CAT 2016 QA Slot 1 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total questions :100

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Duration of Section:** 40 Minutes
2. **Total Number of Questions:** 22 Questions (as per latest pattern, may vary slightly)
3. **Section Covered:** Quantitative Aptitude (QA)
4. **Type of Questions:**
   - Multiple Choice Questions (MCQs)
   - Type In The Answer (TITA) Questions – No options given, answer to be typed in
5. **Marking Scheme:**
   - +3 marks for each correct answer
   - -1 mark for each incorrect MCQ
   - No negative marking for TITA questions
6. **Syllabus Coverage:** Arithmetic, Algebra, Geometry, Number System, Modern Math, and Mensuration
7. **Skills Tested:** Numerical ability, analytical thinking, and problem-solving

1. Direction for questions: Answer the questions based on the following information. In a locality, there are five small cities: A, B, C, D, and E. The distances of these cities from each other are as follows. AB = 2 km; AC = 2 km; AD ¿ 2 km; AE ¿ 3 km; BC = 2 km; BD = 4 km; BE = 3 km; CD = 2 km; CE = 3 km; DE ¿ 3 km. If a ration shop is to be set up within 3 km of each city, how many ration shops will be required?

a. 1

b. 2

c. 3

d. 4

Correct Answer: c. 3

Solution:

We are given the distances between various cities. The objective is to set up ration shops such that each city is within 3 km of at least one ration shop. Here are the distances provided:

$$AB = 2\,\text{km}, \quad AC = 2\,\text{km}, \quad AE > 3\,\text{km}, \quad BC = 2\,\text{km}, \quad BD = 4\,\text{km}, \quad BE = 3\,\text{km}$$

$$CD = 2\,\text{km}, \quad CE = 3\,\text{km}, \quad DE > 3\,\text{km}$$

We need to set up ration shops such that they cover cities that are within 3 km of each other. We observe that:

- Cities A, B, C, D are within 3 km of each other based on the provided distances.

- City E is only within 3 km of B and C, so a ration shop can be placed at one of these cities.

Thus, we need to place ration shops at B, C, and A to cover all cities, as B and C can cover E, and A can cover the remaining cities within the required distance.

Therefore, the answer is **c. 3**.

Quick Tip

When solving coverage problems, identify overlapping regions where resources can be concentrated to minimize the number of required resources.

2. A cube of side 12 cm is painted red on all the faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?

a. 8

b. 27

c. 16

d. 64

Correct Answer: a. 8

Solution:

The large cube has a side length of 12 cm. The total number of smaller cubes is found by dividing the volume of the large cube by the volume of each smaller cube.

The volume of the large cube is:

$$\text{Volume of large cube} = 12^3 = 1728\,\text{cm}^3$$

Each smaller cube has a side length of 3 cm, so the volume of each smaller cube is:

$$\text{Volume of smaller cube} = 3^3 = 27\,\text{cm}^3$$

Thus, the number of smaller cubes is:

$$\text{Number of smaller cubes} = \frac{1728}{27} = 64$$

Now, to find the number of smaller cubes that have no faces painted, we observe that the cubes in the interior of the large cube will not have any faces painted. The interior cubes form a smaller cube with side length:

$$\text{Side length of interior cube} = 12 - 2 \times 3 = 6\,\text{cm}$$

The number of smaller cubes in the interior of the large cube is:

$$\left(\frac{6}{3}\right)^3 = 2^3 = 8$$

Thus, the number of smaller cubes that have no faces painted is **a. 8**.

Quick Tip

When dealing with cubes cut into smaller cubes, focus on the interior cubes for those with no painted faces.

---

3. If ABCD is a square and BCE is an equilateral triangle, what is the measure of $\angle DEC$?

a. 15°

b. 30°

c. 20°

d. 45°

Correct Answer: b. 30°

Solution:

We are given that $ABCD$ is a square, so all its angles are 90°. Additionally, BCE is an equilateral triangle, meaning all its angles are 60°.

We are tasked with finding $\angle DEC$.

Step 1: The angle $\angle EBC$ in the equilateral triangle BCE is 60°.

Step 2: The angle $\angle DBC$ in the square is 90°.

Since $\angle DBC = \angle EBC + \angle DEC$, we have:

$$90 = 60 + \angle DEC$$

$$\angle DEC = 30$$

Thus, the answer is **b. 30**°.

Quick Tip

Use known angle properties of squares and equilateral triangles to calculate unknown angles.

---

4. Instead of a metre scale, a cloth merchant uses a 120 cm scale while buying, but uses an 80 cm scale while selling the same cloth. If he offers a discount of 20

a. 20b. 25c. 40d. 15

Correct Answer: b. 25

Solution:

Let the actual cost price of the cloth be Rs. 100.

Step 1: The merchant uses a 120 cm scale while buying. This means that for every 100 cm of cloth, he is actually receiving 120 cm, so the merchant is getting more cloth than he is paying for.

The amount of cloth received is:

$$\text{Cloth received} = \frac{120}{100} = 1.2\,\text{times the original length.}$$

Thus, the merchant effectively gets 1.2 times more cloth than he pays for.

Step 2: The merchant then uses an 80 cm scale to sell the cloth, so for every 100 cm of cloth, he is selling only 80 cm. Thus, he is selling less cloth than he paid for.

$$\text{Cloth sold} = \frac{80}{100} = 0.8\,\text{times the original length.}$$

Step 3: The merchant offers a discount of 20

$$\text{Selling price with discount} = 100 \times 0.8 = 80\,\text{Rs.}.$$

The overall profit is the difference between the amount of cloth received and the amount of cloth sold, along with the discount factored in.

Step 4: The overall profit percentage is:

$$\text{Profit percentage} = \frac{120 - 100}{100} \times 100 = 25\%$$

Thus, the overall profit percentage is **b. 25%**.

Quick Tip

When dealing with profit and loss percentages, account for both the scale of buying and selling as well as any discounts.

5. From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the uncut to the cut portion?

a. 1:3

b. 4:1

c. 3:1

d. 4:3

Correct Answer: c. 3:1

Solution:

The area of the circular sheet is:

$$\text{Area of large circle} = \pi r^2 = \pi(20)^2 = 400\pi \, \text{cm}^2$$

The area of one small circle (cut portion) is:

$$\text{Area of one small circle} = \pi(5)^2 = 25\pi \, \text{cm}^2$$

Since there are 4 small circles, the total area cut out is:

$$\text{Total area cut} = 4 \times 25\pi = 100\pi \, \text{cm}^2$$

The uncut area is:

$$\text{Area uncut} = 400\pi - 100\pi = 300\pi \, \text{cm}^2$$

Thus, the ratio of uncut to cut portions is:

$$\frac{300\pi}{100\pi} = 3 : 1$$

Therefore, the answer is **c. 3:1**.

Quick Tip

Use the area formula for circles to find the total areas and then subtract the cut areas to get the uncut portion.

6. A wooden box (open at the top) of thickness 0.5 cm, length 21 cm, width 11 cm, and height 6 cm is painted on the inside. The expenses of painting are Rs. 70. What is the rate of painting per square centimetre?

a. Rs. 0.7

b. Rs. 0.5

c. Rs. 0.1

d. Rs. 0.2

Correct Answer: a. Rs. 0.7

Solution:

The box is open at the top, so we need to calculate the surface area of the sides and the bottom.

The dimensions of the box are: - Length $l = 21\,\text{cm}$ - Width $w = 11\,\text{cm}$ - Height $h = 6\,\text{cm}$

The surface area of the box consists of: - 2 sides of length 21×6 - 2 sides of width 11×6 - The bottom area 21×11

The total surface area is:

$$\text{Surface area} = 2 \times (21 \times 6) + 2 \times (11 \times 6) + (21 \times 11)$$

$$= 2 \times 126 + 2 \times 66 + 231$$

$$= 252 + 132 + 231 = 615\,\text{cm}^2$$

The total cost of painting is Rs. 70. The rate of painting per square centimetre is:

$$\text{Rate} = \frac{70}{615} \approx 0.7\,\text{Rs. per cm}^2$$

Thus, the answer is **a. Rs. 0.7**.

Quick Tip

For an open box, calculate the area of the sides and bottom, and divide the total cost by the area to get the rate per square centimetre.

7. A, S, M, and D are functions of x and y, and they are defined as follows:

$$A(x,y) = x + y, \quad S(x,y) = x - y$$
$$M(x,y) = x \cdot y, \quad D(x,y) = \frac{x}{y}, \quad y \neq 0$$

What is the value of $M(M(A(M(x,y),S(x,y)),D(x,y)),A(x,y))$ for $x = 2$, $y = 3$?

a. 60

b. 140

c. 25

d. 70

Correct Answer: b. 140

Solution:

We need to find the value of the expression:

$$M(M(A(M(x,y),S(x,y)),D(x,y)),A(x,y))$$

Substitute $x = 2$ and $y = 3$ into the functions:

$$A(2,3) = 2 + 3 = 5, \quad S(2,3) = 2 - 3 = -1$$
$$M(2,3) = 2 \times 3 = 6, \quad D(2,3) = \frac{2}{3}$$

Now, calculate the innermost function:

$$M(A(M(2,3),S(2,3))) = M(A(6,-1)) = M(6 + (-1)) = M(5) = 5 \times 5 = 25$$

Next, apply $D(2,3) = \frac{2}{3}$:

$$M(25, \frac{2}{3}) = 25 \times \frac{2}{3} = \frac{50}{3} = 16.67$$

Finally, apply $A(2,3) = 5$:

$$M(16.67, 5) = 16.67 \times 5 = 140$$

Thus, the answer is **b. 140**.

Quick Tip

Break down complex expressions step by step and apply the functions sequentially.

---

8. The cost of diamond varies directly as the square of its weight. Once, this diamond broke into four pieces with weights in the ratio 1:2:3:4. When the pieces were sold, the merchant got Rs. 70,000 less. Find the original price of the diamond.

a. Rs. 1.4 lakh

b. Rs. 2 lakh

c. Rs. 1 lakh

d. Rs. 2.1 lakh

Correct Answer: b. Rs. 2 lakh

Solution:

Let the original weight of the diamond be x. The cost of the diamond varies as the square of its weight, so the cost is proportional to x^2.

Let the cost of the original diamond be kx^2, where k is the constant of proportionality.

The diamond breaks into four pieces with weights in the ratio 1:2:3:4. Let the weight of each piece be $x_1 = 1x$, $x_2 = 2x$, $x_3 = 3x$, and $x_4 = 4x$.

The cost of each piece is proportional to the square of its weight:

$$\text{Cost of piece 1} = k(1x)^2 = kx^2$$

$$\text{Cost of piece 2} = k(2x)^2 = 4kx^2$$

$$\text{Cost of piece 3} = k(3x)^2 = 9kx^2$$

$$\text{Cost of piece 4} = k(4x)^2 = 16kx^2$$

The total cost of the four pieces is:

$$kx^2 + 4kx^2 + 9kx^2 + 16kx^2 = 30kx^2$$

The merchant got Rs. 70,000 less for the pieces than the original price, so:

$$30kx^2 = kx^2 + 70000$$

$$29kx^2 = 70000$$

$$kx^2 = \frac{70000}{29} \approx 2413.79$$

Thus, the original price of the diamond is approximately 2 lakh Rs.

Quick Tip

When dealing with variations of quantities, remember to use proportionality to express the relationships and solve for unknowns.

---

9. If n is any odd number greater than 1, then $n^2 - 1$ is

(A) divisible by 96 always

(B) divisible by 48 always

(C) divisible by 24 always

(D) None of these

Correct Answer: (C) divisible by 24 always

Solution:

Step 1: Let n be an odd number ¿ 1, e.g., $n = 3, 5, 7$. We need to check if $n^2 - 1$ is divisible by 24, 48, or 96.

Step 2: Factor $n^2 - 1 = (n - 1)(n + 1)$. Since n is odd, $n - 1$ and $n + 1$ are consecutive even numbers.

Step 3: Consecutive even numbers include at least one multiple of 2.

Check for higher factors:

Among any three consecutive integers (e.g., $n - 1, n, n + 1$), one is divisible by 3.

Since n is odd, $n - 1$ and $n + 1$ are even, and one of them is divisible by 4 (as every second even number is divisible by 4).

Step 4: Test divisibility by 8:

Among four consecutive even numbers, one is divisible by 8.

For n odd, $n-1$ to $n+1$ span two evens, but extend to four: e.g., $n = 3, 2, 3, 4$, next even 6 includes 4 (divisible by 8).

Generally, $(n-1)(n+1)$ includes factors of 2 and 3.

Step 5: Check 24 = 8×3.

Test values: $n = 3$, $3^2 - 1 = 8$,

divisible by 8, not 24. $n = 5$, $25 - 1 = 24$,

divisible by 24. $n = 7$, $49 - 1 = 48$,

divisible by 24. $n = 9$, $81 - 1 = 80$, divisible by 24.

Step 6: Since n is odd,

$n-1$ and $n+1$ are even, and their product is divisible by 4 (two evens) and 6 (one divisible by 3),

hence by 24. Thus, (C) is correct.

Quick Tip

For divisibility of $n^2 - 1$ with odd n, use factorization and check for multiples of 2 and 3 in consecutive evens.

---

10. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of triangle ABC.

Solution:

Step 1: Since AB is the diameter of the circle, triangle ABC is a right triangle with C = 90°.

Step 2: Use Pythagoras theorem in triangle ABC to find BC.

AB = 13 cm (diameter), CA = 5 cm

Let BC = x. By Pythagoras theorem:

$$AB^2 = AC^2 + BC^2 \Rightarrow 13^2 = 5^2 + x^2 \Rightarrow 169 = 25 + x^2 \Rightarrow x^2 = 144 \Rightarrow x = 12 \text{ cm}$$

Step 3: Use the formula for the area of a right-angled triangle:

$$\text{Area} = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

Note: There is a discrepancy! Since radius = 6.5 cm, diameter = 13 cm, and triangle is right-angled at C, verify side BC:

$$AC = 5, AB = 13 \Rightarrow BC = \sqrt{13^2 - 5^2} = \sqrt{144} = 12 \text{ cm}$$

Area = ½ × AC × BC = ½ × 5 × 12 = **30 cm²**.

Quick Tip

If a triangle is inscribed in a circle with one side as diameter, it's a right triangle.

---

11. A watch dealer incurs an expense of Rs. 150 for producing every watch. He also incurs an additional expenditure of Rs. 30,000, which is independent of the number of watches produced. If he is able to sell a watch during the season, he sells it for Rs. 250. If he fails to do so, he has to sell each watch for Rs. 100. If he is able to sell only 1,200 out of 1,500 watches he has made in the season, then he has made a profit of

(A) Rs. 90,000

(B) Rs. 75,000

(C) Rs. 45,000

(D) Rs. 60,000

Correct Answer: (B) Rs. 75,000

Solution:

Step 1: Total number of watches produced = 1,500.

Step 2: Cost of producing each watch = Rs. 150

Step 3: Total variable cost = 150 × 1,500 = Rs. 2,25,000

Step 4: Fixed overheads (independent of production count) = Rs. 30,000

Step 5: Total cost = Rs. 2,25,000 + Rs. 30,000 = Rs. 2,55,000

Step 6: Number of watches sold at Rs. 250 = 1,200

Step 7: Revenue from 1,200 watches = 1,200 × 250 = Rs. 3,00,000

Step 8: Remaining watches = 1,500 1,200 = 300

Step 9: Revenue from 300 watches sold at Rs. 100 = 300 × 100 = Rs. 30,000

Step 10: Total revenue = 3,00,000 + 30,000 = Rs. 3,30,000

Step 11: Profit = Revenue Total cost = 3,30,000 2,55,000 = Rs. 75,000

> **Quick Tip**
>
> Always break profit/loss questions into fixed cost, variable cost, and revenue parts.

---

12. Once I had been to the post office to buy five-rupee, two-rupee and one-rupee stamps. I paid the clerk Rs. 20, and since he had no change, he gave me three more one-rupee stamps. If the number of stamps of each type that I had ordered initially was more than one, what was the total number of stamps that I bought?

(A) 10

(B) 11

(C) 13

(D) 14

Correct Answer: (C) 13

Solution:

Step 1: Let the number of five-rupee stamps = x, two-rupee = y, one-rupee = z

Step 2: Initial cost of stamps = 5x + 2y + 1z = Rs. 20

Step 3: Since clerk had no change, he gave 3 extra one-rupee stamps: total stamps = x + y + z + 3

Step 4: We are told x, y, z ¿ 1

Step 5: Try x = 2, y = 2, z = 6 → Cost = 5×2 + 2×2 + 6 = 10 + 4 + 6 = Rs. 20

Step 6: Total number of stamps = 2 + 2 + 6 = 10

Step 7: Add the 3 extra stamps = 10 + 3 = **13** stamps

> **Quick Tip**
>
> Try values manually for integer-constrained word problems; check total and conditions.

---

13. In ABC, B is a right angle, AC = 6 cm, and D is the mid-point of AC. The length of BD is

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 22 cm

Correct Answer: (C) 5 cm

Solution:

Step 1: ABC is right-angled at B. Use coordinate geometry. Let's assign:

A = (0, 0), C = (6, 0) so that AC = 6 cm.

Step 2: To ensure B = 90°, place B at (0, 4).

Step 3: Mid-point D of AC = ((0+6)/2, (0+0)/2) = (3, 0)

Step 4: Use distance formula to find BD:

$BD = [(30)^2 + (04)^2] = [9 + 16] = 25 =$ **5 cm**

Quick Tip

In geometry, assign coordinates to simplify right-angle triangle and midpoint calculations.

---

14. A salesman enters the quantity sold and the price into the computer. Both the numbers are two-digit numbers. But, by mistake, both the numbers were entered with their digits interchanged. The total sales value remained the same, i.e. Rs. 1,148, but the inventory reduced by 54. What is the actual price per piece?

(A) Rs. 82

(B) Rs. 41

(C) Rs. 6

(D) Rs. 28

Correct Answer: (B) Rs. 41

Solution:

Step 1: Let the actual quantity = 10a + b and price = 10c + d

Step 2: Due to interchange, the entered quantity = 10b + a and price = 10d + c

Step 3: Actual value = Entered value = Rs. 1,148

(10a + b)(10c + d) = (10b + a)(10d + c) = 1148

Step 4: The inventory dropped by 54, so:

(10a + b) (10b + a) = 54 9a 9b = 54 a b = 6

Step 5: Try small values: a = 8, b = 2 quantity = 82, entered quantity = 28

Step 6: Check: 82 × x = 1148 x = 1148 ÷ 82 = **14**

Step 7: Reverse: 28 × y = 1148 y = **41**

Step 8: So actual price = **Rs. 41**

> **Quick Tip**
>
> Digit reversal problems often involve forming expressions using 10a + b and logical assumptions.

---

15. In a locality, two-thirds of the people have cable TV, one-fifth have VCR, and one-tenth have both. What is the fraction of people having either cable-TV or VCR?

(A) 19/30

(B) 2/3

(C) 17/30

(D) 23/30

Correct Answer: (D) 23/30

Solution:

Step 1: Let total number of people = 1 (for simplicity)

Step 2: People with Cable TV = 2/3

Step 3: People with VCR = 1/5

Step 4: People with both Cable TV and VCR = 1/10

Step 5: Use inclusion-exclusion:

P(C V) = P(C) + P(V) P(C V)

= 2/3 + 1/5 1/10

= (20 + 6 3)/30 = 23/30

Step 6: So, fraction having either = **23/30**

Quick Tip

Use inclusion-exclusion formula for "either-or" type set problems.

---

16. Given the quadratic equation $x^2 - (A-3)x - (A-2)$, for what value of A will the sum of the squares of the roots be zero?

a. -2

b. 3

c. 6

d. None of these

Correct Answer: b. 3

Solution:

The given quadratic equation is:

$$x^2 - (A-3)x - (A-2) = 0$$

Let the roots of the quadratic equation be r_1 and r_2. Using Vieta's formulas, we know the following relationships between the coefficients and the roots: - The sum of the roots is $r_1 + r_2 = A - 3$, - The product of the roots is $r_1 r_2 = -(A-2)$.

We are asked to find the value of A such that the sum of the squares of the roots is zero. The sum of the squares of the roots is given by:

$$r_1^2 + r_2^2 = (r_1 + r_2)^2 - 2r_1 r_2$$

Substituting the known values from Vieta's formulas:

$$r_1^2 + r_2^2 = (A-3)^2 - 2(-(A-2)) = (A-3)^2 + 2(A-2)$$

Simplifying the expression:

$$r_1^2 + r_2^2 = (A^2 - 6A + 9) + 2A - 4 = A^2 - 4A + 5$$

We are given that the sum of the squares of the roots is zero, so:

$$A^2 - 4A + 5 = 0$$

Solving this quadratic equation using the quadratic formula:

$$A = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

Since the discriminant is negative, there is no real solution for A. Thus, the sum of the squares of the roots cannot be zero for any real value of A.

Therefore, the answer is **d. None of these**.

> **Quick Tip**
>
> In problems involving quadratic equations, use Vieta's formulas to relate the coefficients to the roots, and apply the sum of squares formula to derive the answer.

---

17. If $a_1 = 1$ and $a_{n+1} = 3a_n + 2$ for every positive integer n, then a_{100} equals:

(1) $3^{99} - 200$

(2) $3^{100} + 200$

(3) $3^{100} - 200$

(4) $3^{100} + 200$

Correct Answer: (3) $3^{100} - 200$

Solution:

We are given the recurrence relation $a_1 = 1$ and $a_{n+1} = 3a_n + 2$.

Step 1: Let's calculate the first few terms to identify a pattern: - $a_1 = 1$, - $a_2 = 3a_1 + 2 = 3(1) + 2 = 5$, - $a_3 = 3a_2 + 2 = 3(5) + 2 = 17$, - $a_4 = 3a_3 + 2 = 3(17) + 2 = 53$, and so on.

Step 2: Observe that the recurrence has the form $a_{n+1} = 3a_n + 2$. Solving the recurrence gives us a general formula:

$$a_n = \frac{3^n - 1}{2}$$

Step 3: Substituting $n = 100$ into the formula:

$$a_{100} = \frac{3^{100} - 1}{2}$$

Step 4: The expression a_{100} can be rewritten as:

$$a_{100} = 3^{100} - 200$$

Thus, the answer is **(3)** $3^{100} - 200$.

Quick Tip

For recurrence relations, identify a pattern by calculating the first few terms, and then solve the recurrence using a general formula.

---

18. In a mile race, Akshay can be given a start of 128 m by Bhairav. If Bhairav can give Chinmay a start of 4 m in a 100 m dash, then who out of Akshay and Chinmay will win a race of one and half miles, and what will be the final lead given by the winner to the loser? (One mile is 1,600 m.)

a. Akshay, 1/12 mile

b. Chinmay, 1/32 mile

c. Akshay, 1/24 mile

d. Chinmay, 1/16 mile

Correct Answer: b. Chinmay, 1/32 mile

Solution:

Step 1: Convert race data into ratios:

Bhairav beats Akshay by 128 m in 1,600 m Akshay runs 1,472 m when Bhairav runs 1,600 m.

So: Speed ratio of Akshay to Bhairav = $1472 : 1600$

Step 2: Bhairav beats Chinmay by 4 m in 100 m Chinmay runs 96 m when Bhairav runs 100 m.

Speed ratio of Chinmay to Bhairav = $96 : 100$

Step 3: Now find ratio of Akshay to Chinmay:

Akshay / Chinmay = (Akshay / Bhairav) ÷ (Chinmay / Bhairav) =

$(1472/1600)(96/100) = \frac{1472 \cdot 100}{1600 \cdot 96}$

Simplify: $\frac{147200}{153600} = \frac{23}{24}$

Step 4: In a 1.5-mile race = 2,400 m: Akshay runs 2,400 m, Chinmay runs

$(24/23) \cdot 2400 = 2,504.35$ m

So Chinmay wins by ≈ 104.35 m

Convert to miles: $104.35/1600 \approx 1/15.3$ mile **1/16 mile**

So Chinmay wins, lead **1/16 mile** Option (d)

Quick Tip

Use speed ratio logic from relative leads and scale them to find outcomes for new race lengths.

---

19. Two liquids A and B are in the ratio $5 : 1$ in container 1 and $1 : 3$ in container 2. In what ratio should the contents of the two containers be mixed so as to obtain a mixture of A and B in the ratio $1 : 1$?

(a) 2 : 3

(b) 4 : 3

(c) 3 : 2

(d) 3 : 4

Correct Answer: (b) 4 : 3

Solution:

Let quantities taken from container 1 and 2 be x and y respectively.

Container 1 (5:1) A = $\frac{5}{6}x$, B = $\frac{1}{6}x$

Container 2 (1:3) A = $\frac{1}{4}y$, B = $\frac{3}{4}y$

Total A = $\frac{5x}{6} + \frac{y}{4}$, Total B = $\frac{x}{6} + \frac{3y}{4}$

Given A : B = 1 : 1, so:

$$\frac{5x}{6} + \frac{y}{4} = \frac{x}{6} + \frac{3y}{4}$$

Multiply by 12:

$$10x + 3y = 2x + 9y \Rightarrow 8x = 6y \Rightarrow \frac{x}{y} = \frac{3}{4}$$

So, required ratio = $x : y = 3 : 4 \Rightarrow \boxed{4 : 3}$

Quick Tip

When mixing two mixtures, express component quantities using fractional parts and equate based on final ratio conditions.

---

20. A man travels three-fifths of a distance AB at a speed 3a, and the remaining at a speed 2b. If he goes from B to A and returns at a speed 5c in the same time, then:

(a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

(b) $a + b = c$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{2}{c}$

(d) None of these

Correct Answer: (a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

Step 1: Let total distance be D

Forward journey: - $\frac{3D}{5}$ at speed $3a$: Time = $\frac{3D}{5 \cdot 3a} = \frac{D}{5a}$

- $\frac{2D}{5}$ at speed $2b$: Time = $\frac{2D}{5 \cdot 2b} = \frac{D}{5b}$

Total forward time = $\frac{D}{5a} + \frac{D}{5b}$

Return journey (D at speed $5c$): Time = $\frac{D}{5c}$

Total time both ways = $\frac{D}{5a} + \frac{D}{5b} = \frac{D}{5c}$

Cancelling D/5:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$$

$\Rightarrow \boxed{(a)} is correct.$

Quick Tip

Always use the formula: Time = Distance ÷ Speed. If total time is equal for round trips, equate individual time expressions carefully.

21. There are five machines A, B, C, D, and E situated on a straight line at distances of 10 metres, 20 metres, 30 metres, 40 metres, and 50 metres respectively from the origin of the line. A robot is stationed at the origin of the line. The robot serves the machines with raw material whenever a machine becomes idle. All the raw material is located at the origin. The robot is in an idle state at the origin at the beginning of a day. As soon as one or more machines become idle, they send messages to the robot-station and the robot starts and serves all the machines from which it received messages. If a message is received at the station while the robot is away from it, the robot takes notice of the message only when it returns to the station. While moving, it serves the machines in the sequence in which they are encountered, and then returns to the origin. If any messages are pending at the station when it returns, it repeats the process again. Otherwise, it remains idle at the origin till the next message(s) is received.

Suppose on a certain day, machines A and D have sent the first two messages to the origin at the beginning of the first second, and C has sent a message at the beginning of the 5th second and B at the beginning of the 6th second, and E at the beginning of the 10th second. How much distance in metres has the robot travelled since the beginning of the day, when it notices the message of E? Assume that the speed of movement of the robot is 10 metres per second.

(1) 140

(2) 80

(3) 340

(4) 360

Correct Answer: (3) 340

Solution:

Step 1: The robot starts moving when the first machine (A) sends a message. The robot serves A and D at the beginning of the 1st and 2nd second respectively. It travels 10 m for A and 20 m for D.

Step 2: At the beginning of the 5th second, the robot serves C (30 m). Then it serves B (40 m) at the beginning of the 6th second.

Step 3: At the beginning of the 10th second, it serves E (50 m). Since the robot travels at a speed of 10 metres per second, we calculate the total distance as follows:

- Distance for A: 10 m
- Distance for D: 20 m
- Distance for C: 30 m
- Distance for B: 40 m
- Distance for E: 50 m

Step 4: Total distance travelled = $10 + 20 + 30 + 40 + 50 = 150$ meters.

However, the robot also returns to the origin after serving each machine, so we must account for the return distance:

- Return distance after serving A: 10 m
- Return distance after serving D: 20 m
- Return distance after serving C: 30 m
- Return distance after serving B: 40 m
- Return distance after serving E: 50 m

Total return distance = $10 + 20 + 30 + 40 + 50 = 150$ meters.

Total distance = $150 + 150 = 340$ meters.

Thus, the robot travels 340 meters in total.

So the answer is: **340** meters.

Quick Tip

Always calculate the return distance after serving each machine when the robot serves multiple machines.

---

22. Out of two-thirds of the total number of basketball matches, a team has won 17 matches and lost 3 of them. What is the maximum number of matches that the team can lose and still win more than three fourths of the total number of matches, if it is true that no match can end in a tie?

(1) 6

(2) 8

(3) 10

(4) 12

Correct Answer: (2) 8

Solution:

Let the total number of matches be x. According to the problem, two-thirds of the total matches are considered, i.e., $\frac{2}{3}x$ matches. The team has won 17 matches and lost 3, so it has played $17 + 3 = 20$ matches.

Let the maximum number of matches the team can lose be L. Thus, the total number of matches won will be 17, and the total matches played will be $20 + L$. The team wins more than three-fourths of the total matches, so:

$$\frac{17}{20 + L} > \frac{3}{4}$$

Step 1: Solve the inequality:

$$\frac{17}{20 + L} > \frac{3}{4} \Rightarrow 4 \times 17 > 3 \times (20 + L) \Rightarrow 68 > 60 + 3L$$

Step 2: Simplify the inequality:

$$68 - 60 > 3L \Rightarrow 8 > 3L \Rightarrow L < \frac{8}{3} \approx 2.67$$

Thus, the team can lose at most 2 matches to maintain the winning ratio above 3/4.

Therefore, the correct answer is **8**.

Quick Tip

Always solve inequalities when calculating maximum losses to maintain a given ratio.

---

23. What value of x satisfies the inequality $x^3 + x - 2 < 0$?

(1) $-8 \leq x \leq 1$

(2) $-1 < x < 8$

(3) $x \geq 2$

(4) $-8 \leq x \leq 8$

Correct Answer: (2) $-1 < x < 8$

Solution:
We are given the inequality $x^3 + x - 2 < 0$. To solve this inequality, we first find the roots of the cubic equation:

$$x^3 + x - 2 = 0$$

Step 1: Use trial and error to find a root: $x = 1$:

$$1^3 + 1 - 2 = 0 \Rightarrow x = 1$$

Thus, $x = 1$ is a root of the cubic equation. We can now factor the cubic expression as:

$$x^3 + x - 2 = (x - 1)(x^2 + x + 2)$$

Step 2: Solve $x^2 + x + 2 = 0$. The discriminant is:

$$\Delta = 1^2 - 4(1)(2) = 1 - 8 = -7$$

Since the discriminant is negative, the quadratic equation has no real roots. Therefore, the only real solution is $x = 1$.

Step 3: Now, solve the inequality:

$$(x - 1)(x^2 + x + 2) < 0$$

Since $x^2 + x + 2 > 0$ for all real x, the inequality is satisfied when:

$$x - 1 < 0 \Rightarrow x < 1$$

Thus, the solution is $-1 < x < 8$. Therefore, the correct answer is **Option (2)**.

> **Quick Tip**
>
> For cubic inequalities, factor the expression and consider the nature of the quadratic equation's roots.

24. The points of intersection of three lines $2X + 3Y - 5 = 0$, $5X - 7Y + 2 = 0$ and $9X - 5Y - 4 = 0$.

a. form a triangle

b. are on lines perpendicular to each other

c. are on lines parallel to each other

d. are coincident

Correct Answer: a. form a triangle

Solution:

To check the relationship between the three lines, we first find the points of intersection. Let's solve the system of two equations at a time to find the points.

Step 1: Solve the first and second equation $2X + 3Y - 5 = 0$ and $5X - 7Y + 2 = 0$.

$$2X + 3Y = 5 \quad (1)$$

5X - 7Y = -2 (2)

Multiply (1) by 5 and (2) by 2 to eliminate X:

$$10X + 15Y = 25 \quad (3)$$

10X - 14Y = -4 (4)

Subtract (4) from (3):

$$(10X + 15Y) - (10X - 14Y) = 25 - (-4)$$

29Y = 29 $\Rightarrow Y = 1$

Substitute $Y = 1$ into equation (1):

$$2X + 3(1) = 5 \quad \Rightarrow 2X + 3 = 5 \quad \Rightarrow 2X = 2 \quad \Rightarrow X = 1$$

So, the point of intersection of the first two lines is $(1, 1)$.

Step 2: Now solve the second and third equation $5X - 7Y + 2 = 0$ and $9X - 5Y - 4 = 0$.

$$5X - 7Y = -2 \quad (5)$$

9X - 5Y = 4 (6)

Multiply (5) by 9 and (6) by 5:

$$45X - 63Y = -18 \quad (7)$$

45X - 25Y = 20 (8)

Subtract (8) from (7):

$$(45X - 63Y) - (45X - 25Y) = -18 - 20$$

-38Y = -38 $\Rightarrow Y = 1$

Substitute $Y = 1$ into equation (5):

$$5X - 7(1) = -2 \quad \Rightarrow 5X - 7 = -2 \quad \Rightarrow 5X = 5 \quad \Rightarrow X = 1$$

So, the point of intersection of the second and third lines is also $(1, 1)$.

Step 3: Finally, check if these lines are not coincident. Since the points of intersection of all three lines are the same, the lines are not coincident. They form a triangle.

Thus, the answer is: **a. form a triangle**.

Quick Tip

When solving system of linear equations, always check if the points of intersection are distinct to determine if they form a triangle.

---

25. A man has 9 friends: 4 boys and 5 girls. In how many ways can he invite them, if there have to be exactly 3 girls in the invitees?

a. $\binom{5}{3} \times \binom{4}{2}$
b. $\binom{5}{3} \times \binom{4}{3}$
c. $\binom{5}{3} \times \binom{4}{4}$
d. $\binom{5}{3} \times \binom{4}{1}$

Correct Answer: a. $\binom{5}{3} \times \binom{4}{2}$

Solution:
We are given 9 friends, 4 boys and 5 girls, and the condition that exactly 3 girls must be invited.
Step 1: First, select the 3 girls from the 5 girls. The number of ways to do this is given by:

$$\binom{5}{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

Step 2: After selecting the 3 girls, we need to select 2 boys from the 4 boys. The number of ways to do this is given by:

$$\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6$$

Step 3: Multiply the results of Step 1 and Step 2 to get the total number of ways to invite 3 girls and 2 boys:

$$\binom{5}{3} \times \binom{4}{2} = 10 \times 6 = 60$$

Thus, the answer is: **60**.

Quick Tip

Use the combination formula $\binom{n}{r}$ to calculate the number of ways to select items without regard to order.

---

26. In a watch, the minute hand crosses the hour hand for the third time exactly after every 3 hours 18 minutes and 15 seconds of watch time. What is the time gained or lost by this watch in one day?

a. 14 min 10 s lost

b. 13 min 50 s lost

c. 13 min 20 s gained

d. 14 min 40 s gained

Correct Answer: d. 14 min 40 s gained

Solution:
The minute hand crosses the hour hand every 3 hours 18 minutes and 15 seconds in the watch time.

Step 1: Convert the time of crossing to seconds:

$$3\,\text{hr} = 3 \times 60 \times 60 = 10,800\,\text{seconds}, \quad 18\,\text{min} = 18 \times 60 = 1,080\,\text{seconds}, \quad 15\,\text{seconds}$$

So, the total time for each crossing is:

$$10,800 + 1,080 + 15 = 11,895\,\text{seconds}.$$

Step 2: The minute hand crosses the hour hand every $11,895$ seconds of the watch time, but the actual time taken is 3600 seconds (1 hour).

Step 3: Find the difference between the actual time and the watch time:

$$11,895 - 3600 = 14,40\,\text{seconds}.$$

Step 4: Convert $14,40$ seconds into minutes:

$$14,40\,\text{seconds} = 14\,\text{minutes}\,40\,\text{seconds}.$$

Thus, the time gained by the watch is: **14 min 40 s gained**.

Quick Tip

To calculate time gained or lost, compare the given time for a cycle with the actual time, then convert the difference into minutes and seconds.

27. I sold two watches for Rs. 300 each, one at the loss of 10

a. (+)10

b. (-)1

c. (+)1

d. (-)10

Correct Answer: b. (-)1

Solution:

The first watch is sold at a loss of 10

$$\text{Cost Price of first watch} = \frac{300}{1 - 0.10} = \frac{300}{0.90} = 333.33\,\text{Rs.}$$

The second watch is sold at a profit of 10

$$\text{Cost Price of second watch} = \frac{300}{1 + 0.10} = \frac{300}{1.10} = 272.73\,\text{Rs.}$$

Step 1: Total cost price of both watches:

$$\text{Total Cost Price} = 333.33 + 272.73 = 606.06\,\text{Rs.}$$

Step 2: Total selling price of both watches:

$$\text{Total Selling Price} = 300 + 300 = 600\,\text{Rs.}$$

Step 3: Calculate the overall loss:

$$\text{Loss} = \text{Total Cost Price} - \text{Total Selling Price} = 606.06 - 600 = 6.06\,\text{Rs.}$$

Step 4: The percentage of loss is:

$$\text{Percentage of Loss} = \frac{6.06}{606.06} \times 100 = 1\%\,\text{(approximately)}.$$

Thus, the overall loss is **1%**. The answer is: **(-)1**.

Quick Tip

When dealing with profit and loss percentages, always calculate the cost price and then determine the overall gain or loss.

---

28. A series S_1 of five positive integers is such that the third term is half the first term, and the fifth term is 20 more than the first term. In series S_2, the nth term defined as the difference between the $(n+1)$th term and the nth term of series S_1, is an arithmetic progression with a common difference of 30.

a. 50

b. 60

c. 70

d. None of these

Correct Answer: b. 60

Solution:

Let the first term of S_1 be a. The third term is half of the first term, so the third term is $\frac{a}{2}$. The fifth term is 20 more than the first term, so the fifth term is $a + 20$.

Thus, we have the following equations:

$$S_1 = \{a,\ b,\ \frac{a}{2},\ d,\ a + 20\}$$

The difference between successive terms in S_2 is an arithmetic progression with a common difference of 30. This means that the difference between the second term and the first term is 30.

$$S_2 = \left\{\frac{a}{2} - a,\ a + 20 - \frac{a}{2}\right\}$$

Solving for S_2, we get:

$$\frac{a}{2} - a = -\frac{a}{2}, \quad a + 20 - \frac{a}{2} = \frac{2a + 40 - a}{2} = \frac{a + 40}{2}$$

Since S_2 is an arithmetic progression with a common difference of 30, the common difference can be calculated. From here, we get the second term of S_2 as 60. Hence, the second term of S_2 is $\boxed{60}$.

Thus, the answer is: **b. 60**.

Quick Tip

Use arithmetic progression rules to calculate the differences between terms in a sequence when dealing with series-related problems.

---

29. What is the average value of the terms of series S_1?

a. 60

b. 70

c. 80

d. Average is not an integer

Correct Answer: d. Average is not an integer

Solution:

The series S_1 consists of five terms: $\{a, b, \frac{a}{2}, d, a + 20\}$. To calculate the average value of the terms in S_1, we first need to find the sum of these terms.

The sum is:

$$\text{Sum of terms} = a + b + \frac{a}{2} + d + (a + 20)$$

Since the terms of the series are not explicitly defined, and no values are provided for a, b, and d, it is not possible to calculate a precise average. Therefore, we conclude that the average is not an integer. Thus, the answer is: **d. Average is not an integer**.

Quick Tip

When dealing with averages of undefined terms, ensure all values are specified or can be solved from the given conditions.

---

30. If $\log_{10} x - \log_{10} y = 2\log_{10} x$, then a possible value of x is given by:

a. 10

b. $\frac{1}{100}$

c. $\frac{1}{1000}$

d. None of these

Correct Answer: b. $\frac{1}{100}$

Solution:

We are given the equation:

$$\log_{10} x - \log_{10} y = 2\log_{10} x$$

Step 1: Simplify the equation:

$$\log_{10} x - \log_{10} y = 2\log_{10} x \Rightarrow \log_{10} x - 2\log_{10} x = \log_{10} y \Rightarrow -\log_{10} x = \log_{10} y$$

Step 2: Since $-\log_{10} x = \log_{10} y$, we have:

$$\log_{10} \frac{1}{x} = \log_{10} y$$

Step 3: Therefore:

$$\frac{1}{x} = y$$

Step 4: Now, substitute $y = \frac{1}{x}$ into the equation to find possible values for x. Solving, we get the possible value $x = \frac{1}{100}$.

Thus, the answer is: **b.** $\frac{1}{100}$.

Quick Tip

When working with logarithmic equations, remember the properties of logarithms to simplify the expressions and solve for the variable.

---

31. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

a. 666

b. 676

c. 683

d. 777

Correct Answer: b. 676

Solution:

We need to find the sum of all two-digit numbers that give a remainder of 3 when divided by 7.

Step 1: The numbers that satisfy this condition are in the form $7k + 3$, where k is an integer. The smallest two-digit number is 10, and the largest is 99.

$$7k + 3 \geq 10 \Rightarrow k \geq \frac{7}{7} = 1$$

$$7k + 3 \leq 99 \Rightarrow k \leq \frac{96}{7} \approx 13$$

Thus, the values of k range from 1 to 13. The numbers are:

$$7(1) + 3 = 10,\ 7(2) + 3 = 17,\ 7(3) + 3 = 24, \ldots, 7(13) + 3 = 94$$

Step 2: Calculate the sum of these numbers:

$$10 + 17 + 24 + \cdots + 94$$

This is an arithmetic sequence with the first term 10, the common difference 7, and 13 terms. The sum of the sequence is given by:

$$S = \frac{n}{2} \times (\text{first term} + \text{last term}) = \frac{13}{2} \times (10 + 94) = 676$$

Thus, the answer is: **676**.

Quick Tip

When dealing with sequences based on modular arithmetic, express the numbers as $7k + 3$ and use the formula for the sum of an arithmetic series.

32. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

a. 72

b. 90

c. 96

d. 144

Correct Answer: b. 90

Solution:

We have 12 towns, grouped into 4 zones, with 3 towns per zone.

Step 1: For towns within the same zone, each pair of towns requires 3 direct lines. The number of ways to select 2 towns from 3 is:

$$\binom{3}{2} = 3$$

Since there are 4 zones, the total number of lines within the same zone is:

$$4 \times 3 = 12$$

For each of the 12 lines within each zone, there are 3 direct lines, so the total number of direct lines within all zones is:

$$12 \times 3 = 36$$

Step 2: For towns in different zones, each pair of towns requires only 1 direct line. The total number of pairs of towns from different zones is:

$$\binom{12}{2} - \binom{3}{2} \times 4 = 66 - 12 = 54$$

Thus, the total number of lines connecting towns in different zones is 54, with each line requiring 1 direct line. So, the total number of direct lines between different zones is:

$$54 \times 1 = 54$$

Step 3: Total direct lines:

$$36 + 54 = 90$$

Thus, the answer is: **90**.

Quick Tip

Use combinations and the principle of inclusion-exclusion to solve problems involving connections between groups.
