CBSE Class X Mathematics Standard Set 1 (30/1/1)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper consists of 38 questions. All questions are compulsory.
2. This paper is divided into five sections - A, B, C, D and E.
3. Section A Nos. 1 to 20 are Multiple Choice Questions. Each carries 1 mark.
4. Section B Nos. 21 to 26 are Very Short Answer type questions. Each carries 2 marks. Answer to these questions should be in the range of 30 to 50 words.
5. Section C Nos. 27 to 33 are Short Answer (SA) type questions. Each carries 3 marks. Answer to these questions should be in the range of 50 to 80 words.
6. Section D Nos. 34 to 36 are Long Answer type questions. Each carries 5 marks. Answer to these questions should be in the range of 80 to 120 words.
7. Section E Nos. 36 to 38 are of 3 Case Study based questions carrying 4 marks each with sub-parts.
8. There is no overall choice. However, an internal choice has been provided in some sections. Only one of the alternatives has to be attempted in such questions.
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

Question 1:

If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x + 1$ is $\sqrt{2}$, then the value of k is:

Options:

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) $\frac{1}{2}$

Correct Answer: (B)

Solution:

The sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is given by $-\frac{b}{a}$. For $p(x) = 2x^2 - k\sqrt{2}x + 1$, the sum of zeroes is:

$$-\frac{-k\sqrt{2}}{2} = \frac{k\sqrt{2}}{2}$$

Equating this to $\sqrt{2}$:

$$\frac{k\sqrt{2}}{2} = \sqrt{2} \implies k = 2$$

Correct Answer: (B)

Quick Tip

For quadratic equations, use $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$ to find the sum of roots.

Question 2:

If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:

Options:

(A) 1.79

(B) 0.31

(C) 0.21%

(D) 0.21

Correct Answer: (D)

Solution:

The probability of losing is given by:

$$1 - \text{Probability of winning} = 1 - 0.79 = 0.21$$

Correct Answer: (D)

Quick Tip

The sum of probabilities of all possible outcomes is always 1. Use this property to find the complementary probability.

Question 3:

If the roots of the equation $ax^2 + bx + c = 0, a \neq 0$ are real and equal, then which of the following relations is true?

Options:

(A) $a = \frac{b^2}{c}$

(B) $b^2 = ac$

(C) $ac = \frac{b^2}{4}$

(D) $c = \frac{b^2}{a}$

Correct Answer: (C)

Solution:

For real and equal roots, the discriminant of the quadratic equation must be zero:

$$\Delta = b^2 - 4ac = 0 \implies b^2 = 4ac$$

Thus:

$$ac = \frac{b^2}{4}$$

Correct Answer: (C)

Quick Tip

For real and equal roots, always set $\Delta = 0$ and simplify to derive the correct relationship.

Question 4:

In an A.P., if the first term $a = 7$, nth term $a_n = 84$, and the sum of the first n terms $S_n = \frac{2093}{2}$, then n is equal to:

Options:

(A) 22

(B) 24

(C) 23

(D) 26

Correct Answer: (C)

Solution:

The nth term of an A.P. is:

$$a_n = a + (n-1)d \implies 84 = 7 + (n-1)d \implies 77 = (n-1)d \implies d = \frac{77}{n-1}$$

The sum of the first n terms is:

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{2093}{2}$$

Substituting $a = 7$ and solving for n, we find:

$$n = 23$$

Correct Answer: (C)

Quick Tip

Use the formulas $a_n = a + (n-1)d$ and $S_n = \frac{n}{2}(2a + (n-1)d)$ to solve A.P. problems systematically.

Question 5:

If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^k$, where a and b are prime numbers, then LCM(p, q) is:

Options:

(A) $2a^2b^2$

(B) $180a^2b^2$

(C) $12a^2b^2$

(D) $180a^3b^4$

Correct Answer: (D)

Solution:

The LCM is determined by taking the highest powers of all prime factors:

$$p = 18a^2b^4 = 2 \cdot 3^2 \cdot a^2b^4, \quad q = 20a^3b^k = 2^2 \cdot 5 \cdot a^3b^k$$

The LCM is:

$$\text{LCM}(p, q) = 2^2 \cdot 3^2 \cdot 5 \cdot a^3b^4 = 180a^3b^4$$

Correct Answer: (D)

Quick Tip

To find the LCM, always take the highest powers of all prime factors present in the given numbers.

Question 6:

AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$, and $C(0, 0)$. Length of AD is equal to:

Options:

(A) $\sqrt{68}$ units

(B) $2\sqrt{15}$ units

(C) $\sqrt{101}$ units

(D) 10 units

Correct Answer: (A)

Solution:

To find the length of AD:

1. First, find the midpoint of BC:

$$M = \left(\frac{6+0}{2}, \frac{4+0}{2}\right) = (3, 2)$$

2. Calculate the distance between $A(5, -6)$ and $M(3, 2)$ using the distance formula:

$$AD = \sqrt{(5-3)^2 + (-6-2)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{4+64} = \sqrt{68}$$

Correct Answer: (A)

Quick Tip

The median connects a vertex of a triangle to the midpoint of the opposite side. Use the midpoint and distance formulas systematically for accurate results.

Question 7:

If $\sec\theta - \tan\theta = m$**, then the value of** $\sec\theta + \tan\theta$ **is:**

Options:

(A) $1 - \frac{1}{m}$

(B) $m^2 - 1$

(C) $\frac{1}{m}$

(D) $-m$

Correct Answer: (C)

Solution:

Given:

$$\sec\theta - \tan\theta = m$$

We know:

$$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$$

Thus:

$$\sec\theta + \tan\theta = \frac{1}{m}$$

Correct Answer: (C)

Quick Tip

Use the identity $\sec^2\theta - \tan^2\theta = 1$ to quickly solve problems involving $\sec\theta$ and $\tan\theta$.

Question 8:

From the data $1, 4, 7, 9, 16, 21, 25$, if all the even numbers are removed, then the probability of getting a prime number from the remaining is:

Options:

(A) $\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{7}$

(D) $\frac{2}{7}$

Correct Answer: (B)

Solution:

1. Remove all even numbers: $4, 16$.
2. Remaining numbers: $1, 7, 9, 21, 25$.
3. Prime numbers in the remaining set: 7.
4. Probability:

$$P(\text{prime}) = \frac{\text{Number of primes}}{\text{Total numbers}} = \frac{1}{5}$$

Correct Answer: (B)

Quick Tip

Remove all non-prime numbers systematically. Remember, 1 is neither prime nor composite.

Question 9:

For some data $x_1, x_2, \ldots, x_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the value of $\sum_{i=1}^{n} f_i (x_i - \overline{x})$ is equal to:

Options:

(A) $n\overline{x}$

(B) 1

(C) $\sum f_i$

(D) 0

Correct Answer: (D)

Solution:

We know that the sum of deviations from the mean, weighted by frequencies, is always zero:

$$\sum_{i=1}^{n} f_i(x_i - \overline{x}) = 0$$

Correct Answer: (D)

Quick Tip

The sum of deviations from the mean is zero because the mean balances all values in a dataset.

Question 10:

The zeroes of a polynomial $x^2 - px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is:

Options:

(A) $-\frac{5}{2}$

(B) $\frac{5}{2}$

(C) -5

(D) 10

Correct Answer: (A)

Solution:

The given polynomial $4x^2 - 5x - 6$ can be solved to find its zeroes using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a = 4,\ b = -5,\ c = -6.$$

Substitute the values:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)} = \frac{5 \pm \sqrt{25 + 96}}{8} = \frac{5 \pm \sqrt{121}}{8}.$$

Simplify:

$$x = \frac{5 + 11}{8} \quad \text{or} \quad x = \frac{5 - 11}{8}.$$

$$x = \frac{16}{8} = 2 \quad \text{or} \quad x = \frac{-6}{8} = -\frac{3}{4}.$$

Thus, the zeroes of $4x^2 - 5x - 6$ are 2 and $-\frac{3}{4}$.

The zeroes of the polynomial $x^2 + px + q$ are twice these zeroes. Therefore, the zeroes are:

$$2 \times 2 = 4 \quad \text{and} \quad 2 \times \left(-\frac{3}{4}\right) = -\frac{3}{2}.$$

The sum of the zeroes is:

$$\text{Sum of zeroes} = 4 + \left(-\frac{3}{2}\right) = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}.$$

For a polynomial $x^2 + px + q$, the sum of the zeroes is given by:

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -p.$$

Thus:

$$-p = \frac{5}{2} \implies p = -\frac{5}{2}.$$

Correct Answer: (a) $-\frac{5}{2}$.

Quick Tip

For finding the roots of quadratics, use the quadratic formula and remember the relationships between roots and coefficients.

Question 11:

If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

Options:

(A) $12, -18$

(B) $-12, 18$

(C) $18, 5$

(D) $-9, -12$

Correct Answer: (B)

Solution:

The distance formula is:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points are $(3, -5)$ and $(x, -5)$. Using the formula:

$$15 = \sqrt{(x-3)^2 + (-5-(-5))^2} = \sqrt{(x-3)^2}$$

Squaring both sides:

$$15^2 = (x-3)^2 \implies 225 = (x-3)^2$$

Taking square roots:

$$x - 3 = 15 \quad \text{or} \quad x - 3 = -15 \implies x = 18 \quad \text{or} \quad x = -12$$

Correct Answer: (B)

Quick Tip

For horizontal or vertical distances, the distance formula simplifies to the absolute difference of coordinates in one direction.

Question 12:

If $\cos(\alpha + \beta) = 0$, then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is:

Options:

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) 0

(D) $\sqrt{2}$

Correct Answer: (A)

Solution:

Given:

$$\cos(\alpha + \beta) = 0$$

This implies:

$$\alpha + \beta = (2n + 1)\frac{\pi}{2} \quad \text{for } n \in \mathbb{Z}$$

Thus:

$$\frac{\alpha + \beta}{2} = (2n + 1)\frac{\pi}{4}$$

The value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ for $(2n + 1)\frac{\pi}{4}$ alternates between:

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Correct Answer: (A)

Quick Tip

For trigonometric equations, simplify step by step and remember periodicity to identify values.

Question 13:

A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of the two hemispheres taken together is:

Options:

(A) $1 : 1$

(B) $1 : 4$

(C) $2 : 3$

(D) $3 : 2$

Correct Answer: (C)

Solution:

The surface area of a sphere is:

$$4\pi r^2$$

Each hemisphere has:

$$2\pi r^2 \text{ (curved surface area)} + \pi r^2 \text{ (base area)} = 3\pi r^2$$

Two hemispheres together:

$$2 \times 3\pi r^2 = 6\pi r^2$$

Ratio of the sphere to the two hemispheres:

$$4\pi r^2 : 6\pi r^2 = 2 : 3$$

Correct Answer: (C)

Quick Tip

Always add the base area to the curved surface area when dealing with hemispheres.

Question 14:

The middle most observation of every data arranged in order is called:

Options:

(A) Mode

(B) Median

(C) Mean

(D) Deviation

Correct Answer: (B)

Solution:

The median is defined as the middle value of a dataset when arranged in ascending or descending order. It divides the data into two equal halves.

Correct Answer: (B)

Quick Tip

To find the median, arrange the data and pick the middle value or average of two middle values for even-sized datasets.

Question 15:

The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

Options:

(A) $\frac{4\pi}{3}$ cu cm

(B) $\frac{5\pi}{3}$ cu cm

(C) $\frac{8\pi}{3}$ cu cm

(D) $\frac{2\pi}{3}$ cu cm

Correct Answer: (D)

Solution:

The largest cone will have its height equal to the edge of the cube ($h = 2$) and the diameter of the base also equal to the edge ($r = 1$):

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1)^2(2) = \frac{2\pi}{3} \text{ cu cm}$$

Correct Answer: (D)

Quick Tip

For the largest cone in a cube, use the edge of the cube as the cone's height and base diameter.

Question 16:

Two dice are rolled together. The probability of getting the sum of numbers on the two dice as 2, 3, or 5, is:

Options:

(A) $\frac{7}{36}$

(B) $\frac{11}{36}$

(C) $\frac{5}{36}$

(D) $\frac{4}{9}$

Correct Answer: (A)

Solution:

List all combinations for sums 2, 3, and 5:

1. Sum 2: $(1,1)$ (1 way)
2. Sum 3: $(1,2), (2,1)$ (2 ways)
3. Sum 5: $(1,4), (2,3), (3,2), (4,1)$ (4 ways)

Total favorable outcomes:

$$1 + 2 + 4 = 7$$

Total outcomes when rolling two dice:

$$6 \times 6 = 36$$

Probability:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{7}{36}$$

Correct Answer: (A)

Quick Tip

For dice problems, list all combinations carefully for the desired sum and count the favorable outcomes.

Question 17:

The center of a circle is at $(2,0)$. If one end of a diameter is at $(6,0)$, then the other end is at:

Options:

(A) $(0,0)$

(B) $(4,0)$

(C) $(-2, 0)$

(D) $(-6, 0)$

Correct Answer: (C)

Solution:

The center of the circle is the midpoint of the diameter:

$$\text{Midpoint formula: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (2, 0)$$

Given one end $(6, 0)$, let the other end be $(x, 0)$. Using the formula:

$$\frac{6 + x}{2} = 2 \implies 6 + x = 4 \implies x = -2$$

The other end is:

$$(-2, 0)$$

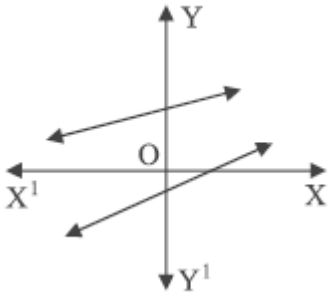
Correct Answer: (C)

Quick Tip

For diameter problems, use the midpoint formula and substitute the center coordinates to find the unknown endpoint.

Question 18:

In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:



Options:

(A) Consistent with unique solution.

(B) Consistent with infinitely many solutions.

(C) Inconsistent.

(D) Inconsistent but can be made consistent by extending these lines.

Correct Answer: (D)

Solution:

From the figure, the two lines are not intersecting at any point in the given plane, indicating that the equations are currently inconsistent. However, the lines are not parallel and can be extended to meet at a point. This makes the pair of equations inconsistent but can be made consistent by extending these lines.

Correct Answer: (D) Inconsistent but can be made consistent by extending these lines.

Quick Tip

If two lines do not intersect in the given plane but are not parallel, they can be extended to meet at a point, making the equations consistent.

Question 19:

Assertion (A): The tangents drawn at the endpoints of a diameter of a circle are parallel.

Reason (R): Diameter of a circle is the longest chord.

Options:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (B)

Solution:

Tangents drawn at the endpoints of a diameter are parallel because they are perpendicular to the radius, and the radii are collinear. While the diameter is indeed the longest chord, it does not explain the parallelism of the tangents.

Correct Answer: (B)

Quick Tip

Tangents at the ends of a diameter are parallel because they are perpendicular to the same straight line (the diameter).

Question 20:

Assertion (A): If the graph of a polynomial touches the x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree n ($n \geq 1$) can have at most n zeroes.

Options:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (D)

Solution:

The graph of a quadratic polynomial can touch the x-axis at one point (e.g., x^2 has one repeated root at $x = 0$). Thus, Assertion (A) is false. Reason (R) is correct, as it describes the fundamental property of polynomials.

Correct Answer: (D)

Quick Tip

If a polynomial graph touches the x-axis, it implies a repeated root, but it can still be a quadratic polynomial.

Section - B

Question 21:

Solve the following system of linear equations $7x - 2y = 5$ and $8x + 7y = 15$ and verify your answer.

Correct Answer: $x = 1, y = 1$

Solution:

Using substitution or elimination method:

1. Solve $7x - 2y = 5 \implies y = \frac{7x-5}{2}$.
2. Substitute y in $8x + 7y = 15$:

$$8x + 7\left(\frac{7x-5}{2}\right) = 15 \implies 16x + 49x - 35 = 30 \implies 65x = 65 \implies x = 1$$

3. Substitute $x = 1$ in $y = \frac{7x-5}{2}$:

$$y = \frac{7(1) - 5}{2} = \frac{2}{2} = 1$$

4. Verification:

$$7(1) - 2(1) = 5 \quad \text{and} \quad 8(1) + 7(1) = 15$$

Solution: $x = 1, y = 1$

Quick Tip

To verify, substitute the values back into the original equations. If satisfied, your solution is correct.

Question 22:

In a pack of 52 playing cards, one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is the queen of hearts, if the lost card is a black card.

Correct Answer: $\frac{1}{51}$

Solution:

If the lost card is black, the total remaining cards are:

$$52 - 1 = 51$$

The queen of hearts is still in the pack, so the probability is:

$$P(\text{Queen of Hearts}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{51}$$

Answer: $\frac{1}{51}$

Quick Tip

For probability questions, count the favorable outcomes and divide by the total possible outcomes after the given condition.

Question 23: (A)

Evaluate: $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$**.**

Correct Answer: 4

Solution:

Substitute values of trigonometric functions:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right) + 2\sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{2} \cdot \frac{1}{2\sqrt{2}} + 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 1 + 3 = 4$$

Answer: 4

Quick Tip

Simplify trigonometric expressions by substituting known values step by step.

Question 23: (B)

If $A = 60^\circ$ **and** $B = 30^\circ$**, verify that:**

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

Correct Answer: Verified

Solution:

The left-hand side (LHS) of the equation is:

$$\sin(A + B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1.$$

The right-hand side (RHS) of the equation is:

$$\sin A \cos B + \cos A \sin B.$$

Substitute $A = 60^\circ$ and $B = 30^\circ$:

$$\sin A = \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos B = \cos 30^\circ = \frac{\sqrt{3}}{2},$$

$$\cos A = \cos 60^\circ = \frac{1}{2}, \quad \sin B = \sin 30^\circ = \frac{1}{2}.$$

Substitute the values:

$$\text{RHS} = \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2}\right).$$

Simplify:

$$\text{RHS} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$$

Since LHS = RHS, the equation is verified.

Answer: Verified.

Quick Tip

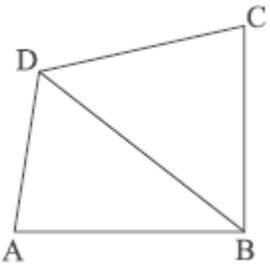
The sum of angles identity for sine is $\sin(A+B) = \sin A \cos B + \cos A \sin B$. Use known trigonometric values for standard angles to simplify.

Question 24:

In the given figure, $ABCD$ is a quadrilateral. Diagonal BD bisects $\angle B$ and $\angle D$. Prove that:

(i) $\triangle ABD \sim \triangle CBD$

(ii) $AB = BC$



Correct Answer: Proved

Solution:

(i) Since BD bisects $\angle B$ and $\angle D$, we have:

$$\angle ABD = \angle CBD \quad \text{and} \quad \angle ADB = \angle CDB$$

Thus, by AA similarity criterion:

$$\triangle ABD \sim \triangle CBD$$

(ii) From $\triangle ABD \sim \triangle CBD$, the sides opposite the equal angles are proportional:

$$\frac{AB}{BC} = 1 \implies AB = BC$$

Proof Complete.

Quick Tip

For similarity, use AA, SSS, or SAS criteria and compare corresponding sides to prove equalities.

Question 25: (A)

Prove that $5 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is irrational.

Correct Answer: Proved

Solution:

Assume $5 - 2\sqrt{3}$ is rational. Then:

$$2\sqrt{3} = 5 - (\text{rational number})$$

Since the sum of a rational and an irrational number is irrational, $5 - 2\sqrt{3}$ must be irrational, contradicting our assumption.

Proof Complete.

Quick Tip

The sum or difference of a rational number and an irrational number is always irrational.

Question 25: (B)

Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.

Correct Answer: Proved

Solution:

Factorize the given expression:

$$5 \times 11 \times 17 + 3 \times 11 = 11(5 \times 17 + 3) = 11(85 + 3) = 11 \times 88$$

Since 11 and 88 are both greater than 1, the given number is a composite number.

Proof Complete.

Quick Tip

A composite number is one that has more than two distinct factors. Factorization helps identify them quickly.

Section - C

Question 26: (A)

Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$. Also, find the value of y.

Correct Answer: The point divides the line segment in the ratio $3 : 2$ and $y = \frac{13}{5}$.

Solution:

Let the ratio be $m : n$. Using the section formula for the x-coordinate:

$$x = \frac{mx_2 + nx_1}{m + n}$$

Substitute $x = \frac{8}{5}$, $x_1 = 1$, $x_2 = 2$:

$$\frac{8}{5} = \frac{m(2) + n(1)}{m + n} \implies \frac{8}{5}(m + n) = 2m + n$$

$$8m + 8n = 10m + 5n \implies 3n = 2m \implies \frac{m}{n} = \frac{3}{2}$$

Thus, the ratio is:

$$m : n = 3 : 2$$

For the y-coordinate:

$$y = \frac{my_2 + ny_1}{m + n}$$

Substitute $m : n = 3 : 2$, $y_1 = 2$, $y_2 = 3$:

$$y = \frac{3(3) + 2(2)}{3 + 2} = \frac{9 + 4}{5} = \frac{13}{5}$$

Answer: The point divides the line segment in the ratio $3:2$ and $y = \frac{13}{5}$.

Quick Tip

Use the section formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ for any division point.

Question 26: (B)

ABCD is a rectangle formed by the points $A(-1,-1)$, $B(-1,6)$, $C(3,6)$, and $D(3,-1)$. $P, Q, R,$ and S are midpoints of sides $AB, BC, CD,$ and DA respectively. Show that diagonals of the quadrilateral $PQRS$ bisect each other.

Correct Answer: Proved

Solution:

Midpoints of the sides are:

$$P = \left(\frac{-1+(-1)}{2}, \frac{-1+6}{2}\right) = (-1, \frac{5}{2}), \quad Q = \left(\frac{-1+3}{2}, \frac{6+6}{2}\right) = (1,6)$$

$$R = \left(\frac{3+3}{2}, \frac{6+(-1)}{2}\right) = (3, \frac{5}{2}), \quad S = \left(\frac{3+(-1)}{2}, \frac{-1+(-1)}{2}\right) = (1,-1)$$

Diagonals of $PQRS$:

- Diagonal PR: Midpoint:

$$\text{Midpoint} = \left(\frac{-1+3}{2}, \frac{\frac{5}{2}+\frac{5}{2}}{2}\right) = \left(1, \frac{5}{2}\right)$$

- Diagonal QS: Midpoint:

$$\text{Midpoint} = \left(\frac{1+1}{2}, \frac{6+(-1)}{2}\right) = \left(1, \frac{5}{2}\right)$$

Since the midpoints of both diagonals are the same, the diagonals bisect each other.

Proof Complete.

Quick Tip

For midpoint calculations, use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and compare midpoints of diagonals.

Question 27:

In a teachers' workshop, the number of teachers teaching French, Hindi, and English are 48, 80, and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Correct Answer: 17 rooms

Solution:

To find the minimum number of rooms, we need to calculate the greatest common divisor (GCD) of 48, 80, and 144. Perform prime factorization:

$$48 = 2^4 \cdot 3, \quad 80 = 2^4 \cdot 5, \quad 144 = 2^4 \cdot 3^2$$

The GCD is:

$$2^4 = 16$$

Thus, each room can accommodate 16 teachers. The number of rooms required for each subject is:

$$\frac{48}{16} = 3, \quad \frac{80}{16} = 5, \quad \frac{144}{16} = 9$$

Total number of rooms:

$$3 + 5 + 9 = 17$$

Answer: 17 rooms.

Quick Tip

For similar problems, use the GCD to determine the maximum number of teachers per room and divide the total accordingly.

Question 28:

Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$**.**

Correct Answer:Proved

Solution:

Start with the left-hand side:

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

Substitute $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$:

$$\text{LHS} = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

Simplify each term:

$$\text{LHS} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$\text{LHS} = \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

Combine terms:

$$\text{LHS} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = 1 + \sec\theta\csc\theta$$

Proof Complete.

Quick Tip

Simplify using trigonometric identities like $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.

Question 29:

Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twice as old as Nazma. How old are Rashmi and Nazma now?

Correct Answer:Rashmi is 60 years old, and Nazma is 22 years old.

Solution:

Let the present ages of Rashmi and Nazma be R and N respectively. From the problem:

$$R - 3 = 3(N - 3) \quad (1)$$

$$R + 10 = 2(N + 10) \quad (2)$$

Simplify equation (1):

$$R - 3 = 3N - 9 \implies R = 3N - 6$$

Substitute $R = 3N - 6$ into equation (2):

$$3N - 6 + 10 = 2N + 20 \implies 3N - 2 = 2N + 20 \implies N = 22$$

Substitute $N = 22$ into $R = 3N - 6$:

$$R = 3(22) - 6 = 60$$

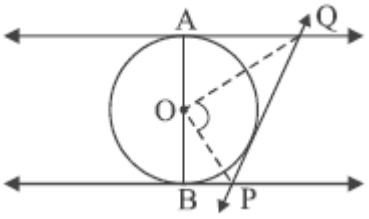
Answer: Rashmi is 60 years old, and Nazma is 22 years old.

Quick Tip

Translate word problems into equations carefully and solve step by step.

Question 30: (A)

In the given figure, AB is a diameter of the circle with center O. $AQ, BP,$ and PQ are tangents to the circle. Prove that $\angle POQ = 90°$.



Correct Answer:Proved

Solution:

Step 1. AB is the diameter, so by the property of a circle, $\angle APB = 90°$ (angle subtended by the diameter in a semicircle).

Step 2. Since AQ and BP are tangents to the circle: - The radius drawn to the tangents at A and B is perpendicular to the tangents. Hence:

$$\angle OAQ = 90° \quad \text{and} \quad \angle OBP = 90°$$

Step 3. In $\triangle POQ$: - $\angle OAQ$ and $\angle OBP$ are perpendicular, and both lines meet at P and Q, forming a right angle at O.

$$\angle POQ = 90°$$

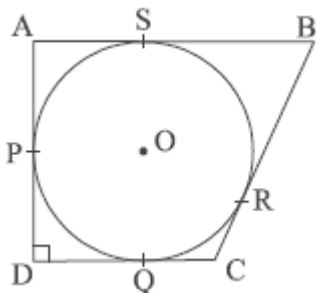
Correct Answer: $\angle POQ = 90°$ is proved.

Quick Tip

To prove such geometry problems, use circle theorems like ”Angle subtended by the diameter is a right angle” and ”Tangents are perpendicular to the radius.”

Question 30: (B)

A circle with center O and radius 8 cm is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, $BC = 30$ cm, and $BS = 24$ cm, then find the length DC.



Correct Answer: 14 **cm**

Solution:

Step 1: Understanding the properties of tangents For tangents drawn to a circle from an external point:

$$AS = AP, \quad BS = BR, \quad CR = CQ, \quad DP = DQ.$$

The radius of the circle is given as:

$$OP = OQ = OR = OS = 8\,\text{cm}.$$

Step 2: Calculate the lengths From the given data:

$$BC = BR + RC, \quad \text{and} \quad BS = BR.$$

Substitute $BS = 24\,\text{cm}$:

$$RC = BC - BS = 30 - 24 = 6\,\text{cm}.$$

Thus:

$$QC = 6\,\text{cm}. \quad \cdots (i)$$

Step 3: Analyze quadrilateral $OPDQ$ Given $AD \perp DC$, quadrilateral $OPDQ$ forms a square because:

$$\angle POQ = \angle QOD = \angle ODP = 90^\circ \quad \text{(angle between radius and tangent)}.$$

Hence:

$$DP = DQ = OP = OQ = 8\,\text{cm}. \quad \cdots (ii)$$

Step 4: Find the length of DC The length of DC is:

$$DC = DQ + QC.$$

Substitute values from equations (i) and (ii):

$$DC = 8 + 6 = 14\,\text{cm}.$$

Final Answer: **The length of DC is 14 cm.**

Quick Tip

For quadrilaterals with an inscribed circle, use tangent properties and radius relations to calculate unknown lengths.

Question 31:

The difference between the outer and inner radii of a hollow right circular cylinder of length 14 cm is 1 cm. If the volume of the metal used in making the cylinder is $176\,\text{cm}^3$, find the outer and inner radii of the cylinder.

Correct Answer:Outer radius $R = \frac{5}{2}$ cm, Inner radius $r = \frac{3}{2}$ cm.

Solution:

Let the outer and inner radii be R and r, respectively. Given $R - r = 1$ and the volume of the metal is:

$$\text{Volume} = \pi h(R^2 - r^2)$$

Substitute $h = 14$ and Volume $= 176$:

$$176 = \pi(14)(R^2 - r^2) \implies R^2 - r^2 = \frac{176}{14\pi} = 4$$

From $R^2 - r^2 = (R - r)(R + r)$:

$$4 = (1)(R + r) \implies R + r = 4$$

Solve $R - r = 1$ and $R + r = 4$:

$$2R = 5 \implies R = \frac{5}{2}, \quad r = \frac{3}{2}$$

Answer: Outer radius $R = \frac{5}{2}$ cm, Inner radius $r = \frac{3}{2}$ cm.

Quick Tip

Use the relationship $R^2 - r^2 = (R - r)(R + r)$ to simplify calculations.

Section - D

Question 32:

An arc of a circle of radius 21 cm subtends an angle of 60° at the center. Find:

(i) The length of the arc.

(ii) The area of the minor segment of the circle made by the corresponding chord.

Correct Answer:

- (i) $\frac{44\pi}{2} = 22\pi$ cm
- (ii) Length of arc = 22π cm

  Area of minor segment = $73.5\pi - 220.5\sqrt{3}\,\text{cm}^2$

Solution:

(i) Length of the arc:

The length of an arc is given by:

$$\text{Length of arc} = \frac{\theta}{360^\circ} \cdot 2\pi r$$

Substitute $\theta = 60^\circ$ and $r = 21$:

$$\text{Length of arc} = \frac{60}{360} \cdot 2\pi(21) = \frac{1}{6} \cdot 2\pi(21) = \frac{44\pi}{6} = 22\pi\,\text{cm}$$

(ii) Area of the minor segment:

The area of the sector is given by:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \cdot \pi r^2$$

$$\text{Area of sector} = \frac{60}{360} \cdot \pi(21)^2 = \frac{1}{6} \cdot \pi(441) = 73.5\pi\,\text{cm}^2$$

The area of the triangle formed by the chord and the two radii can be calculated using the formula:

$$\text{Area of triangle} = \frac{1}{2} \cdot r^2 \cdot \sin\theta$$

Substitute $r = 21$ and $\theta = 60^\circ$:

$$\text{Area of triangle} = \frac{1}{2} \cdot (21)^2 \cdot \sin 60^\circ = \frac{1}{2} \cdot 441 \cdot \frac{\sqrt{3}}{2} = 220.5\sqrt{3}\,\text{cm}^2$$

The area of the minor segment is:

$$\text{Area of minor segment} = \text{Area of sector} - \text{Area of triangle}$$

$$\text{Area of minor segment} = 73.5\pi - 220.5\sqrt{3}\,\text{cm}^2$$

Answer:

Length of arc = 22π cm

Area of minor segment = $73.5\pi - 220.5\sqrt{3}$ cm^2

Quick Tip

Use Length of arc $= \frac{\theta}{360^\circ} \cdot 2\pi r$ and Area of sector $= \frac{\theta}{360^\circ} \cdot \pi r^2$ for arc-related calculations.

Question 33: (A)

The sum of the first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

Correct Answer: -160

Solution:

Let the first term of the A.P. be a and the common difference be d.

The general formula for the n-th term of an A.P. is:

$$a_n = a + (n-1)d.$$

Step 1: Write the equations for the given conditions The first term is $a_1 = a$, and the eighth term is:

$$a_8 = a + 7d.$$

From the problem:

$$a_1 + a_8 = 32 \quad \text{and} \quad a_1 \cdot a_8 = 60.$$

Substitute $a_1 = a$ and $a_8 = a + 7d$:

$$a + (a + 7d) = 32 \implies 2a + 7d = 32 \quad \cdots (1),$$

$$a \cdot (a + 7d) = 60 \implies a^2 + 7ad = 60 \quad \cdots (2).$$

Step 2: Solve the equations From equation (1):

$$2a + 7d = 32 \implies a = \frac{32 - 7d}{2} \quad \cdots (3).$$

Substitute a from equation (3) into equation (2):

$$\left(\frac{32 - 7d}{2}\right)^2 + 7 \cdot \frac{32 - 7d}{2} \cdot d = 60.$$

Simplify:

$$\frac{(32-7d)^2}{4}+\frac{7(32-7d)d}{2}=60.$$

Multiply through by 4 to eliminate fractions:

$$(32-7d)^2+14(32-7d)d=240.$$

Expand:

$$1024-448d+49d^2+448d-98d^2=240.$$

Combine terms:

$$49d^2-98d^2+1024=240 \implies -49d^2+1024=240.$$

Simplify:

$$-49d^2=-784 \implies d^2=16 \implies d=4 \quad \text{or} \quad d=-4.$$

Step 3: Find a Substitute $d=4$ into equation (3):

$$a=\frac{32-7(4)}{2}=\frac{32-28}{2}=2.$$

If $d=-4$, then:

$$a=\frac{32-7(-4)}{2}=\frac{32+28}{2}=30.$$

Thus, the possible values are:

$$a=2, d=4 \quad \text{or} \quad a=30, d=-4.$$

Step 4: Sum of the first 20 terms The sum of the first n terms of an A.P. is:

$$S_n=\frac{n}{2}[2a+(n-1)d].$$

Substitute $n=20$, $a=2$, and $d=4$:

$$S_{20}=\frac{20}{2}[2(2)+19(4)]=10[4+76]=10\cdot 80=800.$$

If $a=30$ and $d=-4$:

$$S_{20}=\frac{20}{2}[2(30)+19(-4)]=10[60-76]=10\cdot(-16)=-160.$$

Answer:

First term = $a=2$, Common difference = $d=4$, Sum of first 20 terms = 800.

Alternatively, $a = 30$, $d = -4$, Sum of first 20 terms = -160.

Quick Tip

For solving A.P. problems, use the general term formula $a_n = a + (n-1)d$ and the sum formula $S_n = \frac{n}{2}[2a + (n-1)d]$.

Question 33: (B)

In an A.P. of 40 terms, the sum of the first 9 terms is 153 and the sum of the last 6 terms is 687. Determine the first term and common difference of the A.P. Also, find the sum of all the terms of the A.P.

Correct Answer: 2540

Solution:

Let the first term of the A.P. be a and the common difference be d.

The formula for the sum of n terms of an A.P. is:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

Step 1: Use the given condition for S_9 For the first 9 terms:

$$S_9 = \frac{9}{2}[2a + (9-1)d] = \frac{9}{2}[2a + 8d].$$

Substitute $S_9 = 153$:

$$153 = \frac{9}{2}[2a + 8d].$$

Simplify:

$$2a + 8d = 34 \quad \cdots (i).$$

Step 2: Use the given condition for the sum of the last 6 terms The sum of the last 6 terms of the A.P. is given by:

$$S_{\text{last 6}} = \frac{6}{2}[2a_{40} - 5d],$$

where the last term $a_{40} = a + 39d$.

Substitute $a_{40} = a + 39d$ into the equation:

$$S_{\text{last 6}} = 3[2(a + 39d) - 5d].$$

Simplify:

$$S_{\text{last 6}} = 3[2a + 78d - 5d] = 3[2a + 73d].$$

Substitute $S_{\text{last } 6} = 687$:

$$687 = 3[2a + 73d].$$

Simplify:

$$2a + 73d = 229 \quad \cdots (ii).$$

Step 3: Solve equations (i) and (ii) From equation (i):

$$2a + 8d = 34 \implies a = 17 - 4d \quad \cdots (iii).$$

Substitute $a = 17 - 4d$ from equation (iii) into equation (ii):

$$2(17 - 4d) + 73d = 229.$$

Simplify:

$$34 - 8d + 73d = 229 \implies 65d = 195 \implies d = 3.$$

Substitute $d = 3$ into equation (iii):

$$a = 17 - 4(3) = 17 - 12 = 5.$$

Step 4: Find the sum of all terms The sum of 40 terms is:

$$S_{40} = \frac{40}{2}[2a + (40 - 1)d].$$

Substitute $a = 5$ and $d = 3$:

$$S_{40} = 20[2(5) + 39(3)] = 20[10 + 117] = 20 \cdot 127 = 2540.$$

Final Answer: **First term:** $a = 5$**, Common difference:** $d = 3$**, Sum of all terms:** $S_{40} = 2540$**.**

Quick Tip

To solve A.P. problems, use the sum formula $S_n = \frac{n}{2}[2a + (n - 1)d]$ and systematically eliminate variables by solving equations.

Question 34: (A)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Correct Answer: Proof Complete

Solution:

Let $\triangle ABC$ have a line $DE \parallel BC$ intersecting AB at D and AC at E.

From the property of similar triangles:

If a line is parallel to one side of a triangle, it divides the other two sides proportionally.

Thus:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Proof:

Since $DE \parallel BC$, by the Basic Proportionality Theorem (or Thales' theorem):

$$\triangle ADE \sim \triangle ABC$$

Hence:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

This proves that DE divides AB and AC in the same ratio.

Proof Complete.

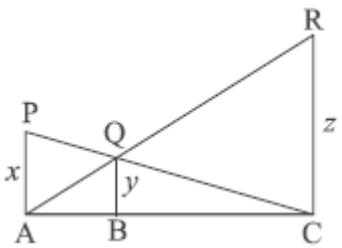
Quick Tip

When solving similar triangle problems, identify corresponding sides and apply proportionality rules.

Question 34: (B)

In the given figure, PA, QB, RC are each perpendicular to AC. If $AP = x$, $BQ = y$, and $CR = z$, prove that:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$



Correct Answer: Proof Complete

Solution:

From the given figure, triangles $\triangle PAQ, \triangle QBR, \triangle RCA$ are similar because they all share the same right angles and a common angle.

For $\triangle PAQ$ and $\triangle QBR$:

$$\frac{AP}{BQ} = \frac{AQ}{BR}$$

$$\frac{x}{y} = \frac{AQ}{BR} \implies BR = \frac{y \cdot AQ}{x}.$$

For $\triangle QBR$ and $\triangle RCA$:

$$\frac{BQ}{CR} = \frac{BR}{RC}$$

$$\frac{y}{z} = \frac{BR}{RC} \implies RC = \frac{z \cdot BR}{y}.$$

Substitute BR into RC:

$$RC = \frac{z \cdot (y \cdot AQ/x)}{y} = \frac{z \cdot AQ}{x}.$$

Using the proportionality of segments:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$

Proof Complete.

Quick Tip

Use similarity properties of triangles to express ratios between the given sides and simplify accordingly.

Question 35:

A pole 6 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point P on the ground is 60°, and the angle of depression of the point P from the top of the tower is 45°. Find the height of the tower and the distance of point P from the foot of the tower. (Use $\sqrt{3} = 1.73$)

Correct Answer: Height of the tower: 8.20 m.

Distance of point P from the foot of the tower: 14.20 m.

Solution:

Let the height of the tower be h m and the distance of point P from the foot of the tower be PR. The total height of the pole and tower is $6 + h$ m.

Step 1: Using the angle of depression (45°) From $\triangle QAR$:

$$\tan 45^\circ = \frac{\text{QR}}{\text{AR}}.$$

Since $\tan 45^\circ = 1$:

$$1 = \frac{h}{\text{AR}} \implies \text{AR} = h. \quad \cdots (i)$$

Step 2: Using the angle of elevation (60°) From $\triangle PAR$:

$$\tan 60^\circ = \frac{\text{PR}}{\text{AR}}.$$

Substitute PR $= 6 + h$ and AR $= h$ (from equation (i)):

$$\sqrt{3} = \frac{6 + h}{h}.$$

Rearrange:

$$\sqrt{3}h = 6 + h.$$

Simplify:

$$\sqrt{3}h - h = 6 \implies h(\sqrt{3} - 1) = 6.$$

Solve for h:

$$h = \frac{6}{\sqrt{3} - 1}.$$

Step 3: Rationalize the denominator Rationalize:

$$h = \frac{6(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{6(\sqrt{3} + 1)}{3 - 1} = \frac{6(\sqrt{3} + 1)}{2}.$$

Simplify:

$$h = 3(\sqrt{3} + 1).$$

Substitute $\sqrt{3} = 1.73$:

$$h = 3(1.73 + 1) = 3(2.73) = 8.19\,\text{m}.$$

Thus, the height of the tower is:

$$h = 8.20\,\text{m}.$$

Step 4: Find the distance of point P from the foot of the tower The distance PR is:

$$PR = 6 + h.$$

Substitute $h = 8.20$:

$$PR = 6 + 8.20 = 14.20\,\text{m}.$$

Final Answer: **Height of the tower:** 8.20 **m.**

Distance of point P **from the foot of the tower:** 14.20 **m.**

Quick Tip

For problems involving elevation and depression, use trigonometric ratios ($\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$) and simplify using given relationships step by step.

Section - E

Question 36:

A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) (a) Find the value of x, the length of side of a tile by factorization.

(b) Solve the quadratic equation for x, using the quadratic formula.

Correct Answer:

(i) $200 \cdot x^2 = 128 \cdot (x+1)^2 (ii) 9x^2 - 32x - 16 = 0) (iii)(a) x = 4 (iii)(b) 4$ units

Solution:

(i) Forming the quadratic equation:

Let the side of the original square tile be x units. The total area of the floor is:

$$\text{Area of the floor} = 200 \cdot x^2$$

If the side length of the tile is increased by 1 unit, the side becomes $(x + 1)$, and the number of tiles required is 128. Hence:

$$\text{Area of the floor} = 128 \cdot (x + 1)^2$$

Equating both expressions for the area:

$$200 \cdot x^2 = 128 \cdot (x + 1)^2$$

(ii) Simplifying to standard form:

Expand and simplify:

$$200x^2 = 128(x^2 + 2x + 1) \implies 200x^2 = 128x^2 + 256x + 128$$

$$200x^2 - 128x^2 - 256x - 128 = 0 \implies 72x^2 - 256x - 128 = 0$$

Divide through by 8 to simplify:

$$9x^2 - 32x - 16 = 0$$

Standard form:

$$9x^2 - 32x - 16 = 0$$

(iii) (a) Solving by factorization:

Multiply $9 \cdot (-16) = -144$. Find two numbers whose product is -144 and sum is -32: -36 and 4. Rewrite the middle term:

$$9x^2 - 36x + 4x - 16 = 0$$

Group terms:

$$(9x^2 - 36x) + (4x - 16) = 0 \implies 9x(x - 4) + 4(x - 4) = 0$$

Factorize:

$$(9x + 4)(x - 4) = 0$$

Solutions:

$$x = -\frac{4}{9} \quad \text{(not valid, side length cannot be negative)}, \qquad x = 4$$

Answer: The original side length of the tile is $x = 4$ units.

(iii) (b) Solving using the quadratic formula:

The quadratic equation is:

$$9x^2 - 32x - 16 = 0$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 9$, $b = -32$, $c = -16$. Substitute:

$$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(9)(-16)}}{2(9)}$$

$$x = \frac{32 \pm \sqrt{1024 + 576}}{18} = \frac{32 \pm \sqrt{1600}}{18} = \frac{32 \pm 40}{18}$$

Calculate:

$$x = \frac{32 + 40}{18} = \frac{72}{18} = 4, \qquad x = \frac{32 - 40}{18} = \frac{-8}{18} = -\frac{4}{9}.$$

Reject $x = -\frac{4}{9}$ as it is not valid. Thus:

$$x = 4.$$

Final Answer: The original side length of the tile is 4 units.

Quick Tip

For problems involving quadratic equations, use both factorization and the quadratic formula to cross-verify results.

Question 37:

BINGO is a game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/her card says BINGO and wins the game.

The table given below shows the data of one such game where 48 balls were used before Tara said "BINGO":

Numbers Announced	Number of Times
$0 - 15$	8
$15 - 30$	9
$30 - 45$	10
$45 - 60$	12
$60 - 75$	9

Based on the above information, answer the following:

(i) Write the median class.

(ii) When the first ball was picked up, what was the probability of calling out an even number?

(iii) (a) Find the median of the given data.

(b) Find the mode of the given data.

Correct Answer:

(i) $30 - 45$ (ii) $\frac{37}{75}$ (iii)(a) **Median:** 40.5 (iii)(b) **Mode:** 51

Solution:

(i) Median Class:

The total frequency is:

$$N = 8 + 9 + 10 + 12 + 9 = 48$$

The cumulative frequency (CF) is:

Class Interval	Cumulative Frequency (CF)
$0 - 15$	8
$15 - 30$	17
$30 - 45$	27
$45 - 60$	39
$60 - 75$	48

The median class is the class where $\frac{N}{2} = \frac{48}{2} = 24$ lies. From the table, the median class is:

30-45.

(ii) Probability of calling out an even number:

The total numbers from 1 to 75 are 75. Half of them are even numbers (2, 4, 6, ..., 74). The total even numbers are:

$$\frac{75}{2} = 37.5 \implies 37 \text{ (rounded down to nearest whole number).}$$

Thus, the probability of calling out an even number is:

$$P(\text{Even Number}) = \frac{\text{Number of Even Numbers}}{\text{Total Numbers}} = \frac{37}{75}.$$

(iii) (a) Median of the data:

The formula for the median is:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \cdot h$$

Where:

- $L = 30$ (lower boundary of the median class),
- $CF = 17$ (cumulative frequency before the median class),
- $f = 10$ (frequency of the median class),
- $h = 15$ (class width).

Substitute:

$$\text{Median} = 30 + \left(\frac{24 - 17}{10} \right) \cdot 15 = 30 + \left(\frac{7}{10} \right) \cdot 15 = 30 + 10.5 = 40.5.$$

Median: 40.5.

iii)(a) Find the length PC in terms of x and hence find the value of x:

Given:

$$BC = 15\,\text{m}, \quad BQ = (7 - x)\,\text{m}.$$

Using the property of tangents to a circle:

$$QC = 15 - (7 - x) = (8 + x)\,\text{m}.$$

Since $PC = QC$, we have:

$$PC = (8 + x)\,\text{m}.$$

In $\triangle ABC$, applying the Pythagoras theorem:

$$AC^2 = AB^2 + BC^2.$$

Substitute the values:

$$AC^2 = 7^2 + 15^2 = 49 + 225 = 274 \implies AC = \sqrt{274} \approx 16.55\,\text{m}.$$

Now, using the property of tangents:

$$AP + PC = AC.$$

Substitute $AP = x$ and $PC = 8 + x$:

$$x + (8 + x) = 16.55 \implies 2x + 8 = 16.55 \implies 2x = 8.55 \implies x = 4.275 \approx 4.28\,\text{m}.$$

iii)(b) Find x and hence find the radius r of the circle:

From part (iii)(a), we get:

$$x = 4.28\,\text{m}.$$

From part (ii), we know that $BQOR$ is a square. Hence:

$$BQ = OQ.$$

Substitute:

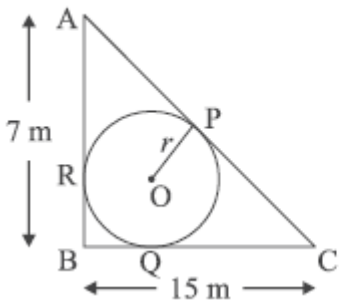
$$BQ = 7 - x \implies r = 7 - 4.28 = 2.72\,\text{m}.$$

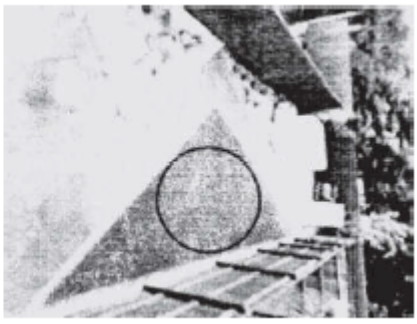
Final Answer:

- $PC = 8 + x = 12.28\,\text{m}$.
- Radius of the circle $r = 2.72\,\text{m}$.

Question 38:

A backyard is in the shape of a triangle ABC with right angle at B. $AB = 7$ m and $BC = 15$ m. A circular pit was dug inside it such that it touches the walls AC, BC, and AB at P, Q, and R respectively, such that $AP = x$ m.





Based on the above information, answer the following questions:

(i) Find the length of AR in terms of x.

(ii) Write the type of quadrilateral $BQOR$.

(iii) (a) Find the length PQ in terms of x and hence find the value of x.

(b) Find x and hence find the radius r of the circle.

Correct Answer:

(i) $AR = x$ m (ii) Quadrilateral BQOR is a square (iii)(a) ≈ 4.28**m**$(iii)(b) 2.72$m

Solution:

(i) $AR = x$ m.

(ii) Quadrilateral BQOR is a square.

(iii) **(a) Find the length PC in terms of x and hence find the value of x:**

$$PC = 8 + x$$

Using Pythagoras theorem in $\triangle ABC$:

$$AC^2 = AB^2 + BC^2$$

Substituting the values:

$$AC^2 = 7^2 + 15^2 = 49 + 225 = 274$$

$$AC = \sqrt{274}$$

Also:

$$AP + PC = AC$$

Substituting $AP = x$ and $PC = 8 + x$:

$$x + (8 + x) = \sqrt{274}$$

$$2x + 8 = \sqrt{274}$$

$$2x = \sqrt{274} - 8$$

$$x = \frac{\sqrt{274} - 8}{2} \approx 4.28\,\text{m}.$$

(iii) **(b) Find x and hence find the radius r of the circle:**

From part (ii), quadrilateral BQOR is a square. Hence:

$$r = 7 - x$$

Substituting $x = 4.28$:

$$r = 7 - 4.28 = 2.72\,\text{m}.$$

Final Answer:

- $PC = 8 + x = 12.28\,\text{m}$.
- Radius of the circle $r = 2.72\,\text{m}$.

Quick Tip

For problems involving inradius and tangents:

- Use the inradius formula $r = \frac{AB+BC-AC}{2}$.
- Apply tangent properties to find segment lengths.
- Use the Pythagoras theorem to compute side lengths.
