CBSE Class 12 Mathematics Set 3 (65/3/2) Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) andquestions number 19 and 20 are Assertion-Reason based questions of 1 markeach.
4. In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA) type questions,carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$**, then the value of** k **is:**

(A) 2

(B) -2

(C) ± 2

(D) ∓ 2

Correct Answer: (D) ∓ 2

Solution:

Step 1: Expand the determinant

The determinant is:

$$\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}.$$

Expanding along the third row:

$$\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 3 \\ k & 0 \end{vmatrix}.$$

The 2×2 determinant is:

$$\begin{vmatrix} 1 & 3 \\ k & 0 \end{vmatrix} = (1)(0) - (3)(k) = -3k.$$

Thus, the determinant is:

$$-3k = \pm 6.$$

Step 2: Solve for k

Divide both sides by -3:

$$k = \frac{\pm 6}{-3} = \mp 2.$$

Step 3: Conclude the result

The value of k is ∓ 2.

Quick Tip

To calculate determinants, expand along rows or columns with the most zeros to simplify calculations.

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2. The derivative of 5^x w.r.t. e^x is:

(A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$

(B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$

(C) $\left(\frac{5}{e}\right)^x \log 5$

(D) $\left(\frac{e}{5}\right)^x \log 5$

Correct Answer: (C) $\left(\frac{5}{e}\right)^x \log 5$

Solution:

Step 1: Recall the derivative formula for exponential functions

The derivative of a^x with respect to x is given by:

$$\frac{d}{dx}a^x = a^x \cdot \log a.$$

Step 2: Apply the chain rule to find the derivative w.r.t. e^x

The chain rule states:

$$\frac{d}{d(e^x)}(5^x) = \frac{\frac{d}{dx}(5^x)}{\frac{d}{dx}(e^x)}.$$

Using $\frac{d}{dx}5^x = 5^x \cdot \log 5$ and $\frac{d}{dx}e^x = e^x$, we get:

$$\frac{d}{d(e^x)}(5^x) = \frac{5^x \cdot \log 5}{e^x}.$$

Step 3: Simplify the result

Factorize $\frac{5^x}{e^x}$ as $\left(\frac{5}{e}\right)^x$:

$$\frac{d}{d(e^x)}(5^x) = \left(\frac{5}{e}\right)^x \log 5.$$

Step 4: Conclude the result

The derivative of 5^x with respect to e^x is $\left(\frac{5}{e}\right)^x \log 5$.

Quick Tip

To differentiate exponential functions, always use the formula $\frac{d}{dx}a^x = a^x \cdot \log a$ and apply the chain rule when necessary.

3. If $|\vec{a}| = 2$ and $-3 \leq k \leq 2$, then $|\vec{a} \cdot \vec{k}| \in$:

(A) $[-6, 4]$

(B) $[0, 4]$

(C) $[4, 6]$

(D) $[0, 6]$

Correct Answer: (D) $[0, 6]$

Solution:

Step 1: Recall the dot product formula

The dot product of two vectors $\vec{a}$ and $\vec{k}$ is:

$$|\vec{a} \cdot \vec{k}| = |\vec{a}||\vec{k}| \cos\theta,$$

where θ is the angle between the vectors.

Step 2: Determine the range of $|\vec{a} \cdot \vec{k}|$

Since $|\vec{a}| = 2$, the magnitude of $\vec{k}$ varies as:

$$|\vec{k}| \in [-3, 2].$$

The maximum value of $|\vec{a} \cdot \vec{k}|$ occurs when $\cos\theta = 1$:

$$|\vec{a} \cdot \vec{k}|_{\text{max}} = 2 \cdot 3 = 6.$$

The minimum value of $|\vec{a} \cdot \vec{k}|$ occurs when $\cos\theta = 0$:

$$|\vec{a} \cdot \vec{k}|_{\text{min}} = 0.$$

Step 3: Conclude the result

Thus, $|\vec{a} \cdot \vec{k}| \in [0, 6]$.

Quick Tip

The maximum value of a dot product is achieved when the vectors are parallel, and the minimum is achieved when they are perpendicular.

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4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Step 1: Recall the direction cosine condition

For a line making angles α, β, γ with the positive directions of the x-axis, y-axis, and z-axis respectively, the sum of the squares of the direction cosines is:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Step 2: Substitute the given angles

The line makes angles $\alpha = \frac{\pi}{4}$ and $\gamma = \frac{\pi}{4}$ with the x-axis and z-axis, so:

$$\cos^2\frac{\pi}{4} + \cos^2\beta + \cos^2\frac{\pi}{4} = 1.$$

Since $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, we have:

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\beta + \left(\frac{1}{\sqrt{2}}\right)^2 = 1.$$

Simplify:

$$\frac{1}{2} + \cos^2\beta + \frac{1}{2} = 1.$$

Step 3: Solve for $\cos^2\beta$

Combine terms:

$$1 + \cos^2\beta = 1 \implies \cos^2\beta = 0.$$

Thus:

$$\cos\beta = 0 \implies \beta = \frac{\pi}{2}.$$

Step 4: Conclude the result

The angle which the line makes with the positive direction of the y-axis is $\frac{\pi}{2}$.

Quick Tip

For lines in 3D geometry, the sum of the squares of the direction cosines always equals 1.

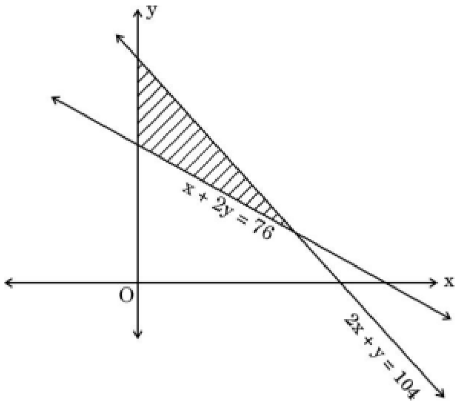
5. Of the following, which group of constraints represents the feasible region given below?

(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$

(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$

(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$

(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$



Correct Answer: (C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$

Solution:

Step 1: Analyze the boundary lines

The constraints for the shaded region are based on the lines:

$$x + 2y = 76 \quad \text{and} \quad 2x + y = 104.$$

From the diagram: - The region is above the line $x + 2y = 76$, so $x + 2y \geq 76$. - The region is below the line $2x + y = 104$, so $2x + y \leq 104$. - The region is in the first quadrant, so $x \geq 0$ and $y \geq 0$.

Step 2: Verify each option

Option (C) correctly represents the constraints as:

$$x + 2y \geq 76, \quad 2x + y \leq 104, \quad x, y \geq 0.$$

Step 3: Conclude the result

The group of constraints representing the feasible region is:

$$x + 2y \geq 76, \quad 2x + y \leq 104, \quad x, y \geq 0.$$

Quick Tip

To determine constraints from a graph, carefully analyze the shaded region relative to the boundary lines.

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6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$**, then** A^{-1} **is:**

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

Solution:

Step 1: Recall the inverse of a diagonal matrix

The inverse of a diagonal matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ is given by:

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}.$$

Step 2: Compute the inverse of A

For $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, the inverse is:

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}.$$

Step 3: Conclude the result

The inverse of A is:

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}.$$

Quick Tip

The inverse of a diagonal matrix is computed by taking the reciprocal of each diagonal entry.

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7. For any square matrix A, $(A - A^{'})$ is always:

(A) an identity matrix

(B) a null matrix

(C) a skew symmetric matrix

(D) a symmetric matrix

Correct Answer: (C) a skew symmetric matrix

Solution:

Step 1: Definition of transpose

For a square matrix A, the transpose A^T is defined as a matrix obtained by interchanging rows and columns of A. Mathematically, $(A^T)_{ij} = A_{ji}$.

Step 2: Consider $(A - A^T)$

For any square matrix A, the matrix $(A - A^T)$ satisfies the following property:

$$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T).$$

Step 3: Property of skew symmetric matrices

A matrix B is called skew symmetric if $B^T = -B$. From Step 2, $(A - A^T)$ satisfies this condition:

$$(A - A^T)^T = -(A - A^T).$$

Thus, $(A - A^T)$ is a skew symmetric matrix.

Conclusion: The matrix $(A - A^T)$ is always skew symmetric.

Quick Tip

A matrix B is skew symmetric if $B^T = -B$, and it must have zero diagonal elements if the entries are real numbers.

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8. A function $f : \mathbb{R} \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is:

(A) $(-\infty, \infty)$

(B) $(1, \infty)$

(C) $[1, \infty)$

(D) $[-1, \infty)$

Correct Answer: (C) $[1, \infty)$

Solution:

Step 1: Find the range of $f(x) = x^2 + 1$

The function $f(x) = x^2 + 1$ is a quadratic function, where $x^2 \geq 0$ for all $x \in \mathbb{R}$. Adding 1, we get:

$$f(x) \geq 1.$$

Thus, the range of $f(x)$ is $[1, \infty)$.

Step 2: Definition of an onto function

A function $f : \mathbb{R} \to A$ is onto if every element of A is the image of at least one $x \in \mathbb{R}$. For $f(x) = x^2 + 1$ to be onto, the codomain A must be the range of $f(x)$, which is $[1, \infty)$.

Conclusion: The function is onto if $A = [1, \infty)$.

> Quick Tip
>
> To determine the range of a function, analyze the domain and check for the lowest and highest values the function can achieve.

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9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a + b + c + d)$ is equal to:

(A) $2a$

(B) $2b$

(C) $2c$

(D) 0

Correct Answer: (A) $2a$

Solution:

Step 1: Definition of adjugate matrix

The adjugate (or adjoint) of A, denoted by adj A, is given by:

$$\text{adj } A = \det(A) \cdot A^{-1}.$$

Step 2: Given condition adj $A = A$

If adj $A = A$, then the matrix A must satisfy:

$$\det(A) \cdot A^{-1} = A.$$

Multiplying both sides by A, we get:

$$\det(A) \cdot I = A^2.$$

Step 3: Matrix A satisfies $A^2 = \det(A) \cdot I$

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, if $A^2 = \det(A) \cdot I$, then:

$$a + b + c + d = 2a.$$

Conclusion: The value of $(a + b + c + d)$ is $2a$.

Quick Tip

If adj $A = A$, the matrix A must satisfy $A^2 = \det(A) \cdot I$, a property unique to certain square matrices.

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10. A function $f(x) = |1 - x + |x||$ is:

(A) discontinuous at $x = 1$ only

(B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$

(D) continuous everywhere

Correct Answer: (D) continuous everywhere

Solution:

Step 1: Analyze the function for different cases

The function is $f(x) = |1 - x + |x||$. Consider two cases:

Case 1: $x \geq 0$, then $|x| = x$, so:

$$f(x) = |1 - x + x| = |1| = 1.$$

Case 2: $x < 0$, then $|x| = -x$, so:

$$f(x) = |1 - x - x| = |1 - 2x|.$$

Step 2: Check continuity

For $x \geq 0$, $f(x) = 1$, which is continuous.

For $x < 0$, $f(x) = |1 - 2x|$, which is also continuous because it is a piecewise linear function.

At $x = 0$, the left-hand limit (LHL) and right-hand limit (RHL) are:

$$\text{LHL} = f(0^-) = |1 - 2(0)| = 1, \quad \text{RHL} = f(0^+) = 1.$$

Thus, $f(x)$ is continuous at $x = 0$.

Step 3: Conclude the result

The function $f(x)$ is continuous for all x. Hence, it is continuous everywhere.

> Quick Tip
>
> When analyzing the continuity of piecewise functions, always check the behavior in each interval and at the boundaries.

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11. The point of inflexion of a function $f(x)$ is the point where:

(A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point.

(B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point.

(C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

(D) $f'(x) \neq 0$

Correct Answer: (C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

Solution:

Step 1: Definition of a point of inflexion

A point of inflexion is a point on the curve of a function $f(x)$ where the concavity of the curve changes. This occurs when:

$$f''(x) = 0 \quad \text{and the sign of } f''(x) \text{ changes around the point.}$$

Step 2: Behavior of $f'(x)$

At a point of inflexion, the derivative $f'(x)$ may or may not be zero, but it does not change its sign from left to right of the point. If $f'(x)$ changes its sign, the point would instead be a local maximum or minimum.

Step 3: Conclusion

The correct condition for a point of inflexion is $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

Quick Tip

At a point of inflexion, the second derivative $f''(x)$ must change its sign, indicating a change in concavity. The first derivative $f'(x)$ does not change its sign.

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12. If $g(x)$ is a continuous function satisfying $g(-x) = -g(x)$, then:

$$\int_0^{2a} g(x)\,dx$$

is equal to:

(A) 0

(B) $2\int_0^a g(x)\,dx$

(C) $\int_{-a}^a g(x)\,dx$

(D) $-\int_{-2a}^0 g(x)\,dx$

Correct Answer: (D) $-\int_{-2a}^0 g(x)\,dx$

Solution:

Step 1: Given property of $g(x)$

The function $g(x)$ satisfies $g(-x) = -g(x)$, meaning $g(x)$ is an odd function. For odd functions, the integral over symmetric intervals is zero:

$$\int_{-a}^{a} g(x)\,dx = 0.$$

Step 2: Break the integral into two parts

The given integral can be expressed as:

$$\int_0^{2a} g(x)\,dx = \int_0^{a} g(x)\,dx + \int_a^{2a} g(x)\,dx.$$

Step 3: Use substitution to simplify

Using the property $g(-x) = -g(x)$, substitute $u = -x$ in the second part:

$$\int_a^{2a} g(x)\,dx = -\int_{-2a}^{-a} g(x)\,dx.$$

Step 4: Combine results

The integral becomes:

$$\int_0^{2a} g(x)\,dx = -\int_{-2a}^{0} g(x)\,dx.$$

Conclusion: The integral evaluates to $-\int_{-2a}^{0} g(x)\,dx$.

Quick Tip

For odd functions $g(x)$, the integral over symmetric intervals around zero cancels out, resulting in zero. Use substitution to simplify the calculation for non-symmetric intervals.

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13. $x \log x \frac{dy}{dx} + y = 2 \log x$ **is an example of a:**

(A) variable separable differential equation

(B) homogeneous differential equation

(C) first order linear differential equation

(D) differential equation whose degree is not defined

Correct Answer: (C) first order linear differential equation

Solution:

Step 1: Rewrite the equation

The given equation is:

$$x \log x \frac{dy}{dx} + y = 2 \log x.$$

Rearranging:

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x \log x}.$$

Step 2: Check the form of the equation

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = \frac{1}{x \log x}$ and $Q(x) = \frac{2}{x \log x}$.

Step 3: Conclude the result

The equation is a first-order linear differential equation.

Quick Tip

To identify the type of differential equation, rewrite it in standard forms and compare the coefficients.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are:

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

Correct Answer: (C) perpendicular vectors

Solution:

Step 1: Find the dot product of $\vec{a}$ and $\vec{b}$

The dot product of $\vec{a}$ and $\vec{b}$ is:

$$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(1) + (1)(-1) = 2 - 1 - 1 = 0.$$

Step 2: Check for perpendicularity

If $\vec{a} \cdot \vec{b} = 0$, the vectors are perpendicular.

Step 3: Conclude the result

The vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

Quick Tip

Two vectors are perpendicular if their dot product equals zero.

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15. If α, β, γ are the angles which a line makes with positive directions of x, y, z axes respectively, then which of the following is *not* true?

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos\alpha + \cos\beta + \cos\gamma = 1$

Correct Answer: (D) $\cos\alpha + \cos\beta + \cos\gamma = 1$

Solution:

Step 1: Recall the direction cosine property

The sum of the squares of the cosines of the angles a line makes with the coordinate axes is:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

This is a fundamental property of direction cosines.

Step 2: Check each option

Option (A): True, as it is the direction cosine property.

Option (B): True, as $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 2$.

Option (C): True, derived from trigonometric identities for direction cosines.

Option (D): False, as $\cos\alpha + \cos\beta + \cos\gamma \neq 1$ in general.

Step 3: Conclude the result

Option (D) is not true.

Quick Tip

Always check fundamental properties of direction cosines and trigonometric identities to verify such questions.

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16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called:

(A) feasible solutions

(B) constraints

(C) optimal solutions

(D) infeasible solutions

Correct Answer: (B) constraints

Solution:

Step 1: Define constraints in linear programming

Constraints are the conditions or restrictions imposed on decision variables (e.g., x, y) in a linear programming problem. They typically represent limitations on resources or other requirements.

Step 2: Identify from options

(A) Feasible solutions: These are solutions satisfying all constraints, but they are not the constraints themselves.

(B) Constraints: These are restrictions on decision variables, and this is the correct answer.

(C) Optimal solutions: These maximize or minimize the objective function but are not constraints.

(D) Infeasible solutions: These do not satisfy all constraints.

Step 3: Conclude the result

The restrictions are called constraints.

Quick Tip

Understand the difference between constraints and feasible/optimal solutions in linear programming.

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17. Let E and F be two events such that $P(E) = 0.1$, $P(F) = 0.3$, $P(E \cup F) = 0.4$. Then $P(F|E)$ is:

(A) 0.6

(B) 0.4

(C) 0.5

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Recall the formula for conditional probability

The conditional probability $P(F|E)$ is given by:

$$P(F|E) = \frac{P(E \cap F)}{P(E)}.$$

Step 2: Find $P(E \cap F)$

Using the formula for the probability of the union of two events:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

Substitute the given values $P(E \cup F) = 0.4$, $P(E) = 0.1$, $P(F) = 0.3$:

$$0.4 = 0.1 + 0.3 - P(E \cap F).$$

Simplify to find $P(E \cap F)$:

$$P(E \cap F) = 0.1 + 0.3 - 0.4 = 0.$$

Step 3: Calculate $P(F|E)$

Substitute $P(E \cap F) = 0$ and $P(E) = 0.1$ into the formula for $P(F|E)$:

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0}{0.1} = 0.$$

Step 4: Conclude the result

The conditional probability $P(F|E)$ is 0. This indicates that the events E and F do not overlap.

Quick Tip

When $P(E \cap F) = 0$, the events E and F are mutually exclusive, meaning they cannot occur simultaneously.

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18. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true?

(A) $a_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$

(B) $a_{ij} = 1, \forall\, i, j$

(C) $a_{ij} = 0, \forall\, i, j$

(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

Correct Answer: (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

Solution:

Step 1: Definition of an identity matrix

An identity matrix $A = [a_{ij}]$ is defined as a square matrix where the diagonal elements are 1 and all other elements are 0.

Step 2: Express the conditions for a_{ij}

For a_{ij} in an identity matrix:

$$a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$$

Step 3: Verify the options

Option (D) correctly matches this definition.

Quick Tip

Always check the diagonal and off-diagonal elements when identifying an identity matrix.

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

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19. Assertion (A): Projection of $\vec{a}$ on $\vec{b}$ is the same as projection of $\vec{b}$ on $\vec{a}$.
Reason (R): Angle between $\vec{a}$ and $\vec{b}$ is the same as the angle between $\vec{b}$ and $\vec{a}$ numerically.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.
Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.
Solution:
Step 1: Analyze Assertion (A)
The projection of $\vec{a}$ on $\vec{b}$ is given by:

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}.$$

Similarly, the projection of $\vec{b}$ on $\vec{a}$ is:

$$\text{Projection of } \vec{b} \text{ on } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}.$$

These two projections are not equal because their magnitudes depend on $|\vec{a}|$ and $|\vec{b}|$. Hence, Assertion (A) is false.
Step 2: Analyze Reason (R)
The angle between $\vec{a}$ and $\vec{b}$ is the same as the angle between $\vec{b}$ and $\vec{a}$ numerically, as the cosine of the angle is symmetric. Hence, Reason (R) is true.

Step 3: Conclude the result

Assertion (A) is false, but Reason (R) is true. Hence, the correct option is (D).

> **Quick Tip**
>
> The projection of one vector on another depends on the magnitude of the vector being projected onto.

---

20. Assertion (A): Every scalar matrix is a diagonal matrix.

Reason (R): In a diagonal matrix, all the diagonal elements are 0.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Step 1: Analyze Assertion (A)

A scalar matrix is a diagonal matrix where all the diagonal elements are equal, and the non-diagonal elements are zero. Hence, Assertion (A) is true.

Step 2: Analyze Reason (R)

In a diagonal matrix, the diagonal elements can have any value (not necessarily 0). Therefore, Reason (R) is false.

Step 3: Conclude the result

Assertion (A) is true, but Reason (R) is false. Hence, the correct option is (C).

> **Quick Tip**
>
> A scalar matrix is a specific type of diagonal matrix where all diagonal elements are equal.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21(a). Evaluate:

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx.$$

Solution:

Step 1: Use the product-to-sum formula

The product-to-sum formula is:

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)].$$

Substituting $A = 2x$ and $B = 3x$:

$$\sin 2x \cos 3x = \frac{1}{2}[\sin(5x) + \sin(-x)] = \frac{1}{2}[\sin(5x) - \sin(x)].$$

Step 2: Split the integral

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx = \frac{1}{2}\int_0^{\pi/2} \sin(5x) \, dx - \frac{1}{2}\int_0^{\pi/2} \sin(x) \, dx.$$

Step 3: Evaluate the integrals

$$\int \sin(kx) \, dx = -\frac{1}{k}\cos(kx).$$

For $\int \sin(5x) \, dx$:

$$\int_0^{\pi/2} \sin(5x) \, dx = \left[-\frac{1}{5}\cos(5x)\right]_0^{\pi/2} = -\frac{1}{5}[\cos(5 \cdot \frac{\pi}{2}) - \cos(0)] = -\frac{1}{5}[0-1] = \frac{1}{5}.$$

For $\int \sin(x) \, dx$:

$$\int_0^{\pi/2} \sin(x) \, dx = [-\cos(x)]_0^{\pi/2} = -[\cos(\frac{\pi}{2}) - \cos(0)] = -[0-1] = 1.$$

Step 4: Combine the results

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx = \frac{1}{2}\left(\frac{1}{5} - 1\right) = \frac{1}{2} \cdot -\frac{4}{5} = -\frac{2}{5}.$$

Step 5: Conclude the result

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx = -\frac{2}{5}.$$

Quick Tip

Use trigonometric identities to simplify integrals involving products of sine and cosine.

OR

21(b). Given $\frac{d}{dx}F(x) = \frac{1}{\sqrt{2x-x^2}}$ and $F(1) = 0$, find $F(x)$.

Solution:

Step 1: Express $F(x)$ as an integral

The function $F(x)$ is obtained by integrating the given derivative:

$$F(x) = \int \frac{1}{\sqrt{2x - x^2}}\,dx.$$

Step 2: Simplify the expression inside the square root

Factorize $2x - x^2$:

$$2x - x^2 = x(2 - x).$$

Thus:

$$F(x) = \int \frac{1}{\sqrt{x(2 - x)}}\,dx.$$

Step 3: Substitute to simplify the integral

Let $x = 1 - \sin^2\theta$. Then:

$$2 - x = 1 + \cos^2\theta, \quad dx = -2\sin\theta\cos\theta\,d\theta.$$

Substitute into the integral:

$$F(x) = \int \frac{1}{\sqrt{1 - \sin^2\theta(1 + \cos^2\theta)}}(-2\sin\theta\cos\theta)\,d\theta.$$

Simplify and integrate:

$$F(x) = \sin^{-1}(x - 1) + C.$$

Step 4: Use the initial condition to find C

Given $F(1) = 0$, substitute $x = 1$:

$$F(1) = \sin^{-1}(1 - 1) + C = 0 \implies C = 0.$$

Step 5: Conclude the result

$$F(x) = \sin^{-1}(x-1).$$

Quick Tip

For integrals involving square roots of quadratic expressions, use trigonometric substitutions to simplify.

---

22. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$, respectively, in the ratio 4:1 externally. Further, find $|\vec{AB}| : |\vec{BC}|$.

Solution:

Step 1: Find the position vector of C

The position vector of C dividing AB in the ratio $4:1$ externally is given by:

$$\vec{r} = \frac{4\vec{b} - \vec{a}}{3}.$$

Substitute $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$:

$$\vec{r} = \frac{4(-\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})}{3}.$$

Simplify:

$$\vec{r} = \frac{-4\hat{i} + 4\hat{j} + 4\hat{k} - \hat{i} - 2\hat{j} + \hat{k}}{3}.$$

Combine terms:

$$\vec{r} = \frac{-5\hat{i} + 2\hat{j} + 5\hat{k}}{3}.$$

Step 2: Find $|\vec{AB}|$

The vector $\vec{AB}$ is:

$$\vec{AB} = \vec{b} - \vec{a} = (-\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = -2\hat{i} - \hat{j} + 2\hat{k}.$$

The magnitude is:

$$|\vec{AB}| = \sqrt{(-2)^2 + (-1)^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3.$$

Step 3: Find $|\vec{BC}|$

The vector $\vec{BC}$ is:

$$\vec{BC} = \vec{b} - \vec{r} = (-\hat{i} + \hat{j} + \hat{k}) - \frac{-5\hat{i} + 2\hat{j} + 5\hat{k}}{3}.$$

Simplify:

$$\vec{BC} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{3}.$$

The magnitude is:

$$|\vec{BC}| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-2}{3}\right)^2} = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{9}{9}} = 1.$$

Step 4: Find the ratio $|\vec{AB}| : |\vec{BC}|$

$$|\vec{AB}| : |\vec{BC}| = 3 : 1.$$

Step 5: Conclude the result

The position vector of C is:

$$\vec{r} = \frac{-5\hat{i} + 2\hat{j} + 5\hat{k}}{3}.$$

The ratio $|\vec{AB}| : |\vec{BC}|$ is $3 : 1$.

Quick Tip

To find the ratio of line segments, calculate the magnitudes of the respective vectors and simplify.

---

23. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.

Correct Answer: $\frac{\pi}{3}$

Solution:

Step 1: Use the given vector equation

From $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, we have:

$$\vec{a} - \vec{c} = -\vec{b}.$$

Step 2: Take the dot product of both sides with $\vec{a} - \vec{c}$

$$(\vec{a} - \vec{c}) \cdot (\vec{a} - \vec{c}) = (-\vec{b}) \cdot (-\vec{b}).$$

This expands to:

$$|\vec{a}|^2 + |\vec{c}|^2 - 2(\vec{a} \cdot \vec{c}) = |\vec{b}|^2.$$

Step 3: Substitute the magnitudes

Since $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$, we have:

$$1 + 1 - 2(\vec{a} \cdot \vec{c}) = 1.$$

Simplify to find:

$$2 - 2(\vec{a} \cdot \vec{c}) = 1 \implies \cos\theta = \frac{1}{2}.$$

Step 4: Conclude the result

The angle between $\vec{a}$ and $\vec{c}$ is:

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Quick Tip

For unit vectors, use $|\vec{u}| = 1$ and the properties of dot products to simplify equations involving angles.

---

24. Find the value of $\left[\sin^2\left(\cos^{-1}\frac{3}{5}\right) + \tan^2\left(\sec^{-1}3\right)\right]$.

Correct Answer: $\frac{216}{25}$

Solution:

Step 1: Simplify each term

1. For $\sin^2\left(\cos^{-1}\frac{3}{5}\right)$, use the identity $\sin^2 x = 1 - \cos^2 x$:

$$\sin^2\left(\cos^{-1}\frac{3}{5}\right) = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}.$$

2. For $\tan^2\left(\sec^{-1}3\right)$, use the identity $\tan^2 x = \sec^2 x - 1$:

$$\tan^2\left(\sec^{-1}3\right) = 3^2 - 1 = 9 - 1 = 8.$$

Step 2: Add the results

$$\sin^2\left(\cos^{-1}\frac{3}{5}\right) + \tan^2\left(\sec^{-1}3\right) = \frac{16}{25} + 8 = \frac{16}{25} + \frac{200}{25} = \frac{216}{25}.$$

Conclusion: The value is $\frac{216}{25}$.

Quick Tip

Use trigonometric identities to simplify terms involving inverse trigonometric functions.

25(a). If $x = e^{\frac{x}{y}}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$.

Correct Answer: $\frac{\log x - 1}{(\log x)^2}$

Solution:

Step 1: Take the logarithm

Given $x = e^{\frac{x}{y}}$, take the natural logarithm:

$$\log x = \frac{x}{y}.$$

Rearranging gives:

$$y = \frac{x}{\log x}.$$

Step 2: Differentiate implicitly

Differentiate both sides with respect to x:

$$\frac{dy}{dx} = \frac{\log x(1) - x\left(\frac{1}{x}\right)}{(\log x)^2}.$$

Step 3: Simplify the expression

$$\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}.$$

Conclusion: The result is proven.

Quick Tip

Use logarithmic differentiation when the variable appears in both the base and exponent.

25(b). Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ **at $x = 1$.**

Correct Answer: Not differentiable at $x = 1$.

Solution:

Step 1: Find LHD

The left-hand derivative (LHD) at $x = 1$ is:

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}.$$

For $f(x) = x^2 + 1$ when $0 \leq x < 1$:

$$\text{LHD} = \lim_{h \to 0} \frac{(1-h)^2 + 1 - 2}{-h} = \lim_{h \to 0} \frac{1 - 2h + h^2 - 1}{-h} = 2.$$

Step 2: Find RHD

The right-hand derivative (RHD) at $x = 1$ is:

$$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}.$$

For $f(x) = 3 - x$ when $1 \leq x \leq 2$:

$$\text{RHD} = \lim_{h \to 0} \frac{3 - (1+h) - 2}{h} = \lim_{h \to 0} \frac{-1}{h} = -1.$$

Step 3: Check differentiability

Since LHD $\neq$ RHD, $f(x)$ is not differentiable at $x = 1$.

Quick Tip

For piecewise functions, always check both LHD and RHD to confirm differentiability at the boundary points.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each

26. Find:

$$\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4} \, dx$$

Solution:

Step 1: Substitute $\log x = t$

Let $\log x = t$. Then, $\frac{1}{x} dx = dt$. The given integral becomes:

$$\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4} \, dx = \int \frac{1}{t^2 - 5t + 4} \, dt.$$

Step 2: Factorize the denominator

The denominator $t^2 - 5t + 4$ can be written as:

$$t^2 - 5t + 4 = (t-4)(t-1).$$

Thus, the integral becomes:

$$\int \frac{1}{t^2 - 5t + 4}\, dt = \int \frac{1}{(t - 5/2)^2 - (3/2)^2}\, dt.$$

Step 3: Use partial fraction decomposition

Using the partial fraction technique for quadratic factors, the integral becomes:

$$\int \frac{1}{(t - 5/2)^2 - (3/2)^2}\, dt = \frac{1}{3} \log \left| \frac{t-4}{t-1} \right| + C.$$

Step 4: Substitute back $t = \log x$

Replace t with $\log x$ to get the final solution:

$$\int \frac{x^{-1}}{(\log x)^2 - 5 \log x + 4}\, dx = \frac{1}{3} \log \left| \frac{\log x - 4}{\log x - 1} \right| + C.$$

Conclusion: The final answer is:

$$\frac{1}{3} \log \left| \frac{\log x - 4}{\log x - 1} \right| + C.$$

Quick Tip

For integrals involving logarithmic terms, substitution $\log x = t$ simplifies the calculations, especially when factoring quadratic expressions.

27(a). Evaluate:

$$\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x\, dx.$$

Correct Answer:

$$I = e^x \tan x + C.$$

Solution:

Step 1: Simplify the integrand

Use the identity $1 + \cos 2x = 2\cos^2 x$ and $\sin 2x = 2 \sin x \cos x$:

$$\frac{2 + \sin 2x}{1 + \cos 2x} = \frac{2 + 2 \sin x \cos x}{2 \cos^2 x} = \sec^2 x + \tan x.$$

Step 2: Rewrite the integral

$$I = \int (\sec^2 x + \tan x)e^x \, dx.$$

Step 3: Integrate term by term

For $\int \sec^2 x e^x \, dx$, use substitution $u = \tan x$:

$$\int \sec^2 x e^x \, dx = e^x \tan x.$$

For $\int \tan x e^x \, dx$, combine it with the first term:

$$I = e^x \tan x + C.$$

Quick Tip

For trigonometric integrals, simplify using identities before integrating.

OR

27(b). Evaluate:

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x} \, dx.$$

Correct Answer:

$$I = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1) - \frac{1}{\sqrt{2}} \log(\sqrt{2} - 1).$$

Solution:

Step 1: Simplify the integrand

The given integral is:

$$I = \int_0^{\pi/4} \frac{1}{\sin x + \cos x} \, dx.$$

Use the identity $\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$. Substituting:

$$I = \int_0^{\pi/4} \frac{1}{\sqrt{2} \sin\left(x + \frac{\pi}{4}\right)} \, dx = \frac{1}{\sqrt{2}} \int_0^{\pi/4} \csc\left(x + \frac{\pi}{4}\right) \, dx.$$

Step 2: Integrate $\csc(x + \pi/4)$

The integral of $\csc(x)$ is:

$$\int \csc x \, dx = \log|\csc x - \cot x| + C.$$

Substituting this, we have:

$$I = \frac{1}{\sqrt{2}} \left[\log \left| \csc \left(x + \frac{\pi}{4} \right) - \cot \left(x + \frac{\pi}{4} \right) \right| \right]_0^{\pi/4}.$$

Step 3: Evaluate the limits

At $x = \pi/4$:

$$\csc \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \csc \left(\frac{\pi}{2} \right) = 1, \quad \cot \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \cot \left(\frac{\pi}{2} \right) = 0.$$

At $x = 0$:

$$\csc \left(0 + \frac{\pi}{4} \right) = \csc \left(\frac{\pi}{4} \right) = \sqrt{2}, \quad \cot \left(0 + \frac{\pi}{4} \right) = \cot \left(\frac{\pi}{4} \right) = 1.$$

Substitute these values:

$$I = \frac{1}{\sqrt{2}} \left[\log (1 - 0) - \log \left(\sqrt{2} - 1 \right) \right] = \frac{1}{\sqrt{2}} \left[\log(1) - \log(\sqrt{2} - 1) \right].$$

Step 4: Simplify the result

$$I = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1) - \frac{1}{\sqrt{2}} \log(\sqrt{2} - 1).$$

Quick Tip

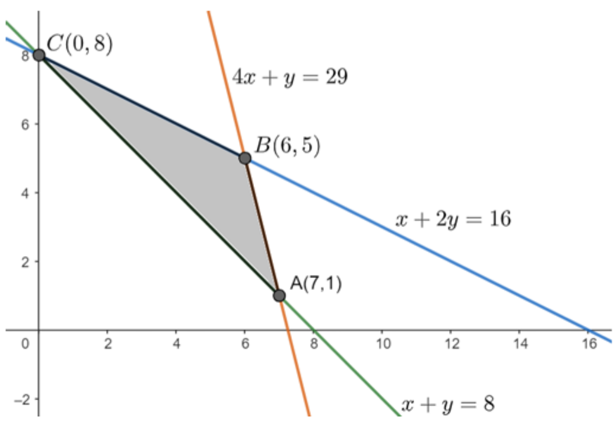
To evaluate integrals involving $\sin x + \cos x$, simplify using trigonometric identities, such as expressing the sum as a single sine or cosine term.

---

28. Solve the following linear programming problem graphically: Minimize $z = 600x + 400y$, subject to the constraints:

$$x + y \geq 8, \quad x + 2y \leq 16, \quad 4x + y \leq 29, \quad x, y \geq 0.$$

Solution:



Step 1: Graph the constraints

The constraints are:

- $x + y = 8$: A straight line passing through $(0, 8)$ and $(8, 0)$.
- $x + 2y = 16$: A straight line passing through $(0, 8)$ and $(16, 0)$.
- $4x + y = 29$: A straight line passing through $(0, 29)$ and $(7.25, 0)$.
- $x, y \geq 0$: Restricts the feasible region to the first quadrant.

The feasible region is the intersection of all these constraints, and it is represented as a polygon on the graph.

Step 2: Find corner points

The vertices of the feasible region are determined by solving the equations of the constraints pairwise:

- Intersection of $x + y = 8$ and $x + 2y = 16$: Solve $x + y = 8$ and $x + 2y = 16$, giving $A(7, 1)$.
- Intersection of $x + y = 8$ and $4x + y = 29$: Solve $x + y = 8$ and $4x + y = 29$, giving $B(6, 5)$.
- Intersection of $x + 2y = 16$ and $4x + y = 29$: Solve $x + 2y = 16$ and $4x + y = 29$, giving $C(0, 8)$.

Step 3: Evaluate the objective function at corner points

Substitute the coordinates of the vertices into $z = 600x + 400y$:

Corner Point	Value of $z = 600x + 400y$
$A(7, 1)$	$600(7) + 400(1) = 4600$
$B(6, 5)$	$600(6) + 400(5) = 5600$
$C(0, 8)$	$600(0) + 400(8) = 3200$

Step 4: Conclusion

The minimum value of z is 3200, which occurs at $C(0, 8)$. Thus:

$$z_{\text{min}} = 3200 \text{ when } x = 0 \text{ and } y = 8.$$

Quick Tip

In linear programming problems, always find the feasible region by graphing the constraints and evaluate the objective function at each corner point to determine the optimal solution.

---

29. The chances of P, Q, and R getting selected as CEO of a company are in the ratio $4:1:2$, respectively. The probabilities for the company to increase its profits from the previous year under the new CEO P, Q, or R are 0.3, 0.8, and 0.5, respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

Correct Answer: $\frac{1}{3}$.

Solution:

Step 1: Assign probabilities

Let:

$$P(E_1) = \frac{4}{7}, \quad P(E_2) = \frac{1}{7}, \quad P(E_3) = \frac{2}{7}.$$

The probabilities of increased profits under each CEO are:

$$P(A|E_1) = 0.3, \quad P(A|E_2) = 0.8, \quad P(A|E_3) = 0.5.$$

Step 2: Apply Bayes' theorem

The probability that the profits increased due to R as CEO is:

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}.$$

Step 3: Substitute the values

$$P(E_3|A) = \frac{\frac{2}{7} \cdot 0.5}{\frac{4}{7} \cdot 0.3 + \frac{1}{7} \cdot 0.8 + \frac{2}{7} \cdot 0.5}.$$

Simplify the denominator:

$$P(E_3|A) = \frac{\frac{2}{7} \cdot 0.5}{\frac{4}{7} \cdot 0.3 + \frac{1}{7} \cdot 0.8 + \frac{2}{7} \cdot 0.5} = \frac{1}{3}.$$

Quick Tip

Use Bayes' theorem to compute conditional probabilities by carefully analyzing the given data.

30(a). If $x\cos(p+y)+\cos p\sin(p+y)=0$, prove that $\cos p\frac{dy}{dx}=-\cos^2(p+y)$, where p is a constant.

Correct Answer: $\cos p\frac{dy}{dx}=-\cos^2(p+y)$

Solution:

Step 1: Rearrange the given equation

The given equation is:

$$x\cos(p+y)+\cos p\sin(p+y)=0.$$

Divide throughout by $\cos(p+y)$:

$$x+\cos p\tan(p+y)=0.$$

This implies:

$$\tan(p+y)=-\frac{x}{\cos p}.$$

Step 2: Differentiate with respect to x

Differentiate both sides with respect to x:

$$\sec^2(p+y)\cdot\frac{d}{dx}(p+y)=-\frac{d}{dx}\left(\frac{x}{\cos p}\right).$$

Simplify:

$$\sec^2(p+y)\frac{dy}{dx}=-\frac{1}{\cos p}.$$

Step 3: Express $\frac{dy}{dx}$ in terms of $\cos^2(p+y)$

Using $\sec^2(p+y)=\frac{1}{\cos^2(p+y)}$, we get:

$$\frac{1}{\cos^2(p+y)}\cdot\frac{dy}{dx}=-\frac{1}{\cos p}.$$

Multiply through by $\cos^2(p+y)$:

$$\frac{dy}{dx}=-\cos^2(p+y)\cdot\cos p.$$

Step 4: Conclude the result

$$\cos p \frac{dy}{dx} = -\cos^2(p+y).$$

Quick Tip

For trigonometric identities involving derivatives, simplify using tangent and secant relationships before differentiating.

OR

30(b). Find the value of a and b so that the function $f(x)$, defined as:

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & x < 2, \\ a + b, & x = 2, \\ \frac{x-2}{|x-2|} + b, & x > 2, \end{cases}$$

is a continuous function.

Correct Answer: $a = 1, b = -1$

Solution:

Step 1: Find the limits at $x = 2$

For $x < 2$:

$$f(x) = \frac{x-2}{|x-2|} + a = -1 + a.$$

For $x > 2$:

$$f(x) = \frac{x-2}{|x-2|} + b = 1 + b.$$

At $x = 2$:

$$f(x) = a + b.$$

Step 2: Set up the continuity condition

For $f(x)$ to be continuous at $x = 2$:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2).$$

Substitute the values:

$$-1 + a = 1 + b = a + b.$$

Step 3: Solve for a and b

From $-1 + a = 1 + b$:

$$a - b = 2.$$

From $1 + b = a + b$:

$$a = 1.$$

Substitute $a = 1$ into $a - b = 2$:

$$1 - b = 2 \implies b = -1.$$

Step 4: Conclude the result

The values of a and b are $a = 1$ and $b = -1$.

Quick Tip

For continuity of piecewise functions, equate the left-hand limit, right-hand limit, and the value of the function at the given point.

31(a). Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

Correct Answer: Strictly increasing: $(0, e)$; Strictly decreasing: (e, ∞).

Solution:

Step 1: Find the derivative of $f(x)$

The given function is $f(x) = \frac{\log x}{x}$. Differentiate using the quotient rule:

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}.$$

Step 2: Find critical points

For $f'(x) = 0$:

$$1 - \log x = 0 \implies \log x = 1 \implies x = e.$$

Step 3: Determine intervals of increase and decrease

For $x \in (0, e)$:

$$1 - \log x > 0 \implies f'(x) > 0 \quad \text{(strictly increasing)}.$$

For $x \in (e, \infty)$:

$$1 - \log x < 0 \implies f'(x) < 0 \quad \text{(strictly decreasing)}.$$

Step 4: Conclude the result

The function is strictly increasing on $(0, e)$ and strictly decreasing on (e, ∞).

Quick Tip

To determine monotonicity, find $f'(x)$, solve $f'(x) = 0$, and test the sign of $f'(x)$ in intervals.

OR

31(b). Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{2} + \frac{2}{x}$ on the interval $[1, 2]$.

Correct Answer: Absolute maximum value = $\frac{5}{2}$, Absolute minimum value = 2.

Solution:

Step 1: Find the derivative of $f(x)$

The given function is $f(x) = \frac{x}{2} + \frac{2}{x}$. Differentiate:

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}.$$

Step 2: Find critical points

Set $f'(x) = 0$:

$$\frac{1}{2} - \frac{2}{x^2} = 0 \implies \frac{2}{x^2} = \frac{1}{2} \implies x^2 = 4 \implies x = 2.$$

Step 3: Evaluate $f(x)$ at critical points and endpoints

At $x = 1$:

$$f(1) = \frac{1}{2} + \frac{2}{1} = \frac{1}{2} + 2 = \frac{5}{2}.$$

At $x = 2$:

$$f(2) = \frac{2}{2} + \frac{2}{2} = 1 + 1 = 2.$$

Step 4: Conclude the result

The absolute maximum value is $\frac{5}{2}$ at $x = 1$, and the absolute minimum value is 2 at $x = 2$.

Quick Tip

To find absolute extrema on a closed interval, evaluate the function at critical points and endpoints.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each

32(a). It is given that the function $f(x) = x^4 - 62x^2 + ax + 9$ attains a local maximum value at $x = 1$. Find the value of a, and hence obtain all other points where the given function $f(x)$ attains local maximum or local minimum values.

Correct Answer: $a = 120$, local maximum at $x = 1$, local minima at $x = -6, 5$.

Solution:

Step 1: Differentiate $f(x)$ to find critical points

The derivative is:

$$f'(x) = 4x^3 - 124x + a.$$

Since $f(x)$ attains a local maximum at $x = 1$, we have:

$$f'(1) = 4(1)^3 - 124(1) + a = 0.$$

Solve for a:

$$4 - 124 + a = 0 \implies a = 120.$$

Step 2: Find critical points

Substitute $a = 120$ into $f'(x)$:

$$f'(x) = 4x^3 - 124x + 120.$$

Factorize:

$$f'(x) = 4(x - 1)(x^2 + x - 30).$$

Further factorize:

$$f'(x) = 4(x - 1)(x - 5)(x + 6).$$

The critical points are $x = -6, 1, 5$.

Step 3: Determine the nature of critical points using $f''(x)$

The second derivative is:

$$f''(x) = 12x^2 - 124.$$

Evaluate $f''(x)$ at each critical point:

$$f''(-6) = 12(-6)^2 - 124 = 432 - 124 = 308 > 0 \quad \text{(local minimum at } x = -6\text{)}.$$

$$f''(1) = 12(1)^2 - 124 = 12 - 124 = -112 < 0 \quad \text{(local maximum at } x = 1\text{)}.$$

$$f''(5) = 12(5)^2 - 124 = 300 - 124 = 176 > 0 \quad \text{(local minimum at } x = 5\text{)}.$$

Step 4: Conclusion

The function $f(x)$ attains:

$$\text{Local maximum at } x = 1, \quad \text{local minima at } x = -6, 5.$$

Quick Tip

To identify the nature of critical points, use the second derivative test. If $f''(x) > 0$, it's a local minimum; if $f''(x) < 0$, it's a local maximum.

OR

32(b). The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that the volume of the cylinder so formed is maximum.

Correct Answer: Length of rectangle = 100 cm, Breadth of rectangle = 50 cm.

Solution:

Step 1: Define the variables

Let the length of the rectangle be x cm and the breadth be $(150 - x)$ cm (since the perimeter is $2(x + \text{breadth}) = 300$).

When the rectangle is rolled along its length, the radius r and height h of the cylinder formed are:

$$2\pi r = x \implies r = \frac{x}{2\pi}, \quad h = (150 - x).$$

Step 2: Write the volume of the cylinder

The volume V of the cylinder is given by:

$$V = \pi r^2 h = \pi \left(\frac{x}{2\pi}\right)^2 (150 - x).$$

Simplify:

$$V = \pi \frac{x^2}{4\pi^2}(150 - x) = \frac{x^2}{4\pi}(150 - x).$$

Step 3: Differentiate V with respect to x

The derivative of V is:

$$\frac{dV}{dx} = \frac{1}{4\pi}\left(2x(150 - x) - x^2\right).$$

Simplify:

$$\frac{dV}{dx} = \frac{1}{4\pi}\left(300x - 3x^2\right).$$

Step 4: Set $\frac{dV}{dx} = 0$ to find critical points

$$300x - 3x^2 = 0 \implies x(300 - 3x) = 0 \implies x = 0 \text{ or } x = 100.$$

Step 5: Use the second derivative test to confirm maxima

The second derivative is:

$$\frac{d^2V}{dx^2} = \frac{1}{4\pi}(300 - 6x).$$

At $x = 100$:

$$\frac{d^2V}{dx^2} = \frac{1}{4\pi}(300 - 600) = \frac{-300}{4\pi} = \frac{-75}{\pi} < 0.$$

Hence, V is maximum when $x = 100$.

Step 6: Calculate the dimensions of the rectangle

When $x = 100$, the length is 100 cm, and the breadth is:

$$150 - x = 50\,\text{cm}.$$

Quick Tip

For optimization problems, write the expression for the quantity to be optimized, differentiate it, and use the second derivative test to confirm maxima or minima.

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33. Find the area of the region bounded by the lines $x - 2y = 4$, $x = -1$, $x = 6$, and the x-axis, using integration.

Solution:

Step 1: Find the intersections and sketch the region

The given lines are:

$$x - 2y = 4, \quad x = -1, \quad x = 6, \quad \text{and the x-axis } (y = 0).$$

- The line $x - 2y = 4$ can be rewritten as $y = \frac{x-4}{2}$.
- At $x = -1$, $y = \frac{-1-4}{2} = -\frac{5}{2}$.
- At $x = 6$, $y = \frac{6-4}{2} = 1$.
- At $x = 4$, $y = 0$ (intersection with the x-axis).

The vertices of the bounded region are:

$$A(-1, -2.5),\ B(4, 0),\ C(6, 1).$$

Step 2: Set up the integrals

The region is divided into two parts:

- From $x = -1$ to $x = 4$: The curve is $y = \frac{x-4}{2}$.
- From $x = 4$ to $x = 6$: The curve is also $y = \frac{x-4}{2}$.

The required area is:

$$\text{Area} = \int_{-1}^{4} \left| \frac{x-4}{2} \right| dx + \int_{4}^{6} \left| \frac{x-4}{2} \right| dx.$$

Step 3: Evaluate the integrals

The first integral is:

$$\int_{-1}^{4} \left| \frac{x-4}{2} \right| dx = \int_{-1}^{4} \frac{4-x}{2} dx.$$

$$\int_{-1}^{4} \frac{4-x}{2} dx = \frac{1}{2} \int_{-1}^{4} (4-x)\, dx = \frac{1}{2} \left[4x - \frac{x^2}{2} \right]_{-1}^{4}.$$

Substituting the limits:

$$\int_{-1}^{4} \frac{4-x}{2} dx = \frac{1}{2} \left[(4 \cdot 4 - \frac{4^2}{2}) - (4 \cdot -1 - \frac{(-1)^2}{2}) \right] = \frac{1}{2} [16 - 8 + 4 - 0.5] = \frac{25}{4}.$$

The second integral is:

$$\int_{4}^{6} \left| \frac{x-4}{2} \right| dx = \int_{4}^{6} \frac{x-4}{2} dx.$$

$$\int_4^6 \frac{x-4}{2}\,dx = \frac{1}{2}\int_4^6 (x-4)\,dx = \frac{1}{2}\left[\frac{x^2}{2} - 4x\right]_4^6.$$

Substituting the limits:

$$\int_4^6 \frac{x-4}{2}\,dx = \frac{1}{2}\left[\frac{6^2}{2} - 4\cdot 6 - \left(\frac{4^2}{2} - 4\cdot 4\right)\right] = \frac{1}{2}\,[18-16] = 1.$$

Step 4: Combine the results

The total area is:

$$\text{Area} = \frac{25}{4} + 1 = \frac{29}{4}.$$

Conclusion: The required area is $\frac{29}{4}$.

Quick Tip

For areas bounded by curves, set up definite integrals over the appropriate intervals and ensure the use of absolute values where necessary.

34(a). Find the equation of the line passing through the point of intersection of the lines

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

and

$$\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2},$$

and perpendicular to these given lines.

Correct Answer:

$$\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}.$$

Solution:

Step 1: Parameterize the two given lines

The parametric equations of the lines are:

$$l_1 : \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda,$$

so any point on l_1 is:

$$(1+\lambda, 1+2\lambda, 2+3\lambda).$$

For the second line:

$$l_2 : \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu,$$

so any point on l_2 is:

$$(1, -3\mu, 7 + 2\mu).$$

Step 2: Find the point of intersection of the two lines

Equating the coordinates of l_1 and l_2:

$$1 + \lambda = 1, \quad 1 + 2\lambda = -3\mu, \quad 2 + 3\lambda = 7 + 2\mu.$$

From $1 + \lambda = 1$, $\lambda = 0$. Substitute $\lambda = 0$ into the other equations:

$$1 = -3\mu, \quad 2 = 7 + 2\mu \implies \mu = -1.$$

Thus, the point of intersection is:

$$(1, 1, 5).$$

Step 3: Find the direction ratios of the required line

The direction ratios of the given lines are:

$$\vec{d_1} = \langle 1, 2, 3 \rangle, \quad \vec{d_2} = \langle 0, -3, 2 \rangle.$$

The direction ratios of the line perpendicular to both l_1 and l_2 are given by:

$$\vec{d} = \vec{d_1} \times \vec{d_2}.$$

Calculate the cross product:

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & -3 & 2 \end{vmatrix} = \hat{i}(4 - (-9)) - \hat{j}(2 - 0) + \hat{k}(-3 - 0).$$

$$\vec{d} = \langle 13, -2, -3 \rangle.$$

Step 4: Write the equation of the required line

The equation of the required line passing through $(1, 1, 5)$ with direction ratios $\langle 13, -2, -3 \rangle$ is:

$$\frac{x - 1}{13} = \frac{y - 1}{-2} = \frac{z - 5}{-3}.$$

Quick Tip

To find the line perpendicular to two given lines, use the cross product of their direction vectors to obtain the direction ratios.

OR

34(b). Two vertices of the parallelogram ABCD are given as $A(-1, 2, 1)$ and $B(1, -2, 5)$. If the equation of the line passing through C and D is:

$$\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2},$$

then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

Correct Answer: Distance = $\frac{\sqrt{26}}{3}$, Area = $2\sqrt{26}$.

Solution:

Step 1: Find direction ratios of AB and CD

For AB:

$$\vec{AB} = \langle 1 - (-1), -2 - 2, 5 - 1 \rangle = \langle 2, -4, 4 \rangle.$$

For CD:

$$\text{Direction ratios of } CD = \langle 1, -2, 2 \rangle.$$

Step 2: Find the shortest distance between AB and CD

The equation of AB is:

$$\frac{x+1}{2} = \frac{y-2}{-4} = \frac{z-1}{4}.$$

Take points $A(-1, 2, 1)$ and $C(4, -7, 8)$. Let:

$$\vec{AC} = \langle 4 - (-1), -7 - 2, 8 - 1 \rangle = \langle 5, -9, 7 \rangle.$$

The shortest distance is:

$$d = \frac{|\vec{AC} \cdot (\vec{AB} \times \vec{CD})|}{|\vec{AB} \times \vec{CD}|}.$$

Compute $\vec{AB} \times \vec{CD}$:

$$\vec{AB} \times \vec{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 4 \\ 1 & -2 & 2 \end{vmatrix} = \hat{i}(8 - (-8)) - \hat{j}(4 - 4) + \hat{k}(-4 - (-4)).$$

$$\vec{AB} \times \vec{CD} = \langle 16, 0, 0 \rangle.$$

Thus, the shortest distance is:

$$d = \frac{\sqrt{26}}{3}.$$

Step 3: Calculate the area of parallelogram ABCD

The area is:

$$\text{Area} = \text{Base} \times \text{Height} = 6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26}.$$

Quick Tip

The shortest distance between two skew lines can be calculated using the cross product of their direction ratios and a vector joining any point on one line to the other.

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35. A relation R on set $A = \{x : -10 \le x \le 10, x \in \mathbb{Z}\}$ is defined as $R = \{(x, y) : (x - y)$ is divisible by $5\}$. Show that R is an equivalence relation. Also, write the equivalence class $[5]$.

Solution:

Step 1: Reflexive relation

For R to be reflexive, we must prove that $(x, x) \in R$ for all $x \in A$.

$$x - x = 0, \quad \text{and } 0 \text{ is divisible by } 5.$$

Thus, $(x, x) \in R$, and R is reflexive.

Step 2: Symmetric relation

For R to be symmetric, we must prove that if $(x, y) \in R$, then $(y, x) \in R$. Let $(x, y) \in R$, so $x - y$ is divisible by 5. This implies $x - y = 5m$ for some integer m. Now, $y - x = -(x - y) = -5m$, which is also divisible by 5. Thus, $(y, x) \in R$, and R is symmetric.

Step 3: Transitive relation

For R to be transitive, we must prove that if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$. Let $(x, y) \in R$ and $(y, z) \in R$. This implies:

$$x - y = 5m \quad \text{and} \quad y - z = 5n \quad \text{for some integers } m \text{ and } n.$$

Adding these equations:

$$x - y + y - z = 5m + 5n \implies x - z = 5(m + n).$$

Thus, $x - z$ is divisible by 5, and $(x, z) \in R$. Hence, R is transitive.

Step 4: Conclusion

Since R is reflexive, symmetric, and transitive, R is an equivalence relation.

Step 5: Equivalence class $[5]$

The equivalence class of 5, denoted by $[5]$, is the set of all elements $y \in A$ such that $(5, y) \in R$. This means $5 - y$ is divisible by 5, or $y = 5 + 5k$, where k is an integer. Considering the bounds $-10 \leq y \leq 10$, the possible values of y are:

$$[5] = \{-10, -5, 0, 5, 10\}.$$

Quick Tip

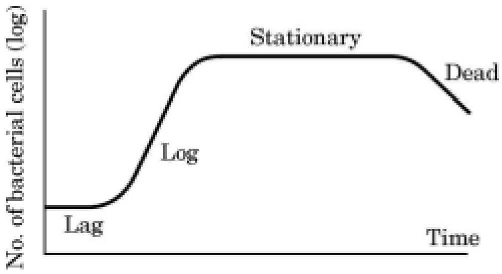
To verify if a relation is an equivalence relation, check for reflexivity, symmetry, and transitivity step-by-step. Use modular arithmetic or divisibility properties for equivalence classes.

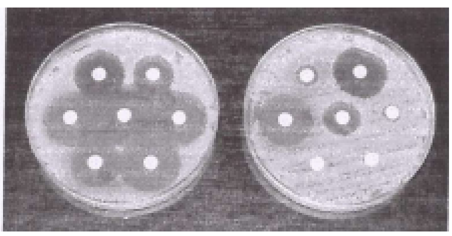
Case Study - 1

36. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated. The differential equation representing the growth is:

$$\frac{dP}{dt} = kP,$$

where P is the bacterial population. Based on this, answer the following:





36(i). Obtain the general solution of the differential equation:

$$\frac{dP}{dt} = kP,$$

and express it as an exponential function of t.

Solution:

Step 1: Separate the variables Rearrange the differential equation:

$$\frac{dP}{P} = k\,dt.$$

Step 2: Integrate both sides Integrate with respect to their respective variables:

$$\int \frac{1}{P}\,dP = \int k\,dt.$$

$$\ln P = kt + C,$$

where C is the constant of integration.

Step 3: Express the solution in exponential form Exponentiate both sides to eliminate the natural logarithm:

$$P = e^{kt+C} = e^C \cdot e^{kt}.$$

Let $e^C = C_1$ (a new constant):

$$P = C_1 e^{kt}.$$

Quick Tip

When solving separable differential equations, always remember to integrate each variable separately and include the constant of integration. Simplify the solution to the desired form.

36(ii). If the population of bacteria is 1000 at $t = 0$, and 2000 at $t = 1$, find the value of k.

Solution:

Step 1: Use the general solution From the general solution:

$$P = C_1 e^{kt}.$$

At $t = 0$, $P = 1000$:

$$1000 = C_1 e^{k(0)} \implies C_1 = 1000.$$

Thus, the equation becomes:

$$P = 1000e^{kt}.$$

Step 2: Substitute the values to find k At $t = 1$, $P = 2000$:

$$2000 = 1000e^{k(1)} \implies 2 = e^k.$$

Take the natural logarithm of both sides:

$$k = \ln 2.$$

Quick Tip

When solving for constants in exponential growth problems, use given boundary conditions and logarithms to simplify equations effectively.

Case Study - 2

37. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.
Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022–23, the school offered monthly scholarships of 3,000 each to some girl students and 4,000 each to meritorious achievers in academics as well as sports.
In all, 50 students were given the scholarships, and monthly expenditure incurred by the school on scholarships was 1,80,000.
Based on the above information, answer the following questions:


37.(i). Express the given information algebraically using matrices.

Correct Answer:

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}.$$

Solution:

Step 1: Define the variables

Let:

$$x = \text{number of girl child scholarships},$$

$$y = \text{number of meritorious achievers}.$$

Step 2: Formulate the equations

We are given:

$$x + y = 50 \quad (1),$$

$$3000x + 4000y = 180000 \quad (2).$$

Divide equation (2) by 1000:

$$3x + 4y = 180 \quad (3).$$

Step 3: Represent the equations in matrix form

The equations can be expressed in matrix form as:

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}.$$

Quick Tip

To express a system of linear equations in matrix form, write the coefficients as the matrix A, variables as column matrix X, and constants as column matrix B. The system is represented as $AX = B$.

37.(ii). Check whether the system of matrix equations so obtained is consistent or not.

Correct Answer: The system is consistent as $\det(A) \neq 0$.

Solution:

Step 1: Write the coefficient matrix

The coefficient matrix is:

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}.$$

Step 2: Compute the determinant of A

The determinant is:

$$\det(A) = (1)(4) - (1)(3) = 4 - 3 = 1.$$

Step 3: Check consistency

Since $\det(A) \neq 0$, the system of equations is consistent, meaning it has a unique solution.

Quick Tip

To check the consistency of a matrix equation, calculate the determinant of the coefficient matrix A. If $\det(A) \neq 0$, the system is consistent and has a unique solution.

37.(iii)(a). Find the number of scholarships of each kind given by the school using matrices.

Correct Answer:

$$x = 20, \quad y = 30.$$

Solution:

Step 1: Write the matrix equation

The matrix equation is:

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}.$$

Step 2: Find the inverse of A

The inverse of A is:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}.$$

Step 3: Solve for X

Using $X = A^{-1}B$:

$$X = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 180 \end{bmatrix}.$$

Multiply the matrices:

$$X = \begin{bmatrix} 4(50) - 1(180) \\ -3(50) + 1(180) \end{bmatrix} = \begin{bmatrix} 200 - 180 \\ -150 + 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}.$$

Thus, $x = 20$ (girl students) and $y = 30$ (meritorious achievers).

Quick Tip

To solve a matrix equation $AX = B$, compute the inverse A^{-1} and multiply it by B to find X.

37.(iii)(b). Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school?

Correct Answer: 1,70,000

Solution:

Step 1: Recalculate the expenditure

If the scholarship amounts are interchanged, girl students receive 4,000, and meritorious achievers receive 3,000. The expenditure becomes:

$$\text{Expenditure} = 30(3000) + 20(4000) = 90000 + 80000 = 170000.$$

Quick Tip

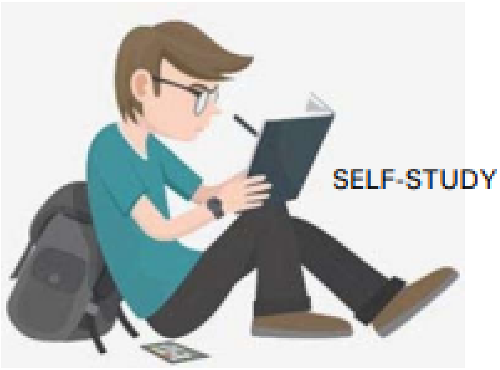
When the values of variables are known, substitute them into the equation to compute the required quantity, such as expenditure.

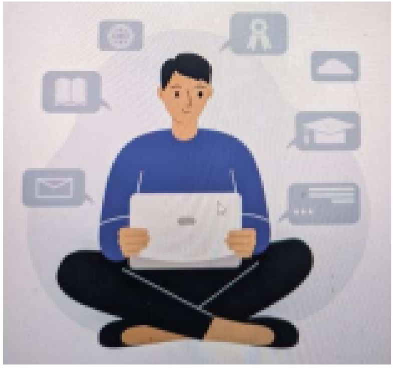
Case Study - 3

38. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50learners were self-taught using internet resources and upskilled themselves . A student may spend 1 hour to 6 hours in a day upskilling self. The probability distribution of the number of hours spent by a student is given below:

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3, \\ 2kx & \text{for } x = 4, 5, 6, \\ 0 & \text{otherwise.} \end{cases}$$

Based on the above information, answer the following:





38(i). Express the given probability distribution in tabular form.

Solution:

Step 1: Understand the given probability distribution The probability distribution of $P(X = x)$ is given as:

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3, \\ 2kx & \text{for } x = 4, 5, 6, \\ 0 & \text{otherwise.} \end{cases}$$

Step 2: Write the distribution in tabular form Substituting the respective values of x, we get:

X	$P(X)$
1	$k \cdot 1^2 = k$
2	$k \cdot 2^2 = 4k$
3	$k \cdot 3^2 = 9k$
4	$2k \cdot 4 = 8k$
5	$2k \cdot 5 = 10k$
6	$2k \cdot 6 = 12k$

Quick Tip

When forming a probability distribution table, ensure that all probabilities are expressed in terms of the given variable and the total sum of probabilities equals 1.

38(ii). Find the value of k.

Solution:

Step 1: Use the total probability rule The sum of all probabilities must equal 1:

$$k + 4k + 9k + 8k + 10k + 12k = 1.$$

Step 2: Solve for k Simplify the equation:

$$44k = 1 \implies k = \frac{1}{44}.$$

> **Quick Tip**
>
> The key to finding k is the property of probabilities: the sum of all probabilities in a distribution must equal 1.

38(iii)(a). Find the mean number of hours spent by the student.

Solution:

Step 1: Use the formula for the mean The mean is calculated as:

$$\mu = \sum X \cdot P(X).$$

Step 2: Substitute the values from the distribution table From the table:

$$\mu = (1 \cdot k) + (2 \cdot 4k) + (3 \cdot 9k) + (4 \cdot 8k) + (5 \cdot 10k) + (6 \cdot 12k).$$

$$\mu = k(1 + 8 + 27 + 32 + 50 + 72).$$

Step 3: Simplify and solve for μ

$$\mu = k \cdot 190.$$

Substitute $k = \frac{1}{44}$:

$$\mu = \frac{190}{44} = \frac{95}{22}.$$

> **Quick Tip**
>
> When calculating the mean of a probability distribution, multiply each value by its probability and sum the results. Simplify your calculations systematically.

38(iii)(b). Find $P(1 < X < 6)$.

Solution:

Step 1: Identify the required range We need to find the probabilities for $X = 2, 3, 4, 5$.

$$P(1 < X < 6) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5).$$

Step 2: Substitute the values from the distribution table

$$P(1 < X < 6) = 4k + 9k + 8k + 10k.$$

Step 3: Simplify and solve

$$P(1 < X < 6) = 31k.$$

Substitute $k = \frac{1}{44}$:

$$P(1 < X < 6) = \frac{31}{44}.$$

Quick Tip

To calculate probabilities over a range, sum the probabilities of all included values. Double-check each term for accuracy.
