Vector Algebra JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Vector Algebra

1. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to : **(+4, -1)**

a. $\frac{1}{2}$

b. $-\frac{3}{2}$

c. $-\frac{1}{2}$

d. -1

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2. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}, \pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is : - **(+4, -1)**

a. $\frac{5\pi}{12}$

b. $\frac{5\pi}{6}$

c. $\frac{2\pi}{3}$

d. $\frac{\pi}{4}$

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3. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of a for which the vectors $\vec{u} = (\alpha - 2)\vec{a} + \vec{b}$ and $\vec{v} = (2 + 3\alpha)\vec{a} - 3\vec{b}$ are collinear is : **(+4, -1)**

a. $\frac{3}{2}$

b. $\frac{2}{3}$

c. $-\frac{3}{2}$

d. $-\frac{2}{3}$

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4. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors Then which one of the following statements is TRUE ? **(+4, -1)**

a. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

b. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

c. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

d. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

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5. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$ If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is **(+4, -1)**

a. -60

b. -48

c. -84

d. -24

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6. Let A be a point on the x-axis Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$ If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to **(+4, -1)**

a. 76

b. 81

c. 72

d. 64

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7. If the four points, whose position vectors are $3\hat{i} - 4j + 2\hat{k}, l + 2j - \hat{k}_4 - 2\hat{k} - j + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then a is equal to **(+4, -1)**

a. $\frac{107}{17}$

b. $\frac{73}{17}$

c. $-\frac{73}{17}$

d. $-\frac{107}{17}$

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8. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is : **(+4, -1)**

a. $3\sqrt{2}$

b. 1

c. $2\sqrt{3}$

d. $\sqrt{6}$

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9. Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$ Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5 , is ____ **(+4, -1)**

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10. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$ If the minimum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _____ **(+4, -1)**

Answers

1. **Answer: c**

Explanation:

$\vec{b} \times \vec{c} - \vec{b} \times \vec{a} = \vec{0}$
$\vec{b} \times (\vec{c} - \vec{a}) = \vec{0}$
$\vec{b} = \lambda (\vec{c} - \vec{a}) \quad \ldots (i)$
$\vec{a} \cdot \vec{b} = \lambda \left(\vec{a} \cdot \vec{c} - \vec{a}^2\right)$
$4 = \lambda (0 - 6) \Rightarrow \lambda = \frac{-4}{6} = \frac{-2}{3}$
from $(i)\ \vec{b} = \frac{-2}{3} (\vec{c} - \vec{a})$
$\vec{c} = \frac{-3}{2}\vec{b} + \vec{a} = \frac{-1}{2} \left(\hat{i} + \hat{j} + \hat{k}\right)$

$\vec{b} \cdot \vec{c} = -\frac{1}{2}$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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2. **Answer: c**

Explanation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$
$\cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$
$\cos^2\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

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3. Answer: b

Explanation:

Since, $\vec{u}$ and $\vec{v}$ are collinear, therefore $\overrightarrow{ku} + \vec{v} = 0$
$\Rightarrow [k(\alpha - 2) + 2 + 3\alpha]\overrightarrow{a} + (k-3)\overrightarrow{b} = 0 \ldots (i)$
Since $\vec{u}$ and $\vec{v}$ are non-collinear, then for some constant m and n,
$m\vec{a} + n\vec{b} = 0 \Rightarrow m = 0, n = 0$
Hence from equation (i)
$k - 3 = 0 \Rightarrow k = 3$
And $k(\alpha - 2) + 2 + 3\alpha = 0$
$\Rightarrow 3(\alpha - 2) + 2 + 3\alpha = 0 \Rightarrow \alpha = \frac{2}{3}$

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4. Answer: a

Explanation:

The correct option is(A): Projection of$\vec{a}$ on $\vec{b}\ is\ \frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

$a = 5\hat{i} - \hat{j} - 3\hat{k}$
$b = \hat{i} - 3\hat{j} + 5\hat{k}$
$a \cdot \hat{b} = 355 - 3 - 15 = -35 - 13$

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5. Answer: b

Explanation:

$\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$
$\vec{b} \cdot \vec{c} = \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}$
$= -3|b|^2$
$= -48$
So, the correct answer is (B) : -48

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6. Answer: c

Explanation:

The correct answer is (C) : 72

$y = mx + \frac{4}{m}$

$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = 2\sqrt{2}$

$\therefore m = \pm 1$

$y = \pm x \pm 4$ Point of contact on parabola

Let $m = 1, \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$R(4, 8)$

Point of contact on circle $Q(-2, 2)$

$\therefore (QR)^2 = 36 + 36 = 72$

Concepts:

1. Functions:

A function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output. Let A & B be any two non-empty sets, mapping from A to B will be a function only when every element in set A has one end only one image in set B.

Kinds of Functions

The different types of functions are -

One to One Function: When elements of set A have a separate component of set B, we can determine that it is a one-to-one function. Besides, you can also call it injective.

Many to One Function: As the name suggests, here more than two elements in set A are mapped with one element in set B.

Moreover, if it happens that all the elements in set B have pre-images in set A, it is called an onto function or surjective function.

Also, if a function is both one-to-one and onto function, it is known as a bijective. This means, that all the elements of A are mapped with separate elements in B, and A holds a pre-image of elements of B.

Read More: Relations and Functions

7. **Answer: b**

Explanation:

The correct answer is (B) : $\frac{73}{17}$
Let $A : (3, -4, 2) C : (-2, -1, 3)$
$\mathrm{B} : (1, 2, -1)\ \mathrm{D} : (5, -2\alpha, 4)$
A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$\Rightarrow \alpha = \frac{73}{17}$

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8. **Answer: a**

Explanation:

$\vec{b} = \lambda\vec{a} \times (\vec{a} \times \hat{j})$
$\Rightarrow \vec{b} = \lambda(-2\hat{i} - 2\hat{j} + 2\hat{k})$
$|\vec{b}| = |\vec{a}| \therefore \sqrt{6} = \sqrt{12}|\lambda| \Rightarrow \lambda = \pm\frac{1}{\sqrt{2}}$

$\left(\lambda = \frac{1}{\sqrt{2}} \text{ rejected } \because \vec{b} \text{ makes acute angle with y axis }\right)$
$\vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$
$\frac{(3a+2b)-c}{|c|} = 3\sqrt{2}$

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9. Answer: 120 - 120

Explanation:

The correct answer is 120
x+y=5λ
Cases :

x	y	Number of ways
5λ	5λ	20
5λ+1	5λ+4	25
5λ+2	5λ+3	25
5λ+3	5λ+2	25
5λ+4	5λ+1	25

∴ Total number of ways are 120

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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10. Answer: 3501 - 3501

Explanation:

The correct answer is 3501

$[\vec{u}\vec{v}\vec{w}] = \vec{u}.(\vec{v} \times \vec{w})$

$min.(|u||\vec{v}\times\vec{w}|cos\theta) = -\alpha\sqrt{3401}$

$\Rightarrow cos\theta = -1$

$|u| = \alpha(Given)$

$|\vec{v}\times\vec{w}| = \sqrt{3401}$

$$\vec{v}\times\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$\vec{v}\times\vec{w} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$|\vec{v}\times\vec{w}| = \sqrt{1+25\alpha^2+9\alpha^2} = \sqrt{3401}$

$34\alpha^2 = 3400$

$\alpha^2 = 100$

$\alpha = 10(as\ \alpha > 0)$

$so\ \vec{u} = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$

$\vec{u} = \sqrt{\lambda^2 + 25\alpha^2\lambda^2 + 9\alpha^2\lambda}$

$\alpha^2 = \lambda^2(1+25\alpha^2+9\alpha^2)$

$100 = \lambda^2(1+34\times100)$

$\lambda^2 = \frac{100}{3401} = \frac{m}{n}$

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