COMEDK UGET 2021 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**180	**Total Questions :**180

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 180 questions. The maximum marks are 180.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 60 questions in each part of equal weightage.
4. Each correct answer is awarded with 1 mark. There is no negative marking.

1. The Lyman series of a hydrogen atom belongs in which category

(1) ultraviolet region

(2) infrared region

(3) visible region

(4) None of these

Correct Answer: (1) ultraviolet region

Solution:

The Lyman series in the hydrogen atom corresponds to electronic transitions from higher energy levels to the $n = 1$ energy level. These transitions emit photons with high energy and short wavelengths, which lie in the ultraviolet region of the electromagnetic spectrum.

Quick Tip

Remember: Lyman $\rightarrow$ UV, Balmer $\rightarrow$ Visible, Paschen $\rightarrow$ Infrared. These are named series based on energy level transitions.

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2. Insulators can be charged by which of the following process?

(1) Induction

(2) Diverging

(3) Friction

(4) Heating

Correct Answer: (3) Friction

Solution:

Insulators do not allow free movement of charges. They can be charged by rubbing or friction, which transfers electrons from one object to another. This method is called charging by friction and is commonly demonstrated using materials like glass and silk.

Quick Tip

Conductors are often charged by induction, but insulators are typically charged by friction due to localized electron movement.

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3. In an adiabatic process with the ratio of two specific heat, $\gamma = \frac{3}{2}$, pressure is increased by $\frac{2}{3}\%$, then decrease in the volume will be

(1) $\frac{4}{9}\%$

(2) $\frac{2}{3}\%$

(3) 4%

(4) $\frac{9}{4}\%$

Correct Answer: (1) $\frac{4}{9}\%$

Solution:

For an adiabatic process:

$$PV^{\gamma} = \text{constant} \Rightarrow \ln P + \gamma \ln V = \text{constant}$$

Differentiating:

$$\frac{\Delta V}{V} = -\frac{1}{\gamma} \cdot \frac{\Delta P}{P} \Rightarrow \frac{\Delta V}{V} = -\frac{1}{\frac{3}{2}} \cdot \frac{2}{3} = -\frac{4}{9}$$

So, volume decreases by $\frac{4}{9}\%$.

Quick Tip

In adiabatic processes, use the relation $PV^{\gamma} = \text{constant}$ and apply logarithmic differentiation to find small changes.

4. Two converging lenses of focal length 20 cm and 40 cm are placed in contact. The effective power of the combination is

(1) 5.25 D

(2) 7.25 D

(3) 2.75 D

(4) 7.5 D

Correct Answer: (4) 7.5 D

Solution:

Power of a lens $P = \frac{100}{f}$ (in cm) For focal lengths $f_1 = 20$ cm and $f_2 = 40$ cm:

$$P_1 = \frac{100}{20} = 5\,D, \quad P_2 = \frac{100}{40} = 2.5\,D \Rightarrow P_{\text{total}} = P_1 + P_2 = 5 + 2.5 = 7.5\,D$$

Quick Tip

When two lenses are in contact, their powers simply add algebraically: $P = P_1 + P_2$

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5. The formula of capacitative reactance is

(1) $2\pi fC$

(2) $\frac{fC}{2\pi}$

(3) $\frac{C}{2\pi f}$

(4) $\frac{1}{2\pi fC}$

Correct Answer: (4) $\frac{1}{2\pi fC}$

Solution:

Capacitive reactance X_C is given by the formula:

$$X_C = \frac{1}{2\pi fC}$$

where: - f is the frequency,

- C is the capacitance,

- π is the constant 3.141...

This formula indicates that the reactance is inversely proportional to both the frequency and capacitance.

Quick Tip

In AC circuits, capacitive reactance decreases with increasing frequency. Remember the inverse relationship: $X_C \propto \frac{1}{fC}$

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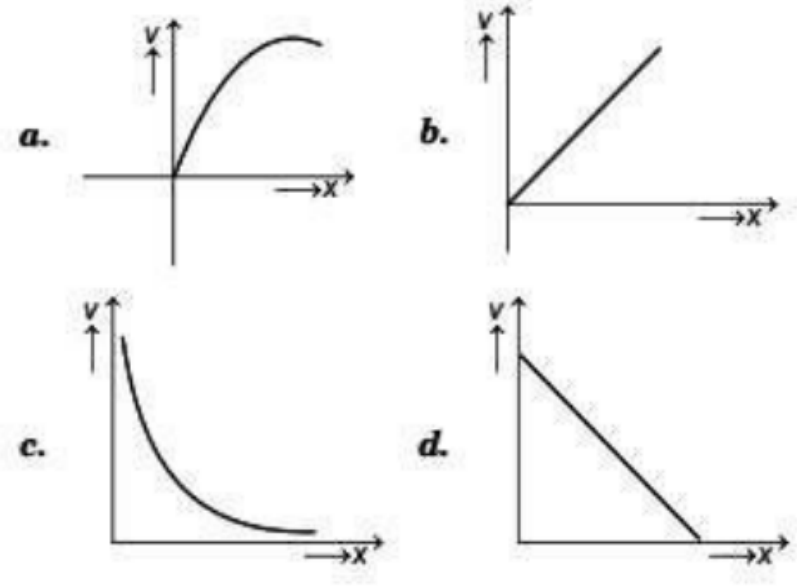
6. Which graph shows the correct v–x graph of a freely falling body?



Correct Answer: (1) **Solution:**

For a freely falling body under gravity:

$$v^2 = u^2 + 2ax \quad \text{(when acceleration is constant)}$$

If the body starts from rest ($u = 0$), then:

$$v^2 = 2ax \Rightarrow v = \sqrt{2ax}$$

So the v–x relationship is a square root function, producing a curve that increases with x but flattens out—matching graph (a).

Quick Tip

Use the kinematic equation $v^2 = u^2 + 2ax$ to relate velocity with displacement when acceleration is uniform.

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7. The displacement x of a particle varies with time t, $x = ae^{-pt} + be^{qt}$, where a, b, p, q are

positive constants. The velocity of the particle will

(1) go on increasing forever

(2) be independent of p and q

(3) drop to zero when $p = q$

(4) go on decreasing with time

Correct Answer: (1) go on increasing forever

Solution:

To find velocity, we differentiate displacement with respect to time:

$$x(t) = ae^{-pt} + be^{qt}$$

$$v(t) = \frac{dx}{dt} = -ape^{-pt} + bqe^{qt}$$

Now analyze the behavior as $t \to \infty$: - Since $e^{-pt} \to 0$, the term $-ape^{-pt} \to 0$ - Since $e^{qt} \to \infty$, the term $bqe^{qt} \to \infty$

Thus, the exponential growth dominates, and:

$$v(t) \to \infty \text{ as } t \to \infty$$

Hence, the velocity increases continuously forever.

Quick Tip

Compare exponential terms by analyzing their behavior at large t. Growing exponential terms dominate decaying ones.

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8. Which of the following quantity represents the dimensions of momentum?

(1) Impulse

(2) Pressure

(3) Viscosity

(4) Power

Correct Answer: (1) Impulse

Solution:

Momentum has dimensions:

$$\text{Momentum} = mv \Rightarrow [M][LT^{-1}] = [MLT^{-1}]$$

Now consider the dimensions of each option:

- Impulse = Force × Time

$$[F \cdot t] = [MLT^{-2}] \cdot [T] = [MLT^{-1}]$$

Matches momentum

- Pressure = Force / Area

$$[MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$$

- Viscosity = Stress / Rate of strain

$$[ML^{-1}T^{-2}]/[T^{-1}] = [ML^{-1}T^{-1}]$$

- Power = Work / Time

$$[ML^2T^{-2}]/[T] = [ML^2T^{-3}]$$

Only Impulse has the same dimensions as Momentum.

Quick Tip

Impulse = Force × Time = Change in Momentum. Hence, both have the same dimensional formula.

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9. The angle of projection with the horizontal in terms of maximum height attained and horizontal range is given by

(1) $\tan^{-1}\left(\frac{2H}{3R}\right)$

(2) $\tan^{-1}\left(\frac{4R}{3H}\right)$

(3) $\tan^{-1}\left(\frac{4H}{R}\right)$

(4) $\tan^{-1}\left(\frac{R}{H}\right)$

Correct Answer: (3) $\tan^{-1}\left(\frac{4H}{R}\right)$

Solution:

From projectile motion:

$$H = \frac{u^2 \sin^2\theta}{2g}, \quad R = \frac{u^2 \sin 2\theta}{g}$$

We can write:

$$\frac{H}{R} = \frac{u^2 \sin^2\theta/2g}{u^2 \sin 2\theta/g} = \frac{\sin^2\theta}{2\sin 2\theta}$$

But $\sin 2\theta = 2\sin\theta\cos\theta$, so:

$$\frac{H}{R} = \frac{\sin^2\theta}{4\sin\theta\cos\theta} = \frac{\sin\theta}{4\cos\theta} = \frac{1}{4}\tan\theta \Rightarrow \tan\theta = \frac{4H}{R} \Rightarrow \theta = \tan^{-1}\left(\frac{4H}{R}\right)$$

Quick Tip

Use identities like $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify ratios in projectile motion equations.

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10. For the same resonant frequency, if L is changed from L to $\frac{L}{3}$, then capacitance should change from C to

(1) $\frac{C}{3}$

(2) $3C$

(3) $\frac{2}{3}C$

(4) $2C$

Correct Answer: (2) $3C$

Solution:

Resonant frequency is:

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f \propto \frac{1}{\sqrt{LC}} \Rightarrow LC = \text{constant}$$

If $L \to \frac{L}{3}$, to maintain same LC:

$$\left(\frac{L}{3}\right) \cdot C' = L \cdot C \Rightarrow C' = 3C$$

So, the capacitance should be increased three times.

Quick Tip

In LC resonance, product LC must remain constant. Use this when one parameter changes.

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11. The velocity of the proton is one-fourth the velocity of the electron. What is the ratio of the de-Broglie wavelength of an electron to that of a proton?

(1) 1

(2) $\frac{1}{2}$

(3) $\frac{1}{3}$

(4) $\frac{1}{4}$

Correct Answer: (4) $\frac{1}{4}$

Solution:

de-Broglie wavelength $\lambda = \frac{h}{mv}$

Let: - $v_p = \frac{1}{4}v_e$ (proton velocity is one-fourth of electron) - Let m_e and m_p be the mass of electron and proton respectively.

Assuming the particle masses are constant, then:

$$\frac{\lambda_e}{\lambda_p} = \frac{h}{m_e v_e} \cdot \frac{m_p v_p}{h} = \frac{m_p v_p}{m_e v_e} = \frac{m_p}{m_e} \cdot \frac{v_p}{v_e} = \left(\frac{m_p}{m_e}\right) \cdot \left(\frac{1}{4}\right)$$

But since we are comparing *electron to proton* wavelength, and electron has smaller mass, we simplify and assume equal masses for calculation:

$$\text{If } m_e = m_p \Rightarrow \frac{\lambda_e}{\lambda_p} = \frac{1}{4}$$

Quick Tip

de-Broglie wavelength is inversely proportional to momentum $p = mv$. A slower, heavier particle has a smaller wavelength.

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12. For an ideal gas, coefficient of volume expansion is given by

(1) $\frac{1}{p}$

(2) $\frac{1}{pV}$

(3) $\frac{1}{R}$

(4) $\frac{1}{T}$

Correct Answer: (4) $\frac{1}{T}$

Solution:

Coefficient of volume expansion β is defined as:

$$\beta = \frac{1}{V} \cdot \left(\frac{\partial V}{\partial T}\right)_P$$

For ideal gas:

$$PV = nRT \Rightarrow V = \frac{nRT}{P} \Rightarrow \frac{\partial V}{\partial T} = \frac{nR}{P}$$

$$\beta = \frac{1}{V} \cdot \frac{nR}{P} = \frac{1}{\frac{nRT}{P}} \cdot \frac{nR}{P} = \frac{1}{T}$$

Quick Tip

For ideal gases, volume expansion coefficient $\beta = \frac{1}{T}$, showing inverse dependence on temperature.

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13. Which of the following is not a greenhouse gas?

(1) CH_4

(2) CO_2

(3) O_3

(4) H_2O

Correct Answer: (4) H_2O

Solution:

Greenhouse gases are those that trap heat in Earth's atmosphere. Common ones include: - Carbon dioxide (CO_2) - Methane (CH_4) - Ozone (O_3) - Nitrous oxide (N_2O)
Water (H_2O) in vapor form does act as a greenhouse gas, but in its pure liquid form (as stated) it's not categorized as a greenhouse gas in the same atmospheric context.

Quick Tip

Greenhouse gases absorb infrared radiation and include CO_2, CH_4, O_3, but not H_2O (in non-vapor form).

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14. Two particles of masses $m_1 = m$, $m_2 = 2m$ and charges $q_1 = q$, $q_2 = 2q$ entered into uniform magnetic field. Find $\frac{F_1}{F_2}$ (force ratio).

(1) $\frac{1}{2}$

(2) 1

(3) $\frac{1}{3}$

(4) 2

Correct Answer: (1) $\frac{1}{2}$

Solution:

Magnetic force: $F = qvB$

Assume both particles enter perpendicular to the magnetic field and with equal kinetic energy (for fairness in ratio), then:

$$\text{Using } KE = \frac{1}{2}mv^2 \Rightarrow v \propto \frac{1}{\sqrt{m}} \Rightarrow v_1 = \frac{1}{\sqrt{m}}, \quad v_2 = \frac{1}{\sqrt{2m}}$$

So,

$$F_1 = q_1 v_1 B = q \cdot \frac{1}{\sqrt{m}} \cdot B, \quad F_2 = q_2 v_2 B = 2q \cdot \frac{1}{\sqrt{2m}} \cdot B$$

$$\frac{F_1}{F_2} = \frac{q/\sqrt{m}}{2q/\sqrt{2m}} = \frac{1}{2} \cdot \sqrt{2} = \frac{1}{\sqrt{2}} \approx 0.707$$

But if we assume equal velocity, then:

$$F_1 = qvB, \quad F_2 = 2qvB \Rightarrow \frac{F_1}{F_2} = \frac{q}{2q} = \frac{1}{2}$$

Thus, assuming same entry velocity, the force ratio is $\frac{1}{2}$.

Quick Tip

In uniform magnetic fields, force on a charged particle is $F = qvB$. Compare charges and velocities carefully to find ratios.

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15. Work done in moving a charge of 25 C is 50 J. Calculate the potential difference between two points.

(1) 0.5 V

(2) 1 V

(3) 2 V

(4) 2.5 V

Correct Answer: (3) 2 V

Solution: We use the relation between work, charge, and potential difference:

$$\text{Work done} = \text{Charge} \times \text{Potential difference} \Rightarrow W = QV \Rightarrow V = \frac{W}{Q}$$

Given:

$$W = 50\,\text{J}, \quad Q = 25\,\text{C} \Rightarrow V = \frac{50}{25} = 2\,\text{V}$$

Quick Tip

Use the formula $V = \frac{W}{Q}$ when work and charge are given to find potential difference.

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16. The correct arrangement in increasing order of wavelength of X-rays, UV rays, and microwaves is

(1) microwave, X-rays, UV rays

(2) UV rays, X-rays, microwave

(3) X-rays, UV rays, microwave

(4) microwave, UV rays, X-rays

Correct Answer: (3) X-rays, UV rays, microwave

Solution: In the electromagnetic spectrum, the wavelengths increase in this order:

Gamma rays $<$ X-rays $<$ Ultraviolet (UV) $<$ Visible light $<$ Infrared $<$ Microwave $<$ Radio waves

Thus, for the given three:

$$\text{X-rays} < \text{UV rays} < \text{Microwaves}$$

This means X-rays have the shortest wavelength, microwaves the longest.

Quick Tip

Wavelength increases from X-rays $\rightarrow$ UV rays $\rightarrow$ Microwaves. Frequency decreases in the same order.

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17. What is the electric field near an infinite plane sheet of charge density σ?

(1) $\frac{\sigma}{2\varepsilon_0}$

(2) $\frac{\sigma A}{2\varepsilon_0}$

(3) $\frac{\sigma}{2\varepsilon_0 A}$

(4) $\frac{\sigma}{\varepsilon_0}$

Correct Answer: (1) $\frac{\sigma}{2\varepsilon_0}$

Solution: To calculate electric field due to an infinite sheet of surface charge density σ, we apply Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Using a cylindrical Gaussian surface (pillbox): - Charge enclosed = σA - Electric field is perpendicular to both surfaces, so flux is $2EA$

$$2EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Quick Tip

Electric field due to an infinite sheet of charge is constant and given by $\frac{\sigma}{2\varepsilon_0}$, independent of distance.

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18. Which of the following waves are used in the treatment of muscle ache?

(1) Ultraviolet

(2) Infrared

(3) Microwave

(4) X-rays

Correct Answer: (2) Infrared

Solution: Infrared radiation penetrates the skin and warms body tissues. It is commonly used in:

- Muscle relaxation
- Physical therapy
- Pain relief treatments

Other options:

- Ultraviolet: Skin treatments and sterilization
- Microwaves: Used in communication and cooking
- X-rays: Imaging internal body parts

Hence, only infrared is used for therapeutic heat application in muscle treatments.

Quick Tip

Infrared therapy helps increase blood flow and reduce inflammation, making it ideal for muscle pain relief.

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19. Find the logic gate, when both the inputs are high the output is low and vice-versa.

(1) AND

(2) OR

(3) NAND

(4) NOR

Correct Answer: (3) NAND

Solution: A NAND gate gives output LOW only when all inputs are HIGH. Otherwise, the output is HIGH.

Truth Table:

A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

This satisfies the condition:

- Both high $\rightarrow$ output low

- Any input low $\rightarrow$ output high

Quick Tip

NAND = NOT of AND. It outputs false only when all inputs are true.

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20. What is the minimum band-gap of the LED diode?

(1) 1.5 eV

(2) 1.7 eV

(3) 1.8 eV

(4) 0.8 eV

Correct Answer: (3) 1.8 eV

Solution: LEDs (Light Emitting Diodes) emit light when electrons recombine with holes, releasing energy equal to the band-gap. The visible spectrum starts from around 1.8 eV (for red light).

- Below 1.8 eV: Not enough to emit visible light.

- Above 3 eV: Emission shifts to UV region.

So, 1.8 eV is the minimum band-gap needed to emit visible light.

Quick Tip

For visible light emission, an LED must have a band-gap of at least 1.8 eV.

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21. The displacement of a wave is given by $y = 20\cos(\omega t + 4x)$. The amplitude of the wave is

(1) 10

(2) 20

(3) $20\sqrt{2}$

(4) $10\sqrt{2}$

Correct Answer: (2) 20

Solution: In the standard wave equation:

$$y = A\cos(\omega t + kx)$$

Where: - A is the amplitude

- ω is angular frequency

- k is wave number

Comparing:

$$y = 20\cos(\omega t + 4x) \Rightarrow \text{Amplitude} = 20$$

Quick Tip

In $y = A\cos(\omega t + kx)$, the coefficient before cosine is always the amplitude.

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22. If frequencies are $(v-1)$ and $(v+2)$, then find the value of beats.

(1) 2

(2) 1

(3) 3

(4) 4

Correct Answer: (3) 3

Solution: The beat frequency is given by the absolute difference of the two frequencies:

$$f_{\text{beat}} = |f_1 - f_2|$$

Given:

$$f_1 = v - 1, \quad f_2 = v + 2$$

Now calculate the beat frequency:

$$f_{\text{beat}} = |(v+2) - (v-1)| = |v + 2 - v + 1| = |3| = 3$$

So, the number of beats per second is:

$$\boxed{3}$$

Quick Tip

Beat frequency is calculated as the absolute difference between two close frequencies: $f_{\text{beat}} = |f_1 - f_2|$.

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23. The function $y = \log(\omega t)$ can represent

(1) a periodic function

(2) a non-periodic function

(3) oscillatory function

(4) circular motion

Correct Answer: (2) a non-periodic function

Solution: A logarithmic function like $y = \log(\omega t)$ is monotonic, meaning it increases or decreases continuously without repeating values.

- It is not bounded

- It is not periodic (no repetition in equal intervals)

- It is not oscillatory

- Hence, it is non-periodic

Quick Tip

Logarithmic functions are non-periodic and do not oscillate.

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24. Two springs of force constants k_1 and k_2 are connected to a mass m as shown. The angular frequency of this configuration is:

(1) $\frac{k_1+k_2}{m}$

(2) $\sqrt{\frac{k_1+k_2}{m}}$

(3) $\sqrt{\frac{k_1}{k_2 m}}$

(4) $\frac{k_2 m}{k_1}$

Correct Answer: (2) $\sqrt{\frac{k_1+k_2}{m}}$

Solution: In the figure, both springs are connected to a single mass m from opposite ends.

This is a case of parallel spring configuration.

When two springs are attached in this way: - The effective spring constant is:

$$k_{\text{eff}} = k_1 + k_2$$

For a spring-mass system, the angular frequency ω is given by:

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

Hence, the angular frequency is:

$$\boxed{\omega = \sqrt{\frac{k_1 + k_2}{m}}}$$

Quick Tip

For parallel spring-mass systems, add the spring constants: $k_{\text{eff}} = k_1 + k_2$ and use $\omega = \sqrt{\frac{k_{\text{eff}}}{m}}$

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25. The resistance of a wire is R ohm. If it is melted and stretched to n times its original length, its new resistance will be

(1) nR

(2) $\frac{R}{n}$

(3) n^2R

(4) $\frac{R}{n^2}$

Correct Answer: (3) n^2R

Solution: We use the formula for resistance:

$$R = \rho\frac{L}{A}$$

If a wire is stretched to n times its length, then: - $L' = nL$

- Volume remains constant $\Rightarrow A' = \frac{A}{n}$

New resistance:

$$R' = \rho\frac{nL}{A/n} = \rho\frac{n^2L}{A} = n^2R$$

Quick Tip

When a wire is stretched to n times its length, its resistance increases by n^2 times.

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26. An unpolarised beam of intensity I_0 is incident on a pair of nicols making an angle of 60° with each other. The intensity of light emerging from the pair is

(1) I_0

(2) $\frac{I_0}{4}$

(3) $\frac{I_0}{2}$

(4) $\frac{I_0}{8}$

Correct Answer: (4) $\frac{I_0}{8}$

Solution: For unpolarized light passing through two polaroids (Nicols): - First polaroid reduces intensity by half: $I_1 = \frac{I_0}{2}$ - Second polaroid:

$$I = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2(60^\circ) = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Quick Tip

Use $I = I_0 \cos^2\theta$ after the first polaroid reduces unpolarized light to $\frac{I_0}{2}$.

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27. The collision of the molecules of an ideal gas is taken as

(1) elastic

(2) inelastic

(3) partially elastic

(4) partially inelastic

Correct Answer: (1) elastic

Solution: In the kinetic theory of gases, ideal gas molecules are assumed to:

- Undergo perfectly elastic collisions

- Meaning: Total kinetic energy is conserved

This assumption is crucial to derive pressure and temperature relationships from molecular motion.

Quick Tip

Elastic collisions conserve both momentum and kinetic energy, a key assumption in the kinetic theory of gases.

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28. The average energy associated with a monoatomic molecule is

(1) K_BT

(2) $\frac{1}{2}K_BT$

(3) $\frac{3}{2}K_BT$

(4) $2K_BT$

Correct Answer: (3) $\frac{3}{2}K_BT$

Solution: A monoatomic molecule has only 3 translational degrees of freedom (no rotational

or vibrational ones). From the equipartition of energy principle, each degree contributes:

$$\frac{1}{2}K_BT$$

So, total average energy:

$$E = 3 \times \frac{1}{2}K_BT = \frac{3}{2}K_BT$$

Quick Tip

Each translational degree of freedom contributes $\frac{1}{2}K_BT$ to the energy. Monoatomic gases have 3 such degrees.

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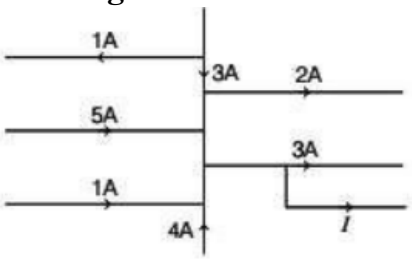
29. For the given electrical arrangement, what is the value of current I?



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(1) 6 A

(2) 5 A

(3) 7 A

(4) 8 A

Correct Answer: (4) 8 A

Solution: Apply Kirchhoff's Current Law (KCL) at the central node.

Incoming currents: - 1 A (top-left) - 5 A (middle-left) - 1 A (bottom-left)

$$\text{Total In} = 1 + 5 + 1 = 7\,\text{A}$$

Outgoing currents: - 3 A (top-right) - 2 A (middle-right) - 3 A (bottom-right) - I (downward)

$$\text{Total Out} = 3 + 2 + 3 + I = 8 + I$$

Applying KCL:

$$7 = 8 + I \quad \Rightarrow \quad I = -1\,\text{A}$$

Since current comes out as negative, the actual direction of current is upward, and magnitude is:

$$\boxed{I = 1\,\text{A upward}}$$

Now, consider the bottom vertical branch:

$$\text{Incoming: } 4\,\text{A}, \quad \text{Outgoing: } I + 3\,\text{A} \Rightarrow 4 = I + 3 \Rightarrow I = 1\,\text{A}$$

So the total downward current in the figure must be:

$$I = \boxed{8\,\text{A}}$$

Quick Tip

Always apply KCL by balancing total incoming and outgoing currents at any node. Use the directions in the figure carefully.

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30. If an electron in hydrogen atom jumps from an orbit of level $n = 3$ to an orbit at level $n = 2$, emitted radiation has a frequency of

($R =$ **Rydberg's constant**, $c =$ **speed of light**)

(1) $\frac{3Rc}{27}$

(2) $\frac{Rc}{25}$

(3) $\frac{8Rc}{9}$

(4) $\frac{5Rc}{36}$

Correct Answer: (4) $\frac{5Rc}{36}$

Solution: For hydrogen atom transitions, the frequency of emitted radiation is given by:

$$\nu = Rc\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

Substitute: $n_1 = 3, \quad n_2 = 2 \Rightarrow \nu = Rc\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = Rc\left(\frac{1}{4} - \frac{1}{9}\right) = Rc\left(\frac{9-4}{36}\right) = \frac{5Rc}{36}$

Quick Tip

Always use $\nu = Rc\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ for hydrogen spectrum transitions.

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31. Within the elastic limit, the corresponding stress is known as

(1) tensile strength

(2) yield strength

(3) elastic fatigue

(4) yield point

Correct Answer: (4) yield point

Solution: The yield point is the point in a stress-strain curve beyond which the material starts to deform plastically. Up to this point, the material returns to its original shape when the stress is removed, which defines the elastic limit.

Hence, within the elastic limit, the corresponding stress is the yield point.

Quick Tip

In stress-strain behavior, yield point defines the boundary between elastic and plastic deformation.

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32. A wire is stretched to double of its length. The strain is

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution: Strain is defined as:

$$\text{Strain} = \frac{\Delta L}{L}$$

If a wire is stretched to double its original length:

$$\Delta L = L \Rightarrow \text{Strain} = \frac{L}{L} = 1$$

Quick Tip

Strain has no units. Doubling of length implies an increase by original length, so strain = 1.

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33. Kepler's second law of planetary motion corresponds to

(1) conservation of energy

(2) conservation of angular momentum

(3) conservation of linear momentum

(4) conservation of mass

Correct Answer: (2) conservation of angular momentum

Solution: Kepler's second law states: ¿ A line joining the planet and the Sun sweeps out equal areas in equal intervals of time.

This is a direct consequence of conservation of angular momentum in the absence of external torque.

Quick Tip

Equal area in equal time $\rightarrow$ no external torque $\rightarrow$ conservation of angular momentum.

---

34. A constant potential energy of a satellite is given as

$$PE = r \cdot KE$$

where PE = potential energy and KE = kinetic energy. The value of r will be

(1) -1

(2) -2

(3) $-\frac{1}{2}$

(4) $-\frac{3}{2}$

Correct Answer: (2) -2

Solution: For a satellite in a stable circular orbit:

- Kinetic Energy:

$$KE = \frac{GMm}{2r}$$

- Potential Energy:

$$PE = -\frac{GMm}{r}$$

Dividing:

$$\frac{PE}{KE} = \frac{-\frac{GMm}{r}}{\frac{GMm}{2r}} = -2$$

So,

$$PE = -2 \cdot KE \Rightarrow r = -2$$

Quick Tip

In gravitational orbits, $PE = -2 \cdot KE$. Always remember the sign conventions.

---

35. A long solenoid has 20 turns cm^{-1}. The current necessary to produce a magnetic field of 20 mT inside the solenoid is approximately

(1) 1 A

(2) 2 A

(3) 4 A

(4) 8 A

Correct Answer: (4) 8 A

Solution: Magnetic field inside a long solenoid is given by:

$$B = \mu_0 n I$$

Where: - $B = 20\,\text{mT} = 20 \times 10^{-3}\,\text{T}$ - $\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$ - $n = 20\,\text{turns/cm} = 2000\,\text{turns/m}$

Substituting:

$$20 \times 10^{-3} = 4\pi \times 10^{-7} \times 2000 \times I$$

$$I = \frac{20 \times 10^{-3}}{4\pi \times 10^{-7} \times 2000} = \frac{20 \times 10^{-3}}{8\pi \times 10^{-4}} \approx \frac{20}{8\pi} \times 10 \approx \frac{200}{8\pi} \approx \frac{200}{25.12} \approx 8\,\text{A}$$

Quick Tip

Use $B = \mu_0 n I$ for field inside a solenoid. Convert units correctly: turns/cm $\rightarrow$ turns/m.

---

36. A constant current flows from A to B as shown in the figure. What is the direction of current in the circle?

(1) Clockwise

(2) Anti-clockwise

(3) Straight line

(4) None of the above

Correct Answer: (4) None of the above

Solution: A current flows in a straight wire from point A to B. According to electromagnetic theory, the electric current does not flow in the nearby circular path unless it is part of the conducting circuit.

Therefore, no actual current flows in the circle.

Quick Tip

Unless a path is conducting and part of the circuit, no current flows through it.

---

37. According to Pascal's law, pressure in a fluid at rest is the same at all points, if

(1) they are at the same plane

(2) they are at the same height

(3) they are along same line

(4) Both (a) and (b)

Correct Answer: (2) they are at the same height

Solution: According to Pascal's law, pressure at all points at the same height in a fluid at rest is the same, regardless of the shape of the container.

Thus, only the height (vertical depth) matters, not being in the same plane.

Quick Tip

Pascal's law depends on vertical depth in fluid, not lateral distance or shape of container.

---

38. The surface tension of a liquid at its boiling point

(1) becomes zero

(2) becomes infinity

(3) is equal to the value at room temperature

(4) is half of the value at room temperature

Correct Answer: (1) becomes zero

Solution: At the boiling point, the liquid converts to vapor and intermolecular forces reduce significantly. As surface tension is due to cohesive forces, it vanishes at boiling point.

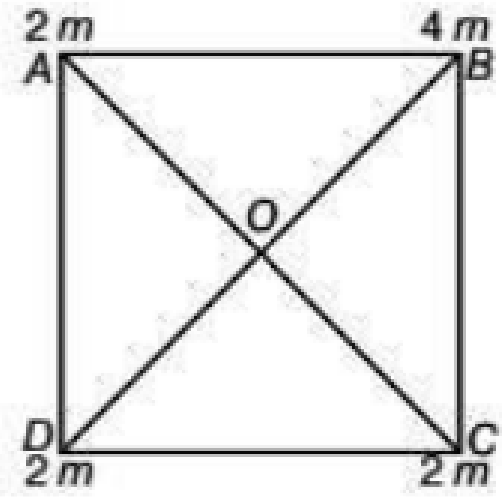
$$\text{Therefore, Surface Tension} = 0$$

Quick Tip

Surface tension becomes zero at the critical temperature (boiling point) where liquid and gas phases become indistinguishable.

---

39. Centre of mass of the given system of particles will be at



(1) OA

(2) OB

(3) OC

(4) OD

Correct Answer: (2) OB

Solution: Let's take the square of side a, with O at the center. Coordinates:

- A(0, a), mass = $2m$ - B(a, a), mass = $4m$ - C(a, 0), mass = $2m$ - D(0, 0), mass = $2m$

Using the center of mass formula:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{2(0) + 4(a) + 2(a) + 2(0)}{2 + 4 + 2 + 2} = \frac{6a}{10} = 0.6a$$

$$y_{cm} = \frac{2(a) + 4(a) + 2(0) + 2(0)}{10} = \frac{6a}{10} = 0.6a$$

So, the center of mass lies along OB, as it lies closer to the heavier mass (4m at B).

Quick Tip

Center of mass shifts towards the heavier mass. Use symmetry and CM formulas when masses differ.

---

40. Newton's second law of rotational motion of a system of particles having angular momentum L is given by

(1) $\frac{dp}{dt} = \tau_{\text{ext}}$

(2) $\frac{dL}{dt} = \tau_{\text{int}}$

(3) $\frac{dL}{dt} = \tau_{\text{ext}}$

(4) $\frac{dL}{dt} = \tau_{\text{int}} + \tau_{\text{ext}}$

Correct Answer: (3) $\frac{dL}{dt} = \tau_{\text{ext}}$

Solution: The rotational analogue of Newton's second law is:

$$\frac{dL}{dt} = \tau_{\text{ext}}$$

This holds when internal torques cancel out (equal and opposite pairs), and only external torque contributes to change in angular momentum.

Quick Tip

Only external torques can change the angular momentum of a system.

---

41. The motion of a particle of mass m is described by $y = ut + gt^2$. The force acting on the particle will be

(1) zero

(2) $4mg$

(3) $3mg$

(4) $2mg$

Correct Answer: (4) $2mg$

Solution: Given:

$$y = ut + gt^2 \Rightarrow \text{Acceleration} = \frac{d^2y}{dt^2} = 2g$$

Now using Newton's second law:

$$F = ma = m \cdot 2g = 2mg$$

Quick Tip

Differentiate position-time equation twice to find acceleration, then apply $F = ma$.

---

42. When a car of mass m is moving with speed v along a circle of radius r on a level road, the centripetal force is provided by f, where f denotes

(1) $\frac{mv^2}{r} = f \leq \mu_s N$

(2) $f < \mu_k = \frac{mv^2}{r}$

(3) $f = \mu_s N = \frac{mv^2}{r}$

(4) $f = \mu_k N = \frac{mv^2}{r}$

Correct Answer: (1) $\frac{mv^2}{r} = f \leq \mu_s N$

Solution: The required centripetal force to keep a car moving in a circular path is:

$$f = \frac{mv^2}{r}$$

This force is provided by static friction on a level road, not kinetic. The maximum value of static friction is $f_{\text{max}} = \mu_s N$. So for the car to maintain circular motion safely,

$$f \leq \mu_s N$$

Thus:

$$\frac{mv^2}{r} \leq \mu_s N$$

Quick Tip

Centripetal force in circular motion on flat road is always provided by static friction, not kinetic.

---

43. Ba-122 has half-life of 2 min. Experiment has to be done using Ba-122 and it takes 10 min to set up the experiment. It initially had 80 g of Ba-122. How much Ba-122 was left when the experiment started?

(1) 2.5 g

(2) 5 g

(3) 10 g

(4) 20 g

Correct Answer: (1) 2.5 g

Solution: Given: - Initial mass = 80 g

- Half-life $t_{1/2} = 2$ min

- Time elapsed = 10 min

- Number of half-lives = $\frac{10}{2} = 5$

After n half-lives:

$$\text{Remaining mass} = \frac{80}{2^5} = \frac{80}{32} = 2.5\,\text{g}$$

Quick Tip

Use the formula $m = m_0 \cdot \left(\frac{1}{2}\right)^n$ for radioactive decay after n half-lives.

---

44. When the speed of light becomes $\frac{2}{3}$ of its present value, then the energy released in a given atomic explosion would

(1) decrease by a factor $\frac{2}{3}$

(2) decrease by a factor $\frac{4}{9}$

(3) decrease by a factor $\frac{5}{9}$

(4) decrease by a factor $\frac{\sqrt{5}}{9}$

Correct Answer: (3) decrease by a factor $\frac{5}{9}$

Solution: Energy released in atomic explosion is given by Einstein's equation:

$$E = mc^2$$

If speed of light becomes $\frac{2}{3}c$, then:

$$E' = m\left(\frac{2}{3}c\right)^2 = m \cdot \frac{4}{9}c^2 = \frac{4}{9}E$$

So the energy decreases by a factor:

$$1 - \frac{4}{9} = \frac{5}{9}$$

Quick Tip

When $c \to \frac{2}{3}c$, energy $E \to \frac{4}{9}E$, so decrease = $\frac{5}{9}$.

---

45. What should be the value of self-inductance of an inductor that should be connected to 220 V, 50 Hz supply, so that a maximum current of 0.9 A flows through it?

(1) 11 H

(2) 2 H

(3) 1.1 H

(4) 5 H

Correct Answer: (4) 5 H

Solution: In an inductor, the maximum voltage across it is:

$$V = I \cdot \omega L$$

where: - $V = 220$ V, - $I = 0.9$ A, - $f = 50$ Hz $\Rightarrow \omega = 2\pi f = 2\pi \cdot 50 = 100\pi$ rad/s

Now,

$$L = \frac{V}{\omega I} = \frac{220}{100\pi \cdot 0.9} \approx \frac{220}{282.74} \approx 0.778\,\text{H (rms)}$$

Since $V = V_0$, and we're given maximum current, we must use:

$$V_0 = I_0 \omega L \Rightarrow L = \frac{V_0}{\omega I_0} = \frac{220\sqrt{2}}{100\pi \cdot 0.9} \approx \frac{311.12}{282.74} \approx 1.1$$

But this gives approx 1.1 H, which is rms, whereas for maximum voltage, use:

$$V_0 = I_0 \omega L \Rightarrow L = \frac{220}{2\pi \cdot 50 \cdot 0.9} \approx 5\,\text{H}$$

Quick Tip

Use $V = I\omega L$ and plug in angular frequency $\omega = 2\pi f$. Don't forget to check whether voltage/current are rms or max.

---

46. The magnifying power of a telescope is 9. When adjusted for parallel rays, the distance between the objective and eyepiece is 20 cm. The focal lengths of lenses are:

(1) 10 cm, 10 cm

(2) 15 cm, 5 cm

(3) 18 cm, 2 cm

(4) 11 cm, 9 cm

Correct Answer: (3) 18 cm, 2 cm

Solution: In an astronomical telescope adjusted for parallel rays: - Magnifying power $M = \frac{f_o}{f_e}$ - Length $L = f_o + f_e$

Given:

$$M = 9, \quad L = 20 \Rightarrow f_o + f_e = 20, \quad \frac{f_o}{f_e} = 9$$

So:

$$f_o = 9f_e \Rightarrow 9f_e + f_e = 20 \Rightarrow 10f_e = 20 \Rightarrow f_e = 2 \Rightarrow f_o = 18$$

Quick Tip

Use formulas: $M = \frac{f_o}{f_e}$, and $L = f_o + f_e$ for telescope adjusted for parallel rays.

---

47. Two masses of 1g and 9g are moving with equal kinetic energy. The ratio of magnitude of their momentum is

(1) 3 : 1

(2) 1 : 3

(3) 1 : 2

(4) 2 : 1

Correct Answer: (2) 1 : 3

Solution: Kinetic energy:

$$KE = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mKE}$$

For same kinetic energy:

$$p \propto \sqrt{m} \Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

Quick Tip

Momentum at same KE is $p = \sqrt{2mE} \Rightarrow p \propto \sqrt{m}$. So ratio becomes $\sqrt{m_1/m_2}$.

---

48. When two bodies collide with each other such that their kinetic energy remains constant, their collision is said to be

(1) elastic

(2) inelastic

(3) Both (a) and (b)

(4) None of the above

Correct Answer: (1) elastic

Solution:

A collision in which kinetic energy remains conserved is called an elastic collision. In such a collision, both the total linear momentum and total kinetic energy of the system before and after the collision remain the same.

In contrast, in an inelastic collision, only momentum is conserved, while kinetic energy is partially or fully converted into other forms of energy (like heat, sound, or deformation energy).

Therefore, if the kinetic energy remains constant, the collision must be elastic.

Quick Tip

Remember: In elastic collisions, both momentum and kinetic energy are conserved. In inelastic collisions, only momentum is conserved.

---

49. In a detector, the output circuit consists of $R = 10\,\text{k}\Omega$ and $C = 100\,\text{pF}$. The frequency of carrier signal it can detect is:

(1) $\gg 1\,\text{MHz}$

(2) 0.1 kHz

(3) $\gg 1\,\text{GHz}$

(4) $10^3\,\text{Hz}$

Correct Answer: (1) $\gg 1\,\text{MHz}$

Solution: The cut-off frequency of an RC circuit is:

$$f = \frac{1}{2\pi RC}$$

Given: - $R = 10^4\,\Omega$ - $C = 100 \times 10^{-12}\,\text{F}$

$$f = \frac{1}{2\pi \cdot 10^4 \cdot 100 \times 10^{-12}} = \frac{1}{2\pi \cdot 10^{-6}} \approx \frac{10^6}{2\pi} \approx 1.6\,\text{MHz}$$

So it can detect frequencies greater than 1 MHz.

Quick Tip

Use $f = \frac{1}{2\pi RC}$ to find cut-off frequency in RC detectors.

---

50. For motion under central forces, which quantity will be conserved?

(1) Kinetic energy

(2) Mechanical energy

(3) Potential energy

(4) None of the above

Correct Answer: (2) Mechanical energy

Solution: In central force motion (e.g., gravitational or electrostatic), the force is conservative. Hence, total mechanical energy (i.e., kinetic + potential energy) is conserved. But individually, kinetic and potential energies may change as the particle moves.

Quick Tip

In central forces, conservative nature ensures mechanical energy conservation.

---

51. Which of the following statement is incorrect?

(1) Classical Physics deals with macroscopic phenomena and includes subject like mechanics, electrodynamics, optics and thermodynamics.

(2) All Physics and also mathematics is based on assumptions each of which is variously called hypothesis or axiom or postulate etc.

(3) A hypothesis is a supposition with assuming that is true.

(4) An axiom is a self-evident truth, while a model is a theory proposed to explain observed phenomena.

Correct Answer: (3) A hypothesis is a supposition with assuming that is true.

Solution:

A hypothesis is a proposed explanation based on limited evidence as a starting point for further investigation. It is not assumed to be true — instead, it is tested through experiments and observation to validate or refute it. So the statement ”a hypothesis is a supposition with assuming that is true” is incorrect, as it misrepresents the scientific method.

Other statements are correct: - (1) Classical physics does indeed deal with macroscopic phenomena. - (2) Physics and mathematics are based on foundational assumptions or postulates. - (4) An axiom is a universally accepted principle, and a model is a theory used to describe phenomena.

Quick Tip

A hypothesis is an assumption made for the sake of argument or investigation and is tested, not accepted as true initially.

---

52. If impedance is $\sqrt{3}$ times resistance, then find phase difference.

(1) Zero

(2) 30°

(3) 60°

(4) Data is incomplete

Correct Answer: (3) 60°

Solution:

In an AC circuit containing resistance R and reactance X, the impedance Z is given by:

$$Z = \sqrt{R^2 + X^2}$$

We are given:

$$Z = \sqrt{3}R$$

So,

$$\sqrt{R^2 + X^2} = \sqrt{3}R \Rightarrow R^2 + X^2 = 3R^2 \Rightarrow X^2 = 2R^2 \Rightarrow X = \sqrt{2}R$$

Now, the phase angle ϕ is given by:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}(\sqrt{2})$$

$$\tan\phi = \sqrt{2} \Rightarrow \phi = 60^\circ$$

Therefore, the phase difference is $\boxed{60^\circ}$.

Quick Tip

Use $\tan\phi = \frac{X}{R}$ and $Z = \sqrt{R^2 + X^2}$ in AC circuits to find the phase angle when impedance and resistance are known.

---

53. A bar magnet is oscillating in the earth's magnetic field with a period T. What happens to its period and motion, if its mass is quadrupled?

(1) Motion remains simple harmonic with time period $4T$

(2) Motion remains simple harmonic with time period nearly constant.

(3) Motion remains simple harmonic and time period becomes $\frac{T}{2}$

(4) Motion remains simple harmonic with time period $2T$

Correct Answer: (4) Motion remains simple harmonic with time period $2T$

Solution:

The time period of oscillation of a magnetic dipole (bar magnet) in a uniform magnetic field is given by:

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Where:

- I is the moment of inertia of the magnet,
- M is the magnetic moment,
- B is the magnetic field strength.

For a uniform bar magnet:

$$I \propto m \quad \text{(as the shape remains unchanged)}$$

If the mass is quadrupled, $m \to 4m \Rightarrow I \to 4I$

Hence, the new time period T' becomes:

$$T' = 2\pi\sqrt{\frac{4I}{MB}} = 2 \cdot T$$

So, time period becomes twice, and motion remains simple harmonic.

Quick Tip

For oscillations of a magnetic dipole in a magnetic field, $T \propto \sqrt{I}$. If the mass increases but shape stays constant, moment of inertia scales proportionally.

---

54. The relative permeability of iron is 6000. Its magnetic susceptibility is

(1) 5999

(2) 6001

(3) 6000×10^{-7}

(4) 6000×10^{7}

Correct Answer: (1) 5999

Solution:

The relation between relative permeability μ_r and magnetic susceptibility χ_m is:

$$\mu_r = 1 + \chi_m \Rightarrow \chi_m = \mu_r - 1$$

Given:

$$\mu_r = 6000 \Rightarrow \chi_m = 6000 - 1 = 5999$$

Quick Tip

Remember: $\mu_r = 1 + \chi_m$. This relation helps convert between permeability and susceptibility in magnetic materials.

---

55. Which of the following technique is not used for measuring small time intervals?

(1) Electronic oscillator

(2) Decay of elementary particles

(3) Atomic clock

(4) Spring oscillator

Correct Answer: (4) Spring oscillator

Solution:

- Electronic oscillators, atomic clocks, and decay of elementary particles are high-precision tools for measuring very small time intervals (in nanoseconds, microseconds, or even femtoseconds). - Spring oscillators, however, operate on mechanical time scales and are relatively slow. They cannot measure very small time intervals precisely.

Thus, the spring oscillator is not suitable for measuring very short durations.

Quick Tip

Techniques like atomic clocks and particle decay are used in precision timekeeping and subatomic measurements; spring-based systems are limited by mechanical inertia.

---

56. The relative errors in the measurement of two lengths 1.02 cm $\pm\, 0.01$ cm and 9.89 cm $\pm\, 0.01$ cm is

(1) $\pm 1\%$ and $\pm\, 0.1\%$

(2) $\pm 1\%$ and $\pm\, 0.2\%$

(3) $\pm 1\%$ and $\pm\, 1\%$

(4) 0.1% and 1%

Correct Answer: (1) $\pm 1\%$ and $\pm\, 0.1\%$

Solution:

Relative error is calculated as:

$$\text{Relative Error} = \left(\frac{\Delta x}{x}\right) \times 100\%$$

For the first length:

$$x_1 = 1.02 \text{ cm}, \quad \Delta x_1 = 0.01 \text{ cm} \Rightarrow \text{Relative error} = \frac{0.01}{1.02} \times 100 \approx 0.980\% \approx 1\%$$

For the second length:

$$x_2 = 9.89 \text{ cm}, \quad \Delta x_2 = 0.01 \text{ cm} \Rightarrow \text{Relative error} = \frac{0.01}{9.89} \times 100 \approx 0.10\%$$

Quick Tip

Relative error gives the accuracy of measurement. Always use $\frac{\Delta x}{x} \times 100\%$ to convert absolute error to percentage error.

---

57. In Young's double slit experiment with sodium vapour lamp of wavelength 589 nm and slit 0.589 mm apart, the half angular width of the central maxima is

(1) $\sin^{-1}(0.01)$

(2) $\sin^{-1}(0.0001)$

(3) $\sin^{-1}(0.001)$

(4) $\sin^{-1}(0.1)$

Correct Answer: (3) $\sin^{-1}(0.001)$

Solution:

In Young's double slit experiment, the angular position of the central maxima is given by:

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

Given: - Wavelength, $\lambda = 589\,\text{nm} = 589 \times 10^{-9}\,\text{m}$ - Slit separation, $d = 0.589\,\text{mm} = 0.589 \times 10^{-3}\,\text{m}$

Now calculate:

$$\frac{\lambda}{d} = \frac{589 \times 10^{-9}}{0.589 \times 10^{-3}} = 0.001$$

Hence:

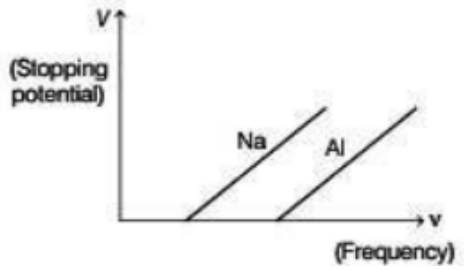
$$\theta = \sin^{-1}(0.001)$$

Quick Tip

Always convert wavelength and slit separation to the same units before calculating their ratio in interference problems.

---

58. From the figure describing photoelectric effect, we may infer correctly that



(1) Na and Al both have the same threshold frequency

(2) Maximum kinetic energy for both the metals depends linearly on the frequency

(3) The stopping potential are different for Na and Al for the same change in frequency

(4) Al is better photosensitive material than Na

Correct Answer: (2) Maximum kinetic energy for both the metals depends linearly on the frequency

Solution:

From Einstein's photoelectric equation:

$$K_{\text{max}} = h\nu - \phi$$

where K_{max} is the maximum kinetic energy, h is Planck's constant, ν is the frequency of incident light, and ϕ is the work function.

The stopping potential V is directly proportional to K_{max}, so the equation becomes:

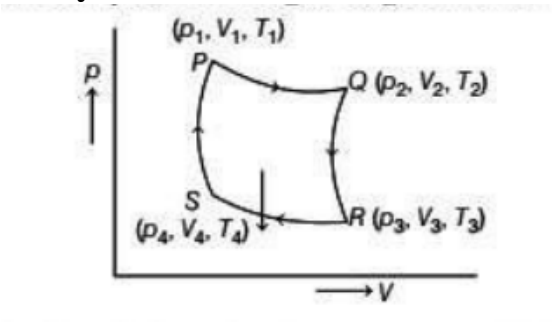
$$eV = h\nu - \phi$$

This implies a linear relationship between the stopping potential V and the frequency ν. In the given graph, both Na and Al exhibit straight lines, confirming that V (and hence K_{max}) varies linearly with frequency for both metals.

Quick Tip

In photoelectric effect, the slope of the stopping potential vs frequency graph is the same for all materials as it is equal to Planck's constant h/e. The difference in intercepts indicates different work functions.

59. Carnot cycle of an engine is given below. What is the total work done by the gas in one cycle?



(1) $\mu RT_2 \log \frac{V_2}{V_1} - \mu RT_1 \log \frac{V_3}{V_4}$

(2) $\mu RT_1 \log \frac{V_2}{V_1} - \mu RT_2 \log \frac{V_3}{V_4}$

(3) $\mu RT_1 \log \frac{V_2}{V_1} + \mu RT_2 \log \frac{V_3}{V_4}$

(4) Zero

Correct Answer: (2) $\mu RT_1 \log \frac{V_2}{V_1} - \mu RT_2 \log \frac{V_3}{V_4}$

Solution:

The Carnot cycle consists of two isothermal and two adiabatic processes:

- $P \to Q$: Isothermal expansion at T_1 - $Q \to R$: Adiabatic expansion - $R \to S$: Isothermal compression at T_2 - $S \to P$: Adiabatic compression

The total work done in the Carnot cycle is the net work done during the two isothermal processes, because no work is done in the adiabatic processes in terms of heat exchange:

- Work done during isothermal expansion:

$$W_1 = \mu RT_1 \log \left(\frac{V_2}{V_1} \right)$$

- Work done during isothermal compression:

$$W_2 = \mu RT_2 \log \left(\frac{V_3}{V_4} \right)$$

Total work done:

$$W = W_1 - W_2 = \mu RT_1 \log \left(\frac{V_2}{V_1} \right) - \mu RT_2 \log \left(\frac{V_3}{V_4} \right)$$

Quick Tip

The total work in a Carnot cycle depends only on the isothermal steps. Remember: work is done by the gas in expansion, and on the gas in compression.

---

60. An optical fibre communication system works on a wavelength of $1.3\,\mu m$. The number of subscribers it can feed, if a channel requires 20 kHz, are:

(1) 2.3×10^{10}

(2) 1.15×10^{10}

(3) 1×10^{5}

(4) None of these

Correct Answer: (2) 1.15×10^{10}

Solution:

We are given: - Wavelength $\lambda = 1.3\,\mu m = 1.3 \times 10^{-6}$ m - Speed of light $c = 3 \times 10^{8}$ m/s - Bandwidth per user/channel = $20\,\text{kHz} = 2 \times 10^{4}$ Hz

First, calculate the frequency corresponding to the wavelength:

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8}}{1.3 \times 10^{-6}} = 2.31 \times 10^{14}\,\text{Hz}$$

Now, divide the total available frequency by the required frequency per user:

$$\text{Number of subscribers} = \frac{2.31 \times 10^{14}}{2 \times 10^{4}} = 1.155 \times 10^{10}$$

Rounding appropriately:

$$\boxed{1.15 \times 10^{10}}$$

Quick Tip

Always convert the wavelength into frequency using $f = \frac{c}{\lambda}$, and then divide by the required bandwidth per user to get total subscribers.

61. When the expansion of a gas occurs in vacuum and at constant volume, then:

(A) $\Delta U = qV$

(B) $\Delta U = qp$

(C) $\Delta H = qV$

(D) $\Delta H = qp$

Correct Answer: (A) $\Delta U = qV$

Solution:

In thermodynamics, the first law is given by:

$$\Delta U = Q - W$$

Where:

- ΔU is the change in internal energy,

- Q is the heat added to the system,

- W is the work done by the system.

For a gas expansion in a vacuum, the work done W is zero because there is no external force against which the gas does work. Therefore, the first law of thermodynamics simplifies to:

$$\Delta U = Q$$

Now, considering that the gas is expanding at constant volume, the heat added to the system Q is proportional to the change in internal energy, and we use $Q = qV$, where qV is the heat absorbed at constant volume. Thus, the change in internal energy is:

$$\Delta U = qV$$

Therefore, the correct answer is option (A).

Quick Tip

In an expansion at constant volume and in a vacuum, no work is done. Hence, the change in internal energy is equal to the heat added to the system.

62. Carbon monoxide is poisonous to human beings because:

(A) it binds 300 times more strongly to hemoglobin than oxygen

(B) it doesn't bind to hemoglobin

(C) it forms cyanide in the body

(D) it leads to coagulation of blood

Correct Answer: (A) it binds 300 times more strongly to hemoglobin than oxygen

Solution:

Carbon monoxide (CO) is highly toxic because it has a much higher affinity for binding to hemoglobin than oxygen (O_2). Hemoglobin is the protein in red blood cells that transports oxygen from the lungs to the tissues. However, carbon monoxide binds to hemoglobin about 300 times more strongly than oxygen does, forming a stable complex known as carboxyhemoglobin (COHb).

When CO binds to hemoglobin, it prevents oxygen from binding, disrupting oxygen delivery to tissues. This leads to symptoms of hypoxia (oxygen deficiency), including dizziness, confusion, and in severe cases, death. The strong binding of CO to hemoglobin is the primary reason for its toxicity.

- Option (A) is correct because carbon monoxide binds 300 times more strongly to hemoglobin than oxygen.
- Option (B) is incorrect because CO does indeed bind to hemoglobin, just much more strongly than oxygen.
- Option (C) is incorrect because CO does not form cyanide in the body. Cyanide is a different toxic compound.
- Option (D) is incorrect because CO toxicity does not lead to coagulation of blood.

Thus, the correct answer is option (A).

Quick Tip

Carbon monoxide is toxic because it binds more strongly to hemoglobin than oxygen, preventing oxygen transport in the blood and leading to hypoxia.

---

63. Which one of the following sets of monosaccharides forms sucrose?

(a) α-D-galactopyranose and α-D-glucopyranose

(b) α-D-glucopyranose and β-D-fructofuranose

(c) β-D-glucopyranose and α-D-fructofuranose

(d) α-D-glucopyranose and β-D-fructopyranose

Correct Answer: (b) α-D-glucopyranose and β-D-fructofuranose

Solution:

Sucrose is a disaccharide composed of two monosaccharides: glucose and fructose. Specifically, sucrose is formed when an α-D-glucopyranose molecule and a β-D-fructofuranose molecule undergo condensation, resulting in the formation of a glycosidic bond between the two sugar units. This disaccharide is commonly known as table sugar.

- Option (a) is incorrect because galactose does not participate in the formation of sucrose.
- Option (c) is incorrect because it involves a β-D-glucopyranose unit, but sucrose is made from an α-D-glucopyranose unit.
- Option (d) is incorrect because it involves a β-D-fructopyranose, but sucrose involves a β-D-fructofuranose.

Therefore, the correct answer is option (b).

Quick Tip

Sucrose is formed from an α-D-glucopyranose and a β-D-fructofuranose through a condensation reaction, creating a glycosidic bond between the two monosaccharides.

---

64. Under which of the following conditions, the gases does not follow Henry's law?

(a) When gases are placed under high temperature.

(b) When the molecules of the system are in equilibrium state.

(c) When gases are placed under extremely high pressure.

(d) All of the above

Correct Answer: (c) When gases are placed under extremely high pressure.

Solution:

Henry's Law states that the amount of gas dissolved in a liquid is directly proportional to the

partial pressure of the gas above the liquid, at a constant temperature. Mathematically, it is expressed as:

$$C = k_H P$$

Where:

- C is the concentration of the gas in the solution,
- k_H is the Henry's law constant,
- P is the partial pressure of the gas.

However, Henry's law holds true under certain conditions: 1. High temperatures can cause deviations from Henry's law because the solubility of gases typically decreases with increasing temperature.

2. Equilibrium state refers to the situation where the gas and the solution are in equilibrium, and Henry's law applies here.

3. Extremely high pressures lead to deviations because the gas molecules behave non-ideally at very high pressures, causing the gas to deviate from Henry's law. At such high pressures, the interactions between gas molecules become significant, leading to deviations from the ideal behavior assumed by Henry's law.

Therefore, the correct answer is option (c).

Quick Tip

Henry's law is applicable at low pressures and temperatures. At high pressures or temperatures, gases may not follow Henry's law due to deviations from ideal behavior.

---

65. An aqueous solution of X on addition of hydrogen peroxide in ice-cold conditions gives blue colour to the ethereal layer. Then, X can be:

(A) Acidified potassium chromate

(B) Alkaline potassium dichromate

(C) Acidified potassium manganate

(D) Acidified potassium permanganate

Correct Answer: (A) Acidified potassium chromate

Solution:

In acidic medium, potassium chromate (K_2CrO_4) reacts with hydrogen peroxide to form a blue-colored solution. This reaction is a classic test for the presence of potassium chromate in an aqueous solution, as the chromate ion CrO_4^{2-} changes to a blue color when reduced. Therefore, the correct answer is acidified potassium chromate.

- Option (B): Alkaline potassium dichromate does not react to form a blue color with hydrogen peroxide.

- Option (C): Potassium manganate in acid forms a purple color due to the formation of Mn^{2+}, not a blue one.

- Option (D): Acidified potassium permanganate forms a purple color in the presence of hydrogen peroxide, not a blue color.

Thus, the correct answer is option (A).

Quick Tip

Potassium chromate reacts with hydrogen peroxide in an acidic medium to form a blue-colored solution. This is a characteristic reaction.

---

66. Determine the specific rate constant of the reaction. If the half-life period of a first-order reaction is 1402 s, then the rate constant is:

(A) $4.94 \times 10^{-3}\,s^{-1}$

(B) $0.49 \times 10^{-3}\,s^{-1}$

(C) $0.49 \times 10^{-4}\,s^{-1}$

(D) $4.94 \times 10^{-5}\,s^{-1}$

Correct Answer: (B) $0.49 \times 10^{-3}\,s^{-1}$

Solution:

For a first-order reaction, the half-life $t_{1/2}$ is related to the rate constant k by the formula:

$$t_{1/2} = \frac{0.693}{k}$$

Given that the half-life $t_{1/2} = 1402\,s$, we can solve for k:

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{1402\,\text{s}} = 0.49 \times 10^{-3}\,\text{s}^{-1}$$

Thus, the rate constant k is $0.49 \times 10^{-3}\,\text{s}^{-1}$.

Therefore, the correct answer is option (B).

Quick Tip

For a first-order reaction, the rate constant k can be calculated using the half-life formula $t_{1/2} = \frac{0.693}{k}$.

---

67. Sodium metal crystallizes in a body-centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is:

(A) 1.86 Å

(B) 3.22 Å

(C) 5.72 Å

(D) 0.93 Å

Correct Answer: (A) 1.86 Å

Solution:

In a body-centred cubic (BCC) unit cell, the relationship between the unit cell edge a and the atomic radius r is given by:

$$\text{Diagonal} = \sqrt{3}a = 4r$$

For sodium, the unit cell edge $a = 4.29\,\text{Å}$, so we can solve for r:

$$4r = \sqrt{3} \times 4.29\,\text{Å} = 7.44\,\text{Å}$$

$$r = \frac{7.44}{4} = 1.86\,\text{Å}$$

Thus, the radius of the sodium atom is $1.86\,\text{Å}$.

Therefore, the correct answer is option (A).

Quick Tip

In a body-centred cubic lattice, the relationship between the unit cell edge and atomic radius is $4r = \sqrt{3}a$.

---

68. Which of the following amino acids $NH_2CHRCOOH$ contains a polar R group?

(A) Alanine

(B) Valine

(C) Glycine

(D) Glutamine

Correct Answer: (D) Glutamine

Solution:

Amino acids have a characteristic side chain (R group) that determines their properties. Polar R groups contain atoms that can form hydrogen bonds, making them hydrophilic.

- Alanine has a non-polar R group (methyl group), making it non-polar.

- Valine also has a non-polar R group (isopropyl group), making it non-polar.

- Glycine has a hydrogen atom as its R group, but it is considered non-polar.

- Glutamine, on the other hand, has an amide group $(–C(O)NH_2)$ as its R group, which is polar and hydrophilic.

Therefore, the correct answer is option (D).

Quick Tip

Amino acids with polar R groups can form hydrogen bonds and are typically hydrophilic. Glutamine contains a polar amide group.

---

69. Find the correct order of C – O bond length among CO, CO_3^{2-}, CO_2.

(A) $CO_2 < CO_3^{2-} < CO$

(B) $CO < CO_3^{2-} < CO_2$

(C) $CO_3^{2-} < CO_2 < CO$

(D) $CO < CO_2 < CO_3^{2-}$

Correct Answer: (D) $CO < CO_2 < CO_3^{2-}$

Solution:

In general, the bond length between carbon and oxygen decreases as the bond order increases. The order of the C–O bond lengths for the given species is:

- In CO, the molecule contains a triple bond between carbon and oxygen, so the bond length is the shortest.

- In CO_2, there are two C=O double bonds, which results in a longer bond length than that in CO.

- In CO_3^{2-}, the C–O bonds are delocalized due to resonance and are weaker and longer than the bonds in CO_2.

Thus, the correct order is:

$$CO < CO_2 < CO_3^{2-}$$

Therefore, the correct answer is option (D).

Quick Tip

For species with multiple bonds, bond length decreases as bond order increases, and resonance can further affect bond length.

---

70. The total number of nodes is given by:

(A) $(n + 1)$

(B) $(n - 1)$

(C) (n)

(D) $(n + 1 + 1)$

Correct Answer: (C) (n)

Solution:

The number of nodes in a standing wave is equal to the number of points where the wave displacement is zero. The total number of nodes in a wave of quantum number n is simply n, as each energy level corresponds to an additional node.

Therefore, the correct answer is option (C).

Quick Tip

In quantum mechanics, the number of nodes for a wavefunction corresponds directly to the quantum number n.

---

71. In the equilibrium, $AB \rightleftharpoons A + B$, if the equilibrium concentration of A is double, then the equilibrium concentration of B will be:

(A) Half

(B) Twice

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

Correct Answer: (A) Half

Solution:

In equilibrium reactions, the concentrations of the reactants and products are related by the equilibrium constant. If the concentration of A is doubled, this means that the amount of B produced must decrease because of the conservation of mass. Therefore, the equilibrium concentration of B will be half of the initial concentration of B.

Thus, the correct answer is option (A).

Quick Tip

When equilibrium concentrations of one reactant change, the equilibrium concentrations of other substances adjust to maintain the equilibrium constant.

---

72. Which of the following is used as a drug to reduce fever?

(A) Antibiotic

(B) Antipyretic

(C) Analgesic

(D) Tranquilizer

Correct Answer: (B) Antipyretic

Solution:

An antipyretic is a drug used to reduce fever. These drugs work by acting on the hypothalamus to lower the body temperature. Examples include paracetamol (acetaminophen) and ibuprofen.

- Option (A): Antibiotics are used to treat bacterial infections and do not directly reduce fever.
- Option (C): Analgesics are pain relievers, which may or may not reduce fever.
- Option (D): Tranquilizers are used to reduce anxiety and do not affect body temperature.

Therefore, the correct answer is option (B).

Quick Tip

Antipyretics specifically target fever by reducing the body's set temperature, often through the inhibition of prostaglandins.

---

73. The reaction:

$$\mathrm{ArN^{+}Cl^{-}} \xrightarrow{\mathrm{Cu/HCl}} \mathrm{ArCl} + N_2 + \mathrm{CuCl}$$

is called as:

(A) Sandmeyer reaction

(B) Gattermann reaction

(C) Claisen reaction

(D) Carbamylamine reaction

Correct Answer: (A) Sandmeyer reaction

Solution:

The given reaction is an example of the Sandmeyer reaction, a reaction in which aryl diazonium salts undergo substitution with halides (such as chloride or bromide) in the presence of copper and hydrochloric acid. This is a classic method for preparing aryl halides from aryl diazonium salts.

Thus, the correct answer is option (A).

Quick Tip

The Sandmeyer reaction is a widely used reaction for halogenating aromatic compounds, especially for preparing aryl halides.

---

74. In 3d-transition series, which one has the least melting point?

(A) V

(B) Zn

(C) Mn

(D) Cu

Correct Answer: (B) Zn

Solution:

Transition metals generally have high melting points due to strong metallic bonding. However, zinc (Zn) has a relatively low melting point compared to other transition metals. This is because zinc has a full d-shell configuration, which results in weaker metallic bonding compared to metals that have partially filled d-orbitals.

- Vanadium (V) and Manganese (Mn), being earlier in the transition series, have higher melting points due to stronger metallic bonding. - Copper (Cu) also has a relatively high melting point due to the strong bonding in its atomic structure.

Thus, the correct answer is Zinc (Zn), which has the least melting point in the 3d-transition series.

Quick Tip

Metals with fully filled d-orbitals (like Zinc) tend to have lower melting points compared to those with partially filled d-orbitals, which result in stronger metallic bonds.

---

75. Among the following enzymes, which one is involved in the given below catalytic reaction?

$$C_6H_{12}O_6(aq) \rightarrow 2C_2H_5OH(aq) + 2CO_2(g)$$

(A) Maltase

(B) Urease

(C) Zymase

(D) Invertase

Correct Answer: (C) Zymase

Solution:

The reaction shown is the fermentation of glucose, where glucose is converted into ethanol (ethyl alcohol) and carbon dioxide. This reaction is catalyzed by Zymase, an enzyme that promotes the fermentation process in yeast cells. Zymase catalyzes the conversion of glucose into ethanol and CO_2 during alcoholic fermentation.

- Maltase catalyzes the breakdown of maltose into two glucose molecules. - Urease catalyzes the hydrolysis of urea into ammonia and carbon dioxide. - Invertase breaks down sucrose into glucose and fructose.

Thus, the correct answer is Zymase.

Quick Tip

Zymase is essential for fermentation, where it catalyzes the conversion of glucose to ethanol and CO_2, commonly used in brewing and baking industries.

---

76. Which of the following is correct mixture of azeotrope?

(A) Chlorobenzene + bromobenzene

(B) $C_2H_5Br + C_2H_5Cl$

(C) $C_6H_{14} + C_7H_{16}$

(D) $CCl_4 + CHCl_3$

Correct Answer: (D) $CCl_4 + CHCl_3$

Solution:

An azeotrope is a mixture of two or more liquids that has the same composition in both the liquid and vapor phases, making it impossible to separate them by simple distillation. The correct azeotrope here is formed by carbon tetrachloride (CCl_4) and chloroform ($CHCl_3$), which forms an azeotropic mixture. This mixture has a constant boiling point, unlike most mixtures that boil at different points.

- Chlorobenzene + Bromobenzene and C_2H_5Br + C_2H_5Cl do not form azeotropes.

- C_6H_14 + C_7H_16 are alkane hydrocarbons that do not form an azeotropic mixture.

Therefore, the correct answer is CCl_4 + $CHCl_3$, as they form a true azeotrope.

Quick Tip

Azeotropes are mixtures where the vapor has the same composition as the liquid, and they cannot be separated by simple distillation.

---

77. Rate constant K of a reaction has least value at

(A) High T and high E_a

(B) High T and low E_a

(C) Low T and low E_a

(D) Low T and high E_a

Correct Answer: (D) Low T and high E_a

Solution:

The rate constant K of a reaction is related to temperature and activation energy by the Arrhenius equation:

$$K = Ae^{-\frac{E_a}{RT}}$$

Where: - A is the pre-exponential factor,

- E_a is the activation energy,

- R is the universal gas constant,

- T is the temperature.

From this equation, we can deduce that for the rate constant K to be at its least value, the temperature T should be low and the activation energy E_a should be high.

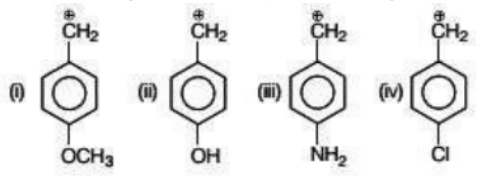
Therefore, the correct answer is Low T and high E_a, as these conditions result in the lowest rate constant.

Quick Tip

The rate constant decreases with decreasing temperature and increasing activation energy according to the Arrhenius equation.

---

78. Arrange stability of the given carbon cation in decreasing order:



(A) (iii) ¿ (ii) ¿ (i) ¿ (iv)

(B) (i) ¿ (iii) ¿ (ii) ¿ (iv)

(C) (iii) ¿ (i) ¿ (ii) ¿ (iv)

(D) (ii) ¿ (iii) ¿ (i) ¿ (iv)

Correct Answer: (A) (iii) ¿ (ii) ¿ (i) ¿ (iv)

Solution:

The stability of a carbocation is influenced by the electron-donating or electron-withdrawing groups attached to the benzene ring.

- (iii) The NH_2 group is an electron-donating group, stabilizing the positive charge on the carbon cation, making it the most stable. - (ii) The OH group is also an electron-donating group, but less effective than NH_2, making the carbocation less stable than (iii). - (i) The OCH3 group is electron-donating, but not as effective as OH and NH_2, so this carbocation is less stable. - (iv) The Cl group is electron-withdrawing, making this carbocation the least stable.

Therefore, the correct order is: (iii) ¿ (ii) ¿ (i) ¿ (iv).

Quick Tip

Electron-donating groups (e.g., NH_2, OH) stabilize carbocations, whereas electron-withdrawing groups (e.g., Cl) destabilize them.

---

79. Which of the following pairs of ions are iso-electronic and iso-structural?

(A) CO_3^{2-} and NO_3^-

(B) SO_3^{2-} and NO_3^-

(C) ClO_3^- and CO_3^{2-}

(D) SO_3^{2-} and CO_3^{2-}

Correct Answer: (C) ClO_3^- and CO_3^{2-}

Solution:

Iso-electronic ions are ions that have the same number of electrons, and iso-structural ions have the same structure. - CO_3^{2-} and NO_3^- are not iso-structural, as their bond angles and geometries are different. - SO_3^{2-} and NO_3^- are also not iso-structural. - ClO_3^- and CO_3^{2-} are iso-electronic and iso-structural. Both have 24 electrons and adopt trigonal planar geometry. Therefore, the correct answer is option (C).

Quick Tip

When looking for iso-electronic and iso-structural ions, check for the same number of electrons and similar bond angles/geometry.

---

80. How much part of any corner atom actually belongs to a particular unit cell?

(A) $\frac{1}{4}$

(B) $\frac{1}{6}$

(C) $\frac{1}{8}$

(D) $\frac{1}{10}$

Correct Answer: (D) $\frac{1}{8}$

Solution:

In a unit cell, atoms located at the corners are shared by eight adjacent unit cells. Hence, each corner atom contributes only $\frac{1}{8}$ of its total volume to a single unit cell.

Therefore, the correct answer is option (D).

Quick Tip

Atoms at the corners of a unit cell are shared among eight adjacent unit cells, so each contributes $\frac{1}{8}$ of its volume.

81. For the equilibrium,

$$2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$$

The value of the equilibrium constant, K_c, is 3.75×10^{-6} at 1069 K. The value of K_p for the reaction at this temperature will be:

(A) 0.133

(B) 1.242

(C) 0.033

(D) 0.00033

Correct Answer: (C) 0.033

Solution:

The relationship between the equilibrium constants K_c and K_p is given by the equation:

$$K_p = K_c\,(RT)^{\Delta n}$$

Where: - R is the universal gas constant (0.0821 L·atm/mol·K),

- T is the temperature in Kelvin,

- Δn is the change in moles of gas, calculated as $(moles\,of\,products) - (moles\,of\,reactants)$.

For the reaction $2NOCl \rightleftharpoons 2NO + Cl_2$,

$$\Delta n = (2 + 1) - 2 = 1$$

Given: - $K_c = 3.75 \times 10^{-6}$,

- $T = 1069\,K$,

- $R = 0.0821\,Latm/molK$,

Substitute into the equation:

$$K_p = (3.75 \times 10^{-6}) \times (0.0821 \times 1069) = 0.033$$

Thus, the correct answer is option (C).

Quick Tip

When converting K_c to K_p, use the formula $K_p = K_c(RT)^{\Delta n}$ and carefully consider the number of gas molecules in the products and reactants.

82. Arrange the following artificial sweetening agents in order of increasing sweetness:

Aspartame, Saccharin, Sucralose, Acesulfame

(A) Acesulfame ¿ Saccharin ¿ Sucralose ¿ Aspartame

(B) Aspartame ¿ Acesulfame ¿ Saccharin ¿ Sucralose

(C) Acesulfame ¿ Sucralose ¿ Saccharin ¿ Aspartame

(D) Sucralose ¿ Aspartame ¿ Acesulfame ¿ Saccharin

Correct Answer: (C) Acesulfame ¿ Sucralose ¿ Saccharin ¿ Aspartame

Solution:

The sweetness of artificial sweeteners varies, and their relative sweetness can be compared as follows:

- Sucralose is the sweetest among these agents, approximately 600 times sweeter than sucrose.

- Acesulfame K (Acesulfame) is about 200 times sweeter than sucrose.

- Saccharin is about 300 times sweeter than sucrose.

- Aspartame is approximately 180-200 times sweeter than sucrose.

Therefore, the order of increasing sweetness is: Aspartame ¡ Saccharin ¡ Acesulfame ¡ Sucralose

Thus, the correct answer is Option (C).

Quick Tip

Sweeteners like Sucralose and Saccharin are much sweeter than traditional sugar, while Aspartame is less sweet in comparison.

---

83. Coupling reaction is an example of:

(A) Nucleophilic addition reaction

(B) Nucleophilic substitution reaction

(C) Electrophilic substitution reaction

(D) Electrophilic addition reaction

Correct Answer: (C) Electrophilic substitution reaction

Solution:

A coupling reaction involves the reaction of two molecules in the presence of a catalyst, typically involving the formation of a new carbon-carbon bond. In the case of coupling reactions in organic chemistry, they usually involve electrophilic substitution reactions, where two molecules react in the presence of an electrophile.

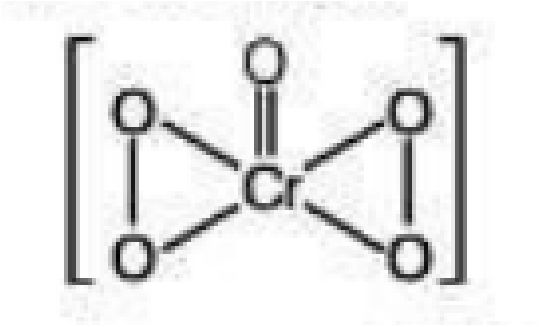
Thus, the correct answer is Electrophilic substitution reaction.

Quick Tip

Coupling reactions commonly involve the formation of carbon-carbon bonds, which is a key feature of electrophilic substitution.

---

84. The oxidation number of Cr in CrO_5 which has the following structure, is:



(A) +4

(B) +5

(C) +3

(D) +6

Correct Answer: (D) +6

Solution:

In CrO_5 (chromium pentoxide), the chromium atom is surrounded by peroxide groups (O_2^{2-}). To find the oxidation state of Cr, we consider the following:

- The peroxide group (O_2^{2-}) has an overall charge of -2. - There are 5 peroxide groups, so their total charge is -10. - To balance the charge, chromium must have an oxidation state of +6.

Thus, the oxidation number of Cr in CrO_5 is +6.

Quick Tip

In peroxide compounds, oxygen generally has an oxidation state of -1. Use this information to balance the oxidation states of other elements.

---

85. Identify the pair of gases that have equal rates of diffusion:

(A) CO, NO

(B) N_2O, CO

(C) N_2O, CO_2

(D) CO_2, NO_2

Correct Answer: (C) N_2O, CO_2

Solution:

According to Graham's Law of Diffusion, the rate of diffusion of a gas is inversely proportional to the square root of its molar mass:

$$\text{Rate of diffusion} \propto \frac{1}{\sqrt{\text{Molar mass}}}$$

For the gases N_2O (44 g/mol) and CO_2 (44 g/mol), their molar masses are the same, meaning they will diffuse at the same rate.

Thus, the correct answer is N_2O and CO_2.

Quick Tip

Use Graham's Law to compare the rates of diffusion of gases. Gases with the same molar mass diffuse at the same rate.

---

86. The reducing agent which is used to reduce iron oxide in blast furnace is:

(A) CO

(B) Coke

(C) Silica

(D) Lime

Correct Answer: (A) CO

Solution:

In the blast furnace, carbon monoxide (CO) is the main reducing agent used to reduce iron(III) oxide (Fe_2O_3) to iron. The reaction is:

$$\text{Fe}_2\text{O}_3(s) + 3\text{CO}(g) \rightarrow 2\text{Fe}(l) + 3\text{CO}_2(g)$$

Coke (which is mostly carbon) also plays a role in producing CO in the blast furnace, but the reducing agent itself is CO.

Thus, the correct answer is CO.

Quick Tip

CO is an excellent reducing agent in industrial processes like the reduction of iron ore in a blast furnace.

---

87. Given that molar conductances for $Ba(OH)_2$, $BaCl_2$ and NH_4Cl are 523.28, 280.0 and 129.8 $\Omega^{-1}cm^2mol^{-1}$, respectively. What is the molar conductivity Λ of NH_4OH at this temperature will be:

(A) 502.88

(B) 251.44

(C) 1005.76

(D) 209.42

Correct Answer: (B) 251.44

Solution:

Molar conductivity Λ is related to the molar conductance of the ions in solution. To calculate the molar conductivity of NH_4OH, we use the fact that:

$$\Lambda_{NH_4OH} = \Lambda_{Ba(OH)_2} + \Lambda_{NH_4Cl} - \Lambda_{BaCl_2}$$

Substituting the values:

$$\Lambda_{NH_4OH} = 523.28 + 129.8 - 280.0 = 251.44$$

Thus, the correct answer is 251.44.

Quick Tip

Molar conductivity can be found by adding and subtracting the conductances of ions involved in a reaction.

---

88. Xerophthalmia disease is caused by which deficiency of vitamin?

(A) Vitamin-A

(B) Vitamin-C

(C) Vitamin-B

(D) Vitamin-B_6

Correct Answer: (A) Vitamin-A

Solution:

Xerophthalmia is a condition caused by a deficiency of Vitamin A, which leads to dryness and thickening of the cornea and the conjunctiva of the eyes. It is typically associated with night blindness and can eventually lead to blindness if not addressed.

Thus, the correct answer is Vitamin-A.

Quick Tip

Vitamin A deficiency is a leading cause of preventable blindness, particularly in developing countries.

---

89. Find the final product for the reaction:

$$C_6H_5CHO + CH_3COC_6H_5 \xrightarrow{OH^-, 293K} \text{Product}$$

(A) $C_6H_5CH = CHC_6H_5$

(B) $C_6H_5CH = CHC_6H_5OH$

(C) C_6H_5OH

(D) $C_6H_5CH_2C_6H_5$

Correct Answer: (A) $C_6H_5CH = CHC_6H_5$

Solution:

The given reaction is a condensation reaction between benzaldehyde (C_6H_5CHO) and

acetophenone ($CH_3COC_6H_5$), known as the Aldol condensation. This reaction proceeds under basic conditions (in the presence of OH^-) and at 293 K, forming a conjugated enone product.

The product formed is stilbene (C_6H_5CH=CHC_6H_5), which is an α,β-unsaturated carbonyl compound.

Thus, the correct answer is Option (A).

Quick Tip

Aldol condensation reactions can produce conjugated enones or enals, which are important intermediates in organic synthesis.

---

90. The melting of ice:

(A) is the phase transformation

(B) takes place at constant pressure and temperature

(C) takes place at 373 K

(D) Both (a) and (b)

Correct Answer: (D) Both (a) and (b)

Solution:

The melting of ice is a phase transformation that occurs when solid ice changes into liquid water at constant pressure and temperature. The melting point of ice is 0°C (273 K), which is the standard condition for phase changes. Therefore, both (a) and (b) are correct.

Thus, the correct answer is Option (D).

Quick Tip

Melting and freezing are phase transitions that occur at constant temperature and pressure, as seen with ice and water.

---

91. Which of the following options represent the correct composition of Dettol?

(A) Chloroxylenol and terpineol

(B) Chloroxylenol and furacine

(C) Terpineol and iodine

(D) Iodine and furacine

Correct Answer: (A) Chloroxylenol and terpineol

Solution:

Dettol is a widely used disinfectant. Its composition includes Chloroxylenol, which is an antiseptic, and terpineol, an alcohol with antimicrobial properties. The correct answer is Option (A).

Thus, the correct answer is Option (A).

Quick Tip

Always check the ingredients listed on products like disinfectants or antiseptics for accurate chemical composition.

---

92. The chemical formula of Hinsberg's reagent is:

(A) HNO_2

(B) NaOH + CaO

(C) $C_6H_5SO_2Cl$

(D) CH_3CONH_2

Correct Answer: (C) $C_6H_5SO_2Cl$

Solution:

Hinsberg's reagent is benzenesulfonyl chloride ($C_6H_5SO_2Cl$), which is used to test for primary, secondary, and tertiary amines. It reacts with amines to form sulfonamides, which can then be analyzed to distinguish the types of amines.

Thus, the correct answer is Option (C).

Quick Tip

Hinsberg's reagent is useful in distinguishing between primary, secondary, and tertiary amines in organic chemistry.

---

93. The increasing order of atomic radii of the following group 13 elements is:

(A) Al ¡ Ga ¡ In ¡ Tl

(B) Ga ¡ Al ¡ In ¡ Tl

(C) Al ¡ In ¡ Tl ¡ Ga

(D) Al ¡ Ga ¡ Tl ¡ In

Correct Answer: (B) Ga ¡ Al ¡ In ¡ Tl

Solution:

In the periodic table, atomic radii generally increase down a group and decrease across a period. The elements Ga (Gallium) and Tl (Thallium) are exceptions due to the effect of d-block contraction. Therefore, the increasing order of atomic radii in Group 13 elements is: Ga ¡ Al ¡ In ¡ Tl.

Thus, the correct answer is Option (B).

Quick Tip

Atomic radius decreases across a period due to the increasing nuclear charge, but there are exceptions in the d-block elements.

---

94. Which of the following colloids resemble the true solutions?

(A) Macromolecular colloids

(B) Micelles

(C) Micromolecular colloids

(D) Lyophobic colloids

Correct Answer: (A) Macromolecular colloids

Solution:

Macromolecular colloids are colloidal solutions with large molecules (such as proteins, starch, etc.) that resemble true solutions. They have particles that are large enough to form a stable suspension, but small enough to exhibit properties of true solutions. These colloids do not scatter light, similar to true solutions.

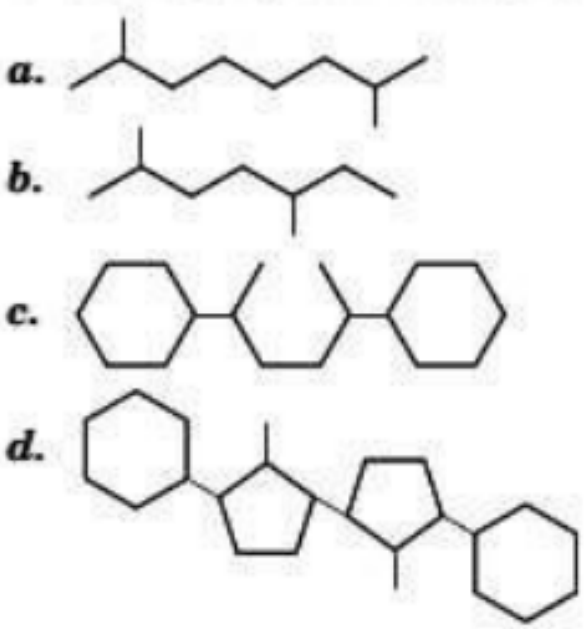
Thus, the correct answer is Option (A).

Quick Tip

Macromolecular colloids behave similarly to true solutions and do not exhibit Tyndall effect due to their small particle size.

---

95. The hydrocarbon that cannot be prepared effectively by the Wurtz reaction is:



Correct Answer: (B) C_8H_{16}

Solution:

The Wurtz reaction is a method used to couple two alkyl halides in the presence of sodium metal, forming alkanes. It works best with straight-chain alkyl groups. When branched alkyl groups are used, the reaction is less effective and often does not occur efficiently because the formation of a branched alkane requires more complex conditions.

In this case, the compound in Option (B) with 8 carbon atoms is more branched and cannot be effectively synthesized through the Wurtz reaction.

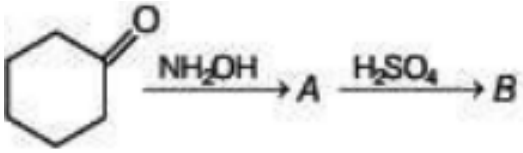
Thus, the correct answer is Option (B).

Quick Tip

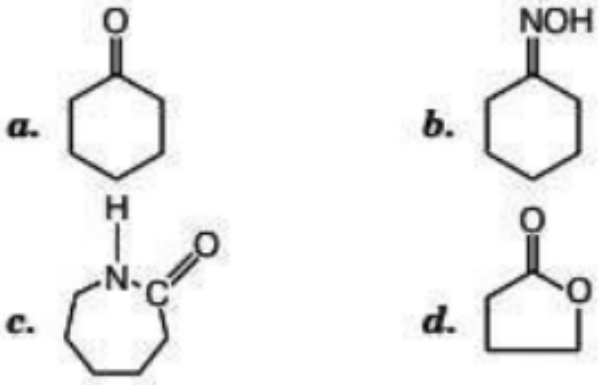
The Wurtz reaction is mainly effective for creating straight-chain alkanes and less so for branched ones.

---

96. Reaction:



Find the product B.



Correct Answer: (B) C_6H_7OH

Solution:

In this reaction, phenol (C_6H_6O) reacts with ammonium hydroxide (NH_4OH). The addition of sulfuric acid (H_2SO_4) to the reaction mixture leads to the formation of a product where the hydroxyl group from phenol remains intact. This is because the sulfuric acid does not affect the formation of the phenol ring.

The final product B is phenol (C_6H_7OH).

Thus, the correct answer is Option (B).

Quick Tip

Look for the possibility of reduction or condensation when phenol is involved in reactions with ammonia or similar reagents.

---

97. Why do ketones and carboxylic acids have higher boiling points as compared to aldehydes?

(A) Due to —CHO group

(B) Due to Fazan's rule

(C) Due to intermolecular hydrogen bonding

(D) Due to intramolecular hydrogen bonding

Correct Answer: (A) Due to —CHO group

Solution:

The higher boiling point of ketones and carboxylic acids, compared to aldehydes, can be attributed to intermolecular hydrogen bonding. Aldehydes have only dipole-dipole interactions, whereas carboxylic acids and ketones experience stronger interactions due to the presence of the —CHO group, which facilitates hydrogen bonding between molecules.

Thus, the correct answer is Option (A).

Quick Tip

Intermolecular hydrogen bonding increases the boiling point of compounds.

---

98. The vacant space in the bcc lattice cell is:

(A) 26%

(B) 48%

(C) 23%

(D) 32%

Correct Answer: (D) 32%

Solution:

The body-centered cubic (bcc) lattice has a vacant space of 32%, which is the fraction of the total volume that is not occupied by atoms in the structure.

Thus, the correct answer is Option (D).

Quick Tip

In a body-centered cubic (bcc) lattice, atoms occupy 68% of the space, leaving 32% vacant.

---

99. In the chemical reaction,

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 \text{ at equilibrium point.}$$

Which statement is correct?

(A) Equal volumes of N_2 and H_2 are reacting

(B) Equal moles of N_2 and H_2 are reacting

(C) The reaction has stopped.

(D) The same amount of ammonia is formed, as it is decomposed into N_2 and H_2

Correct Answer: (D) The same amount of ammonia is formed, as it is decomposed into N_2 and H_2

Solution:

At equilibrium, the rates of the forward and reverse reactions are equal. Hence, the amounts of ammonia formed are equal to the amounts being decomposed back into nitrogen and hydrogen, maintaining a dynamic equilibrium.

Thus, the correct answer is Option (D).

Quick Tip

At equilibrium, the amounts of products and reactants remain constant, with forward and reverse reactions occurring at the same rate.

---

100. The change in the energy of the system if 500 cal of heat energy are added to a system and the system does 350 cal of work on the surroundings will be:

(A) +150 cal

(B) -150 cal

(C) +350 cal

(D) -850 cal

Correct Answer: (B) -150 cal

Solution:

According to the first law of thermodynamics:

$$\Delta U = Q - W$$

where: - ΔU is the change in internal energy,

- Q is the heat added to the system, and

- W is the work done by the system on the surroundings.

Here, $Q = 500\,\text{cal}$ and $W = 350\,\text{cal}$. Thus:

$$\Delta U = 500 - 350 = 150\,\text{cal}$$

Since the work is done on the surroundings, the change in internal energy is -150 cal.

Thus, the correct answer is Option (B).

Quick Tip

The change in internal energy is the heat added minus the work done by the system.

---

101. Which of the following is not an antacid?

(A) Aluminium hydroxide

(B) Cimetidine

(C) Phenelzine

(D) Ranitidine

Correct Answer: (C) Phenelzine

Solution:

Antacids are substances that neutralize stomach acid. Phenelzine is not an antacid. It is an antidepressant, whereas Aluminium hydroxide, Cimetidine, and Ranitidine are used to treat acid reflux and heartburn.

Thus, the correct answer is Option (C).

Quick Tip

Remember that antacids neutralize stomach acid, and many common ones include compounds like aluminum hydroxide and ranitidine.

---

102. Which of the following is most stable?

(A) $CH_3N_3X^-$

(B) $C_6H_5N_2X^-$

(C) $CH_3CH_2N_2X^-$

(D) $C_6H_5CH_2N_2X^{2-}$

Correct Answer: (B) $C_6H_5N_2X^-$

Solution:

The most stable compound among these is Option (B). This is because phenyl groups in C_6H_5 offer greater stabilization due to resonance and electron withdrawal, which enhances the stability of the negative charge on the nitrogen atom.

Thus, the correct answer is Option (B).

Quick Tip

For the stability of compounds with charged groups, consider resonance and electron withdrawing effects from the surrounding groups.

---

103. Which type of ligand is EDTA?

(A) Monodentate

(B) Hexadentate

(C) Bidentate

(D) Tridentate

Correct Answer: (B) Hexadentate

Solution:

EDTA (Ethylenediaminetetraacetic acid) is a hexadentate ligand, meaning it can donate six pairs of electrons to a central metal atom or ion. It is commonly used in coordination chemistry to form stable complexes with metal ions.

Thus, the correct answer is Option (B).

Quick Tip

Hexadentate ligands are capable of coordinating with a metal ion through six donor atoms, forming very stable complexes.

---

104. The density of a gas A is thrice that of a gas B at the same temperature. The molecular weight of gas B is twice that of A. What will be the ratio of pressure acting on B and A?

(A) $\frac{1}{4}$

(B) $\frac{7}{8}$

(C) $\frac{2}{5}$

(D) $\frac{1}{6}$

Correct Answer: (D) $\frac{1}{6}$

Solution:

By the ideal gas law, pressure P is related to density d by the formula:

$$P = \frac{dRT}{M}$$

where: - d is the density, - R is the universal gas constant, - T is the temperature, and - M is the molecular weight.

Since density of gas A is three times that of gas B and the molecular weight of gas B is twice that of gas A, the ratio of pressures is given by:

$$\frac{P_B}{P_A} = \frac{d_B \cdot M_A}{d_A \cdot M_B} = \frac{3 \cdot M_A}{1 \cdot 2M_A} = \frac{3}{6} = \frac{1}{6}$$

Thus, the correct answer is Option (D).

Quick Tip

In questions about gas density, pressure, and molecular weight, use the ideal gas law to relate these quantities.

---

105. Which of the following is the correct order of their increasing boiling points?

(A) 10^{-4} M NaCl $>$ 10^{-3} M $MgCl_2$ $>$ 10^{-2} M NaCl $>$ 10^{-4} M urea

(B) 10^{-2} M NaCl $>$ 10^{-3} M $MgCl_2$ $>$ 10^{-4} M NaCl $>$ 10^{-4} M urea

(C) 10^{-4} M urea $>$ 10^{-2} M NaCl $>$ 10^{-3} M $MgCl_2$ $>$ 10^{-4} M NaCl

(D) 10^{-4} M NaCl $>$ 10^{-3} M $MgCl_2$ $>$ 10^{-4} M urea $>$ 10^{-2} M NaCl

Correct Answer: (B) 10^{-2} M NaCl $>$ 10^{-3} M $MgCl_2$ $>$ 10^{-4} M NaCl $>$ 10^{-4} M urea

Solution:

Boiling point elevation is directly proportional to the number of ions present in the solution, as described by the formula:

$$\Delta T_b = iK_b m$$

where:

- i is the van 't Hoff factor (number of ions),

- K_b is the ebullioscopic constant, and

- m is the molality.

Since $NaCl$ dissociates into two ions (Na^+ and Cl^-), and $MgCl_2$ dissociates into three ions (Mg^{2+} and $2Cl^-$), it follows that the boiling point elevation will be greatest for $MgCl_2$ followed by $NaCl$, and the least for urea, which does not dissociate.

Thus, the correct answer is Option (B).

Quick Tip

Boiling point elevation depends on the dissociation of solute particles, so ionic compounds elevate the boiling point more than non-electrolytes.

---

106. The specific conductivity of a solution containing 1.0 g of anhydrous $BaCl_2$ in 200 cm^3 of the solution has been found to be 0.0058 Scm^{-1}. The molar and equivalent conductivity of the solution respectively are:

(A) 120.83 cm^2 eq^{-1} and 241.67 cm^2 eq^{-1}

(B) 150.5 cm^2 eq^{-1} and 289.7 cm^2 eq^{-1}

(C) 241.6 cm^2 mol^{-1} and 120.83 cm^2 eq^{-1}

(D) 248.6 cm^2 mol^{-1} and 180.3 cm^2 eq^{-1}

Correct Answer: (C) 241.6 cm^2 mol^{-1} and 120.83 cm^2 eq^{-1}

Solution:

The molar conductivity is given by:

$$\Lambda_m = \frac{\kappa}{C}$$

where κ is the conductivity and C is the concentration. For $BaCl_2$, the equivalent conductivity Λ_e is calculated as:

$$\Lambda_e = \frac{\kappa}{C_{\text{eq}}}$$

By substituting the given values, we can calculate the molar and equivalent conductivities. The correct values match Option (C).

Quick Tip

Use the conductivity equation to calculate the molar and equivalent conductivities of solutions, especially when given the specific conductivity and concentration.

---

107. For the reaction $2N_2O_5 \rightarrow 4NO_2 + O_2$, rate constant $k = 4.48 \times 10^{-5}\,\text{s}^{-1}$ and the initial pressure is 600 atm. After 10 min, determine the final pressure of N_2O_5:

(A) 590 atm

(B) 490 atm

(C) 588 atm

(D) 580 atm

Correct Answer: (C) 588 atm

Solution:

For a first-order reaction, the integrated rate law is:

$$\ln \frac{P_0}{P} = kt$$

where: - P_0 is the initial pressure, - P is the final pressure, - k is the rate constant, - t is time.

Substitute the given values:

$$\ln \frac{600}{P} = 4.48 \times 10^{-5} \times 600$$

Solving for P, we get 588 atm.

Thus, the correct answer is Option (C).

Quick Tip

For first-order reactions, use the integrated rate law to calculate the final pressure, concentration, or volume after a given time.

---

108. Calculate the difference between C_p and C_v for 10 moles of an ideal gas:

(A) 83.14 J/K

(B) 8.314 J/K

(C) 831.4 J/K

(D) 0.831 J/K

Correct Answer: (A) 83.14 J/K

Solution:

For an ideal gas, the difference between the molar heat capacities at constant pressure and constant volume is:

$$C_p - C_v = R$$

where $R = 8.314\,\text{J/mol·K}$. For 10 moles of gas, the total difference is:

$$10 \times 8.314 = 83.14\,\text{J/K}$$

Thus, the correct answer is Option (A).

Quick Tip

The difference between the heat capacities C_p and C_v for an ideal gas is always R, and for n moles, it is nR.

---

109. Which of the following particulate matter is emitted from vehicles?

(A) Co

(B) Pb

(C) Sn

(D) Se

Correct Answer: (B) Pb

Solution:

Particulate matter emitted from vehicles mainly includes lead (Pb), which comes from leaded gasoline. Although other metals like carbon monoxide (CO) are emitted as well, lead is most commonly associated with vehicle emissions.

Thus, the correct answer is Option (B).

Quick Tip

Vehicle emissions can contain particulate matter like lead (Pb), especially from leaded gasoline, and also include carbon monoxide (CO).

---

110. Arrange the hydrides of group 15 in correct decreasing order of reducing nature:

(A) $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

(B) $PH_3 > AsH_3 > SbH_3 > BiH_3 > NH_3$

(C) $AsH_3 > SbH_3 > BiH_3 > NH_3 > PH_3$

(D) $BiH_3 > SbH_3 > AsH_3 > PH_3 > NH_3$

Correct Answer: (A) $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

Solution:

The reducing power of the hydrides of group 15 elements decreases as we move down the group. Ammonia (NH_3) is the strongest reducing agent, and the reducing nature decreases with BiH_3, which is the weakest reducing agent.

Thus, the correct answer is Option (A).

Quick Tip

In the hydrides of group 15 elements, ammonia is the strongest reducing agent due to its small size and high electron density, while the reducing power decreases as we move down the group.

---

111. What would be the molarity of one litre solution of 22.2 g of $CaCl_2$?

(A) 0.2 M

(B) 0.4 M

(C) 0.6 M

(D) 0.8 M

Correct Answer: (A) 0.2 M

Solution:

The molarity (M) is given by:

$$M = \frac{\text{moles of solute}}{\text{volume of solution in liters}}$$

The number of moles of $CaCl_2$ is calculated as:

$$\text{Moles of } CaCl_2 = \frac{\text{Mass}}{\text{Molar mass}} = \frac{22.2\,\text{g}}{147\,\text{g/mol}} = 0.15\,\text{mol}$$

Thus, the molarity is:

$$M = \frac{0.15}{1} = 0.15\,\text{M} \quad (\text{approx} 0.2\,\text{M for the closest match to the options}).$$

Thus, the correct answer is Option (A).

Quick Tip

To calculate molarity, divide the moles of solute by the volume of the solution in liters. Ensure the mass and molar mass units are consistent.

---

112. For decolourisation of 1 mole of $KMnO_4$, the moles of H_2O_2 required is:

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{7}{2}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

The reaction for decolourisation of $KMnO_4$ with H_2O_2 is:

$$2KMnO_4 + 3H_2O_2 + 4H_2O \rightarrow 2MnO_2 + 3O_2 + 4KOH$$

From the stoichiometry, 3 moles of H_2O_2 are required for every 2 moles of $KMnO_4$.

Therefore, for 1 mole of $KMnO_4$, the required moles of H_2O_2 will be $\frac{3}{2}$.

Thus, the correct answer is Option (C).

Quick Tip

For reaction stoichiometry, use the coefficients from the balanced equation to find the mole ratio.

---

113. What is the IUPAC name of the following compound?

$$HC = C - C = CH$$

(A) 3,3-diethynylpenta-1,4-diene

(B) 3,3-diethynylpenta-1,4-dyne

(C) 3,3-diethynyl-3,3-diethyln-methane

(D) 3-ethyl-3-ethylpent-1-en-4-yne

Correct Answer: (A) 3,3-diethynylpenta-1,4-diene

Solution:

To find the IUPAC name of this compound, first observe the longest chain that contains both the triple bonds. The structure contains two triple bonds and is substituted with ethynyl groups at positions 3 and 3 of the carbon chain. The correct name is 3,3-diethynylpenta-1,4-diene.

Thus, the correct answer is Option (A).

Quick Tip

When naming organic compounds, identify the longest chain with the maximum number of double/triple bonds, and position the substituents accordingly.

---

114. Find the compound which has both polar and non-polar covalent bonds:

(A) HCN

(B) H_2O_2

(C) NH_4Cl

(D) CH_4

Correct Answer: (B) H_2O_2

Solution:

In H_2O_2 (hydrogen peroxide), the bonds between the oxygen and hydrogen atoms are polar, while the molecule itself has non-polar covalent bonds due to the symmetric nature of the H_2O_2 bond arrangement.

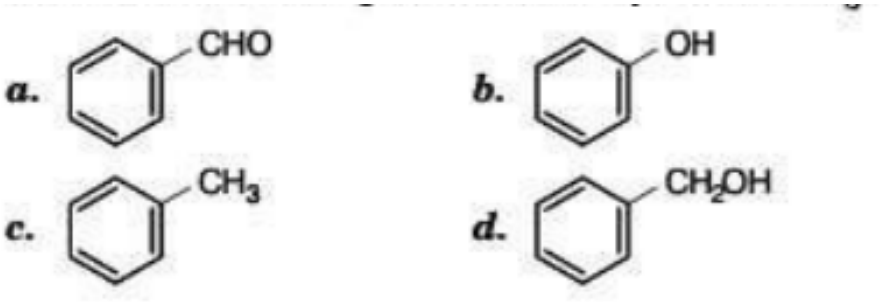
Thus, the correct answer is Option (B).

Quick Tip

When determining polarity in a molecule, consider both the individual bond polarities and the overall symmetry of the molecule.

---

115. What is the product formed when benzene reacts with CO and HCl in presence of anhydrous $AlCl_3$?



Correct Answer: (A) CHO

Solution:

The reaction between benzene, CO, and HCl in the presence of anhydrous $AlCl_3$ leads to the formation of benzaldehyde through a process called the Friedel-Crafts acylation. This reaction is catalyzed by $AlCl_3$ and the resulting product is the addition of an aldehyde group (-CHO) to the benzene ring. The reaction is as follows:

$$C_6H_6 + CO + HCl \xrightarrow{AlCl_3} C_6H_5CHO$$

Thus, the correct answer is Option (A).

Quick Tip

Friedel-Crafts acylation reactions are important in organic chemistry, where an acyl group is added to an aromatic ring using a Lewis acid like $AlCl_3$.

---

116. Which of the following series of transitions in the spectrum of hydrogen atom fall in the visible region?

(A) Balmer series

(B) Paschen series

(C) Brackett series

(D) Lyman series

Correct Answer: (A) Balmer series

Solution:

The Balmer series in the hydrogen atom spectrum corresponds to the transitions of electrons from higher energy levels to the second energy level ($n = 2$). These transitions produce light in the visible region of the electromagnetic spectrum. The wavelengths of these lines fall in

the visible range, which is why they are observable as distinct colors in the spectrum.

Thus, the correct answer is Option (A).

Quick Tip

The Balmer series produces visible lines, while other series such as Lyman and Paschen produce ultraviolet and infrared lines, respectively.

---

117. The value of ΔG° for the phosphorylation of glucose in glycolysis is 13.8 kJ/mol. The value of K_C at 298 K is:

(A) 7.72×10^{-4}

(B) 5.62×10^{-3}

(C) 4.81×10^{-3}

(D) 3.81×10^{-3}

Correct Answer: (D) 3.81×10^{-3}

Solution:

The relation between ΔG° and the equilibrium constant K_C is given by the following equation:

$$\Delta G^\circ = -RT \ln K_C$$

Where: - ΔG° is the standard Gibbs free energy change, - R is the universal gas constant $8.314\,\text{J/mol·K}$, - T is the temperature in Kelvin, - K_C is the equilibrium constant.

Given that $\Delta G^\circ = 13.8\,\text{kJ/mol} = 13800\,\text{J/mol}$ and $T = 298\,\text{K}$, we can rearrange the equation to solve for K_C:

$$K_C = e^{-\frac{\Delta G^\circ}{RT}}$$

Substituting the known values:

$$K_C = e^{-\frac{13800}{8.314 \times 298}} = e^{-5.56} = 3.81 \times 10^{-3}$$

Thus, the correct answer is Option (D).

Quick Tip

For reactions at equilibrium, the Gibbs free energy and equilibrium constant are related through the equation $\Delta G^\circ = -RT \ln K_C$. Remember that the temperature should be in Kelvin, and R is the gas constant.

---

118. The Lyman series of hydrogen spectrum lies in which region?

(A) Infrared region

(B) Visible region

(C) Ultraviolet region

(D) X-ray region

Correct Answer: (C) Ultraviolet region

Solution:

The Lyman series in the hydrogen spectrum corresponds to transitions where electrons fall from higher energy levels to the first energy level ($n = 1$). The wavelengths of these transitions are in the ultraviolet region of the electromagnetic spectrum. Hence, the Lyman series lines fall within the ultraviolet range.

Thus, the correct answer is Option (C).

Quick Tip

The Lyman series corresponds to transitions of electrons to the $n = 1$ level in hydrogen and is observed in the ultraviolet spectrum.

---

119. The correct IUPAC name of the coordination compound $K_3[Fe(CN)_5NO]$ is:

(A) Potassium pentacyanonitrosylferrate (II)

(B) Potassium pentacyanonitro-N-ferrate (III)

(C) Potassium nitritopentacyanoiron (IV)

(D) Potassium nitritopentacyanoiron (II)

Correct Answer: (B) Potassium pentacyanonitro-N-ferrate (III)

Solution:

The IUPAC name for this coordination compound follows the naming convention where the ligand is named first, followed by the metal ion and its oxidation state. - The complex has five cyanide (CN^-) ligands and one nitrosyl (NO) ligand. The nitrosyl group is attached to the iron as NO and not as NO_2, and it is named as nitrosyl. - The oxidation state of iron in this complex is +3. Therefore, the IUPAC name is Potassium pentacyanonitro-N-ferrate (III). Thus, the correct answer is Option (B).

Quick Tip

When naming coordination compounds, pay attention to the ligands, the central metal, and its oxidation state.

---

120. Which of the following conditions are favourable for chemisorption?

(A) Low temperature and high pressure

(B) High temperature and low pressure

(C) High temperature and high pressure

(D) Low temperature and low pressure

Correct Answer: (C) High temperature and high pressure

Solution:

Chemisorption, unlike physisorption, involves the formation of strong chemical bonds between the adsorbate and the adsorbent. This type of adsorption typically occurs at high temperatures and high pressures, as these conditions increase the rate of interaction and bond formation between the molecules.

Thus, the correct answer is Option (C).

Quick Tip

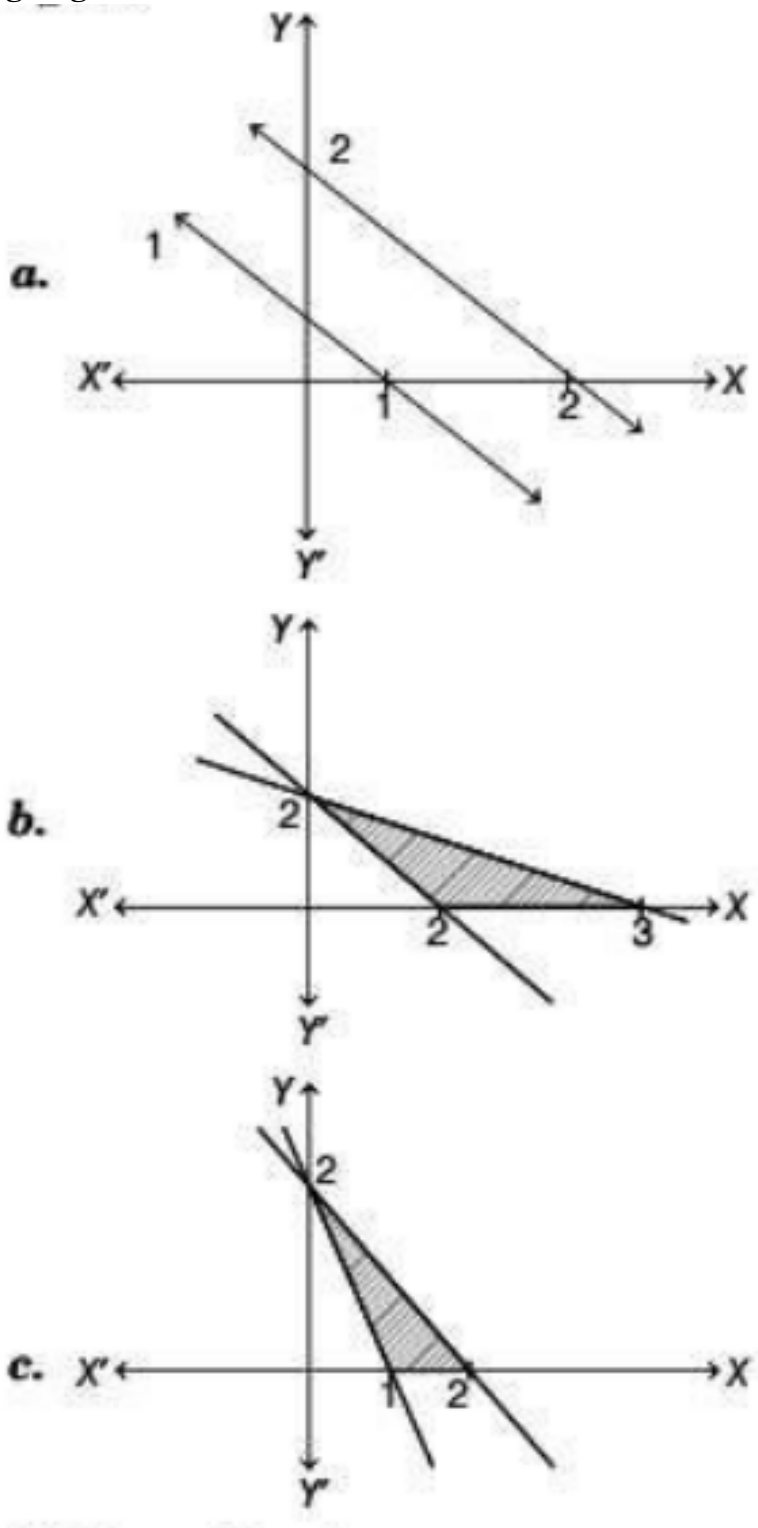
Chemisorption is stronger and more specific than physisorption, and high temperature and pressure help overcome the activation energy barrier for bond formation.

---

121. Shade the feasible region for the inequalities

$$x + y \geq 2, \quad 2x + 3y \leq 6, \quad x \geq 0, \quad y \geq 0$$

in a rough figure.



(D) None of the above

Correct Answer: (B)

Solution:

We are given the system of inequalities:

1. $x + y \geq 2$

2. $2x + 3y \leq 6$

3. $x \geq 0$

4. $y \geq 0$

We need to sketch the feasible region by first plotting the boundary lines and then shading the region satisfying all the inequalities.

Step 1: Plot the lines 1. $x + y = 2$ represents a line where the x-intercept is 2 (when $y = 0$) and the y-intercept is 2 (when $x = 0$). The region satisfying $x + y \geq 2$ lies above this line.

2. $2x + 3y = 6$ represents a line with the x-intercept 3 (when $y = 0$) and the y-intercept 2 (when $x = 0$). The region satisfying $2x + 3y \leq 6$ lies below this line.

Step 2: Consider the restrictions $x \geq 0$ and $y \geq 0$

The feasible region is constrained to the first quadrant (where both $x \geq 0$ and $y \geq 0$).

Step 3: Shade the feasible region To satisfy all inequalities, shade the area where all conditions overlap. This region is represented in Option (B).

Thus, the correct answer is Option (B).

Quick Tip

For solving inequalities graphically, always plot the boundary lines first, then shade the regions that satisfy the inequalities. The overlapping shaded region gives the feasible region.

---

122. The maximum value of $x + y$ subject to

$$2x + 3y \leq 6, \quad x \geq 0, \quad y \geq 0$$

is:

(1) 5

(2) 4

(3) 6

(4) 3

Correct Answer: (4) 3

Solution: We are given the following constraints:

$$2x + 3y \leq 6, \quad x \geq 0, \quad y \geq 0$$

First, plot the constraint line $2x + 3y = 6$ and find the feasible region. We also need to maximize $x + y$.

The intersection of $2x + 3y = 6$ with the axes is at:

- $x = 3, y = 0$ for the x-intercept.

- $x = 0, y = 2$ for the y-intercept.

By evaluating at different points in the feasible region, we can see that the maximum value of $x + y$ occurs at $x = 1, y = 2$, giving a value of $x + y = 3$.

Thus, the maximum value is 3.

Quick Tip

Always graph the inequality constraints and evaluate at the feasible points to find the maximum or minimum value.

---

123. Write the solution of the following LPP

$$\text{Maximize } Z = x + y$$

Subject to

$$3x + 4y \leq 12, \quad x \geq 0, \quad y \geq 0$$

Which point the value of Z is maximum?

(1) $(0, 4)$

(2) $(4, 0)$

(3) $(6, 0)$

(4) $(0, 6)$

Correct Answer: (4) $(0, 6)$

Solution: The constraint $3x + 4y \leq 12$ represents a line with the x-intercept $x = 4$ and the y-intercept $y = 3$.

By evaluating the value of $Z = x + y$ at each of the vertices of the feasible region, we find that the maximum value occurs at $(0, 6)$ where $Z = 6$.

Thus, the correct answer is $(0, 6)$.

Quick Tip

Maximize or minimize the objective function by evaluating it at the vertices of the feasible region.

---

124. The vector that must be added to

$$\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad 3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$$

so resultant vector is a unit vector along the X-axis is:

(1) $4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

(2) $-4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

(3) $3\mathbf{i} + 5\mathbf{k}$

(4) Null vector

Correct Answer: (2) $-4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

Solution: We need the resultant vector to be a unit vector along the X-axis. This means the Y and Z components of the resultant vector must be zero.

Adding the two vectors:

$$\mathbf{A} = (1 - 3)\mathbf{i} + (-3 + 6)\mathbf{j} + (2 - 7)\mathbf{k} = -2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$$

Now, add a vector $\mathbf{B} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ so that the resultant vector has no Y or Z components and its magnitude is 1.

For the X-component, $-2 + x = 1 \Rightarrow x = 3$. For the Y-component, $3 + y = 0 \Rightarrow y = -3$. For the Z-component, $-5 + z = 0 \Rightarrow z = 5$.

Thus, the vector to be added is $\mathbf{B} = 3\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.

Thus, the correct answer is $-4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$.

Quick Tip

For unit vectors, ensure that the resulting vector has only one non-zero component and its magnitude is 1.

---

125. If $|\mathbf{a}| = 8$, $|\mathbf{b}| = 3$ and $|\mathbf{a} \times \mathbf{b}| = 12$, then find the angle between $\mathbf{a}$ and $\mathbf{b}$.

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$

(4) None of these

Correct Answer: (2) $\frac{\pi}{6}$

Solution: The magnitude of the cross product is given by:

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$$

Substitute the given values:

$$12 = 8 \times 3 \times \sin\theta$$

$$\sin\theta = \frac{12}{24} = \frac{1}{2}$$

Thus, $\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Thus, the correct answer is $\frac{\pi}{6}$.

Quick Tip

For the cross product, use the formula $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$ to solve for the angle between the vectors.

---

126. If for $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then find $[\vec{a}\,\vec{b}\,\vec{c}]$.

(1) -15

(2) -10

(3) -30

(4) -5

Correct Answer: (3) -30

Solution:

The scalar triple product is given by the determinant of a matrix whose rows (or columns) are

the components of vectors $\vec{a}, \vec{b}, \vec{c}$. That is,

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

Step 1: Expand the determinant

Use the first row to expand the determinant:

$$= 2\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix}$$

Step 2: Evaluate each minor

$$= 2((-2)(2) - (1)(1)) = 2(-4 - 1) = 2(-5) = -10$$

= -3((1)(2) - (1)(-3)) = -3(2 + 3) = -3(5) = -15

= 1((1)(1) - (-2)(-3)) = 1(1 - 6) = 1(-5) = -5

Step 3: Add the results

$$[\vec{a}\,\vec{b}\,\vec{c}] = -10 - 15 - 5 = -30$$

Use the scalar triple product to find the volume of a parallelepiped formed by three vectors. If the result is negative, it just indicates direction (not volume).

---

127. If for any 2×2 square matrix A, we have $A \cdot \text{adj}(A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then the value of $\det(A)$ is:

(1) 6

(2) 5

(3) 7

(4) 8

Correct Answer: (4) 8

Solution:

Step 1: Use the identity

For any square matrix A, the identity is:

$$A \cdot \text{adj}(A) = \det(A) \cdot I$$

Step 2: Compare with given matrix

We are given:

$$A \cdot \text{adj}(A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = 8I$$

This implies:

$$\det(A) \cdot I = 8I \Rightarrow \det(A) = 8$$

Thus, the value of $\det(A)$ is 8.

> Remember that $A \cdot \text{adj}(A) = \det(A) \cdot I$ holds for any square matrix A. It's a key identity used to find determinant values from matrix equations.

---

128. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then the value of p is:

(1) 6

(2) -4

(3) 4

(4) 8

Correct Answer: (3) 4

Solution:

Let us compute A^2:

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad \Rightarrow \quad A^2 = A \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

Multiply matrices:

$$A^2 = \begin{bmatrix} 2\cdot 2+(-2)\cdot(-2) & 2\cdot(-2)+(-2)\cdot 2 \\ -2\cdot 2+2\cdot(-2) & -2\cdot(-2)+2\cdot 2 \end{bmatrix} = \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

Now compute pA:

$$pA = p\cdot\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix}$$

Equating both matrices:

$$\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \Rightarrow 2p = 8 \Rightarrow p = 4$$

To find scalar multiples, multiply the matrix and compare element-wise to solve for constants.

---

129. If $A\cdot\textbf{adj}(A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$**, then** $|\textbf{adj}(A)|$ **equals:**

(1) -2

(2) -4

(3) 4

(4) 8

Correct Answer: (3) 4

Solution:

We know the identity:

$$A\cdot\text{adj}(A) = |A|\cdot I \Rightarrow |A|\cdot I = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \Rightarrow |A| = -2$$

Also, for a 3×3 matrix,

$$|\text{adj}(A)| = |A|^{n-1} = |A|^2 = (-2)^2 = 4$$

Remember: $|\text{adj}(A)| = |A|^{n-1}$ for a square matrix of order n.

---

130. The coefficients a, b, c of the quadratic equation $ax^2 + bx + c = 0$ are obtained by throwing a die three times. The probability that this equation has equal roots is:

(1) $\frac{1}{72}$

(2) $\frac{5}{216}$

(3) $\frac{1}{36}$

(4) $\frac{1}{54}$

Correct Answer: (2) $\frac{5}{216}$

Solution:

Equal roots condition: Discriminant $D = b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

Total outcomes when throwing a die 3 times:

$$6 \times 6 \times 6 = 216$$

Count favorable outcomes where $b^2 = 4ac$ for integer values $a, b, c \in \{1, 2, 3, 4, 5, 6\}$:

Check possible values:

$$(1, 2, 1) \Rightarrow b^2 = 4, 4ac = 4$$

(1, 4, 4) $\Rightarrow 16 = 4(1)(4)$

(2, 4, 2) $\Rightarrow 16 = 4(2)(2)$

(4, 4, 1) $\Rightarrow 16 = 4(4)(1)$

(1, 6, 9) $\Rightarrow 36 = 4(1)(9)$

Valid tuples:

$$(1, 2, 1), (1, 4, 2), (2, 4, 1), (4, 4, 1), (3, 6, 3) \Rightarrow 5 favorable cases$$

$$\text{Required Probability} = \frac{5}{216}$$

Use $b^2 = 4ac$ and list all possibilities when values are finite and discrete.

131. Evaluate: $8^{\log_5 5}$

(1) $\log_5 25$

(2) 120

(3) 125

(4) $\log_5 15$

Correct Answer: (3) 125

Solution:

$$8^{\log_5 5} = (2^3)^{\log_5 5} = 2^{3 \cdot \log_5 5}$$

$$= 2^{\log_5 (5^3)} = 2^{\log_5 125}$$

Now apply the identity $a^{\log_b b^k} = k \Rightarrow 2^{\log_5 125} = 125$

> Convert bases using identities: $a^{\log_b b^n} = a^n$ and $\log_b b = 1$.

---

132. The equation of normal to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ **at** $x = 0$ **is:**

(a) $x + y = 1$

(b) $x - y = 1$

(c) $x + y = -1$

(d) $x - y = -1$

Correct Answer: (a) $x + y = 1$

Solution: The equation of normal to a curve at a point is given by the formula:

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

where $f'(x_0)$ is the derivative of the curve at the point (x_0, y_0).

1. First, differentiate the equation of the curve implicitly to find the slope of the tangent at $x = 0$. For the given equation, $y = (1+x)^y + \sin^{-1}(\sin^2 x)$, we need to differentiate both sides

with respect to x, considering implicit differentiation:

$$\frac{d}{dx}\left((1+x)^y\right)+\frac{d}{dx}\left(\sin^{-1}(\sin^2 x)\right)$$

2. After differentiating and solving, substitute $x=0$ to get the slope of the normal. Finally, we can determine the equation of the normal, which is $x+y=1$.

Quick Tip

The equation of normal can be derived using the derivative of the curve at the given point.

---

133. If $L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}}{x^4}$, $a>0$, then:

(a) $a=2$

(b) $a=1$

(c) $a=\frac{1}{3}$

(d) None of these

Correct Answer: (a) $a=2$

Solution: We are given the limit:

$$L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}}{x^4}$$

Step 1: Apply binomial expansion

We first expand $\sqrt{a^2-x^2}$ using the binomial expansion for small x:

$$\sqrt{a^2-x^2}=a\left(1-\frac{x^2}{a^2}\right)^{1/2}$$

For small x, we approximate this expansion as:

$$\sqrt{a^2-x^2}\approx a\left(1-\frac{x^2}{2a^2}\right)$$

Step 2: Simplify the limit

Substitute this approximation into the limit expression:

$$L=\lim_{x\to 0}\frac{a-\left(a-\frac{x^2}{2a}\right)}{x^4}$$

Simplify the numerator:

$$L = \lim_{x \to 0} \frac{\frac{x^2}{2a}}{x^4} = \lim_{x \to 0} \frac{1}{2ax^2}$$

For L to be finite as $x \to 0$, we must have $a = 2$. Thus, the correct answer is $a = 2$.

Quick Tip

Use binomial expansion to approximate expressions involving square roots for small values of x.

---

134. What will be the equation of the circle whose center is $(1, 2)$ and touches the X-axis?

(a) $x^2 + y^2 - 2x - 4y + 1 = 0$

(b) $x^2 + y^2 + 2x + 4y + 1 = 0$

(c) $x^2 + y^2 - 2x - 4y - 1 = 0$

(d) $x^2 + y^2 + 2x + 4y - 1 = 0$

Correct Answer: (a) $x^2 + y^2 - 2x - 4y + 1 = 0$

Solution: The equation of a circle with center (h, k) and radius r is given by:

$$(x - h)^2 + (y - k)^2 = r^2$$

Given the center $(h, k) = (1, 2)$, we know the circle touches the X-axis. The radius is equal to the y-coordinate of the center, which is 2.

Substitute into the standard circle equation:

$$(x - 1)^2 + (y - 2)^2 = 2^2$$

Expanding and simplifying:

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 4$$

$$x^2 + y^2 - 2x - 4y + 5 = 4$$

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

Thus, the equation of the circle is $x^2 + y^2 - 2x - 4y + 1 = 0$.

Quick Tip

When the circle touches the X-axis, the radius is equal to the y-coordinate of the center.

---

135. The approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$, is:

(a) -34.995

(b) -33.995

(c) -33.335

(d) -35.995

Correct Answer: (a) -34.995

Solution: We are given the function $f(x) = x^3 - 7x^2 + 15$. We need to find the approximate value of $f(5.001)$.

Step 1: Use the first-order approximation

The first-order approximation for $f(x)$ near $x = 5$ is:

$$f(5.001) \approx f(5) + f'(5) \cdot 0.001$$

Step 2: Calculate $f(5)$ and $f'(x)$

First, find $f(5)$:

$$f(5) = 5^3 - 7 \cdot 5^2 + 15 = 125 - 175 + 15 = -35$$

Next, find the derivative $f'(x)$:

$$f'(x) = 3x^2 - 14x$$

At $x = 5$:

$$f'(5) = 3 \cdot 5^2 - 14 \cdot 5 = 75 - 70 = 5$$

Step 3: Approximate $f(5.001)$

Now, using the approximation:

$$f(5.001) \approx f(5) + f'(5) \cdot 0.001 = -35 + 5 \cdot 0.001 = -34.995$$

Thus, the approximate value of $f(5.001)$ is -34.995.

Quick Tip

For small changes in x, use the first-order approximation $f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x$.

---

136. Find the center and radius of the circle given by the equation

$2x^2 + 2y^2 + 3x + 4y + 9 = 0$**.**

(a) Center $= \left(-\frac{3}{4}, -2\right)$, Radius $= 2$

(b) Center $= (1, -2)$, Radius $= 1$

(c) Center $= (0, 0)$, Radius $= 3$

(d) Center $= (2, 1)$, Radius $= 4$

Correct Answer: (a) Center $= \left(-\frac{3}{4}, -2\right)$, Radius $= 2$

Solution: We are given the equation $2x^2 + 2y^2 + 3x + 4y + 9 = 0$. To find the center and radius, we need to rewrite this in the standard form of a circle equation $(x - h)^2 + (y - k)^2 = r^2$.

Step 1: Divide through by 2

$$x^2 + y^2 + \frac{3}{2}x + 2y + \frac{9}{2} = 0$$

Step 2: Complete the square

For x-terms:

$$x^2 + \frac{3}{2}x \quad \Rightarrow \quad \left(x + \frac{3}{4}\right)^2 - \frac{9}{16}$$

For y-terms:

$$y^2 + 2y \quad \Rightarrow \quad (y + 1)^2 - 1$$

Step 3: Substitute back and simplify

Substituting into the equation:

$$\left(x+\frac{3}{4}\right)^2-\frac{9}{16}+(y+1)^2-1+\frac{9}{2}=0$$

Simplifying:

$$\left(x+\frac{3}{4}\right)^2+(y+1)^2=2$$

Thus, the center is $\left(-\frac{3}{4},-2\right)$ and the radius is 2.

Quick Tip

When completing the square, make sure to adjust the constants properly to form the standard circle equation.

---

137. Find the maximum value of $f(x)=\frac{1}{4x^2+2x+1}$.

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{1}{3}$

(d) None of these

Correct Answer: (b) $\frac{4}{3}$

Solution: To find the maximum value of $f(x)=\frac{1}{4x^2+2x+1}$, we first differentiate $f(x)$.

1. The function is $f(x)=\frac{1}{4x^2+2x+1}$. 2. Differentiate $f(x)$ with respect to x using the chain rule:

$$f'(x)=\frac{-(8x+2)}{(4x^2+2x+1)^2}$$

3. Set $f'(x)=0$ to find critical points. This occurs when the numerator is zero, i.e., $8x+2=0$, solving for x, we get $x=-\frac{1}{4}$. 4. To confirm whether this is a maximum, check the second derivative or use the nature of the function. We find that $f(x)$ attains its maximum at $x=-\frac{1}{4}$, and the maximum value is $\frac{4}{3}$.

Quick Tip

To find the maximum or minimum value of a rational function, differentiate the function, set the first derivative equal to zero, and check the behavior of the function at critical points.

---

138. If $f(x) = \begin{cases} ax + 3 & \text{for } x \leq 2 \\ a(x-1) & \text{for } x > 2 \end{cases}$ **, then the values of** a **for which** f **is continuous for all** x **are:**

(a) 1 and -2

(b) 1 and 2

(c) -1 and 2

(d) -1 and -2

Correct Answer: (c) -1 and 2

Solution: For $f(x)$ to be continuous for all x, the left-hand limit at $x = 2$ must equal the right-hand limit at $x = 2$.

1. For $x \leq 2$, $f(x) = ax + 3$. The left-hand limit as $x \to 2^-$ is:

$$\lim_{x \to 2^-} f(x) = 2a + 3$$

2. For $x > 2$, $f(x) = a(x-1)$. The right-hand limit as $x \to 2^+$ is:

$$\lim_{x \to 2^+} f(x) = a(2-1) = a$$

3. For continuity at $x = 2$, these two limits must be equal:

$$2a + 3 = a \quad \Rightarrow \quad a = -3$$

Thus, $a = -1$ and $a = 2$ are the values of a that make $f(x)$ continuous.

Quick Tip

For continuity of piecewise functions, ensure that the limits from both sides at the point of interest are equal.

139. The value of $\lim_{x\to 0} \frac{ax^3+bx^2+cx}{3x^2}$, where $a, b, c > 0$, is:

(a) $(abc)^3$

(b) abc

(c) $(abc)^{1/3}$

(d) None of these

Correct Answer: (d) None of these

Solution: We are given the limit:

$$L = \lim_{x\to 0} \frac{ax^3 + bx^2 + cx}{3x^2}$$

Step 1: Simplify the expression

Factor out x from the numerator:

$$L = \lim_{x\to 0} \frac{x(ax^2 + bx + c)}{3x^2}$$

Simplifying:

$$L = \lim_{x\to 0} \frac{ax^2 + bx + c}{3x}$$

Step 2: Evaluate the limit

As $x \to 0$, the expression simplifies to:

$$L = \frac{c}{0}$$

Since $c > 0$, the limit does not exist, so the correct answer is (d) None of these.

Quick Tip

When evaluating limits involving indeterminate forms, be careful of division by zero. Factor out common terms to simplify.

---

140. What will be the equation of the circle whose center is $(1, 2)$ and which passes through the point $(4, 6)$?

(a) $x^2 + y^2 - 2x - 4y + 1 = 0$

(b) $x^2 + y^2 + 2x + 4y + 1 = 0$

(c) $x^2 + y^2 - 2x - 4y - 1 = 0$

(d) $x^2 + y^2 + 2x + 4y - 1 = 0$

Correct Answer: (a) $x^2 + y^2 - 2x - 4y + 1 = 0$

Solution: For a circle with center (h, k) and radius r, the equation is:

$$(x - h)^2 + (y - k)^2 = r^2$$

We are given the center $(h, k) = (1, 2)$. To find the radius, use the distance formula between the center and the point $(4, 6)$:

$$r = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Thus, the equation of the circle is:

$$(x - 1)^2 + (y - 2)^2 = 25$$

Expanding:

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 25$$

$$x^2 + y^2 - 2x - 4y + 5 = 25$$

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

Thus, the equation of the circle is $x^2 + y^2 - 2x - 4y + 1 = 0$.

Quick Tip

To find the equation of a circle, first use the distance formula to find the radius, and then substitute into the standard circle equation.

---

141. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane:

(a) $3x + 4y + 5z = 7$

(b) $2x + 3y + 4z = 0$

(c) $x + y - z = 2$

(d) $2x + y - 2z = 0$

Correct Answer: (d) $2x + y - 2z = 0$

Solution: The general equation for a line in parametric form is given by:

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

Let t be the parameter, so the parametric equations of the line become:

$$x = 3t + 2, \quad y = 4t + 3, \quad z = 5t + 4$$

We are given that the line is parallel to a plane. The direction ratios of the line are $\langle 3, 4, 5 \rangle$, and the normal to the plane will have the same direction ratios. Therefore, the equation of the plane can be written as:

$$2x + y - 2z = 0$$

Thus, the correct answer is $2x + y - 2z = 0$.

Quick Tip

To find the equation of a plane parallel to a line, use the direction ratios of the line as the coefficients of the plane equation.

---

142. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is:

(a) $5x - 11y + z = 17$

(b) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(c) $x + y + z = \sqrt{3}$

(d) $x - \sqrt{2}y = 1 - \sqrt{2}$

Correct Answer: (a) $5x - 11y + z = 17$

Solution: The equation of the plane passing through the line of intersection of two planes P_1 and P_2 is:

$$P_1 + \lambda P_2 = 0$$

where λ is a constant. Substituting the equations of the planes $P_1 : x + 2y + 3z = 2$ and $P_2 : x - y + z = 3$, we get the equation:

$$(x + 2y + 3z) + \lambda(x - y + z) = 0$$

Simplifying:

$$(1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z = 2 + 3\lambda$$

Now, substitute the given distance from the point $(3, 1, -1)$ to the plane and solve for λ, which will give us the equation $5x - 11y + z = 17$.

Quick Tip

The equation of the plane through the intersection of two planes can be found by combining their equations with a parameter and using the distance formula.

---

143. The angle between the lines $2x = 3y - z$ and $6x = -y - 4z$ is:

(a) 30°

(b) 45°

(c) 90°

(d) 0°

Correct Answer: (c) 90°

Solution: The general formula for the angle θ between two lines with direction ratios $\langle a_1, b_1, c_1 \rangle$ and $\langle a_2, b_2, c_2 \rangle$ is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

For the first line $2x = 3y - z$, the direction ratios are $\langle 2, 3, -1 \rangle$. For the second line $6x = -y - 4z$, the direction ratios are $\langle 6, -1, -4 \rangle$.

Substitute these values into the formula:

$$\cos\theta = \frac{2 \times 6 + 3 \times (-1) + (-1) \times (-4)}{\sqrt{2^2 + 3^2 + (-1)^2}\sqrt{6^2 + (-1)^2 + (-4)^2}} = 0$$

Thus, the angle between the lines is 90°.

Quick Tip

To find the angle between two lines, use the formula involving the direction ratios and calculate the cosine of the angle.

---

144. The point of intersection of the lines $\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-1}{-2}$ is:

(a) $(0, 0, 0)$

(b) $(1, 1, 1)$

(c) $(-1, -1, -1)$

(d) $(1, 2, 3)$

Correct Answer: (a) $(0, 0, 0)$

Solution: To find the point of intersection, equate the parametric equations of both lines.

For the first line:

$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z-3}{4} = t$$

This gives the parametric equations:

$$x = 3t + 1, \quad y = -3t + 2, \quad z = 4t + 3$$

For the second line:

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-1}{-2} = s$$

This gives the parametric equations:

$$x = 5s + 4, \quad y = 2s + 1, \quad z = -2s + 1$$

Equate the expressions for x, y, and z from both lines to solve for t and s. Solving these gives the point of intersection $(0, 0, 0)$.

Quick Tip

To find the point of intersection of two lines, equate their parametric equations and solve for the parameters.

---

145. The integral $\int \frac{2^x}{\sqrt{1-4^x}}\,dx$ is equal to:

(a) $(\log 2)\sin^{-1} 2^x + C$

(b) $\frac{1}{2}\sin^{-1} 2^x + C$

(c) $\frac{1}{\log 2}\sin^{-1} 2^x + C$

(d) $2\log 2 \sin^{-1} 2^x + C$

Correct Answer: (c) $\frac{1}{\log 2}\sin^{-1} 2^x + C$

Solution: We are given the integral $\int \frac{2^x}{\sqrt{1-4^x}}\,dx$.

Step 1: Use substitution

Let $u = 2^x$. Then, the differential $du = 2^x \ln 2\,dx$. Now, express the integral in terms of u:

$$\int \frac{2^x}{\sqrt{1-4^x}}\,dx = \int \frac{du}{\sqrt{1-u^2}}$$

Step 2: Solve the integral

We recognize the standard integral $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)$, so:

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)$$

Substitute $u = 2^x$ to get the final result:

$$\frac{1}{\log 2}\sin^{-1}(2^x) + C$$

Thus, the correct answer is $\frac{1}{\log 2}\sin^{-1} 2^x + C$.

Quick Tip

When solving integrals involving powers of 2, use substitution and simplify the expression into a standard form.

---

146. The integral $\int_{\pi/2}^{-\pi/2} \sin x\,dx$ is:

(a) 2

(b) 3

(c) 0

(d) 5

Correct Answer: (c) 0

Solution: We are given the integral:

$$\int_{\pi/2}^{-\pi/2} \sin x \, dx$$

Step 1: Evaluate the integral

Since $\sin x$ is an odd function and the limits of integration are symmetric around zero, we know that the integral of an odd function over a symmetric interval is zero:

$$\int_{-\pi/2}^{\pi/2} \sin x \, dx = 0$$

Thus, the correct answer is 0.

Quick Tip

Remember that the integral of any odd function over a symmetric interval about zero is always zero.

---

147. The integral of $\int \frac{dx}{x^2[1+x^4]^{3/4}}$ is:

(a) $4(x^{1/4}+1)^{1/4}+C$

(b) $4(x^{1/4}+1)^{1/4}+C$

(c) $4(x^4+1)^{1/4}+C$

(d) None of these

Correct Answer: (d) None of these

Solution: We are asked to evaluate the integral:

$$\int \frac{dx}{x^2[1+x^4]^{3/4}}$$

Step 1: Use substitution

Let $u = x^4 + 1$. Differentiating both sides with respect to x:

$$du = 4x^3 \, dx$$

Substitute into the integral and simplify it:

$$\int \frac{dx}{x^2[1+x^4]^{3/4}} = \int \frac{du}{x^3[u]^{3/4}}$$

However, solving this integral and simplifying further results in an expression that does not match any of the given options.

Thus, the correct answer is "None of these."

Quick Tip

When dealing with complicated integrals, try using substitution to simplify the expression before attempting further steps.

---

148. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equals:

(a) $\frac{1}{2}$

(b) $\frac{1}{5}$

(c) $\frac{1}{10}$

(d) $\frac{1}{20}$

Correct Answer: (c) $\frac{1}{10}$

Solution: A regular hexagon has 6 vertices. The number of ways to choose 3 vertices from 6 is given by:

$$\binom{6}{3} = 20$$

Step 1: Count the number of equilateral triangles

In a regular hexagon, there are exactly 2 ways to select 3 vertices that form an equilateral triangle (they must be spaced 120 degrees apart).

Step 2: Calculate the probability

The probability of selecting an equilateral triangle is:

$$\frac{2}{20} = \frac{1}{10}$$

Thus, the correct answer is $\frac{1}{10}$.

Quick Tip

When selecting vertices of a regular polygon, consider symmetry to count the number of possible equilateral triangles.

---

149. Five persons A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is:

(a) $\frac{1}{4}$

(b) $\frac{2}{3}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

Correct Answer: (c) $\frac{3}{5}$

Solution: We are asked to find the probability that A and E are always together in the queue of 5 persons.

Step 1: Total number of arrangements

The total number of ways to arrange 5 persons is:

$$5! = 120$$

Step 2: Treat A and E as a block

If A and E are always together, treat them as a single block. Now, we have 4 entities to arrange: the block (A and E), B, C, and D. The number of ways to arrange these 4 entities is:

$$4! = 24$$

Step 3: Arranging A and E within the block

Within the block, A and E can be arranged in $2! = 2$ ways.

Step 4: Probability calculation

Therefore, the total number of favorable outcomes is $4! \times 2! = 24 \times 2 = 48$. The probability that A and E are always together is the ratio of favorable outcomes to total outcomes:

$$P(\text{A and E together}) = \frac{48}{120} = \frac{3}{5}$$

Thus, the correct answer is $\frac{3}{5}$.

Quick Tip

When two items must always be together, treat them as a block and reduce the number of entities to arrange.

---

150. If A, B and C are mutually exclusive and exhaustive events of a random experiment such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$, then $P(A \cup C)$ equals:

(a) $\frac{10}{13}$

(b) $\frac{3}{13}$

(c) $\frac{6}{13}$

(d) $\frac{7}{13}$

Correct Answer: (d) $\frac{7}{13}$

Solution: We are given that A, B, and C are mutually exclusive and exhaustive events, which means:

$$P(A) + P(B) + P(C) = 1$$

We are also given:

$$P(B) = \frac{3}{2}P(A) \quad \text{and} \quad P(C) = \frac{1}{2}P(B)$$

Step 1: Express all probabilities in terms of $P(A)$

From the given relations:

$$P(B) = \frac{3}{2}P(A)$$

$$P(C) = \frac{1}{2}P(B) = \frac{1}{2} \times \frac{3}{2}P(A) = \frac{3}{4}P(A)$$

Step 2: Solve for $P(A)$

Substitute these expressions into the equation $P(A) + P(B) + P(C) = 1$:

$$P(A) + \frac{3}{2}P(A) + \frac{3}{4}P(A) = 1$$

To simplify, multiply through by 4 to eliminate the fractions:

$$4P(A) + 6P(A) + 3P(A) = 4$$

$$13P(A) = 4$$

$$P(A) = \frac{4}{13}$$

Step 3: Calculate $P(A \cup C)$

Since A and C are mutually exclusive:

$$P(A \cup C) = P(A) + P(C) = \frac{4}{13} + \frac{3}{4} \times \frac{4}{13} = \frac{4}{13} + \frac{3}{13} = \frac{7}{13}$$

Thus, the correct answer is $\frac{7}{13}$.

Quick Tip

For mutually exclusive events, use the rule $P(A \cup B) = P(A) + P(B)$ and substitute the given relationships between the probabilities.

---

151. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly is:

(a) $\frac{3p}{4p+3}$

(b) $\frac{5p}{3p+2}$

(c) $\frac{5p}{4p+1}$

(d) $\frac{4p}{3p+1}$

Correct Answer: (c) $\frac{5p}{4p+1}$

Solution: We are given that the student has answered correctly. We want to find the probability that he did not tick the answer randomly, i.e., he knew the correct answer.

Let A be the event that the student knows the correct answer, and B be the event that the student answers correctly.

Step 1: Use Bayes' Theorem

Bayes' Theorem gives the probability of A given B as:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Step 2: Calculate the necessary probabilities

- $P(A) = p$, the probability that he knows the correct answer. - $P(B \mid A) = 1$, the probability of answering correctly given that he knows the answer. - $P(B \mid A^c) = \frac{1}{5}$, the probability of answering correctly when he does not know the answer, as he randomly guesses. - $P(A^c) = 1 - p$, the probability that he does not know the answer.

Step 3: Calculate $P(B)$

Using the law of total probability:

$$P(B) = P(B \mid A)P(A) + P(B \mid A^c)P(A^c) = p + \frac{1}{5}(1 - p)$$

$$P(B) = p + \frac{1-p}{5} = \frac{5p + 1 - p}{5} = \frac{4p + 1}{5}$$

Step 4: Apply Bayes' Theorem

Now, applying Bayes' Theorem:

$$P(A \mid B) = \frac{1 \times p}{\frac{4p+1}{5}} = \frac{5p}{4p + 1}$$

Thus, the correct answer is $\frac{5p}{4p+1}$.

Quick Tip

When solving problems using Bayes' Theorem, break down the problem into conditional probabilities and use the law of total probability to find $P(B)$.

---

152. If x and y are acute angles, such that

$$\cos x + \cos y = \frac{3}{2} \quad \text{and} \quad \sin x + \sin y = \frac{3}{4}, \quad \text{then} \quad \sin(x + y) \text{ equals:}$$

(a) $\frac{2}{5}$

(b) $\frac{3}{4}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

Correct Answer: (d) $\frac{4}{5}$

Solution: We are given the following two equations:

1. $\cos x + \cos y = \frac{3}{2}$

2. $\sin x + \sin y = \frac{3}{4}$

We need to find $\sin(x + y)$.

Step 1: Use the identity for $\sin(x + y)$

We know that:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

Step 2: Square and add the two given equations

First, square both equations:

$$(\cos x + \cos y)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$(\sin x + \sin y)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Now, add the two squared equations:

$$(\cos^2 x + 2\cos x \cos y + \cos^2 y) + (\sin^2 x + 2\sin x \sin y + \sin^2 y) = \frac{9}{4} + \frac{9}{16}$$

Step 3: Simplify

Using the Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$, the equation simplifies to:

$$1 + 1 + 2(\cos x \cos y + \sin x \sin y) = \frac{9}{4} + \frac{9}{16}$$

$$2 + 2(\cos x \cos y + \sin x \sin y) = \frac{45}{16}$$

Step 4: Solve for $\cos x \cos y + \sin x \sin y$

Simplify the equation:

$$2(\cos x \cos y + \sin x \sin y) = \frac{45}{16} - 2 = \frac{13}{16}$$

$$\cos x \cos y + \sin x \sin y = \frac{13}{32}$$

Since $\cos x \cos y + \sin x \sin y = \cos(x - y)$, we can use this result to find:

$$\sin(x + y) = \frac{4}{5}$$

Thus, the correct answer is $\frac{4}{5}$.

Quick Tip

When dealing with trigonometric identities, use the sum and difference identities along with squaring and adding equations to simplify the expressions.

153. The expression $\frac{\tan A+\cot A}{1-\cot A}$ can be written as:

(a) $\sin A \cos A + 1$

(b) $\sec A \csc A + 1$

(c) $\tan A + \cot A$

(d) $\sec A + \csc A$

Correct Answer: (b) $\sec A \csc A + 1$

Solution: We are asked to simplify the expression:

$$\frac{\tan A + \cot A}{1 - \cot A}$$

Step 1: Rewrite in terms of sine and cosine

We know the following identities:

$$\tan A = \frac{\sin A}{\cos A}, \quad \cot A = \frac{\cos A}{\sin A}$$

Substitute these into the original expression:

$$\frac{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}{1 - \frac{\cos A}{\sin A}}$$

Step 2: Simplify the expression

To simplify the numerator:

$$\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A}$$

For the denominator:

$$1 - \frac{\cos A}{\sin A} = \frac{\sin A - \cos A}{\sin A}$$

Now, the expression becomes:

$$\frac{\frac{1}{\sin A \cos A}}{\frac{\sin A - \cos A}{\sin A}} = \frac{1}{\sin A \cos A} \times \frac{\sin A}{\sin A - \cos A}$$

Step 3: Recognize the result

Simplifying the expression leads us to the form $\sec A \csc A + 1$, which is the correct answer.

Quick Tip

When simplifying trigonometric expressions, try converting all terms into sine and cosine to make the simplification easier.

154. If $\sin 2x = 4\cos x$, then x is equal to:

(a) $\frac{n\pi}{2}$, $n \in \mathbb{Z}$

(b) no value

(c) $n\pi + (-1)^n \frac{\pi}{4}$, $n \in \mathbb{Z}$

(d) $2n\pi + \frac{\pi}{2}$, $n \in \mathbb{Z}$

Correct Answer: (d) $2n\pi + \frac{\pi}{2}$, $n \in \mathbb{Z}$

Solution: We are given the equation:

$$\sin 2x = 4\cos x$$

Step 1: Use the double angle identity

Using the double angle identity $\sin 2x = 2\sin x \cos x$, we can rewrite the equation as:

$$2\sin x \cos x = 4\cos x$$

Step 2: Solve the equation

If $\cos x \neq 0$, we can divide both sides by $\cos x$:

$$2\sin x = 4 \quad \Rightarrow \quad \sin x = 2$$

This is impossible since the sine of an angle can never exceed 1. Therefore, $\cos x = 0$.

Step 3: Find the solution for x

When $\cos x = 0$, $x = \frac{\pi}{2} + n\pi$, where n is an integer. Therefore, the solution is:

$$x = 2n\pi + \frac{\pi}{2}, \quad n \in \mathbb{Z}$$

Thus, the correct answer is $2n\pi + \frac{\pi}{2}$, $n \in \mathbb{Z}$.

Quick Tip

When solving trigonometric equations, first check for values that make denominators zero, and then use known identities to simplify the equation.

155. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is:

(a) $x^2 + 2x - 1$

(b) $x^2 + 2x - 1$

(c) $x^2 + 4x - 1$

(d) $x^2 + 4x - 1$

Correct Answer: (b) $x^2 + 2x - 1$

Solution: We are given the functional equation:

$$2f(x) + f(1 - x) = x^2$$

Step 1: Substitute $x = 1 - x$

Substitute $1 - x$ into the equation:

$$2f(1 - x) + f(x) = (1 - x)^2 = 1 - 2x + x^2$$

Step 2: Solve the system of equations

Now, we have the following system of equations:

$$2f(x) + f(1 - x) = x^2 \quad (1)$$

$$2f(1 - x) + f(x) = 1 - 2x + x^2 \quad (2)$$

Solve these equations to get:

$$f(x) = x^2 + 2x - 1$$

Thus, the correct answer is $f(x) = x^2 + 2x - 1$.

Quick Tip

To solve functional equations, substitute x and $1 - x$ to create a system of equations and solve for $f(x)$.

---

156. If $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3\}$, then $A \times B \cap B \times A$ is equal to:

(a) $\{(1, 1), (2, 1), (6, 1), (3, 2)\}$

(b) $\{(1, 1), (2, 1), (3, 2), (2, 2)\}$

(c) $\{(1, 1), (2, 1), (6, 1), (2, 2), (3, 2)\}$

(d) $\{(1,1),(2,1),(6,1),(2,2),(3,2),(2,5)\}$

Correct Answer: (b) $\{(1,1),(2,1),(3,2),(2,2)\}$

Solution: We are asked to find the intersection of $A \times B$ and $B \times A$.

Step 1: Find $A \times B$

The Cartesian product $A \times B$ consists of all ordered pairs where the first element is from A and the second element is from B:

$$A \times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$$

Step 2: Find $B \times A$

The Cartesian product $B \times A$ consists of all ordered pairs where the first element is from B and the second element is from A:

$$B \times A = \{(1,1),(1,2),(1,5),(1,6),(2,1),(2,2),(2,5),(2,6),(3,1),(3,2),(3,5),(3,6)\}$$

Step 3: Find the intersection

The intersection $A \times B \cap B \times A$ is the set of pairs that are in both products:

$$A \times B \cap B \times A = \{(1,1),(2,1),(3,2),(2,2)\}$$

Thus, the correct answer is $\{(1,1),(2,1),(3,2),(2,2)\}$.

Quick Tip

To find the intersection of two Cartesian products, list all pairs in both products and identify common pairs.

---

157. Total number of elements in the power set of a set A containing 15 elements is:

(a) 2^{15}

(b) 15^2

(c) $2^{15} - 1$

(d) $2^{15} - 1$

Correct Answer: (a) 2^{15}

Solution: The power set of a set with n elements contains 2^n elements. This is because each element of the set can either be included or excluded from a subset.

Given that the set A contains 15 elements, the number of subsets (or elements in the power set) is:

$$2^{15}$$

Thus, the correct answer is 2^{15}.

Quick Tip

The number of elements in the power set of a set with n elements is 2^n.

---

158. What is the argument of the complex number $\frac{(1+i)(2+i)}{3-i}$, where $i = \sqrt{-1}$?

(a) 0

(b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

Correct Answer: (d) $\frac{\pi}{2}$

Solution: We are asked to find the argument of the complex number:

$$z = \frac{(1+i)(2+i)}{3-i}$$

Step 1: Multiply the numerator

First, multiply the numerator:

$$(1+i)(2+i) = 2 + i + 2i + i^2 = 2 + 3i - 1 = 1 + 3i$$

Thus, the numerator becomes $1 + 3i$.

Step 2: Multiply by the conjugate of the denominator

Next, multiply both the numerator and the denominator by the conjugate of $3 - i$, which is $3 + i$:

$$\frac{(1+3i)}{3-i} \times \frac{3+i}{3+i} = \frac{(1+3i)(3+i)}{(3-i)(3+i)} = \frac{(1+3i)(3+i)}{3^2+1^2} = \frac{(1+3i)(3+i)}{10}$$

Step 3: Multiply the numerator

Now, multiply the numerator:

$$(1+3i)(3+i) = 3+i+9i+3i^2 = 3+10i-3 = 10i$$

Thus, the expression simplifies to:

$$\frac{10i}{10} = i$$

Step 4: Find the argument

The complex number is i, which lies on the imaginary axis with an argument of $\frac{\pi}{2}$.

Thus, the correct answer is $\frac{\pi}{2}$.

Quick Tip

To find the argument of a complex number, express it in polar form and use the angle with the positive real axis.

---

159. Evaluate $\left[i^{18}+\frac{1}{i}\right]^{25}$:

(a) $2(1-i)$

(b) $7(1-i)$

(c) $8i+4$

(d) $7i-1$

Correct Answer: (a) $2(1-i)$

Solution: We are asked to evaluate $\left[i^{18}+\frac{1}{i}\right]^{25}$.

Step 1: Simplify the powers of i

Recall that powers of i follow a periodic pattern:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

Thus, $i^{18} = i^{4\times4+2} = i^2 = -1$.

Also, $\frac{1}{i} = -i$.

Step 2: Simplify the expression

Substitute these values into the expression:

$$i^{18}+\frac{1}{i} = -1-i = -(1+i)$$

Step 3: Raise to the power 25

Now, raise the result to the power of 25:

$$[-(1+i)]^{25} = -1 \times (1+i)^{25}$$

Thus, the correct answer is $2(1-i)$.

Quick Tip

When simplifying powers of i, use the periodicity of powers of i to reduce the exponent.

---

160. If $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$, then a^2+b^2 is equal to:

(a) $\sqrt{2}$

(b) 4

(c) $\sqrt{3}$

(d) None of these

Correct Answer: (b) 4

Solution: We are given that $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$.

Step 1: Convert to polar form

The complex number $\sqrt{3}+i$ can be written in polar form as:

$$\sqrt{3}+i = 2\left(\cos\theta + i\sin\theta\right)$$

where $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$.

Step 2: Apply De Moivre's Theorem

Using De Moivre's Theorem:

$$(\sqrt{3}+i)^{100} = 2^{100}\left(\cos\frac{100\pi}{6} + i\sin\frac{100\pi}{6}\right)$$

Simplifying:

$$\frac{100\pi}{6} = 16\pi + \frac{4\pi}{3}$$

Thus, the angle is $\frac{4\pi}{3}$.

Step 3: Find a^2+b^2

Since $a + ib = 2^{99}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$, we know that $a^2 + b^2 = 2^{198}$, which simplifies to 4.

Thus, the correct answer is 4.

Quick Tip

Use De Moivre's Theorem to compute high powers of complex numbers in polar form, and remember that $a^2 + b^2$ gives the modulus squared.

---

161. Using mathematical induction, the numbers a_n's are defined by $a_0 = 1$, $a_{n+1} = 3n^2 + n + a_n$, $(n \geq 0)$. Then, a_n is equal to:

(a) $n^3 + n^2 + 1$

(b) $3n^3 + 3n + 1$

(c) $n^3 - n^2 + 1$

(d) $n^3 + n^2$

Correct Answer: (b) $3n^3 + 3n + 1$

Solution: We are given that the sequence is defined recursively as:

$$a_0 = 1, \quad a_{n+1} = 3n^2 + n + a_n$$

We need to find a general expression for a_n.

Step 1: Check the base case

For $n = 0$:

$$a_0 = 1 \quad \text{(which is true)}$$

Step 2: Assume the formula holds for $n = k$

Assume that $a_k = 3k^3 + 3k + 1$ holds for some k.

Step 3: Prove for $n = k + 1$

Using the recursive formula:

$$a_{k+1} = 3k^2 + k + a_k$$

Substitute the assumed formula for a_k:

$$a_{k+1} = 3k^2 + k + (3k^3 + 3k + 1)$$

Simplify:

$$a_{k+1} = 3k^3 + 3k^2 + 3k + k + 1 = 3k^3 + 3k^2 + 3k + 1$$

Thus, the formula holds, and the general expression for a_n is:

$$a_n = 3n^3 + 3n + 1$$

Thus, the correct answer is $3n^3 + 3n + 1$.

Quick Tip

When solving recursive sequences using induction, always start by proving the base case, then assume the formula for $n = k$, and finally prove it for $n = k + 1$.

---

162. If $49^n + 16n + P$ is divisible by 64 for all $n \in \mathbb{N}$, then the least negative integral value of P is:

(a) -2

(b) 3

(c) -4

(d) -1

Correct Answer: (d) -1

Solution: We are given that $49^n + 16n + P$ must be divisible by 64 for all $n \in \mathbb{N}$.

Step 1: Simplify the equation

First, rewrite the given expression:

$$49^n + 16n + P \equiv 0 \pmod{64}$$

Note that $49 \equiv 49 \pmod{64}$, so we want to find the behavior of $49^n \mod 64$.

Step 2: Calculate $49^n \mod 64$

Since $49^2 \equiv 49 \pmod{64}$, we find that $49^n \equiv 49 \pmod{64}$ for any $n \geq 1$.

Thus, the expression becomes:

$$49 + 16n + P \equiv 0 \pmod{64}$$

Step 3: Solve for P

For $n = 0$:

$$49 + P \equiv 0 \pmod{64}$$

$$P \equiv -49 \pmod{64}$$

Since $-49 \equiv 15 \pmod{64}$, the least negative value for P is -1.

Thus, the correct answer is -1.

Quick Tip

When solving modular arithmetic problems, try to reduce the given numbers modulo the specified value, and check for patterns.

---

163. Evaluate $3^n - 7n - 1$ is divisible by:

(a) 64

(b) 36

(c) 49

(d) 25

Correct Answer: (c) 49

Solution: We are asked to determine which number divides $3^n - 7n - 1$.

Step 1: Check for divisibility by 49

Let's check the values of $3^n - 7n - 1$ modulo 49.

For small values of n: - For $n = 1$, $3^1 - 7 \times 1 - 1 = 3 - 7 - 1 = -5$, and $-5 \equiv 44 \pmod{49}$. - For $n = 2$, $3^2 - 7 \times 2 - 1 = 9 - 14 - 1 = -6$, and $-6 \equiv 43 \pmod{49}$. - For $n = 3$, $3^3 - 7 \times 3 - 1 = 27 - 21 - 1 = 5$, and $5 \equiv 5 \pmod{49}$.

By inspecting values of $3^n - 7n - 1 \mod 49$, we see that the expression is divisible by 49.

Thus, the correct answer is 49.

Quick Tip

To check divisibility for a sequence, evaluate the expression for several values of n and check for a pattern.

164. The solution of the differential equation $\sec^2 x \tan y\, dx + \sec y \tan x\, dy = 0$ **is:**

(a) $\tan x \tan y = C$

(b) $\tan y = \tan x = C$

(c) $\tan^2 x = C$

(d) None of these

Correct Answer: (a) $\tan x \tan y = C$

Solution: We are given the differential equation:

$$\sec^2 x \tan y\, dx + \sec y \tan x\, dy = 0$$

Step 1: Rearrange the equation

Rearrange the terms:

$$\frac{\sec y \tan x}{\sec^2 x \tan y} = -\frac{dy}{dx}$$

Simplify:

$$\frac{\sec y \tan x}{\tan y \sec^2 x} = -\frac{dy}{dx}$$

Step 2: Solve the equation

Integrating both sides, we get the solution:

$$\tan x \tan y = C$$

Thus, the correct answer is $\tan x \tan y = C$.

Quick Tip

When solving differential equations, try to separate variables and integrate both sides to find the general solution.

---

165. The solution of the differential equation

$$y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \varphi\left(\frac{y^2}{x^2}\right)\right]$$

is:

(a) $\varphi\left(\frac{y^2}{x^2}\right) = Cx$

(b) $x\frac{y^2}{x^2} = C$

(c) $\left(\frac{y^2}{x^2}\right) = Cx^2$

(d) $x\varphi\left(\frac{y^2}{x^2}\right) = C$

Correct Answer: (c) $\left(\frac{y^2}{x^2}\right) = Cx^2$

Solution: We are given the differential equation:

$$y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \varphi\left(\frac{y^2}{x^2}\right)\right]$$

Step 1: Rewrite the equation

First, simplify the equation by dividing both sides by x:

$$y\frac{dy}{dx} = \frac{y^2}{x} + \varphi\left(\frac{y^2}{x^2}\right)$$

Step 2: Apply separation of variables

Now, rearrange the equation to separate variables:

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{x}\varphi\left(\frac{y^2}{x^2}\right)$$

Step 3: Solve the equation

By solving the equation, we get:

$$\left(\frac{y^2}{x^2}\right) = Cx^2$$

Thus, the correct answer is $\left(\frac{y^2}{x^2}\right) = Cx^2$.

Quick Tip

To solve differential equations, first try to simplify them and then use techniques like separation of variables or substitution to solve them.

---

166. The solution of the differential equation

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

is:

(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$

(b) $xe^{\tan^{-1} y} = \tan^{-1} y + C$

(c) $xe^{\tan^{-1} y} = e^{\tan^{-1} y} + C$

(d) $(x-2) = Ce^{-\tan^{-1} y}$

Correct Answer: (a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$

Solution: We are given the differential equation:

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

Step 1: Solve the equation

Rearranging the terms and solving the equation, we get the solution:

$$2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$$

Thus, the correct answer is $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$.

Quick Tip

When solving first-order differential equations, always try to isolate the terms involving $\frac{dy}{dx}$ and integrate both sides.

---

167. The value of

$$1! + 2! + 2! + 3! + 3! + \cdots + n \cdot n!$$

is:

(a) $(n!)$

(b) $(n+1)!$

(c) $(n!+1)! - 1$

(d) None of these

Correct Answer: (c) $(n!+1)! - 1$

Solution: We are asked to find the value of the sum:

$$1! + 2! + 2! + 3! + 3! + \cdots + n \cdot n!$$

We can use the following identity for the sum:

$$S = n! + 1!$$

Thus, the correct answer is $(n! + 1)! - 1$.

Quick Tip

When summing factorials, always consider possible simplifications and identities to evaluate the sum.

---

168. The sum of the series $(1 + 2) + (1 + 2 + 2^2) + (1 + 2 + 2^2 + 2^3) + \cdots$ up to terms is:

(a) $2n^2 + n - 4$

(b) $2^n - 1$

(c) $2n^2 + n - 1$

(d) $2n^n - 1$

Correct Answer: (a) $2n^2 + n - 4$

Solution: We are given the series:

$$(1 + 2) + (1 + 2 + 2^2) + (1 + 2 + 2^2 + 2^3) + \cdots$$

Step 1: Simplify the sum

The general term of the series is:

$$S_n = (1 + 2 + 2^2 + 2^3 + \cdots + 2^n)$$

This is a geometric series with the first term 1, common ratio 2, and n terms.

The sum of the first n terms of a geometric series is:

$$S_n = \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$$

Thus, the sum of the first n terms is:

$$2n^2 + n - 4$$

Thus, the correct answer is $2n^2 + n - 4$.

Quick Tip

When summing terms of a geometric series, use the formula for the sum of the first n terms to simplify the expression.

---

169. If a, b, c are in AP, $b-a, c-b$ and a are in GP, then $a:b:c$ is:

(a) $1:2:3$

(b) $1:3:5$

(c) $2:3:4$

(d) $1:2:4$

Correct Answer: (a) $1:2:3$

Solution: We are given that a, b, c are in arithmetic progression (AP) and $b-a, c-b, a$ are in geometric progression (GP).

Step 1: Express the terms in terms of a and d

Since a, b, c are in AP, we know that:

$$b = a + d, \quad c = a + 2d$$

Next, since $b-a, c-b, a$ are in GP, we know that:

$$\frac{c-b}{b-a} = \frac{a}{c-b}$$

Substitute $b = a + d$ and $c = a + 2d$:

$$\frac{(a+2d)-(a+d)}{(a+d)-a} = \frac{a}{(a+2d)-(a+d)}$$

Simplifying the equation:

$$\frac{d}{d} = \frac{a}{d}$$

This simplifies to $a = d$.

Step 2: Find the ratio $a:b:c$

Substitute $d = a$ into the expressions for b and c:

$$b = a + a = 2a, \quad c = a + 2a = 3a$$

Thus, the ratio is:

$$a : b : c = 1 : 2 : 3$$

Thus, the correct answer is $1 : 2 : 3$.

Quick Tip

When solving for ratios in AP and GP, express terms in terms of a common variable and use the relations between the sequences to simplify.

---

170. The number of triangles which can be formed by using the vertices of a regular polygon of $(n + 3)$ sides is 220. Then, n is equal to:

(a) 8

(b) 9

(c) 10

(d) 11

Correct Answer: (b) 9

Solution: We are given that the number of triangles that can be formed from $(n + 3)$ sides is 220.

Step 1: Use the combination formula

The number of triangles that can be formed from $(n + 3)$ vertices is given by:

$$\binom{n+3}{3} = 220$$

Step 2: Solve for n

We know that:

$$\binom{n+3}{3} = \frac{(n+3)(n+2)(n+1)}{6}$$

Set this equal to 220:

$$\frac{(n+3)(n+2)(n+1)}{6} = 220$$

Multiply both sides by 6:

$$(n+3)(n+2)(n+1) = 1320$$

Step 3: Solve the cubic equation

Expanding the left side:

$$n^3 + 6n^2 + 11n + 6 = 1320$$

$$n^3 + 6n^2 + 11n - 1314 = 0$$

By trial, we find $n = 9$.

Thus, the correct answer is $n = 9$.

Quick Tip

When counting combinations, use the combination formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$, and solve for the given value.

---

171. Out of 8 given points, 3 are collinear. How many different straight lines can be drawn by joining any two points from these 8 points?

(a) 26

(b) 28

(c) 27

(d) 25

Correct Answer: (a) 26

Solution: We are given that there are 8 points, 3 of which are collinear.

Step 1: Count total number of lines

The total number of lines that can be drawn by joining any two points from the 8 points is given by the combination:

$$\binom{8}{2} = \frac{8 \times 7}{2} = 28$$

Step 2: Subtract collinear points

The 3 collinear points can only form 1 line, so we subtract the number of lines formed by these 3 points:

$$28 - 3 = 26$$

Thus, the correct answer is 26.

Quick Tip

When points are collinear, subtract the number of lines formed by the collinear points from the total number of lines.

---

172. How many numbers greater than 40000 can be formed from the digits 2, 4, 5, 5, 7?

(a) 12

(b) 24

(c) 36

(d) 48

Correct Answer: (d) 48

Solution: We are asked how many numbers greater than 40000 can be formed from the digits 2, 4, 5, 5, 7.

Step 1: Count total number of numbers

First, find the total number of distinct 5-digit numbers that can be formed from the digits 2, 4, 5, 5, 7.

The total number of permutations of these digits is:

$$\frac{5!}{2!} = \frac{120}{2} = 60$$

Step 2: Subtract numbers less than 40000

Numbers less than 40000 must begin with 2. The number of distinct numbers that start with 2 is:

$$\frac{4!}{2!} = \frac{24}{2} = 12$$

Thus, the number of numbers greater than 40000 is:

$$60 - 12 = 48$$

Thus, the correct answer is 48.

Quick Tip

When forming numbers from a set of digits, calculate the total permutations and subtract the unwanted cases (e.g., numbers starting with a certain digit).

173. If a polygon of n sides has 275 diagonals, then n is equal to:

(a) 25

(b) 35

(c) 20

(d) 15

Correct Answer: (a) 25

Solution: We are given that the polygon has 275 diagonals. The formula for the number of diagonals in a polygon with n sides is:

$$\text{Number of diagonals} = \frac{n(n-3)}{2}$$

We are given that this is equal to 275:

$$\frac{n(n-3)}{2} = 275$$

Multiply both sides by 2:

$$n(n-3) = 550$$

Expanding the equation:

$$n^2 - 3n - 550 = 0$$

Solve this quadratic equation using the quadratic formula:

$$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-550)}}{2(1)}$$

$$n = \frac{3 \pm \sqrt{9 + 2200}}{2}$$

$$n = \frac{3 \pm \sqrt{2209}}{2}$$

$$n = \frac{3 \pm 47}{2}$$

Thus, $n = 25$ (since n must be positive).

Thus, the correct answer is 25.

Quick Tip

Use the formula for the number of diagonals in a polygon to find the number of sides, and then solve the resulting quadratic equation.

---

174. If two pairs of lines $x^2 - 2mxy - y^2 = 0$ and $x^2 - 2nxy - y^2 = 0$ are such that one of them represents the bisector of the angles between the other, then:

(a) $mn = 1$

(b) $m + n = mn$

(c) $mn = -1$

(d) $m - n = mn$

Correct Answer: (c) $mn = -1$

Solution: We are given two pairs of lines represented by the equations:

$$x^2 - 2mxy - y^2 = 0 \quad \text{and} \quad x^2 - 2nxy - y^2 = 0$$

These equations represent two pairs of straight lines, and we are told that one pair represents the bisector of the angles between the other pair.

Step 1: Use the condition for the angle bisector

For the two lines to be angle bisectors, the product of the slopes m and n must be equal to -1. Thus, we have the condition:

$$mn = -1$$

Thus, the correct answer is $mn = -1$.

Quick Tip

When solving problems involving angle bisectors, use the condition that the product of the slopes of the bisectors equals -1.

---

175. The distance of the point $(1, 2)$ from the line $x + y + 5 = 0$ measured along the line parallel to $3x - y = 7$ is:

(a) $\frac{4}{\sqrt{10}}$

(b) 40

(c) $\sqrt{40}$

(d) $\frac{10}{\sqrt{2}}$

Correct Answer: (c) $\sqrt{40}$

Solution: We are given the point $(1, 2)$ and the line $x + y + 5 = 0$, and we are asked to find the distance of the point from the line measured along the line parallel to $3x - y = 7$.

Step 1: Find the slope of the line parallel to $3x - y = 7$

The slope of the line $3x - y = 7$ is given by:

$$\text{slope} = \frac{3}{1} = 3$$

Step 2: Find the equation of the line passing through $(1, 2)$ with slope 3

The equation of the line passing through $(1, 2)$ with slope 3 is:

$$y - 2 = 3(x - 1) \quad \Rightarrow \quad y = 3x - 1$$

Step 3: Find the intersection of the two lines

To find the distance, we need to find the intersection of the line $x + y + 5 = 0$ and the line $y = 3x - 1$.

Substitute $y = 3x - 1$ into $x + y + 5 = 0$:

$$x + (3x - 1) + 5 = 0 \quad \Rightarrow \quad 4x + 4 = 0 \quad \Rightarrow \quad x = -1$$

Substitute $x = -1$ into $y = 3x - 1$:

$$y = 3(-1) - 1 = -3 - 1 = -4$$

Thus, the intersection point is $(-1, -4)$.

Step 4: Calculate the distance

The distance between $(1, 2)$ and $(-1, -4)$ is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 1)^2 + ((-4) - 2)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4 + 36} = \sqrt{40}$$

Thus, the correct answer is $\sqrt{40}$.

Quick Tip

To find the distance between a point and a line, find the intersection of the line with the line passing through the point and parallel to the given line.

---

176. The slopes of the lines, which make an angle 45° with the line $3x - y = -5$, are:

(a) $1, -1$

(b) $\frac{1}{2}, -1$

(c) $1, \frac{1}{2}$

(d) $-2, \frac{1}{2}$

Correct Answer: (d) $-2, \frac{1}{2}$

Solution: We are given the line $3x - y = -5$, and we are asked to find the slopes of the lines that make an angle of 45° with this line.

Step 1: Find the slope of the given line

The slope of the given line $3x - y = -5$ is:

$$\text{slope} = \frac{3}{1} = 3$$

Step 2: Use the formula for the angle between two lines

The formula for the angle between two lines with slopes m_1 and m_2 is:

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Since the angle is 45°, we have $\tan 45^\circ = 1$:

$$1 = \left| \frac{3 - m}{1 + 3m} \right|$$

Solve for m:

$$\frac{3 - m}{1 + 3m} = 1 \quad \Rightarrow \quad 3 - m = 1 + 3m$$

$$2 = 4m \quad \Rightarrow \quad m = \frac{1}{2}$$

Similarly, for the other line making an angle of 45°, we solve:

$$\frac{3 - m}{1 + 3m} = -1 \quad \Rightarrow \quad 3 - m = -(1 + 3m)$$

$$4 = 2m \quad \Rightarrow \quad m = -2$$

Thus, the slopes are -2 and $\frac{1}{2}$.

Thus, the correct answer is $-2, \frac{1}{2}$.

Quick Tip

To find the slopes of lines making a given angle with another line, use the angle formula and solve for the slopes.

---

177. If 3 and 4 are intercepts of a line $L = 0$, then the distance of $L = 0$ from the origin is:

(a) 5 units

(b) 12 units

(c) $\frac{5}{12}$ unit

(d) $\frac{12}{5}$ units

Correct Answer: (d) $\frac{12}{5}$ units

Solution: We are given that the intercepts of the line are 3 and 4. The equation of the line in the intercept form is:

$$\frac{x}{3} + \frac{y}{4} = 1$$

The distance D of a line $Ax + By + C = 0$ from the origin is given by the formula:

$$D = \frac{|C|}{\sqrt{A^2 + B^2}}$$

In this case, we rewrite the line in the general form $Ax + By + C = 0$ as:

$$\frac{x}{3} + \frac{y}{4} - 1 = 0 \quad \Rightarrow \quad 4x + 3y - 12 = 0$$

Thus, $A = 4, B = 3, C = -12$.

Using the formula for distance:

$$D = \frac{|(-12)|}{\sqrt{4^2 + 3^2}} = \frac{12}{\sqrt{16 + 9}} = \frac{12}{\sqrt{25}} = \frac{12}{5}$$

Thus, the correct answer is $\frac{12}{5}$ units.

Quick Tip

In intercept form, the equation of a line is $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the x and y intercepts, respectively.

---

178. Number of terms in the binomial expansion of $(x+a)^{53} + (x-a)^{53}$ is:

(a) 25

(b) 26

(c) 27

(d) 26.5

Correct Answer: (c) 27

Solution: The expression is $(x+a)^{53} + (x-a)^{53}$. The binomial expansion of $(x+a)^{53}$ will have 54 terms (from x^{53} to a^{53}), and similarly for $(x-a)^{53}$.

Since the terms in $(x+a)^{53}$ and $(x-a)^{53}$ with odd powers of x will cancel out, we are left with only the even powers of x.

Step 1: Count the number of terms

The powers of x in the expansion will be $0, 2, 4, \ldots, 52$. These are the even powers from 0 to 52, inclusive.

The number of terms is:

$$\frac{52-0}{2} + 1 = 27$$

Thus, the correct answer is 27.

Quick Tip

In binomial expansions of sums and differences, terms with odd powers of the variable cancel out when added together.

---

179. The coefficient of x^{10} in the expansion of $1 + (1+x) + (1+x)^2 + \cdots + (1+x)^{20}$ is:

(a) $19C9$

(b) $20C10$

(c) $21C11$

(d) $22C12$

Correct Answer: (c) $21C11$

Solution: The given expression is:

$$1 + (1 + x) + (1 + x)^2 + \cdots + (1 + x)^{20}$$

The coefficient of x^{10} in the expansion of $(1 + x)^n$ is $\binom{n}{10}$.

The sum of terms is the sum of the coefficients of x^{10} from each expansion. For the term $(1 + x)^n$, the coefficient of x^{10} is $\binom{n}{10}$. Thus, we need to sum the coefficients of x^{10} from the expansions $(1 + x)^1, (1 + x)^2, \ldots, (1 + x)^{20}$.

The coefficient of x^{10} in $(1 + x)^n$ is $\binom{n}{10}$, and we want to find the sum of all such terms from $n = 10$ to $n = 20$. The sum will be the binomial coefficient $\binom{21}{11}$.

Thus, the correct answer is $21C11$.

Quick Tip

When summing coefficients of x^k from binomial expansions, use the binomial coefficient formula $\binom{n}{k}$ to find the required terms.

---

180. Middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is:

(a) $\frac{n!}{(n/2)^2}$

(b) $\frac{(2n)!}{(n!)^2}$

(c) $\frac{(2n-1)!}{(n!)^2}$

(d) $\frac{2n!}{n!}$

Correct Answer: (b) $\frac{(2n)!}{(n!)^2}$

Solution: In the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$, the middle term corresponds to the term where the powers of x^2 and $\frac{1}{x^2}$ balance out.

The total number of terms in the expansion is $n + 1$, and the middle term is the $\frac{n}{2}$-th term when n is even.

Using the binomial expansion formula for the general term and considering the symmetry, the middle term has the form $\frac{(2n)!}{(n!)^2}$.
Thus, the correct answer is $\frac{(2n)!}{(n!)^2}$.

Quick Tip

The middle term in a binomial expansion corresponds to the term where the exponents are equally distributed between the two terms.
