NEST 24 June 2023 Shift 1 Question Paper

Time Allowed :3 Hours	**Maximum Marks :**200	**Total questions :**68

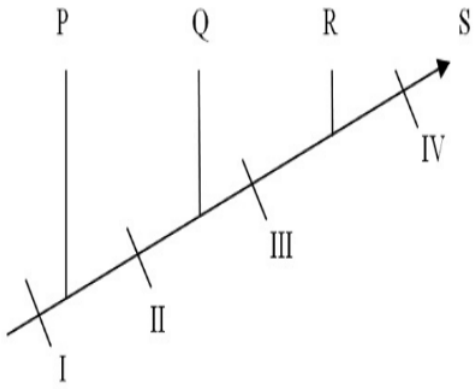
General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Conducting Bodies:** National Institute of Science Education and Research (NISER) and University of Mumbai - Department of Atomic Energy Centre for Excellence in Basic Sciences (UM-DAE CEBS).
2. **Exam Mode:** Online (CBT)
3. **Total Marks:** 200
4. **Total Questions:** 68

BIOLOGY

1. A cladogram representing the evolution of some animals (P, Q, R, and S) is shown. The features marked I to IV in the cladogram represent different features that have evolved. What is the correct statement with respect to the given cladogram?



(1) If Q is a bullfrog, II could be placenta.

(2) If features III and IV are mammary glands and hair, respectively, then R represents kangaroo and S represents humans.

(3) If Q, R, and S represent bullfrog, koala, and human, respectively, then feature II could be two pairs of limbs.

(4) If feature I is lungs, then P and Q are shark and bullfrog respectively.

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2. A 1000 base pair double-stranded DNA (B form) has a melting temperature (T_m) of 58°C. If a duplex RNA (A form) of the same length and sequence is constructed, then the T_m of this new RNA duplex with respect to the DNA (B form) would be:

(1) higher due to greater stability of A form of RNA duplex.

(2) lower due to lower stability of A form of RNA duplex.

(3) lower because of unfavorable enthalpy of formation of RNA duplex.

(4) identical, as the number of hydrogen bonds remain the same.

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3. A biochemist wants to purify a protein X (molecular weight = 30.2 kDa and pI = 7.5) from a solution containing proteins X and Y (molecular weight = 30.9 kDa and pI = 3.5)

using ion exchange chromatography. In this technique, an anion exchange resin is positively charged and a cation exchange resin is negatively charged. The most appropriate resin where protein X, but not Y will remain bound is

(1) cation exchanger at pH = 7.5.

(2) anion exchanger at pH = 2.5.

(3) cation exchanger at pH = 5.0.

(4) anion exchanger at pH = 7.5.

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4. Bacterial endospores are highly resistant to environmental stresses such as heat, UV radiation, and oxidizing agents. They can remain dormant for a prolonged period. During the period of dormancy, they prevent the accumulation of potentially harmful mutations in their DNA. This is primarily due to:

(1) decreased water content and reduced enzyme activity.

(2) decreased salt concentration and enhanced enzyme activity.

(3) decreased respiration and decreased DNA supercoiling.

(4) increased enzyme activity and increased membrane permeability.

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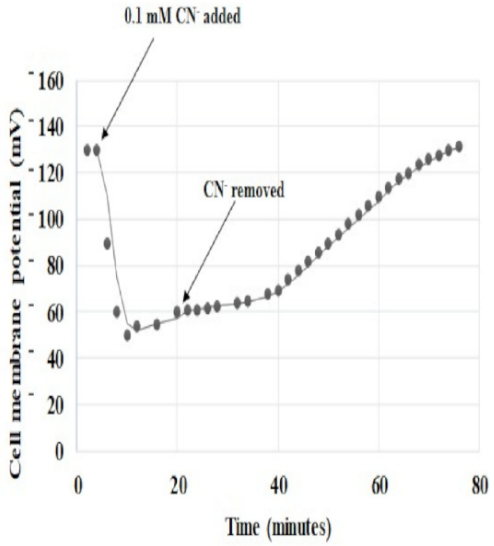
5. Pea seeds were allowed to germinate for 4 days, and segments of the epicotyl were treated with cyanide (CN^-) at a concentration of 0.1 mM. The membrane potential was recorded before and after the addition and removal of cyanide. The graph of cell membrane potential (mV) against time (minutes) is shown below. Based on these experimental observations, the correct option is:



(1) addition of cyanide will decrease the membrane potential because of the depletion of ATP.

(2) addition of cyanide will decrease the pH of the intermembrane space.

(3) addition of excess oxygen will increase the membrane potential in the presence of cyanide.

(4) addition of cyanide will cause a rapid but irreversible depolarization of membrane potential.

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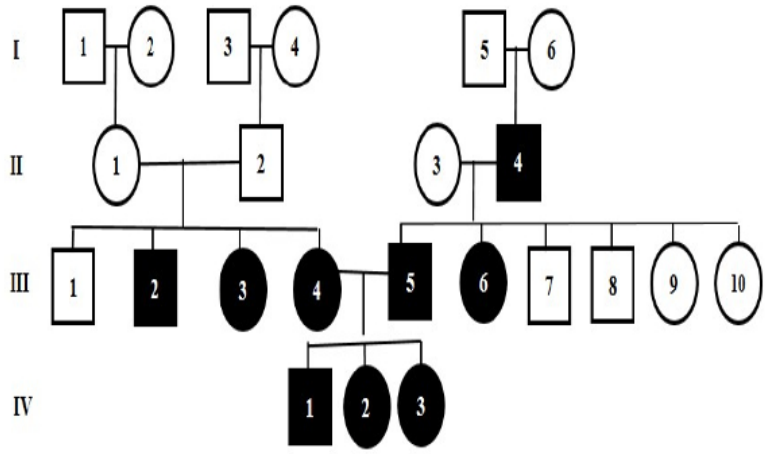
6. In the given pedigree, circles represent females and squares represent males. Filled shapes indicate affected individuals while unfilled shapes indicate unaffected individuals. Based on the pedigree information provided, the correct inheritance pattern is:



Based on these results, the correct option is:

(1) autosomal dominant.

(2) autosomal recessive.

(3) X-linked dominant.

(4) X-linked recessive.

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7. A population has a single locus with three alleles (X1, X2, X3). The frequencies of these alleles are p, q, and r, respectively, and if $p + q + r = 1$, then the correct statement is:

(1) the population is in Hardy-Weinberg equilibrium.

(2) the population is not in Hardy-Weinberg equilibrium because it has three alleles.

(3) this information is insufficient to state whether the population is in Hardy-Weinberg equilibrium.

(4) the population will be in Hardy-Weinberg equilibrium if $r = 0$ and $p + q = 1$.

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8. Mendelian theory of inheritance was crucial for Darwin-Wallace's theory of natural selection because

(1) it explained how new variations will persist in a population over time despite their low frequency, which is essential for natural selection.

(2) the Mendelian theory explained how characters acquired over an organism's lifetime can be passed on to the next generation, which is essential for natural selection to operate.

(3) the Mendelian theory explained how variation appears in the biological system which is crucial for evolution by the process of natural selection.

(4) the Mendelian theory first explained that linkage is essential for natural selection.

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9. In a closed ecosystem, red algae, crabs, and green algae interact with each other. The following experimental treatments were imposed:

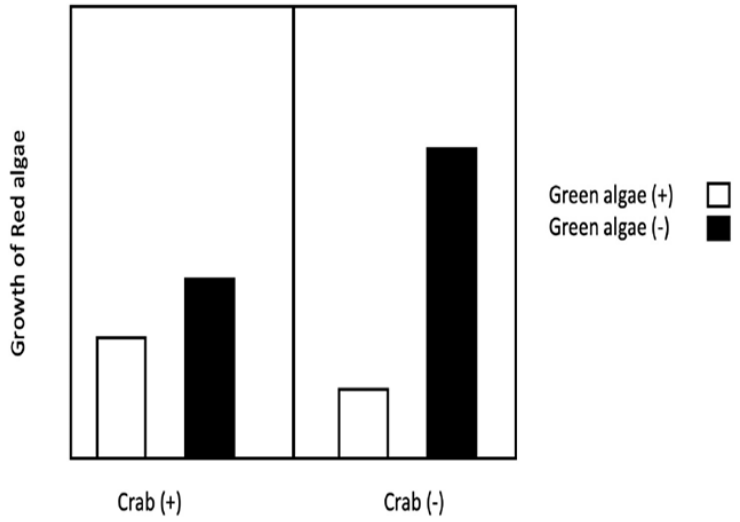
i. All crabs removed

ii. All green algae removed

iii. All crabs and green algae removed

iv. All crabs, red, and green algae are retained (Control)

The results of red algae growth from these experiments are shown in the graph. Plus (+) indicates presence and minus (-) indicates absence. Red algae growth in all the treatments was significantly different from each other.



Based on these results, the correct option is:

(1) in the absence of crabs, green algae positively affected red algae growth.

(2) in the presence of crabs, green algae positively affected red algae growth.

(3) in the absence of green algae, crabs negatively affected red algae growth.

(4) in the presence of green algae, crabs negatively affected red algae growth.

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10. Synthesis of ATP takes place when protons flow back to the matrix side through an enzyme complex called ATP synthase. ATP synthase is made up of two operational units: a rotatory and a stationary component. Assume that the ATP synthase does ~ 100 **rotations per second, and each rotation results in the generation of 3 ATP molecules. Suppose that a cell needs** $\sim 3 \times 10^{10}$ **molecules of ATP for one round of cell division that takes 30 minutes with the ATP synthase functioning at 50% efficiency. The minimum number of ATP synthase required for one round of cell division in this cell is:**

(1) 222222

(2) 55555.5

(3) 111111

(4) 277777

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11. The permeability of molecules across a lipid bilayer depends on their size and solubility in a nonpolar solvent relative to their solubility in water. For the species $O_2, H_2O, K^+, glycerol$ and $glucose$, the order of their permeability across a lipid bilayer in the absence of any protein transporters is

(1) $O_2 > H_2O > Glycerol > Glucose > K^+$

(2) $H_2O > O_2 > Glycerol > Glucose > K^+$

(3) $O_2 > H_2O > Glucose > Glycerol > K^+$

(4) $K^+ > O_2 > H_2O > Glycerol > Glucose$

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12. A monoclonal (antibody specific for a given epitope of an antigen) IgG antibody (M) was produced in the laboratory that binds a surface protein (P) of a virus (V1) and neutralizes 100% of V1. It was observed that M is also effective in neutralizing other viruses V2, V3 and V4 at 40%, 90% and 32% efficiency as compared to V1. If there are no other confounding factors, then the most accurate inference is:

(1) V1, V2, V3, and V4 have identical surface protein P.

(2) the genome of V1 and V3 has to be 90% identical for the observed binding efficiency.

(3) the sequence of P is more similar in V2 and V4 than in V1 and V2.

(4) a segment of sequence in P is highly conserved in V1 and V3 whereas the same segment is less conserved in V2 and V4.

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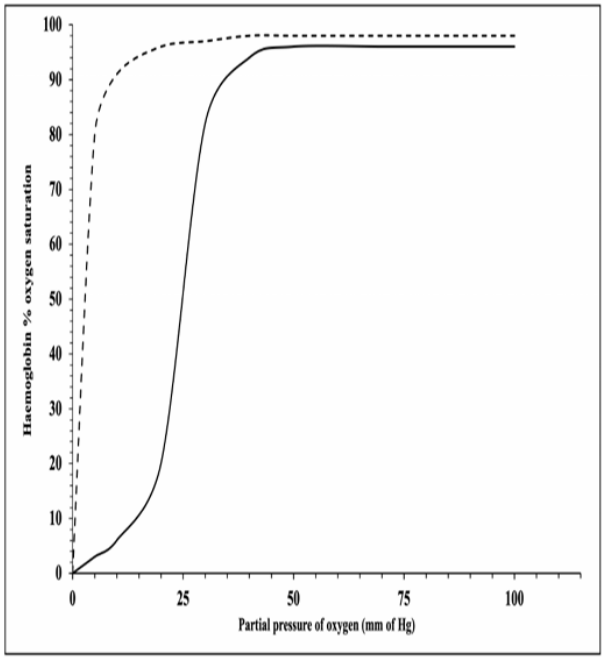
13. A genetic engineering experiment resulted in a new haemoglobin variant which exists primarily as $\alpha\beta$ dimers in solution. Based on the oxygen saturation curves of the variant (dashed line) and normal haemoglobin (solid line), it can be inferred that:



(1) the variant has a higher affinity to oxygen and shows non-cooperative behaviour.

(2) the variant has a higher affinity to oxygen and shows cooperative behaviour.

(3) the oxygen dissociation of the variant is less than 10% at 20 mm of Hg.

(4) at sea level (~ 150 mm of Hg), animals with this variant of haemoglobin can survive better than the ones with normal haemoglobin.

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14. A hospital witnessed an outbreak of a nosocomial (hospital-acquired) infection caused by *Klebsiella pneumoniae*. It showed resistance to antibiotics generally used for its treatment. Under the given scenario, the antibiotic resistance of this organism can be attributed to:

(1) the presence of antibiotic resistance gene acquired through horizontal transfer from other prevalent strains in the hospital.

(2) silent mutations in target sites of antibiotics.

(3) neutralisation of antibiotics by enzymes that target and inactivate these antibiotics.

(4) reduced permeability of these antibiotics due to the formation of a protective barrier.

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15. In the ABC model, the acquisition of floral organ identity is regulated by the floral homeotic genes named A, B, and C. A wild-type flower is depicted below.

Consider the following mutants:

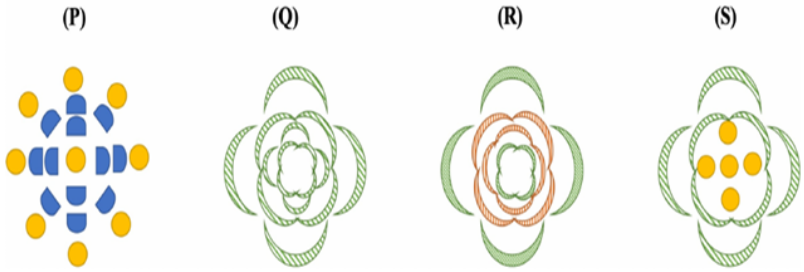
i. loss of gene C

ii. loss of gene A

iii. loss of gene B

iv. loss of genes B and C

Possible floral phenotypes (P-S) resulting from these mutations are given below.



Based on this ABC model, the correct option(s) is(are):

(1) (i) corresponds to R and (iv) corresponds to Q.

(2) (i) corresponds to S and (ii) corresponds to Q.

(3) (ii) corresponds to P and (iii) corresponds to S.

(4) (iii) corresponds to P and (iv) corresponds to R.

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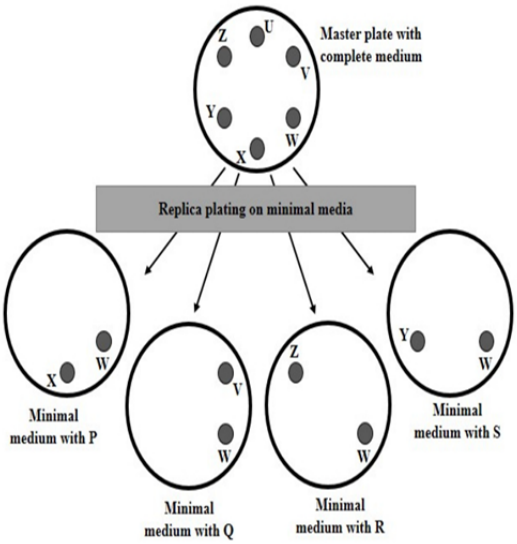
16. Auxotrophs are organisms with mutations that have lost the ability to synthesise essential organic compounds, and hence need supplements for growth in minimal media. Prototrophs are organisms that are able to synthesize essential compounds and hence can grow in minimal media without supplements. Colonies from six different strains of *E. coli* (U, V, W, X, Y, Z) are grown separately on a single master plate containing complete medium. No strain carries more than a single auxotrophic

mutation. Replica plates are then made from this master plate. Each of this replica plate contains minimal medium to which a single supplement (either P, Q, R, or S) is added. In the diagram below, filled circles represent colonies that grew on the medium and absence denotes failure to grow.



Based on the scheme, the correct interpretation(s) is(are):

(1) strain W is a prototrophic strain and strain V is auxotrophic for Q.

(2) strain Z is auxotrophic for R and strain Y is auxotrophic for S.

(3) strain U is a prototrophic strain and strain X is auxotrophic for Q.

(4) strain Y is auxotrophic for P and strain Z is auxotrophic for R.

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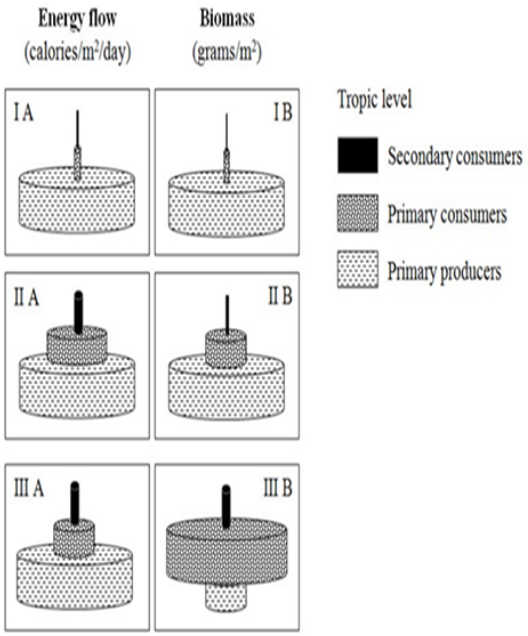
17. Energy diagrams allow ecologists to compare patterns of energy flow through the trophic levels of different ecosystems while biomass diagrams allow them to compare the amount of material present in living organisms at different trophic levels. The energy flow and biomass diagrams for a few ecosystems are given.



Choose the correct statement(s):

(1) III B cannot be the biomass pyramid of an open ocean since a biomass pyramid cannot be inverted at any level.

(2) I B and II B are the biomass pyramids of a grassland and a forest ecosystem respectively, since most of the biomass in a grassland is found in the green parts and hence the efficiency of energy transfer from producers to primary consumers is higher in grassland as compared to a forest ecosystem.

(3) III A could be the energy flow pyramid for an open ocean.

(4) I A could be the energy flow pyramid for a forest ecosystem, since the majority of the biomass is tied up in wood and is not available to most herbivores.

Chemistry

18. Pyrophosphoric acid ($H_4P_2O_7$) and pyrophosphorous acid ($H_4P_2O_5$) are oxyacids of phosphorus. The number of P–OH bonds present in pyrophosphoric acid and pyrophosphorous acid, respectively, are:

(1) 4, 3

(2) 4, 4

(3) 4, 2

(4) 2, 4

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19. The pair of molecules having the same shape is:

(1) PF_5 and BrF_5

(2) O_3 and I_3^-

(3) BF_3 and ClF_3

(4) XeO_3 and PF_3

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20. Boron on reaction with fluorine forms P, which on reaction with NaH at a high temperature produces Q along with a sodium salt. P on reaction with NH_3 in 1:1 ratio gives R. Q on reaction with NH_3 in 1:2 ratio at an elevated temperature gives S. The correct statement is:

(1) B–N bond in R is covalent in nature.

(2) All B–H bonds in Q are equivalent.

(3) The shape of S is tetrahedral.

(4) Compound Q when dissolved in water gives an acid.

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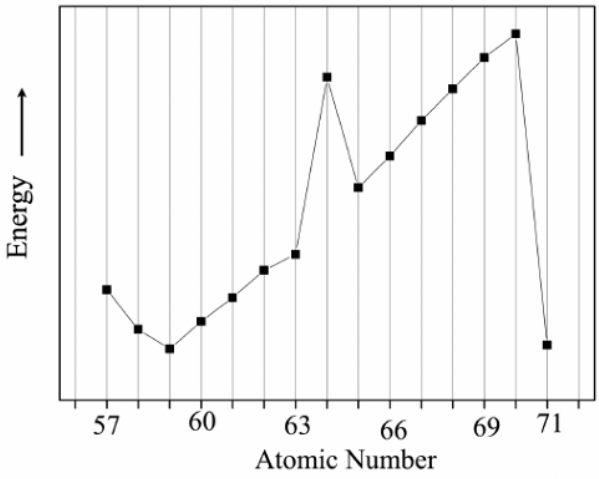
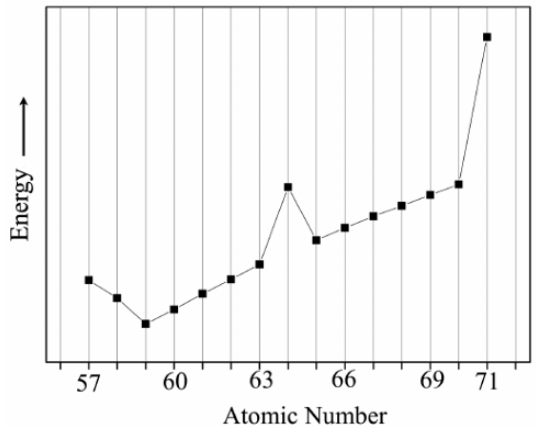
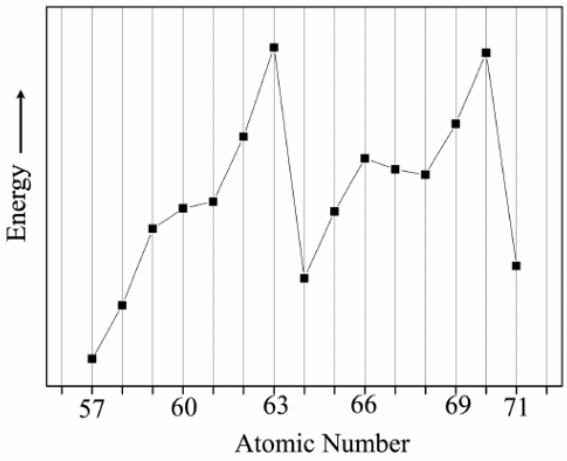
21. Among the following graphs, the one that represents the correct trend in the third ionization energy of lanthanum and the lanthanides is: (1)



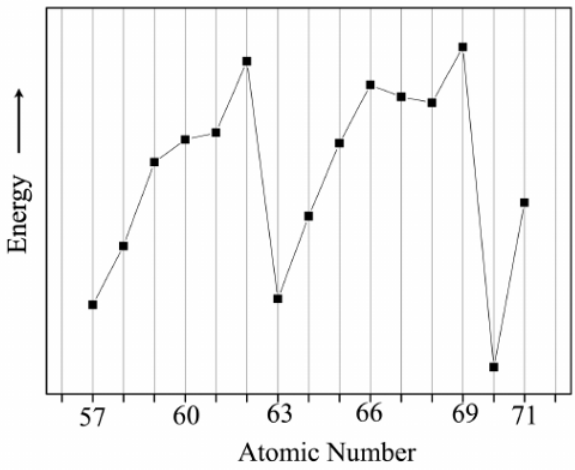
(2)



(3)



(4)



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22. The ground state electronic energy of He atom (E_{He}) can be expressed in terms of the ground state energy of the hydrogen atom (E_H) and the electron-electron interaction energy (E_{ee}) in the He atom. If E_{ee} is equal to x times the magnitude of E_H, then E_{He} is given by:

(1) $E_{He} = 4E_H + xE_H$

(2) $E_{He} = 2(4E_H + xE_H)$

(3) $E_{He} = 8E_H - xE_H$

(4) $E_{He} = 8E_H + xE_H$

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23. Consider a metal crystal with simple cubic, fcc, and bcc structures. Assume that the nearest neighbour atoms (spheres) touch each other in the unit cells. The correct statement is

(1) The percentage of the void space in different crystal structures follow the order: fcc $<$ bcc $<$ simple cubic.

(2) The number of atoms in the unit cell are 14, 9 and 8 for fcc, bcc and simple cubic structures, respectively.

(3) The lowest percentage of the void space among the three crystal structures is approximately 48%.

(4) The percentage of the void space in any of the above crystal structures will depend on the lattice parameter and hence cannot be predicted without the knowledge of the lattice parameter.

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24. A certain amount (n moles) of a monoatomic ideal gas changes from an initial state $X(P_1, V_1, T_1)$ to a final state $Y(P_2, V_2, T_2)$. Let ΔU, ΔH, and ΔS represent the changes in internal energy, enthalpy, and entropy, respectively in this process. The correct expression is:

(1) $\Delta H = \Delta U + P_2(V_2 - V_1)$

(2) $\Delta H = \Delta U + P_2(V_2 - V_1) + V_2(P_2 - P_1)$

(3) $\Delta S = C_v \ln(T_2/T_1) + nR \ln(V_2/V_1)$

(4) $\Delta S = (3/2)(P_1V_1/T_1) \ln(T_2/T_1) + (P_2V_2/T_2) \ln(V_2/V_1)$

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25. A mixture of 0.1 mol of a weak acid HX and 0.2 mol of another weak acid HY is dissolved in 1 kg of water. The degrees of ionization of the two acids HX and HY in the

final solution are 0.1 and 0.2, respectively. Assuming Raoult's law to be valid, the elevation of boiling point (ΔT_b), in terms of the boiling point elevation constant (K_b), is given by:

(1) $\Delta T_b = 0.25K_b$

(2) $\Delta T_b = 0.35K_b$

(3) $\Delta T_b = 0.30K_b$

(4) $\Delta T_b = 0.32K_b$

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26. When levorotatory 2-methylbutan-1-ol (P) is heated with concentrated HCl, dextrorotatory 1-chloro-2-methylbutane (Q) is formed. The correct statement is:

(1) P and Q have identical absolute configuration.

(2) The reaction takes place at the asymmetric centre.

(3) The reaction involves a carbocationic intermediate.

(4) If the concentration of P is doubled, keeping the concentration of HCl unchanged, the rate of the reaction does not change.

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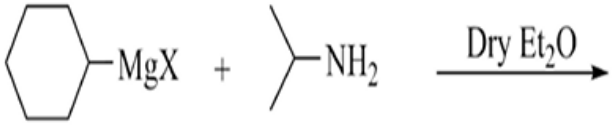
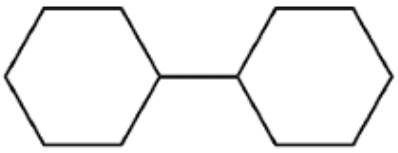
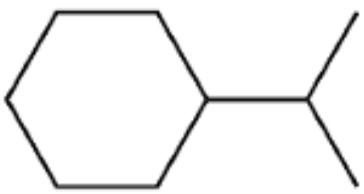
27. In the reaction shown below,



the major hydrocarbon product is



(1)



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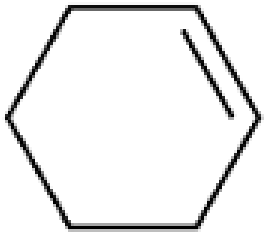

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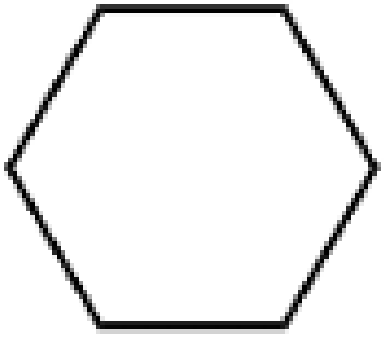

(4)

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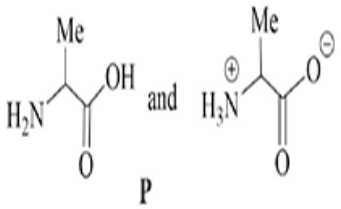
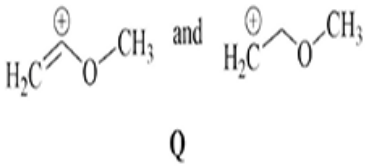
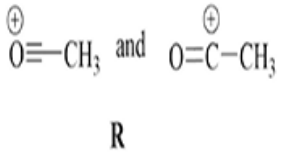
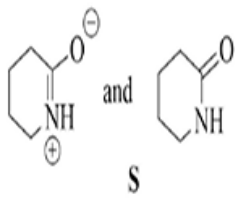
28. Among the following pairs, the pairs that represent resonance structures are:









(1) P, Q and R

(2) Q, R and S

(3) Q and S

(4) R and S

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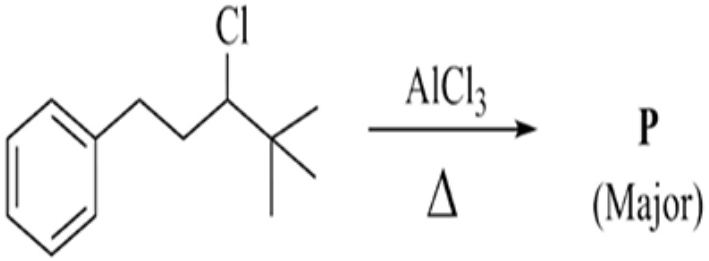
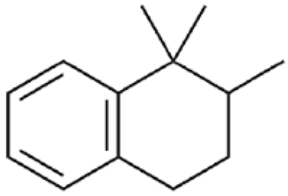
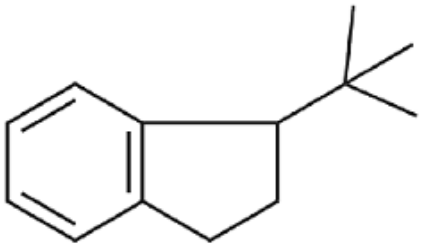
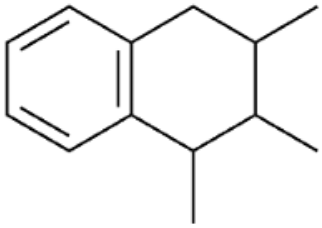
29. In the reaction shown below,

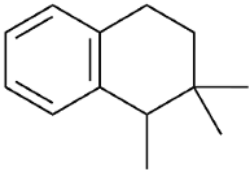


The product P is:

(1) 

(2) 

(3) 

(4) 

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30. The reaction of the alkali metals with oxygen results in the formation of different alkali metal oxides, monoxides, peroxides and superoxides. These oxides have interesting chemical properties and applications. The ease of formation and stability of these oxides depend mainly on the charge and size of the alkali metals in relation to the size of the anions. In an application like in a space suit, a chemical may be used which

gives oxygen and also absorbs carbon dioxide from exhalation. In this context, the correct statement(s) is(are):

(1) The order of the stability of alkali metal monoxides is $Li_2O < Na_2O < K_2O$.

(2) Superoxide formation decreases down the group.

(3) Na_2O_2 on reaction with CO_2 gives Na_2CO_3 and itself gets oxidized to oxygen.

(4) Potassium superoxide can be used in a space suit.

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31. Negatively charged monodentate strong field ligand (X^-) and weak field ligand (Y^-) form complexes $[MnX_6]^{4-}$ and $[MnY_6]^{4-}$, respectively, under certain reaction conditions. Let the crystal field splitting energy for $[MnX_6]^{4-}$ and $[MnY_6]^{4-}$ be $\Delta_0 1$ and $\Delta_0 2$, respectively. The correct statement(s) is(are):

(1) Electron pairing energy in $[MnX_6]^{4-}$ is smaller than $\Delta_0 1$.

(2) $[MnY_6]^{4-}$ is more stabilized than $[MnX_6]^{4-}$.

(3) The $t_{2}g$ orbitals in $[MnX_6]^{4-}$ are stabilized by $2\Delta_0 1$ as compared to degenerate d orbitals.

(4) $[MnY_6]^{4-}$ is intense in colour as compared to $[MnX_6]^{4-}$.

---

32. The bonding in the species HeH can be studied in a way similar to that of H_2. The correct statement(s) is(are):

(1) The dipole moment of HeH is larger than that of H_2^+.

(2) Among HeH, HeH^+, and HeH_2^{2+}, the species that is most stable is HeH_2^{2+}.

(3) Among HeH, HeH^-, and HeH^+, the system with the smallest bond length is HeH^+.

(4) HeH^+ has a symmetrical distribution of electrons about the bond axis.

---

33. The hydrolysis of sucrose in excess acid solution follows first-order kinetics and results in D-glucose and D-fructose. Since, sucrose, D-glucose, and D-fructose are optically active compounds, the progress of the reaction can be monitored by measuring the angle of rotation of the polarized light in a polarimeter at different times. In a certain experiment, 1 L of 0.1 M sucrose solution is hydrolyzed and the angle of

rotation (R_t) is measured at different times (t). Let $R_0 = 25°$, $R_\infty = -15°$, and $R_{30} = 5°$ be the angles of rotation at times $t = 0$ min, $t = \infty$, and $t = 30$ min respectively. The concentration of sucrose at time t is proportional to the change in the angle of rotation ($R_t - R_\infty$). Molecular mass of sucrose is 342.3 g/mol. The correct statement(s) is(are):

(1) The half-life of the reaction is 15 min.

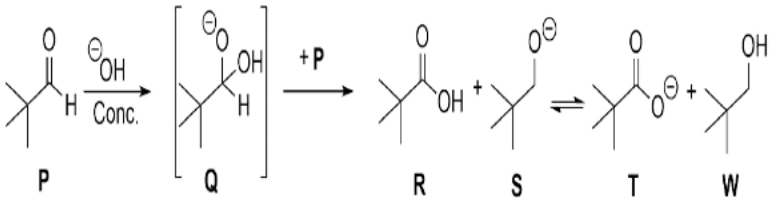
(2) The rate constant (k) is $3.85 \times 10^{-4}\,\text{s}^{-1}$.

(3) The mass of sucrose hydrolyzed in 60 min is 25.6 g.

(4) The rate of the reaction at 30 min is $1.925 \times 10^{-5}\,\text{M s}^{-1}$.

---

34. Consider the following mechanism of a reaction.



The correct statement(s) is(are):

(1) The same mechanism will operate, if compound P does not have one of the methyl groups.

(2) The reaction involves both oxidation and reduction of P.

(3) The equilibrium is favoured towards products T and W because T and W are weaker conjugate base and acid as compared to R and S respectively.

(4) Hydride is a nucleophile in the reaction of Q with P.

Mathematics

35. Let $g : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $g(x)g'(x) > 0$ for all $x \in \mathbb{R}$. Then:

(1) g is increasing.

(2) g is decreasing.

(3) $|g|$ is increasing.

(4) $|g|$ is decreasing.

---

36. The number of real roots of $f(x) = x^6 + x^3 - 1$ is:

(1) 0

(2) 2

(3) 4

(4) 6

---

37. In a throw of a (biased single) dice, the probability of the outcome being a number n is $\frac{1}{4}$ if n is even, and $\frac{1}{12}$ if n is odd. If the dice is thrown twice, then the probability that the sum of the two outcomes is an even number is:

(1) $\frac{3}{8}$

(2) $\frac{1}{2}$

(3) $\frac{5}{8}$

(4) $\frac{3}{4}$

---

38. Let sgn(x) be the function defined as:

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ -1, & \text{if } x < 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = (x - \sqrt{5})$ sgn$(x^2 - 5)$. Then the number of discontinuities of f is:

(1) 0

(2) 1

(3) 2

(4) 3

---

39. Let S be the set of all natural numbers x such that

(i) $100 \leq x \leq 999$**,**

(ii) 0 appears at least once as a digit in the decimal expansion of x**,and**

(iii) the sum of the digits of x **is 10. The number of elements in** S **is:**

(1) 18.

(2) 20.

(3) 27.

(4) 30.

---

40. The horizontal line $y = k$ **intersects the parabola** $y = 2(x-4)(x-6)$ **at points** A **and** B**. If the length of** AB **is 8, then the value of** k **is:**

(1) 30

(2) 10

(3) 20

(4) 8

---

41. Let $S(n) = \frac{1}{n^4}\sum_{l=1}^{n}(l+2)(l+4)(l+6)$**. The value of** $\lim_{n\to\infty} S(n)$ **is:**

(1) $\frac{1}{6}$

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) 1

---

42. Let α **be a complex number such that** $\alpha \neq 1$ **and** $\alpha^5 = 1$**. Let** $A = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \alpha & 0 \\ \alpha & 0 & 0 \end{pmatrix}$ **and** I **denote the identity matrix. Then the value of** $I + A + A^2 + A^3 + A^4$ **is:**

(1) $(1+\alpha^2+\alpha^4)\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

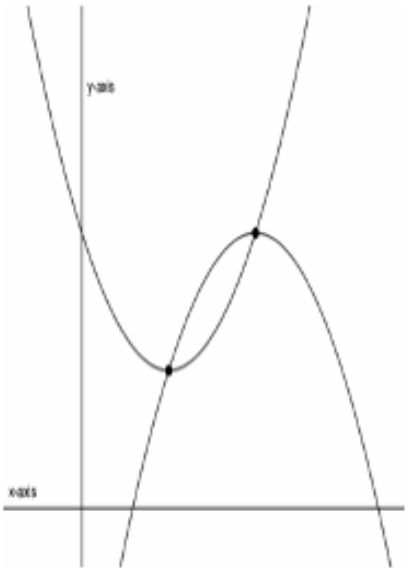
(2) $\alpha(1+\alpha^2)\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

(3) $(1+\alpha^2+\alpha^4)\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

(4) $(1+\alpha^2+\alpha^4)\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

---

43. Let P and Q be the vertices of the parabolas $y = x^2 + bx + c$ and $y = -x^2 + dx + e$, respectively.



If P and Q are the points of intersection of the parabolas, then the slope of the line through P and Q is:

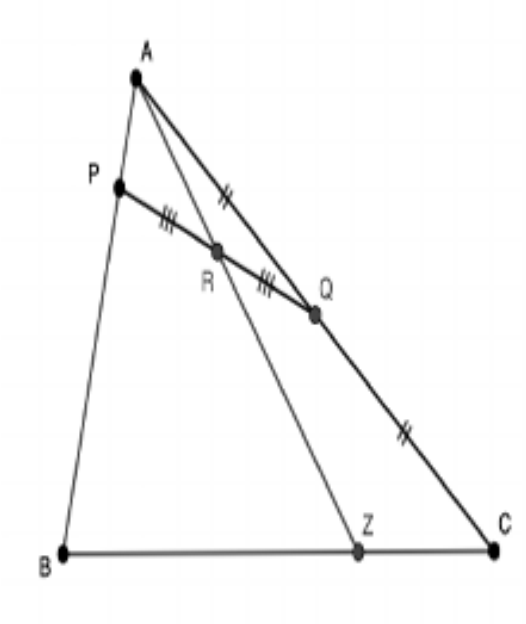
(1) $\frac{c+e}{2}$

(2) $\frac{c+d}{2}$

(3) $\frac{b+d}{2}$

(4) $\frac{b+e}{2}$

44. Let ABC be a triangle with $AC = 2048$, $AB = 512$, and $BC = 2000$. Let P be a point on the segment AB such that $AP = 1$, and Q be a point on the segment AC such that $AQ = 1024$. Let R be the midpoint of PQ. Let Z be the point of intersection of AR and BC. Then the length of ZC is:



(1) $\frac{2000}{256}$

(2) $\frac{2000}{257}$

(3) $\frac{1000}{256}$

(4) $\frac{1000}{257}$

---

45. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(0) = 1$ and

$$|f(x) - f(y)| \leq |\sin((x-y)^2)| \quad \text{for all } x, y \in \mathbb{R},$$

and let g be the function defined by $g(x) = x^2 f(x^2)$ for all $x \in \mathbb{R}$. Then the value of $g'(2)$ is

(1) 2

(2) 4

(3) 6

(4) 0

---

46. Let $n \geq 3$ be an integer. Let $P_1, P_2, \ldots, P_{2n}$ be points in the plane, which are the vertices of a regular $2n$-gon. The number of obtuse-angled triangles with vertices contained in the set $\{P_1, P_2, \ldots, P_{2n}\}$ is:

(1) $n(n-1)(n-2)$

(2) $\frac{n^2(n-1)(n-2)}{3}$

(3) $\frac{n(n-1)^2}{2}$

(4) $2n(2n-1)(2n-2)$

---

47. If A, B, C are 3×3 matrices with entries in $\mathbb{R}$, satisfying the condition $AB = AC$, then:

(1) the determinant of AB is 0.

(2) either A is the zero matrix or $B = C$.

(3) either $B = C$ or A is not an invertible matrix.

(4) either A is the zero matrix or the determinant of $B - C$ is 0.

---

48. Let X, Y, Z be sets and $f : X \to Y$ and $g : Y \to Z$ be functions. Then:

(1) $g \circ f$ being injective implies f injective.

(2) $g \circ f$ being surjective implies g surjective.

(3) $g \circ f$ being injective implies g injective.

(4) g being surjective implies $g \circ f$ surjective.

---

49. Let $f : (0,3) \cup (6,9) \to \mathbb{R}$ be a differentiable function such that $f'(x) = \frac{1}{2}$ for all $x \in (0,3) \cup (6,9)$. Then:

(1) f is an increasing function.

(2) f is a one-to-one function.

(3) $f(8) - f(7) = f(2) - f(1)$.

(4) There exists a number $c \in \mathbb{R}$ such that $f(x+6) = f(x) + c$ for all $x \in (0,3)$.

---

50. Let A and B be two points on the parabola $y - 2x^2 = 0$ and O be the origin $(0,0)$. If:

(1) OAB is an isosceles triangle, then the y-coordinates of A and B are equal.

(2) OAB is an equilateral triangle, then the length of each side is $\sqrt{3}$.

(3) OAB is an isosceles triangle and the two equal sides are of length $\sqrt{3}$, then OAB is an equilateral triangle.

(4) OAB is an equilateral triangle, then its altitude is $\sqrt{3}$.

---

51. Let $f : [0,1] \to \mathbb{R}$ be a continuous function and P be a polynomial of degree 4 with coefficients in $\mathbb{R}$. If $P(f(x)) = 0$ for all $x \in [0,1]$, then

(1) $f(x) = 0$ for all $x \in \mathbb{R}$.

(2) f is a constant function.

(3) for all continuous functions g, there exists $x \in [0,1]$ such that $P(g(x)) = 0$.

(4) P has at most two roots which do not belong to $\mathbb{R}$.

Physics

52. A thin spherical copper shell of radius R, completely filled with a viscous fluid, is rotating about the vertical axis with a constant angular speed ω_0. Due to a leak at the bottom of the shell, the fluid starts dripping steadily and vertically from the shell. The net change in angular speed ($\delta\omega$) when the shell gets empty is

(1) proportional to R^2.

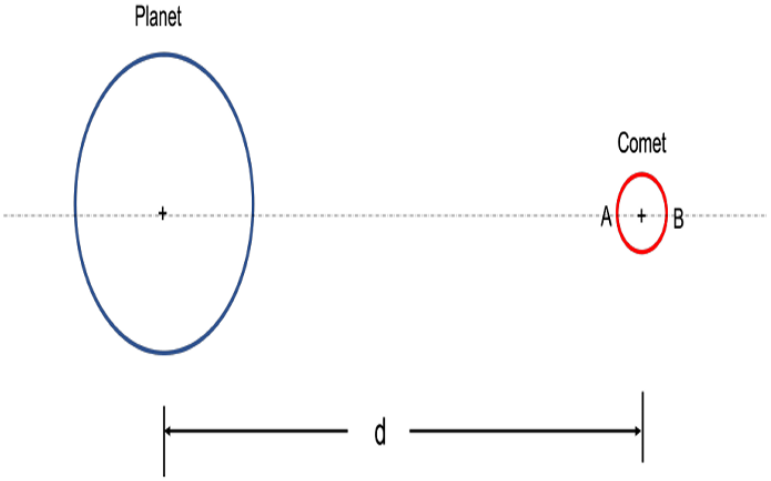
(2) proportional to R.

(3) proportional to R^3.

(4) independent of R.

53. A spherical comet having mass M_s and radius r is moving towards a planet of mass M_p as shown in the figure. At a separation distance d, equal gravitational force is experienced by the two identical test masses m which are placed at diametrically

opposite ends (A and B) of the comet. Assuming $d \gg r$, the correct choice about the separation distance d is:



(1) d is proportional to $M_p^{1/3}$.

(2) d is proportional to $M_s^{2/3}$.

(3) d is independent of r.

(4) d is inversely proportional to $M_s^{2/3}$.

---

54. A metal rod, connected between two high voltage electrodes, attains steady-state temperature through a balance between radiated power loss and Joule heating. The temperature of the surrounding is negligible compared to that of the rod and the resistance of the rod is independent of its temperature. Assuming the current through the rod to be I, the dominant wavelength of radiation (λ) is given by $\lambda \propto I^{\alpha}$. Then, the value of α is:

(1) 1.0

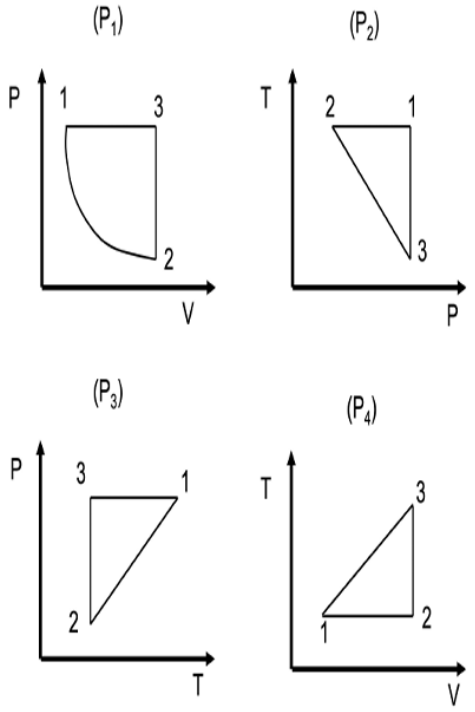
(2) -0.5

(3) 2.0

(4) -1.0

---

55. Thermodynamic processes (P_1, P_2, P_3, P_4) in which an ideal gas passes through states 1, 2, and 3 are shown in the figure where P, V, T are pressure, volume, and temperature, respectively. The process(processes) that could be identical to the process P_1 is (are):



(1) P_2 only.

(2) P_2 and P_3 only.

(3) P_3 and P_4 only.

(4) P_2 and P_4 only.

---

56. A proton accelerated from rest by a potential difference of V volts has a de Broglie wavelength of 0.20Å (1.0 Å = 10^{-10} m). A fully ionized Helium atom is similarly accelerated by a potential difference of 2V volts. Its de Broglie wavelength (in Å) is closest to:

(1) 0.05

(2) 0.07

(3) 0.10

(4) 0.20

---

57. Consider the Bohr model of the hydrogen atom with Bohr radius a_B. If the mass of the electron and that of the proton become twice the present values, then the new Bohr

radius will:

(1) remain unchanged.

(2) change to $2a_B$.

(3) change to $\frac{a_B}{2}$.

(4) change to $4a_B$.

---

58. Two radioactive samples X and Y have the same number of atoms initially $[N_X(t=0) = N_Y(t=0)]$. The half-life $\tau_{1/2}^X$ of X is half the mean life of Y. Then $N_Y(t)$ is seven times $N_X(t)$ when $t/\tau_{1/2}^X$ is closest to:

(1) 1

(2) 2

(3) 5

(4) 10

---

59. An elastic conducting ring of mass m is extended radially with constant speed v in an uniform magnetic field of strength B, which is perpendicular to the plane of the ring. Take the resistance R of the ring to be a constant. The magnetic moment (μ) of the ring in terms of the instantaneous radius of the ring (r) is given by $\mu = Kr^\alpha$, where K and α are constants. Then, the value of α is:

(1) 0

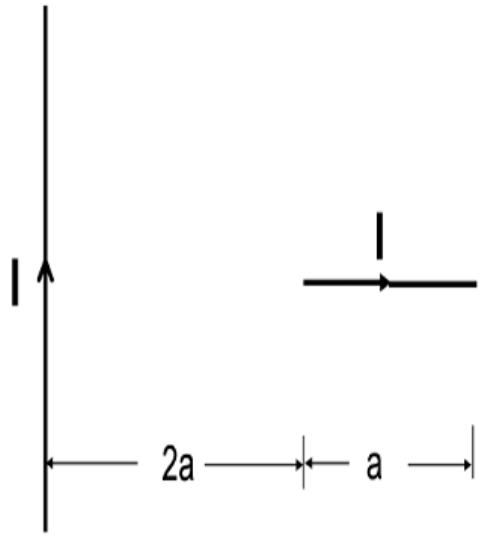
(2) 1

(3) 4

(4) 3

---

60. A horizontal straight wire of length a is placed perpendicular to a long current carrying straight vertical wire at a distance of $2a$ and lies in the same plane as shown in the figure. Both wires carry steady current I. The magnitude of the force on the horizontal wire due to the vertical wire is:



(1) $\frac{\mu_0 I^2 \ln(3/2)}{2\pi}$

(2) 0

(3) $\frac{\mu_0 I^2 \ln 3}{\pi}$

(4) $\frac{3\mu_0 I^2 \ln 2}{2\pi}$

---

61. A double convex lens of the objective is changed to plano-convex. The objective is made of a plastic material with refractive index 1.3. Then:

(1) the numerical aperture of the double convex lens is 1.3.

(2) the numerical aperture of the plano-convex lens is 0.39.

(3) the diameter to focal length ratio of the plano-convex lens is 1.2.

(4) the critical angle of the double convex lens is 60°.

---

62. An elastic wave generates a stress of magnitude N while propagating in a wire. The relation between its frequency ω and the wavevector k is given by $\omega = \omega_0\sqrt{1-\cos(ka)}$, where ω_0 and a are constants. In the long wavelength approximation ($\lambda \gg a$), the linear density of the wire is:

(1) $\frac{2N}{\omega_0 a}$

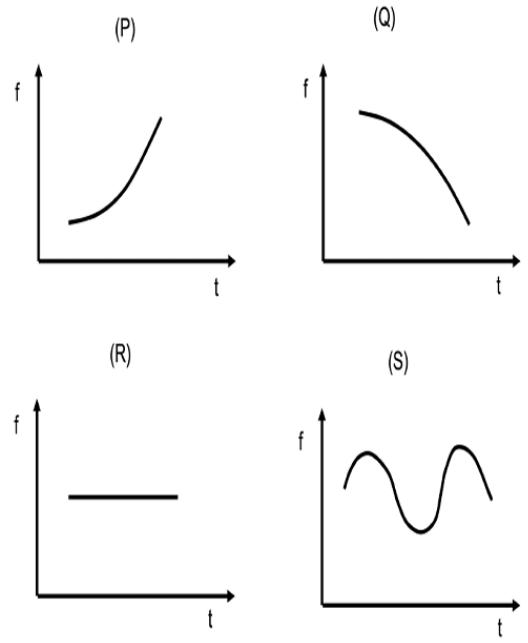
(2) $\frac{\omega_0 a}{N}$

(3) $\frac{2N}{(\omega_0 a)^2}$

(4) $\frac{N}{\omega_0^2 a}$

---

63. A slow steady stream of water is falling into a tall cylindrical bucket. Let $f(t)$ denote the dominant frequency of the sound of the fall onto the horizontal water surface in the bucket. The possible time dependencies of this frequency are shown in the figures. The graph which best describes the variation of f with t qualitatively is:



(1) (P).

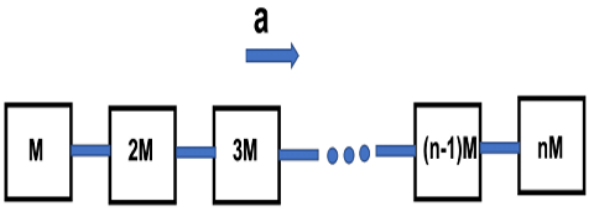
(2) (Q).

(3) (R).

(4) (S).

---

64. An accelerating train (acceleration $a = 1\,\text{m/s}^2$) of n blocks has a mass distribution as shown in the figure. The rightmost block of mass nM is the engine. The blocks are connected through an Aluminum cable of cross-section $10\,\text{cm}^2$. The maximum allowed strain in the connecting cables is 0.001. Taking $M = 1000\,\text{kg}$ and Young's modulus of Aluminum to be $7 \times 10^{10}\,\text{Pa}$, the correct choice(s) is (are):



(1) All connecting cables will have equal strain.

(2) The difference of stress in any two consecutive cables is in arithmetic progression.

(3) The train can have at most 12 blocks.

(4) The maximum stress is between blocks having masses M and $2M$.

---

65. A point object P of mass m and charge q is placed at a distance r from a stationary object Q of mass M and charge $-q$. Let T_g be the time for P to reach Q if it is moving only under gravitational attraction. Similarly, let T_e be the time for P to reach Q if it is moving only under electrostatic attraction. Then:

(1) T_g depends on M but not on m.

(2) T_e depends on m but not on M.

(3) T_g^2 is directly proportional to G.

(4) T_e^2 is directly proportional to ϵ_0.

---

66. The correct statement(s) about an ideal gas is(are)

(1) for an adiabatic process, the work-done does not depend upon the path.

(2) for an isobaric process, the change in enthalpy is equal to the net heat input to the system.

(3) the total potential energy of an ideal gas can be a function of time.

(4) for an isothermal process, the entropy of an ideal gas remains constant.

---

67. The electrostatic potential in the region between two long coaxial cylinders of radii a and b is given by $\phi = \alpha \ln(r/a) + \beta$, where α and β are constants. Here, $\alpha > 0$ and r denotes radial distance from the axis such that $a < r < b$. Then, the correct option(s) is (are):

(1) The charge per unit length on the inner cylinder is $-2\pi\epsilon_0\alpha$.

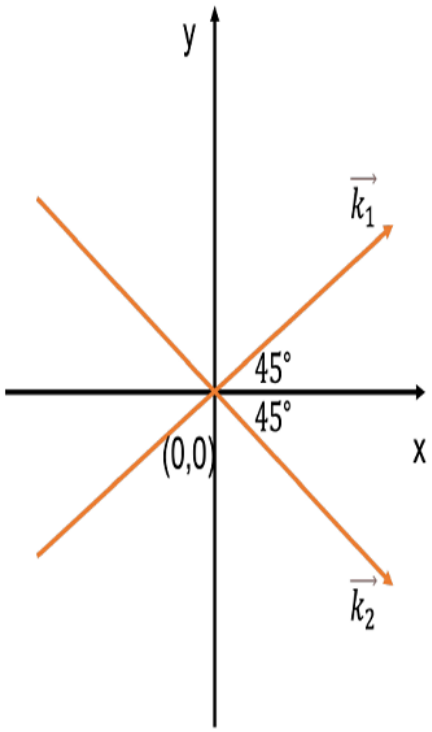
(2) Capacitance per unit length is $\ln(b/a)/(2\pi\epsilon_0)$.

(3) The charge density on the outer cylinder is $-(\epsilon_0\alpha)/b^2$.

(4) The electric field in the region between two cylinders is $-(\alpha/r)\hat{r}$.

---

68. Two plane waves having amplitude E_0 are described by $\vec{E}_1 = E_0 \cos\left(\vec{k}_1 \cdot \vec{r} - \omega t\right) \hat{z}$ and $\vec{E}_2 = E_0 \cos\left(\vec{k}_2 \cdot \vec{r} - \omega t\right) \hat{z}$. The wavevectors $\vec{k}_1$ and $\vec{k}_2$ pass through the origin making an angle of 45° with the x-axis, as shown in the figure. At $t = 0$ and $x = 0$ plane, the correct option(s) is (are):



(1) The periodicity of interference pattern in the yz-plane is $\sqrt{2}\lambda$.

(2) The amplitude of resultant wave is $2E_0$.

(3) The direction of polarization of resultant wave will change.

(4) The maximum intensity is E_0^2.
