Circles JEE Main PYQ -1

Total Time: 20 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Circles

1. The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$ The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is : **(+4)**

a. $x^2 + y^2 + 3x + 3y + 4 = 0$

b. $x^2 + y^2 + 5x + 5y + 12 = 0$

c. $x^2 + y^2 + 3x + 5y + 8 = 0$

d. $x^2 + y^2 - 5x - 5y + 12 = 0$

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2. The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$ The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is : **(+4)**

a. $x^2 + y^2 + 3x + 3y + 4 = 0$

b. $x^2 + y^2 + 5x + 5y + 12 = 0$

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d. $x^2 + y^2 - 5x - 5y + 12 = 0$

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3. Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$ If the distance of L from the point $(5, 3, 8)$ is α, then $3\alpha^2$ is equal to_______ **(+4)**

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4. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2(3 - \sqrt{3})x + x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6}, k > 0$ touch internally at the point $P(\alpha, \beta)$, then $(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2$ is equal to _______ **(+4)**

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5. The locus of the mid points of the chords of the circle $C_1 : (x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to **(+4)**

a. $\frac{\pi}{2}$

b. $\frac{\pi}{4}$

c. $\frac{\pi}{6}$

d. $\frac{3\pi}{4}$

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6. The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c, m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is : **(+4)**

a. $5\sqrt{3}$

b. $\frac{14}{3}$

c. $\frac{1}{3}$

d. 5

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7. Circle in 1st quadrant touches both the axes at A & B. If length of perpendicular from $P(\alpha, \beta)$ on circle to chord AB is equal to 11. Find $\alpha.\beta$ **(+4)**

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8. Two circles having radius r_1 and r_2 touch both the coordinate axes. Line $x + y = 2$ makes intercept 2 on both the circles. The value of $r_1^2 + r_2^2 - r_1 r_2$ is: **(+4)**

a. $\frac{9}{2}$

b. 6

c. 7

d. 8

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9. Let a circle of $x^2 + y^2 = 16$ and line passing through (1,2) cuts the circle at A and B then the locus of the mid-point of AB is: **(+4)**

a. $x^2 + y^2 + x + y = 0$

b. $x^2 + y^2 - x + 2y = 0$

c. $x^2 + y^2 - x - 2y = 0$

d. $x^2 + y^2 + x + 2y = 0$

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10. From O(0, 0), two tangents OA and OB are drawn to a circle $x^2 + y^2 - 6x + 4y + 8 = 0$, then the equation of circumcircle of $\triangle$ OAB. **(+4)**

a. $x^2 + y^2 - 3x + 2y = 0$

b. $x^2 + y^2 + 3x - 2y = 0$

c. $x^2 + y^2 + 3x + 2y = 0$

d. $x^2 + y^2 - 3x - 2y = 0$

Answers

1. **Answer: b**

Explanation:

Only possibility $a = 0, � = 1$
? equation of circle $x^2 + y^2 - x - y = 0$
Image of circle in $x + y + 2 = 0$ is
$x^2 + y^2 + 5x + 5y + 12 = 0$

Concepts:

1. **Circle:**

A **circle** can be geometrically defined as a combination of all the points which lie at an equal distance from a fixed point called the centre. The concepts of the circle are very important in building a strong foundation in units likes mensuration and coordinate geometry. We use **circle formulas** in order to calculate the area, diameter, and circumference of a circle. The length between any point on the circle and its centre is its radius.

Any line that passes through the centre of the circle and connects two points of the circle is the diameter of the circle. The radius is half the length of the diameter of the circle. The area of the circle describes the amount of space that is covered by the circle and the circumference is the length of the boundary of the circle.

Also Check:

Areas Related to Circles	Perimeter and Area of Circle	Circles Revision Notes

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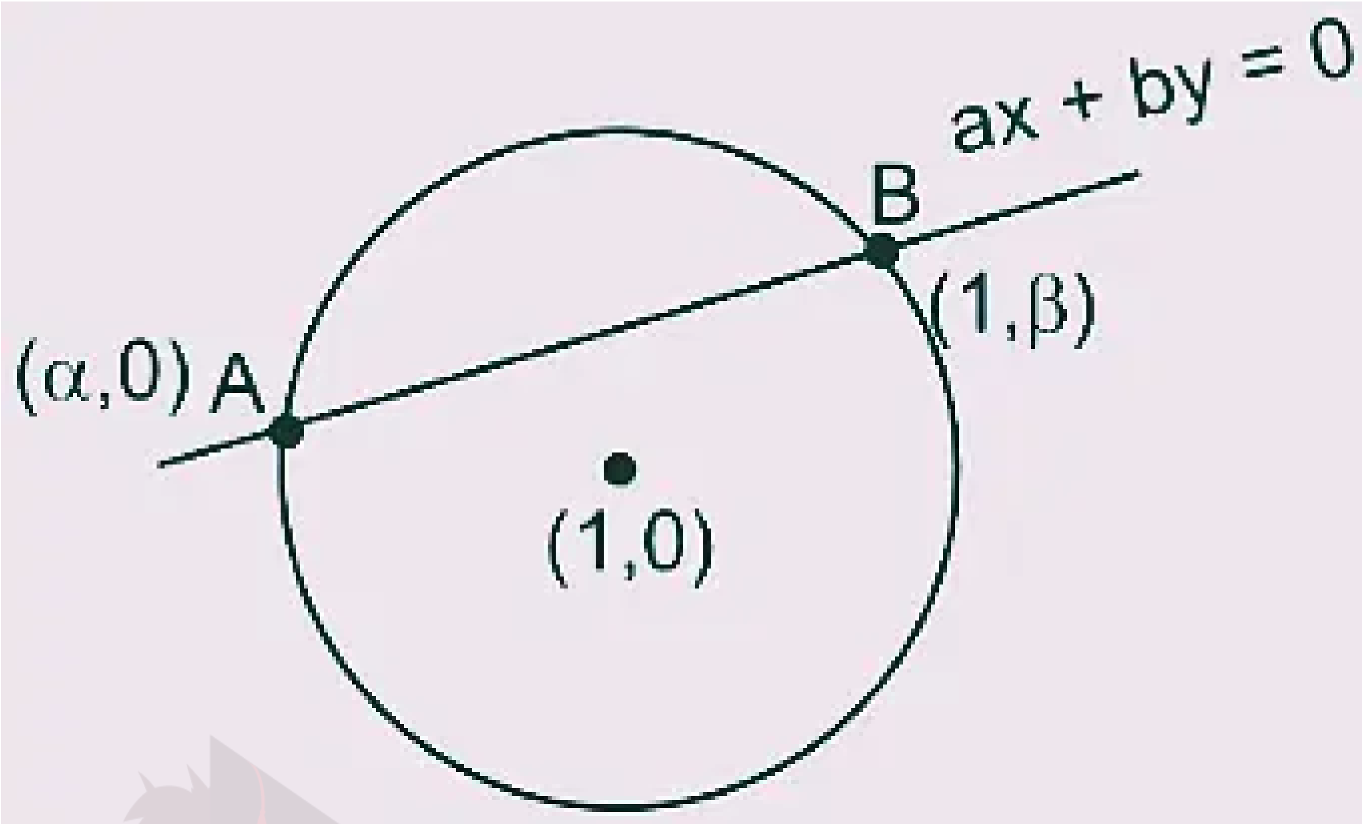
2. **Answer: b**

Explanation:



Only possibility $\alpha = 0, \beta = 1$

$\therefore$ equation of circle $x^2 + y^2 - x - y = 0$

Image of circle in $x + y + 2 = 0$ is

$x^2 + y^2 + 5x + 5y + 12 = 0$

Hence, the correct option is(B): $x^2 + y^2 + 5x + 5y + 12 = 0$

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3. **Answer: 158 - 158**

Explanation:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

$\therefore$ Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be $(5, 3, 8)$ and foot of $\perp$ from Q on this line be R.

Now, $R \equiv (k+2, -k+3, -k+1)$

DR of QR are $(k-3, -k, -k-7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

So, the correct answer is 158.

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4. **Answer: 25 - 25**

Explanation:

The correct answer is 25

$C_1(-3, -4)$

$r_1 = \sqrt{25 - 16} = 3$

$C_2 = (-3 + \sqrt{3}, -4 + \sqrt{6})$

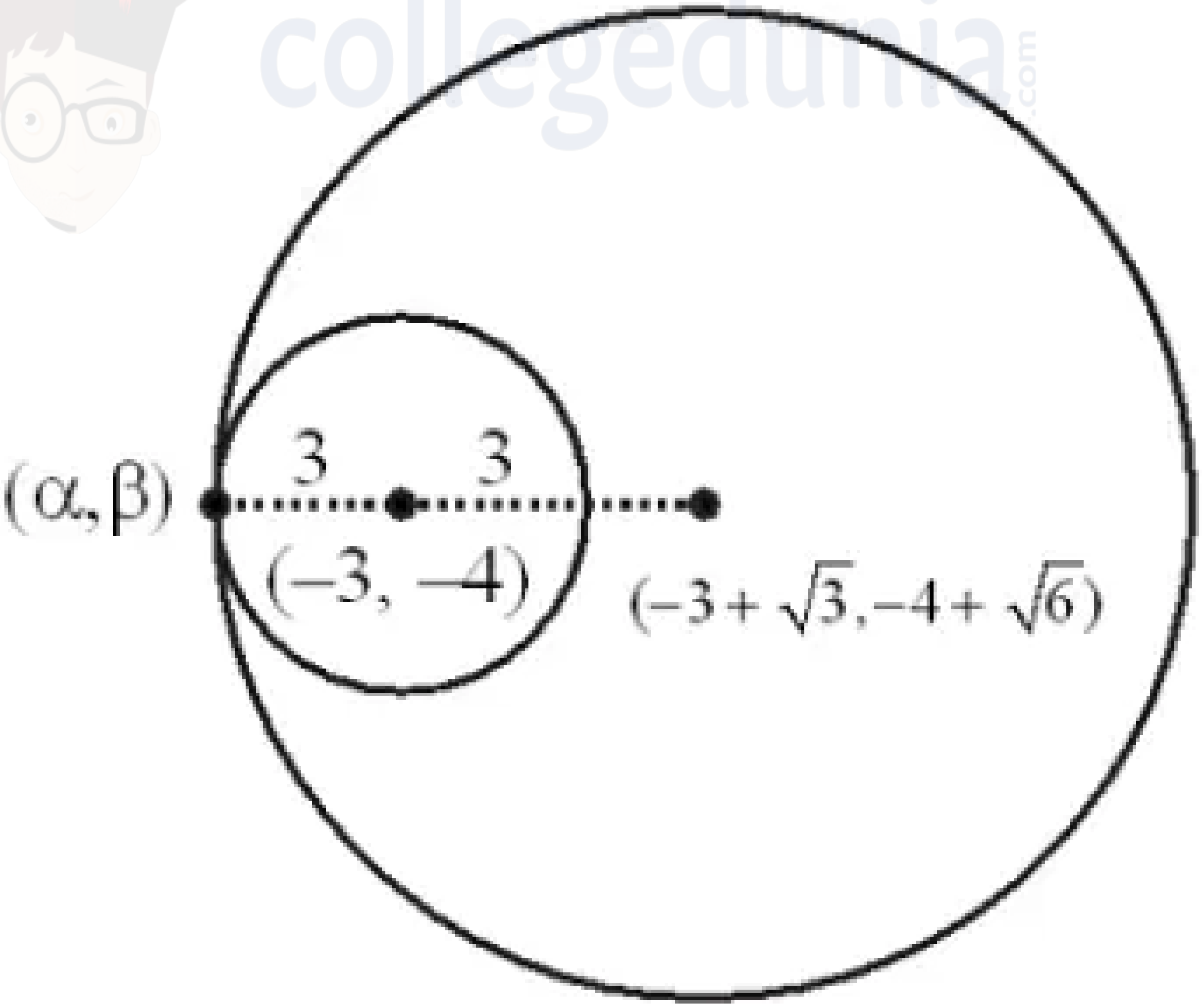
$r_2 = \sqrt{34 + k}$

$C_1C_2 = |r_1 - r_2|$

$C_1C_2 = \sqrt{3 + 6} = 3$

$3 = |3 - \sqrt{34 + k}| \Rightarrow k = 2$

$r_2 = 6$



$(\alpha, \beta) = (-\sqrt{3} - 3, -4 - \sqrt{6})$
$(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2 = 9 + 16 = 25$

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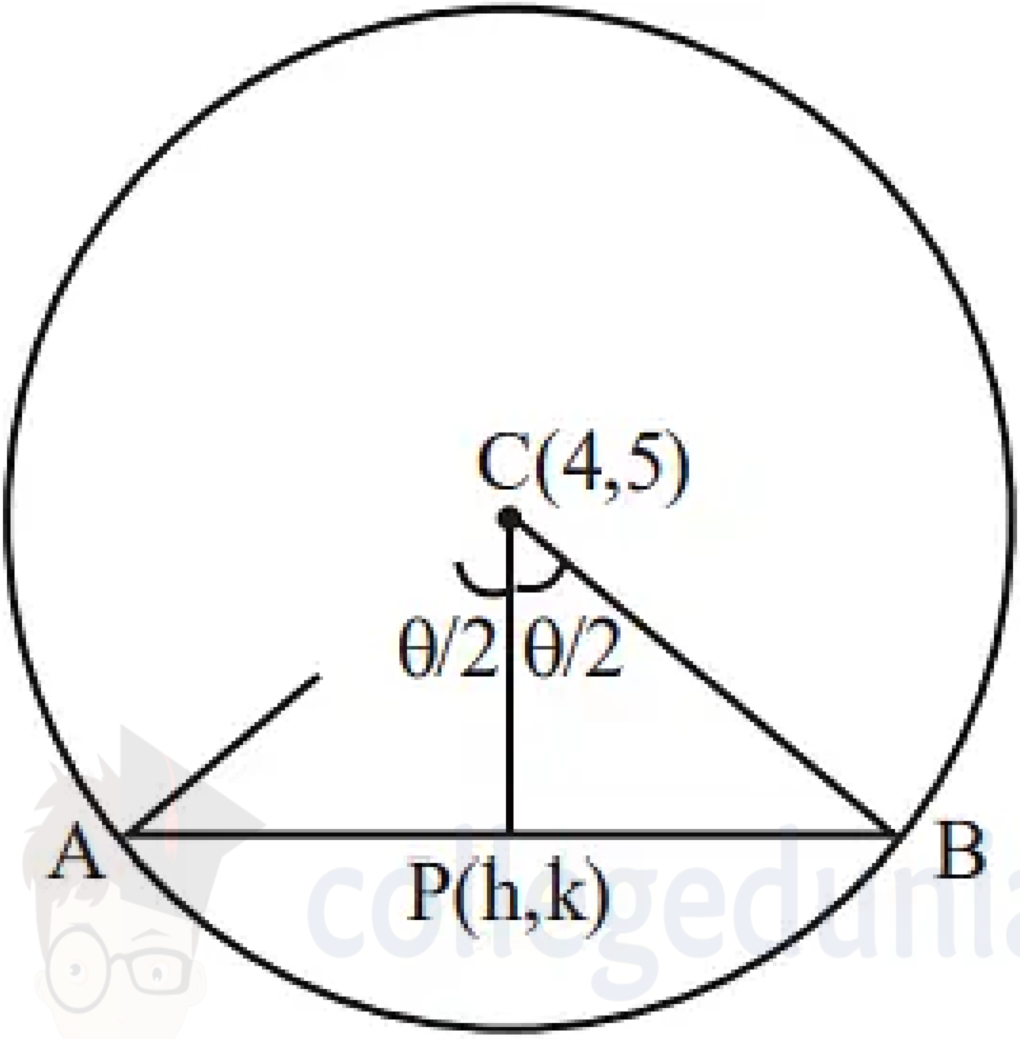
5. **Answer: a**

Explanation:

The correct answer is (A) : $\frac{\pi}{2}$
In $\triangle CPB$



$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h-4)^2 + (k-5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x-4)^2 + (y-5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \theta_2 = \frac{\pi}{2}$

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6. **Answer: d**

Explanation:

For
$y^2 = \frac{x}{2}, T : y = mx + \frac{1}{8m}$
For tangent to $y^2 + 1 = x$
$\Rightarrow \left(mx + \frac{1}{8m}\right)^2 + 1 = x$
$D = 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$
$\therefore T : x - 2\sqrt{2}y + 1 = 0$
$d = \left|\frac{6+8+1}{\sqrt{9}}\right| = 5$

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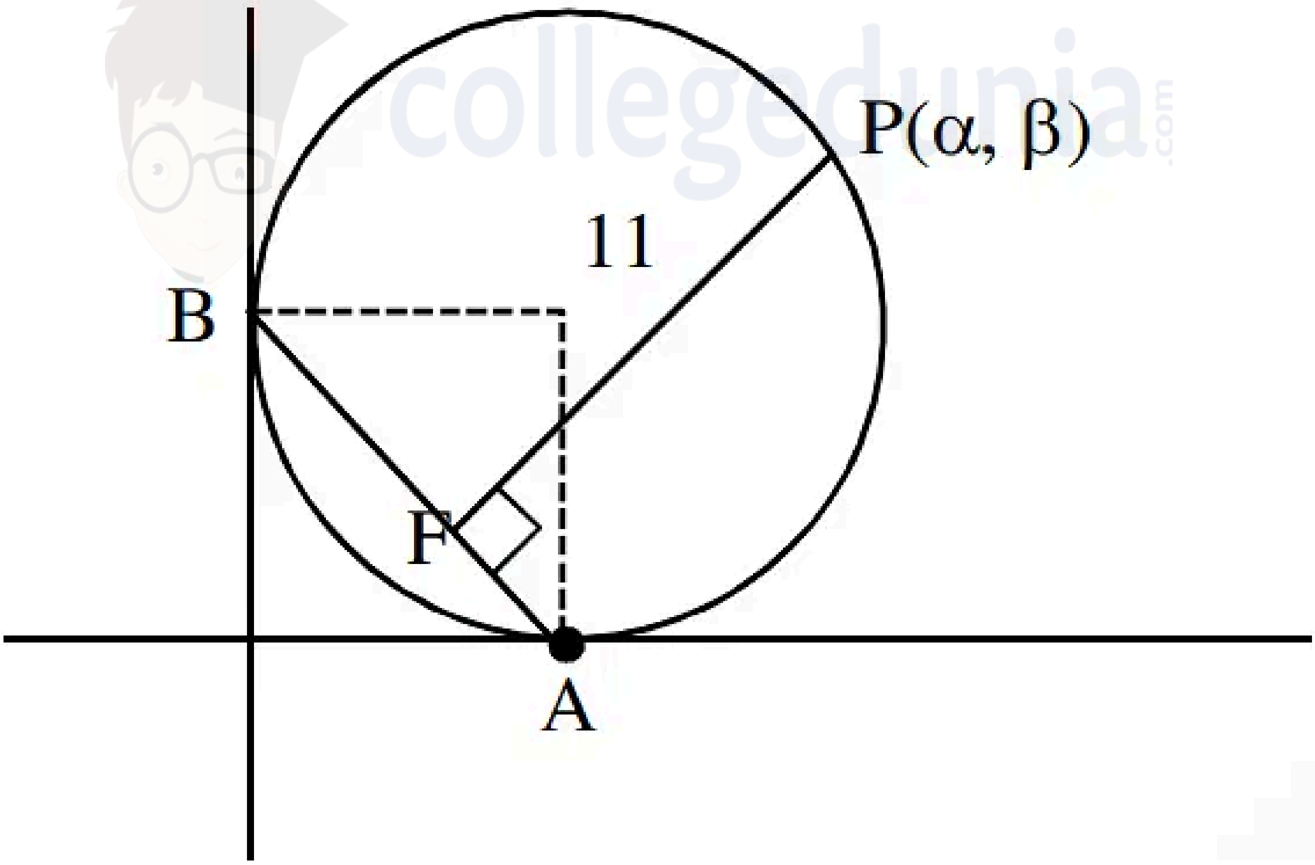
7. **Answer: 121 - 121**

Explanation:

C : $(x - r)^2 + (y-r)^2 = r^2$;
$\alpha^2 + \beta^2 - 2r(\alpha + \beta) + r^2 = 0$



$\alpha^2 + \beta^2 - 2r(11\sqrt{2} + r) + r2 = 0$
$\alpha^2 + \beta^2 - 22\sqrt{2}r - r^2 = 0$
PF = $\frac{\alpha+\beta-r}{\sqrt{2}}$ = 11

$\alpha + B = 11\sqrt{2} + r$
$\alpha^2 + \beta^2 + 2\alpha\beta = 242 + r^2 + 22r\sqrt{2}$
$\alpha\beta = 121$

So, the answer is 121.

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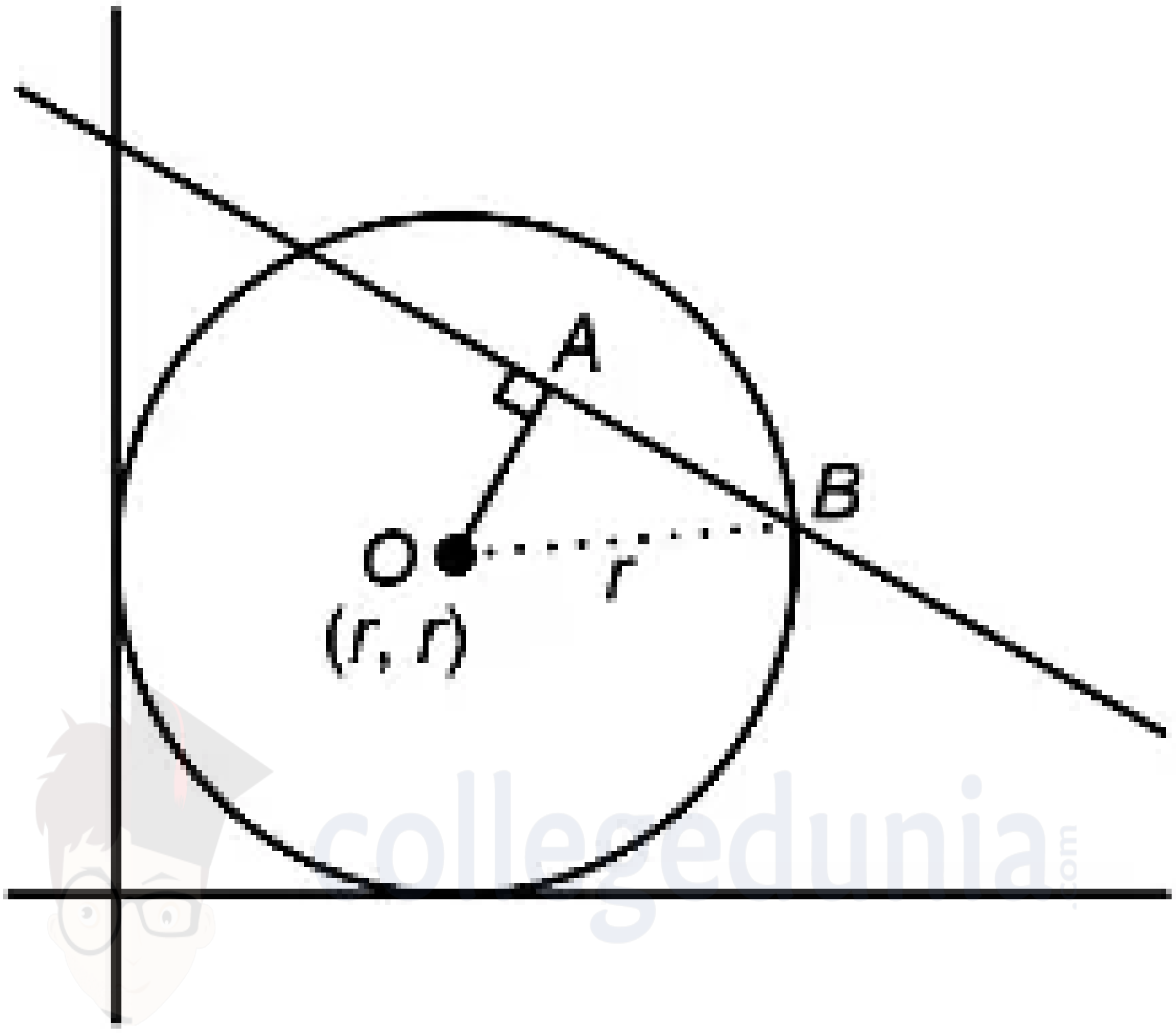
8. Answer: c

Explanation:

The correct option is (C): 7


A
B
O
(r, r)
r

$$AB = 1$$

$$OA = \sqrt{r^2 - 1}$$

$$\Rightarrow \left|\frac{2r - 2}{\sqrt{2}}\right| = \sqrt{r^2 - 1}$$

$$\Rightarrow \sqrt{2}(r - 1) = \sqrt{r^2 - 1}$$

$$\Rightarrow 2(r - 1)^2 = r^2 - 1$$

$$\Rightarrow 2r^2 - 4r + 2 = r^2 - 1$$

$$\Rightarrow r^2 - 4r + 3 = 0$$

$$\Rightarrow (r-1)(r-3)=0$$

$$\Rightarrow r = 1, 3$$

$$\therefore r_1 = 1 \text{ and } r_2 = 3$$

$$\therefore r_1^2 + r_2^2 - r_1 \cdot r_2$$

$$= 1 + 9 - 3$$

$$= 7$$

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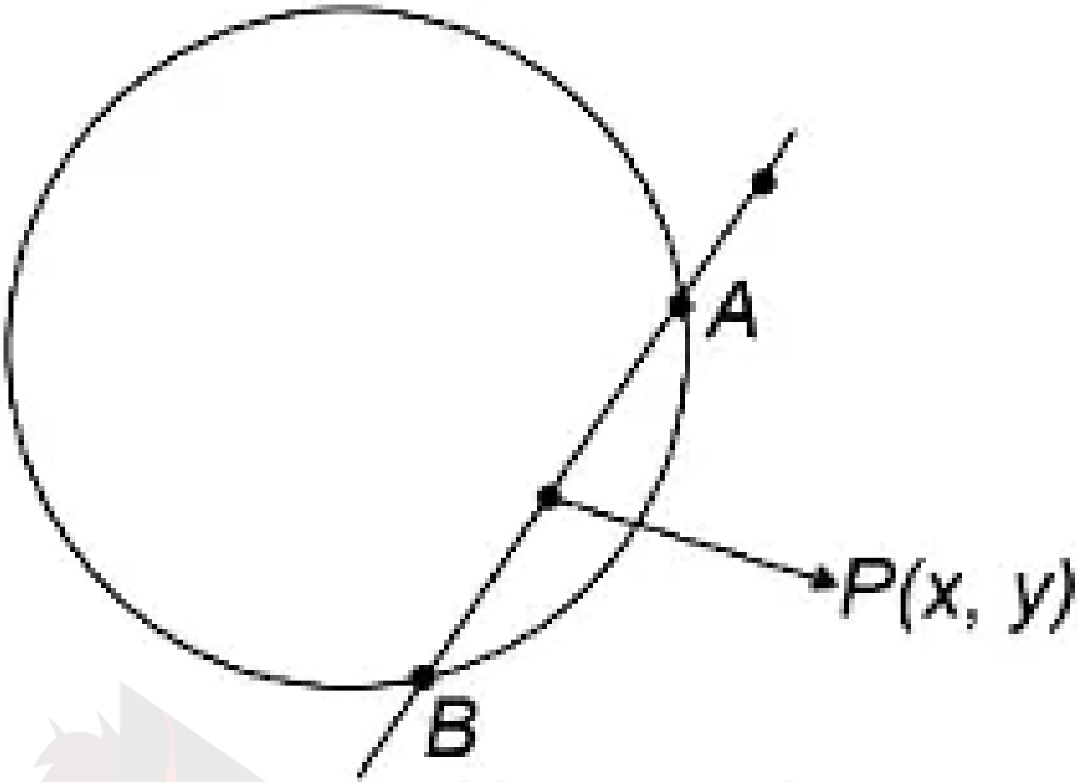
9. **Answer: c**

Explanation:

The correct option is (C): $x^2 + y^2 - x - 2y = 0$



Let $P(x_1 y_1)$ be the mid-point of AB

Then $T = S_1$

$$x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16$$

$$\Rightarrow \quad xx_1 + yy_1 = x_1^2 + y_1^2 \quad \ldots(i)$$

$\because$ (i) passes through (1, 2)

$$\therefore \quad x_1 + 2y_1 = x_1^2 + y_1^2$$

∴ required locus

$$x^2 + y^2 - x - 2y = 0$$

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10. Answer: a

Explanation:

The correct option is (A): $x^2 + y^2 - 3x + 2y = 0$

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