Vector Algebra JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Vector Algebra

1. From a point A with position vector $p(\hat{i}+\hat{j}+\hat{k})$, AB and AC are drawn perpendicular to the lines $\vec{r}=\hat{k}+\lambda(\hat{i}+\hat{j})$ and $\vec{r}=-\hat{k}+\mu(\hat{i}-\hat{j})$, respectively. A value of p is equal to : **(+4, -1)**

a. -1

b. $\sqrt{2}$

c. 2

d. -2

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2. If the volume of a parallelepiped whose coterminous edges are $\vec{a}=\hat{i}+\hat{j}+2\,\hat{k}, \vec{b}=2\,\hat{i}+\lambda\,\hat{j}+\hat{k}$ and $\vec{c}=2\,\hat{i}+2\hat{j}+\lambda\hat{k}$ is 35 cu.m, then a value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}-\vec{c}\cdot\vec{a}$ is : **(+4, -1)**

a. -10

b. 2

c. 22

d. -14

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3. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1,0,2) is : **(+4, -1)**

a. $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

b. $\vec{r}\cdot(3\hat{i}+7\hat{j}+3\hat{k})=7$

c. $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=7$

d. $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

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4. If , , are non-coplanar vectors and is a real number, then [(+) 2] = [+] for **(+4, -1)**

a. (A) No value of

b. (B) Exactly one value of

c. (C) Exactly two values of

d. (D) Exactly three values of

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5. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}, \vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}, \vec{a}\cdot\vec{c}=7, 2\vec{b}\cdot\vec{c}+43=0, \vec{a}\times\vec{c}=\vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to **(+4, -1)**

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6. Let $\Delta, \nabla \in \{\Lambda, V\}$ be such that $(p\rightarrow q)\Delta(p\nabla q)$ is a tautology. Then **(+4, -1)**

a. $\Delta=\Lambda, \nabla=\Lambda$

b. $\Delta=V, \nabla=V$

c. $\Delta=V, \nabla=\wedge$

d. $\Delta=\Lambda, \nabla=V$

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7. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}\cdot\vec{c}=0$ and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}-\vec{c}}{2}$ If $\vec{d}$ be a vector such that $\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$, then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ is equal to **(+4, -1)**

a. $\frac{3}{4}$

b. $\frac{1}{2}$

c. $-\frac{1}{4}$

d. $\frac{1}{4}$

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8. If ABCD is a parallelogram, vector AB = 2i + 4j - 5k and vector AD = i + 2j + 3k, then the unit vector in the direction of BD is **(+4, -1)**

a. $\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

b. $\frac{1}{69}(\hat{i}+2\hat{j}-8\hat{k})$

c. $\frac{1}{\sqrt{69}}(-\hat{i}-2\hat{j}+8\hat{k})$

d. $\frac{1}{69}(-\hat{i}-2\hat{j}+8\hat{k})$

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9. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$, If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to : **(+4, -1)**

a. $\sqrt{22}$

b. 4

c. $\sqrt{32}$

d. 6

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10. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=\sqrt{14}, |\vec{b}|=\sqrt{6}$ and $|\vec{a}\times\vec{b}|=\sqrt{48}$ Then $(\vec{a}\cdot\vec{b})^2$ is equal to **(+4, -1)**

Answers

1. **Answer: b**

Explanation:

The Correct Option is (B): $\sqrt{2}$

Concepts:

1. Vectors:

The quantities having magnitude as well as direction are known as Vectors or Vector quantities. Vectors are the objects which are found in accumulated form in vector spaces accompanying two types of operations. These operations within the vector space include the addition of two vectors and multiplication of the vector with a scalar quantity. These operations can alter the proportions and order of the vector but the result still remains in the vector space. It is often recognized by symbols such as U ,V, and W

Representation of a Vector :

A line having an arrowhead is known as a directed line. A segment of the directed line has both direction and magnitude. This segment of the directed line is known as a vector. It is represented by a or commonly as AB. In this line segment AB, A is the starting point and B is the terminal point of the line.

Types of Vectors:

Here we will be discussing different types of vectors. There are commonly 10 different types of vectors frequently used in maths. The 10 types of vectors are:

1. Zero vector
2. Unit Vector
3. Position Vector
4. Co-initial Vector
5. Like and Unlike Vectors
6. Coplanar Vector
7. Collinear Vector

8. Equal Vector
9. Displacement Vector
10. Negative of a Vector

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2. Answer: c

Explanation:

Answer (c) 22

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3. Answer: c

Explanation:

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$ point (1,0,2) Eq ^{n} of plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1+\lambda\{r.(\hat{i}-2\hat{j})+2\}=0$ $\vec{r}\cdot\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\}-1+2\lambda=0$ Point $\hat{i}+0\hat{j}+2\hat{k}=\vec{r}$ $\therefore(\hat{i}+2\hat{k})\cdot\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\}-1+2\lambda=0$ $1+\lambda+2-1+2\lambda=0$ $\lambda=-\frac{2}{3}$ $\therefore\ \vec{r}\cdot\left[\hat{i}\left(\frac{1}{3}\right)+\hat{j}\left(\frac{7}{3}\right)+\hat{k}\right]=\frac{7}{3}$ $r\cdot[\hat{i}+7\hat{j}+3\hat{k}]=7$

Concepts:

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4. Answer: a

Explanation:

Explanation:
Given:Non- coplanar vectors , , and [(+) ^{2}] = [+]We have to find the value of .since, , , are non-coplanar vectors [ ] ≠ 0Now,
[(+) ^{2}] = [+]Using the definition of scalar triple product, we get
(+) (^{2} ×) = +)×) = (× + ×) = (+) (^{2} ×) = (0 + ×)
(×)[Using properties of cross product-2] ^{4}((×)) + (×) = (×)
^{4}([ ] + [ ]) = − [ ][Using properties of scalar triple product-3]

4([ ]) = − [ ] 4 = − 1Which is not true for any real value of .Hence, the correct option is (A).

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5. **Answer: 8 - 8**

Explanation:

The correct answer is 8

$\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k},\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k},\vec{a}\cdot\vec{c}=7$

$\vec{a}\times\vec{c}-\vec{b}\times\vec{c}=\vec{0}$

$(\vec{a}-\vec{b})\times\vec{c}=0\Rightarrow(\vec{a}-\vec{b})$ is paralleled to $\vec{c}$

$\vec{a}-\vec{b}=\mu\vec{c}$, where μ is a scalar

$-2\hat{i}+7\hat{j}+2\lambda\hat{k}=\mu\cdot\overrightarrow{c}$

Now $\vec{a}\cdot\vec{c}=7$ gives $2\lambda^2+12=7\mu$

And $\vec{b}\cdot\vec{c}=-\frac{43}{2}$ gives $4\lambda^2+82=43\mu$

$\mu=2$ and $\lambda^2=1$

$|\vec{a}\cdot\vec{b}|=8$

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6. Answer: b

Explanation:

The correct answer is (B) : $\Delta = V, \nabla = V$
Given (p→q)Δ(p∇q)
Option I Δ=∧,∇=∨

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Option 2 $\Delta = \vee, \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Option 3 $\Delta = \vee, \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

Hence, it is tautology.
Option 4Δ=∧,∇=∧

Concepts:

1. Types of Vectors:

In general, vectors are used in Maths and Science and are categorized into 10 different types of vectors such as:-

1. **Unit Vector** - When a vector has a magnitude of 1 unit length, called a Unit Vector.
2. **Co-Initial Vector** - Two or more vectors that have the same initial point are known to be Co-Initial Vectors.
3. **Coplanar Vector** - Vectors that lie either in the same plane or are parallel to the same plane are called Coplanar vectors.
4. **Equal Vector** - When two vectors have equal direction as well as magnitude, they are Equal Vectors, even if the initial point is different for both vectors.
5. **Negative of a Vector** - When two vectors have the same magnitude but have exactly different directions.
6. **Zero Vector** - When a vector has the same starting and ending point and has zero magnitudes is called a zero vector. The starting point needs to coincide with the terminal point. It is denoted by 0. It is also known as the null vector.
7. **Position Vector** - A vector that indicates the location or the position of a point in a plane (three-dimensional Cartesian system) w.r.t. its origin. If A is a reference

origin and there's an arbitrary point B in the plane then AB will be called the position vector of the point.

8. **Like and Unlike Vectors** - Like vectors are the vectors that have the same direction. And unlike vectors are the vectors that have opposite directions.
9. **Collinear Vector** - Vectors either lying in the same line or which are parallel to the same line are Collinear vectors.
10. **Displacement Vector** - The vector AB will be known as a displacement vector if a point is displaced from position B to position A.

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7. Answer: d

Explanation:

$$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{\vec{b}-\vec{c}}{2}$$
$$\vec{a}\cdot\vec{c}=\frac{1}{2},\vec{a}\cdot\vec{b}=\frac{1}{2}$$
$$\therefore\vec{b}\cdot\vec{d}=\frac{1}{2}$$
$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\vec{a}\cdot(\vec{b}\times(\vec{c}\times\vec{d}))$$
$$=\vec{a}\cdot((\vec{b}\cdot\vec{d})\vec{c}-(\vec{b}\cdot\vec{c})\vec{d})$$
$$=(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})=\frac{1}{4}$$

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8. Answer: c

Explanation:

The correct answer is option C) $\frac{1}{\sqrt{69}}(-\hat{i} - 2\hat{j} + 8\hat{k})$

Q. Sol^n (10)

Given that;–

$\vec{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{AD} = \hat{i} + 2\hat{j} + 3\hat{k}$

Then unit vector in the direction of BD will be;–

$\vec{BD} = \vec{BA} + \vec{AD} = \vec{AD} - \vec{AB}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 4\hat{j} - 5\hat{k})$

$= -\hat{i} - 2\hat{j} + 8\hat{k}$

$\therefore$ unit vector $= \dfrac{-\hat{i} - 2\hat{j} + 8\hat{k}}{\sqrt{(-1)^2 + (-2)^2 + 8^2}}$

$= \dfrac{-\hat{i} - 2\hat{j} + 8\hat{k}}{\sqrt{69}}$

$= \dfrac{1}{\sqrt{69}}(-\hat{i} - 2\hat{j} + 8\hat{k})$ (Ans)

9. **Answer: d**

Explanation:

Projection of $\vec{b}\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $\left(\vec{a} + \vec{b}\right) \perp \vec{c} \Rightarrow \left(\vec{a} + \vec{b}\right).\vec{c} = 0$

$\Rightarrow 5b_1 + b_2 = -10$ ....(2)

from (1) and (2) $\Rightarrow$ $b_1 = -3$ and $b_2 = 5$

then $\left|\vec{b}\right| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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10. **Answer: 36 - 36**

Explanation:

The correct answer is 36.

$|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6} \quad |\vec{a} \times \vec{b}| = \sqrt{48}$

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 \times |\vec{b}|^2$

$\Rightarrow (\vec{a} \cdot \vec{b})^2 = 84 - 48 = 36$

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