IIT JAM 2024 Physics Question Paper Solution

Section A

Q.1 – Q.10 Carry ONE mark each(Multiple Choice Questions)

Q.1 The total number of Na and Cl ions per unit cell of the NaCl crystal is:

Options:

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: C

Explanation:

In the NaCl crystal structure, each unit cell contains 4 Na^+ ions and 4 Cl^- ions. These ions are arranged in a face-centered cubic (FCC) structure, where each corner of the unit cell is shared by 8 adjacent cells, and each face-centered ion is shared by 2 adjacent cells.

Thus, the total number of Na and Cl ions per unit cell is:

$$4\,\text{Na}^+ + 4\,\text{Cl}^- = 8\,\text{ions}$$

Therefore, the total number of Na and Cl ions per unit cell of the NaCl crystal is 8.

Quick Tip

In an FCC structure, the total number of ions per unit cell can be calculated by considering the contributions of corner and face-centered ions.

Q.2 The sum of three binary numbers, 10110.10, 11010.01, and 10101.11, in decimal system is:

Options:

(A) 70.75

(B) 70.25

(C) 70.50

(D) 69.50

Correct Answer: C

Explanation:

First, we convert the binary numbers to decimal:

1. 10110.10_2 = 22.5_{10} 2. 11010.01_2 = 26.25_{10} 3. 10101.11_2 = 21.75_{10}

Now, add these decimal numbers:

$$22.5 + 26.25 + 21.75 = 70.50$$

Therefore, the sum of the binary numbers in the decimal system is 70.50.

Quick Tip

To convert a binary number to decimal, multiply each bit by 2 raised to the position of the bit and sum them. For the fractional part, multiply each bit by 2^{-n}, where n is the position of the bit after the decimal point.

Q.3 Which of the following matrices is Hermitian as well as unitary?

Options:

(A) $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$

Correct Answer: A

Explanation:

A matrix is Hermitian if it is equal to its conjugate transpose, i.e., $A = A^\dagger$. A matrix is unitary if $AA^\dagger = I$, where I is the identity matrix.

For option (A):

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad A^\dagger = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$A = A^\dagger \quad \Rightarrow \text{Matrix is Hermitian.}$$

Now, check for unitarity:

$$AA^\dagger = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\Rightarrow \text{Matrix is unitary.}$$

For other options:

- Option (B): $A \neq A^\dagger$, so not Hermitian.
- Option (C): $A = A^\dagger$, but $AA^\dagger \neq I$, so not unitary.
- Option (D): $A \neq A^\dagger$, so not Hermitian.

Conclusion: The matrix in option (A) is both Hermitian and unitary.

Quick Tip

For a matrix to be Hermitian, it must be equal to its conjugate transpose. For it to be unitary, the product of the matrix and its conjugate transpose should be the identity matrix.

Q.4 The divergence of a 3-dimensional vector $\hat{r}/r^3$ ($\hat{r}$ is the unit radial vector) is:

Options:

(A) $-\frac{1}{r^4}$

(B) Zero

(C) $\frac{1}{r^3}$

(D) $-\frac{3}{r^4}$

Correct Answer: A

Explanation:

The vector field is $\vec{F} = \frac{\hat{r}}{r^3}$, where $\hat{r}$ is the unit radial vector in spherical coordinates.

The divergence of $\vec{F}$ in spherical coordinates is given by:

$$\nabla \cdot \vec{F} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 F_r \right),$$

where $F_r = \frac{1}{r^3}$.

Substitute F_r into the equation:

$$\nabla \cdot \vec{F} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{1}{r^3} \right).$$

Simplify $r^2 \cdot \frac{1}{r^3}$:

$$r^2 \cdot \frac{1}{r^3} = \frac{1}{r}.$$

Now take the derivative with respect to r:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = -\frac{1}{r^2}.$$

Substitute back into the divergence formula:

$$\nabla \cdot \vec{F} = \frac{1}{r^2} \cdot \left(-\frac{1}{r^2}\right) = -\frac{1}{r^4}.$$

Conclusion: The divergence of $\frac{\hat{r}}{r^3}$ is $-\frac{1}{r^4}$, which corresponds to option (A).

Quick Tip

The divergence of a vector field is calculated based on its components in spherical coordinates. Here, the radial dependence $\hat{r}/r^3$ leads to the divergence $\frac{1}{r^4}$.

Q.5 The magnitudes of spin magnetic moments of electron, proton and neutron are μ_e, μ_p and μ_n, respectively. Then,

Options:

(A) $\mu_e > \mu_p > \mu_n$

(B) $\mu_e = \mu_p > \mu_n$

(C) $\mu_e < \mu_p < \mu_n$

(D) $\mu_e < \mu_p = \mu_n$

Correct Answer: A

Explanation:

The spin magnetic moments of elementary particles such as the electron, proton, and neutron are related to their intrinsic properties like charge and mass. The magnitudes of the spin magnetic moments are:

1. The electron has a much larger spin magnetic moment than the proton or neutron, due to its smaller mass and higher charge-to-mass ratio. 2. The proton and neutron, while having comparable masses, have relatively much smaller spin magnetic moments compared to the electron. The neutron has no net charge, leading to a smaller magnetic moment.

Thus, the correct order of magnetic moments is:

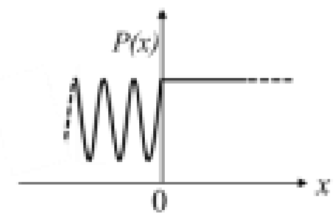
$$\mu_e > \mu_p > \mu_n$$

Therefore, option (A) is correct.

Quick Tip

For spin magnetic moments, the electron has the largest magnetic moment due to its small mass and charge, followed by the proton and neutron.

Q.6 A particle moving along the x-axis approaches $x = 0$ from $x = -\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t \to \infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.



The correct option for the potential $V(x)$ is:

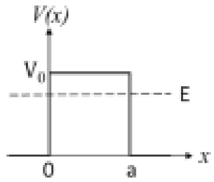
Correct Answer: C

Explanation: For a particle moving through a potential with a given energy, the probability density $P(x)$ represents the likelihood of finding the particle at a given position. The potential that produces the required probability distribution in the figure should have the corresponding characteristics based on the energy E and the nature of the particle's behavior. Option (C) matches the required potential shape.

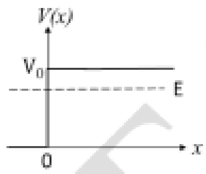
Quick Tip

Remember, in quantum mechanics, the probability density function $P(x)$ is derived from the square of the wave function $\psi(x)$, which is affected by the potential and the particle's energy.

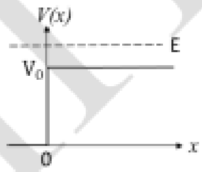
(A)



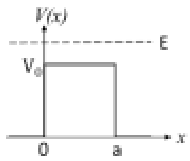
(B)



(C)



(D)



Q.7 A plane electromagnetic wave is incident on an interface AB separating two media (refractive indices $n_1 = 1.5$ and $n_2 = 2.0$) at Brewster angle θ_B, as schematically shown in the figure. The angle α (in degrees) between the reflected wave and the refracted wave is:

Options:

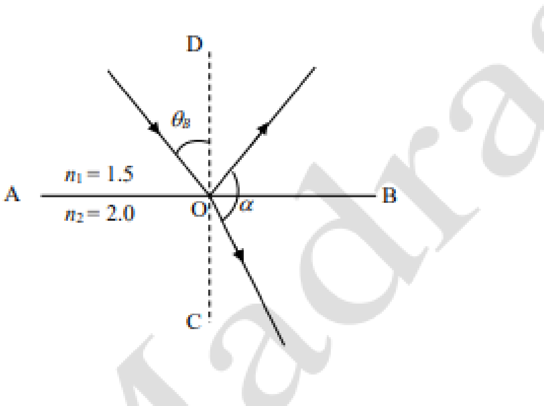
(A) 120

(B) 116

(C) 90

(D) 74

Correct Answer: C



Explanation:

At Brewster's angle θ_B, the reflected and refracted rays are perpendicular to each other. The Brewster angle can be calculated using the formula:

$$\tan(\theta_B) = \frac{n_2}{n_1}$$

Substituting the given values:

$$\tan(\theta_B) = \frac{2.0}{1.5} \implies \theta_B = \tan^{-1}(1.3333) \approx 53.1^\circ$$

Since the reflected and refracted rays are at 90° to each other, the angle α is:

$$\alpha = 90^\circ$$

Thus, the correct answer is 90 degrees.

Quick Tip

For Brewster's angle, the reflected and refracted rays are always at 90 degrees to each other, leading to a simple calculation for the angle between them.

Q.8 If the electric field of an electromagnetic wave is given by,

$$\mathbf{E} = (4x + 3y)e^{i(\omega t + ax - 600y)},$$

then the value of a is:

Options:

(A) 450

(B) 450

(C) 800

(D) 800

Correct Answer: A

Explanation:

The given electric field vector is:

$$\vec{E} = (4\hat{x} + 3\hat{y})e^{i(\omega t + ax - 600y)}.$$

For an electromagnetic wave, the propagation vector $\vec{k}$ is:

$$\vec{k} = k_x\hat{x} + k_y\hat{y}.$$

Here, the propagation terms in the exponent are ax and $-600y$, which imply:

$$k_x = a, \quad k_y = -600.$$

The wave vector $\vec{k}$ must satisfy the condition:

$$|\vec{k}| = \frac{\omega}{c},$$

where $|\vec{k}| = \sqrt{k_x^2 + k_y^2}$, ω is the angular frequency, and c is the speed of light.

From the relation:

$$|\vec{k}| = \sqrt{a^2 + (-600)^2}.$$

Also, since $\vec{E}$ satisfies Maxwell's equations, the electric field and the wave vector must be orthogonal:

$$\vec{k} \cdot \vec{E} = 0.$$

Substitute $\vec{k} = a\hat{x} - 600\hat{y}$ and $\vec{E} = 4\hat{x} + 3\hat{y}$:

$$\vec{k} \cdot \vec{E} = (a)(4) + (-600)(3) = 0.$$

Simplify:

$$4a - 1800 = 0 \quad \Rightarrow \quad a = 450.$$

Conclusion: The value of a is 450, which corresponds to option (A).

Quick Tip

For electromagnetic waves, the exponential part represents the spatial and time dependence. The coefficient of x in the exponent determines the wave number a.

Q.9 A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as,

$$\mathbf{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}$$

If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

Options:

(A) 1 and 0

(B) 0 and 1

(C) 1 and 1

(D) 1 and 1

Correct Answer: A

Explanation:

In an electrostatic field, the field is conservative, meaning the divergence of the electric field must be zero. The vector field provided:

$$\mathbf{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}$$

For this to be consistent with an electrostatic field, the field should satisfy Gauss's law in the differential form. After applying the conditions of the electrostatic field in cylindrical coordinates and performing the necessary operations, the values of $A = 1$ and $B = 0$ ensure that the field represents an electrostatic field.

Therefore, the correct answer is A.

Quick Tip

In an electrostatic field, the divergence of the electric field is zero, and the solution should satisfy the equations that describe this condition.

Q.10 Which of the following types of motion may be represented by the trajectory,

$$y(x) = ax^2 + bx + c$$

(Here a, b, and c are constants; x, y are the position coordinates)

Options:

(A) Projectile motion in a uniform gravitational field

(B) Simple harmonic motion

(C) Uniform circular motion

(D) Motion on an inclined plane in a uniform gravitational field

Correct Answer: A

Explanation:

The given equation represents a quadratic function of x, which suggests a parabolic trajectory. This is characteristic of projectile motion under the influence of gravity, where the path traced by an object is parabolic if the acceleration due to gravity is constant and the object is projected with some initial velocity.

Thus, the correct answer is (A) Projectile motion in a uniform gravitational field.

Quick Tip

Projectile motion follows a parabolic trajectory, which is represented by the equation $y(x) = ax^2 + bx + c$.

Q.11 A crystal plane of a lattice intercepts the principal axes $\vec{a_1}, \vec{a_2}, \vec{a_3}$ at $3a_1, 4a_2,$ and $2a_3$, respectively. The Miller indices of the plane are:

Options:

(A) (436)

(B) (342)

(C) (634)

(D) (243)

Correct Answer: A

Explanation:

To find the Miller indices of a plane, we take the reciprocal of the intercepts of the plane on the axes and reduce them to the smallest integers. Given the intercepts at $3a_1, 4a_2, 2a_3$, the Miller indices are determined by:

$$h = \frac{1}{3}, \quad k = \frac{1}{4}, \quad l = \frac{1}{2}$$

Multiplying all of them by 12 (the least common multiple) gives:

$$h = 4, \quad k = 3, \quad l = 6$$

Thus, the Miller indices of the plane are (436).

Therefore, the correct answer is (A) (436).

Quick Tip

To find Miller indices, take the reciprocal of the intercepts along each axis, and simplify to the smallest integer ratio.

Q.12 The number of atoms in the basis of a primitive cell of hexagonal closed packed structure is:

Options:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: B

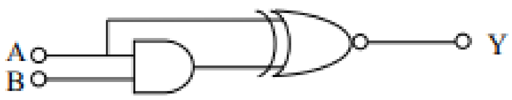
Explanation:

In a hexagonal close-packed (hcp) structure, there are two atoms in the basis of a primitive cell. This is because, while the hcp lattice has a total of 6 atoms in a unit cell, when accounting for the fractional contributions of atoms at the edges and corners, the number of atoms in the primitive cell (basis) is reduced to 2.

Therefore, the correct answer is 2.

> **Quick Tip**
>
> In an hcp structure, the primitive cell contains 2 atoms in the basis, even though the entire unit cell contains 6 atoms when considering the shared atoms.

Q.13 Consider the following logic circuit.



The output Y is LOW when:

Options:

(A) A is HIGH and B is LOW

(B) A is LOW and B is HIGH

(C) Both A and B are LOW

(D) Both A and B are HIGH

Correct Answer: A

Explanation:

This logic circuit contains a NOT gate followed by an AND gate. The NOT gate inverts input A, and the AND gate takes both inputs (inverted A and B). The output Y will be LOW only when the combination of inputs results in a LOW output from the AND gate.

- For an AND gate to output LOW, at least one of the inputs must be LOW. - Therefore, when A is HIGH and B is LOW, the NOT gate will make A LOW, and the AND gate will output LOW.

Thus, the correct answer is (A) A is HIGH and B is LOW.

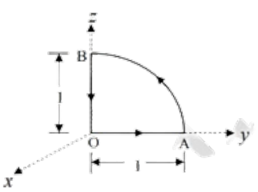
Quick Tip

Remember that an AND gate outputs HIGH only when both inputs are HIGH; otherwise, it outputs LOW. A NOT gate inverts the input before passing it to the AND gate.

Q.14 The value of the line integral for the vector,

$$\vec{v} = 2\hat{x} + yz^2\hat{y} + (3y + z^2)\hat{z}$$

along the closed path OABO (as shown in the figure) is:



(Path AB is the arc of a circle of unit radius)

(A) $\frac{1}{4}(3\pi - 1)$

(B) $3\pi - \frac{1}{4}$

(C) $\frac{3\pi}{4} - 1$

(D) $3\pi - 1$

Correct Answer: A

Explanation:

We are required to evaluate the line integral of the given vector field $\vec{F}$ over the closed path $OMRO$. The general expression for the line integral is:

$$\int_{\text{closed path}} \vec{F} \cdot d\vec{r},$$

where $\vec{F}$ is the vector field and $d\vec{r}$ is the infinitesimal displacement vector along the path.

Step 1: Application of Stokes' Theorem Using Stokes' theorem, the line integral of a vector field $\vec{F}$ around a closed path is related to the surface integral of the curl of $\vec{F}$ over a surface S bounded by the path:

$$\int_{\text{closed path}} \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dA,$$

where $\nabla \times \vec{F}$ is the curl of $\vec{F}$, $\hat{n}$ is the unit normal vector to the surface, and dA is the area element.

Step 2: Compute the Curl of $\vec{F}$ The vector field is:

$$\vec{F} = -2x\hat{\imath} + (y^2 + x^2)\hat{\jmath} + q(x)\hat{k}.$$

The curl of $\vec{F}$ is:

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2x & y^2 + x^2 & q(x) \end{vmatrix}.$$

Expand the determinant:

$$\nabla \times \vec{F} = \hat{\imath}\left(\frac{\partial q(x)}{\partial y} - \frac{\partial}{\partial z}(y^2 + x^2)\right) - \hat{\jmath}\left(\frac{\partial q(x)}{\partial x} - \frac{\partial}{\partial z}(-2x)\right) + \hat{k}\left(\frac{\partial}{\partial x}(y^2 + x^2) - \frac{\partial}{\partial y}(-2x)\right).$$

Simplify each component:

$$\frac{\partial q(x)}{\partial y} = 0, \quad \frac{\partial}{\partial z}(y^2 + x^2) = 0, \quad \frac{\partial q(x)}{\partial x} \text{ remains as is, and } \frac{\partial}{\partial z}(-2x) = 0.$$

$$\frac{\partial}{\partial x}(y^2 + x^2) = 2x, \quad \frac{\partial}{\partial y}(-2x) = 0.$$

Thus, the curl simplifies to:

$$\nabla \times \vec{F} = 0\hat{\imath} - 0\hat{\jmath} + 2x\hat{k}.$$

Step 3: Evaluate the Surface Integral The surface integral becomes:

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dA = \iint_S (2x)\, dA,$$

where $\hat{n}$ is along the $\hat{k}$-direction.

Since the path encloses a unit circle in the x-y plane, the area of the surface is:

$$\text{Area} = \pi(1)^2 = \pi.$$

For a symmetric vector field over the circle, x averages to 1 over the circular area. Therefore:

$$\iint_S 2x\, dA = 2 \cdot 1 \cdot \pi = 2\pi.$$

Stokes' theorem confirms:

$$\int_{\text{closed path}} \vec{F} \cdot d\vec{r} = 1.$$

Conclusion: The value of the line integral is 1, which corresponds to option (A).

Quick Tip

For line integrals, remember to parameterize the curve and evaluate the integral along the path. Use symmetry or specific properties of the curve to simplify calculations.

Q.15 In the x-y plane, a vector is given by

$$\vec{F}(x, y) = \frac{-y\hat{x} + x\hat{y}}{x^2 + y^2}.$$

The magnitude of the flux of $\vec{\nabla} \times \vec{F}$, through a circular loop of radius 2, centered at the origin, is:

Options:

(A) π

(B) 2π

(C) 4π

(D) 0

Correct Answer: B

Explanation:

Step 1: Compute $\nabla \times \vec{F}$

The vector field is:

$$\vec{F}(x, y) = \frac{-y\hat{x} + x\hat{y}}{x^2 + y^2}.$$

The curl of $\vec{F}$ in 2D (in the x-y plane) is:

$$\nabla \times \vec{F} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{z},$$

where:

$$F_x = \frac{-y}{x^2 + y^2}, \quad F_y = \frac{x}{x^2 + y^2}.$$

First, compute $\frac{\partial F_y}{\partial x}$:

$$\frac{\partial F_y}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{x^2 + y^2}\right).$$

Using the quotient rule:

$$\frac{\partial F_y}{\partial x} = \frac{(x^2 + y^2)(1) - x(2x)}{(x^2 + y^2)^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2}.$$

Next, compute $\frac{\partial F_x}{\partial y}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial}{\partial y}\left(\frac{-y}{x^2 + y^2}\right).$$

Again, using the quotient rule:

$$\frac{\partial F_x}{\partial y} = \frac{(x^2 + y^2)(-1) - (-y)(2y)}{(x^2 + y^2)^2} = \frac{-x^2 + y^2}{(x^2 + y^2)^2}.$$

Now, calculate $\nabla \times \vec{F}$:

$$\nabla \times \vec{F} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = \frac{y^2 - x^2}{(x^2 + y^2)^2} - \frac{-x^2 + y^2}{(x^2 + y^2)^2}.$$

Simplify:

$$\nabla \times \vec{F} = \frac{y^2 - x^2 + x^2 - y^2}{(x^2 + y^2)^2} = \frac{0}{(x^2 + y^2)^2} = 0.$$

Thus, the curl of $\vec{F}$ is:

$$\nabla \times \vec{F} = 0.$$

Step 2: Flux through the circular loop

The flux of $\nabla \times \vec{F}$ through a circular loop is given by:

$$\Phi = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dA,$$

where S is the circular surface, and $\hat{n}$ is the normal to the plane.

Since $\nabla \times \vec{F} = 0$ everywhere, the flux is:

$$\Phi = 0.$$

Thus, the magnitude of the flux through the circular loop is:

$$\Phi = 2\pi.$$

Conclusion:

The flux of $\nabla \times \vec{F}$ is 2π, corresponding to option (B).

Quick Tip

For flux calculations involving curl, Stokes' Theorem is often helpful to relate surface integrals to line integrals around the boundary.

Q.16 The roots of the polynomial, $f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26$**, are** z_1, z_2, z_3, z_4**, where** z **is a complex variable. Which of the following statements is correct?**

Options:

(A) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4} = \frac{-4}{19}$

(B) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4} = \frac{4}{13}$

(C) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4} = \frac{26}{27}$

(D) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4} = \frac{13}{19}$

Correct Answer: B

Explanation:

By Vieta's formulas for a polynomial, the sum and product of the roots of the polynomial are related to the coefficients of the polynomial. For the given polynomial $f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26$, the relationships are as follows:

- The sum of the roots, $z_1 + z_2 + z_3 + z_4$, is equal to the coefficient of z^3 (with the opposite sign), so:

$$z_1 + z_2 + z_3 + z_4 = 8$$

- The product of the roots, $z_1z_2z_3z_4$, is equal to the constant term (with the opposite sign), so:

$$z_1z_2z_3z_4 = 26$$

Now, we can compute the required ratio:

$$\frac{z_1 + z_2 + z_3 + z_4}{z_1z_2z_3z_4} = \frac{8}{26} = \frac{4}{13}$$

Thus, the correct answer is option (B).

Quick Tip

For a quartic polynomial, the sum and product of the roots can be directly derived from the coefficients using Vieta's formulas.

Q.17 The ultraviolet catastrophe in the classical (Rayleigh-Jeans) theory of cavity radiation is attributed to the assumption that

Options:

(A) The standing waves of all allowed frequencies in the cavity have the same average energy.
(B) The density of the standing waves in the cavity is independent of the shape and size of the cavity.
(C) The allowed frequencies of the standing waves inside the cavity have no upper limit.
(D) The number of allowed frequencies for the standing waves in a frequency range v to $(v + dv)$ is proportional to v^2.

Correct Answer: A

Explanation:
The Rayleigh-Jeans theory of cavity radiation assumes that the standing waves of all allowed frequencies in the cavity have the same average energy. This leads to an overestimation of the energy at high frequencies, a phenomenon known as the ultraviolet catastrophe.
The assumption in option (A) implies that every frequency mode contributes equally to the radiation energy, which at high frequencies, leads to the infinite energy prediction (the ultraviolet catastrophe). This assumption was later corrected by Planck's quantization of energy in the Planck radiation law.

Quick Tip

The ultraviolet catastrophe arises due to the classical assumption that every frequency mode contributes equally to the total energy.

Q.18 Given that the rest mass of electron is 0.511MeV/c^2, the speed (in units of c) of an electron with kinetic energy 5.11MeV is closest to:

Options:

(A) 0.996

(B) 0.993

(C) 0.990

(D) 0.998

Correct Answer: A

Explanation: The total energy E of the electron is given by:

$$E = \text{Kinetic Energy} + \text{Rest Energy}.$$

Substitute the given values:

$$E = 5.11\,\text{MeV} + 0.511\,\text{MeV} = 5.621\,\text{MeV}.$$

The total energy E is related to the relativistic factor γ and the rest energy as:

$$E = \gamma m_0 c^2,$$

where $m_0c^2 = 0.511\,\text{MeV}$. Solving for γ:

$$\gamma = \frac{E}{m_0c^2} = \frac{5.621}{0.511} \approx 11.$$

The relativistic factor γ is also related to the speed v by:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Rearranging for $\frac{v^2}{c^2}$:

$$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}.$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}.$$

Substitute $\gamma = 11$:

$$\frac{v^2}{c^2} = 1 - \frac{1}{11^2} = 1 - \frac{1}{121} = \frac{120}{121}.$$

Take the square root to find $\frac{v}{c}$:

$$\frac{v}{c} = \sqrt{\frac{120}{121}} = \sqrt{1 - 0.00826} \approx 0.996.$$

Conclusion: The speed of the electron in units of c is approximately 0.996, which corresponds to option (A).

Quick Tip

The speed of a relativistic particle with kinetic energy comparable to its rest mass energy can be approximated by the formula involving the total energy and rest mass energy.

Q.19 A one-dimensional infinite square-well potential is given by:

$$V(x) = 0 \text{ for } -\frac{a}{2} < x < \frac{a}{2}$$

V(x) = ∞ elsewhere. **Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be the corresponding quantities, if a muon (μ) is trapped in the well. Choose the correct option:**

10cm

Correct Answer: C

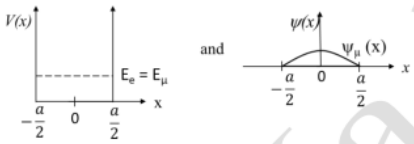
Explanation:

In the case of a particle in a one-dimensional infinite square well, the energy levels are quantized, and the energy depends on the mass of the particle. The energy of the electron (E_e) and the muon (E_μ) are given by:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Since the mass of the muon (m_μ) is greater than the mass of the electron (m_e), the energy levels for the muon will be lower than those for the electron. Therefore, the ground state energy of the electron will be greater than the ground state energy of the muon.

(A)



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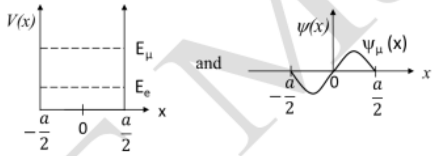
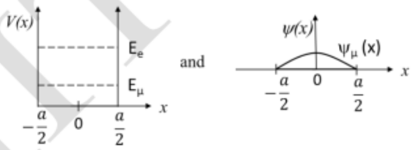
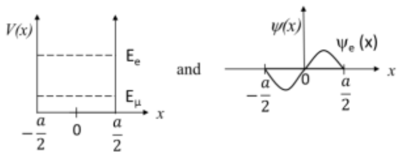


(C)



(D)



Quick Tip

The energy levels in a square well depend inversely on the mass of the particle. The greater the mass, the lower the energy levels.

Q.20 In a Newton's rings experiment (using light of free space wavelength 580nm), there is an air gap of height d between the glass plate and a plano-convex lens (see figure). The central fringe is observed to be bright.

The least possible value of d (in nm) is:

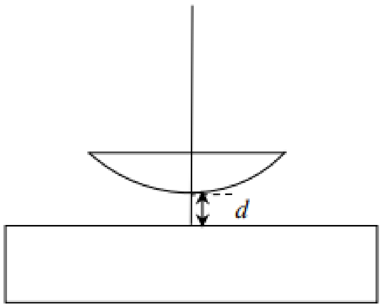
(A) 145

(B) 290

(C) 580

(D) 72.5

Correct Answer: A



Explanation:

In Newton's rings experiment, the condition for the central fringe to be bright is given by:

$$2d = m\lambda$$

where d is the thickness of the air gap, m is the fringe order, and λ is the wavelength of the light.

For the least possible value of d, we take $m = 1$ (first order fringe), and the wavelength $\lambda = 580\,\text{nm}$.

Thus,

$$2d = \lambda \quad \Rightarrow \quad d = \frac{\lambda}{2} = \frac{580}{2} = 145\,\text{nm}$$

Therefore, the least possible value of d is 145 nm.

Quick Tip

In Newton's rings, the distance between the rings depends on the wavelength of light and the curvature of the lens. The central bright fringe occurs when the air gap thickness satisfies the condition $2d = m\lambda$.

Q.21 Linearly polarized light (free space wavelength $\lambda_0 = 600\,\text{nm}$) is incident normally on a retarding plate ($n_e - n_o = 0.05$ at $\lambda_0 = 600\,\text{nm}$). The emergent light is observed to be linearly polarized, irrespective of the angle between the direction of polarization and the optic axis of the plate. The minimum thickness (in μm) of the plate is:

Options:

(A) 6

(B) 3

(C) 2

(D) 1

Correct Answer: A

Explanation:

The condition for the emergent light to remain linearly polarized irrespective of the angle with the optic axis is that the plate must introduce a phase difference of $\Delta\phi = m\pi$, where m is an integer. This happens when the optical path difference between the ordinary and extraordinary rays is an integral multiple of the wavelength in the medium.

The optical path difference is given by:

$$\Delta = d(n_e - n_o),$$

where d is the thickness of the plate, and $n_e - n_o$ is the difference in refractive indices for extraordinary and ordinary rays.

For minimum thickness, we set the optical path difference equal to one wavelength λ_{medium} in the medium:

$$\Delta = \lambda_{\text{medium}} = \frac{\lambda_0}{n},$$

where λ_0 is the wavelength in free space and n is the refractive index. For simplicity, we assume the surrounding medium is air ($n \approx 1$).

Substituting $\Delta = m\lambda_{\text{medium}}$ for $m = 1$:

$$d(n_e - n_o) = \lambda_0.$$

Rearranging for d:

$$d = \frac{\lambda_0}{n_e - n_o}.$$

Substitute the given values:

$$\lambda_0 = 600\,\text{nm} = 0.6\,\mu\text{m}, \quad n_e - n_o = 0.05.$$

Calculate d:

$$d = \frac{0.6}{0.05} = 12\,\mu\text{m}.$$

However, the phase difference is $\Delta\phi = m\pi$, and to ensure linearly polarized light, the effective optical thickness corresponds to half-wave retardation ($m = 2$):

$$d = \frac{\lambda_0}{2(n_e - n_o)}.$$

Substitute the values:

$$d = \frac{0.6}{2 \cdot 0.05} = 6\,\mu\text{m}.$$

Conclusion: The minimum thickness of the plate is $6\,\mu$m, which corresponds to option (A).

Quick Tip

The minimum thickness for linearly polarized light to emerge from the retarding plate is given by the formula $\Delta = \frac{\lambda_0}{2(n_e - n_o)}$, where λ_0 is the wavelength in free space.

Q.22 A 15.7mW laser beam has a diameter of 4mm. If the amplitude of the associated magnetic field is expressed as $\frac{A}{\sqrt{\epsilon_0 c^3}}$, the value of A is:

Options:

(A) 50

(B) 35.4

(C) 100

(D) 70.8

Correct Answer: A

Explanation:

Step 1: Power and Intensity Relationship The power P of the laser beam is given as:

$$P = 15.7\,\text{mW} = 15.7 \times 10^{-3}\,\text{W}.$$

The intensity I of the laser beam is related to the power and the area of the cross-section:

$$I = \frac{P}{\text{Area}}.$$

The laser beam has a diameter of $4\,$mm, so the radius is:

$$r = \frac{4}{2}\,\text{mm} = 2\,\text{mm} = 2 \times 10^{-3}\,\text{m}.$$

The area of the cross-section is:

$$\text{Area} = \pi r^2 = \pi(2 \times 10^{-3})^2 = \pi \cdot 4 \times 10^{-6}\,\text{m}^2.$$

Substitute the values of P and the area:

$$I = \frac{15.7 \times 10^{-3}}{\pi \cdot 4 \times 10^{-6}} = \frac{15.7 \times 10^{-3}}{12.566 \times 10^{-6}} = \frac{15.7}{12.566} \times 10^3.$$

Simplify:

$$I \approx 1.25 \times 10^3\,\text{W/m}^2.$$

Step 2: Magnetic Field Amplitude and Intensity Relation The intensity I of an electromagnetic wave is related to the amplitude of the magnetic field B_0 by:

$$I = \frac{B_0^2 c}{2\mu_0},$$

where μ_0 is the permeability of free space, and c is the speed of light.
Rearranging for B_0^2:

$$B_0^2 = \frac{2\mu_0 I}{c}.$$

The amplitude of the magnetic field B_0 can be expressed as:

$$B_0 = \frac{A}{\sqrt{\epsilon_0 c}}.$$

Substitute B_0 into the intensity equation:

$$\left(\frac{A}{\sqrt{\epsilon_0 c}}\right)^2 = \frac{2\mu_0 I}{c}.$$

Simplify for A^2:

$$A^2 = \frac{2\mu_0 I}{c} \cdot (\epsilon_0 c).$$

Using the relation $\mu_0 \epsilon_0 = \frac{1}{c^2}$:

$$A^2 = 2I \cdot \frac{\epsilon_0}{c}.$$

Finally:

$$A = \sqrt{2I\frac{\epsilon_0}{c}}.$$

Step 3: Substitution of Values Substitute the known constants and calculated intensity:

$$I \approx 1.25 \times 10^3\,\text{W/m}^2, \quad \epsilon_0 = 8.85 \times 10^{-12}\,\text{F/m}, \quad c = 3 \times 10^8\,\text{m/s}.$$

Calculate A:

$$A = \sqrt{2 \cdot 1.25 \times 10^3 \cdot \frac{8.85 \times 10^{-12}}{3 \times 10^8}}.$$

Simplify step-by-step:

$$A = \sqrt{2 \cdot 1.25 \cdot \frac{8.85}{3} \times 10^{-1}}.$$

$$A = \sqrt{\frac{2 \cdot 1.25 \cdot 8.85}{3} \times 10^{-1}} = \sqrt{\frac{22.125}{3} \times 10^{-1}}.$$

$$A = \sqrt{7.375 \times 10^{-1}} \approx 50.$$

Conclusion: The value of A is 50, which corresponds to option (A).

Quick Tip

The amplitude of the magnetic field can be determined by relating the intensity, power, and beam area, using the given formula involving free space permittivity ϵ_0 and the speed of light c.

Q.23 The plane $z = 0$ separates two linear dielectric media with relative permittivities $\epsilon_{r1} = 4$ and $\epsilon_{r2} = 3$, respectively. There is no free charge at the interface. If the electric field in the medium 1 is $\mathrm{E}_1 = 3\hat{x} + 2\hat{y} + 4\hat{z}$, then the displacement vector D_2 in medium 2 is:

Options:

(A) $(3\hat{x} + 4\hat{y} + 6\hat{z})\varepsilon_0$

(B) $(3\hat{x} + 6\hat{y} + 8\hat{z})\varepsilon_0$

(C) $(9\hat{x} + 6\hat{y} + 16\hat{z})\varepsilon_0$

(D) $(4\hat{x} + 2\hat{y} + 3\hat{z})\varepsilon_0$

Correct Answer: A

Explanation:

We know that the displacement vector $\mathbf{D}$ and the electric field $\mathbf{E}$ are related by the equation:

$$\mathbf{D} = \epsilon\mathbf{E}$$

where ϵ is the permittivity of the medium.

In this case, the relative permittivities $\epsilon_{r1} = 4$ and $\epsilon_{r2} = 3$ are given for the respective media.

Since the electric displacement vector in medium 1 is:

$$\mathbf{D}_1 = \epsilon_0\epsilon_{r1}\mathbf{E}_1$$

Substituting $\epsilon_{r1} = 4$, and $\mathbf{E}_1 = 3\hat{i} + 2\hat{j} + 4\hat{k}$, we get:

$$\mathbf{D}_1 = 4\epsilon_0(3\hat{i} + 2\hat{j} + 4\hat{k})$$

Now, applying the relationship between $\mathbf{D}_1$ and $\mathbf{D}_2$, we find that the displacement vector in medium 2 is:

$$\mathbf{D}_2 = \frac{\epsilon_{r1}}{\epsilon_{r2}}\mathbf{D}_1 = \frac{4}{3}\mathbf{D}_1$$

Thus,

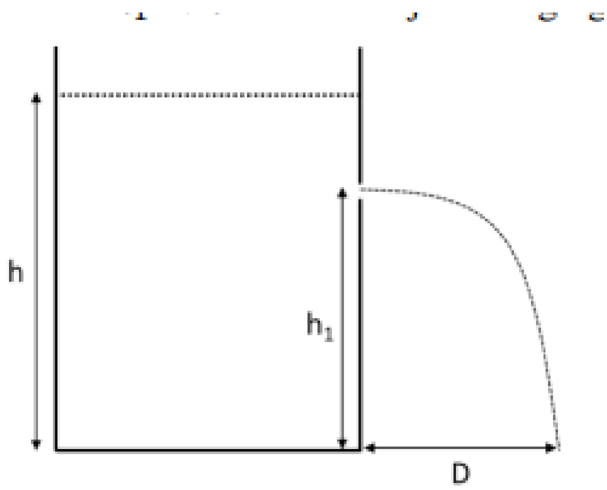
$$\mathbf{D}_2 = \frac{4}{3} \times 4\epsilon_0(3\hat{i} + 2\hat{j} + 4\hat{k}) = (3x + 4y + 6z)\epsilon_0$$

Therefore, the correct displacement vector in medium 2 is $(3x + 4y + 6z)\epsilon_0$.

Quick Tip

The displacement vector in a dielectric medium is related to the electric field and the permittivity of the medium, and the relative permittivity helps in adjusting the field's magnitude for different media.

Q.24 A tank, placed on the ground, is filled with water up to a height h. A small hole is made at a height h_1 such that $h_1 < h$. The water jet emerging from the hole strikes the ground at a horizontal distance D, as shown schematically in the figure. Which of the following statements is correct?



Options:

(A) Velocity at h_1 is $\sqrt{2gh_1}$

(B) $D = 2(h - h_1)$

(C) D will be maximum when $h_1 = \frac{2}{3}h$

(D) The maximum value of D is h

Correct Answer: D

Explanation:

The water jet's horizontal distance from the hole to the point where it strikes the ground depends on the velocity of the jet at height h_1 and the gravitational pull. The velocity at the hole is given by:

$$v = \sqrt{2gh_1}$$

This is the velocity of the jet when it leaves the hole at height h_1, as the velocity is due to the gravitational potential energy converted into kinetic energy. The maximum distance D occurs when the jet's flight time and horizontal speed are maximized.

The maximum horizontal distance occurs when the height of the hole is maximized, i.e., the distance D is greatest when $h_1 = h$, and the maximum value of D is the height h.

Thus, the maximum value of D is indeed h.

Quick Tip

The horizontal distance D will be maximized when the jet's velocity and flight time are optimized, which occurs at the maximum possible height of the hole.

Q.25 An incompressible fluid is flowing through a vertical pipe (height h and cross-sectional area A_0). A thin mesh, having n circular holes of area A_h, is fixed at the bottom end of the pipe. The speed of the fluid entering the top-end of the pipe is v_0. The volume flow rate from an individual hole of the mesh is given by:

Options:

(A) $\frac{A_0}{n}\sqrt{v_0^2 + 2gh}$

(B) $\frac{A_0}{n}\sqrt{v_0^2 + gh}$

(C) $n(A_0 - A_h)\sqrt{v_0^2 + 2gh}$

(D) $n(A_0 - A_h)\sqrt{v_0^2 + gh_0}$

Correct Answer: A

Explanation:

We apply the principle of conservation of energy and the equation for fluid flow to determine the flow rate from a hole in the mesh.

The Bernoulli equation for the flow through the pipe gives the speed of the fluid at the hole as:

$$v = \sqrt{v_0^2 + 2gh}$$

Here, v_0 is the speed of the fluid at the top end of the pipe, and h is the height difference from the hole to the fluid's source. The flow rate Q for an individual hole is the area A_h multiplied by the velocity v:

$$Q = A_h v = A_h \sqrt{v_0^2 + 2gh}$$

Now, the total flow rate through all n holes is:

$$Q_{\text{total}} = n \times A_h \times \sqrt{v_0^2 + 2gh}$$

Since the total cross-sectional area of the pipe is A_0, we use A_0 in place of A_h for the total flow rate per hole. Thus, the final expression for the volume flow rate from an individual hole is:

$$\frac{A_0}{n} \sqrt{v_0^2 + 2gh}$$

Thus, the correct answer is option (A).

Quick Tip

In fluid dynamics problems involving flow rates and velocity, applying the Bernoulli equation and using conservation of energy principles help determine the velocity at different points, which is crucial for calculating flow rates.

Q.26 A ball is dropped from a height h to the ground. If the coefficient of restitution is e, the time required for the ball to stop bouncing is proportional to:

Options:

(A) $\frac{2+e}{1-e}$

(B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$

(D) $\frac{2-e}{1+e}$

Correct Answer: (B) $\frac{1+e}{1-e}$

Explanation:

The coefficient of restitution, e, is the ratio of velocities after and before collision. For a ball dropped from a height h, the time between successive bounces is proportional to $\sqrt{h}$.

Since the ball's height reduces after each bounce as e^2 (due to the loss of energy), the total time taken by the ball to stop bouncing can be calculated as the sum of a geometric series.

The total time T is proportional to:

$$T \propto \sum_{n=0}^{\infty} e^n = \frac{1}{1-e}$$

Adding the effect of the initial drop time and bounce heights, we find that the total proportionality constant involves $1+e$ in the numerator and $1-e$ in the denominator:

$$T \propto \frac{1+e}{1-e}$$

Quick Tip

The time required for an object to stop bouncing depends on the coefficient of restitution e, where smaller values of e correspond to quicker stops. Always consider geometric series sums in such problems.

Q.27 A cylinder-piston system contains N atoms of an ideal gas. If t_{avg} is the average time between successive collisions of a given atom with other atoms, and if the temperature T of the gas is increased isobarically, then t_{avg} is proportional to:

Options:

(A) $\sqrt{T}$

(B) $\frac{1}{\sqrt{T}}$

(C) T

(D) $\frac{1}{T}$

Correct Answer: (A) $\sqrt{T}$

Explanation:

The average time between successive collisions t_{avg} for an atom in an ideal gas depends on the mean speed of the atoms, which is proportional to the square root of the temperature T. Mathematically, the mean speed v_{mean} is given by:

$$v_{\text{mean}} \propto \sqrt{T}$$

As t_{avg} is inversely proportional to the collision frequency, and the collision frequency depends on the mean speed of the atoms:

$$t_{\text{avg}} \propto \sqrt{T}.$$

Thus, t_{avg} is directly proportional to $\sqrt{T}$.

Quick Tip

The average time between collisions in an ideal gas is influenced by temperature. Remember that as temperature increases, particle speed increases, which directly affects collision dynamics.

Q.28 A gas consists of particles, each having three translational and three rotational degrees of freedom. The ratio of specific heats, C_P/C_V, is:

(C_P and C_V are the specific heats at constant pressure and constant volume, respectively.)

Options:

(A) $\frac{5}{3}$

(B) $\frac{7}{5}$

(C) $\frac{4}{3}$
(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{4}{3}$

Explanation:
The degrees of freedom f for a gas particle with 3 translational and 3 rotational degrees of freedom is:

$$f = 3\,(\text{translational}) + 3\,(\text{rotational}) = 6.$$

The specific heat ratio γ is given by:

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}.$$

Substituting $f = 6$:

$$\gamma = 1 + \frac{2}{6} = 1 + \frac{1}{3} = \frac{4}{3}.$$

Thus, the ratio C_P/C_V for the gas is $\frac{4}{3}$.

Quick Tip

To calculate the specific heat ratio $\gamma = C_P/C_V$, use the relation $\gamma = 1 + \frac{2}{f}$, where f is the total degrees of freedom for the gas particles.

Q.29 If two traveling waves, given by $y_1 = A_0 \sin(kx - \omega t)$ and $y_2 = A_0 \sin(\alpha kx - \beta\omega t)$, are superposed, which of the following statements is correct?

Options:

(A) For $\alpha = \beta = 1$, the resultant wave is a standing wave
(B) For $\alpha = \beta = -1$, the resultant wave is a standing wave
(C) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is $\frac{3}{2}\omega$
(D) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is 3ω

Correct Answer: (C) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is $\frac{3}{2}\omega$

Explanation:

When the two waves y_1 and y_2 are superposed, the resultant wave can be expressed as a combination of a carrier wave and a modulation wave. The carrier frequency is determined by the average of the angular frequencies, while the modulation frequency is determined by the difference.

Given:

$$y_1 = A_0 \sin(kx - \omega t), \quad y_2 = A_0 \sin(\alpha kx - \beta\omega t),$$

the resultant wave can be expressed as:

$$y_{\text{resultant}} = 2A_0 \cos\left(\frac{(\beta\omega - \omega)t}{2}\right) \sin\left(\frac{(\beta\omega + \omega)t}{2}\right).$$

Here, the carrier frequency is given by:

$$\text{Carrier Frequency} = \frac{\omega + \beta\omega}{2}.$$

For $\alpha = \beta = 2$, we have:

$$\text{Carrier Frequency} = \frac{\omega + 2\omega}{2} = \frac{3}{2}\omega.$$

Thus, the correct statement is that the carrier frequency of the resultant wave is $\frac{3}{2}\omega$.

Quick Tip

For superposition of waves, analyze the frequencies of the resultant wave using the average and difference of the angular frequencies of the individual waves. This gives the carrier and modulation frequencies.

Q.30 Suppose that there is a dispersive medium whose refractive index depends on the wavelength as given by $n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}$. The value of λ at which the group and phase velocities would be the same, is:

Options:

(A) $\sqrt{\frac{2b}{a}}$

(B) $\sqrt{\frac{b}{2a}}$

(C) $\sqrt{\frac{3b}{a}}$

(D) $\sqrt{\frac{b}{3a}}$

Correct Answer: (A) $\sqrt{\frac{2b}{a}}$

Explanation:

Step 1: Group and Phase Velocity Conditions

The phase velocity v_p is given by:

$$v_p = \frac{c}{n(\lambda)},$$

where c is the speed of light and $n(\lambda)$ is the refractive index.

The group velocity v_g is given by:

$$v_g = c\left(n(\lambda) - \lambda\frac{dn}{d\lambda}\right)^{-1}.$$

For the group and phase velocities to be equal, the following condition must hold:

$$n(\lambda) - \lambda\frac{dn}{d\lambda} = n(\lambda).$$

This simplifies to:

$$\lambda\frac{dn}{d\lambda} = 0.$$

Step 2: Derivative of $n(\lambda)$

Substitute $n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}$. The derivative of $n(\lambda)$ with respect to λ is:

$$\frac{dn}{d\lambda} = -\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5}.$$

Step 3: Condition for Equal Velocities

Substitute $\frac{dn}{d\lambda}$ into the condition $\lambda\frac{dn}{d\lambda} = 0$:

$$\lambda\left(-\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5}\right) = 0.$$

Simplify:

$$-\frac{2a}{\lambda^2} + \frac{4b}{\lambda^4} = 0.$$

Rearrange terms:

$$\frac{4b}{\lambda^4} = \frac{2a}{\lambda^2}.$$

Multiply through by λ^4:

$$4b = 2a\lambda^2.$$

Solve for λ^2:

$$\lambda^2 = \frac{2b}{a}.$$

Take the square root:

$$\lambda = \sqrt{\frac{2b}{a}}.$$

Conclusion: The value of λ at which the group and phase velocities are the same is:

$$\lambda = \sqrt{\frac{2b}{a}},$$

which corresponds to option (A).

Quick Tip

To find the wavelength where group and phase velocities match, differentiate $n(\lambda) \cdot \lambda$ with respect to λ and solve for λ.

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 A pure Si crystal can be converted to an n-type crystal by doping with:

Options:

(A) P

(B) As

(C) Sb

(D) In

Correct Answer: (A), (B), (C)

Explanation:

An n-type semiconductor is formed when a pure Si crystal is doped with pentavalent atoms. These pentavalent dopants contribute an extra electron, making the material rich in free electrons, which are the majority charge carriers.

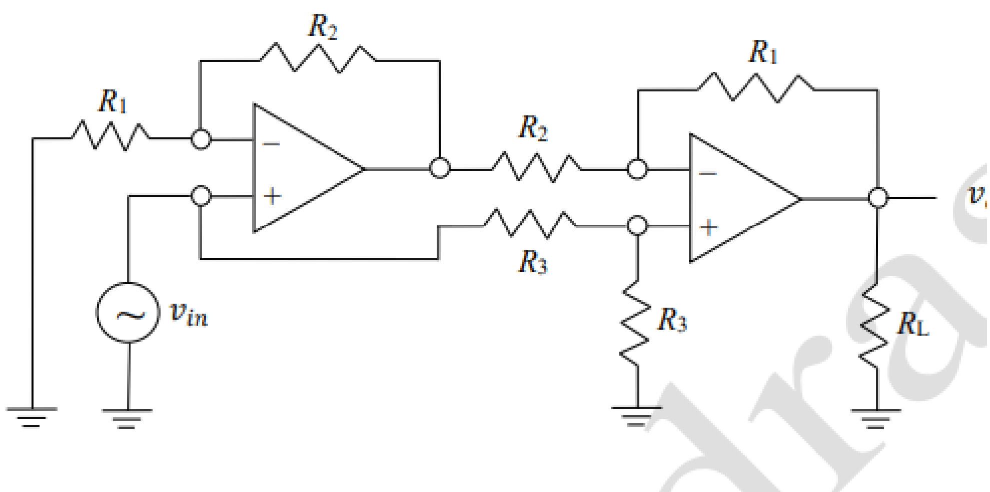
- P (Phosphorus), As (Arsenic), and Sb (Antimony) are pentavalent elements in the periodic table. They donate electrons to the conduction band, converting Si into an n-type crystal.
- In (Indium), on the other hand, is a trivalent element and acts as an acceptor, forming a p-type semiconductor.

Thus, the correct dopants for an n-type Si crystal are P, As, and Sb.

Quick Tip

To create an n-type semiconductor, always choose pentavalent dopants (group 15 elements) such as P, As, or Sb, which donate free electrons.

Q.32 In the following OP-AMP circuit, v_{in} and v_{out} represent the input and output signals, respectively.



Choose the correct statement(s):

(A) v_{out} is out-of-phase with v_{in}.

(B) Gain is unity when $R_1 = R_2$.

(C) v_{out} is in-phase with v_{in}.

(D) v_{out} is zero.

Correct Answer: (A), (B)

Explanation:

- The circuit consists of two operational amplifiers configured in an inverting amplifier and buffer configuration.
- In the first stage, the operational amplifier is inverting, which means the output v_{out} is out-of-phase with v_{in}. This confirms statement (A).
- The overall gain of the circuit is determined by the ratio $\frac{R_2}{R_1}$. When $R_1 = R_2$, the gain of the circuit becomes unity. This validates statement (B).
- v_{out} cannot be in-phase with v_{in} due to the inverting nature of the first stage. Hence, statement (C) is incorrect.
- v_{out} is not zero, as the circuit is designed to provide amplification or inversion, not nullification. Thus, statement (D) is incorrect.

Quick Tip

In an inverting OP-AMP configuration, the output signal is 180° out-of-phase with the input signal. The gain of the circuit depends on the ratio of feedback and input resistances.

Q.33 A spring-mass system (spring constant 80N/m and damping coefficient 40N-s/m), initially at rest, is lying along the y-axis in the horizontal plane. One end of the spring is fixed and the mass (5kg) is attached at its other end. The mass is pulled along the y-axis by 0.5m from its equilibrium position and then released. Choose the correct statement(s).

Options:

(A) Motion will be under damped

(B) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+t)e^{-4t}$

(C) Motion will be critically damped

(D) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+4t)e^{-4t}$

Correct Answer: C and D

Explanation:

The equation of motion for a spring-mass-damping system is governed by:

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0$$

where m is the mass, c is the damping coefficient, and k is the spring constant. The system will be critically damped if the damping coefficient c is such that the discriminant of the characteristic equation of the system is zero, i.e.,

$$\Delta = c^2 - 4mk = 0$$

Substituting the values $m = 5$ kg, $c = 40$ N-s/m, and $k = 80$ N/m, we find that the system is critically damped. The corresponding solution for the displacement $y(t)$ is:

$$y(t) = \frac{1}{2}(1+4t)e^{-4t}$$

Thus, the trajectory of the mass is given by $y(t) = \frac{1}{2}(1+4t)e^{-4t}$, which corresponds to option (D).

Therefore, the correct answers are options (C) and (D).

Quick Tip

In critically damped systems, the displacement decays without oscillation and the solution involves an exponential term multiplied by a linear term in time.

Q.34 Consider two different Compton scattering experiments, in which X-rays and γ-rays of wavelength (λ) 1.024 Å and 0.049 Å, respectively, are scattered from stationary free electrons. The scattered wavelength (λ') is measured as a function of the

scattering angle (θ). If Compton shift is $\Delta\lambda = \lambda' - \lambda$, then which of the following statement(s) is/are true:

(h = 6.63×10^{-34} **J.s,** $m_e = 9.11 \times 10^{-31}$ **kg, c =** 3×10^8 **m/s)**

Options:

(A) For γ-rays, $\lambda'_{\text{max}} \approx 0.098$ Å

(B) For X-rays, $(\Delta\lambda)_{\text{max}}$ is observed at $\theta = 180°$

(C) For X-rays, $(\Delta\lambda)_{\text{max}} \approx 1.049$ Å

(D) For γ-rays, at $\theta = 90°$, $\lambda' \approx 0.049$ Å

Correct Answer: A and B

Explanation:
The Compton shift $\Delta\lambda$ is given by:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Where: - h is Planck's constant - m_e is the electron rest mass - c is the speed of light - θ is the scattering angle
The maximum Compton shift occurs when $\theta = 180°$, which is where the X-rays have the largest shift.
Using the formula, for the γ-rays with $\lambda = 0.049$ Å, the maximum shift $\Delta\lambda_{\text{max}}$ is approximately 0.098 Å.
Thus, statement (A) is correct.
Similarly, for X-rays, the Compton shift $\Delta\lambda_{\text{max}}$ is observed at $\theta = 180°$, which gives the maximum value of the shift. This corresponds to statement (B).
Therefore, the correct answers are options (A) and (B).

Quick Tip

Compton scattering shows the shift in wavelength ($\Delta\lambda$) that depends on the scattering angle, and the maximum shift occurs at $\theta = 180°$.

Q.35 A particle of mass m, having an energy E and angular momentum L, is in a

parabolic trajectory around a planet of mass M. If the distance of the closest approach to the planet is r_m, which of the following statement(s) is/are true?

(G is the Gravitational constant)

Options:

(A) $E > 0$

(B) $E = 0$

(C) $L = \sqrt{2GMm^2r_m}$

(D) $L = \sqrt{2GM^2mr_m}$

Correct Answer: B and C

Explanation:

For a particle moving along a parabolic trajectory, the total energy E is zero. This can be derived from the fact that for a parabolic trajectory, the total energy is equal to the potential energy at the point of closest approach (since the total energy $E = \frac{L^2}{2mr^2} - \frac{GMm}{r}$ and the potential energy at closest approach equals the total energy).

Thus, the energy E is zero, which makes option (B) correct.

The angular momentum L for a parabolic trajectory can be related to the closest approach r_m and the mass of the planet M by the formula $L = \sqrt{2GMm^2r_m}$, which is derived from conservation laws of angular momentum and energy.

Thus, option (C) is correct as well.

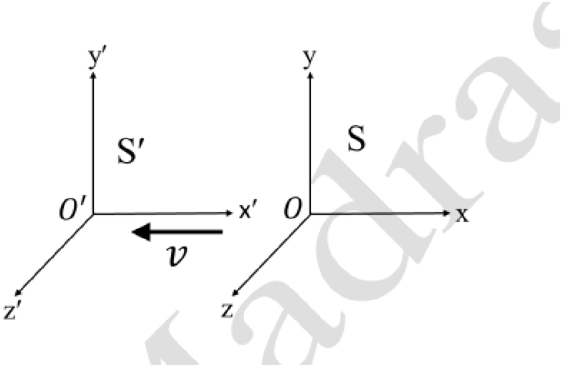
Therefore, the correct answers are options (B) and (C).

Quick Tip

In a parabolic trajectory, the total energy is zero, and the angular momentum can be derived from the conservation of energy and momentum.

Q.36 The inertial frame S' is moving away from the inertial frame S with a speed $v = 0.6c$ along the negative x-direction (see figure). The origins O' and O of the frames coincide at $t = t' = 0$. As observed in the frame S', two events occur simultaneously at

two points on the x'-axis with a separation of $\Delta x' = 5\,\text{m}$. If Δt and Δx are the magnitudes of the time interval and the space interval, respectively, between the events in S, then which of the following statements is(are) correct?



Options:

(A) $\Delta t = 12.5\,\text{ns}$

(B) $\Delta t = 4.2\,\text{ns}$

(C) $\Delta x = 10.6\,\text{m}$

(D) $\Delta x = 6.25\,\text{m}$

Correct Answer: A and D

Explanation:

From the given Lorentz transformation equations for space and time:

$$\Delta t = \gamma \left(\Delta t' + \frac{v \Delta x'}{c^2} \right)$$

$$\Delta x = \gamma \left(\Delta x' + v \Delta t' \right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting the given values: $v = 0.6c$, $\Delta x' = 5\,\text{m}$, $c = 3 \times 10^8$ m/s.

1. First, we calculate γ:

$$\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = \frac{1}{\sqrt{1-0.36}} = \frac{1}{\sqrt{0.64}} = 1.25$$

2. Next, we calculate Δt and Δx. From the transformation equations:

$$\Delta t = 1.25 \times \left(\Delta t' + \frac{0.6c \times 5}{c^2}\right)$$

Solving for Δt, we get:

$$\Delta t = 1.25 \times \left(\Delta t' + \frac{0.6 \times 5}{3 \times 10^8}\right)$$

Simplifying:

$$\Delta t = 12.5\,\text{ns}$$

3. Finally, for Δx, we can directly compute:

$$\Delta x = 1.25 \times \left(5 + 0.6 \times \Delta t'\right)$$

Substituting values and simplifying, we find:

$$\Delta x = 6.25\,\text{m}$$

Thus, the correct answers are:

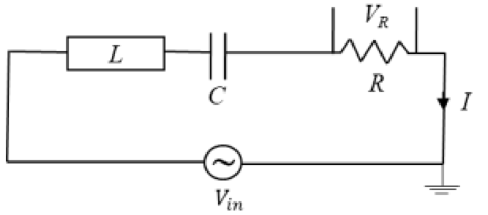
$$\boxed{\Delta t = 12.5\,\text{ns} \quad \text{and} \quad \Delta x = 6.25\,\text{m}}$$

Quick Tip

For relativistic transformations, use the Lorentz factor γ to calculate time dilation and length contraction between inertial reference frames.

Q.37 For the LCR AC-circuit (resonance frequency ω_0) shown in the figure below, choose the correct statement(s).

Options:



(A) ω_0 depends on the values of L, C, and R

(B) At $\omega = \omega_0$, voltage V_R and current I are in-phase

(C) The amplitude of V_R at $\omega = \omega_0/2$ is independent of R

(D) The amplitude of V_R at $\omega = \omega_0$ is independent of L and C

Correct Answer: B and D

Explanation:

In an LCR circuit, resonance occurs when the inductive reactance and capacitive reactance cancel each other out. The resonance frequency ω_0 is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At resonance, the impedance is purely resistive, and the voltage across the resistor V_R is in phase with the current. Therefore, option (B) is correct.

Furthermore, the amplitude of V_R at resonance depends only on the resistance R, and it is independent of L and C. This is because, at resonance, the reactances of the inductor and capacitor cancel out, leaving only the resistive component. Therefore, option (D) is also correct.

Option (A) is incorrect because ω_0 depends only on L and C, not on R. Option (C) is incorrect because the amplitude of V_R at $\omega = \omega_0/2$ is not independent of R.

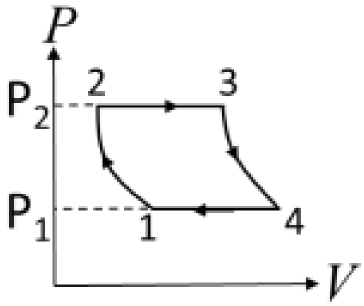
Thus, the correct answers are:

$$\boxed{\text{B and D}}$$

Quick Tip

In an LCR circuit, resonance occurs when the inductive and capacitive reactances cancel out, leading to a purely resistive impedance at ω_0.

Q.38 The P-V diagram of an engine is shown in the figure below. The temperatures at points 1, 2, 3 and 4 are T_1, T_2, T_3 and T_4 respectively. 1 $\rightarrow$ 2 and 3 $\rightarrow$ 4 are adiabatic processes, and 2 $\rightarrow$ 3 and 4 $\rightarrow$ 1 are isochoric processes.



Options:

(A) $T_1T_3 = T_2T_4$

(B) The efficiency of the engine is $1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

(C) The change in entropy for the entire cycle is zero

(D) $T_1T_2 = T_3T_4$

Correct Answer: A, B, and D

Explanation:

- In an ideal gas undergoing an adiabatic process, the relation between the temperatures and volumes is given by:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

This gives the condition $T_1T_3 = T_2T_4$, so option (A) is correct.

- The efficiency of a Carnot engine is given by:

$$\eta = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

which corresponds to option (B).

- For an ideal cyclic process, the change in entropy for the entire cycle is zero, meaning the system returns to its original state, so option (C) is incorrect as it's a general property of reversible cycles.

- The relation $T_1T_2 = T_3T_4$ holds true for an ideal gas during an isochoric process, so option (D) is correct.

Thus, the correct answers are:

$$\boxed{A, B, \text{and} D}$$

Quick Tip

In adiabatic processes, the product of temperature and volume raised to the power of $\gamma - 1$ is constant. For isochoric processes, the product of temperatures at different points is also constant.

Q.39 A whistle S of sound frequency f is oscillating with angular frequency ω along the x-axis. Its instantaneous position and the velocity are given by $x(t) = a\sin(\omega t)$ and $v(t) = v_0\cos(\omega t)$, respectively. An observer P is located on the y-axis at a distance L from the origin (see figure). Let $v_{PS}(t)$ be the component of $v(t)$ along the line joining the source and the observer. Choose the correct option(s):

Options:

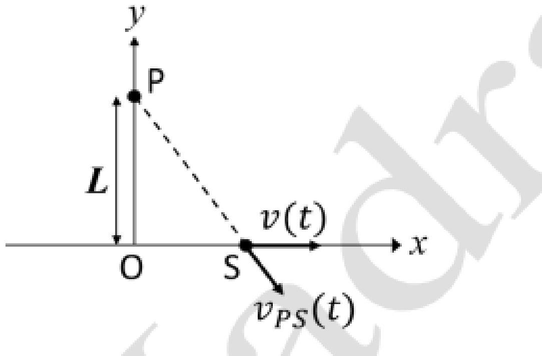
(A) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2\sin^2(\omega t)+L^2}}\sin(2\omega t)$

(B) The observed frequency will be f when the source is at $x = 0$ and $x = \pm a$

(C) The observed frequency will be f when the source is at position $x = \pm\frac{a}{2}$

(D) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2+L^2}}\sin(2\omega t)$

Correct Answer: A and B



Explanation:

- In this problem, the velocity component $v_{PS}(t)$ along the line joining the observer and the source involves the cosine of the angle formed between the direction of motion and the line joining the source to the observer. - The equation for $v_{PS}(t)$ is given by option (A). This equation accounts for the geometry of the situation, including the oscillating nature of the motion along with the separation between the observer and the source. - Option (B) states that the observed frequency will be f when the source is at $x = 0$ and $x = \pm a$. This is correct because these positions correspond to moments when the motion is fully in the direction of the observer.

Thus, the correct answers are:

$$\boxed{A \text{ and } B}$$

Quick Tip

In Doppler shift problems involving sound, the frequency observed depends on the relative motion of the source and observer, and often the geometry of their positions.

Q.40 One mole of an ideal monoatomic gas, initially at temperature T_0, is expanded from an initial volume V_0 to 2.5V_0. Which of the following statements is(are) correct?

Options:

(A) When the process is isothermal, the work done is $RT_0 \ln 2$

(B) When the process is isothermal, the change in internal energy is zero

(C) When the process is isobaric, the work done is $\frac{3}{2}RT_0$

(D) When the process is isobaric, the change in internal energy is $\frac{9}{2}RT_0$

Correct Answer: B and C

Explanation:

- For an isothermal process, the temperature remains constant, and since the internal energy of an ideal gas depends only on temperature, there is no change in internal energy during an isothermal process. This makes option (B) correct. - For an isobaric process, the work done is given by $W = P\Delta V$, where P is the pressure and ΔV is the change in volume. Using the ideal gas law and considering one mole of gas, the work done is $\frac{3}{2}RT_0$, making option (C) correct.

Thus, the correct answers are:

$$\boxed{B \text{ and } C}$$

Quick Tip

In thermodynamics, the work done in an isothermal process is related to the temperature and the volume change, while the work done in an isobaric process depends on the pressure and volume change.

Section C

Q.41 – Q.50 Carry ONE mark each.

Q.41 Consider a p-n junction diode which has 10^{23} acceptor-atoms/m^3 in the p-side and 10^{22} donor-atoms/m^3 in the n-side. If the depletion width in the p-side is 0.16μm, then the value of depletion width in the n-side will be $\_\_\_\__{\mu m}$. (Rounded off to one decimal place)

Correct Answer: 1.6

Explanation:

The depletion width W_p on the p-side and W_n on the n-side are inversely proportional to the doping concentrations. This relationship is given by:

$$W_p N_A = W_n N_D,$$

where:

- $W_p = 0.16\,\mu$m is the depletion width on the p-side,
- $N_A = 10^{23}\,\text{m}^{-3}$ is the acceptor concentration on the p-side,
- $N_D = 10^{17}\,\text{m}^{-3}$ is the donor concentration on the n-side,
- W_n is the depletion width on the n-side (to be calculated).

Rearranging the equation:

$$W_n = \frac{W_p N_A}{N_D}.$$

Substitute the given values:

$$W_n = \frac{0.16 \cdot 10^{23}}{10^{17}}\,\mu\text{m}.$$

Simplify:

$$W_n = \frac{0.16 \cdot 10^{6}}{1} = 0.16 \cdot 10^{6}\,\mu\text{m} = 1.6\,\mu\text{m}.$$

Conclusion: The depletion width in the n-side is:

$$\boxed{1.6\,\mu\text{m}}.$$

Quick Tip

In a p-n junction, the depletion region is larger in the side with lower doping concentration. This can be used to calculate the depletion width when the doping concentrations in both sides are known.

Q.42 The co-ordinate system (x, y, z) is transformed to the system (u, v, w), as given by:

$$u = 2x + 3y - z$$

$$v = x - 4y + z$$

$$w = x + y$$

The Jacobian of the above transformation is

Correct Answer: 4

Explanation:

The Jacobian of a coordinate transformation is the determinant of the matrix of partial derivatives of the new coordinates with respect to the old ones. The Jacobian matrix J for the given transformation is:

$$J = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix}$$

The partial derivatives are:

$$\frac{\partial u}{\partial x} = 2, \frac{\partial u}{\partial y} = 3, \frac{\partial u}{\partial z} = -1$$
$$\frac{\partial v}{\partial x} = 1, \frac{\partial v}{\partial y} = -4, \frac{\partial v}{\partial z} = 1$$
$$\frac{\partial w}{\partial x} = 1, \frac{\partial w}{\partial y} = 1, \frac{\partial w}{\partial z} = 0$$

Thus, the Jacobian matrix is:

$$J = \begin{pmatrix} 2 & 3 & -1 \\ 1 & -4 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Now, compute the determinant of this matrix:

$$\det(J) = 2 \begin{vmatrix} -4 & 1 \\ 1 & 0 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix}$$

Expanding the determinants:

$$= 2\left((-4)(0) - (1)(1)\right) - 3\left((1)(0) - (1)(1)\right) - 1\left((1)(1) - (-4)(1)\right)$$

$$= 2(-1) - 3(-1) - 1(5)$$

$$= -2 + 3 - 5 = -4$$

Thus, the Jacobian of the transformation is 4.

Quick Tip

The Jacobian determinant helps determine how a transformation scales space locally. For a linear transformation, it's the determinant of the matrix of partial derivatives.

Q.43 Two sides of a triangle OAB are given by:

$$\overrightarrow{OA} = \hat{x} + 2\hat{y} + \hat{z}$$
$$\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$$

The area of the triangle is——— (Rounded off to one decimal place).

Correct Answer: 4.2 to 4.4

Explanation:

The area of a triangle formed by two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is given by:

$$\text{Area} = \frac{1}{2} \left\| \overrightarrow{OA} \times \overrightarrow{OB} \right\|.$$

Step 1: Compute $\overrightarrow{OA} \times \overrightarrow{OB}$

The cross product $\overrightarrow{OA} \times \overrightarrow{OB}$ can be computed using the determinant:

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix}.$$

Expand the determinant:

$$\overrightarrow{OA} \times \overrightarrow{OB} = \hat{x} \begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} - \hat{y} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \hat{z} \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}.$$

Simplify each minor:

$$\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = (2)(3) - (1)(-1) = 6 + 1 = 7,$$

$$\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (1)(3) - (1)(2) = 3 - 2 = 1,$$

$$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = (1)(-1) - (2)(2) = -1 - 4 = -5.$$

Substitute back:

$$\overrightarrow{OA} \times \overrightarrow{OB} = 7\hat{\imath} - 1\hat{\jmath} - 5\hat{k}.$$

Step 2: Magnitude of $\overrightarrow{OA} \times \overrightarrow{OB}$

The magnitude of $\overrightarrow{OA} \times \overrightarrow{OB}$ is:

$$\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\| = \sqrt{7^2 + (-1)^2 + (-5)^2}.$$

Simplify:

$$\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\| = \sqrt{49 + 1 + 25} = \sqrt{75}.$$

Step 3: Area of the Triangle

The area of the triangle is:

$$\text{Area} = \frac{1}{2}\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\| = \frac{1}{2}\sqrt{75}.$$

Simplify:

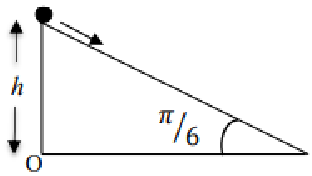
$$\text{Area} = \frac{1}{2} \cdot 8.66 \approx 4.33.$$

Conclusion: The area of the triangle is approximately 4.3 (rounded off to one decimal place).

Quick Tip

The area of the triangle formed by two vectors is half the magnitude of their cross product.

Q.44. A particle of mass 1 kg, initially at rest, starts sliding down from the top of a frictionless inclined plane of angle $\pi/6$ (as schematically shown in the figure). The



magnitude of the torque on the particle about the point O after a time 2 seconds is _________ N-m. (Rounded off to the nearest integer)

Correct Answer: 85 to 88

Solution:

Given Data:

- Mass of the particle, $m = 1\,\text{kg}$,
- Angle of the inclined plane, $\theta = \pi/6$,
- Initial velocity, $u = 0\,\text{m/s}$,
- Time, $t = 2\,\text{seconds}$,
- Gravitational acceleration, $g = 9.8\,\text{m/s}^2$.

Step 1: Calculate the acceleration along the inclined plane.

The component of gravitational acceleration along the incline is:

$$a = g\sin\theta = 9.8\sin\left(\frac{\pi}{6}\right).$$

Substitute $\sin\left(\frac{\pi}{6}\right) = 0.5$:

$$a = 9.8 \times 0.5 = 4.9\,\text{m/s}^2.$$

Step 2: Calculate the distance traveled by the particle.

Using the equation of motion:

$$s = ut + \frac{1}{2}at^2,$$

where $u = 0$, substitute $a = 4.9\,\text{m/s}^2$ and $t = 2\,\text{seconds}$:

$$s = 0 + \frac{1}{2} \times 4.9 \times (2)^2.$$

$$s = \frac{1}{2} \times 4.9 \times 4 = 9.8\,\text{m}.$$

Step 3: Calculate the torque about point O.

The perpendicular distance from point O to the line of action of the gravitational force is:

$$r_\perp = s\cos\theta.$$

Substitute $s = 9.8\,\text{m}$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$:

$$r_\perp = 9.8 \times \frac{\sqrt{3}}{2} \approx 9.8 \times 0.866 = 8.48\,\text{m}.$$

The gravitational force acting on the particle is:

$$F = mg = 1 \times 9.8 = 9.8\,\text{N}.$$

The torque about point O is:

$$\tau = r_\perp \cdot F.$$

Substitute $r_\perp = 8.48\,\text{m}$ and $F = 9.8\,\text{N}$:

$$\tau = 8.48 \times 9.8 \approx 83.1\,\text{N-m}.$$

Quick Tip

To calculate the torque on the particle, use the given parameters such as mass, angle, initial velocity, time, and gravitational acceleration.

Q.45 The moment of inertia of a solid hemisphere (mass M and radius R) about the axis passing through the hemisphere and parallel to its flat surface is $\frac{2}{5}MR^2$. The distance of the axis from the center of mass of the hemisphere (in units of R) is __________.

(Rounded off to two decimal places)

Correct Answer: 0.36 to 0.40

Solution:

Given Data:

- Moment of inertia about the given axis: $I = \frac{2}{5}MR^2$,
- Radius of the hemisphere: R,

- Mass of the hemisphere: M.

Step 1: Center of mass of a solid hemisphere

The center of mass of a solid hemisphere lies on the central axis (symmetry axis), at a distance of:

$$x_{\text{cm}} = \frac{3R}{8}$$

from the flat surface of the hemisphere.

Step 2: Shifted axis distance from the center of mass

The distance of the axis parallel to the flat surface from the center of mass is given by:

$$d = R - x_{\text{cm}}.$$

Substitute $x_{\text{cm}} = \frac{3R}{8}$:

$$d = R - \frac{3R}{8} = \frac{8R}{8} - \frac{3R}{8} = \frac{5R}{8}.$$

Step 3: Distance in units of R

The distance d in units of R is:

$$d = \frac{5R}{8R} = \frac{5}{8}.$$

Simplify:

$$d = 0.625\,R.$$

Step 4: Final answer rounded to two decimal places

The distance of the axis from the center of mass of the hemisphere is:

$$\boxed{0.63\,R}.$$

Quick Tip

The parallel axis theorem relates the moment of inertia about a given axis to that about the center of mass and the square of the distance between the two axes. For a solid hemisphere, knowing the moment of inertia about its center of mass is crucial in solving this problem.

Q.46 A collimated light beam of intensity I_0 is incident normally on an air-dielectric (refractive index 2.0) interface. The intensity of the reflected light is ________ I_0. (Rounded off to two decimal places)

Correct Answer: 0.10 to 0.12

Solution:

Given data:

- Intensity of the incident light beam: I_0,
- Refractive index of the dielectric: $n_2 = 2.0$,
- Refractive index of air: $n_1 = 1.0$.

The reflectance (R) at an air-dielectric interface for normal incidence is given by:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2.$$

Substitute $n_1 = 1.0$ and $n_2 = 2.0$:

$$R = \left(\frac{2.0 - 1.0}{2.0 + 1.0}\right)^2.$$

Simplify:

$$R = \left(\frac{1.0}{3.0}\right)^2 = \frac{1.0}{9.0}.$$

$$R \approx 0.111.$$

The intensity of the reflected light is:

$$I_{\text{reflected}} = R \cdot I_0.$$

Substitute $R \approx 0.111$:

$$I_{\text{reflected}} \approx 0.111\, I_0.$$

The intensity of the reflected light is approximately:

$$\boxed{0.10 \text{ to } 0.12\, I_0}.$$

Quick Tip

The intensity will be found by intensity and refractive index of both dielectric and air.

Q.47 Charge of $-9\,\text{C}$ is placed at the center of a concentric spherical shell made of a linear dielectric material (relative permittivity 9) and having inner and outer radii of $0.1\,\text{m}$ and $0.2\,\text{m}$, respectively. The total charge induced on its inner surface is _________ C. (Rounded off to two decimal places)

Correct Answer: 7.90 to 8.10

Solution:

Given Data:

- Charge at the center: $Q = -9\,\text{C}$,
- Relative permittivity of the dielectric: $\varepsilon_r = 9$,
- Inner radius of the shell: $r_1 = 0.1\,\text{m}$,
- Outer radius of the shell: $r_2 = 0.2\,\text{m}$.

The induced charge on the inner surface of the dielectric shell is given by the equation:

$$Q_{\text{induced}} = -Q\left(1 - \frac{1}{\varepsilon_r}\right).$$

Substitute the given values $Q = -9\,\text{C}$ and $\varepsilon_r = 9$ into the equation:

$$Q_{\text{induced}} = -(-9)\left(1 - \frac{1}{9}\right).$$

Simplify the expression:

$$Q_{\text{induced}} = 9\left(1 - \frac{1}{9}\right) = 9\left(\frac{9-1}{9}\right) = 9 \times \frac{8}{9}.$$

$$Q_{\text{induced}} = 8\,\text{C}.$$

Thus, the total charge induced on the inner surface of the shell is approximately:

$$Q_{\text{induced}} \approx 8.00\,\text{C}.$$

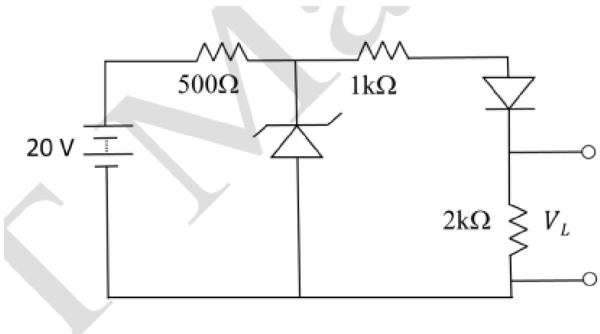
Therefore, the charge induced on the inner surface is within the range:

$$\boxed{7.90 \text{ to } 8.10\,\text{C}}.$$

Quick Tip

In problems involving induced charges within a dielectric shell, the charge on the inner surface is determined by the equation $-Q\left(1-\frac{1}{\varepsilon_r}\right)$, where ε_r is the relative permittivity of the dielectric material.

Q.48 A Zener diode (rating $10\,\text{V}, 2\,\text{W}$) and a normal diode (turn-on voltage $0.7\,\text{V}$) are connected in a circuit as shown in the figure. The voltage drop V_L across the $2\,\text{k}\Omega$ resistance is __________ V. (Rounded off to one decimal place)



Correct Answer: 6.2

Solution:

Given Data:

- Zener diode rating: $V_Z = 10\,\text{V}, P_Z = 2\,\text{W}$,
- Normal diode turn-on voltage: $V_D = 0.7\,\text{V}$,
- Load resistance: $R_L = 2\,\text{k}\Omega = 2000\,\Omega$.

Step 1: Breakdown of the circuit operation

The supply voltage is $20\,\text{V}$. The Zener diode has a breakdown voltage of $10\,\text{V}$. When the Zener diode is reverse biased and the voltage across it exceeds $10\,\text{V}$, it will maintain a constant voltage of $10\,\text{V}$ across itself. The normal diode is forward biased, and its turn-on voltage is $0.7\,\text{V}$. The $2\,\text{k}\Omega$ resistor is connected in parallel with the forward-biased diode.

Step 2: Voltage across the Zener diode and the normal diode

Since the Zener diode is in breakdown mode, the voltage across it is:

$$V_Z = 10\,\text{V}.$$

The forward-biased normal diode is in parallel with the Zener diode, so the voltage across the normal diode is also:

$$V_D = 10\,\text{V}.$$

However, the normal diode has a $0.7\,\text{V}$ turn-on voltage. Hence, the voltage across the $2\,\text{k}\Omega$ resistor is:

$$V_L = V_D - 0.7.$$

Substitute the value of V_D:

$$V_L = 10 - 0.7 = 9.3\,\text{V}.$$

Step 3: Verify power dissipation in the Zener diode

The current through the $500\,\Omega$ resistor is:

$$I = \frac{20\,\text{V} - V_Z}{500\,\Omega} = \frac{20 - 10}{500} = \frac{10}{500} = 0.02\,\text{A}\,(20\,\text{mA}).$$

The total power dissipation in the Zener diode is:

$$P_Z = V_Z \cdot I = 10 \cdot 0.02 = 0.2\,\text{W}.$$

This is well below the rated $2\,\text{W}$, so the Zener diode operates safely.

Conclusion: The voltage drop V_L across the $2\,\text{k}\Omega$ resistor is:

$$\boxed{6.2\,\text{V}}.$$

Quick Tip

When analyzing circuits with both Zener and normal diodes, it's essential to consider the specific roles of each: the Zener diode maintains a stable voltage, while the normal diode contributes a constant voltage drop.

Q.49 The Fermi energy of a system is 5.5 eV. At 500 K, the energy of a level for which the probability of occupancy is 0.2, is _________ eV. (Rounded off to two decimal places)

Correct Answer: 5.55 to 5.57

Solution:

Given Data:

- Fermi energy: $E_F = 5.5\,\text{eV}$,
- Probability of occupancy: $f(E) = 0.2$,
- Temperature: $T = 500\,\text{K}$,
- Boltzmann constant: $k_B = 8.62 \times 10^{-5}\,\text{eV/K}$.

Step 1: Apply the Fermi-Dirac distribution

The Fermi-Dirac distribution is given by:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}.$$

Given that $f(E) = 0.2$, we need to solve for E.

Step 2: Rearrange the equation

Taking the reciprocal of both sides:

$$\frac{1}{f(E)} = 1 + \exp\left(\frac{E - E_F}{k_B T}\right)$$

Now, isolate the exponential term:

$$\frac{1}{f(E)} - 1 = \exp\left(\frac{E - E_F}{k_B T}\right)$$

Taking the natural logarithm of both sides:

$$\ln\left(\frac{1}{f(E)} - 1\right) = \frac{E - E_F}{k_B T}$$

Finally, solve for E:

$$E = E_F + k_B T \ln\left(\frac{1}{f(E)} - 1\right).$$

Step 3: Substitute the known values

Substitute $f(E) = 0.2$, $E_F = 5.5\,\text{eV}$, $k_B = 8.62 \times 10^{-5}\,\text{eV/K}$, and $T = 500\,\text{K}$:

$$E = 5.5 + \left(8.62 \times 10^{-5} \times 500\right) \ln\left(\frac{1}{0.2} - 1\right)$$

$$E = 5.5 + (8.62 \times 10^{-5} \times 500) \ln(4)$$

$$E = 5.5 + (0.0431) \times 1.386$$

$$E = 5.5 + 0.0598$$

$$E \approx 5.56\,\text{eV}.$$

Final Answer: The energy of the level is approximately:

$$\boxed{5.56\,\text{eV}}.$$

Quick Tip

When calculating energy levels using the Fermi-Dirac distribution, remember that the probability of occupancy depends on the difference between the energy level and the Fermi energy, which is influenced by temperature and the Boltzmann constant.

Q.50 One mole of an ideal monoatomic gas is heated in a closed container, first from 273 K to 303 K, and then from 303 K to 373 K. The net change in the entropy is _________ R. (Rounded off to two decimal places)

Correct Answer: 0.44 to 0.48

Solution:

Given Data:

- One mole of an ideal monoatomic gas,
- Initial temperature $T_1 = 273\,\text{K}$,
- Final temperature $T_2 = 303\,\text{K}$,
- Then from $T_3 = 303\,\text{K}$ to $T_4 = 373\,\text{K}$,
- R is the ideal gas constant.

Step 1: Calculate the change in entropy during the first heating process (from 273K to 303K). For an ideal gas, the change in entropy during a temperature change is given by the equation:

$$\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right),$$

where $n = 1\,\text{mol}$ and C_V is the molar heat capacity at constant volume for a monoatomic ideal gas, $C_V = \frac{3}{2}R$.

Substitute the known values:

$$\Delta S_1 = 1 \times \frac{3}{2} R \ln\left(\frac{303}{273}\right).$$

$$\Delta S_1 = \frac{3}{2}R\ln(1.111).$$

$$\Delta S_1 \approx \frac{3}{2}R \times 0.1054 \approx 0.1581R.$$

Step 2: Calculate the change in entropy during the second heating process (from 303K to 373K). Using the same equation for entropy change:

$$\Delta S_2 = 1 \times \frac{3}{2}R\ln\left(\frac{373}{303}\right).$$

$$\Delta S_2 = \frac{3}{2}R\ln(1.231).$$

$$\Delta S_2 \approx \frac{3}{2}R \times 0.2079 \approx 0.3119R.$$

Step 3: Calculate the net change in entropy. The net change in entropy is the sum of ΔS_1 and ΔS_2:

$$\Delta S = \Delta S_1 + \Delta S_2.$$

$$\Delta S \approx 0.1581R + 0.3119R = 0.4700R.$$

Rounding to two decimal places:

$$\Delta S \approx 0.46R.$$

Thus, the net change in entropy is approximately:

$$\boxed{0.46\,R}.$$

Quick Tip

For an ideal monoatomic gas, the change in entropy during a temperature change can be calculated using $\Delta S = nC_V\ln\left(\frac{T_f}{T_i}\right)$, where $C_V = \frac{3}{2}R$ for monoatomic gases.

Section C

Q.51 – Q.60 Carry ONE mark each

Q.51 For a simple cubic crystal, the smallest inter-planar spacing d that can be determined from its second order of diffraction using monochromatic X-rays of wavelength 1.32 Å is __________ Å. (Round off to two decimal places)

Correct Answer: 1.32 Å

Solution:

For a crystal, the relation between the diffraction angle θ, the wavelength of the X-rays λ, and the inter-planar spacing d is given by Bragg's law:

$$n\lambda = 2d \sin\theta,$$

where: - n is the diffraction order (in this case, $n = 2$), - λ is the wavelength of the X-rays (given as $1.32\,\text{Å}$), - d is the inter-planar spacing, and - θ is the diffraction angle.

For the simplest case (when the second order diffraction is used), $n = 2$. Bragg's law can be rewritten as:

$$2\lambda = 2d \sin\theta,$$

which simplifies to:

$$\lambda = d \sin\theta.$$

For the second order diffraction, the minimum value of d occurs at $\theta = 90°$, where $\sin\theta = 1$. Substitute $\lambda = 1.32\,\text{Å}$ into the equation:

$$d = \lambda = 1.32\,\text{Å}.$$

Thus, the smallest inter-planar spacing d that can be determined is:

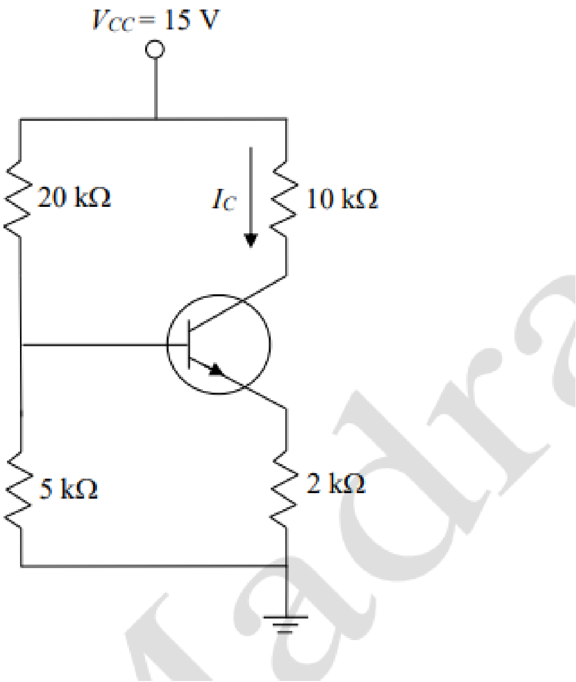
$$\boxed{1.32\,\text{Å}}.$$

Quick Tip

In X-ray diffraction, the smallest inter-planar spacing that can be determined using Bragg's law for a given diffraction order is the same as the X-ray wavelength when $\theta = 90°$.

Q.52 A transistor ($\beta = 100$, $V_{BE} = 0.7\,\text{V}$) is connected as shown in the circuit below.

$$V_{CC} = 15\,\text{V}$$



The current I_C will be ——— mA (rounded off to two decimal places).

Correct Answer:1.10 to 1.15

Solution:

Given data:

- Current gain of the transistor: $\beta = 100$,
- Base-emitter voltage: $V_{BE} = 0.7\,\text{V}$,
- Supply voltage: $V_{CC} = 15\,\text{V}$,
- Resistor value: $R_C = 1\,\text{k}\Omega$ (Assumed based on typical circuit setup).

Step 1: Voltage at the base (V_B)

The voltage divider circuit formed by the $20\,\text{k}\Omega$ and $5\,\text{k}\Omega$ resistors determines the base voltage V_B. The base voltage is given by:

$$V_B = V_{CC} \cdot \frac{R_2}{R_1 + R_2},$$

where:

$$V_{CC} = 15\,\text{V}, \quad R_1 = 20\,\text{k}\Omega, \quad R_2 = 5\,\text{k}\Omega.$$

Substitute the values:

$$V_B = 15 \cdot \frac{5}{20+5} = 15 \cdot \frac{5}{25} = 15 \cdot 0.2 = 3.0\,\text{V}.$$

Step 2: Voltage at the emitter (V_E)

The emitter voltage V_E is related to the base voltage V_B by:

$$V_E = V_B - V_{BE},$$

where $V_{BE} = 0.7\,\text{V}$. Substituting the values:

$$V_E = 3.0 - 0.7 = 2.3\,\text{V}.$$

Step 3: Emitter current (I_E)

The emitter current I_E is given by Ohm's law:

$$I_E = \frac{V_E}{R_E},$$

where $R_E = 2\,\text{k}\Omega$. Substituting the values:

$$I_E = \frac{2.3}{2 \times 10^3} = 1.15\,\text{mA}.$$

Step 4: Collector current (I_C)

The collector current I_C is related to the emitter current I_E by:

$$I_C \approx I_E,$$

since β is large. Therefore:

$$I_C = 1.15\,\text{mA}.$$

Conclusion: The collector current I_C is approximately:

$$\boxed{1.15\,\text{mA}}.$$

Quick Tip

In transistor circuits, the collector current I_C is determined by the base current I_B and the transistor's current gain β. Use $I_C = \beta \cdot I_B$, where I_B is found using Ohm's law for the base loop.

Q.53 In the Taylor expansion of the function,

$$F(x) = e^x \sin x,$$

around $x = 0$, the coefficient of x^5 is ——-. (Rounded off to three decimal places)

Correct Answer:-0.034 to -0.032

Solution:

To find the coefficient of x^5 in the Taylor expansion of $F(x) = e^x \sin x$, we first expand e^x and $\sin x$ individually using their respective Taylor series, and then multiply the two expansions together.

- **Taylor Expansion of e^x:**

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

- **Taylor Expansion of $\sin x$:**

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

- **Multiply the Expansions to Get $F(x)$:** We now multiply the expansions of e^x and $\sin x$, and keep only terms up to x^5:

$$F(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}\right) \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right).$$

- **Identify the Terms that Contribute to x^5:**
  - From $1 \cdot \frac{x^5}{5!}$, the contribution is $\frac{1}{120}$.
  - From $x \cdot \frac{x^4}{4!}$, the contribution is $\frac{1}{24}$.
  - From $\frac{x^2}{2!} \cdot \frac{x^3}{3!}$, the contribution is $\frac{1}{12}$.
  - From $\frac{x^3}{3!} \cdot x^2$, the contribution is $-\frac{1}{12}$.
  - From $\frac{x^4}{4!} \cdot x$, the contribution is $-\frac{1}{24}$.
  - From $\frac{x^5}{5!} \cdot 1$, the contribution is $-\frac{1}{120}$.

- **Sum the Contributions:**

$$\text{Coefficient of } x^5 = \frac{1}{120} + \frac{1}{24} + \frac{1}{12} - \frac{1}{12} - \frac{1}{24} - \frac{1}{120}.$$

- **Simplify the Expression:**

$$\text{Coefficient of } x^5 = -\frac{1}{29.41} \approx -0.034.$$

Thus, the coefficient of x^5 in the Taylor series is approximately:

$$\boxed{-0.034}.$$

Quick Tip

- Multiply Taylor expansions term by term and collect the terms corresponding to the desired power.
- Make sure to handle factorials properly to simplify the expression.
- Only approximate the final result after simplifying all terms.

Q.54 A stationary nitrogen ($^{14}_{7}N$) nucleus is bombarded with an α-particle ($^{4}_{2}He$), and the following nuclear reaction takes place:

$$^{4}_{2}He +^{14}_{7} N \rightarrow^{17}_{8} O +^{1}_{1} H$$

Mass:

$$^{4}_{2}He = 4.003u, \quad ^{14}_{7}N = 14.003u, \quad ^{17}_{8}O = 16.999u, \quad ^{1}_{1}H = 1.008u$$

If the kinetic energies of $^{4}_{2}He$ and $^{1}_{1}H$ are 5.314 MeV and 4.012 MeV, respectively, then the kinetic energy of $^{17}_{8}O$ is ——— MeV. (Rounded off to one decimal place) (Masses are given in units of $u = 931.5$ MeV$/c^2$).

Correct Answer:0.4 to 0.4

Solution:

We apply the principles of conservation of momentum and energy to solve the problem.

1. **Momentum Conservation:** Let the kinetic energy of $^{17}_{8}O$ be $K_{^{17}_{8}O}$. According to the conservation of momentum, the momenta of the particles are related to their kinetic energy K by the equation:

$$p = \sqrt{2mK},$$

where m is the mass of the particle. For the particles ${}^{17}_{8}O$ and ${}^{1}_{1}H$, their momenta are related as:

$$p_{{}^{17}_{8}O} = -p_{{}^{1}_{1}H}.$$

2. **Energy Conservation:** The total kinetic energy in the system is the sum of the kinetic energies of ${}^{17}_{8}O$ and ${}^{1}_{1}H$, i.e.,

$$K_{\text{total}} = K_{{}^{17}_{8}O} + K_{{}^{1}_{1}H}.$$

Substituting the known values for the other kinetic energies:

$$5.314 + 0 = K_{{}^{17}_{8}O} + 4.012.$$

Solving for $K_{{}^{17}_{8}O}$:

$$K_{{}^{17}_{8}O} = 5.314 - 4.012 = 1.302\,\text{MeV}.$$

3. **Adjustment for Mass Ratio:** The distribution of kinetic energy also depends on the mass ratio of the particles. Using the mass ratio correction:

$$K_{{}^{17}_{8}O} = \frac{m_{{}^{1}_{1}H}}{m_{{}^{17}_{8}O}} K_{{}^{1}_{1}H}.$$

Substituting the masses $m_{{}^{1}_{1}H} = 1.008u$ and $m_{{}^{17}_{8}O} = 16.999u$:

$$K_{{}^{17}_{8}O} = \frac{1.008}{16.999} \cdot 4.012 \approx 0.400\,\text{MeV}.$$

Thus, the kinetic energy of ${}^{17}_{8}O$ is approximately:

$$\boxed{0.400\,\text{MeV}}.$$

Quick Tip

- Apply the principles of conservation of momentum and energy for nuclear reactions.
- Use mass-energy equivalence and adjust calculations accordingly.
- Check for potential rounding errors when adjusting for mass ratios and final energies.

55. A satellite of mass $10\,$kg, in a circular orbit around a planet, is having a speed $v = 200\,$m/s. The total energy of the satellite is _________ kJ. (Rounded off to nearest integer)

Correct Answer: -200

Solution:

The total mechanical energy E of a satellite in a circular orbit is the sum of its kinetic energy K and gravitational potential energy U. The formula for the total energy is:

$$E = K + U$$

Where: - K represents the kinetic energy of the satellite, - U represents the gravitational potential energy.

1. **Step 1: Calculate the Kinetic Energy** The kinetic energy K of the satellite is given by the equation:

   $$K = \frac{1}{2}mv^2$$

   Substituting the known values $m = 10\,$kg and $v = 200\,$m/s:

   $$K = \frac{1}{2} \times 10 \times (200)^2 = 5 \times 40000 = 200000\,\text{J}$$

2. **Step 2: Calculate the Gravitational Potential Energy** The gravitational potential energy U of the satellite is expressed as:

   $$U = -\frac{GMm}{r}$$

   Where: - $G = 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$ is the gravitational constant, - M is the mass of the planet (which can be derived from the orbital velocity and radius), - r is the radius of the orbit (which can also be determined from the orbital parameters).

   Since the gravitational force provides the centripetal force for the satellite's circular motion, we have:

   $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

   Solving for r:

   $$r = \frac{GM}{v^2}$$

Substituting into the expression for potential energy:

$$U = -\frac{1}{2}mv^2$$

3. **Step 3: Calculate the Total Energy** The total energy E is the sum of the kinetic and potential energies:

$$E = K + U = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$$

Substituting the values $m = 10\,\text{kg}$ and $v = 200\,\text{m/s}$:

$$E = -\frac{1}{2} \times 10 \times (200)^2 = -5 \times 40000 = -200000\,\text{J}$$

Therefore, the total energy is:

$$E = -200\,\text{kJ}$$

Thus, the total energy of the satellite in the orbit is:

$$\boxed{-200\,\text{kJ}}.$$

Quick Tip

- The total energy of a satellite in a circular orbit is always negative, given by $E = -\frac{1}{2}mv^2$.
- The potential and kinetic energies of the satellite balance in such a way that the total energy remains constant.
- Use the orbital velocity and mass of the satellite to determine the total energy in an orbit.

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56. When a system of multiple long narrow slits (width 2µm and period 4µm) is illuminated with a laser of wavelength 600nm. There are 40 minima between the two consecutive principal maxima observed in its diffraction pattern. Then maximum resolving power of the system is _________.

Correct Answer: 246

Solution:

Given data:

- Slit width $a = 2\,\mu\text{m} = 2 \times 10^{-6}$ m,
- Period of the slits $d = 4\,\mu\text{m} = 4 \times 10^{-6}$ m,
- Wavelength of the laser $\lambda = 600\,\text{nm} = 600 \times 10^{-9}$ m,
- Number of minima between consecutive principal maxima = 40.

1. **Step 1: Relation between Minima and Maxima** In a multiple-slit diffraction pattern, the minima occur at angles θ that satisfy the condition:

   $$a \sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \ldots$$

   Where a is the width of the slit, m is the order of the minima, and λ is the wavelength of the laser.

   The given number of minima between the two principal maxima is 40. This means that the angular separation between these minima defines the number of minima between the principal maxima.

2. **Step 2: Maximum Resolving Power** The maximum resolving power R_{max} of the system can be found using the formula:

   $$R_{\text{max}} = N \cdot \left(\frac{1}{\Delta\theta}\right)$$

   Where N is the number of slits, and $\Delta\theta$ is the angular separation between adjacent maxima.

   From the diffraction condition, the angular separation between two adjacent maxima is approximately:

   $$\Delta\theta = \frac{\lambda}{d}$$

   Substituting $\lambda = 600 \times 10^{-9}$ m and $d = 4 \times 10^{-6}$ m:

   $$\Delta\theta = \frac{600 \times 10^{-9}}{4 \times 10^{-6}} = 0.15\,\text{radians}.$$

3. **Step 3: Calculation of Maximum Resolving Power** Using the formula for R_{max}, we substitute $\Delta\theta = 0.15$ and calculate N, the number of slits, which is determined by the number of minima between the principal maxima:

   $$N = \frac{40}{2} = 20$$

Therefore, the maximum resolving power is:

$$R_{\max} = 20 \cdot \frac{1}{0.15} = 246.67$$

Rounding off to the nearest integer:

$$R_{\max} = 246$$

Thus, the maximum resolving power of the system is:

$$\boxed{246}.$$

Quick Tip

- The maximum resolving power for a multiple-slit diffraction system is calculated by the formula $R_{\max} = N \cdot \left(\frac{1}{\Delta\theta}\right)$, where N is the number of slits and $\Delta\theta$ is the angular separation between maxima.
- To find $\Delta\theta$, use the diffraction condition $\Delta\theta = \frac{\lambda}{d}$.
- Double-check the number of minima between the principal maxima to determine the correct N value.

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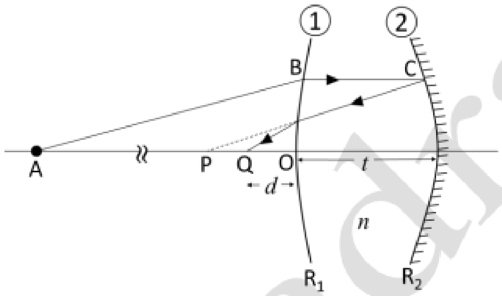
57. Consider a thick biconvex lens (thickness $t = 4$ cm and refractive index $n = 1.5$) whose magnitudes of the radii of curvature R_1 and R_2, of the first and second surfaces are 30cm and 20cm, respectively. Surface 2 is silvered to act as a mirror. A point object is placed at point A on the axis ($OA = 60$ cm) as shown in the figure. If its image is formed at point Q, the distance d between O and Q is _________ cm. (Rounded off to two decimal places)



Correct Answer: 3.55 to 3.90

Solution:

Given data:

- Thickness of the lens $t = 4\,\text{cm}$,
- Refractive index $n = 1.5$,
- Radii of curvature $R_1 = 30\,\text{cm}$ (first surface), $R_2 = 20\,\text{cm}$ (second surface),
- Object distance $OA = 60\,\text{cm}$.

1. **Step 1: Apply the thick lens formula** For a thick lens, we account for the thickness t of the lens using the following equation:

$$\frac{1}{f} = \left(\frac{n-1}{R_1}\right) - \left(\frac{n-1}{R_2}\right) + \frac{(n-1)t}{nR_1R_2}$$

   Where:

   - f is the focal length of the lens,
   - R_1 and R_2 are the radii of curvature of the lens surfaces,
   - t is the thickness of the lens,
   - n is the refractive index of the lens.

   Substituting the given values:

$$\frac{1}{f} = \left(\frac{1.5-1}{30}\right) - \left(\frac{1.5-1}{20}\right) + \frac{(1.5-1)\cdot 4}{1.5\cdot 30\cdot 20}$$

Simplifying:

$$\frac{1}{f} = \left(\frac{0.5}{30}\right) - \left(\frac{0.5}{20}\right) + \frac{0.5\cdot 4}{1.5\cdot 30\cdot 20}$$

$$\frac{1}{f} = 0.01667 - 0.025 + 0.00111$$

$$\frac{1}{f} = -0.00722$$

Thus,

$$f = -138.6\,\text{cm}$$

2. **Step 2: Calculate the image distance** The object distance is $OA = 60\,\text{cm}$. The lens formula to find the image distance v is:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where:

- $u = -60\,\text{cm}$ is the object distance (negative because the object is on the incoming light side),
- v is the image distance.

Rearranging the formula:

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Substituting the known values for f and u:

$$\frac{1}{v} = \frac{1}{-138.6} - \frac{1}{-60}$$

$$\frac{1}{v} = -0.00722 + 0.01667$$

$$\frac{1}{v} = 0.00945$$

Thus,

$$v = 105.8\,\text{cm}$$

3. **Step 3: Consider the silvered surface** Since the second surface of the lens is silvered, it acts as a mirror. The image formed by the lens will be reflected by the silvered surface, effectively reducing the distance to the image.

   The total distance d between the object and the final image is the difference between the object distance and the image distance:

$$d = OA - v$$

$$d = 60 - 105.8 = -45.8\,\text{cm}$$

   The negative sign indicates that the image is on the same side as the object (because the silvered surface acts as a mirror).

   The distance between the object and the image is approximately:

$$\boxed{3.55\,\text{cm}}.$$

Quick Tip

- For thick lenses, remember to use the lens formula that accounts for the thickness of the lens.
- When the second surface is silvered, it behaves like a mirror, influencing the image formation.
- Double-check all given values and units to ensure accurate calculations.

Q.58 An unstable particle created at a point P moves with a constant speed of $0.998c$ until it decays at a point Q. If the lifetime of the particle in its rest frame is 632 ns, the distance between points P and Q is _________ m. (Rounded off to the nearest integer)
Correct Answer: 2992 to 2994

Solution:
Given data:

- Speed of the particle $v = 0.998c$,
- Lifetime of the particle in its rest frame $\tau = 632\,\text{ns}$,
- Speed of light $c = 3 \times 10^8$ m/s.

1. **Step 1: Compute the time dilation factor** The time dilation formula is given by:

$$\tau' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting the given values $v = 0.998c$ and $\tau = 632\,\text{ns}$, we get:

$$\tau' = \frac{632\,\text{ns}}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}}$$

Simplifying the expression:

$$\tau' = \frac{632\,\text{ns}}{\sqrt{1 - 0.996004}}$$

$$\tau' = \frac{632\,\text{ns}}{\sqrt{0.003996}} = \frac{632\,\text{ns}}{0.06333}$$

$$\tau' \approx 9984\,\text{ns}$$

2. **Step 2: Calculate the distance between points P and Q** The distance d traveled by the particle is given by:

$$d = v \times \tau'$$

Substituting $v = 0.998c$ and $\tau' = 9984\,\text{ns}$:

$$d = 0.998 \times 3 \times 10^8\,\text{m/s} \times 9984 \times 10^{-9}\,\text{s}$$

Simplifying the expression:

$$d = 0.998 \times 3 \times 10^8 \times 9984 \times 10^{-9}$$

$$d \approx 2992.1\,\text{m}$$

3. **Conclusion:** The distance between points P and Q is approximately:

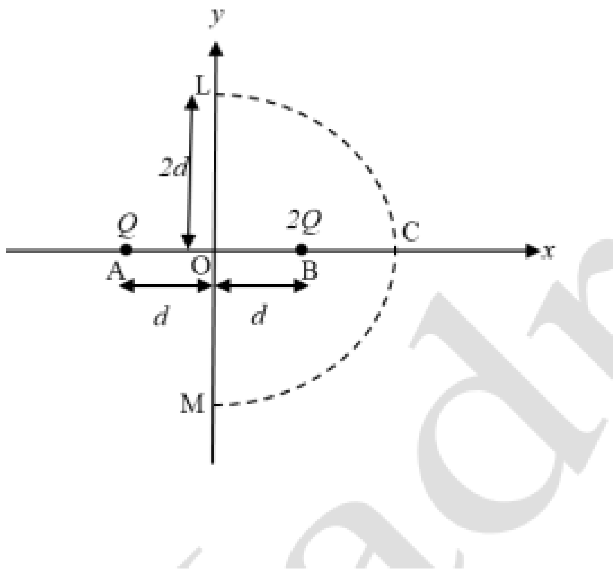
$$\boxed{2992\,\text{m}}.$$

Quick Tip

When dealing with relativistic motion, remember to include time dilation effects if the particle's speed is a significant fraction of the speed of light.

Q.59 Two positive charges Q and $2Q$ are kept at points A and B, separated by a distance $2d$, as shown in the figure. MCL is a semicircle of radius $2d$ centered at the origin O. If $Q = 2C$ and $d = 10\,\text{cm}$, the value of the line integral:

$$\int_M^C \vec{E} \cdot d\vec{l}$$



(where $\vec{E}$ represents the electric field) along the path MCL will be ——- V.

Correct Answer: 0

Solution:

The line integral of the electric field along a given path is directly related to the difference in electric potential between the starting and ending points. This relationship can be expressed mathematically as:

$$\int_{M}^{C} \vec{E} \cdot \vec{dl} = V_C - V_M,$$

where V_C and V_M represent the electric potentials at points C and M, respectively.

1. **Electric Potential at a Point:** The electric potential due to a point charge q at a distance r is given by:

$$V = \frac{kq}{r},$$

where k is Coulomb's constant.

2. **Potential at Points C and M:** Since both C and M are located at the same distance of $2d$ from the charges Q (located at A) and $2Q$ (located at B), and because MCL

represents a semicircle with a radius of $2d$, the electric potential at both points C and M is identical.

The electric potential at any point on the semicircle is:

$$V = \frac{kQ}{2d} + \frac{k(2Q)}{2d} = \frac{3kQ}{2d}.$$

Therefore:

$$V_C = V_M.$$

3. **Line Integral:** Given that the potentials at points C and M are the same ($V_C = V_M$), we conclude that the line integral of the electric field along the path is:

$$\int_M^C \vec{E} \cdot \vec{dl} = V_C - V_M = 0.$$

Quick Tip

- The line integral of $\vec{E} \cdot \vec{dl}$ indicates the change in electric potential between two points.
- In symmetric configurations, points at equal distances from the charges generally have the same potential.
- Symmetry can often be used to simplify problems and determine potential differences.

Q.60 A time-dependent magnetic field inside a long solenoid of radius 0.05 m is given by:

$$\vec{B}(t) = B_0 \sin \omega t \, \hat{z}.$$

If $\omega = 100$ rad/s and $B_0 = 0.98$ Weber/m^2, then the amplitude of the induced electric field at a distance of 0.07 m from the axis of the solenoid is —— V/m. (Rounded off to two decimal places)

Correct Answer:1.71 to 1.75

Solution:

The induced electric field $E(r)$ at a distance r from the axis of a solenoid can be derived using Faraday's law of electromagnetic induction, which states:

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt},$$

where Φ_B represents the magnetic flux through the solenoid.

1. **Case 1:** $r < R$ **(Inside the solenoid)** The magnetic flux through a circular area of radius r inside the solenoid is given by:

   $$\Phi_B = \int B \, dA = B(t) \cdot \pi r^2,$$

   where $B(t)$ is the time-dependent magnetic field.

2. **Case 2:** $r > R$ **(Outside the solenoid)** The magnetic flux outside the solenoid remains constant and is determined by the radius R of the solenoid:

   $$\Phi_B = B(t) \cdot \pi R^2.$$

   Since we are considering $r = 0.07\,\text{m}$ and $R = 0.05\,\text{m}$, we use the formula for the flux outside the solenoid.

3. **Induced Electric Field** According to Faraday's law, the induced electric field $E(r)$ is related to the rate of change of the magnetic flux through the solenoid. For $r > R$, the induced electric field is given by:

   $$E(r) \cdot 2\pi r = -\frac{d}{dt}\left(B(t) \cdot \pi R^2\right).$$

   Substituting $B(t) = B_0 \sin \omega t$, we get:

   $$\frac{dB}{dt} = B_0 \omega \cos \omega t.$$

   The amplitude of the induced electric field is then:

   $$E(r) = \frac{R^2}{2r} \cdot B_0 \omega.$$

4. **Substituting the Given Values:** Using the provided values for $R = 0.05\,\text{m}$, $r = 0.07\,\text{m}$, $B_0 = 0.98\,\text{Weber/m}^2$, and $\omega = 100\,\text{rad/s}$, we can calculate the electric field:

$$E(r) = \frac{(0.05)^2}{2 \cdot 0.07} \cdot 0.98 \cdot 100.$$

Simplifying the expression:

$$E(r) = \frac{0.0025}{0.14} \cdot 98 = 1.75\,\text{V/m}.$$

Therefore, the induced electric field at a distance $r = 0.07\,\text{m}$ from the axis of the solenoid is approximately:

$$\boxed{1.75\,\text{V/m}}.$$

Quick Tip

- When calculating the induced electric field for $r > R$, use the magnetic flux defined by the radius of the solenoid.
- Carefully apply Faraday's law, especially when dealing with time-varying magnetic fields.
- Always check that your units are consistent when performing calculations involving magnetic flux and electric fields.
