JEE (MAIN) 2025

MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

DATE & DAY: 04nd April 2025 & Friday

PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM – 06:00 PM

SUBJECT: PHYSICS

REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):
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PART : PHYSICS

1. A particle of mass m is at a distance 3R from the centre of earth. Find minimum kinetic energy of particle to leave earth field. (R = Radius of earth)

(1) $\frac{mgR}{3}$ (2) 3mgR (3) $\frac{1}{3}$mgR (4) $\frac{mgR}{2}$

Ans. **(1)**

Sol.

Energy conservation

$E_i = E_f$

K.E. + P.E. = 0 + 0

$K.E. + \frac{-GMm}{3R} = 0$

$K.E. = \frac{GMm}{3R}$

$= \frac{GMmR}{3R^2}$

$K.E. = \frac{mgR}{3}$

2. In a YDSE setup, the slits are separated by 1.5 mm and the distance between slits and screen is 2m. On using light of wavelength 400 nm, it is observed that 20 maximum of double slit experiment lie inside the central maxima of single slit diffraction. The width of each slit is _____ μm.

(1) 100 μm (2) 150 μm (3) 200 μm (4) 250 μm

Ans. **(2)**

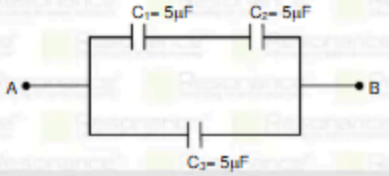
Sol. $20 \times \frac{\lambda D}{d} = \frac{2\lambda D}{b}$

$\frac{10}{d} = \frac{1}{b}$

$b = \frac{d}{10}$

b = 0.15 mm = 150 μm

3. Find equivalent capacitance between A and B after dielectric of dielectric constant K = 4 is filled completely between the space of plates of C_1

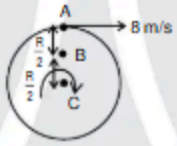


(1) 5 μF (2) 4 μF (3) 9 μF (4) 8 μF

Ans. **(3)**

Sol. $C_{eq} = \frac{(4\times5)\times5}{(4\times5)+5} + 5 = 9\mu F$

4. A disc is performing pure rolling if speed of top point is 8 m/s. Find speed of point B.



(1) 2 m/s (2) 4 m/s (3) 6 m/s (4) 8 m/s

Ans. **(3)**

Sol. Vel, at point (A) $\rightarrow 2V = 8$

$V = 4$ m/sec

$v = \omega R$

$4 = \omega R$

$\omega = \frac{4}{R}$

Vel at point (B) $\rightarrow V_B \rightarrow V + \frac{R}{2}\omega$

$\Rightarrow V + + \frac{R}{2}\times\frac{4}{R}$

$= 4 + 2$

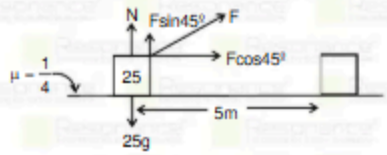
$= 6$ m/s

5. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle 45° with the horizontal. The friction coefficient between the block and the surface is 0.25. The block travels at a uniform velocity, the work done by the applied force during a displacement of 5m of block is :

(1) 735 J (2) 490 J (3) 245 J (4) 970 J

Ans. **(3)**

Sol.



$F\cos45° = \mu_k(25g - F\sin45°)$

$$F = \frac{\mu_k \times 25g}{\cos 45° + \mu_k \sin 45°} \Rightarrow \frac{\frac{1}{4}\times 25\times 10}{\frac{1}{\sqrt{2}} + \frac{1}{4}\times\frac{1}{\sqrt{2}}}$$

work done $\Rightarrow 69.3 \times \cos45° \times 5$ $\quad F = 69.3N$

$69.3 \times \frac{1}{\sqrt{2}} \times 5 \Rightarrow 245J$

6. For N type semiconductor which of the following options are correct

(1) Pentavalent impurity is mixed in intrinsic semiconductor

(2) hole is majority charge carrier

(3) $n_e n_h \geq n_i^2$

(4) trivalent impurity is mixed in intrinsic semiconductor

Ans. **(1)**

7. **Statement-1:** angular momentum and Planck constant have the same dimension

Statement-2: Planck constant depends on angular momentum integrally

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

Ans. **(3)**

8. Two polarises P_1 & P_2 are aligned in such a way that intensity is zero. P_3 polarises is inserted between P_1 and P_2 such that final transmitted ray will have the maximum intensity. Find angle between P=1and P_3.

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{8}$

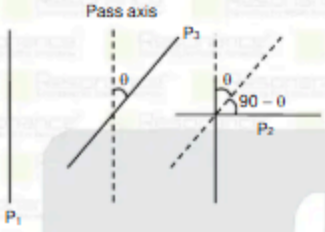
Ans. **(1)**

Sol. Melus Law

$I_1 = I_0 \cos^2\theta$ .....(1)

$I_2 = (I_0 \cos^2\theta) \cos^2 (90 - \theta)$

$I_2 = I_0 \cos^2\theta_1 \sin^2 \theta_1$



$I_2 = \frac{I_0}{4} \sin^2 2\theta_1$

for maximum

$\theta_1 = \frac{\pi}{4}$

9. Ratio of dimension of electric dipole and magnetic dipole moment is $M^pL^qT^r$, then value of p and q

(1) 0, –1 (2) 1, –1 (3) 1, 0 (4) –1, 1

Ans. **(1)**

Sol. $\frac{p}{M} = \frac{q \times d \times B}{\tau}$

$\tau = MB$

$F = qvB$

$\frac{p}{M} = \frac{q \times d \times F}{F \times r \times q}$

$= \frac{1}{LT^{-1}} = \frac{T}{L}$

$= M^0L^0T^1$

$p = 0$

$q = -1$

10. In vernier scale 300 division in 15 cm and 25 vernier scale division is equal to 24 main scale division, than least count will be

(1) $\frac{1}{240}$ cm (2) $\frac{1}{120}$ cm (3) $\frac{1}{380}$ cm (4) $\frac{1}{480}$ cm

Ans. **(4)**

Sol. $300 \times VS = 15$

$VS = \frac{1}{20}$ cm

$25\ VS = 24\ MS$

$MS = \frac{25}{24} VS$

$Lc = MS - VS$

$= \left(\frac{25}{24} - 1\right) \times VS$

$= \frac{1}{24} \times \frac{1}{20}$ cm

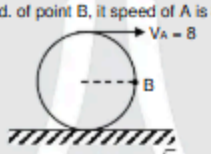
$LC = \frac{1}{480}$ cm

11. Which velocity we provide a thing so that we throw at up and It does hot come back on the earth.

(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{\frac{2GM}{R}}$ (3) $\sqrt{\frac{GM}{3R}}$ (4) $\sqrt{\frac{GM}{2R}}$

Ans. **(2)**

12. Wheel is in pure rolling find speed. of point B, it speed of A is 8 m/s



(1) $2\sqrt{2}$ (2) $2\sqrt{4}$ (3) $2\sqrt{3}$ (4) $4\sqrt{2}$

Ans. **(4)**

Sol. $V_B = V_C\sqrt{2}$

$= \frac{8}{2}\sqrt{2} = 4\sqrt{2}$

13. A medium has relative permittivity $\frac{1}{0.085}$ and relative permeability is $\frac{10}{\pi}$. Find ratio of speed of light in vacuum to the medium.

(1) 6.12 (2) 3.14 (3) 2.28 (4) 1.27

Ans. **(1)**

Sol. $\varepsilon_r = \frac{1}{0.085}$

$\mu_r = \frac{10}{\pi}$

Speed of light in vacuum $c = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r\mu_r}}$

$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$ ....(2)

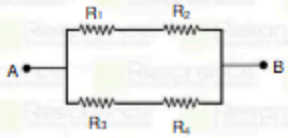
Ratio $= \sqrt{\mu_r\varepsilon_r}$

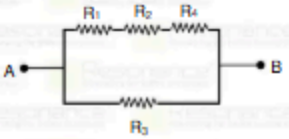
$\Rightarrow \sqrt{\frac{1}{0.085} \times \frac{10}{\pi}}$

$\Rightarrow \sqrt{37.46}$

$= 6.12$

14. Four resistances R_1, R_2, R_3 are of 5Ω each and R_4 is of 10Ω. In which of the following combination equivalent resistance will be 6Ω :

(1) 

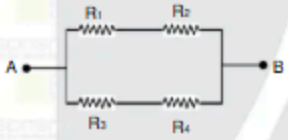
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



$$\Rightarrow \quad R_{eq} = \frac{(5+5)\times(5+10)}{5+5+5+10} = 6\Omega.$$

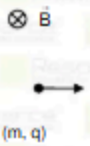
15. Match the following (Where the letters dQ, dU and dW have usual meaning)

Column I	**Column II**
(A) Isochoric process	(P) dQ = W
(B) Adiabatic process	(Q) dQ = dU
(C) Isothermal process	(R) dQ = 0
(D) Isobaric process	(S) dQ = dU + PdV

(1) A-Q, B-R, C-P, D-S
(2) A-S, B-R, C-P, D-Q
(3) A-R, B-Q, C-P, D-S
(4) A-P, B-R, C-Q, D-S

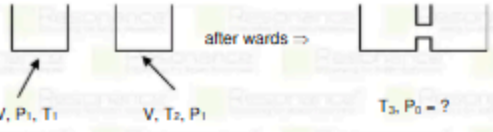
Ans. **(1)**

16. Charge and mass are q and m respectively. Magnetic field is perpendicular to the motion of charge. Find time when the charge will again come at it's own position i.e. its. initial position.



Sol. $T = 2\pi \dfrac{m}{qB}$

17. Find final pressure after connecting two useless.



Ans. $\left(\frac{2P_1}{T_1}+\frac{P_2}{T_2}\right)\times\frac{T_3}{3}-P_0$

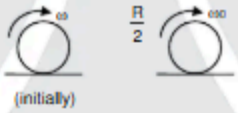
Sol. $n=\frac{PV}{RT}$

$n_1+n_2=n_1^1+n_2^1$

$\frac{P_1\times 2V}{T_1}+\frac{P_2\times V}{T_2}=\frac{P_0\times 3V}{T_3}$

$\left(\frac{2P_1}{T_1}+\frac{P_2}{T_2}\right)\times\frac{T_3}{3}-P_0$

18. Mass sphere is decreasing at a constant rate. Find final ω_0 when its radius becomes $\left(\frac{R}{2}\right)$



(1) 16ω (2) 32ω (3) 64ω (4) 36ω

Ans. **(2)**

Sol. $L=I\omega=\frac{2}{5}mR^2\omega$

$=\frac{2}{5}\rho\times\frac{4}{3}\pi R^3\times R^2\times\omega$

$L=$ constant $\because \tau=0$

$\Rightarrow R^5\omega=$ constant

$\omega_0=\omega\times 2^5=32\,\omega$

19. n identical bulbs each takes power P when connected with main supply separately. If n bulbs are connected in series with same main supply. Then total power developed will be

(1) nP (2) $\frac{P}{n^2}$ (3) $\frac{P}{n}$ (4) n^2P

Ans. **(3)**

Sol. $P=\frac{V^2}{R}$

$P'=\frac{V^2}{nR}$

$P'=\frac{P}{n}$

20. Displacement of a wave is expressed as $x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)$ m. The wavelength of wave if its Velocity is 300 m/s is

(1) 0.5 m (2) 5 m (3) 0.33 (4) 3 m

Ans. (4)

Sol. $V = f\lambda$ and $\omega = 628 = 2\pi f$

$300 = \frac{628}{2\pi}\lambda$

$\lambda = 3m$

21. A metallic ring is uniformly charged as shown in fig. AC & BD are two mutually perpendicular diameters. Electric field due to are AB at 'O' is E in magnitude. What would be the magnitude of Electric field at 'O' due to arc ABC?

(1) $\sqrt{2}E$ (2) Zero (3) E/2 (4) 2E

Ans. (1)

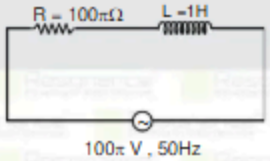
Sol. $E_{AB} = \frac{K\lambda}{R}\sqrt{2}$

$E_{ABC} = \frac{2K\lambda}{R}$

$= \frac{2}{\sqrt{2}}E$

$= \sqrt{2}E$

22. An AC source of 100π volt is connected to the given circuit. Find maximum value of the current in the circuit



(1) $\frac{1}{\sqrt{2}}$ A (2) 1 A (3) $\sqrt{3}$ A (4) 0.5 A

Ans. (2)

Sol. $Z = i\omega L + R$

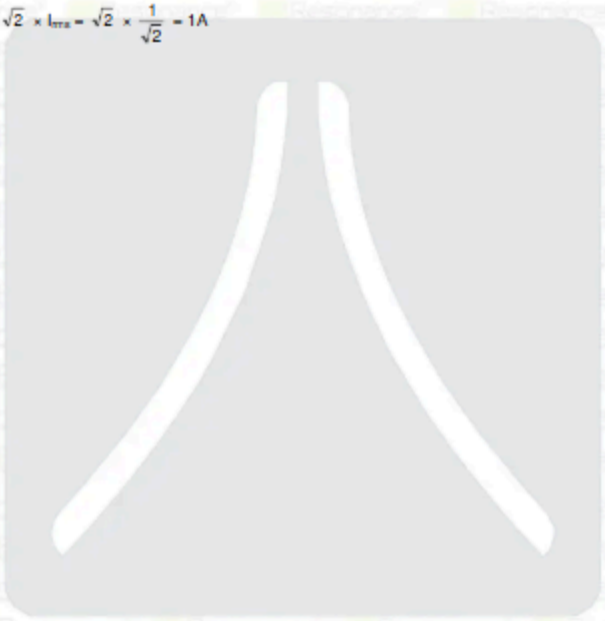
$|Z| = \sqrt{\omega^2 L^2 + R^2}$

$|Z| = 100\sqrt{2}\,\pi\,\Omega$

$I_{rms} = \dfrac{V}{Z} = \dfrac{100\pi}{100\sqrt{2}\pi} = \dfrac{1}{\sqrt{2}}\,A$

$I_{rms} = \dfrac{I_0}{\sqrt{2}}$

$\therefore\ I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 1A$



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MEMORY BASED QUESTIONS & TEXT SOLUTION

SHIFT-2

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PAPER-1

Duration: 3 Hrs.
Time: 03:00 PM – 06:00 PM

SUBJ ECT: CHEMISTRY

PART : CHEMISTRY

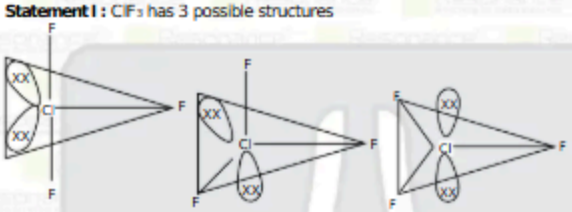
1. Maximum and minimum ionisation energy of group 13 element is for :

(1) B, In (2) B, T/ (3) A/, In (4) Al, T/

Ans. **(1)**

Sol. Order of IE B >T/ >Ga >Al >In

2. **Statement I :** ClF_3 has 3 possible structures



(I) **(II)** **(III)**

Statement II : III is most stable structure due to least LP-BP repulsion

Select correct option

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I & Statement II are correct.

(3) Statement I is incorrect but Statement II is correct.

(4) Both Statement I & Statement II are incorrect.

Ans. **(1)**

Sol. I is most stable structure according to bent's rule

3. Total number of electrons in Cr(z =24) for which the value of azimuthal quantum number (/) is 1 & 2

(1) 18 (2) 15 (3) 17 (4) 20

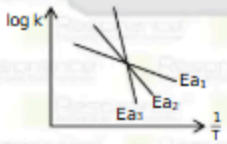
Ans. **(3)**

Sol.

$_{24}Cr$	$1s^2$	$2s^2$	$2p^6$	$3s^2$	$3p^6$	$3d^5$	$4s^1$
/ =	0	0	1	0	1	2	0

6 +6 +5 =17

4. Consider the following graph between rate constant log (k) and $\frac{1}{T}$



Select correct order :

(1) $Ea_1 > Ea_2 > Ea_3$ (2) $Ea_3 > Ea_2 > Ea_1$ (3) $Ea_3 > Ea_1 > Ea_2$ (4) $Ea_1 > Ea_2 > Ea_3$

Ans. **(3)**

Sol. $k = Ae^{-E_a/RT}$

$/nk = /nA - \left(\frac{E_a}{R}\right)\frac{1}{T}$

$y = c + m x \qquad m = \frac{-E_a}{R}$

$|m| = \frac{E_a}{R}$

5. The amount of CaO produced when 150 kg of limestone (75% pure) is strongly heated:
(1) 625 mol (2) 1125 mol (3) 500 mol (4) 1000 mol

Ans. **(1125)**

Sol. $CaCO_3(s) \xrightarrow[75\%]{\Delta} CaO(S) + CO_2(g)$

$\frac{150\times10^3}{100}\text{Mol}\ \frac{150\times10^3}{100}\times\frac{75}{100}$

$=1125$ mol

6. Consider the following
(a) $[Ni(CO)_4]$ (b) $[Ni(CN)_6]^{-2}$ (c) $[FeF_6]^{-3}$ (d) $[CoF_6]^{-3}$
The correct order of number of unpaired electrons is
(1) c >d >a =b (2) c >d >a >b (3) d >c >a =b (4) c >a >d =b

Ans. **(1)**

Sol: (a) diamagnetic
(b) diamagnetic
(c) Fe^{+3} (wf) n=5
(d) Co^{+3} (wf) n =4

7. Consider the following zero order reaction
A → Product
Half life of the reaction is 1 hr, if initial concentration of the reactant is 2M.
Find the half life of the reaction in min, if the initial conc. of the reaction is 0.5 M

Ans. **(15)**

Sol. $t_{1/2} \propto (a)^{1-n}$

$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_1}{a_2}\right)^{1-n} \frac{1}{(t_{1/2})_2} = \frac{2}{0.5}$

$(t_{1/2})_2 = \frac{1}{4}\text{hr} = 15\text{min}$

8. If x mg of $Mg(OH)_2$ is added in 1L of solution to make a solution with pH =12, then find the value of x.
Given molar mass of $Mg(OH)_2 = \frac{58g}{mol}$ & assume $Mg(OH)_2$ dissociates completely in water.

Ans. **(290)**

Sol. $\underset{\frac{10^{-2}}{2}M}{Mg(OH)_2\,aq} \longrightarrow Mg^{+2}_{(aq)} + \underset{10^{-2}M}{2OH^-_{aq}}$

Moles of $Mg(OH)_2 = \frac{10^{-2}}{2}\text{mol} = 29\times10^{-2}\,g = 290\text{mg}$

9. Match the column
(For an ideal gas)

Column-I	**Column-II**
(a) Adiabatic process	(i) $W = O$
(b) Isochoric process	(ii) $\Delta E = W$
(c) Isothermal process	(iii) $\Delta P = O$
(d) Isoboric process	(iv) $\Delta E = O$

(1) a → (ii), b → (i), c → (iv), d → (iii) (2) a → (i), b → (ii), c → (iv), d → (iii)
(3) a → (i), b → (iii), c → (iv), d → (ii) (4) a → (iv), b → (i), c → (ii), d → (iii)

Ans. **(1)**
Sol. Theory based

10. The incorrect relationship in the following pair in relation to ionization enthalpy is :
(1) $Mn^{+} < Mn^{+2}$ (2) $Mn^{+} < Fe^{+2}$ (3) $Fe^{+2} < Fe^{+3}$ (4) $Mn^{+} < Cr^{+}$

Ans. **(2)**
Sol. $_{25}Mn^{+2}$: [Ar] $3d^5$
$_{26}Fe^{+2}$: [Ar] $3d^6$

11. Formula $Co(NH_3)_4Cl_3$ (1 mol) on reaction with excess $AgNO_3$ results in x mol of AgCl. Also x is number of LP on central atom of BrF_5 then determine total number of geometric isomers possible for given formula

Ans. **(2)**
Sol. $\overset{\infty}{B}rF_5$ i.e. x = 1
$[CoCl_2(NH_3)_4]Cl$, it's Cis and trans geometric isomers are possible.

12. At what pH does $[H^+] + [OH^-]$ attain minimum value at 25°C for an aqueous solution

Ans. **(7)**
Sol. pH = 7
$[H^+] + [OH^-]$
$= 10^{-7} + 10^{-7} = 2 \times 10^{-7}$ M

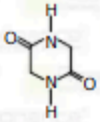
13. **Statement I:** Ionisation energy of Group 14 element is greater than corresponding Group 13 element.
Statement II : MP of Group 13 element is greater than corresponding Group 14 element.
Select correct option:
(1) Statement I is correct but Statement II is incorrect.
(2) Both Statement I & Statement II are correct.
(3) Statement I is incorrect but Statement II is correct.
(4) Both Statement I & Statement II are incorrect.

Ans. **(1)**
Sol. Theory based

14. **Statement I:** $k_f = \frac{RT_f^2}{1000 \times \Delta S_{fusion}}$ is formula for molal depression constant.
Statement II: k_f of water is greater than C_6H_6.
(1) Statement I is correct but Statement II is incorrect.
(2) Both Statement I & Statement II are correct.
(3) Statement I is incorrect but Statement II is correct.
(4) Both Statement I & Statement II are incorrect.

Ans. **(4)**
Sol. $k_f = \frac{RT_f^2}{1000 \times \ell_f}$

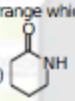
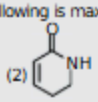
15. A dipeptide 'z' on complete hydrolysis gives 'y' and 'z'. y on treatment with aq. HNO_2 produces lactic acid on the other hand 'z' on heating gives the following the given dipeptide is:



(1) Alanine – Glycine (2) Valine – Glycine (3) Valine – Leucine (4) Alamine – Alamine

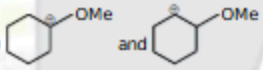
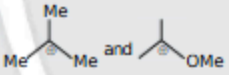
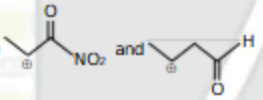
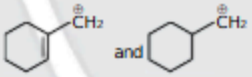
Ans. (1)

16. Arrange which of the following is maximum basic.

(1)  (2)  (3)  (4) 

Ans. (3)

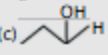
17. In which of the following first cation is more stable than second.

(a)  (b)  (c)  (d) 

(1) (a) & (d) (2) (a), (b) & (c) (3) (c) & (d) (4) (b), (c) & (d)

Ans. (1)

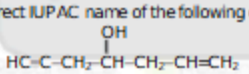
18. Which of the following gives yellow precipitate with I_2/NaOH.

(a)  (b)  (c)  (d) 

(1) a & b (2) b & c (3) a & c (4) b & d

Ans. (1)

19. The correct IUPAC name of the following compounds is



(1) 4-Hydroxyhept-6-en-1-yne (2) Hept-6-en-1-yne-4-ol
(3) 4-Hydroxyhept-1-en-6-yne (4) Hept-1-en-6-yne-4-ol

Ans. (4)

20. Given below are two statements:
Statement-I: Aqueous KOH gives elimination reaction as major product always.
Statement-II: Alcoholic KOH eliminates H^+ from β-carbon atom.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement-I is correct and statement-II is incorrect
(2) Statement-I is incorrect and statement-II is correct.
(3) Both statement-I and statement-Ii are correct
(4) Both statement-I and statement-Ii are incorrect

Ans. (2)

PART : MATHEMATICS

1. $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\ldots\ldots\ldots\infty=?$

(1) $\cot^{-1}\left(\frac{1}{2}\right)$ (2) $\cot^{-1}\left(\frac{1}{3}\right)$ (3) $\cot^{-1}(2)$ (4) $\cot^{-1}(3)$

Ans. **(1)**

Sol. $\sum_{r=1}^{\infty}\cot^{-1}\left(r^2+\frac{3}{4}\right)$

$T_r=\cot^{-1}\left(r^2+\frac{3}{4}\right)=\tan^{-1}\left(\frac{1}{r^2+\frac{3}{4}}\right)=\tan^{-1}\left(\frac{1}{1+r^2-\frac{1}{4}}\right)=\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$

$=\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$

$T_1=\tan^{-1}\left(\frac{3}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$

$T_2=\tan^{-1}\left(\frac{5}{2}\right)-\tan^{-1}\left(\frac{3}{2}\right)$

$T_3=\tan^{-1}\left(\frac{7}{2}\right)-\tan^{-1}\left(\frac{5}{2}\right)$

$\vdots$

$T_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$

$\sum T_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$

$\sum_{n=1}^{\infty}T_n=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}=\cot^{-1}\left(\frac{1}{2}\right)$

2. Let $L_1:\frac{x-1}{3}=\frac{y}{4}=\frac{z}{5}$ & $L_2:\frac{x-p}{2}=\frac{y}{3}=\frac{z}{4}$. If the shortest distance between L_1 & L_2 is $\frac{1}{\sqrt{6}}$. Then possible value of p is –

(1) 3 (2) 5 (3) 2 (4) 7

Ans. **(3)**

Sol. $\vec{r}=\vec{a}+\lambda\vec{p}$ $\vec{r}=\vec{b}+\mu\vec{q}$

Shortest distance $\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$\vec{a}=(1,0,0)$ $\vec{p}=(3,4,5)$

$\vec{b}=(p,0,0)$ $\vec{q}=(2,3,4)$

$\vec{p}\times\vec{q}=\hat{i}-2\hat{j}+\hat{k}$ $(\vec{b}-\vec{a})=(p-1)\hat{i}$

$d=\left|\frac{(p-1).1}{\sqrt{1+4+1}}\right|=\frac{1}{\sqrt{6}}$

$|p-1|=\pm1$ $p=\begin{cases}2\\0\end{cases}$

3. Let mean & variances of observations 2, 3, 3, 4, 5, 7, a, b is 4 & 2 then mean deviation about mole is observations is

Ans. **(1)**

Sol. $\text{mean} = 4 \quad \frac{2+3+3+4+5+7+a+b}{3} = 4$

$a+b+24 = 32$

$a+b = 8 \quad \ldots(1)$

$r^2 = \frac{4+9+9+16+25+49+a^2+b^2}{3} - 16 = 2$

$\frac{a^2+b^2+112}{3} = 18$

$a^2+b^2 = 144-112$

$a^2+b^2 = 32$

$(a+b)^2 - 2ab = 32$

$64 - 2ab = 32$

$2ab = 32$

$ab = 16$

$\therefore \; x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

$x = 4$

$\therefore$ a = b = 4

$\therefore$ observation is 2,3,3,4,5,7,4,4

Mode = 4

$\therefore \; \text{mean deviation} = \frac{\sum|x-\text{mod}|}{x} = \frac{2+1+1+0+1+3+0+0}{8} = 1$

4. $\int \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{\sqrt{1+x^2}-x}\,dx =$

(1) $\frac{1}{2}\left[\frac{(\sqrt{x^2+1}+x)^{21}}{21} + \frac{(\sqrt{x^2+1}+x)^{19}}{19}\right] + c$ (2) $\left[\frac{(\sqrt{x^2+1}+x)^{21}}{21} + \frac{(\sqrt{x^2+1}+x)^{19}}{19}\right] + c$

(3) $\left[\frac{(\sqrt{x^2+1}+x)^{20}}{20} + \frac{(\sqrt{x^2+1}+x)^{19}}{19}\right] + c$ (4) $\left[\frac{(\sqrt{x^2+1}+x)^{20}}{20} + \frac{(\sqrt{x^2+1}+x)^{18}}{18}\right] + c$

Ans. **(1)**

Sol. $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$\int \frac{(\sec\theta+\tan\theta)^{19}}{\sec\theta-\tan\theta} \cdot \sec^2\theta \, d\theta$

$\int (\sec\theta+\tan\theta)^{20}(\sec^2\theta)d\theta$

$\sec\theta+\tan\theta = t$

$\sec\theta-\tan\theta = \frac{1}{t}$

$\tan\theta = \frac{1}{2}\left(t - \frac{1}{t}\right)$

$\sec^2\theta\, d\theta = \frac{1}{2}\left(1+\frac{1}{t^2}\right)dt$

$\frac{1}{2}\int t^{20}\left(1+\frac{1}{t^2}\right)dt = \frac{1}{2}\left[\frac{t^{21}}{21}+\frac{t^{19}}{19}\right]+c$

$= \frac{1}{2}\left[\frac{(\sec\theta+\tan\theta)^{21}}{21}+\frac{(\sec\theta+\tan\theta)^{19}}{19}\right]+c$

$= \frac{1}{2}\left[\frac{(\sqrt{x^2+1}+x)^{21}}{21}+\frac{(\sqrt{x^2+1}+x)^{19}}{19}\right]+c$

5. There are two sets A and B each has 3 elements which are in A.P. sum of the elements in each is 36 & difference of the elements of A & B is d & D respectively. Such that $D = 3+d$ and $\frac{p+q}{p-q} = \frac{19}{5}$. Where p & q are product of the elements of A & B respectively. The value of $p - q$ is

Ans. **(510)**

Sol. $A = \{12-d, 12, 12+d\}$
$A = \{9-d, 12, 15+d\}$

$\frac{p+q}{p-q} = \frac{19}{5}$ $\qquad \frac{p}{q} = \frac{12}{7}$

$\frac{(12-d)(12)(12+d)}{(q-d)(12)(15+d)} = \frac{12}{7}$

$d = 6$

$A = \{6, 12, 18\}$ $\qquad B = \{3, 12, 21\}$

$p - q = 6\times12\times18 - 3\times12\times21$

$= 12\times3\times3\,(12-7) = 540$ Ans.

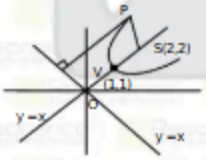
6. If $y = x$ is axis of parabola whose vertex & focus are at distances $\sqrt{2}$ & $2\sqrt{2}$ from origin respectively in first quadrant. If (1, k) lies on parabola then possible value of k is

(1) 7 (2) 8 (3*) 9 (4) 10

Ans. **(3)**

Sol. $y = x$ is axis & distances of vertex & focus from origin are $\sqrt{2}$ & $2\sqrt{2}$ respectively.

$OV = \sqrt{2}$



$V(r\cos\theta, r\sin\theta)$

$V(1, 1)$, $S(2,2)$

equation of directrix is

$x + y + \lambda = 0$ at O, $\lambda = 0$ $\qquad x + y = 0$

$P(1, k)$ lies on parabola

$\therefore\ SP = PM$

$\sqrt{(2-1)^2+(2-k)^2} = \left|\frac{1+k}{\sqrt{2}}\right|$

$1+4-4k+k^2=\frac{1+k^2+2k}{2}$

$\Rightarrow k^2-10k+9=0$

$\Rightarrow (k-1)(k-9)=0 \qquad K=1 \text{ or } 9$

7. Let f(x) and g(x) satisfies the functional equation

$2g(x)+3g\left(\frac{1}{x}\right)=x$ and $2f(x)+3f\left(\frac{1}{x}\right)=x^2+5$

If $\alpha=\int_1^2 f(x)dx \& \beta=\int_1^2 g(x)dx$. Then $30\alpha+5\beta$.

(1) $8+\ell n9$ (2) $8+\ell n8$ (3) $11+\ell n8$ (4) $11+\ell n3$

Ans. **(2)**

Sol. $2g(x)+3g\left(\frac{1}{x}\right)=x \quad ....(1)$

$2g\left(\frac{1}{x}\right)+3g(x)=\frac{1}{x} \quad .....(2)$

$(1)\times 2-(2)\times 3$

$-5g(x)=2x-\frac{3}{x}$

$g(x)=\frac{3}{5x}-\frac{2x}{5}$

Similarly

$f(x)=\frac{3}{5x^2}-\frac{2x^2}{5}+1$

$\alpha=\left[\frac{-3}{5x}-\frac{2x^3}{15}+x\right]_1^2=\frac{-3}{10}-\frac{16}{15}+2-\left(\frac{-3}{5}-\frac{2}{15}+1\right)$

$=\frac{-9-32+60+18+4-30}{30}$

$=\frac{11}{30}$

$\beta=\left[\frac{3}{5}\ell nx-\frac{2x^2}{5\times 2}\right]_1^2=\frac{3}{5}\ell n2-\frac{4}{5}-0+\frac{1}{5}=\frac{3}{5}\ell n2-\frac{3}{5}$

$30\alpha+5\beta=11+3\ell n2-3=8+\ell n8$

8. $\sum_{r=1}^{15} r^2 \cdot {}^{15}C_r=2^m.3^n.5^l$ then $\ell+m+n=$

Ans. **(19)**

Sol. $S=\sum_{r=1}^{15} r^2\ {}^{15}C_r=\sum_{r=1}^{15} r^2.\frac{15}{r}.{}^{14}C_{r-1}$

$=15\sum_{r=1}^{15}((r-1)+1)\ {}^{14}C_{r-1}$

$=15\sum_{r=2}^{15}(r-1).\frac{14}{r-1}.{}^{13}C_{r-2}+15.\sum_{r=1}^{15}{}^{14}C_{r-1}$

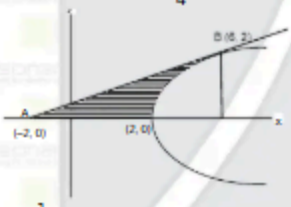
$=15.14.2^{13}+15.2^{14}$
$=3.5.2.7.2^{13}+3.5.2^{14}$
$=3.5.7.2^{14}+3.5.2^{14}$
$=3.5.2^{14}(7+1)$
$=3^1.5^1.2^{17}=2^{17}.3^1.5^1$
$\therefore\ \ell+m+n=17+1+1=19$

9. A line passing through A (–2, 0) touches parabola $y^2=x-2$ at B, then area between parabola, line AB & x-axis is

(1) $\frac{5}{3}$ (2) $\frac{7}{3}$ (3) $\frac{8}{3}$ (4) $\frac{10}{3}$

Ans. **(3)**

Sol. line through A(–2, 0) is
$y-0=m(x+2)$
parabola is $y^2=x-2$
for point of intersection
$m^2(x+2)^2=x-2$
$\Rightarrow\quad m^2(x^2+4x+4)=x-2$
$\Rightarrow\quad m^2x^2+(4m^2-1)x+4m^2+2=0$
$D=0\quad\Rightarrow(4m^2-1)^2-4m^2(4m^2+2)=0$
$\Rightarrow 16m^4-8m^2+1-16m^4-8m^2=0$
$\Rightarrow 16m^2=1\qquad m=\pm\frac{1}{4}$



$y=\frac{1}{4}(x+2)\qquad x-4y+2=0$

T.at B $yy_1=\frac{1}{2}(x+x_1)-2$
$2yy_1=x+x_1-4$
$x-2yy_1+x_1-4=0$
$2y_1=4\ \&\ x_1-4=2$
$y_1=2,\ \ x_1=6$

Required Area $=\int_0^2\left(y^2+2-(4y-2)\right)dy-\left[\frac{y^3}{3}-2y^2+4y\right]_0^2=\frac{8}{3}-8+8=\frac{8}{3}$

10. α is one root of equation $x^2+x+1=0$ and $\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2=20$, then value of n is

Ans. **(11)**

Sol. $x^2+x+1=0 \begin{cases}\omega \\ \omega^2\end{cases}=\alpha$

$$\sum_{k=1}^{n}\left(\omega^k+\frac{1}{\omega^k}\right)^2=20$$

$$\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+\ldots\ldots\ldots=20$$

$$(-1)^2+(-1)^2+(2)^2+\ldots\ldots=20$$

$$(1+1+4)+(1+1+4)+(1+1+4)+1+1=20$$

$$n=11$$

11. The sum $\frac{2.1}{4+3\cdot1^2+1^4}+\frac{2.2}{4\cdot3.2^2+2^4}+\frac{2.3}{4+3\cdot3^2+3^4}+\ldots\ldots\ldots$ up to 20 this is $\frac{m}{n}$ (where m, n are co-prime) then $2m+n$ is equal to

Ans. **(421)**

Sol. Here $t_n=\frac{2n}{4+3n^2+n^4}$

$$=\frac{2n}{(n^2+2)^2-n^2}=\frac{2n}{(n^2+2+n)(n^2+2-n)}$$

$$=\frac{1}{n^2-n+2}-\frac{1}{n^2+n+2}$$

$$T_1=\frac{1}{2}-\frac{1}{4}$$

$$T_2=\frac{1}{4}-\frac{1}{8}$$

$$T_3=\frac{1}{8}-\frac{1}{14}$$

$$T_4=\frac{1}{14}-\frac{1}{22}$$

$$T_{20}=\frac{1}{382}-\frac{1}{422}$$

$$\therefore S_{20}=\frac{1}{2}-\frac{1}{422}=\frac{211-1}{422}=\frac{210}{422}=\frac{105}{211}=\frac{m}{n}$$

Then $2m+n=421$

12. In triangle ABC, A is (2, –3 , 1), B is (3, –1, –5) and C is (1, –4, –4). If G is the centroid of triangle ABC then the value of $6[(AG)^2+(BG)^2+(CG)^2]$ is equal to

(1) 164 (2) 184 (3) 82 (4) 41

Ans. **(1)**

Sol. $AB=\sqrt{41}=c$

$BC=\sqrt{14}=a$

$CA=\sqrt{27}=b$

$AG=2/3$ (median)

$(AG)^2=4/9\,(m_1)^2$

$6[(AG)^2+(BG)^2+(CG)^2]=6\times4/9\,(m_1^2+m_2^2+m_3^2)$

$=\frac{8}{3}\times\frac{3}{4}\,(a^2+b^2+c^2)$

$=2\,(41+14+27)=164$

13. Let A = {– 3,– 2, – 1, 0, 1, 2, 3}& xRy ⇒ 2x – y ∈ {0, 1}. If ℓ is number of elements in given relation, m & n are minimum number of elements to be added to make it reflexive & symmetric respectively then value of $\ell+m+n$ is

Ans. **(17)**

Sol. Let A = {– 3,– 2, – 1, 0, 1, 2, 3}

xRy ⇒ 2x – y ∈ {0, 1}

∴ R = {(0, 0), (0, –1), (2, 3), (1, 1), (1, 2), (–1, 2), (–1, – 3}

n = 7

for reflexive the (–1. –1), (2, 2), (3, 3), (– 3, – 3), (– 2, – 2)

must be added ∴ m = 5

for symmetric (–1, 0), (3, 2), (2, 1), (2, –1), (–3, –1)

must be added ∴ n = 5

∴ $\ell+m+n=7+5+5=17$