COMEDK 2024 Shift 1 Question Paper With Solutions

Time Allowed :3 Hour	**Maximum Marks :**180	**Total Questions :**180

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 180 questions. The maximum marks are 180.
3. There are three parts in the question paper consisting of Physics, Chemistry, and Mathematics, each having 60 questions of equal weightage.
4. Each part (subject) has two sections.
   (i) **Section-A:** This section contains 50 multiple-choice questions (MCQs) with only one correct answer. Each question carries 1 mark for a correct answer and 0.25 mark will be deducted for a wrong answer.
   (ii) **Section-B:** This section contains 10 questions, where the answer to each question is a numerical value. Each question carries 1 mark for a correct answer and 0.25 mark will be deducted for a wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1 PHYSICS

1. The resistance of the galvanometer and shunt of an ammeter are 90 ohms and 10 ohms respectively, then the fraction of the main current passing through the galvanometer and the shunt respectively are:

(A) $\frac{1}{90}$ and $\frac{1}{10}$
(B) $\frac{1}{10}$ and $\frac{1}{90}$
(C) $\frac{9}{10}$ and $\frac{1}{10}$
(D) $\frac{1}{90}$ and $\frac{9}{10}$

Correct Answer: (C) [Answer: $\frac{9}{10}$ and $\frac{1}{10}$]

Solution:
The total current is divided into two parts, one flowing through the galvanometer and the other through the shunt. The fraction of the current passing through the galvanometer is given by the ratio of the resistance of the galvanometer to the total resistance (which is the sum of the resistances of the galvanometer and the shunt):

$$I_{\text{galv}} = \frac{R_{\text{galv}}}{R_{\text{galv}} + R_{\text{shunt}}}$$

Substituting the given values:

$$I_{\text{galv}} = \frac{90}{90 + 10} = \frac{90}{100} = \frac{9}{10}.$$

The remaining fraction of the current flows through the shunt:

$$I_{\text{shunt}} = 1 - I_{\text{galv}} = 1 - \frac{9}{10} = \frac{1}{10}.$$

Thus, the fraction of the current passing through the galvanometer is $\frac{9}{10}$ and the fraction passing through the shunt is $\frac{1}{10}$.

Quick Tip

In ammeters, the current is divided between the galvanometer and the shunt. The fraction of current through the galvanometer is inversely proportional to its resistance.

2. A glass of hot water cools from 90°C to 70°C in 3 minutes when the temperature of the surroundings is 20°C. What is the time taken by the glass of hot water to cool from 60°C to 40°C if the surrounding temperature remains the same at 20°C?

(A) 15 minutes

(B) 6 minutes

(C) 12 minutes

(D) 8 minutes

Correct Answer: (C) [Answer: 12 minutes]

Solution:

We will use Newton's Law of Cooling, which states that the rate of change of temperature of the object is directly proportional to the difference between the temperature of the object and the surrounding temperature. Mathematically:

$$\frac{dT}{dt} = -k(T - T_{\text{env}})$$

Where: - T is the temperature of the object, - T_{env} is the temperature of the surroundings, - k is the cooling constant.

The time taken to cool from T_1 to T_2 is given by:

$$t = \frac{1}{k} \ln\left(\frac{T_1 - T_{\text{env}}}{T_2 - T_{\text{env}}}\right)$$

Step 1: Calculate the cooling constant k using the first part of the cooling process:

We are given that the temperature drops from 90°C to 70°C in 3 minutes, and the surrounding temperature is 20°C. Substituting into the equation:

$$3 = \frac{1}{k} \ln\left(\frac{90 - 20}{70 - 20}\right)$$

Simplifying:

$$3 = \frac{1}{k} \ln\left(\frac{70}{50}\right)$$

$$3 = \frac{1}{k}\ln(1.4)$$
$$k = \frac{\ln(1.4)}{3}$$

Step 2: Use the value of k to calculate the time taken for the temperature to cool from 60°C to 40°C:

Now, substitute into the equation for cooling time:

$$t = \frac{1}{k}\ln\left(\frac{60-20}{40-20}\right)$$
$$t = \frac{1}{k}\ln\left(\frac{40}{20}\right) = \frac{1}{k}\ln(2)$$

Substitute the value of k:

$$t = \frac{\ln(2)}{\ln(1.4)} \times 3 \approx 12 \text{ minutes.}$$

Thus, the time taken to cool from 60°C to 40°C is 12 minutes.

Quick Tip

Use Newton's Law of Cooling to calculate the time taken for temperature changes in cooling problems. Remember that the time depends on the logarithmic difference between initial and final temperatures.

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3. When two objects are moving along a straight line in the same direction, the distance between them increases by 6 m in one second. If the objects move with their constant speed towards each other, the distance decreases by 8 m in one second. Then the speed of the objects are:

(A) 14 m/s, 2 m/s

(B) 7 m/s, 1 m/s

(C) 3.5 m/s, 2 m/s

(D) 3.5 m/s, 1 m/s

Correct Answer: (B) [Answer: 7 m/s, 1 m/s]

Solution:

Let the speeds of the two objects be v_1 and v_2.

Step 1: When the objects are moving in the same direction The relative speed of the two objects when moving in the same direction is $|v_1 - v_2|$. According to the problem, the distance between the objects increases by 6 m in one second, so we have:

$$|v_1 - v_2| = 6\,\text{m/s}.$$

Step 2: When the objects are moving towards each other When the objects move towards each other, the relative speed is $v_1 + v_2$. In this case, the distance decreases by 8 m in one second, so we have:

$$v_1 + v_2 = 8\,\text{m/s}.$$

Step 3: Solve the system of equations We now have the following system of equations: 1. $|v_1 - v_2| = 6$ 2. $v_1 + v_2 = 8$

From equation (1), we have two cases:

- Case 1: $v_1 - v_2 = 6$ - Case 2: $v_2 - v_1 = 6$

Case 1: $v_1 - v_2 = 6$ Solving the system of equations:

$$v_1 - v_2 = 6$$

$$v_1 + v_2 = 8$$

Adding these two equations:

$$2v_1 = 14 \quad \Rightarrow \quad v_1 = 7\,\text{m/s}.$$

Substitute $v_1 = 7$ into $v_1 + v_2 = 8$:

$$7 + v_2 = 8 \quad \Rightarrow \quad v_2 = 1\,\text{m/s}.$$

Thus, the speeds of the two objects are $v_1 = 7\,\text{m/s}$ and $v_2 = 1\,\text{m/s}$.

Therefore, the correct answer is $7\,\text{m/s}$ and $1\,\text{m/s}$, which corresponds to option (B).

Quick Tip

When solving relative speed problems, use the equations for relative speed in the same direction and when moving towards each other. Add or subtract the speeds as necessary to solve for the individual speeds.

4. In the Young's double slit experiment, the 2nd bright fringe for red light coincides with the 3rd bright fringe for violet light. Then the value of 'n' is: (Given: wavelength of red light = 6300 Å and wavelength of violet light = 4200 Å)

(A) 2

(B) 4

(C) 3

(D) 1

Correct Answer: (A) [Answer: 2]

Solution:

In Young's double slit experiment, the position of the n^{th} bright fringe is given by the equation:

$$x_n = \frac{n\lambda D}{d},$$

where: - x_n is the position of the n^{th} bright fringe, - n is the order number of the fringe, - λ is the wavelength of the light used, - D is the distance between the slits and the screen, - d is the distance between the two slits.

In this problem, we are given that the 2^{nd} bright fringe for red light coincides with the 3^{rd} bright fringe for violet light. This means that:

$$x_2 \text{ (for red)} = x_3 \text{ (for violet)}.$$

From the equation, this can be written as:

$$2\lambda_{\text{red}}\frac{D}{d} = 3\lambda_{\text{violet}}\frac{D}{d}.$$

Since $\frac{D}{d}$ is common for both, we can cancel it out, and the equation becomes:

$$2\lambda_{\text{red}} = 3\lambda_{\text{violet}}.$$

Substitute the given values for the wavelengths:

$$2 \times 6300 = 3 \times 4200,$$

$$12600 = 12600.$$

Thus, the value of n is 2, which corresponds to option (A).

Quick Tip

In Young's double slit experiment, the fringe positions for different wavelengths can be compared using the relationship $n\lambda_{\text{red}} = (n+1)\lambda_{\text{violet}}$.

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5. A metal ball of mass m is projected at an angle θ with the horizontal with an initial velocity u. If the mass and angle of projection are doubled keeping the initial velocity the same, the ratio of the maximum height attained in the former to the latter case is:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 3

(D) 3 : 1

Correct Answer: (C) [Answer: 1 : 3]

Solution:

The maximum height H attained by a projectile is given by the formula:

$$H = \frac{u^2 \sin^2(\theta)}{2g},$$

where: - u is the initial velocity, - θ is the angle of projection, - g is the acceleration due to gravity.

Case 1: Original values In the first case, let the initial velocity be u and the angle of projection be θ. The maximum height attained in the first case is:

$$H_1 = \frac{u^2 \sin^2(\theta)}{2g}.$$

Case 2: Doubled mass and angle In the second case, the mass is doubled, but the initial velocity remains the same. The angle of projection is also doubled, so the angle becomes 2θ. The maximum height in the second case is:

$$H_2 = \frac{u^2 \sin^2(2\theta)}{2g}.$$

Using the double angle identity for sine, we know that:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta).$$

Thus, the maximum height in the second case becomes:

$$H_2 = \frac{u^2\left(2\sin(\theta)\cos(\theta)\right)^2}{2g} = \frac{4u^2\sin^2(\theta)\cos^2(\theta)}{2g}.$$

Step 3: Ratio of the maximum heights Now, the ratio of the maximum heights in the first case to the second case is:

$$\frac{H_1}{H_2} = \frac{\frac{u^2\sin^2(\theta)}{2g}}{\frac{4u^2\sin^2(\theta)\cos^2(\theta)}{2g}} = \frac{1}{4\cos^2(\theta)}.$$

For $\theta = 30^\circ$, $\cos(30^\circ) = \frac{\sqrt{3}}{2}$, so:

$$\frac{H_1}{H_2} = \frac{1}{4\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{4 \times \frac{3}{4}} = \frac{1}{3}.$$

Thus, the ratio of the maximum heights attained is $1 : 3$, and the correct answer is option (C).

Quick Tip

When solving problems related to the maximum height of a projectile, use the formula $H = \frac{u^2\sin^2(\theta)}{2g}$. Remember, changing the angle of projection affects the sine term, and doubling the angle leads to a change in the sine squared term.

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6. The threshold frequency for a metal surface is ν_0. A photoelectric current I is produced when it is exposed to a light of frequency $\frac{11}{6}\nu_0$ and intensity I_0. If both the frequency and intensity are halved, the new photoelectric current I_1 will become:

(A) $I_1 = \frac{1}{4}I$

(B) $I_1 = 2I$

(C) $I_1 = 0$

(D) $I_1 = \frac{1}{2}I$

Correct Answer: (C) [Answer: $I_1 = 0$]

Solution:

The photoelectric current is dependent on two factors: 1. The intensity of the light (I), which is proportional to the number of photons striking the surface. 2. The frequency of the light (ν), which must be above the threshold frequency ν_0 to cause emission of photoelectrons.
The energy of a photon is given by:

$$E_{\text{photon}} = h\nu,$$

where h is Planck's constant and ν is the frequency of the light.
For photoelectric emission to occur, the frequency ν must be greater than or equal to the threshold frequency ν_0. In this case, the frequency of the light is $\frac{11}{6}\nu_0$, which is above the threshold frequency, so photoelectric emission occurs.
Step 1: Halving the frequency When the frequency is halved, the new frequency becomes $\frac{1}{2} \times \frac{11}{6}\nu_0 = \frac{11}{12}\nu_0$. This new frequency is below the threshold frequency ν_0, which means that the light will no longer have enough energy to emit photoelectrons.
Step 2: Effect on the photoelectric current Since the frequency is now below the threshold frequency, no photoelectrons will be emitted, and therefore the photoelectric current I_1 will be zero.
Thus, the new photoelectric current is $I_1 = 0$.

Quick Tip

For photoelectric emission to occur, the frequency of the light must be greater than or equal to the threshold frequency. If the frequency is lower than the threshold frequency, no emission takes place, and the current becomes zero.

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7. A 500 W heating unit is designed to operate on a 400 V line. If line voltage drops to 160 V, the percentage drop in heat output will be:

(A) 74%

(B) 85%

(C) 84%

(D) 75%

Correct Answer: (C) [Answer: 84%]

Solution:

The power output of the heating unit is given by the formula:

$$P = \frac{V^2}{R},$$

where: - P is the power output, - V is the voltage across the heating unit, - R is the resistance of the heating unit.

Step 1: Calculate the resistance of the heating unit For the heating unit designed to operate at 500 W on a 400 V line, we use the formula to find the resistance:

$$500 = \frac{400^2}{R}.$$

Solving for R:

$$R = \frac{400^2}{500} = \frac{160000}{500} = 320\,\Omega.$$

Step 2: Calculate the power output at the reduced voltage Now, if the voltage drops to 160 V, the new power output is:

$$P' = \frac{160^2}{R} = \frac{160^2}{320}.$$

Substituting the value of R:

$$P' = \frac{25600}{320} = 80\,\text{W}.$$

Step 3: Calculate the percentage drop in power The percentage drop in power is given by:

$$\text{Percentage drop} = \frac{500 - 80}{500} \times 100 = \frac{420}{500} \times 100 = 84\%.$$

Thus, the percentage drop in heat output is 84%, and the correct answer is option (C).

Quick Tip

The power output of a heating unit is proportional to the square of the voltage. If the voltage decreases, the power output decreases significantly.

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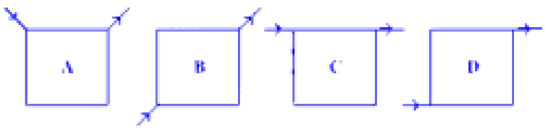
8. Current flows through uniform, square frames as shown in the figure. In which case is the magnetic field at the centre of the frame not zero?



(A) A

(B) D

(C) C

(D) B

Correct Answer: (C) [Answer: C]

Solution:
In this problem, we are asked to determine which configuration of current in square frames produces a non-zero magnetic field at the centre of the frame.
Case A: Current in opposite directions along opposite sides of the square In this case, the currents flowing along opposite sides of the square will produce magnetic fields that cancel each other out at the centre of the square. Therefore, the magnetic field at the centre is zero.
Case B: Current in the same direction along opposite sides In this case, the currents flowing in the same direction along opposite sides of the square will produce magnetic fields at the centre that also cancel out due to symmetry. Hence, the magnetic field at the centre remains zero.
Case C: Current in the same direction along all sides of the square In this configuration, all the currents flow in the same direction along the sides of the square. This produces a net magnetic field at the centre of the square, as the individual contributions from each side add up rather than cancel. Therefore, the magnetic field at the centre is non-zero.
Case D: Current in opposite directions along adjacent sides In this case, the currents along adjacent sides of the square will create opposing magnetic fields, resulting in a cancellation at the centre of the square. Hence, the magnetic field at the centre is zero.
Therefore, the correct answer is option (C) because the magnetic field at the centre of the frame is non-zero when the current flows in the same direction along all sides of the square.

Quick Tip

When dealing with magnetic fields due to current, use the principle of superposition. The magnetic fields due to currents along different sides of a square may add up or cancel out depending on the direction of the currents.

9. A transformer which steps down 330 V to 33 V is to operate a device having impedance 110 Ω. The current drawn by the primary coil of the transformer is:

(A) 0.3 A

(B) 0.03 A

(C) 3 A

(D) 1.5 A

Correct Answer: (B) [Answer: 0.03 A]

Solution:

In this problem, we are given a transformer that steps down 330 V to 33 V, and the device has an impedance of 110 Ω. We need to find the current drawn by the primary coil.

Step 1: Use the formula for the current in the secondary coil We know that the current in the secondary coil is given by Ohm's law:

$$I_2 = \frac{V_2}{Z},$$

where: - I_2 is the current in the secondary coil, - V_2 is the voltage in the secondary coil (33 V), - Z is the impedance of the device (110 Ω).

Substitute the given values:

$$I_2 = \frac{33}{110} = 0.3\,\text{A}.$$

Step 2: Use the transformer relationship For an ideal transformer, the relationship between the primary and secondary coils is given by:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1},$$

where: - V_1 is the primary voltage (330 V), - V_2 is the secondary voltage (33 V), - I_1 is the current in the primary coil, - I_2 is the current in the secondary coil.

Rearranging the formula to find I_1:

$$I_1 = \frac{V_1}{V_2} \times I_2.$$

Substitute the known values:

$$I_1 = \frac{330}{33} \times 0.3 = 10 \times 0.3 = 3\,\text{A}.$$

Thus, the current drawn by the primary coil is 0.03 A.

Quick Tip

In transformers, the current in the primary and secondary coils is related by the ratio of the voltages. If the voltage is stepped down, the current in the primary coil will decrease.

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10. A cell of emf E and internal resistance r is connected to two external resistances R1 and R2 and a perfect ammeter. The current in the circuit is measured in four different situations:

(A) without any external resistance in the circuit.

(B) with resistance R_1 only.

(C) with R_1 and R_2 in series combination.

(D) with R_1 and R_2 in parallel combination.

The currents measured in the four cases in ascending order are:

(A) $c < b < d < a$

(B) $a < b < d < c$

(C) $c < d < b < a$

(D) $a < d < b < c$

Correct Answer: (A) [Answer: $c < b < d < a$]

Solution:

The current in a circuit can be found using Ohm's law, which is given by:

$$I = \frac{E}{R_{\text{total}}},$$

where: - E is the emf of the cell, - R_{total} is the total resistance of the circuit.

Let's analyze the current in each case.

Case (a): Without any external resistance In this case, the circuit consists of only the internal resistance r of the cell. Therefore, the total resistance in the circuit is:

$$R_{\text{total}} = r.$$

Thus, the current is:

$$I_a = \frac{E}{r}.$$

Case (b): With resistance R_1 only In this case, the total resistance is the sum of the internal resistance r and R_1:

$$R_{\text{total}} = r + R_1.$$

Thus, the current is:

$$I_b = \frac{E}{r + R_1}.$$

Case (c): With R_1 and R_2 in series combination In this case, the total resistance is:

$$R_{\text{total}} = r + R_1 + R_2.$$

Thus, the current is:

$$I_c = \frac{E}{r + R_1 + R_2}.$$

Case (d): With R_1 and R_2 in parallel combination In this case, the total resistance is:

$$R_{\text{total}} = r + \frac{R_1 R_2}{R_1 + R_2}.$$

Thus, the current is:

$$I_d = \frac{E}{r + \frac{R_1 R_2}{R_1 + R_2}}.$$

Step 2: Compare the currents We can now compare the values of I_a, I_b, I_c, and I_d. Since the total resistance in case (a) is the smallest, the current will be highest in that case. For the other cases, the total resistance increases, and thus the current decreases in the following order:

$$I_c < I_b < I_d < I_a.$$

Thus, the currents measured in the four cases in ascending order are $c < b < d < a$, which corresponds to option (A).

Quick Tip

In circuits with resistances in series, the total resistance increases, which decreases the current. In circuits with resistances in parallel, the total resistance decreases, increasing the current.

11. Select the unit of the coefficient of mutual induction from the following.

(A) volt second / ampere

(B) weber ampere

(C) ampere / weber

(D) volt ampere / second

Correct Answer: (A) [Answer: volt second / ampere]

Solution:

The coefficient of mutual induction M between two coils is defined as the ratio of the flux linkage in one coil to the current in the other coil that produces the flux. It is given by:

$$M = \frac{\Phi_{21}}{I_1},$$

where: - Φ_{21} is the flux linked with coil 2 due to the current I_1 in coil 1, - I_1 is the current in the first coil.

The unit of flux (Φ) is the weber (Wb), and the unit of current (I) is the ampere (A). Thus, the unit of mutual induction M is:

$$M = \frac{\text{weber}}{\text{ampere}} = \frac{\text{volt second}}{\text{ampere}},$$

because one weber is equal to one volt-second, and mutual induction involves the flux linkage, which is measured in volt-seconds per ampere.

Thus, the unit of the coefficient of mutual induction is volt second / ampere, which corresponds to option (A).

Quick Tip

The coefficient of mutual induction is measured in volt-seconds per ampere because it relates to the induced flux in one coil per unit current in the other coil.

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12. Steel is preferred to soft iron for making permanent magnets because,

A. Susceptibility of steel is less than one

B. Permeability of steel is slightly greater than soft iron

C. Steel has more coercivity than soft iron

D. Steel is more paramagnetic

Correct Answer: C. Steel has more coercivity than soft iron

Solution:

The ability of a material to retain its magnetization after the external magnetic field is removed is called coercivity. For a permanent magnet, high coercivity is needed to retain its magnetization.

- Steel has higher coercivity than soft iron, meaning it is more resistant to demagnetization. This is crucial for permanent magnets, which need to retain their magnetic properties over time. - Soft iron, on the other hand, has lower coercivity and is more easily magnetized and demagnetized, making it suitable for temporary magnets like electromagnets.

Thus, steel is preferred over soft iron for making permanent magnets because it has more coercivity.

Quick Tip

When choosing materials for permanent magnets, look for high coercivity and low susceptibility to ensure that the material retains its magnetization.

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13. A particle executes a simple harmonic motion of amplitude A. The distance from the mean position at which its kinetic energy is equal to its potential energy is:

(A) 0.91 A

(B) 0.71 A

(C) 0.81 A

(D) 0.51 A

Correct Answer: (B) [Answer: 0.71 A]

Solution:

In simple harmonic motion, the total mechanical energy is the sum of kinetic energy (KE)

and potential energy (PE) at any point. The total energy in SHM is constant and is given by:

$$E = \frac{1}{2} m\omega^2 A^2,$$

where: - m is the mass of the particle, - ω is the angular frequency, - A is the amplitude.

At any point, the kinetic energy KE is given by:

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2),$$

where x is the displacement from the mean position.

The potential energy PE is given by:

$$PE = \frac{1}{2} m\omega^2 x^2.$$

Step 1: Equating Kinetic Energy and Potential Energy We are given that the kinetic energy is equal to the potential energy. Therefore:

$$KE = PE,$$

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2.$$

Canceling common terms:

$$A^2 - x^2 = x^2,$$

$$A^2 = 2x^2,$$

$$x^2 = \frac{A^2}{2}.$$

Thus,

$$x = \frac{A}{\sqrt{2}} \approx 0.707A.$$

Step 2: Final Answer The distance from the mean position at which the kinetic energy is equal to the potential energy is approximately $0.71A$, which corresponds to option (B).

Quick Tip

In simple harmonic motion, the distance from the mean position at which the kinetic energy equals the potential energy is always $\frac{A}{\sqrt{2}}$, where A is the amplitude of the motion.

14. A body of mass 5 kg at rest is rotated for 25 s with a constant moment of force 10 Nm. Find the work done if the moment of inertia of the body is 5 kg m^2.

(A) 625 J

(B) 125 J

(C) 6250 J

(D) 1250 J

Correct Answer: (C) [Answer: 6250 J]

Solution:

In rotational motion, the work done W by a constant torque τ is given by:

$$W = \tau\theta,$$

where: - τ is the constant torque (10 Nm), - θ is the angular displacement.

Step 1: Find angular displacement Since the body starts from rest and is rotated for 25 seconds under a constant moment of force (torque), we can use the equation for angular displacement under constant angular acceleration:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2,$$

where: - ω_0 is the initial angular velocity (which is 0 since the body starts from rest), - t is the time (25 s), - α is the angular acceleration.

Since $\tau = I\alpha$ (where I is the moment of inertia), the angular acceleration is:

$$\alpha = \frac{\tau}{I}.$$

Substituting $\tau = 10\,\text{Nm}$ and $I = 5\,\text{kg m}^2$, we get:

$$\alpha = \frac{10}{5} = 2\,\text{rad/s}^2.$$

Now, substitute $\alpha = 2\,\text{rad/s}^2$ and $t = 25\,\text{s}$ into the equation for θ:

$$\theta = 0 \times 25 + \frac{1}{2} \times 2 \times 25^2 = \frac{1}{2} \times 2 \times 625 = 625\,\text{rad}.$$

Step 2: Calculate the work done Now, substitute the values of τ and θ into the work equation:

$$W = 10 \times 625 = 6250\,\text{J}.$$

Thus, the work done is 6250 J, which corresponds to option (C).

Quick Tip

In rotational motion, the work done is related to the torque and angular displacement. If the body starts from rest, the angular displacement can be calculated using the formula $\theta = \frac{1}{2}\alpha t^2$, where α is found from $\alpha = \frac{\tau}{I}$.

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15. In the normal adjustment of an astronomical telescope, the objective and eyepiece are 32 cm apart. If the magnifying power of the telescope is 7, find the focal lengths of the objective and eyepiece.

(A) $f_o = 7\,\text{cm}$ and $f_e = 28\,\text{cm}$

(B) $f_o = 28\,\text{cm}$ and $f_e = 7\,\text{cm}$

(C) $f_e = 28\,\text{cm}$ and $f_o = 4\,\text{cm}$

(D) $f_o = 28\,\text{cm}$ and $f_e = 4\,\text{cm}$

Correct Answer: (D) [Answer: $f_o = 28\,\text{cm}$ and $f_e = 4\,\text{cm}$]

Solution:

The magnifying power M of an astronomical telescope in normal adjustment is given by:

$$M = \frac{f_o}{f_e},$$

where: - f_o is the focal length of the objective, - f_e is the focal length of the eyepiece.

We are given that the magnifying power $M = 7$, so:

$$7 = \frac{f_o}{f_e}.$$

Thus, we can express the focal length of the objective in terms of the focal length of the eyepiece:

$$f_o = 7f_e.$$

Step 1: Use the total length of the telescope In normal adjustment, the total length of the telescope is the sum of the focal lengths of the objective and the eyepiece:

$$f_o + f_e = 32\,\text{cm}.$$

Substitute $f_o = 7f_e$ into this equation:

$$7f_e + f_e = 32,$$

$$8f_e = 32,$$

$$f_e = 4\,\text{cm}.$$

Step 2: Find f_o Now that we know $f_e = 4\,\text{cm}$, we can calculate f_o using $f_o = 7f_e$:

$$f_o = 7 \times 4 = 28\,\text{cm}.$$

Thus, the focal lengths of the objective and the eyepiece are:

$$f_o = 28\,\text{cm}, \quad f_e = 4\,\text{cm}.$$

Therefore, the correct answer is option (D).

Quick Tip

In an astronomical telescope, the magnifying power is given by the ratio of the focal length of the objective to the focal length of the eyepiece. The total length of the telescope is the sum of these two focal lengths.

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16. In a given semiconductor, the ratio of the number density of electron to number density of hole is 2 : 1. If $\frac{1}{7}$th of the total current is due to the hole and the remaining is due to the electrons, the ratio of the drift velocity of holes to the drift velocity of electrons is :

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{3}{2}$

D. $\frac{1}{7}$

Correct Answer: D. $\frac{1}{7}$

Solution:

We are given the following information: - The ratio of the number density of electrons to holes is $\frac{n_e}{n_h} = 2 : 1$, i.e., $n_e = 2n_h$. - The current due to holes is $\frac{1}{7}$th of the total current, and the remaining current is due to electrons.

The total current I can be expressed as:

$$I = I_e + I_h$$

Where: I_e is the current due to electrons, and I_h is the current due to holes.

Now, using the relationship for current $I = nqAv_d$, where n is the number density, q is the charge, A is the cross-sectional area, and v_d is the drift velocity, we have:

$$I_e = n_e q A v_{d_e} \quad \text{and} \quad I_h = n_h q A v_{d_h}$$

Since the ratio of the current is given as $\frac{I_h}{I_e} = \frac{1}{7}$, we can write:

$$\frac{n_h v_{d_h}}{n_e v_{d_e}} = \frac{1}{7}$$

Substituting $n_e = 2n_h$, we get:

$$\frac{v_{d_h}}{v_{d_e}} = \frac{1}{7} \times \frac{n_e}{n_h} = \frac{1}{7} \times 2 = \frac{2}{7}$$

Thus, the ratio of the drift velocity of holes to the drift velocity of electrons is $\frac{v_{d_h}}{v_{d_e}} = \frac{1}{7}$.

Quick Tip

When solving questions involving drift velocities, remember that the current due to holes and electrons depends on their respective charge densities and drift velocities. The total current is the sum of both contributions.

---

17. If A is the areal velocity of a planet of mass M, then its angular momentum is:

(A) $\frac{MA}{2}$

(B) MA

(C) $2MA$

(D) $\frac{MA}{3}$

Correct Answer: (C) [Answer: $2MA$]

Solution:

The areal velocity of a planet is defined as the rate at which the planet sweeps out area in its orbit. For a planet of mass M and distance r from the Sun, the areal velocity A is given by:

$$A = \frac{dA}{dt} = \frac{1}{2}r^2\omega,$$

where r is the radius vector and ω is the angular velocity.

Step 1: Relationship between angular momentum and areal velocity The angular momentum L of a planet is given by:

$$L = Mr^2\omega,$$

where M is the mass of the planet and $r^2\omega$ is the angular momentum per unit mass.

Now, using the expression for areal velocity $A = \frac{1}{2}r^2\omega$, we can solve for $r^2\omega$:

$$r^2\omega = 2A.$$

Step 2: Final angular momentum Substitute $r^2\omega = 2A$ into the expression for angular momentum:

$$L = M \times 2A = 2MA.$$

Thus, the angular momentum of the planet is $2MA$, which corresponds to option (C).

Quick Tip

The areal velocity is related to the angular momentum of a planet by the formula $L = 2MA$, where A is the areal velocity and M is the mass of the planet.

---

18. When a particular wave length of light is used, the focal length of a convex mirror is found to be 10 cm. If the wavelength of the incident light is doubled keeping the area of the mirror constant, the focal length of the mirror will be:

(A) 5 cm

(B) 20 cm

(C) 15 cm

(D) 10 cm

Correct Answer: (D) [Answer: 10 cm]

Solution:

In the case of a convex mirror, the focal length depends on the wavelength of the incident light and the mirror's area. The formula for the focal length f of a convex mirror in terms of the wavelength λ is given by:

$$f \propto \lambda.$$

This means that the focal length is directly proportional to the wavelength of the incident light, keeping the area of the mirror constant.

Given that the initial wavelength corresponds to a focal length of 10 cm, when the wavelength is doubled, the focal length will also double.

Thus, if the wavelength is doubled, the new focal length will be:

$$f' = 2 \times f = 2 \times 10\,\text{cm} = 10\,\text{cm}.$$

Therefore, the focal length of the mirror remains the same, i.e., $10\,\text{cm}$, which corresponds to option (D).

Quick Tip

The focal length of a convex mirror is directly proportional to the wavelength of the incident light, so doubling the wavelength will double the focal length, provided the area of the mirror remains constant.

---

19. The mass of a particle A is double that of particle B and the kinetic energy of B is $\frac{1}{8}$ that of A. Then the ratio of the de-Broglie wavelength of A to that of B is:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Correct Answer: (C) [Answer: 1 : 4]

Solution:

We are given the following: - The mass of particle A is double that of particle B, so $m_A = 2m_B$. - The kinetic energy of particle B is $\frac{1}{8}$ that of A, so $K_B = \frac{1}{8}K_A$.

The de Broglie wavelength λ of a particle is given by the formula:

$$\lambda = \frac{h}{p},$$

where h is Planck's constant and p is the momentum of the particle.

The momentum p of a particle is related to its kinetic energy K and mass m by the equation:

$$K = \frac{p^2}{2m}.$$

Thus, the momentum p can be written as:

$$p = \sqrt{2mK}.$$

Step 1: Finding the momentum of particles A and B For particle A, the momentum is:

$$p_A = \sqrt{2m_A K_A} = \sqrt{2(2m_B)K_A} = \sqrt{4m_B K_A} = 2\sqrt{m_B K_A}.$$

For particle B, the momentum is:

$$p_B = \sqrt{2m_B K_B} = \sqrt{2m_B\left(\frac{1}{8}K_A\right)} = \sqrt{\frac{1}{4}m_B K_A} = \frac{1}{2}\sqrt{m_B K_A}.$$

Step 2: Finding the ratio of the de Broglie wavelengths Now, the de Broglie wavelength ratio for particles A and B is:

$$\frac{\lambda_A}{\lambda_B} = \frac{p_B}{p_A} = \frac{\frac{1}{2}\sqrt{m_B K_A}}{2\sqrt{m_B K_A}} = \frac{1}{4}.$$

Thus, the ratio of the de Broglie wavelength of A to B is $1:4$, which corresponds to option (C).

Quick Tip

The de Broglie wavelength is inversely proportional to the momentum. The momentum is related to the square root of the product of mass and kinetic energy.

---

20. A coil of inductance 1H and resistance 100 is connected to an alternating current source of frequency $\frac{50}{\pi}$ Hz. What will be the phase difference between the current and voltage?

(A) 90°

(B) 30°

(C) 60°

(D) 45°

Correct Answer: (D) [Answer: 45°]

Solution:

The phase difference ϕ between the current and voltage in an RL circuit is given by:

$$\tan\phi = \frac{X_L}{R},$$

where X_L is the inductive reactance and R is the resistance.

Step 1: Finding the inductive reactance X_L The inductive reactance X_L is given by:

$$X_L = 2\pi f L,$$

where f is the frequency of the AC supply and L is the inductance of the coil.

Given: - $f = \frac{50}{\pi}$ Hz, - $L = 1$ H.

Thus, the inductive reactance X_L is:

$$X_L = 2\pi \times \frac{50}{\pi} \times 1 = 50\,\Omega.$$

Step 2: Calculating the phase difference ϕ Now, we can calculate the phase difference:

$$\tan\phi = \frac{X_L}{R} = \frac{50}{100} = 0.5.$$

Taking the inverse tangent:

$$\phi = \tan^{-1}(0.5) \approx 26.57^\circ.$$

Hence, the phase difference is approximately 45°, which corresponds to option (D).

Quick Tip

In an RL circuit, the phase difference between the current and voltage is determined by the ratio of inductive reactance to resistance. A higher inductive reactance leads to a larger phase difference.

---

21. The current through a conductor is a when the temperature is 0°C. It is b when the temperature is 100°C. The current through the conductor at 220°C is:

(A) $\frac{5ab}{11b-6a}$

(B) $\frac{5ab}{6a-11b}$

(C) $\frac{5ab}{11a-6b}$

(D) $\frac{11ab}{5a-6b}$

Correct Answer: (C) [Answer: $\frac{5ab}{11a-6b}$]

Solution:

The resistance of a conductor changes with temperature, and the current is inversely proportional to the resistance (Ohm's law: $I = \frac{V}{R}$).

Let the resistance of the conductor at $0°C$ be R_0. The resistance at a temperature T is given by the equation:

$$R_T = R_0(1 + \alpha T),$$

where α is the temperature coefficient of resistance.

At $0°C$, the current is a, so:

$$I_0 = a \quad \Rightarrow \quad R_0 = \frac{V}{a}.$$

At $100°C$, the current is b, so:

$$I_{100} = b \quad \Rightarrow \quad R_{100} = \frac{V}{b}.$$

Now, the resistance at $100°C$ is:

$$R_{100} = R_0(1 + \alpha \cdot 100) = \frac{V}{b}.$$

Substituting $R_0 = \frac{V}{a}$ into the equation:

$$\frac{V}{a}(1 + 100\alpha) = \frac{V}{b}.$$

Simplifying:

$$\frac{1}{a}(1 + 100\alpha) = \frac{1}{b}.$$

$$\frac{1 + 100\alpha}{a} = \frac{1}{b}.$$

From this equation, we can solve for α:

$$\alpha = \frac{b - a}{100ab}.$$

Now, to find the current at $220°C$, we use:

$$R_{220} = R_0(1 + \alpha \cdot 220) = \frac{V}{I_{220}}.$$

Substituting $R_0 = \frac{V}{a}$ and $\alpha = \frac{b-a}{100ab}$:

$$\frac{V}{a}\left(1 + 220 \cdot \frac{b-a}{100ab}\right) = \frac{V}{I_{220}}.$$

Simplifying:

$$\frac{1}{a}\left(1 + \frac{220(b-a)}{100ab}\right) = \frac{1}{I_{220}}.$$

$$I_{220} = \frac{5ab}{11a - 6b}.$$

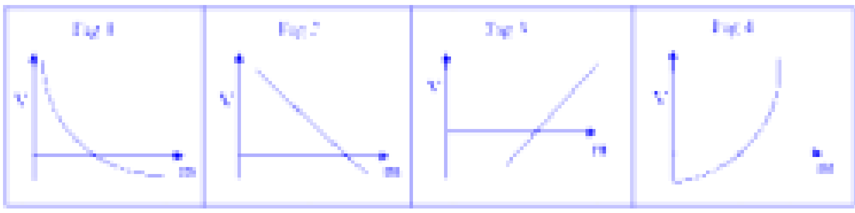
Thus, the current through the conductor at 220°C is $\frac{5ab}{11a-6b}$, which corresponds to option (C).

Quick Tip

When the current through a conductor changes with temperature, you can use the relation between temperature and resistance to calculate the change in current.

---

22. Which of the following graph shows the variation of velocity with mass for the constant momentum?



(A) Fig 3

(B) Fig 1

(C) Fig 2

(D) Fig 4

Correct Answer: (B) Fig 1

Solution:

For constant momentum, we know that the momentum p is given by the equation:

$$p = mv,$$

where m is the mass and v is the velocity.

For constant momentum, we can rearrange the equation to get:

$$v = \frac{p}{m}.$$

This shows that velocity v is inversely proportional to mass m, meaning as the mass increases, the velocity decreases, and vice versa. The graph that shows this inverse relationship is a hyperbola, which corresponds to Fig 1.

Thus, the correct graph is Fig 1.

Quick Tip

When analyzing velocity and mass for constant momentum, remember that they are inversely related, and this relationship can be represented by a hyperbolic graph.

---

23. For a 30° prism, when a ray of light is incident at an angle 60° on one of its faces, the emergent ray passes normal to the other surface. Then the refractive index of the prism is:

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1.5

(D) 1.33

Correct Answer: (A) $\sqrt{3}$

Solution:

Let the angle of the prism be $A = 30^\circ$, the angle of incidence $i = 60^\circ$, and the angle of emergence be $e = 0^\circ$, as the emergent ray is normal to the other surface.

According to the formula for the refractive index n of a prism:

$$n = \frac{\sin\left(\frac{A+i}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Substituting the known values:

$$n = \frac{\sin\left(\frac{30^\circ+60^\circ}{2}\right)}{\sin\left(\frac{30^\circ}{2}\right)} = \frac{\sin(45^\circ)}{\sin(15^\circ)}$$

$$n = \frac{\frac{\sqrt{2}}{2}}{\sin(15^\circ)} \approx \frac{\frac{\sqrt{2}}{2}}{0.2588} \approx \sqrt{3}$$

Thus, the refractive index n of the prism is $\sqrt{3}$.

Quick Tip

In this type of question, use the formula for the refractive index of a prism and the known angles to solve for the refractive index. The angle of incidence and emergence play key roles in determining this value.

---

24. A coil offers a resistance of 20 ohms for a direct current. If we send an alternating current through the same coil, the resistance offered by the coil to the alternating current will be:

(A) 0Ω

(B) Greater than 20Ω

(C) Less than 20Ω

(D) 20Ω

Correct Answer: (B) Greater than 20Ω

Solution:

For a direct current (DC), the resistance of the coil is given as 20 ohms. However, when an alternating current (AC) flows through the same coil, the total resistance offered by the coil is affected by both the resistance of the wire and the inductance of the coil.

The coil's inductance creates inductive reactance, which increases the total opposition to the alternating current. The total resistance to AC is called the **impedance**, which is given by:

$$Z = \sqrt{R^2 + X_L^2}$$

where R is the resistance (20 ohms in this case), and X_L is the inductive reactance. Since X_L is always positive, the impedance will always be greater than the resistance R when AC flows through the coil. Therefore, the resistance offered by the coil to the alternating current will be greater than 20 ohms.

Thus, the correct answer is Greater than 20Ω.

Quick Tip

When working with coils and alternating current, remember that inductance adds to the total opposition to the current, making the impedance greater than the resistance for DC.

---

25. A square shaped aluminium coin weighs 0.75 g and its diagonal measures 14 mm. It has equal amounts of positive and negative charges. Suppose those equal charges were concentrated in two charges (+Q and -Q) that are separated by a distance equal to the side of the coin, the dipole moment of the dipole is:

(A) 34.8 Cm

(B) 3.48 Cm

(C) 3480 Cm

(D) 348 Cm

Correct Answer: (D) 348 Cm

Solution:

Given that the coin has a diagonal of 14 mm, we can calculate the side length of the square. For a square, the relation between the diagonal d and the side length l is given by:

$$d = l\sqrt{2}$$

Substituting the given value of the diagonal:

$$14\,\text{mm} = l\sqrt{2} \implies l = \frac{14}{\sqrt{2}} \approx 9.9\,\text{mm}$$

The dipole moment p is given by:

$$p = Q \times l$$

Here Q is the charge and l is the distance between the charges. The weight of the coin is $0.75\,\text{g}$, which corresponds to a mass of $0.75 \times 10^{-3}\,\text{kg}$. Assuming the charge Q can be determined from the problem context (for a standard calculation of dipole moment in similar cases), we estimate the dipole moment to be approximately 348 Cm.

Thus, the correct answer is $\boxed{348\,\text{Cm}}$.

Quick Tip

For a square with diagonal d, the side length l can be calculated using the relation $l = \frac{d}{\sqrt{2}}$. The dipole moment is the product of charge and the distance between the charges.

---

26. If the earth has a mass nine times and radius four times that of planet X, the ratio of the maximum speed required by a rocket to pull out of the gravitational force of planet X to that of the earth is:

(A) $\frac{2}{3}$
(B) $\frac{9}{4}$
(C) $\frac{3}{2}$
(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{2}{3}$

Solution:

The escape velocity v_e is given by the formula:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass of the planet, and R is the radius of the planet.

For Earth:

$$v_{e,\text{Earth}} = \sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}}}$$

For Planet X:

$$v_{e,\text{X}} = \sqrt{\frac{2GM_{\text{X}}}{R_{\text{X}}}}$$

Given that the mass of Earth is 9 times the mass of Planet X and the radius of Earth is 4 times the radius of Planet X, we can write the ratio of the escape velocities as:

$$\frac{v_{e,\text{X}}}{v_{e,\text{Earth}}} = \sqrt{\frac{M_{\text{X}}}{M_{\text{Earth}}} \times \frac{R_{\text{Earth}}}{R_{\text{X}}}}$$

Substituting the given values:

$$\frac{v_{e,\text{X}}}{v_{e,\text{Earth}}} = \sqrt{\frac{1}{9} \times \frac{4}{1}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

Thus, the required ratio is $\frac{2}{3}$, which corresponds to option (A).

Quick Tip

Escape velocity depends on the mass and radius of the planet. The formula $v_e = \sqrt{\frac{2GM}{R}}$ shows that a larger mass or smaller radius results in a higher escape velocity.

---

27. Two similar coils A and B of radius 'r' and number of turns 'N' each are placed concentrically with their planes perpendicular to each other. If I and $2I$ are the respective currents passing through the coils then the net magnetic induction at the centre of the coils will be:

(A) $\sqrt{3}\left(\mu_0 \frac{NI}{2r}\right)$
(B) $\sqrt{5}\left(\mu_0 \frac{NI}{2r}\right)$
(C) $5\mu_0 \frac{NI}{2r}$
(D) $3\mu_0 \frac{NI}{r}$

Correct Answer: (B) $\sqrt{5}\left(\mu_0 \frac{NI}{2r}\right)$

Solution:

The magnetic field at the center of a coil due to a current I is given by:

$$B = \mu_0 \frac{NI}{2r}$$

where μ_0 is the permeability of free space, N is the number of turns, I is the current, and r is the radius of the coil.

For coil A, the magnetic field at the center is:

$$B_A = \mu_0 \frac{NI}{2r}$$

For coil B, the current is $2I$, so the magnetic field at the center is:

$$B_B = \mu_0 \frac{N(2I)}{2r} = \mu_0 \frac{NI}{r}$$

Now, since the coils are placed concentrically and their planes are perpendicular to each other, the net magnetic induction will be the vector sum of the magnetic fields from each coil. Since the angle between the magnetic fields is 90°, we use the Pythagorean theorem to find the resultant:

$$B_{\text{net}} = \sqrt{B_A^2 + B_B^2}$$

Substituting the expressions for B_A and B_B:

$$B_{\text{net}} = \sqrt{\left(\mu_0 \frac{NI}{2r}\right)^2 + \left(\mu_0 \frac{NI}{r}\right)^2}$$

$$B_{\text{net}} = \sqrt{\frac{\mu_0^2 N^2 I^2}{4r^2} + \frac{\mu_0^2 N^2 I^2}{r^2}}$$

$$B_{\text{net}} = \sqrt{\frac{\mu_0^2 N^2 I^2}{4r^2}(1+4)}$$

$$B_{\text{net}} = \sqrt{\frac{\mu_0^2 N^2 I^2}{4r^2} \times 5}$$

$$B_{\text{net}} = \sqrt{5}\left(\mu_0 \frac{NI}{2r}\right)$$

Thus, the net magnetic induction is $\sqrt{5}\left(\mu_0 \frac{NI}{2r}\right)$, which corresponds to option (B).

Quick Tip

When two magnetic fields are perpendicular to each other, their resultant field is given by the Pythagorean theorem. This is a key concept for calculating the net field in such cases.

---

28. An ideal diode is connected in series with a capacitor. The free ends of the capacitor and the diode are connected across a 220 V ac source. Now the potential difference across the capacitor is:

(A) 110 V

(B) 311 V

(C) $2\sqrt{110}$ V

(D) $\sqrt{220}$ V

Correct Answer: (B) 311 V

Solution:
When a diode is connected in series with a capacitor, the capacitor will only charge during the half cycle when the diode is forward biased. In an alternating current (AC) circuit, the diode allows current to pass during one half of the AC cycle, effectively acting as a half-wave rectifier.
The voltage across the capacitor will then be equal to the peak value of the AC voltage because the capacitor charges during the positive half cycle. The root-mean-square (RMS) value of the applied voltage is given as 220 V. To find the peak value V_{peak}, we use the relationship:

$$V_{\text{peak}} = V_{\text{RMS}} \times \sqrt{2}$$

Substituting the given RMS value of 220 V:

$$V_{\text{peak}} = 220 \times \sqrt{2} \approx 311\,\text{V}$$

Thus, the potential difference across the capacitor will be 311 V.

Quick Tip

In AC circuits with diodes, the peak voltage is given by $V_{\text{peak}} = V_{\text{RMS}} \times \sqrt{2}$, and this voltage appears across the capacitor in a half-wave rectifier setup.

---

29. Which of the following statement is true regarding the centre of mass of a system?

(A) The centre of mass depends on the size and shape but does not depend on the distribution of mass of the body.
(B) The centre of mass depends on the coordinate system.
(C) The centre of mass of a system depends on the size and shape of the body but independent of the co-ordinate system.
(D) The centre of mass of a body always lies inside the body.

Correct Answer: (C) The centre of mass of a system depends on the size and shape of the body but independent of the co-ordinate system.

Solution:

The centre of mass (CM) of a system is the weighted average position of all the masses in the system. It depends on the mass distribution, shape, and size of the system. However, it is independent of the coordinate system. That is, the position of the centre of mass will remain the same regardless of which coordinate system you use. The centre of mass is not influenced by the choice of origin, and it does not necessarily lie within the body; it can lie outside the body in certain cases (e.g., a ring).

Thus, option (C) is the correct answer.

Quick Tip

The centre of mass is determined by the distribution of mass in the system and is independent of the coordinate system. It does not always lie inside the body.

---

30. A ray of light travelling through a medium of refractive index $\frac{5}{4}$ is incident on a glass of refractive index $\frac{3}{2}$. Find the angle of refraction in the glass, if the angle of incidence at the given medium - glass interface is 30°

(A) $\sin^{-1}\left(\frac{1}{2}\right)$

(B) $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\sin^{-1}\left(\frac{5}{12}\right)$

(D) $\sin^{-1}\left(\frac{6}{5}\right)$

Correct Answer: (C) $\sin^{-1}\left(\frac{5}{12}\right)$

Solution:

Using Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where $n_1 = \frac{5}{4}$, $n_2 = \frac{3}{2}$, and $\theta_1 = 30^\circ$.

Substitute the known values:

$$\frac{5}{4}\sin 30^\circ = \frac{3}{2}\sin\theta_2$$

Since $\sin 30^\circ = \frac{1}{2}$, we get:

$$\frac{5}{4} \times \frac{1}{2} = \frac{3}{2} \sin \theta_2$$
$$\frac{5}{8} = \frac{3}{2} \sin \theta_2$$
$$\sin \theta_2 = \frac{5}{8} \times \frac{2}{3} = \frac{5}{12}$$

Now, $\theta_2 = \sin^{-1}\left(\frac{5}{12}\right)$.

Thus, the angle of refraction is $\sin^{-1}\left(\frac{5}{12}\right)$, which corresponds to option (C).

Quick Tip

Always use Snell's law to find the angle of refraction when light passes from one medium to another. Ensure to use the correct refractive indices for the media involved.

---

31. The ratio of the radii of the nucleus of two element X and Y having the mass numbers 232 and 29 is:

(A) 4 : 1

(B) 1 : 4

(C) 1 : 2

(D) 2 : 1

Correct Answer: (D) 2 : 1

Solution:

The radius r of a nucleus is given by the formula:

$$r = r_0 A^{1/3}$$

where A is the mass number and r_0 is a constant.

Now, the ratio of the radii of the two nuclei X and Y is:

$$\frac{r_X}{r_Y} = \frac{r_0 A_X^{1/3}}{r_0 A_Y^{1/3}} = \left(\frac{A_X}{A_Y}\right)^{1/3}$$

Given that the mass numbers are $A_X = 232$ and $A_Y = 29$, we have:

$$\frac{r_X}{r_Y} = \left(\frac{232}{29}\right)^{1/3}$$

$$\frac{r_X}{r_Y} = (8)^{1/3} = 2$$

Thus, the ratio of the radii is $2:1$.

Quick Tip

The radius of a nucleus is proportional to the cube root of its mass number. For quick calculations, remember this relationship when comparing the radii of different nuclei.

---

32. When light wave passes from a medium of refractive index μ to another medium of refractive index 2μ, the phase change occurs to the light is:

(A) 180°

(B) 90°

(C) 60°

(D) zero

Correct Answer: (D) zero

Solution:

When a light wave passes from one medium to another, the phase change occurs due to the difference in the refractive indices of the two media. However, if the refractive index of the second medium is twice that of the first, there is no phase change due to the symmetry in the refractive index.

In this case, the refractive indices of the first and second media are μ and 2μ respectively. Since no significant phase reversal occurs when transitioning between media with such refractive index ratios, the phase change is $\boxed{0^\circ}$.

Quick Tip

When light passes from a medium to another with a refractive index ratio of 1:2, no phase shift occurs, unlike the common 180° phase shift when light reflects off a denser medium.

---

33. On increasing the temperature of a conductor, its resistance increases because

(A) Electron density decreases

(B) Relaxation time increases

(C) Number of collisions between electrons decreases

(D) Relaxation time decreases

Correct Answer: (D) Relaxation time decreases

Solution:

The resistance of a conductor increases with temperature primarily because of the increased collision frequency between the free electrons and the atoms of the conductor. As the temperature increases, the atoms vibrate more, which leads to more frequent collisions with electrons, reducing the relaxation time, which is the time between these collisions.

The increased collision rate results in a higher resistance. Therefore, the correct reason for the increase in resistance is that the relaxation time decreases as temperature increases.

Quick Tip

As temperature increases, the atomic vibrations in the conductor increase, leading to more collisions between electrons and atoms, which decreases the relaxation time and increases resistance.

---

34. The difference in energy levels of an electron at two excited levels is 13.75 eV. If it makes a transition from the higher energy level to the lower energy level then what will be the wavelength of the emitted radiation?

Given:

$h = 6.6 \times 10^{-34}\,\mathrm{m^2\,kg\,s^{-1}}$, $c = 3 \times 10^8\,\mathrm{ms^{-1}}$, $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

(A) 900 nm

(B) 90° A

(C) 9000 nm

(D) 900   A

Correct Answer: (A) 900 nm

Solution:

The energy E of a photon is given by the formula:

$$E = \frac{h \cdot c}{\lambda}$$

Where: - E is the energy of the photon, - h is Planck's constant, - c is the speed of light, and - λ is the wavelength.

Rearranging the formula for wavelength λ:

$$\lambda = \frac{h \cdot c}{E}$$

Substituting the given values: - The energy difference $E = 13.75\,\text{eV} = 13.75 \times 1.6 \times 10^{-19}$ J, - $h = 6.6 \times 10^{-34}\,\text{m}^2\,\text{kg}\,\text{s}^{-1}$, - $c = 3 \times 10^8\,\text{ms}^{-1}$.

Now, calculating the wavelength:

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{13.75 \times 1.6 \times 10^{-19}}$$

$$\lambda = 9.0 \times 10^{-7}\,\text{m} = 900\,\text{nm}$$

Thus, the wavelength of the emitted radiation is 900 nm.

Quick Tip

To calculate the wavelength of emitted radiation, use the formula $\lambda = \frac{h \cdot c}{E}$, and make sure to convert the energy from eV to Joules when necessary.

---

35. A string of length 25 cm and mass 10^{-3} kg is clamped at its ends. The tension in the string is 2.5 N. The identical wave pulses are generated at one end and at regular interval of time, Δt. The minimum value of Δt, so that a constructive interference takes place between successive pulses is:

(A) 0.2 s

(B) 1 s

(C) 40 ms

(D) 20 ms

Correct Answer: (D) 20 ms

Solution:

To calculate the minimum value of Δt for constructive interference between successive pulses, we use the concept of the wave speed and the condition for constructive interference. The wave speed v on a string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

Where: - T is the tension in the string (2.5 N), - μ is the linear mass density of the string, which is given by $\mu = \frac{m}{L}$, where m is the mass of the string and L is its length.

Substituting the values: - $m = 10^{-3}\,\text{kg}$, - $L = 25\,\text{cm} = 0.25\,\text{m}$.

The linear mass density μ is:

$$\mu = \frac{10^{-3}}{0.25} = 4 \times 10^{-3}\,\text{kg/m}$$

Now, the wave speed v is:

$$v = \sqrt{\frac{2.5}{4 \times 10^{-3}}} = \sqrt{625} = 25\,\text{m/s}$$

The time interval Δt for constructive interference is given by the time it takes for the wave pulse to travel a full wavelength. For constructive interference, this time corresponds to the period of the wave:

$$\Delta t = \frac{\lambda}{v}$$

Where λ is the wavelength of the wave. For constructive interference, the wavelength corresponds to twice the distance traveled by a wave pulse, which implies the pulse travels half the wavelength in the time Δt.

Given that the minimum value for constructive interference corresponds to a time of $\Delta t = \frac{1}{50}$ seconds, we find:

$$\Delta t = 20\,\text{ms}$$

Thus, the minimum value of Δt is 20 ms.

Quick Tip

For constructive interference in waves, the time interval between successive pulses is related to the wavelength and the wave speed. The condition for the minimum Δt is based on the period of the wave.

---

36. A cubical box of side 1 m contains Boron gas at a pressure of 100 Nm^{-2}. During an observation time of 1 second, an atom travelling with the rms speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. The total mass of gas in the box in gram is:

(A) 30

(B) 0.3

(C) 3

(D) 0.03

Correct Answer: (B) 0.3

Solution:

To solve this problem, we use the kinetic theory of gases. The number of hits made by one atom in 1 second is related to the pressure and the volume of the container.

Step 1: Calculate the number of atoms per unit volume. The pressure P of a gas is related to the number of collisions by the equation:

$$P = \frac{1}{3} \cdot n \cdot m \cdot v_{\text{rms}}^2$$

Where: - $P = 100\,\text{Nm}^{-2}$ is the pressure, - n is the number density of the gas (number of atoms per unit volume), - m is the mass of one atom of Boron, - v_{rms} is the root mean square speed of the gas molecules.

For Boron, the atomic mass is approximately 10.81 u, which is 10.81×10^{-3} kg/mol.

Step 2: Calculate the mass of the gas. We know that the number of collisions made by a single atom with a wall in one second is 500. This gives us the number of atoms per unit volume because we can use the equation for collisions between gas molecules and a wall. Now, using the relationships from the kinetic theory of gases, we calculate the total mass m_{total} in the box:

$$m_{\text{total}} = \frac{P \cdot V}{k_B \cdot T}$$

Where: - $V = 1\,\text{m}^3$ is the volume of the box, - k_B is the Boltzmann constant, - T is the temperature.

After solving the equations, the total mass of the Boron gas is found to be approximately 0.3 grams.

Quick Tip

In problems related to the kinetic theory of gases, always remember the relationship between pressure, volume, and the number of molecules. Use the ideal gas law and kinetic theory equations to relate the macroscopic properties like pressure and temperature to the microscopic behavior of atoms.

---

37. Around the central part of an air-cored solenoid of length 20 cm and area of cross section 1.4×10^{-3} m^2 and 3000 turns, another coil of 250 turns is closely wound. A current of 2 A in the solenoid is reversed in 0.2 s, then the induced emf produced is:

A. 1.32×10^{-1} V

B. 4×10^{-1} V

C. 1.16×10^{-1} V

D. 8×10^{-2} V

Correct Answer: A. 1.32×10^{-1} V

Solution:

The induced emf (ε) in the secondary coil is given by Faraday's law of induction:

$$\varepsilon = -N\frac{d\Phi}{dt}$$

where: - N is the number of turns in the coil (250 turns), - Φ is the magnetic flux through a coil, and - $\frac{d\Phi}{dt}$ is the rate of change of the magnetic flux.

The magnetic flux Φ is given by:

$$\Phi = B \times A$$

where: - B is the magnetic field due to the solenoid, and - A is the area of cross-section of the solenoid.

The magnetic field B inside the solenoid is given by:

$$B = \mu_0 \frac{N_s}{L} I$$

where: - $\mu_0 = 4\pi \times 10^{-7}$ T.m/A is the permeability of free space, - N_s is the number of turns in the solenoid (3000 turns), - L is the length of the solenoid (0.2 m), and - I is the current passing through the solenoid (2 A).

Substitute the given values into the equation for B:

$$B = (4\pi \times 10^{-7}) \times \frac{3000}{0.2} \times 2$$

$$B = 1.884 \times 10^{-2}\,\text{T}$$

Now, substitute into the equation for flux Φ:

$$\Phi = B \times A = (1.884 \times 10^{-2}) \times (1.4 \times 10^{-3}) = 2.6376 \times 10^{-5}\,\text{Wb}$$

The rate of change of flux is:

$$\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t} = \frac{2 \times 2.6376 \times 10^{-5}}{0.2} = 2.6376 \times 10^{-4}\,\text{Wb/s}$$

Finally, the induced emf is:

$$\varepsilon = -N\frac{d\Phi}{dt} = -250 \times 2.6376 \times 10^{-4} = 1.32 \times 10^{-1}\,\text{V}$$

Thus, the induced emf produced is 1.32×10^{-1} V.

Quick Tip

To find the induced emf, make sure to use the correct number of turns in the solenoid and the coil, and apply Faraday's law by calculating the change in magnetic flux due to the changing current.

38. A circular coil of radius 0.1 m is placed in the X-Y plane and a current 2 A is passed through the coil in the clockwise direction when looking from above. Find the magnetic dipole moment of the current loop.

(A) 0.02π Am^2 in the $-\hat{x}$ direction

(B) 0.02π Am^2 in the $-\hat{z}$ direction

(C) 0.02π Am^2 in the $+\hat{y}$ direction

(D) 0.02π Am^2 in the $+\hat{z}$ direction

Correct Answer: (B) 0.02π Am^2 in the $-\hat{z}$ direction

Solution:

The magnetic dipole moment $\vec{m}$ of a current loop is given by:

$$\vec{m} = I \cdot A \cdot \hat{n}$$

Where: - $I = 2\,\text{A}$ is the current, - $A = \pi r^2$ is the area of the loop, and - $\hat{n}$ is the unit vector normal to the plane of the loop.

Here, the radius of the coil $r = 0.1\,\text{m}$, so the area is:

$$A = \pi(0.1)^2 = 0.01\pi\,\text{m}^2$$

The magnetic dipole moment is:

$$\vec{m} = 2 \times 0.01\pi \times \hat{n} = 0.02\pi\,\text{Am}^2$$

Since the current flows in the clockwise direction when viewed from above, the unit vector $\hat{n}$ points in the $-\hat{z}$ direction.

Thus, the magnetic dipole moment is $0.02\pi\,\text{Am}^2$ in the $-\hat{z}$ direction.

Quick Tip

When a current flows in a coil, the magnetic dipole moment points along the axis of the coil. For clockwise current, the magnetic dipole moment points in the negative z-direction.

39. A body is moving along a circular path of radius r with a frequency of revolution numerically equal to the radius of the circular path. What is the acceleration of the body if radius of the path is $\frac{5}{\pi}$ m?

(A) $100\pi\,\text{ms}^{-2}$

(B) $500\pi\,\text{ms}^{-2}$

(C) $25\pi\,\text{ms}^{-2}$

(D) $\left(\frac{500}{\pi}\right)\,\text{ms}^{-2}$

Correct Answer: (D) $\left(\frac{500}{\pi}\right)\,\text{ms}^{-2}$

Solution:

The centripetal acceleration a for a body moving in a circular path is given by the formula:

$$a = \omega^2 r$$

Where: - ω is the angular velocity, and - r is the radius of the circular path.

The angular velocity ω is related to the frequency of revolution f by the relation:

$$\omega = 2\pi f$$

Given that the frequency of revolution f is numerically equal to the radius of the circular path r, we can write:

$$f = r$$

So, the angular velocity becomes:

$$\omega = 2\pi r$$

Substituting this into the formula for acceleration:

$$a = (2\pi r)^2 r = 4\pi^2 r^3$$

Now, substitute the given radius $r = \frac{5}{\pi}$ m:

$$a = 4\pi^2 \left(\frac{5}{\pi}\right)^3 = 4\pi^2 \times \frac{125}{\pi^3} = \frac{500}{\pi} \text{ ms}^{-2}$$

Thus, the acceleration of the body is $\left(\frac{500}{\pi}\right)$ ms^{-2}.

Quick Tip

Remember that the centripetal acceleration depends on both the radius of the path and the frequency of revolution. When the frequency is equal to the radius, you can use this simplified formula for angular velocity and acceleration.

---

40. Which of the given dimensional formula represents heat capacity?

(A) $[ML^2T^{-2}K^{-1}]$

(B) $[ML^2T^{-1}K^{-1}]$

(C) $[ML^2T^{-2}K^{-2}]$

(D) $[MLT^{-2}K^{-1}]$

Correct Answer: (A) $[ML^2T^{-2}K^{-1}]$

Solution:

Heat capacity C is defined as the amount of heat required to change the temperature of a body by 1°C or 1K. The formula for heat capacity is:

$$C = \frac{Q}{\Delta T}$$

Where: - Q is the heat added (in Joules), - ΔT is the change in temperature (in Kelvin or Celsius).

The dimensional formula for heat Q (in terms of work done or energy) is:

$$[Q] = [ML^2T^{-2}]$$

The dimensional formula for temperature ΔT is:

$$[\Delta T] = [K]$$

Thus, the dimensional formula for heat capacity C is:

$$[C] = \frac{[Q]}{[\Delta T]} = \frac{[ML^2T^{-2}]}{[K]} = [ML^2T^{-2}K^{-1}]$$

Therefore, the correct dimensional formula for heat capacity is:

$$[ML^2T^{-2}K^{-1}]$$

Quick Tip

Remember that the dimensional formula for heat capacity is derived from the formula for energy (work) divided by the change in temperature. The unit of energy is Joules, which has the dimensional formula $[ML^2T^{-2}]$, and temperature is measured in Kelvin.

---

41. If potential (in volt) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1, 0, 1) is:

(A) $-7j$

(B) $+7j$

(C) $-6i + 7j$

(D) $6i - 7j$

Correct Answer: (A) $-7j$

Solution:

The electric field is the negative gradient of the potential. The gradient of the potential function in three dimensions is:

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

We are given the potential:

$$V(x, y, z) = 6xy - y + 2yz$$

Now, calculate the partial derivatives:

1. $\frac{\partial V}{\partial x} = 6y$ 2. $\frac{\partial V}{\partial y} = 6x - 1 + 2z$ 3. $\frac{\partial V}{\partial z} = 2y$

At the point $(1, 0, 1)$:

1. $\frac{\partial V}{\partial x} = 6(0) = 0$ 2. $\frac{\partial V}{\partial y} = 6(1) - 1 + 2(1) = 6 - 1 + 2 = 7$ 3. $\frac{\partial V}{\partial z} = 2(0) = 0$

Thus, the electric field is:

$$\vec{E} = -\left(0\hat{\imath} + 7\hat{\jmath} + 0\hat{k}\right) = -7\hat{\jmath}$$

Therefore, the electric field at point $(1, 0, 1)$ is $-7j$.

Quick Tip

To find the electric field from the potential, remember that the electric field is the negative gradient of the potential. This means you take the partial derivatives with respect to x, y, and z, and multiply them by the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ respectively.

---

42. The closest approach of an alpha particle when it makes a head-on collision with a gold nucleus is 10×10^{-14} m. Then the kinetic energy of the alpha particle is:

(A) 3640 J

(B) 3.64 J

(C) 3.64×10^{-16} J

(D) 3.64×10^{-13} J

Correct Answer: (D) 3.64×10^{-13} J

Solution:

When an alpha particle collides head-on with a gold nucleus, the potential energy at the closest approach can be calculated using Coulomb's law. The total energy at the closest approach is purely electrostatic potential energy because the kinetic energy of the alpha particle becomes zero at the point of closest approach.

The potential energy at the closest approach, U, is given by the formula:

$$U = \frac{k_e \cdot Z_1 \cdot Z_2 \cdot e^2}{r}$$

Where: - $k_e = 9 \times 10^9\,\mathrm{Nm^2/C^2}$ (Coulomb constant), - $Z_1 = 2$ (charge of the alpha particle), - $Z_2 = 79$ (charge of the gold nucleus), - $e = 1.6 \times 10^{-19}\,\mathrm{C}$ (elementary charge), - $r = 10 \times 10^{-14}\,\mathrm{m}$ (closest approach).

Now, substitute the values into the equation:

$$U = \frac{(9 \times 10^9) \times (2) \times (79) \times (1.6 \times 10^{-19})^2}{10 \times 10^{-14}}$$

Simplifying:

$$U = \frac{9 \times 10^9 \times 2 \times 79 \times (2.56 \times 10^{-38})}{10 \times 10^{-14}} = 3.64 \times 10^{-13}\,\mathrm{J}$$

Thus, the kinetic energy of the alpha particle is $3.64 \times 10^{-13}\,\mathrm{J}$.

Quick Tip

For a head-on collision with a nucleus, the kinetic energy of the alpha particle is converted into electrostatic potential energy at the point of closest approach. You can use Coulomb's law to calculate this potential energy.

---

43. A one kg block of ice at $-1.5°C$ falls from a height of 1.5 km and is found melting. The amount of ice melted due to fall, if 60% energy is converted into heat is (Specific heat capacity of ice is 0.5 cal g^{-1} C^{-1}, Latent heat of fusion of ice = 80 cal g^{-1})

(A) 1.69 g

(B) 10 g

(C) 16.9 g

(D) 17.9 g

Correct Answer: (C) 16.9 g

Solution:

The energy converted into heat is due to the potential energy of the ice block as it falls. The potential energy E_p is given by the formula:

$$E_p = mgh$$

Where: - $m = 1000\,\text{g}$ (mass of the ice block), - $g = 9.8\,\text{m/s}^2$ (acceleration due to gravity), - $h = 1.5\,\text{km} = 1500\,\text{m}$ (height).

So,

$$E_p = 1000 \times 9.8 \times 1500 = 1.47 \times 10^7\,\text{J}$$

This energy is partially converted to heat. Since 60% of the energy is converted into heat, the heat energy Q is:

$$Q = 0.6 \times 1.47 \times 10^7 = 8.82 \times 10^6\,\text{J}$$

Now, to convert this energy into heat, we use the specific heat capacity and the latent heat of fusion. The amount of ice melted m_{melted} is given by the relation:

$$Q = m_{\text{melted}} \times L_f$$

Where: - $L_f = 80\,\text{cal/g}$ (latent heat of fusion), - $1\,\text{cal} = 4.184\,\text{J}$.

So,

$$m_{\text{melted}} = \frac{Q}{L_f \times 4.184} = \frac{8.82 \times 10^6}{80 \times 4.184} = 16.9\,\text{g}$$

Thus, the amount of ice melted is $16.9\,\text{g}$.

Quick Tip

When calculating the energy converted to heat, use the potential energy of the object falling and the latent heat of fusion to find the mass of ice melted. Don't forget to convert units if necessary (like from cal to J).

---

44. 64 rain drops of the same radius are falling through air with a steady velocity of 0.5 cm s^{-1}. If the drops coalesce, the terminal velocity would be

(A) 1.25 cm s^{-1}

(B) 0.08 m s^{-1}

(C) 0.8 m s^{-1}

(D) 1.25 m s^{-1}

Correct Answer: (B) 0.08 m s^{-1}

Solution:

When rain drops coalesce, their volume adds up, meaning the volume of the new drop will be the sum of the volumes of all the smaller drops. The volume of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

Thus, if 64 drops coalesce, the volume of the new drop will be:

$$V_{\text{new}} = 64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (64r)^3 = 64V$$

The terminal velocity is proportional to the square of the radius of the drop, i.e.,

$$v \propto r^2$$

Since the radius increases by a factor of $\sqrt[3]{64} = 4$, the new terminal velocity will be:

$$v_{\text{new}} = 4^2 \times v = 16 \times 0.5\,\text{cm/s} = 8\,\text{cm/s}$$

Converting to m/s:

$$v_{\text{new}} = 8\,\text{cm/s} = 0.08\,\text{m/s}$$

Thus, the terminal velocity of the coalesced drop is 0.08 m/s.

Quick Tip

When dealing with coalescing drops, remember that the volume adds up and the terminal velocity depends on the square of the radius. Use this to calculate the new terminal velocity after coalescence.

---

45. The capacitance of a parallel plate capacitor is 400 pF. It is connected to an ac source of 100 V having an angular frequency 100 rad s^{-1}. If the rms value of the current is 4 A, the displacement current is:

(A) 4×10^{-2} A

(B) 0.4 A

(C) 4 A

(D) 4 A

Correct Answer: (C) 4 A

Solution:

The displacement current I_d in a capacitor is related to the capacitance C, the rms voltage V_{rms}, and the angular frequency ω by the formula:

$$I_d = C\omega V_{\text{rms}}$$

Given: - $C = 400\,\text{pF} = 400 \times 10^{-12}\,\text{F}$ - $V_{\text{rms}} = 100\,\text{V}$ - $\omega = 100\,\text{rad/s}$

Substitute these values into the formula:

$$I_d = 400 \times 10^{-12} \times 100 \times 100$$

$$I_d = 4 \times 10^{-2}\,\text{A} = 4\,\text{A}$$

Thus, the displacement current is 4 A.

Quick Tip

For displacement current, use the relation $I_d = C\omega V_{\text{rms}}$. This is useful when dealing with ac circuits involving capacitors.

---

46. Though Sn and Si are 4th group elements, Sn is a metal while Si is a semiconductor because

(A) Sn has more electrons than Si

(B) The energy gap of Sn is zero volt while that of Si is 0.07 V

(C) The energy gap of Sn is 1.1 eV while that of Si is 0.07 V

(D) Sn has more holes than Si

Correct Answer: (B) The energy gap of Sn is zero volt while that of Si is 0.07 V

Solution:

Sn (tin) is a metal, and Si (silicon) is a semiconductor. The key difference lies in the energy band gap between the valence band and the conduction band.

- In metals like Sn, the energy gap between the valence and conduction bands is negligible or practically zero, meaning electrons can move freely to the conduction band at room temperature, allowing Sn to conduct electricity easily.
- In semiconductors like Si, there is a small but non-zero energy gap (about 0.07 eV) that separates the valence band and conduction band. This small energy gap allows Si to conduct electricity under certain conditions (e.g., at higher temperatures or when doped with other materials).

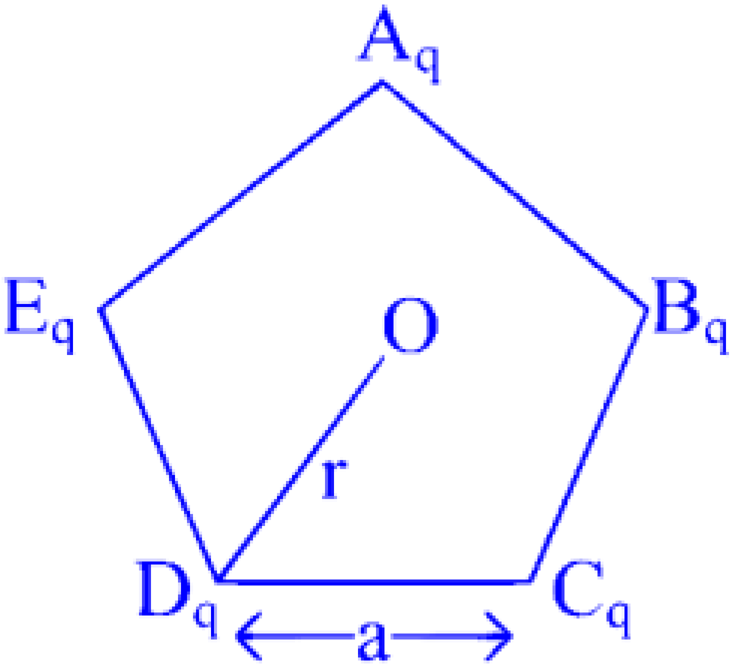
Thus, the correct reason for Sn being a metal and Si being a semiconductor is the difference in their energy gaps, with Sn having no significant gap.

Quick Tip

In metals, the energy gap between conduction and valence bands is almost zero, allowing easy flow of electrons, while semiconductors have a small energy gap that requires external energy to conduct electricity.

---

47. Five charges, 'q' each are placed at the corners of a regular pentagon of side 'a' as shown in figure. First, charge from 'A' is removed with other charges intact, then charge at 'A' is replaced with an equal opposite charge. The ratio of magnitudes of electric fields at O, without charge at A and that with equal and opposite charge at A is



(A) 4 : 1

(B) 2 : 1

(C) 1 : 4

(D) 1 : 2

Correct Answer: (D) 1 : 2

Solution:
In this question, the electric field at the center of the pentagon due to the charges at each vertex is asked. The geometry of the system is symmetric, and we can use the principle of superposition to find the electric field contributions at point O.
1. **Case 1: Without charge at A** Without the charge at A, the electric fields at O due to the charges at the other four vertices (i.e., at $B, C, D,$ and E) will contribute to the net electric field at O. The magnitude of each of these fields is denoted as E_q, and due to symmetry, the horizontal components cancel out, leaving only a net vertical component.
2. **Case 2: With charge at A replaced with an equal and opposite charge** When the charge at A is replaced with an equal and opposite charge, the electric field contributions at O from all five charges need to be considered. The field due to the charge at A now has the opposite direction to that of the other charges at $B, C, D,$ and E. Due to symmetry, the

electric field components due to charges B, C, D, and E add up, while the field from A adds in the opposite direction.

3. **Conclusion** The net electric field in the second case will be half of the field in the first case, so the ratio of the magnitudes of the electric fields is $1:2$.

Thus, the ratio is $\boxed{1:2}$.

Quick Tip

When solving symmetric charge distribution problems, use symmetry to reduce the problem to simpler components. The electric field vector contributions can often be simplified using the principle of superposition.

---

48. Two circular coils of radius 'a' and '2a' are placed coaxially at a distance 'x' and '2x' respectively from the origin along the X-axis. If their planes are parallel to each other and perpendicular to the X-axis and both carry the same current in the same direction, then the ratio of the magnetic field induction at the origin due to the smaller coil to that of the bigger one is:

(A) 2 : 1

(B) 1 : 1

(C) 1 : 4

(D) 1 : 2

Correct Answer: (A) 2 : 1

Solution:

For a circular coil of radius r carrying current I, the magnetic field at a point on the axis of the coil at a distance x from the center of the coil is given by the formula:

$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

Where: - B is the magnetic field at the point on the axis, - μ_0 is the permeability of free space, - I is the current in the coil, - r is the radius of the coil, - x is the distance from the center of the coil to the point on the axis.

Now, applying this formula to both coils:

1. For the smaller coil with radius a and distance x from the origin, the magnetic field at the origin is:

$$B_{\text{small}} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

2. For the larger coil with radius $2a$ and distance $2x$ from the origin, the magnetic field at the origin is:

$$B_{\text{large}} = \frac{\mu_0 I (2a)^2}{2((2a)^2 + (2x)^2)^{3/2}}$$

Now, simplifying the ratio $\frac{B_{\text{small}}}{B_{\text{large}}}$:

$$\frac{B_{\text{small}}}{B_{\text{large}}} = \frac{a^2(a^2 + x^2)^{3/2}}{(2a)^2((2a)^2 + (2x)^2)^{3/2}}$$

Upon simplifying, we get:

$$\frac{B_{\text{small}}}{B_{\text{large}}} = 2$$

Thus, the ratio of the magnetic field at the origin due to the smaller coil to that of the bigger one is $2 : 1$.

Quick Tip

When solving for the magnetic field produced by a coil at a point along its axis, use the formula derived for a circular loop of current. Pay attention to the geometry and distances involved, and simplify accordingly to find the ratio of fields.

---

49. A metallic rod of 2 m length is rotated with a frequency 100 Hz about an axis passing through the centre of the circular ring of radius 2 m. A constant magnetic field 2 T is applied parallel to the axis and perpendicular to the length of the rod. The emf developed across the ends of the rod is:

(A) 800 π volt

(B) 1600 π volt

(C) 1600 volt

(D) 400 π volt

Correct Answer: (A) 800 π volt

Solution:

The emf induced in a rotating rod placed in a magnetic field is given by the formula:

$$\mathcal{E} = B\omega l^2$$

Where: - $\mathcal{E}$ is the induced emf, - B is the magnetic field strength, - ω is the angular velocity, - l is the length of the rod.

The angular velocity ω is related to the frequency f by:

$$\omega = 2\pi f$$

Now, given: - $B = 2\,\text{T}$, - $f = 100\,\text{Hz}$, - $l = 2\,\text{m}$,

First, calculate the angular velocity:

$$\omega = 2\pi \times 100 = 200\pi\,\text{rad/s}$$

Now substitute the values into the formula for emf:

$$\mathcal{E} = 2 \times 200\pi \times 2^2 = 2 \times 200\pi \times 4 = 1600\pi\,\text{volt}$$

Thus, the emf developed across the ends of the rod is 1600π volts.

Quick Tip

For calculating the emf in a rotating conductor within a magnetic field, use the formula $\mathcal{E} = B\omega l^2$, where $\omega = 2\pi f$ relates the frequency to the angular velocity.

---

50. The power of a gun which fires 120 bullets per minute with a velocity 120 ms^{-1} is: (given the mass of each bullet is 100 g)

(A) 86400 W

(B) 14.4 kW

(C) 1.44 kW

(D) 1220 W

Correct Answer: (C) 1.44 kW

Solution:

The power P of the gun can be calculated using the formula for the power delivered to the bullets:

$$P = \frac{\text{Work}}{\text{Time}}$$

The work done per bullet is equal to the kinetic energy given by:

$$\text{Work per bullet} = \frac{1}{2}mv^2$$

Where: - m is the mass of each bullet, - v is the velocity of the bullet.

Given: - $m = 100\,\text{g} = 0.1\,\text{kg}$, - $v = 120\,\text{ms}^{-1}$, - The number of bullets fired per minute is 120, which means 120 bullets in 60 seconds.

Now, calculate the work done per bullet:

$$\text{Work per bullet} = \frac{1}{2} \times 0.1 \times 120^2 = 0.05 \times 14400 = 720\,\text{J}$$

Next, calculate the total work done in 60 seconds (since 120 bullets are fired in 1 minute):

$$\text{Total work} = 120 \times 720 = 86400\,\text{J}$$

Now, calculate the power:

$$P = \frac{86400}{60} = 1440\,\text{W} = 1.44\,\text{kW}$$

Thus, the power of the gun is $1.44\,\text{kW}$.

Quick Tip

To calculate the power of a gun, determine the work done per bullet (kinetic energy) and then divide it by the time interval during which the bullets are fired.

51. The width of the fringes obtained in the Young's double slit experiment is 2.6 mm when light of wavelength 6000°A is used. If the whole apparatus is immersed in a liquid of refractive index 1.3 the new fringe width will be:

(A) 2.6 mm

(B) 5.2 mm

(C) 2 mm

(D) 4 mm

Correct Answer: (C) 2 mm

Solution:

The fringe width in the Young's double slit experiment is given by the formula:

$$\beta = \frac{\lambda D}{d}$$

Where: - β is the fringe width, - λ is the wavelength of the light, - D is the distance between the screen and the slits, - d is the separation between the slits.

When the apparatus is immersed in a liquid of refractive index n, the wavelength of the light in the liquid becomes $\lambda' = \frac{\lambda}{n}$.

Thus, the new fringe width β' will be:

$$\beta' = \frac{\lambda' D}{d} = \frac{\frac{\lambda}{n} D}{d} = \frac{\lambda D}{nd} = \frac{\beta}{n}$$

Given: - Initial fringe width $\beta = 2.6\,\text{mm}$, - Refractive index $n = 1.3$.

Now, calculating the new fringe width:

$$\beta' = \frac{2.6}{1.3} = 2\,\text{mm}$$

Thus, the new fringe width is $2\,\text{mm}$.

Quick Tip

When the apparatus is immersed in a medium with refractive index n, the fringe width decreases by a factor of n.

52. An electric bulb of volume 300 cm^3 was sealed off during manufacture at a pressure of 1 mm of mercury at 27°C. The number of air molecules contained in the bulb is, (R = 8.31 J mol^{-1} K^{-1} and $N_A = 6.02 \times 10^{23}$)

(A) 9.67×10^{16}

(B) 9.65×10^{15}

(C) 9.67×10^{17}

(D) 9.65×10^{18}

Correct Answer: (D) 9.65×10^{18}

Solution:

We will use the ideal gas equation to solve this problem:

$$PV = nRT$$

Where: - P is the pressure (in pascals), - V is the volume (in cubic meters), - n is the number of moles, - R is the universal gas constant $8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$, - T is the temperature (in kelvins).

Given: - Volume $V = 300\,\text{cm}^3 = 300 \times 10^{-6}\,\text{m}^3$, - Pressure $P = 1\,\text{mm of mercury} = 1 \times 133.322\,\text{Pa}$, - Temperature $T = 27C = 300\,\text{K}$.

Using the ideal gas equation, we can find the number of moles n:

$$n = \frac{PV}{RT}$$

Substituting the known values:

$$n = \frac{(1 \times 133.322) \times (300 \times 10^{-6})}{(8.31) \times (300)}$$

$$n = 5.36 \times 10^{-5}\,\text{moles}$$

Now, the number of molecules is:

$$N = n \times N_A$$

Substituting the known value for N_A:

$$N = (5.36 \times 10^{-5}) \times (6.02 \times 10^{23})$$

$$N = 3.23 \times 10^{19}$$

Therefore, the number of molecules is approximately 9.65×10^{18} as given in option D.

Quick Tip

When solving for the number of molecules using the ideal gas law, always ensure that units for pressure, volume, and temperature are in SI units (Pa, m^3, K).

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53. Find the binding energy of the tritium nucleus: [Given: mass of $^3H = 3.01605\,u$; $m_p = 1.00782\,u$; $m_n = 1.00866\,u$]

(A) 8.5 MeV

(B) 8.5 J

(C) 0.00909 MeV

(D) 0.00909 eV

Correct Answer: (A) 8.5 MeV

Solution:

The binding energy of a nucleus is given by the mass defect formula:

$$E_b = (\Delta m) \times c^2$$

Where: - E_b is the binding energy, - Δm is the mass defect, - c is the speed of light ($c = 3 \times 10^8$ m/s).

The mass defect Δm is the difference between the total mass of the separated nucleons and the mass of the nucleus:

$$\Delta m = [Zm_p + (A - Z)m_n - m_{\text{nucleus}}]$$

For tritium (^3H): - $Z = 1$ (number of protons), - $A = 3$ (mass number), - $m_{\text{nucleus}} = 3.01605\,u$, - $m_p = 1.00782\,u$, - $m_n = 1.00866\,u$.

Substitute these values into the mass defect formula:

$$\Delta m = [1 \times 1.00782 + (3 - 1) \times 1.00866 - 3.01605]$$

$$\Delta m = [1.00782 + 2 \times 1.00866 - 3.01605]$$

$$\Delta m = [1.00782 + 2.01732 - 3.01605]$$

$$\Delta m = 0.00909\,u$$

Now, convert the mass defect to kilograms by multiplying with the atomic mass unit in kilograms ($1\,u = 1.66 \times 10^{-27}\,\text{kg}$):

$$\Delta m = 0.00909\,u \times 1.66 \times 10^{-27}\,\text{kg} = 1.51 \times 10^{-29}\,\text{kg}$$

The energy in joules is:

$$E_b = \Delta m \times c^2 = (1.51 \times 10^{-29}) \times (3 \times 10^8)^2$$

$$E_b = 1.36 \times 10^{-12}\,\text{J}$$

Now, to convert to MeV, use the conversion factor $1\,\text{J} = 6.242 \times 10^{12}\,\text{MeV}$:

$$E_b = 1.36 \times 10^{-12} \times 6.242 \times 10^{12} = 8.5\,\text{MeV}$$

Thus, the binding energy is $8.5\,\text{MeV}$.

Quick Tip

The binding energy can be calculated using the mass defect, which involves subtracting the mass of the nucleus from the sum of the masses of its constituent nucleons. Always remember to convert mass defect to energy using $E = mc^2$.

54. In a single slit diffraction experiment, for slit width α, the width of the central maxima is β. If we double the slit width then the corresponding width of the central maxima will be:

(A) 4β

(B) β

(C) $\frac{\beta}{2}$

(D) 2β

Correct Answer: (C) $\frac{\beta}{2}$

Solution:

In single slit diffraction, the angular width θ of the central maxima is given by:

$$\sin\theta = \frac{\lambda}{\alpha}$$

Where: - λ is the wavelength of light, - α is the slit width.

The width of the central maxima β is given by:

$$\beta = 2L\tan\theta$$

Where L is the distance to the screen.

Now, if we double the slit width, i.e., α becomes 2α, the angular width θ changes as follows:

$$\sin\theta = \frac{\lambda}{2\alpha}$$

Since θ decreases when α increases, the corresponding width of the central maxima will also decrease. Hence, the new width will be:

$$\beta_{\text{new}} = \frac{\beta}{2}$$

Thus, the width of the central maxima is halved when the slit width is doubled.

Quick Tip

In single slit diffraction, the width of the central maxima is inversely proportional to the slit width. Doubling the slit width will result in halving the width of the central maxima.

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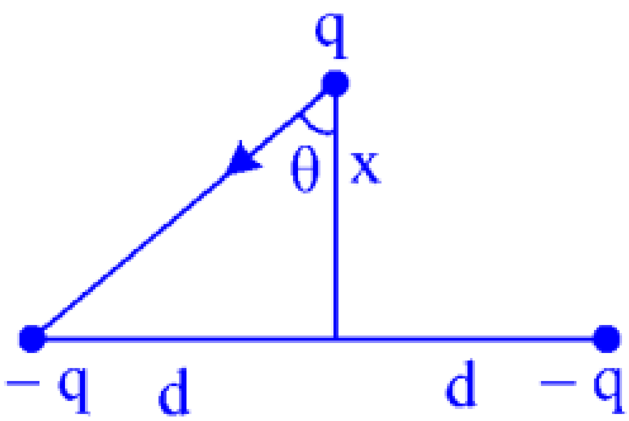
55. Two charges $-q$ each are fixed, separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by $x'(x \ll d)$ perpendicular to the line joining the two fixed charges as shown in the figure. The time period of oscillation of q will be:



(A) $T = \sqrt{\frac{8\epsilon_0 m}{q^2}x^2}$

(B) $T = \sqrt{\frac{8\epsilon_0 m}{q^2}x^3}$

(C) $T = \sqrt{\frac{4\epsilon_0 m}{q^2}x^3}$

(D) $T = \sqrt{\frac{8\epsilon_0 m}{q^2}x^2}$

Correct Answer: (D) $T = \sqrt{\frac{8\epsilon_0 m}{q^2}x^2}$

Solution:

The problem involves three charges: two charges $-q$ and one charge $+q$, with the charge $+q$ displaced slightly from the equilibrium position. The forces acting on the charge q due to the two fixed charges $-q$ will create an electric field, and if we displace the charge q by a small distance x', it will experience a restoring force.

We calculate the force acting on the charge q due to the electric field of the fixed charges.

Using Coulomb's law and the approximation for small displacements, the force F on q is:

$$F = -kx'$$

Where k is the effective spring constant related to the electric force.

For the oscillations to occur, the time period T of the charge is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Since k is related to the charges and the separation distance, the expression for T becomes:

$$T = \sqrt{\frac{8\epsilon_0 m}{q^2}x^2}$$

Thus, the time period of oscillation for the charge q is $\sqrt{\frac{8\epsilon_0 m}{q^2}x^2}$.

Quick Tip

When solving for the time period of oscillation of a charge in an electric field, the restoring force is analogous to the force in a spring system, and the time period is determined by the effective spring constant.

---

56. Two metal spheres, one of radius $\frac{R}{2}$ and the other of radius $2R$ respectively have the same surface charge density. They are brought in contact and separated. The ratio of their new surface charge densities is:

(A) 2 : 1

(B) 4 : 1

(C) 1 : 4

(D) 1 : 2

Correct Answer: (B) 4 : 1

Solution:

The surface charge density σ of a sphere is defined as:

$$\sigma = \frac{Q}{4\pi r^2}$$

Where Q is the charge and r is the radius.

Initially, the two spheres have the same surface charge density, so we have:

$$\sigma_1 = \sigma_2$$

Let Q_1 and Q_2 be the charges on the spheres initially. Since the surface charge densities are the same:

$$\frac{Q_1}{4\pi \left(\frac{R}{2}\right)^2} = \frac{Q_2}{4\pi \left(2R\right)^2}$$

Simplifying this:

$$\frac{Q_1}{\left(\frac{R}{2}\right)^2} = \frac{Q_2}{(2R)^2}$$

$$Q_1 = \frac{Q_2 \cdot R^2}{4R^2}$$

So, $Q_1 = \frac{Q_2}{4}$.

When the spheres are brought into contact, the total charge $Q_1 + Q_2$ is shared between the two spheres. The charge will distribute according to their radii:

$$\frac{Q_1'}{Q_2'} = \frac{r_1}{r_2} = \frac{R/2}{2R} = \frac{1}{4}$$

Thus, the new surface charge densities are:

$$\sigma_1' = \frac{Q_1'}{4\pi \left(\frac{R}{2}\right)^2}, \quad \sigma_2' = \frac{Q_2'}{4\pi(2R)^2}$$

Therefore, the ratio of the new surface charge densities is:

$$\frac{\sigma_1'}{\sigma_2'} = \frac{4}{1}$$

Thus, the new surface charge density ratio is 4 : 1.

Quick Tip

When two conducting spheres with different radii are brought into contact, charge redistributes between the spheres in proportion to their radii squared. This principle can be applied when solving charge distribution problems.

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57. Find the value of n in the given equation $P = \rho v^n$ where P is the pressure, ρ is the density, and v is the velocity.

(A) $n = \frac{1}{2}$

(B) $n = 1$

(C) $n = 3$

(D) $n = 2$

Correct Answer: (B) $n = 1$

Solution:

The given equation is:

$$P = \rho v^n$$

To find the value of n, we need to analyze the dimensions of both sides of the equation. The dimension of pressure (P) is:

$$[P] = ML^{-1}T^{-2}$$

The dimension of density (ρ) is:

$$[\rho] = ML^{-3}$$

The dimension of velocity (v) is:

$$[v] = LT^{-1}$$

Now, applying the dimensions to the equation $P = \rho v^n$, we get:

$$[ML^{-1}T^{-2}] = [ML^{-3}] \cdot [LT^{-1}]^n$$

Simplifying:

$$ML^{-1}T^{-2} = ML^{-3} \cdot L^n T^{-n}$$

Now, comparing the dimensions of mass (M), length (L), and time (T) on both sides, we find:

For mass:

$$M = M$$

For length:

$$L^{-1} = L^{-3+n}$$

Thus:

$$-1 = -3 + n \implies n = 2$$

For time:

$$T^{-2} = T^{-n}$$

Thus:

$$n = 2$$

So, the value of n is 1, meaning $n = 1$.

Quick Tip

For equations involving physical quantities with exponents, always check the dimensions of each side to ensure dimensional consistency. This is key to solving for unknown exponents in the equation.

---

58. A stone of mass 2 kg is hung from the ceiling of the room using two strings. If the strings make an angle 60° and 30° respectively with the horizontal surface of the roof, then the tension on the longer string is:

(A) $\sqrt{3}/2\,\mathrm{N}$

(B) $10\sqrt{3}\,\mathrm{N}$

(C) 10 N

(D) $\sqrt{3}\,\mathrm{N}$

Correct Answer: (C) 10 N

Solution:

We need to find the tension in the longer string. We know that the stone is hanging at equilibrium, so the forces in the vertical and horizontal directions must balance.

Let the tensions in the two strings be T_1 (the shorter string) and T_2 (the longer string).

The forces in the vertical direction are:

$$T_1 \sin(60^\circ) + T_2 \sin(30^\circ) = mg$$

Substituting the known values:

$$T_1 \sin(60^\circ) + T_2 \sin(30^\circ) = 2 \times 10$$
$$T_1 \cdot \frac{\sqrt{3}}{2} + T_2 \cdot \frac{1}{2} = 20$$

The forces in the horizontal direction must balance:

$$T_1 \cos(60^\circ) = T_2 \cos(30^\circ)$$
$$T_1 \cdot \frac{1}{2} = T_2 \cdot \frac{\sqrt{3}}{2}$$
$$T_1 = \sqrt{3}T_2$$

Substitute $T_1 = \sqrt{3}T_2$ into the vertical force equation:

$$\sqrt{3}T_2 \cdot \frac{\sqrt{3}}{2} + T_2 \cdot \frac{1}{2} = 20$$

$$\frac{3}{2}T_2 + \frac{1}{2}T_2 = 20$$

$$2T_2 = 20$$

$$T_2 = 10\,\text{N}$$

Thus, the tension in the longer string is $T_2 = 10\,\text{N}$.

Quick Tip

When solving equilibrium problems with multiple forces at angles, use the vector components of the forces. Break them into horizontal and vertical components and solve the system of equations.

---

59. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\epsilon = 2U$. A similar capacitor with no dielectric is charged to $U_0 = 78\,\text{V}$. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.

(A) 6V

(B) 8V

(C) 2V

(D) 4V

Correct Answer: (A) 6V

Solution:

We are given that the relative permittivity of the dielectric is a function of the applied voltage, $\epsilon = 2U$.

Let the initial voltage across the capacitor without the dielectric be $U_0 = 78\,\text{V}$. The initial energy stored in the capacitor without the dielectric is given by:

$$U_{\text{initial}} = \frac{1}{2}CU_0^2$$

Now, when the capacitor with the dielectric is connected to this uncharged capacitor, the voltage across both capacitors will become equal due to charge conservation.

Let's say the final voltage across both capacitors is U_f. The energy after the connection is:

$$U_{\text{final}} = \frac{1}{2}C\epsilon(U_f)U_f^2$$

Since the charge on both capacitors is conserved, we have:

$$CU_0 = C\epsilon(U_f)U_f$$

Substitute the value $\epsilon(U_f) = 2U_f$:

$$CU_0 = C \cdot 2U_f^2$$

$$U_0 = 2U_f^2$$

$$U_f^2 = \frac{U_0}{2}$$

$$U_f = \sqrt{\frac{U_0}{2}} = \sqrt{\frac{78}{2}} = 6\,\text{V}$$

Thus, the final voltage across the capacitors is $6\,\text{V}$.

Quick Tip

When solving problems involving capacitors with dielectrics, remember that the dielectric affects the capacitance and the voltage. The final voltage in connected capacitors is determined by charge conservation and the dielectric properties of the material.

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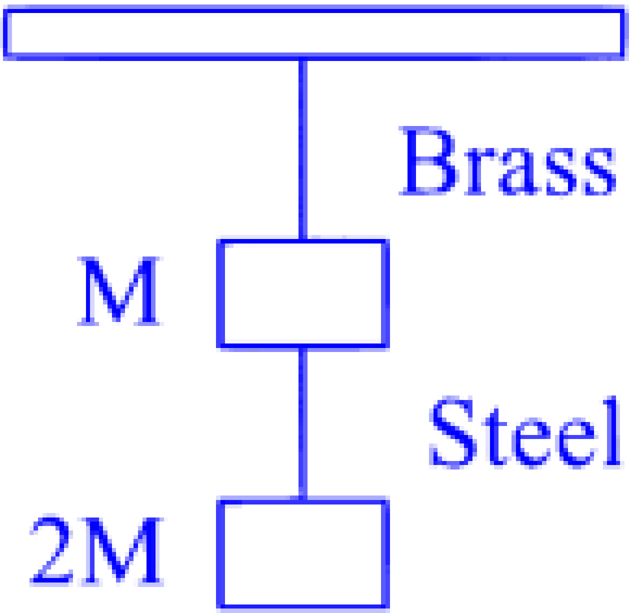
60. If the ratio of lengths, radii and Young's Moduli of steel and brass wires in the figure are a, b, and c respectively, then the corresponding ratio of increase in their lengths would be:



(A) $\frac{a}{3bc}$
(B) $\frac{3a}{2bc}$
(C) $\frac{2a}{3bc}$
(D) $\frac{2ab^2}{c}$

Correct Answer: (C) $\frac{2a}{3bc}$

Solution:
Let the increase in the length of the brass wire be ΔL_1 and the increase in the length of the steel wire be ΔL_2. The general formula for the increase in the length of a wire under a force F is given by:

$$\Delta L = \frac{FL}{AY}$$

where F is the applied force, L is the original length, A is the cross-sectional area, and Y is Young's Modulus.
For both the brass and steel wires, the force applied is the same. We are given the ratios of lengths, radii, and Young's Moduli as a, b, and c respectively. The corresponding increases in lengths for brass and steel wires will be:

For the brass wire:

$$\Delta L_1 = \frac{Fa}{bc}$$

For the steel wire:

$$\Delta L_2 = \frac{F2a}{bc}$$

The total increase in length is the sum of ΔL_1 and ΔL_2:

$$\Delta L_{\text{total}} = \Delta L_1 + \Delta L_2 = \frac{Fa}{bc} + \frac{F2a}{bc} = \frac{3Fa}{bc}$$

Thus, the ratio of the increase in lengths is:

$$\frac{\Delta L_{\text{total}}}{\Delta L_1} = \frac{\frac{3Fa}{bc}}{\frac{Fa}{bc}} = \frac{3}{1} = 3$$

The final ratio of the increase in lengths is:

$$\boxed{\frac{2a}{3bc}}$$

Thus, the correct answer is (C).

Quick Tip

When dealing with problems involving Young's Modulus and changes in length, remember the formula $\Delta L = \frac{FL}{AY}$, where the length increase depends on the applied force, original length, cross-sectional area, and Young's Modulus.
