# Vector Algebra JEE Main PYQ - 3

**Total Time:** 25 Minute **Total Marks:** 40

## Instructions

## Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

### Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

# Vector Algebra

**1.** Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$ If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to _____ **(+4, -1)**

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**2.** If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is $(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to **(+4, -1)**

**a.** $-1$

**b.** 1

**c.** 3

**d.** 5

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**3.** The foot of perpendicular of the point $(2, 0, 5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} - \frac{z+1}{-1}$ is $(a, \beta, \gamma)$. Then which of the following is NOT correct? **(+4, -1)**

**a.** $\frac{\gamma}{\alpha} = \frac{5}{8}$

**b.** $\frac{\beta}{\gamma} = -5$

**c.** $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

**d.** $\frac{\alpha}{\beta} = -8$

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**4.** Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$ Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :- **(+4, -1)**

**a.** $\frac{2}{\sqrt{21}}$

**b.** $2\sqrt{\frac{3}{7}}$

**c.** $\frac{2}{3}\sqrt{\frac{7}{3}}$

**d.** $\frac{2}{3}$

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**5.** Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to **(+4, -1)**

**a.** $3(\hat{i} - \hat{j} - \hat{k})$

**b.** $3(\hat{i} - \hat{j} + \hat{k})$

**c.** $3(\hat{i} + \hat{j} - \hat{k})$

**d.** $3(\hat{i} + \hat{j} + \hat{k})$

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**6.** Let $PQR$ be a triangle. The points $A, B$ and $C$ are on the sides $QR, RP$ and $PQ$ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{A \text{ Area } (\triangle PQR)}{\text{Area } (\triangle ABC)}$ is equal to **(+4, -1)**

**a.** 4

**b.** 3

**c.** 2

**d.** $\frac{5}{2}$

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**7.** Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to **(+4, -1)**

**a.** 1

**b.** 2

**c.** $\frac{3}{2}$

**d.** $-\frac{2}{3}$

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**8.** Let $\overrightarrow{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\overrightarrow{b} = \hat{i} + \hat{k}$ and $\overrightarrow{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors If $\overrightarrow{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to : **(+4, -1)**

**a.** $\frac{11}{7}\sqrt{2}$

**b.** $\frac{11}{7}$

c. $\frac{\sqrt{914}}{7}$

d. $\frac{11}{5}\sqrt{2}$

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**9.** Let $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b} = \hat{i}-2\hat{j}-2\hat{k}$, $\vec{c} = -\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{d}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$, $\vec{a}.\vec{d} = 18$, then find $|\vec{a}\times\vec{d}|^2$ **(+4, -1)**

a. 720

b. 700

c. 360

d. 300

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**10.** If V is volume of parallelepiped whose three coterminous edges are a, b, c, then volume of a parallelepiped whose coterminous edges are a, b+c, a+2b+3c is **(+4, -1)**

a. 6V

b. V

c. 2V

d. 3V

# Answers

1. **Answer: 3 - 3**

## Explanation:

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$
$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$
$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$
$|\vec{a}|^2 = \lambda^2|2\vec{b} + 3\vec{c}|^2$
$|\vec{a}|^2 = \lambda^2(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b}.\vec{c})$
$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$
$\vec{a} = \pm(2\vec{b} + 3\vec{c})$
$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2|\vec{b}\times\vec{c}|}{2\vec{b}.\vec{b}+3\vec{c}.\vec{b}}$
$|\vec{b} \times \vec{c}| = |\vec{b}|^2|\vec{c}|^2 - (\vec{b}.\vec{c})^2 = \frac{3}{4}$
$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2\times\frac{\sqrt{3}}{2}}{2.\frac{1}{4}-\frac{3}{2}} = -\sqrt{3}$
$(\frac{\vec{a}\times\vec{c}}{\vec{a}.\vec{b}})^2 = 3$
So, the correct answer is 3.

## Concepts:

### 1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

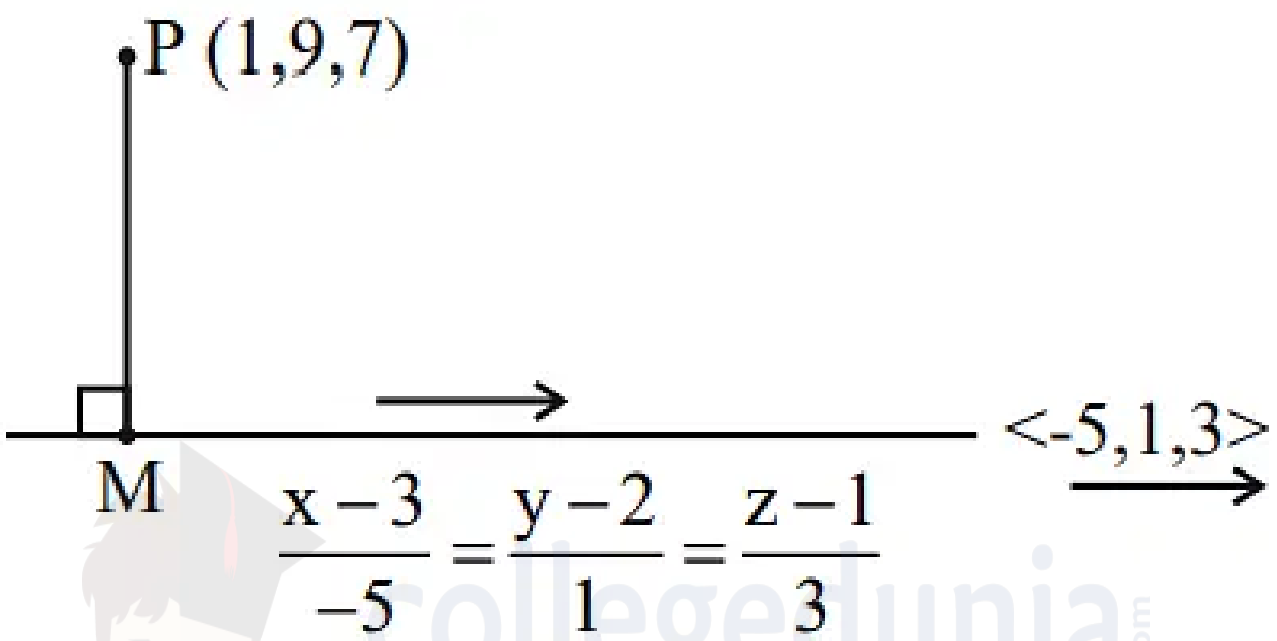
### Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

2. **Answer: d**

**Explanation:**

The correct answer is (D) : 5




Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$

$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{\mathrm{PM}} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

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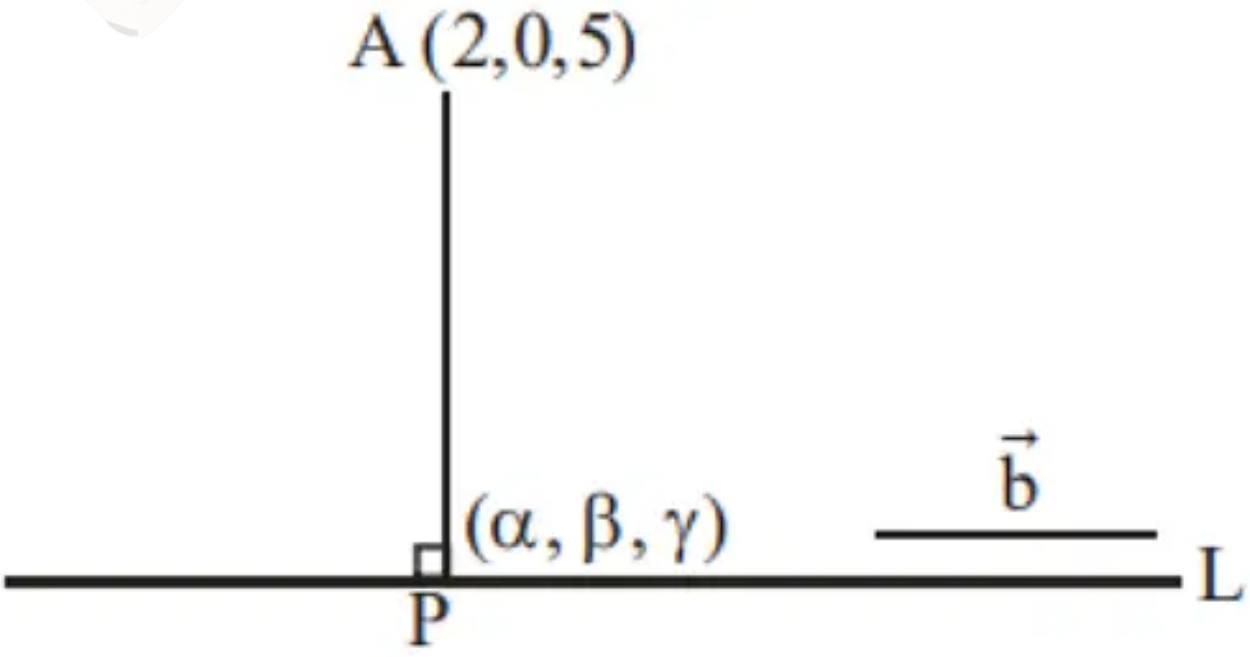
## 3. Answer: b

### Explanation:

The correct answer is (B) : $\frac{\beta}{\gamma} = -5$

$L : \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$




Let foot of perpendicular is

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

Now, $\Rightarrow \overrightarrow{PA} \cdot \overrightarrow{b} = 0$

$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow \lambda = \frac{-1}{6}$

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \alpha = -\frac{4}{3}$

$\Rightarrow \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \beta = \frac{1}{6}$

$\Rightarrow \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \gamma = -\frac{5}{6}$

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## 4. Answer: a

## Explanation:

Projection of $\vec{b}$ on vector $\vec{a} - \vec{b}$ is

$= \frac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$

$= \frac{\vec{a} \cdot \vec{b} - |b|^2}{\sqrt{a^2 + b^2 - 2a \cdot b}} = \frac{3 - b^2}{\sqrt{6 + b^2 - 6}} = \frac{3 - b^2}{b}$

$|\vec{a} \times \vec{b}|^2 = 5$

$a^2 b^2 - (a \cdot b)^2 = 5$

$6b^2 = 14 \Rightarrow b^2 = \frac{7}{3}$
$\therefore \frac{3-b^2}{b} = \frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}} = 2 \times \sqrt{21}$

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## 5. Answer: d

## Explanation:

The correct answer is (D) : $3(\hat{i} + \hat{j} + \hat{k})$
$\vec{a} \times \vec{b} = (\hat{i} - \hat{j})$
Taking cross product with $\vec{a}$
$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$
$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$
$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$
$\Rightarrow 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$
$\Rightarrow \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

## Concepts:

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## 6. Answer: b

### Explanation:

Let $P$ is $\overrightarrow{0}, Q$ is $\vec{q}$ and $R$ is $\vec{r}$
$A$ is $\frac{2\vec{q}+\vec{r}}{3}, B$ is $\frac{2\vec{r}}{3}$ and $C$ is $\frac{\vec{q}}{3}$
Area of $\triangle PQR$ is $= \frac{1}{2}|\overrightarrow{q} \times \overrightarrow{r}|$
Area of $\triangle ABC$ is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$
Area of $\triangle ABC = \frac{1}{6}|\overrightarrow{q} \times r|$ $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)} = 3$

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7. **Answer: a**

## Explanation:

$\overrightarrow{u} = (1, -1, -2), \overrightarrow{v} = (2, 1, -1), \overrightarrow{v} \cdot \overrightarrow{w} = 2$
$\overrightarrow{v} \times \overrightarrow{w} = \overrightarrow{u} + \lambda \overrightarrow{v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$
Taking dot with $\overrightarrow{w}$ in $(1)$
$\overrightarrow{w} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{w} + \lambda \overrightarrow{v} \cdot \overrightarrow{w}$
$\Rightarrow 0 = \overrightarrow{u} \cdot \overrightarrow{w} + 2\lambda$
Taking dot with $\vec{v}$ in (1)
$\overrightarrow{v} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{v} + \lambda \overrightarrow{v} \cdot \overrightarrow{v}$
$\Rightarrow 0 = (2 - 1 + 2) + \lambda \cdot (6)$
$\lambda = -\frac{1}{2}$
$\Rightarrow \overrightarrow{u} \cdot \overrightarrow{w} = -2\lambda = 1$

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Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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8. **Answer: a**

**Explanation:**

$\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$
$\vec{b} = \hat{i} + \hat{k}$
$\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$
$\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$
$\therefore \vec{r} = \vec{c} + \lambda\vec{a}$
$\vec{r} \cdot \vec{b} = 0 \Rightarrow \vec{c} \cdot \vec{b} + \lambda \quad \vec{b} \cdot \vec{a} = 0$
$-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$
$\therefore \vec{r} = \vec{c} + \frac{2\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$
$|\ \vec{r} = \frac{11\sqrt{2}}{7}$

**Concepts:**

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**Vector Algebra Operations:**

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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9. **Answer: a**

**Explanation:**

$\vec{d} = \lambda(\vec{b} \times \vec{c}) = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$
$\vec{a}.\vec{d} = 18$
$\Rightarrow \lambda = 2$
Therefore, $|\vec{a} \times \vec{d}|^2 = \vec{a}^2\vec{d}^2 - (\vec{a}.\vec{d})^2$
$\Rightarrow |\vec{a} \times \vec{d}|^2 = 29 \times 36 - 324 = 1044 - 324 = 720$

### Concepts:

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## 10. Answer: b

### Explanation:

The correct option is (B): V

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