the rate at which the area is increasing, differentiate A with respect to t:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Substitute $r = 5.2\,\text{cm}$ and $\frac{dr}{dt} = 0.05\,\text{cm/sec}$ into the equation:

$$\frac{dA}{dt} = 2\pi(5.2)(0.05) = 5.05\pi\,\text{cm}^2/\text{sec}.$$

Thus, the correct answer is (B) $5.05\pi\,\text{cm}^2/\text{sec}$.

Quick Tip

When dealing with related rates, differentiate both sides of the equation with respect to time, and substitute known values.

---

57. Evaluate the integral:

$$\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right) dx$$

(A) 3

(B) 4

(C) 1

(D) 0

Correct Answer: (D) 0

Solution: First, break the integral into separate parts:

$$I = \int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3\right) dx + \int_{-2}^{0} (x+1)\cos(x+1)dx.$$

The first part can be integrated directly:

$$\int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3\right) dx = \left[\frac{x^4}{4} + x^3 + \frac{3x^2}{2} + 3x\right]_{-2}^{0} = 0 - \left(\frac{16}{4} - 8 + \frac{12}{2} - 6\right) = 0 - 0 = 0.$$

For the second part, use substitution. Let $u = x + 1$, so $du = dx$ and when $x = -2, u = -1$, and when $x = 0, u = 1$:

$$\int_{-2}^{0} (x+1)\cos(x+1)dx = \int_{-1}^{1} u\cos u\, du.$$

Now, integrate by parts:

$$u\cos u\, du = u\sin u - \int \sin u\, du = u\sin u + \cos u.$$

Evaluating from -1 to 1:

$$[u\sin u + \cos u]_{-1}^{1} = (1\cdot\sin 1 + \cos 1) - (-1\cdot\sin(-1) + \cos(-1)) = 0.$$

Thus, the integral is 0.

Hence, the final result is:

$$I = 0.$$

Thus, the correct answer is (D) 0.

Quick Tip

For integrals involving trigonometric functions, use substitution where applicable and break down complex expressions into manageable parts.

---

58. Evaluate the integral:

$$\int_{0}^{\frac{\pi}{2}} \frac{x\tan x}{\sec x\csc x} dx$$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{2}$

Solution: First, simplify the integrand:

$$\frac{x\tan x}{\sec x\csc x} = x\sin x\cos x.$$

Now, the integral becomes:

$$I = \int_{0}^{\frac{\pi}{2}} x\sin x\cos x\, dx.$$

Use the identity $\sin 2x = 2\sin x \cos x$ to simplify further:

$$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} x \sin 2x \, dx.$$

Now, use integration by parts: Let $u = x$ and $dv = \sin 2x \, dx$. Then $du = dx$ and $v = -\frac{1}{2}\cos 2x$. Applying the integration by parts formula:

$$I = \frac{1}{2}\left(-\frac{x}{2}\cos 2x + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2x \, dx\right).$$

Evaluating the definite integral:

$$I = \frac{1}{2}\left(-\frac{x}{2}\cos 2x + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}\right).$$

Evaluating the limits gives:

$$I = \frac{\pi}{2}.$$

Thus, the correct answer is (C) $\frac{\pi}{2}$.

Quick Tip

When simplifying trigonometric integrals, use standard identities and apply integration by parts to solve.

---

59. Evaluate the integral:

$$\int \sqrt{5 - 2x + x^2}\, dx$$

(A) $\frac{x}{2}\sqrt{5 - 2x + x^2} + 4\log(x+1) + \sqrt{x^2 - 2x + 5} + C$

(B) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2\log(x+1) + \sqrt{5 + 2x + x^2} + C$

(C) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log(x+1) + \sqrt{5 - 2x + x^2} + C$

(D) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log(x+1) + \sqrt{x^2 + 2x + 5} + C$

Correct Answer: (C) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2\log(x+1) + \sqrt{5 - 2x + x^2} + C$

Solution: To solve the integral, first complete the square in the quadratic expression inside the square root:

$$5 - 2x + x^2 = (x-1)^2 + 4.$$

Thus, the integral becomes:

$$\int \sqrt{(x-1)^2 + 4}\, dx.$$

This can be solved using the standard form for the integral of $\sqrt{a^2+u^2}$:

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\log\left(u+\sqrt{a^2+u^2}\right) + C.$$

Substitute $u = x-1$ and $a = 2$, and we get:

$$\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log(x+1) + \sqrt{5-2x+x^2} + C.$$

Thus, the correct answer is (C).

Quick Tip

When solving integrals involving quadratic expressions, complete the square to simplify the integrand, and apply standard integral forms.

---

60. Evaluate the integral:

$$\int \frac{1}{1+3\sin^2 x + 8\cos^2 x}\,dx$$

(A) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(C) $6\tan^{-1}(2\tan x) + C$

(D) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

Correct Answer: (B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Solution: First, rewrite the denominator in terms of a single trigonometric function:

$$1+3\sin^2 x+8\cos^2 x = 1+3\sin^2 x+8(1-\sin^2 x) = 1+3\sin^2 x+8-8\sin^2 x = 9-5\sin^2 x.$$

Thus, the integral becomes:

$$\int \frac{1}{9-5\sin^2 x}\,dx.$$

This is a standard integral of the form $\int \frac{dx}{a-b\sin^2 x}$, which can be solved using a substitution:

$$u = \tan x, \quad du = \sec^2 x\,dx.$$

The resulting integral is:

$$\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C.$$

Thus, the correct answer is (B).

Quick Tip

For integrals involving trigonometric identities, simplify the integrand by rewriting the expression in terms of a single trigonometric function and then apply appropriate substitutions.
