NEST 2024 Session 1 Question Paper

Time Allowed :3 Hours	**Maximum Marks :**240	**Total questions :**80

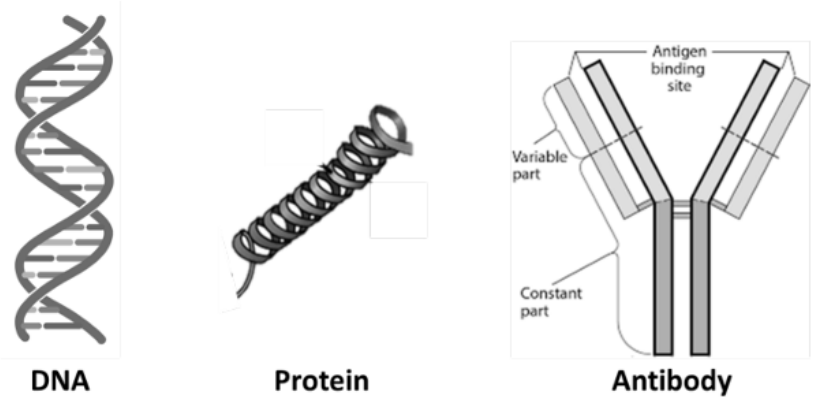
General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Conducting Bodies:** National Institute of Science Education and Research (NISER) and University of Mumbai - Department of Atomic Energy Centre for Excellence in Basic Sciences (UM-DAE CEBS).
2. **Exam Mode:** Online (CBT)
3. **Total Marks:** 240
4. **Total Questions:** 80

BIOLOGY

1. Analyze the images and deduce the correct structural description of the molecules.



(A) DNA – primary; Protein – secondary; Antibody – tertiary

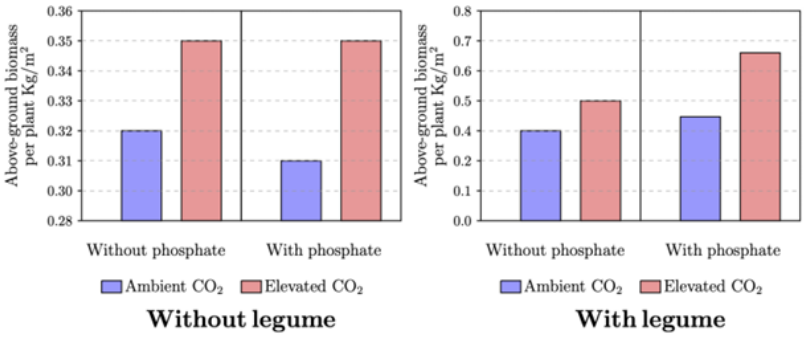
(B) DNA – primary; Protein – secondary; Antibody – quaternary

(C) DNA – secondary; Protein – primary; Antibody – secondary

(D) DNA – secondary; Protein – secondary; Antibody – quaternary

---

2. The effects of elevated CO_2 supply, presence of legume and phosphate availability on the above-ground biomass production of a grassland community is depicted in the graph. Based on this information, the correct option is:



Based on this information, the correct option is:

(A) Nitrogen-deprived plants grow faster under ambient CO_2 condition.

(B) **The growth of plants is enhanced under elevated CO_2 condition in presence of legume.**

(C) In the absence of phosphate, legume provides no growth benefit to the plant.

(D) In the presence of legume, there is no growth benefit on addition of phosphate under elevated CO_2 condition.

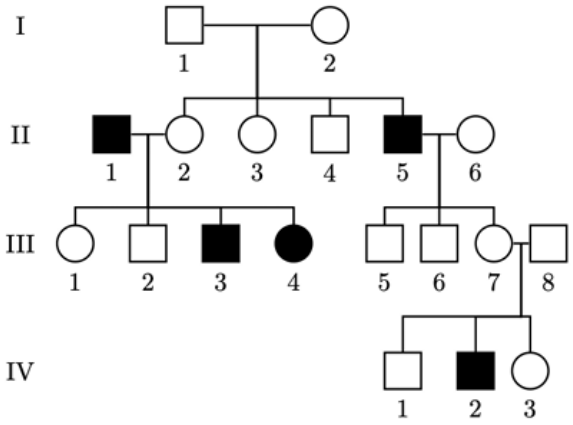
---

3. In a given plant, the flower colour is governed by a single gene locus. The flower can be either white, red, blue or purple colour. However, a single flower will never have different coloured petals. Multiple crosses were carried out between plants with different flower colours (white, red, blue or purple) and the observations on the progeny phenotype are tabulated. Based on this information, the correct option is:

cross	**progeny phenotypes**
White X White	All flowers are white
Red X Red	All flowers are red or some are red and some are white (number of red >> number of white)
Blue X Blue	All flowers are blue or some are blue and some are white (number of blue >> number of white)
Purple X Purple	A mix of red, blue and purple flowers (number of purple flowers > numbers of red and blue flowers and equal number of red and blue flowers)
White X Red	All flowers are red or some are red and some are white (equal number of red and white flowers)
White X Blue	All flowers are blue or some are blue and some are white (equal number of blue and white flowers)
White X Purple	Blue and red flowers (equal number of red and blue flowers)
Red X Blue	All flowers are purple or some are red, some are blue and some are white
Red X Purple	Mostly red and purple flowers and some are blue
Blue X Purple	Mostly blue and purple flowers and some are red

---

4. In the given pedigree, circles represent females and squares represent males. Filled shapes indicate affected individuals, while unfilled shapes indicate unaffected individuals. Based on the pedigree analysis, consider the statements (i) to (iv):



(i) If the inheritance pattern is autosomal recessive, then the individual III-8 is a carrier of the disease.

(ii) If the inheritance pattern is X-linked recessive, then the individuals II-6 and III-7 are carriers of the disease.

(iii) Individuals I-1 and I-2 are homozygous for the allele under study.

(iv) If the inheritance pattern is autosomal recessive, then individuals III-2 and IV-1 must be carriers of the disease.

The correct combination of statements is:

(A) i, ii and iv

(B) **i and ii**

(C) ii, iii, iv

(D) ii, iv

---

5. Katalin Karikó and Drew Weissman were awarded the Nobel Prize in Physiology or Medicine for the year 2023 for their observation that pseudo-uridine (ψ) incorporated mRNA (but not mRNA with normal bases A, U, G, and C) is better for RNA vaccine production. These observations paved the way for developing the RNA vaccine against SARS-CoV-2 (the vaccine produces the spike protein of the virus in a cell). The reason that pseudo-uridine incorporated mRNA worked as a better vaccine candidate is:

A. The innate immune system does not recognise the pseudo-uridine incorporated mRNA allowing translation of the spike protein.

B. Unmodified mRNA encoding spike protein cannot be translated by the ribosome.

C. Antigen-presenting cells can translate only pseudo-uridine incorporated mRNA.

D. Spike protein from unmodified mRNA cannot induce immune response.

---

6. An unknown organism that can utilize NH_4Cl as a nitrogen source was grown for several generations under four different conditions as given below:

(i) medium containing $^{15}NH_4Cl$

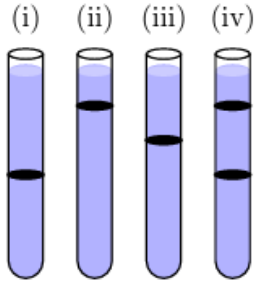
(ii) medium containing $^{14}NH_4Cl$

(iii) medium containing $^{15}NH_4Cl$ followed by culturing in medium containing $^{14}NH_4Cl$ for one generation

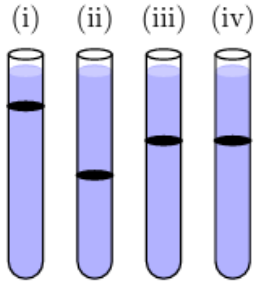
(iv) medium containing $^{15}NH_4Cl$ followed by culturing in medium containing $^{14}NH_4Cl$ for two generations

DNA isolated from the organism grown under the above listed conditions was independently analysed by density gradient centrifugation. Assuming that the mode of DNA replication in this organism is dispersive, the option representing the correct band pattern is:

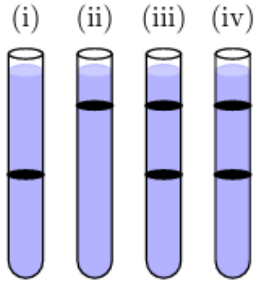
A.



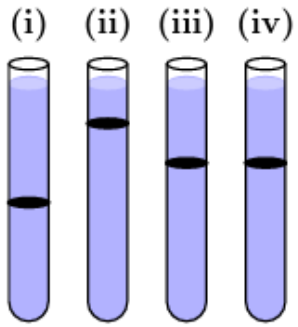
B.



C.



D.



---

7. Consider the following statements about photorespiration in plants:

1. Photorespiration produces one molecule of 3-phosphoglycerate from ribulose bisphosphate.
2. Photorespiration is a wasteful process because neither ATP nor NADPH is produced.

3. Phosphoglycolate is converted to glycolate in peroxisomes.
4. Photorespiration in C3 plants is less compared to C4 plants.

The correct combination of statements is:

A. i and ii

B. ii and iv

C. iii and iv

D. ii and iii

---

8. In order to generate a crop variety with C4 carbon fixation, a researcher expressed PEPcase enzyme from C4 pathway in leaves of the C3 plant. However, the modified C3 plant did not show carbon fixation efficiency as expected for a C4 pathway plant. The most likely reason is:

A. Improper compartmentalisation of the C4 PEPcase in C3 leaves.

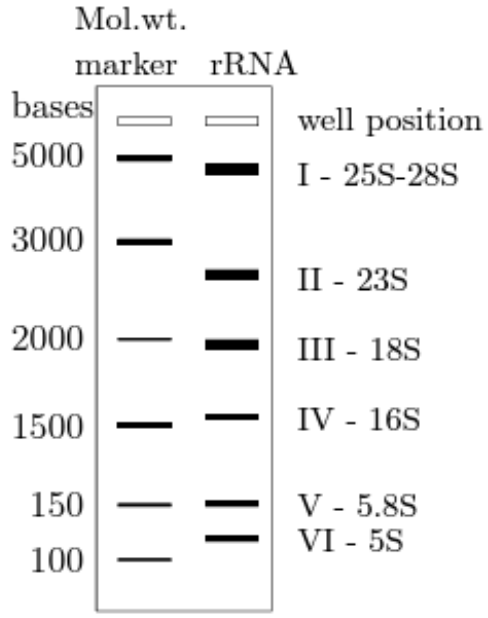
B. The PEPcase from C4 plant is catalytically inactive in the C3 leaves.

C. Carbon-dioxide fixed by C4 pathway PEPcase cannot be utilised by the RuBisCo of C3 leaves.

D. Overexpression of PEPcase from C4 in C3 leaves increases photorespiration.

---

9. An experimenter was analysing homogenized cell lysate (containing mitochondrial + chloroplastic + cytoplasmic contents) of a plant tissue that is heavily infected with a bacterial pathogen. Ribosomal RNA (rRNA) purified from the cell lysate was separated by gel electrophoresis. The band pattern observed after the electrophoresis is depicted in the figure. The correct statement about the observed band pattern is:



The correct statement about the observed band pattern is:

A. I, III and V are from the plant ribosome.

B. I, III and V are from the bacterial ribosome.

C. II, III and VI are from the bacterial ribosome.

D. IV and VI are absent in plant ribosome.

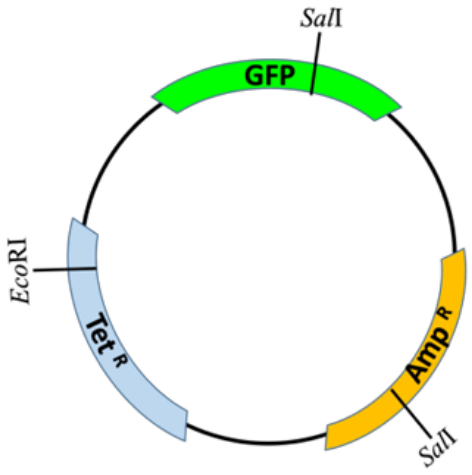
---

10. A researcher incubated algal cells with a fluorescent-labelled cytidine analogue that can enter the cells and get incorporated into nucleic acids. The cells were lysed and different membrane-bound organelles were isolated. The membrane-associated ribosomes were removed and the organelles were then homogenized. The macromolecules thus isolated from each organellar fraction were analysed. The organelle fractions that show incorporation of the fluorescence signal are _____.

---

11. In the vector map, GFP denotes the green fluorescent protein, TetR is a tetracycline resistance gene and AmpR is an ampicillin resistance gene. A DNA fragment containing the gene of interest was generated using SalI digestion. This fragment was ligated with the SalI digested vector. Assume complete digestion by SalI. E. coli was

transformed with the ligation product and transformants were plated on media containing _____. The recombinants having the gene of interest will _____.



(A) ampicillin and are GFP positive

(B) tetracycline and are GFP negative

(C) ampicillin and are GFP negative

(D) tetracycline and are GFP positive

---

12. In an ecological interaction between two species of wild plants (X1 and X2) and a crop plant (Y), wild plants compete with crop plants for nutrients. There is a group of plant pests (P) that infect crop plants and damage them. Further, wild plants X1 support the population of insects (T1) that feast on the plant pest (P). Wild plants X2 support the dwelling of pollinator insects (T2) that promote pollination in the crop plants and thereby promote crop production. The correct statement is:

A. There is direct negative interaction of Y with X1 and X2.

B. The interaction between T1 and T2 insects is commensalism.

C. Removing wild plants X1 and X2 from this ecosystem will improve overall crop (Y) productivity.

D. The interaction between T1 and crop plants Y is parasitism.

---

13. The forelimbs of whales, bats, cheetahs and humans share similar anatomical

structures and have humerus, radius, ulna, carpals, metacarpals and phalanges. Although this suggests that the forelimb bones in these organisms developed from a common ancestor, they were adapted for different functions. The most appropriate option that captures the evolution of these forelimb bones is:

(A) Divergent and orthologous

(B) Convergent and homologous

(C) Divergent and paralogous

(D) Convergent and orthologous

---

14. In the Tree of Life, different life forms are placed on branches of the tree based on their evolutionary relationship with each other. However, viruses are not included in the tree. The reason for this is:

(A) Viruses do not have a cellular structure, hence drawing an evolutionary relationship with other cellular forms is not possible.

(B) Viruses do not evolve from their ancestors, hence evolutionary relationship cannot be deduced.

(C) Evolution of viruses do not follow the principles of Darwinian evolution.

(D) Viruses infect all domains of life, hence drawing an evolutionary relationship is not possible.

---

15. Chloroplasts of certain algal taxa such as Euglenophyta are surrounded by three membranes while, Heterokontophyta and Cryptophyta have four membranes. The chloroplasts present in these organisms is a result of secondary endosymbiotic events. All of them have chlorophyll a. However, Euglenophyta uses chlorophyll b as an accessory pigment, whereas Heterokontophyta and Cryptophyta use chlorophyll c. Assuming gain-of-function and gene addition, the most likely evolutionary scenario best describing the origins of chloroplast in these organisms is:

(A) While Heterokontophyta and Cryptophyta had a common secondary endosymbiont, Euglenophyta evolved from an independent secondary endosymbiont.

(B) Heterokontophyta, Cryptophyta and Euglenophyta all had a common secondary endosymbiont.
(C) Heterokontophyta, Cryptophyta and Euglenophyta all evolved from independent secondary endosymbiotic events.
(D) Heterokontophyta and Cryptophyta were a result of secondary endosymbiotic event from Euglenophyta.

---

16. Infection with Dengue virus can potentially be fatal when the infection results in leakage of blood from blood vessels. One of the disease management clinical strategies in dengue virus infection is platelet transfusion. Considering that thrombocytopenia, a condition of reduced platelet count in blood, is a consequence of the viral infection, the rationale behind administration of platelets is:
(A) Platelets inhibit immune response against the virus.
(B) Clotting factors from platelets stop internal bleeding.
(C) Platelets produce antibodies against the virus.
(D) Platelets have anti-pyretic activity (reduce fever).

---

17. Scientists have recently found that an archaebacteria belonging to the phylum Lokiarchaeota can be grown only in co-culture along with a Methanobacterium and another aerobic proteobacterium. The archaeon catabolizes amino acids that are secreted out from the Methanobacterium, producing formate. The formate released from the archaeon is utilized by the Methanobacterium as an energy source. The proteobacterium, by scavenging the oxygen in the environment, provides an anaerobic environment for the archaeon. This interaction between the archaea and bacteria could have resulted in the evolution of present-day eukaryotic cell. The interactions between Lokiarchaeon with Methanobacterium and Lokiarcheon/Methanobacterium with proteobacterium, respectively, can be described as:
(A) Mutualism and commensalism

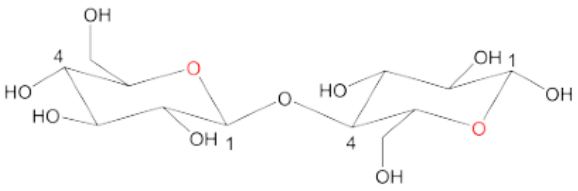
(B) Mutualism and parasitism

(C) Commensalism and mutualism

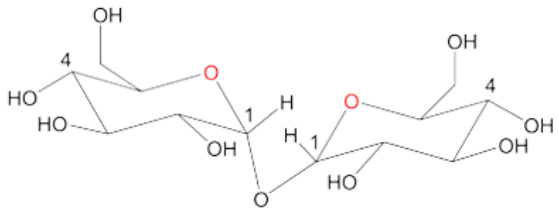
(D) Mutualism and amensalism

---

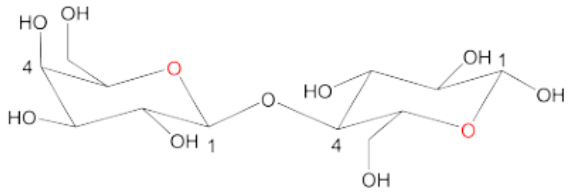
18. Identify the non-reducing disaccharide:



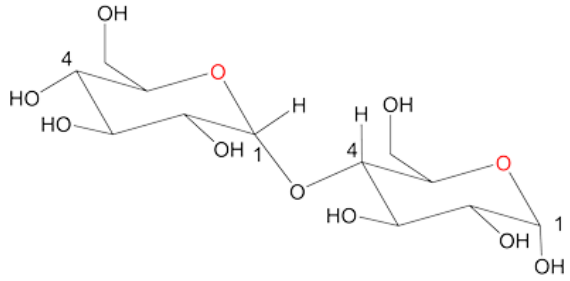
A. Cellobiose (β-D-Glucopyranosyl-(1→ 4)-β-D-glucopyranose)



B. Trehalose (α-D-Glucopyranosyl-(1→ 1)-α-D-glucopyranoside)



C. Lactose (β-D-Galactopyranosyl-(1→ 4)-β-D-glucopyranose)



D. Maltose (α-D-Glucopyranosyl-(1→ 4)-α-D-glucopyranose)

---

19. A researcher found a cottony growth on a plant tissue. On microscopic examination of the sample from there, the researcher observed branched, thread-like structures containing linearly arranged cells with cross walls. Each cell has a distinct nucleus. Staining the cells revealed cell wall structure containing complex polysaccharides. At

the end of some of the branched structures, small spherical structures were seen, which when isolated and placed on another tissue, grew into thread-like structures. Based on these morphological characteristics, the organism can be placed in the broad taxonomical classification of:

(A) Kingdom: Protista, Phylum: Amoebozoa (slime moulds)

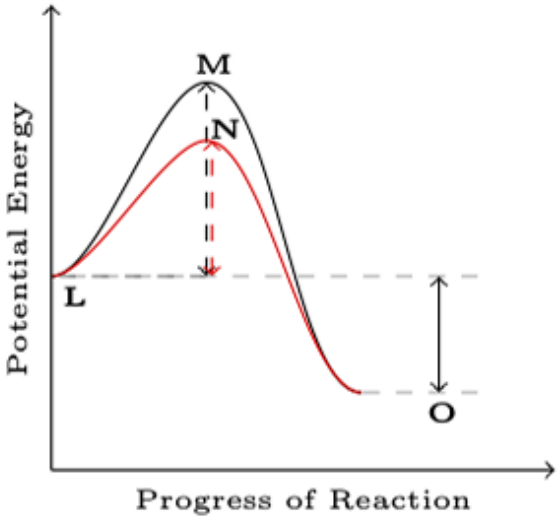
(B) Kingdom: Fungi; Phylum: Ascomycota (sac fungi)

(C) Kingdom: Monera; Phylum: Actinomycetota (filamentous bacteria)

(D) Kingdom: Plantae; Phylum: Charophyta (spirogyra)

---

20. The graph depicts energy diagram (potential energy vs progress of reaction) of an uncatalyzed (black solid line) and enzyme-catalyzed (red solid line) biochemical reaction.



Based on this information, consider the following statements about enzyme catalysis:

1. L, M, N and O represent potential energies of substrate, transition state (TS) in the absence of enzyme, TS in the presence of enzyme, and product, respectively.
2. The rate of the catalyzed reaction increases exponentially to the tune of difference between M and N.
3. The rate of the catalyzed reaction is directly proportional to the energy difference between L and O.

4. L, M, N and O represent potential energies of the substrate, a reaction intermediate, transition state and product of the reaction, respectively.

The correct combination of statements about enzyme catalysis is:

A. i and ii

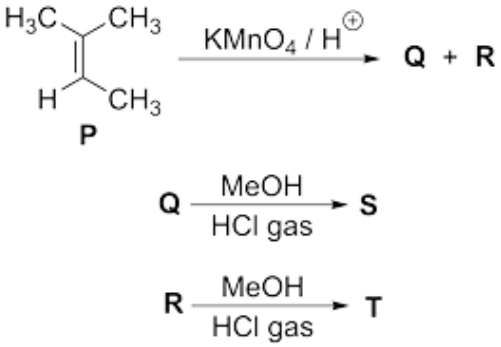
B. ii and iii

C. iii and iv

D. i and iv

CHEMISTRY

1. Consider the following reactions:



In this context, the correct statement is

(a) Q is a ketone and T is an ether.

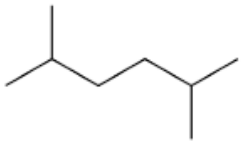
(b) R is an acid and S is a ketal.

(c) R is an aldehyde and S is a ketal.

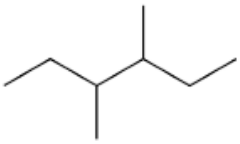
(d) Q is an acid and T is an ester.

---

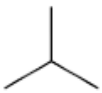
2. Aqueous solution of sodium salt of 2-methylbutanoic acid on Kolbe electrolysis yields P **(major product).** P **is:**



(a)



(b)

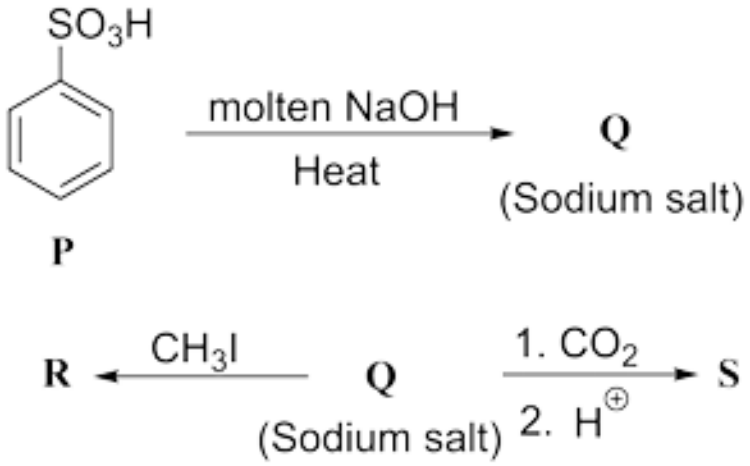


(c)

(d)

---

3. Consider the following reactions:



R and S, respectively, are

(a) Methoxybenzene and 2-Hydroxybenzoic acid

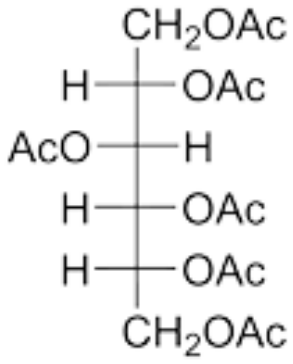
(b) Methylbenzene and Benzoic acid

(c) Methoxybenzene and Benzoic acid

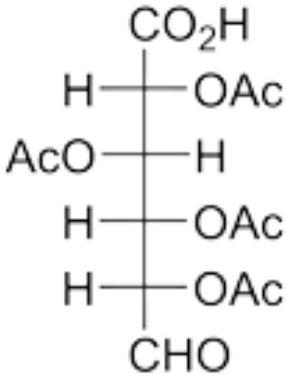
(d) 2-Hydroxy-methylbenzene and 2-Hydroxybenzoic acid

---

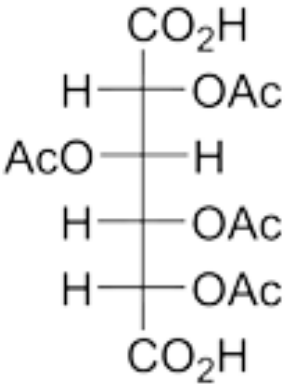
4. Glucose on treatment with bromine water yields X. X upon treatment with excess of acetic anhydride yields Y. The structure of Y is



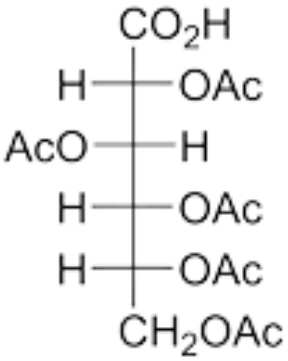
(a)



(b)



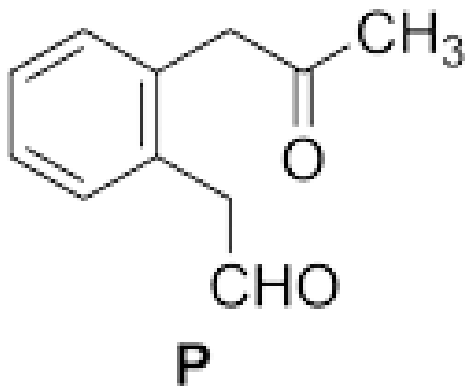
(c)



(d)

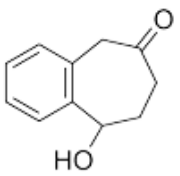
---

5. An intramolecular aldol reaction of P in the presence of dilute NaOH yields a mixture of aldol reaction products.

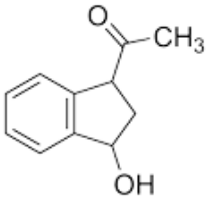


P

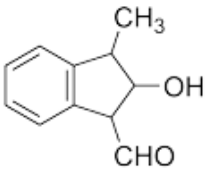
One of the products that can be formed is



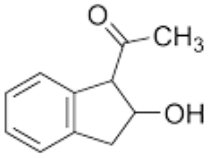
(a)



(b)



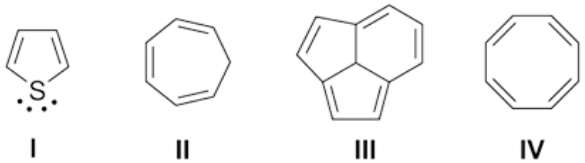
(c)



(d)

---

6. Consider the following compounds.



I II III IV

Among them, the compounds that show aromaticity are

(a) I and IV

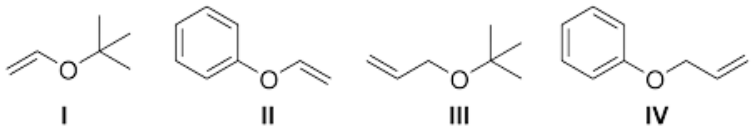
(b) II and III

(c) III and IV

(d) I and III

---

7. Consider the following ethers.



Among them, the ethers that can be synthesized under mild conditions using Williamson's ether synthesis are

(a) I and II

(b) II and III

(c) II and IV

(d) III and IV

---

8. Consider the following reaction

$$x\mathrm{MnO_4^-} + y\mathrm{I^-} + z\mathrm{H^+} \rightarrow p\mathrm{Mn^{2+}} + q\mathrm{IO_3^-} + r\mathrm{H_2O}$$

The correct ratio $x : y$ in the balanced equation is

(a) 6:5

(b) 6:4

(c) 1:1

(d) 5:4

---

9. The spin only magnetic moment of a manganese (atomic number 25) amine complex is 2.83 BM. The oxidation state of Mn in the complex is

(a) +2

(b) +3

(c) +4

(d) +5

---

10. Consider the radioactive decay, $X(radioactive) \rightarrow Y(stable)$. At time $t = 0$, X is present in the pure form and at time $t = 2$ h, the ratio of amounts of X and Y is 1:3. The duration required for the ratio to become 1:15 is

(a) 10 h

(b) 8 h

(c) 6 h

(d) 4 h

---

11. The correct statement regarding chromate and dichromate is

(A) The oxidation states of chromium in chromate and dichromate are, respectively, +4 and +6

(B) Dichromate on reaction with concentrated H_2SO_4 gives chromate

(C) All chromium-oxygen bond distances in dichromate are equal

(D) Total number of electrons involved in the complete reduction of one molecule of dichromate to chromium(III) is six

---

12. Among the elements Cr, Mn, Cu, and Zn, the one having the highest second ionization energy is

(A) Cr

(B) Mn

(C) Cu

(D) Zn

---

13. The compound having highest covalent character in the metal-chlorine bond is

(A) NaCl

(B) $MgCl_2$

(C) $AlCl_3$

(D) $TiCl_4$

---

14. The correct order of basicity is:

(A) $Al(OH)_3 > Ca(OH)_2 > Lu(OH)_3 > La(OH)_3$

(B) $Ca(OH)_2 > La(OH)_3 > Lu(OH)_3 > Al(OH)_3$

(C) $Lu(OH)_3 > La(OH)_3 > Ca(OH)_2 > Al(OH)_3$

(D) $La(OH)_3 > Ca(OH)_2 > Al(OH)_3 > Lu(OH)_3$

---

15. Certain amount (say, n moles) of a monoatomic ideal gas ($C_v = \frac{3}{2}R$) of volume V_1, temperature T_1, and pressure P_1 is expanded against a constant external pressure P_2 until the pressure of the gas becomes P_2. The correct statement is:

(a) If the expansion is carried out adiabatically, the work done is given by $w = \frac{3}{2}V_1(P_2 - P_1)$.

(b) If the expansion is carried out adiabatically, the entropy change is given by $\Delta S = 0$.

(c) If the expansion is carried out isothermally, the work done is given by $w = -nRT_1 \ln\left(\frac{P_1}{P_2}\right)$.

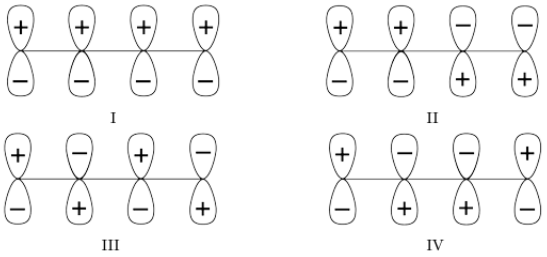
(d) If the expansion is carried out isothermally, the entropy change is given by $\Delta S = -nR \ln\left(\frac{P_1}{P_2}\right)$.

---

17. Consider two sparingly soluble salts PQ and RS_2. At temperature T_0, the solubility products of these two salts are found to be equal. At a higher temperature T_1 ($T_1 > T_0$), the solubility of PQ becomes triple of its solubility at T_0. Also, at another temperature T_2 ($T_2 > T_0$), the solubility of RS_2 becomes double of its solubility at T_0. The ratio of solubility products of PQ at T_1 and RS_2 at T_2 is given by

(a) 1 (b) 9/8 (c) 9/32 (d) 3/2

18. Consider the following molecular orbitals that are formed by sidewise (or lateral) overlap of adjacent 2p atomic orbitals of carbons in a conjugated aliphatic chain.



The correct order of energies of the molecular orbitals is

(a) $IV < II = III < I$

(b) $III < IV < II < I$

(c) $II < IV < III < I$

(d) $IV = II = III < I$

---

19. The hydrogen spectrum consists of an infinite number of spectral lines grouped into different series. A particular series arises from the electronic transitions between various higher energy levels and a particular destination (lower) energy level specific to that particular series. Examples include Lyman series, Balmer series, Paschen series etc. There exists an infinite number of such series and let the lowest frequency of the i^{th} series be denoted by $\nu_{\text{min}}^{(i)}$. The quantity $\sum_{i=1}^{\infty} \nu_{\text{min}}^{(i)}$ expressed in terms of the ground state energy of the hydrogen atom, E_H and the Planck's constant h is

(a) $-E_H/h) \sum_{n=1}^{\infty} \frac{1}{n^2}$

(b) $-E_H/h$

(c) $-E_H$

(d) Infinity

---

20. Consider the first order reaction $\mathbf{X}(g) \rightarrow 2\mathbf{Y}(g) + 2\mathbf{Z}(g)$, involving ideal gases X, Y, and Z inside a reaction chamber of fixed volume and at constant temperature. The

initial ($t = 0$) pressure is 1 atm and after 30 min the total pressure increases to 2.5 atm. The fraction of unreacted X at $t = 1$ h is

(a) 12.5%

(b) 25%

(c) 50%

(d) 75%

Mathematics

1. For positive real numbers a and b, the number of solutions of $a \times 887 - b = 0$ in $\mathbb{R}$ is

(A) 887

(B) 443

(C) 1

(D) 0

---

2. The number of pairs (x, y) of rational numbers such that $x + y = xy$ is

(A) 2

(B) 1

(C) 0

(D) ∞

---

3. The number of positive integers n such that the equation $n^2X^2 + 2024X + (n^2 + 1) = 0$ has a solution in $\mathbb{R}$ is

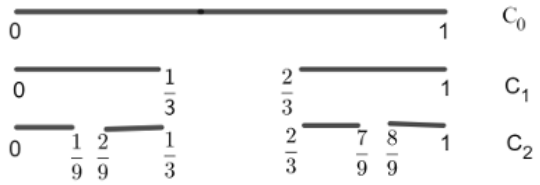
(A) 31

(B) 5

(C) 1012

(D) between 1012 and 2024

4. Consider the closed interval $[0,1]$ and label it as C_0. In the next step, divide the interval $[0,1]$ into three parts $[0,\frac{1}{3}], [\frac{1}{3},\frac{2}{3}], [\frac{2}{3},1]$ and delete the middle interval $(\frac{1}{3},\frac{2}{3})$. Let C_1 be the union of the two intervals $[0,\frac{1}{3}]$ and $[\frac{2}{3},1]$. In the next step, consider the two intervals in C_1, divide each of them into three parts of length $\frac{1}{9}$ and delete the middle intervals $(\frac{1}{9},\frac{2}{9})$ and $(\frac{7}{9},\frac{8}{9})$. The union of the remaining intervals is labelled as C_2. Continuing in this way, the sets C_n are constructed for any $n \in \mathbb{N}$.



Then the sum of the lengths of the intervals in C_4 is

A. $\frac{1}{81}$

B. $\frac{2}{81}$

C. $\frac{14}{81}$

D. $\frac{16}{81}$

---

5. Let S be the set of all functions $f : \mathbb{Z} \to \mathbb{C}$ such that $f(m+n) = f(m)f(n)$ for all $m, n \in \mathbb{Z}$ and $f(0) = 1$. Then

A. there exists $f \in S$ such that $f(-1) = 0$

B. there exists $f \in S$ such that $f(1) = 2 + 3i$

C. there exists $f \in S$ such that $f(1) = 0$

D. $f(1) = 1$ for every $f \in S$

---

6. The number of points x in $\mathbb{R}$ where the function $f(x) = \cos x + \cos(\sqrt{11}x)$ attains its maximum is

A. 0

B. 2

C. 1

D. ∞

---

7. Let P and Q be the points of intersection of the circles

$$C_1 : x^2 + y^2 + 2x - y = 0$$

and

$$C_2 : x^2 + y^2 + 3x + 2y - 3 = 0.$$

Let R be the point $(1, 2)$. The distance between the point R and the line PQ is

(A) $\sqrt{2}$

(B) $\frac{5}{\sqrt{3}}$

(C) $\frac{8}{\sqrt{3}}$

(D) $\frac{4}{\sqrt{10}}$

---

8. Let $n \in \mathbb{N}$ and let $f(x) = x^n$ for all $x \in \mathbb{R}$. Then

$$f(1) + \frac{f^{(1)}(1)}{1!} + \frac{f^{(2)}(1)}{2!} + \cdots + \frac{f^{(n)}(1)}{n!}$$

is equal to

(A) 1

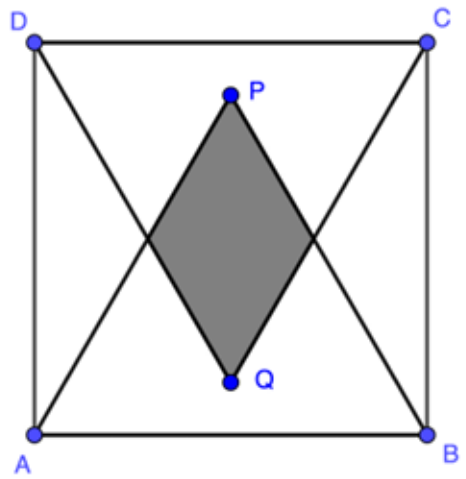
(B) 2^{n-1}

(C) $(n + 1)2^n$

(D) 2^n

---

9. Let $ABCD$ be a square with sides of unit length. Let P and Q be the points in the square such that ABP and CDQ are equilateral triangles.



Then the area of the intersection of the triangles ABP and CDQ (shaded region) is

A. $\frac{2}{\sqrt{3}} - 1$

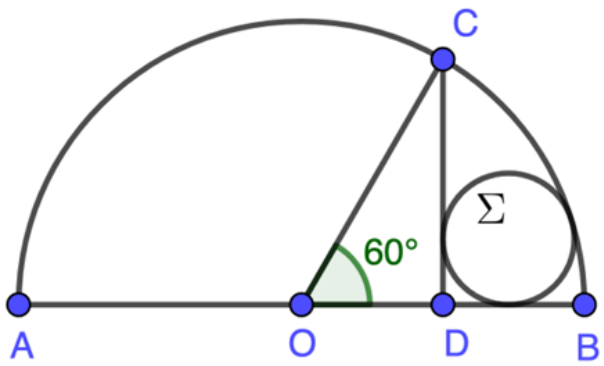
B. Less than $\frac{1}{8}$

C. $\frac{15}{112}$

D. $1 - \frac{1}{\sqrt{3}}$

---

10. Let AB be the diameter of a circle with radius 1 centred at the point O. Let C be a point on the circle such that $\angle BOC = 60°$. Let D be the foot of the perpendicular from C to AB. Let Σ denote the circle that touches segment DB, segment DC and the shorter arc of the circle with endpoints B and C as shown in the figure given below.



Then the radius of the circle Σ is

A. $\sqrt{3} - (3/2)$

B. $\sqrt{3}/8$

C. $(2+\sqrt{3})/16$

D. $(\sqrt{3}-1)/3$

---

11. Let $I = \int_0^1 \frac{x^7}{\sqrt[3]{1+x^8}} dx$. Then

A. $-\frac{1}{4} < I < 0$

B. $0 < I < \frac{1}{8}$

C. $\frac{1}{8} < I < \frac{1}{4}$

D. $\frac{1}{4} < I < \frac{3}{8}$

---

12. Let A and B be two 3×3 matrices such that $AB = BA$ and $A^2 - 5AB + 4B^2 = 0$, where 0 is the zero matrix. Then

A. $A = B$ or $A = 4B$

B. $A - B$ is singular

C. $A - 4B$ is singular

D. A and B are nonsingular if any one of $A - B$ or $A - 4B$ is nonsingular

---

13. The ordinary differential equation

$$y'' - y = 0$$

has A. a unique solution

B. two distinct solutions

C. a non-zero decreasing solution

D. a non-zero periodic solution

---

14. Let $f : [0, 2\pi] \to \mathbb{R}$ be a differentiable, strictly increasing function such that $f(0) < 0 < f(2\pi)$. Let $F(t) = \int_0^t f(x)\,dx$ for all $t \in [0, 2\pi]$. Then

A. F is an increasing function on $[0, 2\pi]$

B. F is a decreasing function on $[0, 2\pi]$

C. F has a minimum in the interval $(0, 2\pi)$

D. F has a maximum in the interval $(0, 2\pi)$

---

15. Let $a > 0$ and f, g be continuous functions on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 3$. Then $\int_0^a f(x)g(x)\,dx$ is

A. $3a$

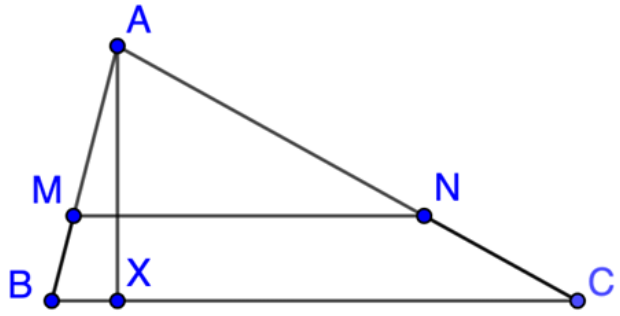
B. $\int_0^a g(x)\,dx$

C. $\int_0^a f(x)\,dx$

D. $\frac{3}{2}\int_0^a f(x)\,dx$

---

16. Let A, B, C be three non-collinear points. Let M and N be points on segments AB and AC respectively such that line MN is parallel to line BC. Suppose that $|MB| = 1$, $|MN| = 3$, $|NC| = 2$, and $|BC| = 5$. Let X be the foot of the perpendicular from A to the side BC. Then the length of the segment AX is:



Then the length of the segment AX is

(A) $1 + \sqrt{2}$

(B) $\frac{5\sqrt{15}}{8}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\dfrac{\sqrt{5}+\sqrt{6}}{2}$

---

17. Let N be the total number of integer solutions to the equation $X_1 + X_2 + X_3 = 30$ where each X_i, $i = 1, 2, 3$, is non-negative and not divisible by 3. Then N is equal to:

(A) 100

(B) 244

(C) 331

(D) 402

---

18. Let R be the equivalence relation on $\mathbb{Z}^2$ defined by $(x_1, y_1)R(x_2, y_2)$ if $2|x_1| + 3|y_1| = 2|x_2| + 3|y_2|$. Then

A. each equivalence class with more than one element has at least 4 elements

B. there are infinitely many equivalence classes with less than 4 elements

C. there exists an equivalence class containing at least 1000 elements

D. there exist equivalence classes with infinitely many elements

---

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying $f' = f$ and $f(0) = 1$. Let $g : \mathbb{R} \to \mathbb{R}$ be the function that satisfies $f(x) + g(x) = x^3$, for all $x \in \mathbb{R}$. Then $\int_0^1 f(x)g(x)\,dx$ is equal to:

(1) $\frac{13}{2} - e + \frac{e^2}{2}$

(2) $\frac{13}{2} - 2e - \frac{e^2}{2}$

(3) $\frac{e^2}{2}$

(4) $e + \frac{e^2}{2}$

---

20. A man has two homes– one in city A and another in city B. He travels between these two cities by flights. The only flight from city A to city B is scheduled at 6:00 AM every

day and the only flight from city B to city A is scheduled at 6:00 PM every day. If the man is in city A at 6:00 AM, the probability that he will take the flight to city B is $\frac{1}{2}$. If he is in city B at 6:00 PM, the probability that he will take the flight to city A is $\frac{2}{3}$. (He may travel twice in a day. So he may travel from city A to city B in the morning and return in the evening.) Suppose he is in city A at 6:00 AM on Monday. The probability that he will be in city B at 5:30 AM on Thursday is:

(A) $\frac{35}{216}$

(B) $\frac{37}{216}$

(C) $\frac{41}{216}$

(D) $\frac{43}{216}$

Physics

1. A particle of unit mass is released from origin with a velocity $v = v_0\hat{i}$ inside a potential well. The potential energy associated with this well is of the form $\varphi(x) = ax^2 \exp(-bx^2)$, where a and b are equal to unity in SI units. The minimum value of v_0, in SI units, for the particle to cross the potential well is closest to:

(A) 0.85

(B) 0.75

(C) 0.95

(D) 0.65

---

2. A body of mass m is subjected to a position-dependent force $F = -k_1x - k_3x^3$, where k_1 and k_3 are positive constants. The motion is bounded with a maximum amplitude A. The motion is analyzed by neglecting the cubic term. Then, the most appropriate condition to do this is:

(A) $k_3 \ll 1$

(B) $\frac{k_3 A}{k_1} \ll 1$

(C) $k_3 \ll k_1$

(D) $\frac{k_3 A^2}{k_1} \ll 1$

---

3. A rigid body is rotating with a constant angular speed of 3 rad.s^{-1} about a fixed axis passing through the points A and B with coordinates (0, 1, 1) and (1, 1, 3) respectively. Assuming all quantities in S.I. units, the instantaneous speed of the point P of the body with coordinates (4, 6, 7) is:

(A) 3

(B) 2

(C) 6

(D) 4

---

4. A planet of mass m is in uniform circular motion about a star of mass M ($M \gg m$). Let $\vec{v}$, $\vec{L}$, $\vec{r}$ be the velocity, angular momentum, and position vector, respectively of the planet about the star. We define a vector $\vec{A} = \vec{v} \times \vec{L}$. Then:

(A) $\vec{A}$ is a zero vector.

(B) the magnitude of $\vec{A}$ is $\frac{GM^2}{m}$.

(C) $\vec{A}$ is in a plane perpendicular to the plane of the planet's orbit.

(D) $\vec{A}$ is along $\vec{r}$.

---

5. A point particle of mass M is oscillating inside a one-dimensional potential energy well of the form $\phi(x) = k|x|$ where k is a positive constant. The particle was initially released from $x = x_0$ with zero initial speed. Then, the time period of oscillation of the particle is

A. $\sqrt{\frac{32Mx_0}{k}}$

B. $\sqrt{\frac{16Mx_0}{k}}$

C. $\sqrt{\frac{8Mx_0}{k}}$

D. $\sqrt{\frac{4Mx_0}{k}}$

---

6. A solid object is dropped from height H under the influence of gravity. The object experiences a drag force whose magnitude is proportional to the square of its instantaneous speed in air. Assume that drag force is negligible compared to gravitational force but contributes exclusively towards the heating of the object. The rise in temperature of the object just before it reaches the ground is proportional to:

(A) H

(B) H^2

(C) H^3

(D) $H^{3/2}$

---

7. A metal wire has radius r and carries a constant current I. It is thermally insulated and its resistivity is temperature independent. The coefficient of thermal expansion is negligible. The rise in temperature of the wire per unit time is found to be proportional to r^n. Here n is:

(A) 0

(B) -2

(C) -4

(D) -6

---

8. An ideal gas inside a container is heated such that its internal energy changes at a slow constant rate K. Due to a leak in the container, the density of gas inside the

container decreases at a slow constant rate Γ. Then, the rate of change of pressure inside the container must:

(A) depend only on Γ.

(B) depend only on K.

(C) depend on both K and Γ.

(D) be zero.

---

9. In an adiabatic compression of a monoatomic ideal gas, the pressure increases to k times the original pressure. Let d_1 be a typical inter-particle distance of the gas before the compression and d_2 be the corresponding value after compression. If the ratio d_1/d_2 is expressed as k^α, then the value of α is:

(A) 0.22

(B) 0.20

(C) 0.26

(D) 0.24

---

10. A solid sphere of uniform density and radius R cools radiatively with a time constant τ as per the following expression: $T(t) = T_s + (T_h - T_s)e^{-t/\tau}$, where T_h is the initial temperature of the object and T_s is the surrounding temperature. Then,

(A) $\tau \propto R^2$

(B) $\tau \propto R$

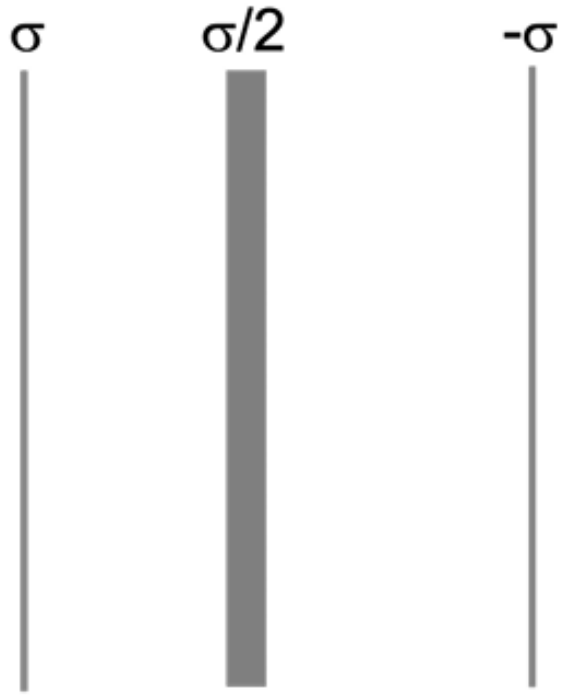
(C) $\tau \propto R^3$

(D) $\tau \propto R^0$

---

11. A large conducting slab having uniform surface charge density is placed between a parallel plate capacitor as shown in the figure. The net surface charge density on the

slab after its placement in between the capacitor plates is $+\sigma/2$. The surface charge density on capacitor plates are $\pm\sigma$. Assuming equal cross-sectional area of capacitor plates and the slab, the induced surface charge density on the conducting slab surface facing the positive terminal of the capacitor is



A. $-\sigma$

B. $-5\sigma/4$

C. $-\sigma/4$

D. $-3\sigma/4$

---

12. A quarter circular loop of radius R and carrying current I is placed in the xz-plane as shown in the figure. The y-component of magnetic field at $(0, R, 0)$ is

A. $\mu_0 I/(16\sqrt{2}R)$

B. $\mu_0 I/(4\sqrt{2}R)$

C. $\mu_0 I/(8\sqrt{2}R)$

D. 0

---

13. The Balmer series of hydrogenic spectral lines refers to an electron transitioning from $n \geq 3$ to $n = 2$, where n is the principal quantum number. A hydrogenic atom with atomic number $Z = 24$ undergoes a Balmer transition of the largest possible wavelength. The emitted photon has energy E_0. Another hydrogenic atom with atomic number $Z = 25$ undergoes a similar Balmer transition with the energy of the emitted photon being E_1. Then $|E_1 - E_0|$, in eV, is closest to:

(A) 9

(B) 27

(C) 54

(D) 90

---

14. A muon has the same charge as the electron and its mass is about 200 times larger than electron mass. It decays in 2×10^{-6} seconds. A muonic hydrogen atom is a bound state of a proton and a muon. The number of revolutions done by the muon in the innermost orbit before it decays is closest to:

(A) 10^{12}

(B) 10^{15}

(C) 10^{16}

(D) 10^{18}

---

15. Select the correct statement regarding the Thompson and Rutherford models of the atom:

(A) The atom in Thompson model is stable.

(B) The atom in Rutherford model is stable.

(C) Using the Thompson model one cannot arrive at the approximate size of the atom.

(D) Using the Rutherford model one can arrive at the approximate size of the atom.

---

16. Consider a linear object of height h placed in front of a convex lens of focal length f which gives rise to an image of height $h_1 = 4.00$ cm on the screen placed at a distance $l = 10.00$ cm from the object. Then the lens is shifted towards the screen by a distance $d = 6.00$ cm and again an image of height $h_2 = 9.00$ cm is observed on the same screen. The ratio of the height to the focal length $\frac{h}{f}$ is:

(A) 3.00

(B) 3.75

(C) 4.00

(D) 4.50

---

17. A powerful point light source is placed at the bottom of a large pool containing water. The depth of the pool is 5 m and the refractive index of the water is 1.33. The area (in m^2) illuminated as viewed from the top is closest to:

(A) 30

(B) 100

(C) 60

(D) 120

---

18. The minimum distance, in cm, between an object and its real image through a convex lens of focal length 10 cm is:

(A) 10

(B) 15

(C) 20

(D) 40

---

19. Consider a monoatomic ideal gas with number of atoms per unit volume, n, at absolute temperature T. The mass of each atom is m. In describing the physics of the system, the wave nature of atoms can be ignored if A. $h/\sqrt{mk_BT} \ll 1$

B. $nh/\sqrt{mk_BT} \ll 1$

C. $n^{1/3}h/\sqrt{mk_BT} \ll 1$

D. $n^{1/2}h/\sqrt{mk_BT} \ll 1$

---

20. Two semi-infinite wires are placed along x and z axes, respectively, as shown in the figure. A square loop of side length a is kept such that its center is at a distance $2a$ from the origin in the yz-plane. If a current $I = kt$ flows in the wires (where t is time and k is constant), then the magnitude of induced emf in the loop is

A. $\mu_0 ka/4\pi$

B. $\mu_0 ka \ln(5/3)/4\pi$

C. 0

D. $\mu_0 ka \ln(3/2)/2\pi$
