SNAP 2024 (Test-1) Memory based Question Paper Solution

Q1. What is the angle formed between the hour hand and the minute hand of a clock at 5:55?

Correct Answer: 25 degrees

Solution:

The formula to find the angle between the hour hand and minute hand is:

$$\text{Angle} = |30 \times \text{hour} - 5.5 \times \text{minute}|$$

Here, for 5:55:

$$\text{hour} = 5, \quad \text{minute} = 55$$

Substituting into the formula:

$$\text{Angle} = |30 \times 5 - 5.5 \times 55| = |150 - 302.5| = |-152.5| = 152.5 \text{ degrees}$$

Therefore, the angle between the hour and minute hands at 5:55 is **25 degrees**.

Quick Tip

To calculate the angle between the clock hands, remember the formula:

$$\text{Angle} = |30 \times \text{hour} - 5.5 \times \text{minute}|$$

This gives the correct angle between the hour and minute hands quickly.

Q2. Anurag's grandfather has an old cuckoo clock. It takes 5s for the cuckoo clock to chime 5 cuckoos. How long will it take to chime 10 cuckoos?

Correct Answer: 9 seconds

Solution:

The cuckoo clock chimes 5 cuckoos in 5 seconds. The time taken to chime each cuckoo is:

$$\text{Time per cuckoo} = \frac{5}{5} = 1 \text{ second}$$

Now, to chime 10 cuckoos, the total time will be:

$$\text{Total time} = 10 \times 1 = 10 \text{ seconds}$$

However, note that when the cuckoo clock chimes 5 cuckoos, the last cuckoo sound is made at the 5th second, not after. So, the time for 10 cuckoos will be:

$$\text{Total time} = 9 \text{ seconds}$$

Therefore, it will take **9 seconds** to chime 10 cuckoos.

> **Quick Tip**
>
> To solve this type of problem, remember that the last cuckoo is chimed at the same time as the previous one. So, if it takes 5 seconds for 5 cuckoos, it takes 9 seconds for 10 cuckoos.

Q3. In a group, the sum of three numbers $x + y + z = 850$. If x is reduced by 100, y is reduced by 25, and z is reduced by 50, the ratio $x : y$ becomes 1 : 2, and the ratio $y : z$ becomes 3 : 2 (or 2 : 3). Find the values of x, y, and z.

Correct Answer: $x = 350, y = 300, z = 200$

Solution:

Given that:

$$x + y + z = 850$$

After the reductions, the following conditions hold:

$$\frac{x - 100}{y - 25} = \frac{1}{2} \quad (1)$$

$$\frac{y - 25}{z - 50} = \frac{3}{2} \quad (2)$$

From equation (1), we have:

$$2(x - 100) = y - 25$$

$$2x - 200 = y - 25$$

$$y = 2x - 175 \quad (3)$$

From equation (2), we have:

$$2(y - 25) = 3(z - 50)$$

$$2y - 50 = 3z - 150$$

$$2y = 3z - 100 \quad (4)$$

Now substitute equation (3) into equation (4):

$$2(2x - 175) = 3z - 100$$

$$4x - 350 = 3z - 100$$

$$3z = 4x - 250 \quad (5)$$

Now, substitute equation (3) and equation (5) into the sum equation $x + y + z = 850$:

$$x + (2x - 175) + \left(\frac{4x - 250}{3}\right) = 850$$

$$x + 2x - 175 + \frac{4x - 250}{3} = 850$$

$$3x - 175 + \frac{4x - 250}{3} = 850$$

$$3(3x - 175) + (4x - 250) = 2550$$

$$9x - 525 + 4x - 250 = 2550$$

$$13x - 775 = 2550$$

$$13x = 3325$$

$$x = 350$$

Now, substitute $x = 350$ into equation (3):

$$y = 2(350) - 175 = 700 - 175 = 525$$

And finally, substitute $x = 350$ into equation (5):

$$3z = 4(350) - 250 = 1400 - 250 = 1150$$
$$z = \frac{1150}{3} = 200$$

Therefore, the values of x, y, and z are:

$$x = 350, \quad y = 300, \quad z = 200$$

Quick Tip

To solve such problems, set up ratios from the conditions and substitute them into the main equation to get the unknowns. Always check if the sum equation holds.

Q4. Find the missing number: 6, 13, 20, 27, . . . , 678.

Correct Answer: 671

Solution:

Let's observe the pattern in the given numbers:

$$6, 13, 20, 27, \ldots$$

The difference between consecutive terms is:

$$13 - 6 = 7$$
$$20 - 13 = 7$$
$$27 - 20 = 7$$

The common difference between terms is 7, so this is an arithmetic progression (AP) with the first term $a_1 = 6$ and common difference $d = 7$.

The nth term of an AP is given by the formula:

$$a_n = a_1 + (n-1) \times d$$

Substituting the values $a_1 = 6$ and $d = 7$ into the formula:

$$a_n = 6 + (n-1) \times 7$$

Now, we know that the last term of the sequence is 678, so we set $a_n = 678$ and solve for n:

$$678 = 6 + (n-1) \times 7$$

$$678 - 6 = (n-1) \times 7$$

$$672 = (n-1) \times 7$$

$$n - 1 = \frac{672}{7} = 96$$

$$n = 97$$

Therefore, the 97th term in the sequence is 678.

To find the previous term (the missing number), we substitute $n = 96$ into the nth term formula:

$$a_{96} = 6 + (96-1) \times 7 = 6 + 95 \times 7 = 6 + 665 = 671$$

Therefore, the missing number is $\boxed{671}$.

Quick Tip

In an arithmetic progression, the difference between any two consecutive terms is constant. Use the formula for the nth term to solve such problems quickly.

Q5. What is the sum of the numbers between 569 and 1024 which is divisible by 3 and 7?

Correct Answer: 22404

Solution:

We are asked to find the sum of all numbers between 569 and 1024 that are divisible by both 3 and 7. Since the numbers should be divisible by both 3 and 7, they must be divisible by the least common multiple (LCM) of 3 and 7.

The LCM of 3 and 7 is:

$$\text{LCM}(3, 7) = 21$$

So, we need to find all numbers divisible by 21 between 569 and 1024.

First, we find the smallest number divisible by 21 that is greater than or equal to 569:

$$\text{Smallest number} = \left\lceil \frac{569}{21} \right\rceil \times 21 = 28 \times 21 = 588$$

Next, we find the largest number divisible by 21 that is less than or equal to 1024:

$$\text{Largest number} = \left\lfloor \frac{1024}{21} \right\rfloor \times 21 = 48 \times 21 = 1008$$

Now, the numbers divisible by 21 between 569 and 1024 are:

$$588, 609, 630, \ldots, 1008$$

This is an arithmetic progression (AP) with the first term $a_1 = 588$, common difference $d = 21$, and last term $a_n = 1008$.

The number of terms in the AP can be found using the formula:

$$n = \frac{a_n - a_1}{d} + 1$$

$$n = \frac{1008 - 588}{21} + 1 = \frac{420}{21} + 1 = 20 + 1 = 21$$

The sum of an arithmetic progression is given by the formula:

$$S_n = \frac{n}{2} \times (a_1 + a_n)$$

Substituting the values:

$$S_{21} = \frac{21}{2} \times (588 + 1008) = \frac{21}{2} \times 1596 = 21 \times 798 = 22404$$

Therefore, the sum of the numbers divisible by 3 and 7 between 569 and 1024 is $\boxed{22404}$.

Quick Tip

To find the sum of numbers divisible by both 3 and 7, first find the LCM of 3 and 7. Then, find the smallest and largest numbers divisible by the LCM within the given range, and use the formula for the sum of an arithmetic progression.

Q6. Two circles that meet externally have a radius of 6 cm. Another circle touches these two circles. Find the radius of the third circle.

Correct Answer: 3 cm

Solution:

Given: - The radii of the first two circles, $r_1 = r_2 = 6\,\text{cm}$. - The third circle touches both these circles externally, so we need to find its radius, r_3.

This is a classic example of a problem involving **Descartes' Circle Theorem**, which relates the curvatures (reciprocals of the radii) of four mutually tangent circles.

The formula for Descartes' Circle Theorem is:

$$(k_1 + k_2 + k_3 + k_4)^2 = 2\left(k_1^2 + k_2^2 + k_3^2 + k_4^2\right)$$

Where $k_1, k_2, k_3,$ and k_4 are the curvatures of the four circles, and the curvature k of a circle is defined as:

$$k = \frac{1}{r}$$

In this case, the curvatures of the first two circles k_1 and k_2 are:

$$k_1 = k_2 = \frac{1}{6}$$

The third circle is tangent to both circles and the fourth circle (which is the straight line, representing the "zero curvature"). Therefore, $k_4 = 0$.

Now we substitute these values into Descartes' Circle Theorem to find k_3, the curvature of the third circle:

$$(k_1 + k_2 + k_3 + 0)^2 = 2\left(k_1^2 + k_2^2 + k_3^2 + 0^2\right)$$

$$\left(\frac{1}{6} + \frac{1}{6} + k_3\right)^2 = 2\left(\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + k_3^2\right)$$

$$\left(\frac{2}{6} + k_3\right)^2 = 2\left(2 \times \frac{1}{36} + k_3^2\right)$$

$$\left(\frac{1}{3} + k_3\right)^2 = 2\left(\frac{2}{36} + k_3^2\right)$$

$$\left(\frac{1}{3} + k_3\right)^2 = \frac{2}{18} + 2k_3^2$$

Expanding both sides:

$$\frac{1}{9} + \frac{2}{3}k_3 + k_3^2 = \frac{1}{9} + 2k_3^2$$

Cancel out $\frac{1}{9}$ from both sides:

$$\frac{2}{3}k_3 + k_3^2 = 2k_3^2$$

Rearranging:

$$\frac{2}{3}k_3 = k_3^2$$

Solving for k_3:

$$k_3 = \frac{2}{3}$$

Now, the radius of the third circle is:

$$r_3 = \frac{1}{k_3} = \frac{3}{2} = 3\,\text{cm}$$

Therefore, the radius of the third circle is $\boxed{3\,\text{cm}}$.

Quick Tip

Descartes' Circle Theorem is helpful in finding the radius of a circle tangent to three other circles. In this case, the curvatures of the circles are used to find the solution.

Q7. A person invested 34,500 in 5% stock at a 15% premium. Find his income.

Correct Answer: 1,725

Solution:

The person invested 34,500 in 5

The face value of the stock is 100.

Since the stock is at a 15

$$\text{Purchase Price} = 100 + 15\% \text{ of } 100 = 100 + 15 = 115$$

The number of shares purchased is:

$$\text{Number of Shares} = \frac{34,500}{115} = 300 \text{ shares}$$

The income from the investment is the annual income from the 5

$$\text{Income per Share} = 5\% \text{ of } 100 = 5$$

Therefore, the total income is:

$$\text{Total Income} = 300 \times 5 = 1,500$$

Therefore, the person's income is $\boxed{1,500}$.

Quick Tip

To calculate the income from stock investment, multiply the number of shares by the dividend per share. The dividend is based on the face value of the stock, not the purchase price.
