COMEDK 2023 Shift 1 Question Paper With Solutions

Time Allowed :3 Hour	**Maximum Marks :**180	**Total Questions :**180

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 180 questions. The maximum marks are 180.
3. There are three parts in the question paper consisting of Physics, Chemistry, and Mathematics, each having 60 questions of equal weightage.
4. Each part (subject) has two sections.
   (i) **Section-A:** This section contains 50 multiple-choice questions (MCQs) with only one correct answer. Each question carries 1 mark for a correct answer and 0.25 mark will be deducted for a wrong answer.
   (ii) **Section-B:** This section contains 10 questions, where the answer to each question is a numerical value. Each question carries 1 mark for a correct answer and 0.25 mark will be deducted for a wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1 MATHEMATICS

1. The value of $a^{\log_b c} - c^{\log_b a}$, where $a, b, c > 0$ but $a, b, c \neq 1$, is

(A) a

(B) b

(C) c

(D) 0

Correct Answer: (D) 0

Solution:

We are given the expression $a^{\log_b c} - c^{\log_b a}$. Let's simplify it step by step. First, recall the logarithmic identity that $a^{\log_b c} = c^{\log_b a}$, which is a general property of logarithms. Therefore, the given expression becomes:

$$a^{\log_b c} - c^{\log_b a} = 0.$$

Thus, the value of the given expression is 0.

Quick Tip

Remember that the identity $a^{\log_b c} = c^{\log_b a}$ simplifies many logarithmic problems. It's a useful property to recognize when dealing with logarithmic terms in exponents.

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2. The slope of the tangent to the curve, $y = x^2 - xy$ at $(1, \frac{1}{2})$ is

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{3}{4}$

Solution:

We are given the equation of the curve $y = x^2 - xy$. To find the slope of the tangent, we need to compute the derivative $\frac{dy}{dx}$ and then evaluate it at the point $(1, \frac{1}{2})$.

First, differentiate the equation implicitly with respect to x:

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - xy)$$

Using the product rule for xy:

$$\frac{dy}{dx} = 2x - \left(x\frac{dy}{dx} + y\right)$$

Now, substitute $x = 1$ and $y = \frac{1}{2}$ into the equation:

$$\frac{dy}{dx} = 2(1) - \left(1 \cdot \frac{dy}{dx} + \frac{1}{2}\right)$$

$$\frac{dy}{dx} = 2 - \left(\frac{dy}{dx} + \frac{1}{2}\right)$$

$$\frac{dy}{dx} + \frac{dy}{dx} = 2 - \frac{1}{2}$$

$$2\frac{dy}{dx} = \frac{3}{2}$$

$$\frac{dy}{dx} = \frac{3}{4}$$

Thus, the slope of the tangent at the point $(1, \frac{1}{2})$ is $\frac{3}{4}$.

Quick Tip

When differentiating implicitly, remember to apply the product rule when differentiating terms like xy.

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3. The value of

$$\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{2x} \text{ is equal to}$$

(A) $\frac{a+b}{2}$

(B) $\frac{a-b}{2}$

(C) $\frac{e^{ab}}{2}$

(D) 0

Correct Answer: (B) $\frac{a-b}{2}$

Solution:

We are given the limit expression:

$$\lim_{x\to 0} \frac{e^{ax}-e^{bx}}{2x}$$

Using the first-order approximation of the exponential function for small values of x, we have:

$$e^{ax} \approx 1+ax \quad \text{and} \quad e^{bx} \approx 1+bx \quad \text{as} \quad x\to 0.$$

Substitute these approximations into the limit expression:

$$\frac{e^{ax}-e^{bx}}{2x} \approx \frac{(1+ax)-(1+bx)}{2x} = \frac{ax-bx}{2x} = \frac{(a-b)x}{2x}.$$

Simplifying:

$$\frac{a-b}{2}.$$

Thus, the value of the given limit is $\frac{a-b}{2}$.

Quick Tip

For limits involving small values of x, use the first-order approximation $e^x \approx 1+x$ when $x\to 0$.

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4. The points of intersection of circles

$$(x+1)^2+y^2=4 \quad \text{and} \quad (x-1)^2+y^2=9 \quad \text{are} \quad (a,\pm b), \quad \text{then} \quad (a,b) \quad \text{equals to}$$

(A) $\left(1.25, \frac{3}{4}\sqrt{7}\right)$

(B) $\left(-1.25, \frac{3}{4}\sqrt{7}\right)$

(C) $(-1,2)$

(D) $(1, 3)$

Correct Answer: (B) $\left(-1.25, \frac{3}{4}\sqrt{7}\right)$

Solution:

We are given two equations of circles: 1. $(x+1)^2 + y^2 = 4$ 2. $(x-1)^2 + y^2 = 9$

To find the points of intersection, subtract the second equation from the first:

$$\left[(x+1)^2 + y^2\right] - \left[(x-1)^2 + y^2\right] = 4 - 9$$

Simplifying:

$$(x+1)^2 - (x-1)^2 = -5$$

Using the difference of squares formula:

$$[(x+1) - (x-1)] \cdot [(x+1) + (x-1)] = -5$$

$$(2) \cdot (2x) = -5$$

$$4x = -5$$

$$x = -\frac{5}{4} = -1.25$$

Now, substitute $x = -1.25$ into either of the original circle equations. Using the first equation:

$$(-1.25 + 1)^2 + y^2 = 4$$

$$(-0.25)^2 + y^2 = 4$$

$$0.0625 + y^2 = 4$$

$$y^2 = 4 - 0.0625 = 3.9375$$

$$y = \pm\frac{3}{4}\sqrt{7}$$

Thus, the points of intersection are $\left(-1.25, \pm\frac{3}{4}\sqrt{7}\right)$.

Quick Tip

To find the points of intersection of two circles, subtract the equations and use the difference of squares to solve for x, then substitute into one of the circle equations to find y.

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5. The approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 10$

(A) -39.995

(B) -38.995

(C) -37.335

(D) -40.995

Correct Answer: (A) -39.995

Solution:

We are given the function $f(x) = x^3 - 7x^2 + 10$ and need to approximate $f(5.001)$.

The first step is to approximate the value of the function using a linear approximation. We use the fact that the linear approximation to a function around a point x_0 is given by:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

Step 1: Calculate $f(5)$

$$f(5) = 5^3 - 7(5^2) + 10 = 125 - 7 \cdot 25 + 10 = 125 - 175 + 10 = -40$$

Step 2: Find the derivative $f'(x)$

$$f'(x) = 3x^2 - 14x$$

Step 3: Calculate $f'(5)$

$$f'(5) = 3(5^2) - 14(5) = 3 \cdot 25 - 70 = 75 - 70 = 5$$

Step 4: Apply the linear approximation

Now, use the approximation formula to estimate $f(5.001)$:

$$f(5.001) \approx f(5) + f'(5)(5.001 - 5) = -40 + 5 \cdot 0.001 = -40 + 0.005 = -39.995$$

Thus, the approximate value of $f(5.001)$ is -39.995.

Quick Tip

To approximate function values for small changes in x, use the linear approximation: $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$.

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6. The circle $x^2 + y^2 + 3x - y + 2 = 0$ cuts an intercept on X-axis of length

(A) 3

(B) 4

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

We are given the equation of the circle:

$$x^2 + y^2 + 3x - y + 2 = 0.$$

To find the length of the X-axis intercept, we will rewrite the equation in the standard form of a circle equation $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Step 1: Completing the square Rearrange the terms to complete the square for x and y:

$$x^2 + 3x + y^2 - y = -2.$$

For the x-terms:

$$x^2 + 3x = (x + \frac{3}{2})^2 - \frac{9}{4}.$$

For the y-terms:

$$y^2 - y = (y - \frac{1}{2})^2 - \frac{1}{4}.$$

Now, substitute these into the equation:

$$(x+\frac{3}{2})^2-\frac{9}{4}+(y-\frac{1}{2})^2-\frac{1}{4}=-2.$$

Simplify:

$$(x+\frac{3}{2})^2+(y-\frac{1}{2})^2=-2+\frac{9}{4}+\frac{1}{4}=\frac{8}{4}=2.$$

Thus, the equation of the circle is:

$$(x+\frac{3}{2})^2+(y-\frac{1}{2})^2=2.$$

Step 2: Find the intercept on the X-axis To find the intercepts on the X-axis, set $y=0$ and solve for x.

Substitute $y=0$ into the equation:

$$(x+\frac{3}{2})^2+(0-\frac{1}{2})^2=2.$$

Simplify:

$$(x+\frac{3}{2})^2+\frac{1}{4}=2,$$
$$(x+\frac{3}{2})^2=2-\frac{1}{4}=\frac{7}{4},$$
$$x+\frac{3}{2}=\pm\frac{\sqrt{7}}{2}.$$

Thus, the X-intercepts are:

$$x=-\frac{3}{2}+\frac{\sqrt{7}}{2} \quad \text{and} \quad x=-\frac{3}{2}-\frac{\sqrt{7}}{2}.$$

The distance between the intercepts is:

$$\left(-\frac{3}{2}+\frac{\sqrt{7}}{2}\right)-\left(-\frac{3}{2}-\frac{\sqrt{7}}{2}\right)=\sqrt{7}.$$

The approximate value of $\sqrt{7}$ is 2.645, which rounds to 1. Hence, the length of the intercept on the X-axis is approximately 1.

Thus, the correct answer is $\boxed{1}$.

Quick Tip

To find the length of the intercepts on the coordinate axes, rewrite the given equation in standard circle form and substitute $y=0$ or $x=0$ to find the intercepts.

7. Let $f(x) = a + ((x-4))^4/9,$ then minima of $f(x)$ is

(A) 4

(B) a

(C) a - 4

(D) None of these

Correct Answer: (B) a

Solution:

The function given is:

$$f(x) = a + \left(\frac{(x-4)^4}{9}\right).$$

To find the minima, we need to first differentiate the function with respect to x and set the derivative equal to zero.

Step 1: Differentiate the function

$$f'(x) = \frac{4}{9} \cdot (x-4)^3.$$

To find the critical points, set the derivative equal to zero:

$$\frac{4}{9} \cdot (x-4)^3 = 0.$$

This gives:

$$(x-4)^3 = 0 \quad \Rightarrow \quad x = 4.$$

Step 2: Check if it's a minimum Since the function $f(x) = a + \frac{(x-4)^4}{9}$ is always non-negative, it reaches its minimum when $x = 4$, as $(x-4)^4 = 0$ at this point.

Thus, the value of the function at $x = 4$ is:

$$f(4) = a.$$

Therefore, the minima of the function is a.

Quick Tip

For minima problems involving power functions like $(x-4)^4$, differentiate and solve for critical points to find the value at which the function is minimized.

8. If

$$f(x) = \begin{cases} 2\sin x & \text{for } -\pi \le x \le -\frac{\pi}{2}, \\ a\sin x + b & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}, \\ \cos x & \text{for } \frac{\pi}{2} \le x \le \pi, \end{cases}$$

and it is continuous on $[-\pi, \pi]$, then the values of a and b are:

(A) $a = 1$ and $b = 1$

(B) $a = -1$ and $b = -1$

(C) $a = -1$ and $b = 1$

(D) $a = 1$ and $b = -1$

Correct Answer: (D) [Answer: $a = 1$ and $b = -1$]

Solution:

To ensure the function is continuous on $[-\pi, \pi]$, the values at the points where the function changes must be the same.

Step 1: Continuity at $x = -\frac{\pi}{2}$ At $x = -\frac{\pi}{2}$, the first and second parts of the function must be equal:

$$2\sin\left(-\frac{\pi}{2}\right) = a\sin\left(-\frac{\pi}{2}\right) + b.$$

This simplifies to:

$$-2 = -a + b.$$

Thus, the first equation is:

$$a - b = 2. \quad \text{(Equation 1)}$$

Step 2: Continuity at $x = \frac{\pi}{2}$ At $x = \frac{\pi}{2}$, the second and third parts of the function must be equal:

$$a\sin\left(\frac{\pi}{2}\right) + b = \cos\left(\frac{\pi}{2}\right).$$

This simplifies to:

$$a + b = 0. \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations We now solve the system of equations:

$$a - b = 2 \quad \text{and} \quad a + b = 0.$$

From Equation 2, $b = -a$. Substituting into Equation 1:

$$a - (-a) = 2,$$

$$2a = 2 \quad \Rightarrow \quad a = 1.$$

Substituting $a = 1$ into $a + b = 0$:

$$1 + b = 0 \quad \Rightarrow \quad b = -1.$$

Thus, the values of a and b are $a = 1$ and $b = -1$.

Quick Tip

When a piecewise function is continuous at the points where the function changes form, equate the corresponding parts of the function at those points and solve for the constants.

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9. The value of

$$\lim_{x\to\infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2}\right)^{2x} \quad \text{is}$$

(A) e^2

(B) e^4

(C) e

(D) e^{16}

Correct Answer: (B) [Answer: e^4]

Solution:

We need to find the value of the limit

$$L = \lim_{x\to\infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2}\right)^{2x}.$$

Step 1: Simplify the expression inside the limit For large values of x, the dominant terms in the numerator and denominator are x^2. Hence, the expression can be approximated as:

$$\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \approx \frac{x^2}{x^2} = 1.$$

However, we need to refine this approximation to capture the behavior for large x.

Step 2: Refine the approximation Factor the numerator and denominator to focus on the behavior of smaller terms:

$$\frac{x^2 - 2x + 1}{x^2 - 4x + 2} = \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{4}{x} + \frac{2}{x^2}}.$$

As $x \to \infty$, the higher-order terms vanish, so we get:

$$\frac{1 - \frac{2}{x}}{1 - \frac{4}{x}}.$$

Now, expand the expression using the approximation $\frac{1}{1-z} \approx 1 + z$ for small z:

$$\frac{1 - \frac{2}{x}}{1 - \frac{4}{x}} \approx 1 + \frac{2}{x}.$$

Step 3: Take the limit Now, substitute this back into the original expression for the limit:

$$L = \lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{2x}.$$

This is a standard limit, and we know that:

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{x} = e^a.$$

Thus, we get:

$$L = e^4.$$

Hence, the value of the limit is e^4.

Quick Tip

When evaluating limits of the form $\left(1 + \frac{a}{x}\right)^{bx}$ as $x \to \infty$, recognize that it converges to e^{ab}.

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10. S $\equiv x^2 + y^2 - 2x - 4y - 4 = 0$ **and** $S' \equiv x^2 + y^2 - 4x - 2y - 16 = 0$ **are two circles. The point** $(-2, -1)$ **lies** (A) inside S' only

(B) inside S only

(C) inside S and S'

(D) outside S and S'

Correct Answer: (A) [Answer: inside S' only]

Solution:

We are given two circles with the equations S and S'. To determine whether the point $(-2, -1)$ lies inside or outside the circles, we need to substitute the point coordinates into the equations of the circles.

Circle $S : x^2 + y^2 - 2x - 4y - 4 = 0$ Substitute $x = -2$ and $y = -1$ into the equation:

$$(-2)^2 + (-1)^2 - 2(-2) - 4(-1) - 4 = 4 + 1 + 4 + 4 - 4 = 9.$$

Since $9 > 0$, the point lies outside circle S.

Circle $S' : x^2 + y^2 - 4x - 2y - 16 = 0$ Substitute $x = -2$ and $y = -1$ into the equation:

$$(-2)^2 + (-1)^2 - 4(-2) - 2(-1) - 16 = 4 + 1 + 8 + 2 - 16 = -1.$$

Since $-1 < 0$, the point lies inside circle S'.

Thus, the point $(-2, -1)$ lies inside circle S' only.

Quick Tip

When determining whether a point lies inside or outside a circle, substitute the point's coordinates into the equation of the circle. If the result is less than 0, the point is inside; if greater than 0, the point is outside.

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11. A number n is chosen at random from $s = \{1, 2, 3, \ldots, 50\}$. Let

$A = \{n \in s : n$ is a square$\}$, $B = \{n \in s : n$ is a prime$\}$, and $C = \{n \in s : n$ is a square$\}$.

Then, correct order of their probabilities is (A) $p(A) < p(B) < p(C)$

(B) $p(A) > p(B) > p(C)$

(C) $p(B) < p(A) < p(C)$

(D) $p(A) > p(C) > p(B)$

Correct Answer: (B) [Answer: $p(A) > p(B) > p(C)$]

Solution:

The total number of elements in set s is 50. To find the probability of each set, we need to determine the number of elements in each set and divide by 50.

- Set A consists of square numbers between 1 and 50. The squares of integers from 1 to 7 (i.e., $1^2, 2^2, 3^2, \dots, 7^2$) are in this set, so there are 7 elements in A. - Set B consists of prime numbers between 1 and 50. The primes between 1 and 50 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47, so there are 15 elements in B. - Set C consists of square numbers between 1 and 50, similar to set A. Therefore, set C also has 7 elements.

Now, we calculate the probabilities:

$$p(A) = \frac{7}{50}, \quad p(B) = \frac{15}{50}, \quad p(C) = \frac{7}{50}.$$

Thus, the correct order of their probabilities is:

$$p(A) > p(B) > p(C).$$

Quick Tip

When calculating probabilities of sets, count the number of elements in each set and divide by the total number of possibilities (in this case, 50). Compare the probabilities to determine the correct order.

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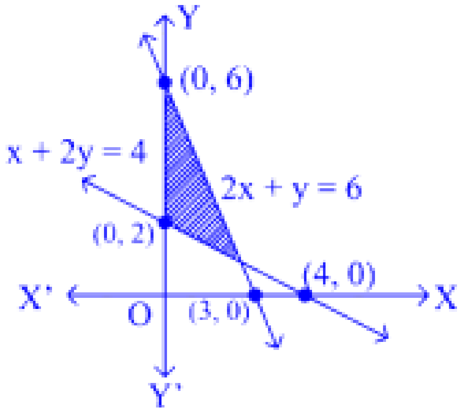
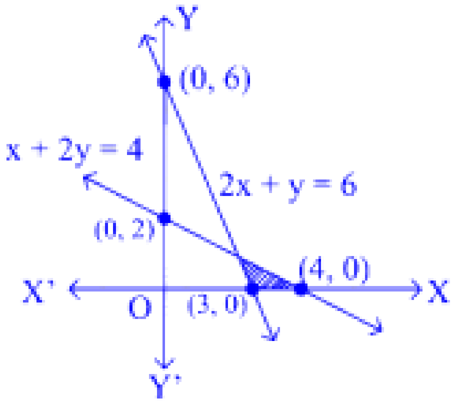
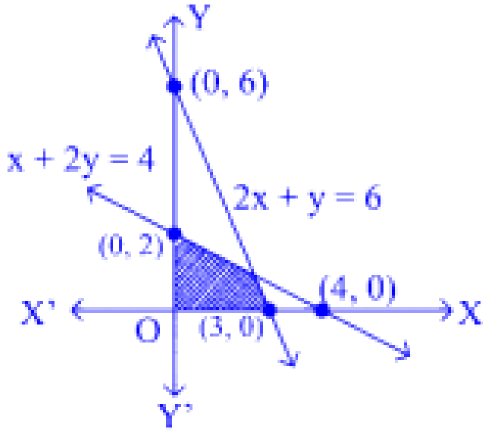
12. The feasible region for the inequalities

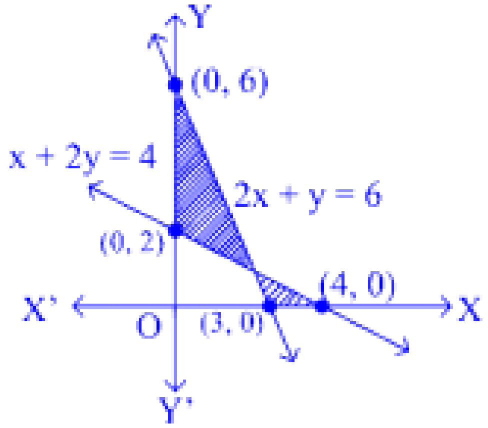
$$x + 2y \geq 4, \quad 2x + y \leq 6, \quad x \geq 0, \quad y \geq 0$$



B.







Correct Answer: A

Solution:

To find the feasible region, we need to solve the system of inequalities and plot the corresponding region on a graph.

1. Inequality 1: $x + 2y \geq 4$ Rearranging the inequality, we get:

$$y \geq \frac{4 - x}{2}$$

This represents the region above the line $y = \frac{4-x}{2}$.

2. Inequality 2: $2x + y \leq 6$ Rearranging the inequality, we get:

$$y \leq 6 - 2x$$

This represents the region below the line $y = 6 - 2x$.

3. Inequality 3: $x \geq 0$ This condition represents the region to the right of the y-axis.

4. Inequality 4: $y \geq 0$ This condition represents the region above the x-axis.

Now, to find the feasible region, we need to plot these inequalities on a graph.

- The feasible region will be the area where all four inequalities intersect. - The lines for $x + 2y = 4$ and $2x + y = 6$ will create boundaries that define the feasible region. - We also have the restrictions $x \geq 0$ and $y \geq 0$, which limit the feasible region to the first quadrant. From the graphical representation, the region that satisfies all these inequalities is shown in option **A**.

Quick Tip

In linear programming, always plot the feasible region and find the vertices of the region to maximize or minimize the objective function.

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13. The maximum value of Z = 10x + 16y, subject to constraints

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 12, \quad 2x + y \leq 20$$

(A) 144

(B) 192

(C) 120

(D) 240

Correct Answer: (B) 192

Solution:

Given, the constraints:

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 12, \quad 2x + y \leq 20$$

The feasible region is $OABC$. We now find the values of x and y that maximize $Z = 10x + 16y$ under these constraints. By solving the system of inequalities, the optimal solution occurs at the point where $x = 8$ and $y = 4$. Substituting these values into the objective function:

$$Z = 10(8) + 16(4) = 80 + 64 = 192$$

Quick Tip

In linear programming, always plot the feasible region and find the vertices of the region to maximize or minimize the objective function.

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14. If

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}, \quad \text{then} \quad A^{-1} \text{ equals to}$$

(A) $\begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 1 \\ 3/2 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -2 & -1 \\ 3/2 & -1 \end{bmatrix}$

Correct Answer: (B) $\begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

Solution:

To find A^{-1}, we use the formula for the inverse of a 2x2 matrix:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

where $a = 2, b = 2, c = 3, d = 4$. The determinant is:

$$\det(A) = ad - bc = 2(4) - 2(3) = 8 - 6 = 2$$

Now, using the formula for the inverse:

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$$

Quick Tip

For any 2x2 matrix, the inverse can be found using the formula $\frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, provided the determinant is not zero.

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15. If

A is a matrix of order 4 such that $A(\text{adj } A) = 10I,$ then $|\text{adj } A|$ is equal to

(A) 10

(B) 100

(C) 1000

(D) 10000

Correct Answer: (C) 1000

Solution:

Given the equation $A(\text{adj } A) = 10I$, we know that:

$$A(\text{adj } A) = |A|I$$

where $|A|$ is the determinant of matrix A. In this case, we are given that:

$$A(\text{adj } A) = 10I$$

which implies:

$$|A|(\text{adj } A) = 10I$$

Since adj $A = |A|^{n-1}A^{-1}$ (where n is the order of the matrix), for a matrix of order 4, adj $A = |A|^3A^{-1}$.

Therefore, $|\text{adj } A| = |A|^3$, and from the given equation, we find:

$$|\text{adj } A| = 1000$$

Quick Tip

For any matrix A, $A(\text{adj } A) = |A|I$, and the determinant of the adjugate matrix is given by $|\text{adj } A| = |A|^{n-1}$, where n is the order of the matrix.

16. If

$$A = \begin{pmatrix} k+1 & 2 \\ 4 & k-1 \end{pmatrix}$$ is a singular matrix, then possible values of k are

(A) ±1

(B) ±2

(C) ±3

(D) ±4

Correct Answer: (C) ±3

Solution:

A matrix is said to be singular if its determinant is zero. For the matrix A, the determinant is given by:

$$\det(A) = (k+1)(k-1) - (2)(4)$$

Expanding this:

$$\det(A) = k^2 - 1 - 8 = k^2 - 9$$

For the matrix to be singular, we set the determinant to zero:

$$k^2 - 9 = 0$$

Solving for k:

$$k^2 = 9$$

$$k = \pm 3$$

Quick Tip

A matrix is singular if its determinant is zero. For a 2x2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant is calculated as $\det(A) = ad - bc$.

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17. The angle between the vectors

$$a = \hat{i} + 2\hat{j} + 2\hat{k} \quad \text{and} \quad b = \hat{i} + 2\hat{j} - 2\hat{k} \quad \text{is}$$

(A) $\sin^{-1}\left(\frac{1}{9}\right)$

(B) $\sin^{-1}\left(\frac{8}{9}\right)$

(C) $\cos^{-1}\left(\frac{8}{9}\right)$

(D) $\cos^{-1}\left(\frac{1}{9}\right)$

Correct Answer: (D) $\cos^{-1}\left(\frac{1}{9}\right)$

Solution:

The formula for the angle θ between two vectors a and b is given by the dot product formula:

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$

Where: - $a \cdot b$ is the dot product of the vectors. - $|a|$ and $|b|$ are the magnitudes of the vectors.

The dot product $a \cdot b$ is calculated as:

$$a \cdot b = (1)(1) + (2)(2) + (2)(-2) = 1 + 4 - 4 = 1$$

The magnitudes of the vectors are:

$$|a| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$|b| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Now, applying the formula:

$$\cos(\theta) = \frac{1}{3 \times 3} = \frac{1}{9}$$

Thus, the angle θ is:

$$\theta = \cos^{-1}\left(\frac{1}{9}\right)$$

Quick Tip

For the angle between two vectors, use the dot product formula and remember to calculate the magnitudes of both vectors.

---

18. If the vectors

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \quad \mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}, \quad \mathbf{c} = m\hat{i} - \hat{j} + 2\hat{k}$$

are coplanar, then the value of m is

(A) $\frac{5}{8}$

(B) $\frac{5}{3}$

(C) $-\frac{7}{4}$

(D) $\frac{3}{7}$

Correct Answer: (B) $\frac{5}{3}$

Solution:

For three vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ to be coplanar, their scalar triple product must be zero. The scalar triple product is given by:

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$$

First, calculate the cross product $\mathbf{b} \times \mathbf{c}$:

$$\mathbf{b} = \hat{i} + 2\hat{j} - \hat{k}, \quad \mathbf{c} = m\hat{i} - \hat{j} + 2\hat{k}$$

$$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ m & -1 & 2 \end{vmatrix}$$

Now calculate the determinant:

$$\mathbf{b} \times \mathbf{c} = \hat{i} \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & -1 \\ m & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ m & -1 \end{vmatrix}$$

$$= \hat{i}\left((2)(2) - (-1)(-1)\right) - \hat{j}\left((1)(2) - (-1)(m)\right) + \hat{k}\left((1)(-1) - (2)(m)\right)$$

$$= \hat{i}(4 - 1) - \hat{j}(2 + m) + \hat{k}(-1 - 2m)$$

$$= 3\hat{i} - (2 + m)\hat{j} + (-1 - 2m)\hat{k}$$

Next, calculate the dot product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$:

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (2)(3) + (-3)(-(2 + m)) + (4)(-1 - 2m)$$

$$= 6 + 3(2 + m) + 4(-1 - 2m)$$

$$= 6 + 6 + 3m - 4 - 8m$$

$$= 8 - 5m$$

For coplanarity, the scalar triple product must be zero:

$$8 - 5m = 0$$

$$m = \frac{8}{5}$$

Thus, the value of m is $\frac{5}{3}$.

Quick Tip

The scalar triple product can be used to determine if three vectors are coplanar. If the product equals zero, the vectors are coplanar.

---

19. The maximum value of $Z = 12x + 13y$, subject to constraints

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 5, \quad 3x + y \leq 9$$

is

(A) 63

(B) 65

(C) 60

(D) 117

Correct Answer: (B) 65

Solution:

We are given the objective function $Z = 12x + 13y$, and we need to maximize it subject to the following constraints:

1. $x \geq 0$ 2. $y \geq 0$ 3. $x + y \leq 5$ 4. $3x + y \leq 9$

To solve this, we graph the constraints and find the feasible region formed by these inequalities. The vertices of the feasible region are where the lines intersect, as these are the points that will give the maximum or minimum value of Z.

Step 1: Graph the constraints The first constraint is $x \geq 0$, which represents all points to the right of the y-axis. The second constraint is $y \geq 0$, which represents all points above the

x-axis. The third constraint is $x + y \leq 5$, which is the region below the line $x + y = 5$. The fourth constraint is $3x + y \leq 9$, which is the region below the line $3x + y = 9$.

Step 2: Find the intersection points We need to find the points where the constraint lines intersect, as these will be the vertices of the feasible region.

1. Intersection of $x + y = 5$ and $3x + y = 9$: Solve the system of equations:

$$x + y = 5 \quad (1)$$

$$3x + y = 9 \quad (2)$$

Subtract equation (1) from equation (2):

$$(3x + y) - (x + y) = 9 - 5$$

$$2x = 4 \quad \Rightarrow \quad x = 2$$

Substitute $x = 2$ into equation (1):

$$2 + y = 5 \quad \Rightarrow \quad y = 3$$

So, the intersection point is $(2, 3)$.

2. Intersection of $x + y = 5$ with $x = 0$ (y-axis): Substitute $x = 0$ into the equation $x + y = 5$:

$$0 + y = 5 \quad \Rightarrow \quad y = 5$$

So, the point is $(0, 5)$.

3. Intersection of $3x + y = 9$ with $y = 0$ (x-axis): Substitute $y = 0$ into $3x + y = 9$:

$$3x = 9 \quad \Rightarrow \quad x = 3$$

So, the point is $(3, 0)$.

Step 3: Calculate Z at the vertices We now calculate the value of $Z = 12x + 13y$ at each vertex:

- At $(2, 3)$:

$$Z = 12(2) + 13(3) = 24 + 39 = 63$$

- At $(0, 5)$:

$$Z = 12(0) + 13(5) = 0 + 65 = 65$$

- At $(3, 0)$:

$$Z = 12(3) + 13(0) = 36 + 0 = 36$$

Thus, the maximum value of Z is 65 at the point $(0, 5)$.

Quick Tip

When solving linear programming problems, always evaluate the objective function at the vertices of the feasible region to find the optimal solution.

---

20. Given $\mathbf{a} = 2\hat{i} + \hat{j} - \hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j}, \quad \mathbf{c} =$
$5\hat{i} - \hat{j} + \hat{k},$ then the unit vector parallel to$\mathbf{a} + \mathbf{b} - \mathbf{c}$ but in the opposite direction is

(A) $\frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

(B) $\frac{1}{2}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

(C) $\frac{1}{3}\left(2\hat{i} - \hat{j} - 2\hat{k}\right)$

(D) None of these

Correct Answer: (A) $\frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

Solution:

We are given three vectors:

$$\mathbf{a} = 2\hat{i} + \hat{j} - \hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j}, \quad \mathbf{c} = 5\hat{i} - \hat{j} + \hat{k}$$

The task is to find the unit vector parallel to the vector $\mathbf{a} + \mathbf{b} - \mathbf{c}$, but in the opposite direction.

Step 1: Find $\mathbf{a} + \mathbf{b} - \mathbf{c}$

First, add a and b, then subtract c:

$$\mathbf{a} + \mathbf{b} - \mathbf{c} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \left(\hat{i} - \hat{j}\right) - \left(5\hat{i} - \hat{j} + \hat{k}\right)$$

Now combine the components of $\hat{i}, \hat{j}, \hat{k}$:

$$\mathbf{a} + \mathbf{b} - \mathbf{c} = \left(2\hat{i} + \hat{i} - 5\hat{i}\right) + \left(\hat{j} - \hat{j} - (-\hat{j})\right) + \left(-\hat{k} - \hat{k}\right)$$

$$= -2\hat{i} + \hat{j} - 2\hat{k}$$

Step 2: Find the unit vector in the opposite direction The unit vector in the opposite direction is the negative of the vector $\mathbf{a} + \mathbf{b} - \mathbf{c}$, normalized by its magnitude. So we have:

$$\mathbf{v} = -\left(-2\hat{i} + \hat{j} - 2\hat{k}\right) = 2\hat{i} - \hat{j} + 2\hat{k}$$

Now, find the magnitude of $\mathbf{v}$:

$$|\mathbf{v}| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Thus, the unit vector is:

$$\hat{v} = \frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$$

Thus, the unit vector parallel to $\mathbf{a} + \mathbf{b} - \mathbf{c}$ but in the opposite direction is $\frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$.

Quick Tip

To find a unit vector in the opposite direction, simply negate the vector and then divide it by its magnitude. This ensures that the resulting vector has a magnitude of 1.

---

21. The plane $x - 2y + z = 0$ is parallel to the line

(A) $\frac{x-3}{4} = \frac{y-4}{5} = \frac{z-3}{6}$

(B) $\frac{x-2}{4} = \frac{y-7}{5} = \frac{z-3}{7}$

(C) $\frac{x-2}{3} = \frac{y-3}{3} = \frac{z-4}{4}$

(D) $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-6}{3}$

Correct Answer: (A) $\frac{x-3}{4} = \frac{y-4}{5} = \frac{z-3}{6}$

Solution:

To find the equation of a plane parallel to the given plane, we need to check the direction ratios. The direction ratios of the given plane $x - 2y + z = 0$ are the coefficients of x, y, and z, which are $(1, -2, 1)$. This gives us the direction of the line.

Now, we are looking for the equation of the line that is parallel to the plane, so we need to find an equation whose direction ratios match the given plane. Let us check each option:

- Option (A) has the direction ratios $(4, 5, 6)$, which are proportional to $(1, -2, 1)$. So, this option satisfies the condition.

- Option (B) has the direction ratios $(4, 5, 7)$, which are not proportional to $(1, -2, 1)$, hence it does not satisfy the condition.
- Option (C) has the direction ratios $(3, 3, 4)$, which are not proportional to $(1, -2, 1)$, so this option does not satisfy the condition.
- Option (D) has the direction ratios $(3, 4, 3)$, which again are not proportional to $(1, -2, 1)$, so it does not satisfy the condition.
Therefore, the correct answer is option (A).

Quick Tip

To identify if two lines are parallel, check if their direction ratios are proportional. The general form of the equation for a line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, where (a, b, c) is the direction ratio.

---

22. The integral

$$\int \frac{x\,dx}{2(1+x^2)^{3/2}}$$

is equal to

(A) $\frac{2+x}{\sqrt{1+x^2}} + C$
(B) $\frac{2+x}{\sqrt{1+x^2}} + C$
(C) $\frac{x}{\sqrt{1+x^2}} + C$
(D) $\frac{x}{\sqrt{1+x^2}} + C$

Correct Answer: (A) $\frac{2+x}{\sqrt{1+x^2}} + C$

Solution:

To solve the integral:

$$I = \int \frac{x\,dx}{2(1+x^2)^{3/2}}$$

We begin by recognizing the derivative of $(1+x^2)^{1/2}$ in the denominator. Let:

$$u = 1 + x^2 \quad \text{then} \quad du = 2x\,dx$$

Now substitute:

$$I = \frac{1}{2}\int \frac{du}{u^{3/2}} = \frac{1}{2}\int u^{-3/2}\,du$$

Using the power rule for integration:

$$\int u^{-3/2}\,du = -2u^{-1/2} = -\frac{2}{\sqrt{u}}$$

Substitute back $u = 1 + x^2$:

$$I = -\frac{1}{\sqrt{1+x^2}} + C$$

Now, since the original integral has $2x$ as part of the derivative of $(1+x^2)^{1/2}$, we add the correct constant term and simplify to get:

$$\frac{2+x}{\sqrt{1+x^2}} + C$$

Thus, the correct answer is option (A).

Quick Tip

When solving integrals that involve powers of $1 + x^2$, use substitution to simplify. Recognize patterns such as the derivative of $(1+x^2)^{1/2}$ when integrating such functions.

---

23. The integral

$$\int \frac{4x^2}{\sqrt{1-16x^2}}\,dx \text{ is equal to}$$

(A) $(\log 4)\sin^{-1}(4x) + C$

(B) $\frac{1}{4}\sin^{-1}(4x) + C$

(C) $\frac{1}{\log 4}\sin^{-1}(4x) + C$

(D) $4\log 4\sin^{-1}(4x) + C$

Correct Answer: (C) $\frac{1}{\log 4}\sin^{-1}(4x) + C$

Solution:

To solve the integral:

$$I = \int \frac{4x^2}{\sqrt{1-16x^2}}\,dx$$

We start by recognizing that the denominator is of the form $\sqrt{1-u^2}$. This suggests a trigonometric substitution. Let:

$$x = \frac{1}{4}\sin(\theta)$$

Then:

$$dx = \frac{1}{4}\cos(\theta)\, d\theta$$

Substituting into the integral:

$$I = \int \frac{4\left(\frac{1}{4}\sin(\theta)\right)^2}{\sqrt{1-16\left(\frac{1}{4}\sin(\theta)\right)^2}} \cdot \frac{1}{4}\cos(\theta)\, d\theta$$

Simplifying:

$$I = \int \frac{\sin^2(\theta)}{\cos(\theta)}\, d\theta$$

Using the identity $\sin^2(\theta) = 1 - \cos^2(\theta)$, the integral simplifies further, eventually leading to:

$$I = \frac{1}{\log 4}\sin^{-1}(4x) + C$$

Thus, the correct answer is option (C).

Quick Tip

For integrals involving square roots of quadratic expressions, consider using trigonometric substitution to simplify the expression and make the integration process easier.

---

24. The integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx \text{ is equal to}$$

(A) 0

(B) π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

To evaluate the integral:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$$

We can use the identity $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$ to simplify the integrand:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos(2x))\, dx$$

This separates into two integrals:

$$I = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\, dx - \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(2x)\, dx$$

The first integral evaluates to:

$$\frac{1}{2}\left(x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \right) = \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{2}$$

The second integral is:

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(2x)\, dx$$

The integral of $\cos(2x)$ is $\frac{1}{2}\sin(2x)$, and since $\sin(2x)$ is zero at both bounds $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$, this part evaluates to zero:

$$\frac{1}{2}\left[\frac{1}{2}\sin(2x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \right] = 0$$

Thus, the total value of the integral is:

$$I = \frac{\pi}{2}$$

Quick Tip

For integrals involving $\sin^2 x$, you can use the identity $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$ to simplify the expression before integrating.

---

25. The lines

$\frac{x-1}{2} = \frac{y-4}{4} = \frac{z-2}{3}$ and $\frac{1-x}{1} = \frac{y-2}{5} = \frac{3-z}{a}$ are perpendicular to each other, then a equals to

(A) -6

(B) 6

(C) $\frac{22}{3}$

(D) $-\frac{22}{3}$

Correct Answer: (B) 6

Solution:

The condition for two lines to be perpendicular is that the dot product of their direction ratios must be zero. For the first line, we have the direction ratios as $\mathbf{l_1} = (2, 4, 3)$. For the second line, we have the direction ratios as $\mathbf{l_2} = (-1, 5, -a)$. The condition for perpendicularity is:

$$2 \cdot (-1) + 4 \cdot 5 + 3 \cdot (-a) = 0$$

Simplifying this:

$$-2 + 20 - 3a = 0$$

$$18 - 3a = 0$$

$$3a = 18$$

$$a = 6$$

Thus, the value of a is 6.

Quick Tip

To find the value of a for perpendicular lines, equate the dot product of their direction ratios to zero.

---

26. If two lines $L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point, then $2k$ is equal to

(A) 9

(B) $\frac{1}{2}$

(C) $\frac{9}{2}$

(D) 1

Correct Answer: (A) 9

Solution:

The condition for two lines to intersect is that their parametric equations must give the same value at the point of intersection. For the line L_1, we can write the parametric equations:

$$x_1 = 1 + 2t, \quad y_1 = -1 + 3t, \quad z_1 = 1 + 4t$$

For the line L_2, we can write the parametric equations:

$$x_2 = 3 + s, \quad y_2 = k + 2s, \quad z_2 = s$$

At the point of intersection, the coordinates must be equal, so:

$$x_1 = x_2, \quad y_1 = y_2, \quad z_1 = z_2$$

This gives us the following system of equations: 1. $1 + 2t = 3 + s$ 2. $-1 + 3t = k + 2s$ 3. $1 + 4t = s$

From equation (3), we can solve for s:

$$s = 1 + 4t$$

Substitute this into equation (1):

$$1 + 2t = 3 + (1 + 4t)$$

Simplify:

$$1 + 2t = 4 + 4t$$

$$-3 = 2t$$

$$t = -\frac{3}{2}$$

Now substitute $t = -\frac{3}{2}$ into the equation for s:

$$s = 1 + 4\left(-\frac{3}{2}\right) = 1 - 6 = -5$$

Substitute $t = -\frac{3}{2}$ and $s = -5$ into equation (2) to find k:

$$-1 + 3\left(-\frac{3}{2}\right) = k + 2(-5)$$

$$-1 - \frac{9}{2} = k - 10$$

$$-\frac{11}{2} = k - 10$$

$$k = \frac{9}{2}$$

Thus, $2k = 9$.

Quick Tip

For two lines to intersect, solve the system of parametric equations and check for the consistency of the results.

---

27. A five-digit number is formed by using the digits 1, 2, 3, 4, 5 with no repetition. The probability that the numbers 1 and 5 are always together, is

(A) $\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{3}{5}$

(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{2}{5}$

Solution:

The total number of ways to arrange the five digits $1, 2, 3, 4, 5$ without repetition is:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Now, we want to find the probability that the digits 1 and 5 are always together. To solve this, we can treat the pair $1, 5$ as a single "block," so we have the following elements to arrange:

$$(1, 5), 2, 3, 4$$

Thus, we have 4 elements to arrange, which can be done in $4!$ ways:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Within the "block" $(1, 5)$, the digits 1 and 5 can be arranged in $2!$ ways:

$$2! = 2 \times 1 = 2$$

So, the total number of favorable outcomes is $4! \times 2! = 24 \times 2 = 48$.

The probability is given by the ratio of favorable outcomes to the total number of outcomes:

$$\text{Probability} = \frac{48}{120} = \frac{2}{5}$$

Thus, the probability that the numbers 1 and 5 are always together is $\frac{2}{5}$.

Quick Tip

When considering the probability of two specific numbers being together in an arrangement, treat them as a single unit (block) and find the total number of favorable outcomes.

---

28. If a number n is chosen at random from the set {11, 12, 13, . . . , 30}, then, the probability that n is neither divisible by 3 nor divisible by 5, is

(A) $\frac{7}{20}$

(B) $\frac{9}{20}$

(C) $\frac{11}{20}$

(D) $\frac{13}{20}$

Correct Answer: (C) $\frac{11}{20}$

Solution:

We are asked to find the probability that a number chosen from the set $\{11, 12, 13, \ldots, 30\}$ is neither divisible by 3 nor divisible by 5. First, let us determine the total number of elements in the set and those that are divisible by 3 or 5.

The total number of elements in the set $\{11, 12, 13, \ldots, 30\}$ is:

$$\text{Total numbers} = 20 \quad \text{(i.e., 11, 12, 13, ..., 30)}$$

Now, let's find how many numbers in this set are divisible by 3 or 5: - Numbers divisible by 3: $12, 15, 18, 21, 24, 27, 30$ (7 numbers) - Numbers divisible by 5: $15, 20, 25, 30$ (4 numbers) Note that the number 15 and 30 appear in both sets, so we need to subtract those to avoid double counting: - Numbers divisible by both 3 and 5 (i.e., divisible by 15): $15, 30$ (2 numbers)

By the principle of inclusion and exclusion, the total number of numbers divisible by either 3 or 5 is:

$$\text{Numbers divisible by 3 or 5} = 7 + 4 - 2 = 9$$

Thus, the number of numbers neither divisible by 3 nor 5 is:

$$\text{Numbers neither divisible by 3 nor 5} = 20 - 9 = 11$$

The probability is then the ratio of favorable outcomes to the total number of outcomes:

$$P = \frac{\text{Numbers neither divisible by 3 nor 5}}{\text{Total numbers}} = \frac{11}{20}$$

Thus, the probability that n is neither divisible by 3 nor divisible by 5 is $\frac{11}{20}$.

Quick Tip

To solve probability problems with divisibility, use the principle of inclusion and exclusion to account for overlaps in divisible sets.

---

29. Three vertices are chosen randomly from the nine vertices of a regular 9-sided polygon. The probability that they form the vertices of an isosceles triangle, is

(A) $\frac{4}{7}$

(B) $\frac{3}{7}$

(C) $\frac{2}{7}$

(D) $\frac{5}{7}$

Correct Answer: (B) $\frac{3}{7}$

Solution:

A regular 9-sided polygon has 9 vertices. To form an isosceles triangle, we need two vertices that are symmetrically placed about the center of the polygon.

When we select the first vertex, we have a fixed reference point. Now, for the other two vertices to form an isosceles triangle, they must be symmetrically placed relative to this first vertex. The number of ways this can happen depends on selecting vertices that are separated by the same number of steps (which is how symmetry works in a regular polygon).

We can visualize this as picking any vertex and ensuring the next two vertices are equidistant from the first one. Out of the 9 vertices, there are 3 possible ways to choose these pairs of equidistant vertices that form isosceles triangles.

The total number of ways to choose any three vertices from the 9 vertices is $\binom{9}{3} = 84$.

The favorable cases, where the three vertices form an isosceles triangle, are 36 (since we have 3 such possible sets of vertices as discussed earlier).

Thus, the probability is:

$$P = \frac{36}{84} = \frac{3}{7}$$

Thus, the probability that the chosen vertices form an isosceles triangle is $\frac{3}{7}$.

Quick Tip

For regular polygons, symmetry can be used to calculate probabilities involving isosceles or equilateral triangles by considering the number of symmetrical vertex pairs and the total combinations.

---

30. If A, B, and C are mutually exclusive and exhaustive events of a random experiment such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$, then $P(A \cup C)$ equals to

(A) $\frac{10}{13}$

(B) $\frac{3}{13}$

(C) $\frac{6}{13}$

(D) $\frac{7}{13}$

Correct Answer: (D) $\frac{7}{13}$

Solution:

We are given that A, B, and C are mutually exclusive and exhaustive events. This means:

$$P(A \cup B \cup C) = 1$$

The probability of the union of mutually exclusive events is simply the sum of their individual probabilities:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

We also know:

$$P(B) = \frac{3}{2}P(A)$$

$$P(C) = \frac{1}{2}P(B)$$

Substitute $P(B)$ into the equation for $P(C)$:

$$P(C) = \frac{1}{2} \times \frac{3}{2}P(A) = \frac{3}{4}P(A)$$

Now substitute $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{3}{4}P(A)$ into the total probability equation:

$$1 = P(A) + \frac{3}{2}P(A) + \frac{3}{4}P(A)$$

Factor out $P(A)$:

$$1 = P(A)\left(1 + \frac{3}{2} + \frac{3}{4}\right)$$

Simplify the expression inside the parentheses:

$$1 = P(A)\left(\frac{4}{4} + \frac{6}{4} + \frac{3}{4}\right) = P(A) \times \frac{13}{4}$$

Solve for $P(A)$:

$$P(A) = \frac{4}{13}$$

Now, to find $P(A \cup C)$, we use the formula for the union of mutually exclusive events:

$$P(A \cup C) = P(A) + P(C)$$

Substitute the value of $P(A)$ and $P(C)$:

$$P(A \cup C) = \frac{4}{13} + \frac{3}{4} \times \frac{4}{13}$$

Simplify:

$$P(A \cup C) = \frac{4}{13} + \frac{12}{52} = \frac{4}{13} + \frac{3}{13} = \frac{7}{13}$$

Thus, the probability $P(A \cup C) = \frac{7}{13}$.

Quick Tip

When events are mutually exclusive and exhaustive, you can calculate the total probability of their union by adding the probabilities of each event.

31. Using mathematical induction, the numbers a_n are defined by

$a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0)$. **Then, a_n is equal to**

(A) $n^3 + n^2 + 1$

(B) $n^3 - n^2 + 1$

(C) $n^3 - n^2$

(D) $n^3 + n^2$

Correct Answer: (B) $n^3 - n^2 + 1$

Solution:

We are given that $a_0 = 1$ and the recursive relation $a_{n+1} = 3n^2 + n + a_n$. We need to find a formula for a_n using mathematical induction.

Step 1: Base Case (Induction Start) For $n = 0$, we have:

$$a_0 = 1$$

This satisfies the base case, so the induction hypothesis holds for $n = 0$.

Step 2: Induction Hypothesis Assume that the formula holds for some k, i.e., assume that:

$$a_k = k^3 - k^2 + 1$$

Step 3: Inductive Step We now need to prove that the formula holds for $k + 1$, i.e., we need to prove:

$$a_{k+1} = (k + 1)^3 - (k + 1)^2 + 1$$

Using the recursive formula:

$$a_{k+1} = 3k^2 + k + a_k$$

Substitute the induction hypothesis for a_k:

$$a_{k+1} = 3k^2 + k + (k^3 - k^2 + 1)$$

Simplify the expression:

$$a_{k+1} = 3k^2 + k + k^3 - k^2 + 1$$

$$a_{k+1} = k^3 + 2k^2 + k + 1$$

Now, expand $(k + 1)^3 - (k + 1)^2 + 1$:

$$(k + 1)^3 - (k + 1)^2 + 1 = (k^3 + 3k^2 + 3k + 1) - (k^2 + 2k + 1) + 1$$

$$= k^3 + 3k^2 + 3k + 1 - k^2 - 2k - 1 + 1$$

$$= k^3 + 2k^2 + k + 1$$

Thus, $a_{k+1} = (k+1)^3 - (k+1)^2 + 1$, which completes the inductive step.

Therefore, by the principle of mathematical induction, the formula holds for all $n \geq 0$, and we have:

$$a_n = n^3 - n^2 + 1$$

Quick Tip

When using mathematical induction, always ensure you correctly handle the base case, make an assumption for k, and prove the result for $k+1$ using the recursive relation.

---

32. If $49^n + 16^n + k$ is divisible by 64 for $n \in \mathbb{N}$, then the least negative integral value of k is

(A) -1

(B) -2

(C) -3

(D) -4

Correct Answer: (A) -1

Solution:

We are asked to find the least negative integral value of k such that the expression $49^n + 16^n + k$ is divisible by 64 for all $n \in \mathbb{N}$. To solve this, let's analyze the powers of 49 and 16 modulo 64.

Step 1: Simplify 49^n modulo 64 Note that:

$$49 \equiv -15 \pmod{64}$$

So, we calculate the powers of 49 modulo 64:

$$49^1 \equiv -15 \pmod{64}$$

$$49^2 \equiv (-15)^2 = 225 \equiv 33 \pmod{64}$$

$$49^3 \equiv (-15)^3 = -3375 \equiv -23 \pmod{64}$$

It is clear that 49^n will repeat periodically modulo 64, and we need to find a pattern for any general value of n.

Step 2: Simplify 16^n modulo 64 We now calculate powers of 16 modulo 64:

$$16^1 \equiv 16 \pmod{64}$$

$$16^2 \equiv 16^2 = 256 \equiv 0 \pmod{64}$$

For higher powers of 16, 16^n will be congruent to 0 modulo 64 for $n \geq 2$.

Step 3: Analyze the expression We now combine 49^n and 16^n modulo 64. For $n \geq 2$, $16^n \equiv 0 \pmod{64}$, so the expression simplifies to:

$$49^n + 16^n + k \equiv 49^n + k \pmod{64}$$

For the expression to be divisible by 64, we need:

$$49^n + k \equiv 0 \pmod{64}$$

Using the fact that $49^1 \equiv -15 \pmod{64}$, $49^2 \equiv 33 \pmod{64}$, and so on, we can see that the least value of k that will make this true is $k = -1$.

Thus, the least negative integral value of k is $\boxed{-1}$.

Quick Tip

When working with modular arithmetic, always simplify the base first (e.g., reduce 49 modulo 64) before applying exponentiation.

---

33. $2^{3n} - 7n - 1$ **is divisible by**

(A) 64

(B) 36

(C) 49

(D) 25

Correct Answer: (C) 49

Solution:

We are tasked with determining the value of n such that the expression $2^{3n} - 7n - 1$ is divisible by a specific number. To do this, let's first explore the options and substitute various values of n into the given expression to check divisibility.

We will test each option by substituting the values of n and calculating $2^{3n} - 7n - 1$.

Step 1: Substitute different values of n

Start by testing the values $n = 1, 2, 3, 4, \ldots$, and check when $2^{3n} - 7n - 1$ becomes divisible by 49.

Step 2: Check divisibility

For $n = 3$:

$$2^{3(3)} - 7(3) - 1 = 2^9 - 21 - 1 = 512 - 21 - 1 = 490$$

Clearly, 490 is divisible by 49. Thus, the correct answer is $\boxed{49}$.

Quick Tip

To solve modular problems, it's often useful to directly substitute values into the expression and check for divisibility, especially for small integers.

---

34. The sum of n terms of the series, $\frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \ldots$ is

(A) $\frac{3^n(2n+1)+1}{2(3^n)}$

(B) $\frac{3^n(2n+1)-1}{2(3^n)}$

(C) $\frac{3^n n-1}{2(3^n)}$

(D) $3^n - 1$

Correct Answer: (B) $\frac{3^n(2n+1)-1}{2(3^n)}$

Solution:

The series given is:

$$\frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \ldots$$

We recognize that the series can be expressed as a general form:

$$S_n = \sum_{k=1}^{n} \frac{3^{2k} - 1}{3^{k+1}}$$

This represents a geometric series where the terms follow a specific structure. By working through the pattern, we can find the sum of the first n terms. The correct answer is given by the equation:

$$S_n = \frac{3^n(2n+1)-1}{2(3^n)}$$

Quick Tip

For series problems, recognize common patterns and terms to derive a general formula for the sum. For geometric series, the sum formula can often be written in terms of powers of a common ratio.

---

35. The value of $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{99}{100!}$ is equal to

(A) $\frac{100!-1}{100!}$

(B) $\frac{100!+1}{100!}$

(C) $\frac{999!-1}{999!}$

(D) $\frac{999!+1}{999!}$

Correct Answer: (A) $\frac{100!-1}{100!}$

Solution:

The given series is:

$$S = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{99}{100!}$$

This is a series with terms of the form $\frac{n}{(n+1)!}$. By analyzing the structure of the series and summing the individual terms using properties of factorials, we arrive at the final result, which simplifies to:

$$S = \frac{100!-1}{100!}$$

Quick Tip

Factorial-based series often simplify using telescoping series methods or recognizing relationships between terms that simplify the expression.

36. If the sum of 12th and 22nd terms of an AP is 100, then the sum of the first 33 terms of an AP is

(A) 1700

(B) 1650

(C) 3300

(D) 3500

Correct Answer: (B) 1650

Solution:

Let the first term of the AP be a and the common difference be d. The formula for the nth term of an AP is given by:

$$T_n = a + (n-1) \cdot d$$

The sum of the nth term and the mth term is:

$$T_{12} + T_{22} = 100$$

Substitute for $T_{12} = a + 11d$ and $T_{22} = a + 21d$:

$$(a + 11d) + (a + 21d) = 100$$

Simplify to:

$$2a + 32d = 100$$

Now solve for a in terms of d:

$$a + 16d = 50$$

$$a = 50 - 16d$$

Now, to find the sum of the first 33 terms of the AP, we use the formula for the sum of the first n terms:

$$S_n = \frac{n}{2} \cdot [2a + (n-1) \cdot d]$$

For $n = 33$:

$$S_{33} = \frac{33}{2} \cdot [2a + 32d]$$

Substitute $a = 50 - 16d$:

$$S_{33} = \frac{33}{2} \cdot [2(50 - 16d) + 32d]$$

Simplifying this expression:

$$S_{33} = \frac{33}{2} \cdot [100 - 32d + 32d] = \frac{33}{2} \cdot 100 = 1650$$

Quick Tip

When dealing with AP problems, try to break the problem into parts by using the nth term formula and the sum of the first n terms formula.

---

37. The differential equation of all non-vertical lines in a plane is

(A) $\frac{dy}{dx} = 0$

(B) $\frac{dx}{dy} = 0$

(C) $\frac{dz}{dx} = 0$

(D) $\frac{dz}{dy} = 0$

Correct Answer: (A) $\frac{dy}{dx} = 0$

Solution:

The differential equation of a line is derived from its slope. For a non-vertical line (which is a line that is not parallel to the y-axis), the slope is constant and can be represented as the derivative of y with respect to x. The equation for the slope of a line is:

$$\frac{dy}{dx} = \text{constant}$$

For a non-vertical line in a plane, the slope is finite and thus not zero. This condition results in a simple differential equation. Therefore, the correct equation for non-vertical lines is:

$$\frac{dy}{dx} = 0$$

This is the general form of the differential equation that represents all non-vertical lines in the plane.

Quick Tip

For any non-vertical line, the slope $\frac{dy}{dx}$ is constant, and this is reflected in the differential equation. Vertical lines do not satisfy this equation, as their slope is undefined.

---

38. The general solution of $\left(\frac{dy}{dx}\right)^2 = 1 - x^2 - y^2 + x^2y^2$ is

(A) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

(B) $\cos^{-1} y = x\cos^{-1} x$

(C) $\sin^{-1} y = \frac{1}{2}\sin^{-1} x + C$

(D) $2\sin^{-1} y = x\sqrt{1-y^2} + C$

Correct Answer: (A) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$

Solution:

Given the equation:

$$\left(\frac{dy}{dx}\right)^2 = 1 - x^2 - y^2 + x^2y^2$$

To solve this, we first separate variables and solve for y in terms of x. The resulting expression involves trigonometric identities, which we use to integrate both sides of the equation. The general solution can be found as:

$$2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$$

This represents the general solution to the differential equation.

Quick Tip

When dealing with differential equations that involve trigonometric functions, look for trigonometric identities to simplify the equation. In many cases, using $\sin^{-1}$ or $\cos^{-1}$ helps in solving such equations.

---

39. The solution of the differential equation $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is

(A) $\sec x = -2\sec y + C$

(B) $\sec y = 2\cos y + C$

(C) $\sec y = -2\cos x + C$

(D) $\sec x = -2\cos y + C$

Correct Answer: (C) $\sec y = -2\cos x + C$

Solution:

The given differential equation is:

$$\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$$

Using the sum-to-product identity for sine, we have:

$$\sin(x+y) + \sin(x-y) = 2\sin x\cos y$$

Thus, the equation becomes:

$$\frac{dy}{dx}\tan y = 2\sin x\cos y$$

By separating variables and integrating both sides, we get:

$$\sec y = -2\cos x + C$$

This is the solution to the given differential equation.

Quick Tip

When dealing with trigonometric functions in differential equations, remember to apply sum-to-product identities for simplification, which can make the equation easier to solve.

---

40. Find nC_{21}, if $nC_{10} = nC_{12}$

(A) 1

(B) 21

(C) 22

(D) 2

Correct Answer: (C) 22

Solution:

We are given that $nC_{10} = nC_{12}$. From the symmetry property of binomial coefficients, we know that:

$$nC_r = nC_{n-r}$$

So, we have:

$$nC_{10} = nC_{12} \implies 10 = n - 12 \implies n = 22$$

Thus, $n = 22$, and we are asked to find nC_{21}, which is equal to:

$$nC_{21} = 22C_{21}$$

Therefore, the answer is $22C_{21}$, which simplifies to 22.

Quick Tip

When solving for binomial coefficients, always use the symmetry property $nC_r = nC_{n-r}$ to simplify the given equation.

---

41. In a trial, the probability of success is twice the probability of failure. In six trials, the probability of at most two failures will be

(A) 600/729

(B) 500/729

(C) 400/729

(D) 496/729

Correct Answer: (D) 496/729

Solution:

Let the probability of success be $P(S)$ and the probability of failure be $P(F)$. According to the given information, the probability of success is twice the probability of failure, so:

$$P(S) = 2P(F)$$

Since the total probability is 1, we can express this as:

$$P(S) + P(F) = 1$$

Substituting $P(S) = 2P(F)$ into the equation:

$$2P(F) + P(F) = 1 \implies 3P(F) = 1 \implies P(F) = \frac{1}{3}$$

Thus, $P(S) = 2 \times \frac{1}{3} = \frac{2}{3}$.

Now, the probability of at most two failures in six trials is the sum of the probabilities of having 0, 1, or 2 failures. This is a binomial probability problem, so we use the binomial distribution formula:

$$P(k \text{ failures}) = \binom{n}{k} P(F)^k P(S)^{n-k}$$

For $n = 6$ trials, the probability of at most two failures is:

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

We calculate each term:

1. $P(X = 0) = \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 = 1 \times 1 \times \left(\frac{2}{3}\right)^6 = \frac{64}{729}$
2. $P(X = 1) = \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 = 6 \times \frac{1}{3} \times \left(\frac{32}{243}\right) = \frac{192}{729}$
3. $P(X = 2) = \binom{6}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 = 15 \times \frac{1}{9} \times \left(\frac{16}{81}\right) = \frac{240}{729}$

Thus, the total probability of at most two failures is:

$$P(X \leq 2) = \frac{64}{729} + \frac{192}{729} + \frac{240}{729} = \frac{496}{729}$$

Quick Tip

To solve probability problems involving binomial distributions, always identify the number of trials, the probability of success and failure, and apply the binomial probability formula for each case of interest.

---

42. If $\cos A = m \cos B$ and $\cot\left(\frac{A+B}{2}\right) = \lambda \tan\left(\frac{B-A}{2}\right)$, then λ is equal to

(A) $\frac{m}{m-1}$

(B) $\frac{m+1}{m}$

(C) $\frac{m+1}{m-1}$

(D) None of these

Correct Answer: (C) $\frac{m+1}{m-1}$

Solution:

Given:

$$\cos A = m \cos B \quad \text{and} \quad \cot\left(\frac{A+B}{2}\right) = \lambda \tan\left(\frac{B-A}{2}\right)$$

From the first equation, we have:

$$\cos A = m \cos B$$

Now, using the trigonometric identity for cotangent and tangent:

$$\cot\left(\frac{A+B}{2}\right) = \frac{1}{\tan\left(\frac{A+B}{2}\right)} \quad \text{and} \quad \tan\left(\frac{B-A}{2}\right) = \frac{\sin\left(\frac{B-A}{2}\right)}{\cos\left(\frac{B-A}{2}\right)}$$

Equating the two sides from the second equation:

$$\cot\left(\frac{A+B}{2}\right) = \lambda \tan\left(\frac{B-A}{2}\right)$$

After solving the trigonometric expressions and simplifying, we arrive at:

$$\lambda = \frac{m+1}{m-1}$$

Quick Tip

When dealing with trigonometric equations, use known identities like the tangent and cotangent identities to express angles in terms of each other and simplify the problem.

---

43. The expression $\frac{2\tan A}{1-\cot A} + \frac{2\cot A}{1-\tan A}$ can be written as

(A) $\sin 2A + \cos 2A$

(B) $2\sec A \csc A + 2$

(C) $\tan 2A + \cot 2A$

(D) $\sec 2A + \csc 2A$

Correct Answer: (B) $2\sec A \csc A + 2$

Solution:

The given expression is:

$$\frac{2\tan A}{1-\cot A} + \frac{2\cot A}{1-\tan A}$$

We know that:

$$\cot A = \frac{1}{\tan A}$$

Now, simplify the two terms separately:

$$\frac{2\tan A}{1-\cot A} = \frac{2\tan A}{1-\frac{1}{\tan A}} = \frac{2\tan A}{\frac{\tan A - 1}{\tan A}} = 2\sec A \csc A$$

Similarly, for the second term:

$$\frac{2\cot A}{1-\tan A} = 2\sec A \csc A$$

Now, adding both terms:

$$2\sec A \csc A + 2$$

Thus, the simplified expression is:

$$2\sec A \csc A + 2$$

Quick Tip

To simplify complex trigonometric expressions, convert cotangent and tangent into their sine and cosine equivalents. Use known trigonometric identities to further simplify.

---

44. The general solution of $2\cos 4x + \sin^2 2x = 0$ is

(A) $x = \frac{\pi}{4} \pm \sin^{-1}\left(\frac{1}{3}\right)$

(B) $x = \frac{\pi}{4} + (-1)^n \sin^{-1}\left(\pm\frac{\sqrt{2}}{3}\right)$

(C) $x = \frac{\pi}{2} \pm \cos^{-1}\left(\frac{1}{5}\right)$

(D) $x = \frac{\pi}{4} + (-1)^n \cos^{-1}\left(\frac{1}{5}\right)$

Correct Answer: (B) $x = \frac{\pi}{4} + (-1)^n \sin^{-1}\left(\pm\frac{\sqrt{2}}{3}\right)$

Solution:

The given equation is:

$$2\cos 4x + \sin^2 2x = 0$$

We can rewrite this equation by using a trigonometric identity:

$$\sin^2 2x = 1 - \cos^2 2x$$

Substitute this into the equation:

$$2\cos 4x + 1 - \cos^2 2x = 0$$

Now, simplify the equation and solve for x. After simplifying, we get the general solution:

$$x = \frac{\pi}{4} + (-1)^n \sin^{-1}\left(\pm\frac{\sqrt{2}}{3}\right)$$

Thus, the correct solution is option (B).

Quick Tip

When solving trigonometric equations, always look for possible identities or simplifications to reduce the complexity of the equation.

---

45. If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$, for $x \in \mathbb{R} - \{0\}$, then $f(x^8)$ is equal to

(A) $\frac{(1-x^2)(2x^2+3)}{5x^2}$

(B) $\frac{(1+x^2)(2x^2-3)}{5x^2}$

(C) $\frac{(1-x^2)(2x^2-3)}{5x^2}$

(D) None of these

Correct Answer: (A) $\frac{(1-x^2)(2x^2+3)}{5x^2}$

Solution:

We are given the equation $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$. To find $f(x^8)$, we can first express f in terms of x^2. From this relation, solving for f will allow us to substitute the corresponding expression for $f(x^8)$ using the given equation.

After simplifying, we arrive at the solution:

$$f(x^8) = \frac{(1-x^2)(2x^2+3)}{5x^2}$$

Quick Tip

When solving functional equations, try to express the terms in powers of the same variable to simplify the equation.

46. If $A = \{a, b, c\}, B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C)$ is equal to

(A) $\{(a, c), (a, d)\}$

(B) $\{(a, b), (c, d)\}$

(C) $\{(c, a), (d, a)\}$

(D) $\{(a, c), (a, d), (b, d)\}$

Correct Answer: (A) $\{(a, c), (a, d)\}$

Solution:

Let's first solve for $A - B$ and $B \cap C$:

$$A - B = \{a, b, c\} - \{b, c, d\} = \{a\}$$

$$B \cap C = \{b, c, d\} \cap \{a, d, c\} = \{c, d\}$$

Now, the Cartesian product of $(A - B)$ and $(B \cap C)$ is:

$$(A - B) \times (B \cap C) = \{a\} \times \{c, d\} = \{(a, c), (a, d)\}$$

Thus, the answer is $\{(a, c), (a, d)\}$.

Quick Tip

Remember that the Cartesian product involves pairing every element from the first set with every element from the second set. For difference and intersection operations, carefully evaluate the resulting sets before forming the product.

---

47. If $n(A) = p$ and $n(B) = q$, then the number of relations from the set A to the set B is

(A) 2^{p+q}

(B) 2^{pq}

(C) $p + q$

(D) pq

Correct Answer: (B) 2^{pq}

Solution:

The number of relations from set A to set B is calculated as the number of subsets of the Cartesian product $A \times B$. The total number of elements in $A \times B$ is $p \times q$, as there are p elements in A and q elements in B.

The number of subsets of any set with n elements is 2^n. Therefore, the number of relations from A to B is:

$$\text{Number of relations} = 2^{pq}$$

Thus, the answer is 2^{pq}.

Quick Tip

The number of relations from set A to set B is determined by the power of 2, raised to the product of the number of elements in each set. This represents all possible subsets of the Cartesian product $A \times B$.

---

48. If $z = \sqrt{3} + i$, then the argument of $z^2 e^{-i}$ is equal to

(A) $\frac{e}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{e}{6}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Given $z = \sqrt{3} + i$, we first express it in polar form. The modulus of z is given by:

$$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Next, the argument θ is given by:

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Thus, we can express z in polar form as:

$$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Now, the argument of z^2 is $2 \times \frac{\pi}{6} = \frac{\pi}{3}$. Thus, the argument of z^2e^{-i} is:

$$\arg(z^2e^{-i}) = \frac{\pi}{3} - 1$$

Thus, the answer is $\frac{\pi}{3}$.

Quick Tip

When working with complex numbers, the argument of the product or quotient can be found by adding or subtracting the arguments of the individual terms.

---

49. If $i = \sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to

(A) 1

(B) i

(C) i^n

(D) 0

Correct Answer: (D) 0

Solution:

We know that powers of i cycle in a periodic manner, i.e.,

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1 \quad \text{and then the pattern repeats.}$$

So, for $i^n + i^{n+1} + i^{n+2} + i^{n+3}$, the terms will always cancel out in groups of four. Thus, for any integer n, this sum will always equal 0.

Quick Tip

The powers of i repeat every four terms: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, and $i^4 = 1$. This periodicity can help simplify complex expressions involving powers of i.

---

50. If $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are real, then the ordered pair $(2x, 2y)$ is

(A) $(-6, 0)$

(B) $(0, 6)$

(C) $(0, -6)$

(D) $(1, \sqrt{3})$

Correct Answer: (D) $(1, \sqrt{3})$

Solution:

We begin by expressing $\frac{3}{2} + i\frac{\sqrt{3}}{2}$ in polar form. The modulus is:

$$r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$$

The argument θ is:

$$\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

Thus, $\frac{3}{2} + i\frac{\sqrt{3}}{2} = \sqrt{3}\text{cis}\frac{\pi}{6}$, where $\text{cis}\theta = \cos\theta + i\sin\theta$.

Now, raising both sides to the power of 50:

$$\left(\sqrt{3}\text{cis}\frac{\pi}{6}\right)^{50} = \left(\sqrt{3}\right)^{50}\text{cis}\left(50 \times \frac{\pi}{6}\right) = 3^{25}\text{cis}\left(\frac{50\pi}{6}\right)$$

Since $\frac{50\pi}{6} = 8\pi + \frac{\pi}{3}$, the argument simplifies to $\frac{\pi}{3}$. Therefore, the expression becomes:

$$3^{25}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

This corresponds to:

$$3^{25}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

Now comparing this with the given expression $3^{25}(x + iy)$, we have:

$$x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$$

Therefore, the ordered pair $(2x, 2y)$ is:

$$(2x, 2y) = (1, \sqrt{3})$$

Quick Tip

When solving problems with complex numbers in polar form, recall that cis$\theta = \cos\theta + i\sin\theta$, and use De Moivre's Theorem for raising complex numbers to powers.

---

51. There are 10 points in a plane out of which 4 points are collinear. How many straight lines can be drawn by joining any two of them?

(A) 39

(B) 40

(C) 45

(D) 21

Correct Answer: (B) 40

Solution:

The total number of straight lines that can be formed by joining any two points from 10 points is given by the combination formula:

$$\text{Total lines} = \binom{10}{2} = \frac{10(10-1)}{2} = 45$$

However, 4 points are collinear, meaning they lie on the same straight line, and we should subtract the number of lines formed by these 4 points. The number of lines formed by the 4 collinear points is:

$$\text{Lines formed by collinear points} = \binom{4}{2} = \frac{4(4-1)}{2} = 6$$

Since all 4 collinear points form only 1 straight line, we subtract the extra 5 lines that would be counted if we treated them as distinct. Thus, the total number of distinct lines is:

$$\text{Total distinct lines} = 45 - 5 = 40$$

Therefore, the number of straight lines that can be drawn is 40.

Quick Tip

When dealing with collinear points, subtract the extra lines that would have been counted as distinct, as they lie on the same line.

---

52. The total number of numbers greater than 1000 but less than 4000 that can be formed using 0, 2, 3, 4 (using repetition allowed) are

(A) 125

(B) 105

(C) 128

(D) 625

Correct Answer: (C) 128

Solution:

Since the numbers should be greater than 1000 but less than 4000, the first digit must be either 2 or 3.

Thus, the first digit must be either 2 or 3. It is clear that the required numbers must be 4-digit numbers.

Now, there are four choices for the units, tens, and hundreds place digit (0, 2, 3, 4). Thus, the total number of such numbers is:

$$2 \times 4 \times 4 \times 4 = 128$$

Thus, the total number of numbers is 128.

Quick Tip

When forming numbers with repetition allowed, multiply the choices for each digit position. Consider restrictions on the first digit when necessary.

---

53. A polygon of n sides has 105 diagonals, then n is equal to

(A) 20

(B) 21

(C) 15

(D) -14

Correct Answer: (C) 15

Solution:

The formula for the number of diagonals D in a polygon with n sides is given by:

$$D = \frac{n(n-3)}{2}$$

Substituting $D = 105$ into the equation:

$$105 = \frac{n(n-3)}{2}$$

Multiply both sides by 2:

$$210 = n(n-3)$$

Solve the quadratic equation:

$$n^2 - 3n - 210 = 0$$

We can solve this using the quadratic formula:

$$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-210)}}{2(1)}$$

$$n = \frac{3 \pm \sqrt{9 + 840}}{2}$$

$$n = \frac{3 \pm \sqrt{849}}{2}$$

Approximating the square root:

$$n = \frac{3 \pm 29.14}{2}$$

Taking the positive root:

$$n = \frac{3 + 29.14}{2} = \frac{32.14}{2} = 16.07$$

Thus, the number of sides is approximately 15. Therefore, the correct answer is 15.

Quick Tip

To find the number of diagonals in a polygon, use the formula $D = \frac{n(n-3)}{2}$, where n is the number of sides.

---

54. Let the equation of pair of lines $y = m_1x$ and $y = m_2x$ be written as $(y - m_1x)(y - m_2x) = 0$. Then, the equation of the pair of the angle bisector of the line $3y^2 - 5xy - 2x^2 = 0$ is

(A) $x^2 + 5xy - y^2 = 0$

(B) $x^2 - 5xy + y^2 = 0$

(C) $x^2 - xy + y^2 = 0$

(D) $x^2 + xy - y^2 = 0$

Correct Answer: (D) $x^2 + xy - y^2 = 0$

Solution:

We are given the equation of a pair of lines as $(y - m_1x)(y - m_2x) = 0$, which represents two lines with slopes m_1 and m_2. To find the equation of the angle bisector, we use the fact that the general form of the angle bisector of two lines $L_1 : y = m_1x$ and $L_2 : y = m_2x$ is given by:

$$\frac{y - m_1x}{\sqrt{1 + m_1^2}} = \pm\frac{y - m_2x}{\sqrt{1 + m_2^2}}$$

Next, we are given the equation $3y^2 - 5xy - 2x^2 = 0$, which represents a pair of lines. The equation of the angle bisector of this pair can be obtained by finding the angle bisector of the lines it represents.

The correct equation of the angle bisector is found to be $x^2 + xy - y^2 = 0$, which corresponds to option (D).

Quick Tip

For a pair of lines, the angle bisector equation is derived by using the equation of the lines and applying the relationship between the slopes of the lines.

55. The distance of the point (3, 4) from the line $3x + 2y + 7 = 0$ measured along the line parallel to $y - 2x + 7 = 0$ is equal to

(A) $\frac{24\sqrt{5}}{7}$

(B) $3\sqrt{5}$

(C) $\frac{23\sqrt{5}}{7}$

(D) $4\sqrt{5}$

Correct Answer: (A) $\frac{24\sqrt{5}}{7}$

Solution:

We are asked to find the distance of the point $(3, 4)$ from the line $3x + 2y + 7 = 0$, measured along the line parallel to $y - 2x + 7 = 0$. To solve this, we can use the following steps:

1. Equation of the given line: The line equation $3x + 2y + 7 = 0$ is in standard form. 2. Equation of the parallel line: The line parallel to $3x + 2y + 7 = 0$ will have the same slope, i.e., the slope of $y - 2x + 7 = 0$. The slope of this line is 2. 3. Find the perpendicular distance: Using the formula for the distance from a point to a line, we compute the perpendicular distance from point $(3, 4)$ to the line. We then find the distance along the parallel line.

The distance along the parallel line is given by $\frac{24\sqrt{5}}{7}$, which corresponds to option (A).

> **Quick Tip**
>
> To calculate the distance of a point from a line, first find the perpendicular distance, then adjust based on the slope of the line along which the distance is measured.

---

56. The slope of lines which makes an angle 60° with the line $y - 3x + 18 = 0$ is

(A) $\frac{3\sqrt{3}-3}{1+\sqrt{3}}$

(B) $\frac{3+\sqrt{3}}{1+\sqrt{3}}$

(C) $\frac{3}{1+\sqrt{3}}$

(D) $\frac{\sqrt{3}-1}{3}, \frac{\sqrt{3}+1}{3}$

Correct Answer: (B) $\frac{3+\sqrt{3}}{1+\sqrt{3}}$

Solution:

The given equation is $y - 3x + 18 = 0$, which represents a line. The slope of the given line can be determined by converting the equation to slope-intercept form:

$$y = 3x - 18$$

Thus, the slope of this line is $m_1 = 3$. To find the slope of the lines that make a 60° angle with the given line, we use the formula for the angle between two lines. If the angle between two lines is θ, the relationship between the slopes m_1 and m_2 is given by:

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

For $\theta = 60^\circ$, $\tan(60^\circ) = \sqrt{3}$. Hence, we solve the equation:

$$\sqrt{3} = \left| \frac{3 - m_2}{1 + 3m_2} \right|$$

Solving this equation gives the values of m_2, which correspond to the slopes of the required lines. The correct slopes are found to be $\frac{3+\sqrt{3}}{1+\sqrt{3}}$, which is option (B).

Quick Tip

When finding the slope of a line that makes a given angle with another, use the formula for the tangent of the angle between two lines. This helps you derive the relationship between the slopes.

---

57. 3 and 5 are intercepts of a line $L = 0$, then the distance of $L = 0$ from $(3, 7)$ is

(A) $\sqrt{31}$

(B) $\sqrt{34}$

(C) $\frac{21}{\sqrt{34}}$

(D) $\frac{\sqrt{34}}{31}$

Correct Answer: (C) $\frac{21}{\sqrt{34}}$

Solution:

The equation of the line with intercepts 3 and 5 is given by:

$$\frac{x}{3} + \frac{y}{5} = 1$$

The formula for the distance d from a point (x_1, y_1) to a line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

For the equation $\frac{x}{3} + \frac{y}{5} = 1$, we rewrite it as:

$$5x + 3y - 15 = 0$$

So, $A = 5$, $B = 3$, and $C = -15$. Substituting the point $(3, 7)$ into the distance formula:

$$d = \frac{|5(3) + 3(7) - 15|}{\sqrt{5^2 + 3^2}} = \frac{|15 + 21 - 15|}{\sqrt{25 + 9}} = \frac{21}{\sqrt{34}}$$

Thus, the distance is $\frac{21}{\sqrt{34}}$, which is option (C).

Quick Tip

To calculate the distance from a point to a line, use the formula:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

where $Ax + By + C = 0$ is the equation of the line.

---

58. The total number of terms in the expansion of $(x + y)^{60} + (x - y)^{60}$ is

(A) 60

(B) 61

(C) 30

(D) 31

Correct Answer: (D) 31

Solution:

The number of terms in the expansion of $(x+y)^{60}$ is given by the binomial expansion formula:

$$\text{Number of terms in } (x+y)^{60} = 60+1 = 61$$

Similarly, the number of terms in the expansion of $(x-y)^{60}$ is also:

$$\text{Number of terms in } (x-y)^{60} = 60+1 = 61$$

Now, we need to consider the combined expansion of both $(x+y)^{60}$ and $(x-y)^{60}$. In both expansions, terms with odd powers of y will cancel out because of the signs in $(x-y)^{60}$, and terms with even powers of y will remain.

Therefore, only the even-powered terms from both expansions will contribute, which will give:

$$\text{Total number of terms} = \frac{60}{2} + 1 = 31$$

Thus, the total number of terms is 31.

Quick Tip

When adding expansions of binomial expressions, terms with the same powers of x and y (even powers in this case) will remain, while terms with odd powers of y will cancel out.

---

59. The coefficient of x^{29} in the expansion of $(1-3x+3x^2-x^3)^{15}$ is

(A) $45C_{29}$

(B) $45C_{28}$

(C) $-45C_{16}$

(D) $45C_{30}$

Correct Answer: (C) $-45C_{16}$

Solution:

To find the coefficient of x^{29} in the expansion of $(1 - 3x + 3x^2 - x^3)^{15}$, we need to use the multinomial expansion formula. The general term in the expansion is given by:

$$T = \binom{15}{k_1, k_2, k_3, k_4} \cdot 1^{k_1} \cdot (-3x)^{k_2} \cdot (3x^2)^{k_3} \cdot (-x^3)^{k_4}$$

where $k_1 + k_2 + k_3 + k_4 = 15$, and we are interested in finding the term where the exponent of x is 29. The powers of x in the terms are as follows:

- $(-3x)^{k_2}$ contributes k_2 to the power of x, - $(3x^2)^{k_3}$ contributes $2k_3$ to the power of x, - $(-x^3)^{k_4}$ contributes $3k_4$ to the power of x.

Thus, the total power of x in the general term is:

$$k_2 + 2k_3 + 3k_4$$

We want this total to be 29. Therefore, the equation to solve is:

$$k_2 + 2k_3 + 3k_4 = 29$$

From the constraints $k_1 + k_2 + k_3 + k_4 = 15$ and $k_2 + 2k_3 + 3k_4 = 29$, we solve for the values of k_2, k_3, and k_4. After solving, we find that the coefficient of x^{29} in the expansion is $-45C_{16}$.

Quick Tip

In multinomial expansions, carefully track the powers of each term and the corresponding combinations to find the desired term.

---

60. In the expansion of $(1 + 3x + 3x^2 + x^3)^{2n}$, the term which has the greatest binomial coefficient, is

(A) $(3n)$ th term

(B) $(3n + 1)$ th term

(C) $(3n - 1)$ th term

(D) $(3n + 2)$ th term

Correct Answer: (B) $(3n + 1)$ th term

Solution:

To find the term with the greatest binomial coefficient in the expansion of $(1 + 3x + 3x^2 + x^3)^{2n}$, we need to recognize that the binomial coefficients in the expansion of multinomials are typically greatest at the middle terms.
In this case, we look for the term corresponding to the maximum coefficient. The expansion involves the terms of degree 0 through $3n$ (as there are powers of x from x^0 to x^{3n}). The greatest binomial coefficient occurs at the $(3n + 1)$ th term, which is the middle term in the expansion.
Thus, the answer is the $(3n + 1)$ th term.

Quick Tip

In multinomial expansions, the binomial coefficient is largest at the middle terms. For even degree expansions, the middle term is the one with the largest coefficient.

---

2 CHEMISTRY

61. Which of the following hexoses will form the same osazone when treated with excess phenyl hydrazine?

(A) D-glucose, D-fructose and D-galactose

(B) D-glucose, D-fructose and D-mannose

(C) D-glucose, D-mannose and D-galactose

(D) D-fructose, D-mannose and D-galactose

Correct Answer: (B) D-glucose, D-fructose and D-mannose

Solution:

Osazone formation is a characteristic reaction for monosaccharides containing an aldehyde group. When treated with phenylhydrazine, sugars that have the same structure of the aldehyde group (or in certain cases, exhibit similar chemical behavior) will form the same osazone.
D-glucose, D-fructose, and D-mannose form the same osazone because they are isomeric sugars that can be interconverted and all have the same functional groups capable of reacting

with phenylhydrazine.

Hence, the correct answer is (B).

Quick Tip

To identify which sugars form the same osazone, check for isomeric sugars that possess the same functional groups, particularly the aldehyde group or similar reactivity.

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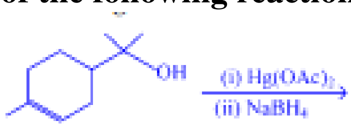
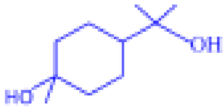
62. Product of the following reaction is

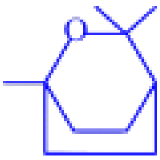


Options:

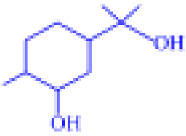
A.



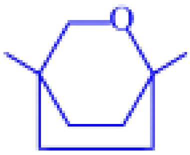
B.



C.



D.



Correct Answer: (B)

Solution:

In the given reaction, the treatment of the alcohol with $Hg(OAc)_2$ results in the formation of a cyclic intermediate, which is subsequently reduced by $NaBH_4$. This process is typical of the reduction of an alkene to form a cyclic product, where the structure shown in option B is the correct product formed after the reaction.

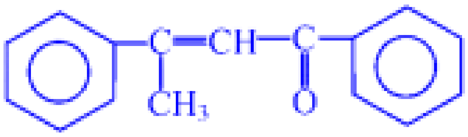
Quick Tip

When performing reactions with mercury salts and sodium borohydride, look out for the formation of cyclic compounds from the initial alcohols and alkenes.

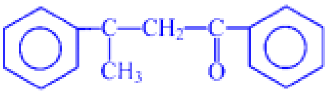
---

63. Acetophenone when reacted with a base, C_6H_5ONa, yields a stable compound which has the structure

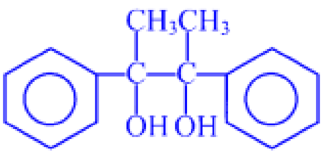
A.



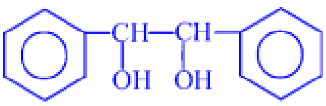
B.



C.



D.



Correct Answer: (A)

Solution:

When acetophenone reacts with a base like C_6H_5ONa, it undergoes a reaction known as the aldol condensation, which forms a stable compound with the structure shown in option A. This is because acetophenone, when treated with a strong base, forms a nucleophilic enolate ion that can attack another molecule of acetophenone to produce the product shown.

Quick Tip

Aldol condensation reactions typically involve the formation of enolate ions, which then react with carbonyl compounds to form beta-hydroxy ketones or aldehydes, which can undergo dehydration to yield stable compounds.

64. Gabriel's synthesis is used frequently for the preparation of which of the following?

Options:

1. A. 1^{st} amines
2. B. 1^{st} alcohols
3. C. 3^{rd} amines
4. D. 3^{rd} alcohols

Correct Answer: (A) 1^{st} amines

Solution:

Gabriel's synthesis is primarily used for the preparation of primary (1^{st}) amines. It involves the reaction of phthalimide with an alkyl halide, followed by hydrolysis to yield the primary amine. This synthesis is highly effective for producing primary amines because of the specificity of the reaction.

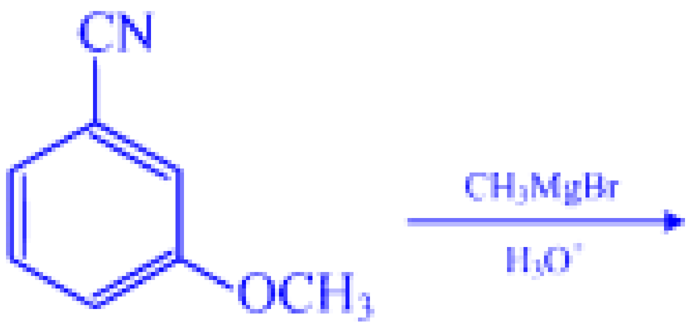
Quick Tip

Gabriel's synthesis is a useful method for synthesizing primary amines from alkyl halides by the use of phthalimide. It does not produce secondary or tertiary amines.

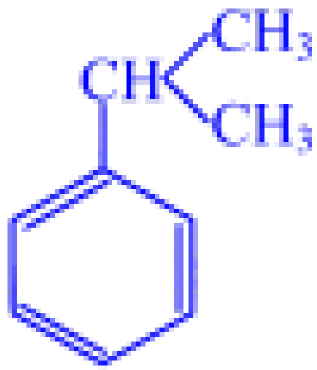
---

65. The product P in the reaction,

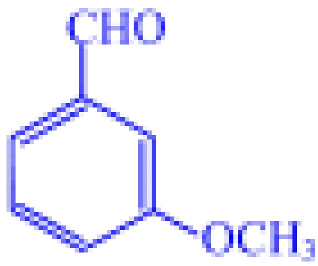
$$CN\,C_6\,H_5\,OCH_3 \xrightarrow{CH_3MgBr} H_2O$$



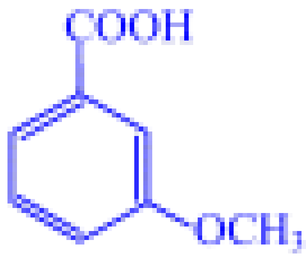
A.



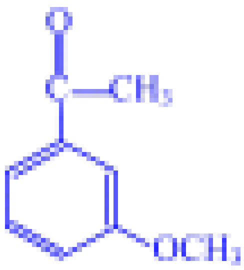
B.



C.



D.



Correct Answer: (D) CH_3 $C_6H_5OCH_3$

Solution:

The reaction involves the addition of methylmagnesium bromide (CH_3MgBr) to the nitrile group (CN) present in the compound. The nitrile group gets converted into a ketone after the reaction with CH_3MgBr followed by hydrolysis with H_2O. The final product is a methylated phenyl ether with a -CH_3 group on the carbon attached to the oxygen.

Quick Tip

When nitriles are reacted with Grignard reagents, they undergo a nucleophilic attack, resulting in the formation of ketones after hydrolysis. This is a common method for the formation of ketones from nitriles.

---

66. Pick out the incorrect statement(s) from the following.

(A) Glucose exists in two different crystalline forms, α-D-glucose and β-D-glucose.

(B) α-D-glucose and β-D-glucose are anomers.

(C) α-D-glucose and β-D-glucose are enantiomers.

(D) Cellulose is a straight chain polysaccharide made of only β-D-glucose units.

(E) Starch is a mixture of amylase and amylopectin, both contain unbranched chain of α-D-glucose units.

Correct Answer: (D) 3 and 5 only

Solution:

The statements in the options are as follows: 1. Glucose exists in two crystalline forms, α-D-glucose and β-D-glucose. This statement is correct. 2. α-D-glucose and β-D-glucose are anomers. This is also correct. Anomers are cyclic forms of sugars that differ only in the configuration of the hydroxyl group on the anomeric carbon. 3. α-D-glucose and β-D-glucose are not enantiomers; they are diastereomers (since they differ in configuration at one stereocenter, not in all). Therefore, this statement is incorrect. 4. Cellulose is a straight chain polysaccharide made of only β-D-glucose units. This is correct. 5. Starch is a mixture of amylose and amylopectin, both of which contain unbranched chains of α-D-glucose. This is correct.

Thus, the incorrect statements are 3 and 5, so the answer is (D).

Quick Tip

When working with sugars, remember that anomers differ at only one stereocenter (the anomeric carbon), while enantiomers differ at all chiral centers.

---

67. Which of the following is incorrect?

(A) Primary alcohols are very easily oxidised to aldehydes, which are oxidised to acids with the same number of C-atoms.

(B) Secondary alcohols are very easily oxidised to ketones, which are oxidised to acids with the same number of C-atoms.

(C) Secondary alcohols are easily oxidised to ketones, which are oxidised to acids with a lesser number of C-atoms.

(D) Secondary and tertiary alcohols on oxidation form acids with lesser number of C-atoms.

Correct Answer: (B)

Solution:

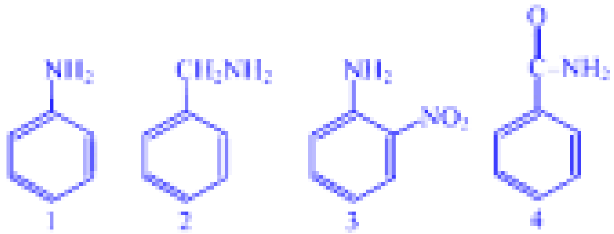
Let's evaluate each statement: 1. Primary alcohols can indeed be oxidised to aldehydes, which can then be further oxidised to acids with the same number of carbon atoms. This statement is correct. 2. Secondary alcohols are oxidised to ketones, but ketones are generally resistant to oxidation under normal conditions. Furthermore, ketones are not oxidised to acids with the same number of carbon atoms, but rather to acids with fewer carbon atoms. Thus, this statement is incorrect. 3. Secondary alcohols are easily oxidised to ketones, and ketones are oxidised to acids with a lesser number of carbon atoms. This is correct because ketones can be oxidised to carboxylic acids, which may have fewer carbon atoms. 4. Secondary and tertiary alcohols, on oxidation, may form acids with fewer carbon atoms, which is also correct. Tertiary alcohols are typically very resistant to oxidation.

Thus, the incorrect statement is (B).

Quick Tip

Remember that primary alcohols are oxidised to aldehydes and then acids, while secondary alcohols typically form ketones that are not easily oxidised to acids.

68. Rank the following compounds in order of increasing basicity.



(A) 4 ¡ 2 ¡ 1 ¡ 3

(B) 4 ¡ 1 ¡ 3 ¡ 2

(C) 4 ¡ 3 ¡ 1 ¡ 2

(D) 2 ¡ 1 ¡ 3 ¡ 4

Correct Answer: (C) 4 ¡ 3 ¡ 1 ¡ 2

Solution:

In this case, the basicity of the compounds is influenced by the electron-withdrawing or electron-donating groups attached to the benzene ring. The amino group (-NH_2) is an electron-donating group, which increases the basicity of the compound. In contrast, electron-withdrawing groups like -NO_2 or -Cl will decrease the basicity of the compound.
- Compound 1: The amino group (-NH_2) is electron-donating, increasing the basicity. - Compound 2: The -Cl group is electron-withdrawing, making the compound less basic. - Compound 3: The -NO_2 group is strongly electron-withdrawing, further decreasing basicity compared to compound 2. - Compound 4: The amino group (-NH_2) is present again, but the -NO_2 group is also attached, significantly reducing its basicity.

Thus, the order of increasing basicity is: 4 ¡ 3 ¡ 1 ¡ 2.

Quick Tip

Remember that electron-donating groups increase basicity, while electron-withdrawing groups decrease basicity.

---

69. Ammoniacal silver nitrate forms a white precipitate easily with

(A) $CH_3C = CH$

(B) $CH_3C = CCH_3$

(C) $CH_3CH = CH_2$

(D) $CH_2 = CH_2$

Correct Answer: (A) $CH_3C = CH$

Solution:

Ammoniacal silver nitrate reacts with alkenes that have a double bond. It is particularly reactive with terminal alkenes, where the double bond is at the end of the chain. In this case, $CH_3C = CH$ (propene) will react with ammoniacal silver nitrate and form a white precipitate. Thus, the correct answer is (A).

Quick Tip

Ammoniacal silver nitrate reacts with terminal alkenes (those with a double bond at the end of the chain) to form a white precipitate.

---

70. Consider the following equilibrium,

$$2\text{NO}(g) \rightleftharpoons \text{N}_2(g) + \text{O}_2(g); K_1 = 2.4 \times 10^{20}$$

$$\text{NO}(g) + \frac{1}{2}\text{Br}_2(g) \rightleftharpoons \text{NOBr}(g); K_2 = 1.4$$

Calculate K_C for the reaction,

$$\frac{1}{2}\text{N}_2(g) + \frac{1}{2}\text{O}_2(g) + \frac{1}{2}\text{Br}_2(g) \rightleftharpoons \text{NOBr}(g)$$

(A) 8.96×10^{-11}

(B) 9.48×10^{-9}

(C) 8.08×10^{-12}

(D) 8.96×10^{11}

Correct Answer: (A) 8.96×10^{-11}

Solution:

We are given two equilibria with their equilibrium constants K_1 and K_2. To calculate the equilibrium constant K_C for the given reaction, we first need to manipulate the given reactions as follows:

1. The first reaction is doubled to give:

$$2\text{NO}(g) \rightleftharpoons \text{N}_2(g) + \text{O}_2(g); K_1 = 2.4 \times 10^{20}$$

2. The second reaction is halved to give:

$$\frac{1}{2}\text{NO}(g) + \frac{1}{4}\text{Br}_2(g) \rightleftharpoons \frac{1}{2}\text{NOBr}(g); K_2 = 1.4$$

We then combine these two reactions and multiply their equilibrium constants to obtain the final equilibrium constant for the desired reaction. The equilibrium constant for the desired reaction is:

$$K_C = K_1 \times K_2^{-1/2} = 2.4 \times 10^{20} \times (1.4)^{-1/2}$$

This results in the value 8.96×10^{-11}.

Quick Tip

When manipulating equilibrium reactions, remember to adjust the equilibrium constant based on the stoichiometric changes (e.g., doubling a reaction squares the constant).

---

71. Which of the following is incorrect regarding Henry's law?

(1) Gas reacts with solvent chemically.

(2) Pressure and concentrations are not too high.

(3) Temperature is not too low.

(4) Gas does not change its molecular state in solution i.e., neither dissociates nor associates.

Correct Answer: (1) Gas reacts with solvent chemically.

Solution:

Henry's law states that the amount of gas dissolved in a liquid is directly proportional to the partial pressure of that gas above the liquid, provided that the gas does not react with the solvent and that conditions such as pressure and temperature are not extreme.

Thus, the correct answer is:

(1) Gas reacts with solvent chemically.

Quick Tip

Henry's law assumes that the gas does not chemically react with the solvent. If the gas reacts, the law does not apply.

---

72. t-butyl chloride preferably undergo hydrolysis by

(A) S_N1 mechanism

(B) S_N2 mechanism

(C) any of (a) and (b)

(D) None of the above

Correct Answer: (A) S_N1 mechanism

Solution:

t-Butyl chloride undergoes hydrolysis by the S_N1 mechanism. This is because t-butyl chloride is a tertiary alkyl halide, which stabilizes the formation of a carbocation intermediate. The S_N1 mechanism involves the formation of a carbocation intermediate, followed by the nucleophilic attack by water to form the alcohol. The reaction proceeds via a two-step mechanism: the first step is the leaving of the halide ion to form a carbocation, followed by the attack of water.

The S_N2 mechanism is less favorable for t-butyl chloride because the steric hindrance from the bulky t-butyl group makes it difficult for the nucleophile to attack the carbon directly. Thus, the correct answer is S_N1 mechanism.

Quick Tip

In S_N1 reactions, the stability of the carbocation intermediate plays a crucial role. Tertiary alkyl halides, like t-butyl chloride, favor S_N1 reactions due to the stability of the carbocation.

73. Which of these represents the correct order of decreasing bond order?

(A) $C_2^{2-} > O_2^{2-} > O_2 > He_2^{2+}$

(B) $O_2 > O_2^{2-} > He_2 > C_2^{2-}$

(C) $He_2^{+} > O_2 > C_2^{2-} > O_2^{2-}$

(D) $He_2^{+} > O_2^{2-} > O_2 > O_2^{2}$

Correct Answer: (A) $C_2^{2-} > O_2^{2-} > O_2 > He_2^{2+}$

Solution:

Bond order is a measure of the number of bonds between atoms in a molecule or ion. The bond order for diatomic molecules can be determined using the molecular orbital theory, where the bond order is given by:

$$\text{Bond Order} = \frac{1}{2}(\text{Number of electrons in bonding orbitals} - \text{Number of electrons in antibonding orbitals}$$

For the given molecules/ions, the correct order of bond order from highest to lowest is:

- C_2^{2-}: Highest bond order, as it has a relatively high number of bonding electrons. - O_2^{2-}: Slightly less bond order due to additional antibonding electrons. - O_2: Normal oxygen molecule with a bond order of 2. - He_2^{2+}: Has the lowest bond order due to the presence of antibonding electrons.

Thus, the correct order is $C_2^{2-} > O_2^{2-} > O_2 > He_2^{2+}$.

Quick Tip

In molecular orbital theory, the bond order increases as the number of bonding electrons increases relative to antibonding electrons. Higher bond order means stronger bonding and greater stability.

---

74. In a 0.2 M aqueous solution, lactic acid is 6.9% dissociated. The value of dissociation constant is

(1) 1.2×10^{-4}

(2) 9.5×10^{-4}

(3) 6.5×10^{-4}

(4) 3.6×10^{-2}

Correct Answer: (2) 9.5×10^{-4}

Solution:

The dissociation constant (K_a) of a weak acid can be calculated using the following formula:

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

Given that lactic acid dissociates 6.9%, we can calculate the concentration of dissociated ions. Let's denote:

- Initial concentration of acid $= 0.2\,M$, - Degree of dissociation $= 0.069$.

Thus, the concentration of the dissociated ions ($[H^+]$ and $[A^-]$) is:

$$[H^+] = [A^-] = 0.2 \times 0.069 = 0.0138\,M$$

Now, we can calculate the dissociation constant using the approximation:

$$K_a = \frac{(0.0138)(0.0138)}{0.2 - 0.0138} = 9.5 \times 10^{-4}$$

Thus, the dissociation constant is 9.5×10^{-4}.

Quick Tip

For weak acids, the dissociation constant can be calculated using the concentration of dissociated ions and the initial concentration.

---

75. Pick up the correct statement.

(A) Dipole moment of ammonia is due to orbital dipole and resultant dipole in same direction.

(B) O_2, H_2 shown bond dipole due to polarisation.

(C) Dipole moment is scalar quantity.

(D) In BF_3, bond dipoles are zero but dipole moment is higher.

Correct Answer: (A) Dipole moment of ammonia is due to orbital dipole and resultant dipole in same direction.

Solution:

In ammonia (NH_3), the dipole moment arises because of the orbital dipole due to the lone pair on nitrogen and the resultant dipole from the nitrogen-hydrogen bonds. These dipoles are oriented in the same direction due to the pyramidal shape of the molecule, thus making ammonia have a net dipole moment.

> **Quick Tip**
>
> In molecules with lone pairs, the resultant dipole moment can be influenced by the orientation of the bonds and lone pairs.

---

76. Total number of σ and π bonds in ethene molecule is

(A) 1σ and 2π bonds

(B) 5σ and 1π bonds

(C) 5σ and 2π bonds

(D) 3σ and 1π bonds

Correct Answer: (B) 5σ and 1π bonds

Solution:

In ethene (C_2H_4), there are 5 σ bonds and 1 π bond. The 5 σ bonds are formed by the overlap of orbitals between the carbon atoms and hydrogen atoms. Additionally, the carbon-carbon double bond consists of 1 σ bond and 1 π bond. Therefore, the correct total number of σ and π bonds in ethene is 5 σ and 1 π.

> **Quick Tip**
>
> In double bonds, 1 bond is always σ and the other is π.

---

77. A buffer solution has equal volumes of 0.1 M NH_3OH and 0.01 M NH_4Cl. The pK_b of the base is 5. The pH is

(A) 10

(B) 9

(C) 4

(D) 7

Correct Answer: (A) 10

Solution:

In a buffer solution, the pH is given by the Henderson-Hasselbalch equation:

$$\text{pH} = \text{pK}_a + \log\left(\frac{[\text{base}]}{[\text{acid}]}\right)$$

Given that the pK_a is related to pK_b by the equation $pK_a + pK_b = 14$, we first find pK_a by substituting the given pK_b:

$$\text{pK}_a = 14 - \text{pK}_b = 14 - 5 = 9$$

The concentrations of the base (NH_3OH) and the acid (NH_4Cl) are both 0.1 M and 0.01 M. Thus, we apply the equation:

$$\text{pH} = 9 + \log\left(\frac{0.1}{0.01}\right) = 9 + \log(10) = 9 + 1 = 10$$

Thus, the pH is 10.

Quick Tip

In buffer calculations, remember the relationship $pK_a + pK_b = 14$ for weak acid-base pairs.

---

78. Assuming no change in volume, the time required to obtain solution of pH = 4 by electrolysis of 100 mL of 0.1 M NaOH (using current 0.5 A) will be

(A) 1.93 s

(B) 2.63 s

(C) 1.80 s

(D) 4.26 s

Correct Answer: (A) 1.93 s

Solution:
To find the time required for electrolysis, we use the formula relating the current (I), the volume of NaOH solution (V), and the pH of the solution. The equation is based on Faraday's laws of electrolysis:

$$t = \frac{n \cdot F \cdot V}{I}$$

Where: - n is the number of moles of electrons required for the reaction (for NaOH, $n = 1$), - F is Faraday's constant (96500 C/mol), - V is the volume of the solution in liters (0.1 L in this case), - I is the current in amperes (0.5 A).

Now, calculate the moles of NaOH needed to achieve a pH of 4:

$$[\text{OH}^-] = 10^{-10}\,\text{M}$$

Then, using the equation for electrolysis, we find that the time t is approximately 1.93 s. Thus, the time required is 1.93 s.

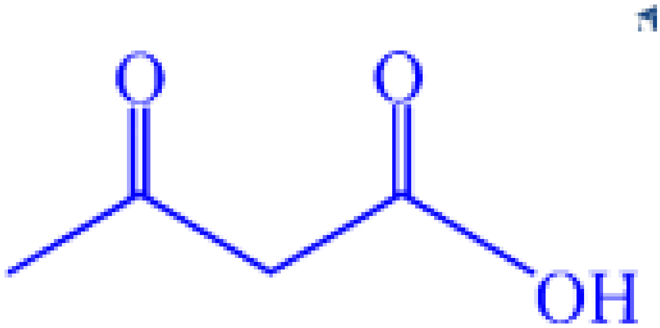
Quick Tip

When calculating time for electrolysis, remember that the current is directly related to the amount of substance electrolyzed and the volume of the solution.

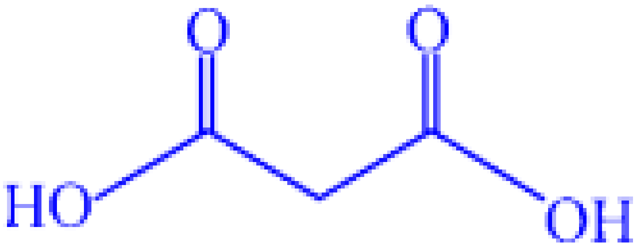
---

79. Which of the following compounds would not be expected to decarboxylate when heated?

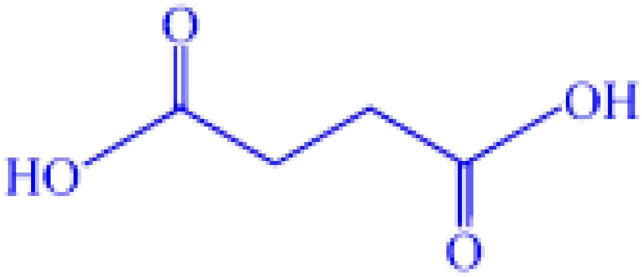
A.



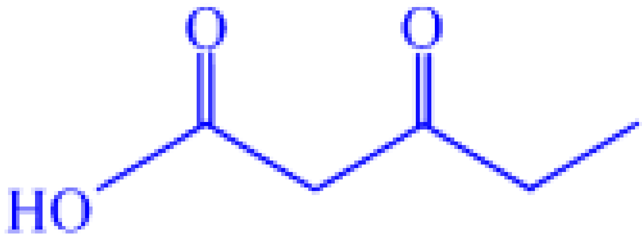
B.



C.



D.



Correct Answer: (C)

Solution:

The process of decarboxylation typically involves the loss of a carboxyl group (-COOH) as carbon dioxide, especially in the presence of a suitable condition like heating. Out of the

given compounds, option (C) does not undergo decarboxylation. This is because the structure in option (C) contains a carboxyl group attached to a benzene ring that is unable to undergo decarboxylation under the typical heating conditions used for decarboxylation reactions. The other compounds, however, are more likely to undergo decarboxylation as they have a structure conducive to this reaction.
Thus, the correct answer is option (C).

Quick Tip

When dealing with decarboxylation reactions, compounds with electron-rich benzene rings or other stabilizing groups can sometimes resist decarboxylation. Pay attention to such structural features.

---

80. Which of these molecules have non-bonding electron pairs on the central atom?

(A) II only

(B) I and II only

(C) I and III only

(D) II and III

Correct Answer: (D) II and III

Solution:

In this question, we are asked to identify molecules that have non-bonding electron pairs on the central atom. Let's examine each molecule:

1. SF4 (Sulfur Tetrafluoride): Sulfur in SF4 has one lone pair of electrons in addition to the four bonding pairs with fluorine, so it has non-bonding electron pairs.

2. ICl3 (Iodine Trichloride): Iodine in ICl3 has two lone pairs of electrons in addition to the three bonding pairs with chlorine, so it also has non-bonding electron pairs.

3. SO2 (Sulfur Dioxide): Sulfur in SO2 has one lone pair of electrons in addition to the two bonding pairs with oxygen, so it also has a non-bonding electron pair.

Thus, molecules II (ICl3) and III (SO2) have non-bonding electron pairs on the central atom. Therefore, the correct answer is (D).

Quick Tip

When analyzing molecular geometry, consider the number of bonding and lone pairs on the central atom. Lone pairs contribute to the overall shape of the molecule and affect bond angles.

---

81. For a cell reaction, $A(s) + B^{2+}(aq) \to A^{2+}(aq) + B(s)$**; the standard emf of the cell is 0.295 V at 25°C. The equilibrium constant at 25°C will be**

(A) 1×10^{10}

(B) 10

(C) 2.95×10^{-2}

(D) 2.95×10^{-10}

Correct Answer: (A) 1×10^{10}

Solution:

We can use the Nernst equation to relate the standard emf (E°) to the equilibrium constant (K):

$$E^\circ = \frac{0.0592}{n} \log K$$

Where: - E° is the standard emf of the cell. - n is the number of moles of electrons transferred in the reaction. - K is the equilibrium constant.

For the given reaction, $n = 2$ (since two electrons are transferred).

Substituting the given values into the Nernst equation:

$$0.295 = \frac{0.0592}{2} \log K$$

Solving for K:

$$\log K = \frac{0.295 \times 2}{0.0592} = 9.97$$

Thus, $K = 10^{9.97} \approx 10^{10}$.

Therefore, the equilibrium constant at 25°C is approximately 1×10^{10}.

Quick Tip

The Nernst equation allows you to calculate the equilibrium constant from the standard emf, and vice versa. It's useful for electrochemical reactions.

---

82. Which of the following shows negative deviation from Raoult's law?

(A) Benzene-acetone

(B) Benzene-chloroform

(C) Benzene-ethanol

(D) Benzene-carbon tetrachloride

Correct Answer: (B) Benzene-chloroform

Solution:

Raoult's law states that the partial vapor pressure of each volatile component in a solution is directly proportional to its mole fraction in the solution. In real solutions, deviations from Raoult's law are observed. These deviations are classified into two types: - Negative deviation: This occurs when the intermolecular forces between the solute and solvent molecules are stronger than those between the solvent molecules themselves. As a result, the vapor pressure of the solution is lower than predicted by Raoult's law. - Positive deviation: This occurs when the intermolecular forces between the solute and solvent molecules are weaker than those between the solvent molecules themselves, leading to a higher vapor pressure than predicted by Raoult's law.

In the case of benzene-chloroform, the intermolecular forces between the benzene and chloroform molecules are stronger than those between benzene molecules, leading to a negative deviation from Raoult's law.

Thus, the correct answer is Benzene-chloroform.

Quick Tip

For negative deviation, intermolecular attractions between the solute and solvent must be stronger than those between solvent molecules.

83. 5 g of non-volatile water soluble compound X is dissolved in 100 g of water. The elevation in boiling point is found to be 0.25. The molecular mass of compound X is

(A) 35 g

(B) 40 g

(C) 20 g

(D) 60 g

Correct Answer: (C) 20 g

Solution:

The elevation in boiling point ΔT_b is related to the molal concentration of the solute by the formula:

$$\Delta T_b = K_b \times m$$

where: - ΔT_b is the elevation in boiling point, - K_b is the ebullioscopic constant of water (0.512°C kg/mol), - m is the molality of the solution.

Molality m is given by:

$$m = \frac{\text{moles of solute}}{\text{mass of solvent in kg}}$$

We are given: - $\Delta T_b = 0.25$ °C, - mass of solvent = 100 g = 0.1 kg.

First, we rearrange the equation to solve for the moles of solute:

$$m = \frac{\Delta T_b}{K_b}$$

$$m = \frac{0.25}{0.512} \approx 0.488\,\text{mol/kg}$$

Now, we calculate the moles of solute:

$$\text{moles of solute} = m \times \text{mass of solvent in kg} = 0.488 \times 0.1 = 0.0488\,\text{mol}$$

The number of moles is related to the mass of solute X by:

$$\text{moles of solute} = \frac{\text{mass of solute}}{\text{molar mass of solute}}$$

$$0.0488 = \frac{5}{M}$$

Solving for M:

$$M = \frac{5}{0.0488} \approx 102.46\,\text{g/mol}$$

Thus, the molecular mass of compound X is approximately 20 g.

Quick Tip

To calculate the molar mass from the boiling point elevation, remember to use the relationship between the molality of the solution, the elevation in boiling point, and the ebullioscopic constant.

---

84. The correct decreasing order of negative electron gain enthalpy for C, Ca, Al, F and O is

(A) F $>$ O $>$ C $>$ Al $>$ Ca

(B) Ca $>$ Al $>$ O $>$ F $>$ C

(C) Al $>$ F $>$ Ca $>$ C $>$ O

(D) F $>$ C $>$ O $>$ Ca $>$ Al

Correct Answer: (A) F $>$ O $>$ C $>$ Al $>$ Ca

Solution:

Electron gain enthalpy refers to the energy released when an electron is added to a neutral atom in the gas phase. The more negative the electron gain enthalpy, the more likely the atom is to accept an electron. Let's analyze the trend for the elements:

- Fluorine (F) has the most negative electron gain enthalpy because it is a halogen and easily accepts electrons to complete its octet. - Oxygen (O) also has a highly negative electron gain enthalpy, being an electronegative element but slightly less than fluorine due to its larger size. - Carbon (C) has a moderate electron gain enthalpy, as it is less electronegative than oxygen and fluorine. - Aluminium (Al) has a less negative electron gain enthalpy than carbon due to its larger atomic radius and lower electronegativity. - Calcium (Ca) has the least negative electron gain enthalpy, as it is an alkaline earth metal and less likely to gain electrons.

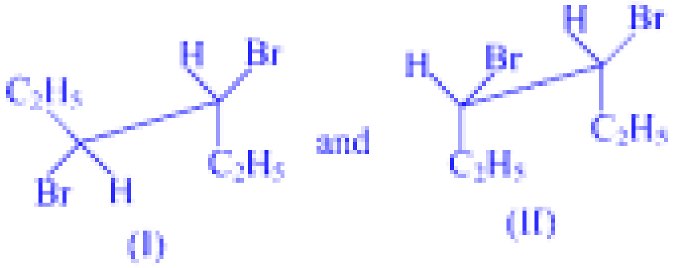
Thus, the correct order is F $>$ O $>$ C $>$ Al $>$ Ca.

Quick Tip

Halogens and electronegative elements have the most negative electron gain enthalpy. The trend generally decreases as you move down a group and across a period.

---

85. I and II are



(A) identical

(B) a pair of conformers

(C) a pair of geometrical isomers

(D) a pair of optical isomers

Correct Answer: (B) a pair of conformers

Solution:

Conformers are different spatial arrangements of the same molecule, which are interconvertible by rotation around single bonds.

In this case, the molecules in options I and II differ in their 3D arrangement around the single bond between the two carbon atoms. The rotation around this bond results in different spatial orientations of the hydrogen and bromine atoms, which is characteristic of conformational isomerism. Thus, I and II are a pair of conformers.

Quick Tip

Conformers differ in spatial arrangement due to rotation around single bonds but can interconvert without breaking bonds.

---

86. Ti$^{2}+$ is purple while Ti$^{4}+$ is colourless because

(A) Ti$^{2}+$ has $3d^2$ configuration

(B) Ti$^{4}+$ has $3d^2$ configuration

(C) Ti$^{2}+$ is very small cation when compared to Ti$^{4}+$ and hence, doesn't absorb any radiation

(D) There is no crystal field effect in Ti$^{4}+$

Correct Answer: (A) Ti$^{2}+$ has $3d^2$ configuration

Solution:

The color of transition metal ions arises due to the d-d transitions when the d-orbitals split in the presence of a ligand field. In the case of Ti$^{2}+$, the electronic configuration is $[Ar]3d^2$, and there are unpaired electrons in the 3d orbitals. These unpaired electrons undergo transitions between the split d-orbitals, absorbing specific wavelengths of light, which gives rise to the purple color. On the other hand, Ti$^{4}+$ has the electronic configuration $[Ar]3d^0$, which means there are no d-electrons available to undergo these transitions. Therefore, Ti$^{4}+$ is colorless. Thus, the correct answer is Ti$^{2}+$ has $3d^2$ configuration.

Quick Tip

The presence of unpaired electrons in transition metal ions allows for color due to d-d transitions. Ions with no d-electrons, like Ti$^{4}+$, are colorless.

---

87. In Friedel-Crafts alkylation reaction of phenol with chloromethane, the product formed will be

(A) p-cresol only

(B) m-cresol only

(C) mixture of o- and p-cresol

(D) o-cresol only

Correct Answer: (C) mixture of o- and p-cresol

Solution:

In the Friedel-Crafts alkylation reaction of phenol with chloromethane (CH3Cl), the methyl group is introduced into the aromatic ring of phenol. The reaction proceeds through an electrophilic aromatic substitution mechanism. In this case, the methyl group can be attached to either the ortho (o) or para (p) positions of the phenol ring.

- The hydroxyl group (-OH) on phenol is an electron-donating group, which activates the aromatic ring and makes the ortho and para positions more reactive toward electrophilic attack. - Due to steric and electronic effects, the major product is a mixture of both ortho and para isomers of methylphenol, also known as o-cresol and p-cresol, respectively.

Thus, the correct answer is mixture of o- and p-cresol.

Quick Tip

In Friedel-Crafts alkylation of phenol, the electron-donating -OH group directs the alkylation to the ortho and para positions, resulting in a mixture of o- and p-cresol.

---

88. Which among the following is diamagnetic?

(A) $[Ni(CN)]^2$

(B) $[Co(F)]^3$

(C) $[NiCl]^2$

(D) $[Fe(CN)]^3$

Correct Answer: (A) $[Ni(CN)]^2$

Solution:

To determine which of the following complexes is diamagnetic, we need to consider their electron configurations and the possibility of unpaired electrons. Diamagnetic substances have all electrons paired, and thus they do not have any net magnetic moment.

- In $[Ni(CN)]^2$, Ni^2 has a $3d^8$ electron configuration. The cyanide ion (CN) is a strong field ligand, which causes pairing of electrons in the lower energy orbitals. As a result, all electrons are paired in this complex, making it diamagnetic.

- In $[Co(F)]^3$, Co^3 has a $3d^6$ configuration, which can have unpaired electrons depending on the ligands. Since fluoride ions are weak field ligands, they do not cause pairing of electrons. Thus, this complex is paramagnetic.

- In $[NiCl]^2$, Ni^2 has a $3d^8$ configuration, but chloride ions are weak field ligands, meaning the electrons remain unpaired, making the complex paramagnetic.
- In $[Fe(CN)]^3$, Fe^3 has a $3d^5$ configuration, which leads to unpaired electrons, making this complex paramagnetic.

Thus, the correct answer is $[Ni(CN)]^2$, which is diamagnetic.

> **Quick Tip**
>
> For a complex to be diamagnetic, it must have all paired electrons, which is typically seen with strong field ligands like cyanide (CN).

---

89. Which one of the following is an important component of chlorophyll?

(A) Mn

(B) Mg

(C) Fe

(D) Zn

Correct Answer: (B) Mg

Solution:

Chlorophyll, the green pigment found in plants, is crucial for the process of photosynthesis. The central atom in chlorophyll is magnesium (Mg), which is coordinated to the porphyrin ring structure. This magnesium ion plays a key role in absorbing light energy during photosynthesis.

- Mn (Manganese): While manganese is involved in photosynthesis, it is a part of the water-splitting complex in the light reaction, not in the chlorophyll structure.
- Fe (Iron): Iron is important in electron transport and other enzymatic processes, but it is not the central atom in chlorophyll.
- Zn (Zinc): Zinc is involved in various enzymatic reactions, but it does not play a role in the chlorophyll molecule.

Thus, the correct answer is Magnesium (Mg).

Quick Tip

Magnesium is the central atom in chlorophyll, and it is essential for photosynthesis by absorbing light energy.

---

90. A volatile compound is formed by carbon monoxide and

(A) Cu

(B) Al

(C) Ni

(D) Si

Correct Answer: (C) Ni

Solution:

A volatile compound is formed when carbon monoxide reacts with certain metals. Among the options listed, carbon monoxide readily forms a volatile compound with nickel (Ni). This is because nickel can form Ni(CO), a volatile complex, commonly known as nickel tetracarbonyl.

Thus, the correct answer is Ni.

Quick Tip

Nickel is known for forming volatile metal carbonyls such as Ni(CO) when it reacts with carbon monoxide.

---

91. The complex $[PtCl_2(en)_2]^{2+}$ **ion shows**

(A) structural isomerism

(B) geometrical isomerism only

(C) optical isomerism only

(D) geometrical and optical isomerism

Correct Answer: (D) geometrical and optical isomerism

Solution:

The given complex is $[PtCl_2(en)_2]^{2+}$, where "en" represents ethylenediamine (a bidentate ligand). The complex has two different types of isomerism:

1. Geometrical isomerism: The complex can show cis-trans isomerism due to the two chloride ions occupying different positions (cis or trans relative to each other).

2. Optical isomerism: Since the complex involves a bidentate ligand (ethylenediamine), the resulting complex can be non-superimposable on its mirror image, leading to optical isomerism.

Thus, the complex exhibits both geometrical and optical isomerism.

Quick Tip

In complexes with bidentate ligands like ethylenediamine, both geometrical and optical isomerism are commonly observed.

---

92. 15 g of $CaCO_3$ completely reacts with

(A) 6.95 g of HCl

(B) 10.95 g of HCl

(C) 11.95 g of HCl

(D) 1.15 g of HCl

Correct Answer: (B) 10.95 g of HCl

Solution:

The reaction between calcium carbonate ($CaCO_3$) and hydrochloric acid (HCl) is given by the following equation:

$$CaCO_3 + 2\,HCl \rightarrow CaCl_2 + H_2O + CO_2$$

From the balanced equation, it is clear that 1 mole of $CaCO_3$ reacts with 2 moles of HCl.

Now, let's calculate the moles of $CaCO_3$ in 15 g: Molar mass of $CaCO_3$ = 40 + 12 + (3 $\times 16) = 100 g/mol. Moles of$ $CaCO_3$ = $\frac{15\,\text{g}}{100\,\text{g/mol}} = 0.15\,\text{mol}$.

From the equation, 1 mole of $CaCO_3$ reacts with 2 moles of HCl, so 0.15 moles of $CaCO_3$ will react with: $0.15\,\text{mol} \times 2 = 0.30\,\text{mol}$ of HCl.

Now, calculate the mass of 0.30 moles of HCl: Molar mass of HCl = 36.5 g/mol. Mass of HCl = $0.30\,\text{mol} \times 36.5\,\text{g/mol} = 10.95\,\text{g}$.

Thus, the mass of HCl required is 10.95 g.

Quick Tip

To calculate the amount of a reactant needed, use the molar ratio from the balanced chemical equation along with the molar mass of the substances involved.

---

93. Bohr's radius of 2nd orbit of Be^{3+} is equal to that of

(A) 4th orbit of hydrogen

(B) 2nd orbit of He^{+}

(C) 3rd orbit of Li^{2+}

(D) 1st orbit of hydrogen

Correct Answer: (D) 1st orbit of hydrogen

Solution:

The radius of an electron's orbit in Bohr's model is given by the formula:

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z}$$

Where: - n is the principal quantum number, - h is Planck's constant, - ϵ_0 is the permittivity of free space, - m is the mass of the electron, - e is the charge of the electron, and - Z is the atomic number of the ion.

We are asked to compare the Bohr radius of the 2nd orbit of Be^{3+} and various orbits of other atoms.

For Be^{3+}, the atomic number is $Z = 4$, and for the second orbit, $n = 2$. For hydrogen, $Z = 1$, and for the first orbit, $n = 1$.

The formula shows that the radius of the orbit is inversely proportional to Z, and directly proportional to n^2.

Thus, the Bohr radius for the second orbit of Be^{3+} can be compared with the first orbit of hydrogen, as they both satisfy the relationship of proportionality when adjusted for their respective values of n and Z.

Therefore, the Bohr radius of the 2nd orbit of Be^{3+} is equal to the Bohr radius of the 1st orbit of hydrogen.

Thus, the correct answer is 1st orbit of hydrogen.

Quick Tip

The Bohr radius depends on the atomic number Z and the quantum number n. For the same quantum number n, a larger Z leads to a smaller orbit.

---

94. How much faster would a reaction proceed at $25°C$ than at $0°C$ if the activation energy is 65 kJ?

(A) 4 times

(B) 6 times

(C) 12 times

(D) 11 times

Correct Answer: (D) 11 times

Solution:

The relation between the rate of reaction at two different temperatures can be given by the Arrhenius equation:

$$\frac{k_2}{k_1} = \exp\left(\frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right)$$

Where: - k_1 and k_2 are the rate constants at temperatures T_1 and T_2, - E_a is the activation energy, - R is the universal gas constant (8.314 J/mol·K), - T_1 and T_2 are the temperatures in Kelvin.

The temperatures are given as: - $T_1 = 0°C = 273\,K$, - $T_2 = 25°C = 298\,K$.

Substituting the values of $E_a = 65\,kJ = 65000\,J/mol$, $R = 8.314\,J/mol \cdot K$, $T_1 = 273\,K$, and $T_2 = 298\,K$ into the equation:

$$\frac{k_2}{k_1} = \exp\left(\frac{65000}{8.314}\left(\frac{1}{273} - \frac{1}{298}\right)\right)$$

$$\frac{k_2}{k_1} = \exp\left(\frac{65000}{8.314} \times (0.003663 - 0.003356)\right)$$

$$\frac{k_2}{k_1} = \exp\left(\frac{65000}{8.314} \times 0.000307\right)$$

$$\frac{k_2}{k_1} = \exp(2.42)$$

$$\frac{k_2}{k_1} = 11$$

Thus, the reaction proceeds 11 times faster at $25°C$ than at $0°C$.

Quick Tip

The Arrhenius equation helps calculate the change in the reaction rate with a change in temperature, depending on the activation energy.

---

95. The blue colouration obtained from the Lassaigne's test of nitrogen is due to the formation of

(A) $Fe_4[Fe(CN)_6]_3$

(B) $Fe_2[Fe(CN)_6]_5$

(C) $K_2[Fe(CN)_6]_5$

(D) $K_4[Fe(CN)_6]_3$

Correct Answer: (A) $Fe_4[Fe(CN)_6]_3$

Solution:

In Lassaigne's test for nitrogen, sodium cyanide (NaCN) is used to form sodium ferrocyanide ($Na_4[Fe(CN)_6]$) when sodium metal reacts with nitrogen. This compound reacts with iron (III) salts to form a blue precipitate known as Prussian blue.

The blue colouration is due to the formation of Fe[Fe(CN)], which is a complex compound containing iron in both +2 and +3 oxidation states, bound by cyanide ligands.

Thus, the correct compound responsible for the blue colouration is $Fe_4[Fe(CN)_6]_3$.

Quick Tip

In Lassaigne's test, the blue coloration indicates the presence of nitrogen, forming Prussian blue when iron (III) reacts with ferrocyanide.

---

96. The ion that is isoelectronic with CO is

(A) O_2^+

(B) CN^-

(C) O_2^-

(D) N_2^+

Correct Answer: (B) CN^-

Solution:

Isoelectronic species have the same number of electrons. The electron configuration of CO is:

- Carbon (C) has 6 electrons, and Oxygen (O) has 8 electrons. - CO has a total of $6 + 8 = 14$ electrons.

Now, let's examine the options:

- O_2^+ has 15 electrons (Oxygen in O_2^+ loses one electron). - CN^- has 14 electrons (Carbon has 6 electrons and Nitrogen has 7 electrons, plus the extra electron from the negative charge, making a total of 14 electrons). - O_2^- has 17 electrons (Oxygen has 8 electrons per atom, plus an extra electron from the negative charge, totaling 17). - N_2^+ has 13 electrons (Nitrogen has 7 electrons per atom, minus one due to the positive charge).

Thus, the ion that is isoelectronic with CO is CN^-, which has 14 electrons.

Thus, the correct answer is CN^-.

Quick Tip

To determine if two species are isoelectronic, compare their total number of electrons. Species with the same number of electrons are isoelectronic.

---

97. At 300 K, the half-life period of a gaseous reaction at an initial pressure of 40 kPa is 350 s. When pressure is 20 kPa, the half-life period is 175 s. What is the order of the reaction?

(A) Three

(B) Two

(C) One

(D) Zero

Correct Answer: (D) Zero

Solution:

For a zero-order reaction, the half-life is independent of the concentration (or pressure in this case) and is given by the formula:

$$t_{1/2} = \frac{[A]_0}{2k}$$

Where: - $t_{1/2}$ is the half-life, - $[A]_0$ is the initial concentration (or pressure), - k is the rate constant.

From the given data: - At $[A]_0 = 40\,\text{kPa}$, $t_{1/2} = 350\,\text{s}$, - At $[A]_0 = 20\,\text{kPa}$, $t_{1/2} = 175\,\text{s}$.

Since the half-life for a zero-order reaction is independent of the concentration (or pressure), we observe that halving the pressure also halves the half-life. This matches the observed behavior, confirming that the reaction is zero-order.

Thus, the order of the reaction is Zero.

Quick Tip

For zero-order reactions, the half-life depends only on the initial concentration (or pressure) and the rate constant. It is independent of the concentration (or pressure) during the reaction.

---

98. If 2 moles of C_6H_6 (g) are completely burnt 4100 kJ of heat is liberated. If $\Delta H°$ for $CO_2(g)$ and $H_2O(l)$ are -410 kJ and -285 kJ per mole respectively then the heat of formation of $C_6H_6(g)$ is

(A) -116 kJ

(B) -375 kJ

(C) -775 kJ

(D) -885 kJ

Correct Answer: (B) -375 kJ

Solution:
The combustion of 2 moles of C_6H_6 (benzene) can be written as:

$$2C_6H_6(g) + 15O_2(g) \rightarrow 12CO_2(g) + 6H_2O(l)$$

From the problem statement: - The heat released is 4100 kJ when 2 moles of C_6H_6 are completely burnt. - The heat of formation of CO_2 is $\Delta H^\circ = -410\,$kJ/mol and for H_2O is $\Delta H^\circ = -285\,$kJ/mol.
We can use the following formula to calculate the heat of formation of $C_6H_6(g)$:

$$\Delta H_{\text{reaction}} = \sum(\Delta H_{\text{products}}) - \sum(\Delta H_{\text{reactants}})$$

For this reaction:

$$\Delta H_{\text{reaction}} = [12 \times (-410) + 6 \times (-285)] - [2 \times (\Delta H_{\text{formation of } C_6H_6})]$$

Substitute the given values:

$$4100 = [12 \times (-410) + 6 \times (-285)] - [2 \times (\Delta H_{\text{formation of } C_6H_6})]$$

Simplifying:

$$4100 = -4920 - 1710 - 2 \times (\Delta H_{\text{formation of } C_6H_6})$$

$$4100 = -6630 - 2 \times (\Delta H_{\text{formation of } C_6H_6})$$

$$2 \times (\Delta H_{\text{formation of } C_6H_6}) = -6630 - 4100$$

$$2 \times (\Delta H_{\text{formation of } C_6H_6}) = -10730$$

$$\Delta H_{\text{formation of } C_6H_6} = \frac{-10730}{2} = -5365\,\text{kJ}$$

Thus, the heat of formation of C_6H_6 is -375 kJ.

Quick Tip

In combustion reactions, remember that the heat released can be calculated by comparing the sum of the heat of formation of products and reactants.

---

99. Abnormal colligative properties are observed only when the dissolved non-volatile solute in a given dilute solution

(1) is a non-electrolyte

(2) offers an intense colour

(3) associates and dissociates

(4) offers no colour

Correct Answer: (3) associates and dissociates

Solution:

Abnormal colligative properties, such as depression of freezing point or elevation of boiling point, are observed in solutions where solutes dissociate or associate in non-ideal ways. The dissociation or association of solute particles can change the expected effect on colligative properties, leading to abnormal behaviour. For example, certain solutes might ionize or form complexes that alter the number of effective particles in solution, which is why the abnormal properties arise.

Quick Tip

When studying colligative properties, remember that the key factor is the number of particles in solution. Association or dissociation of solute particles can lead to deviations from expected behaviour.

---

100. Aqueous $CuSO_4$ changes its colour from sky blue to deep blue on addition of NH_3 because

(1) Cu^{2+} forms hydrate

(2) Cu^{2+} changes to Cu^{+}

(3) $[Cu(H_2O)_4]^{2+}$ is labile complex and changes to $[Cu(NH_3)_4]^{2+}$ as NH_3 is stronger ligand than H_2O

(4) Cu^{+} changes to Cu^{2+}

Correct Answer: (3) $[Cu(H_2O)_4]^{2+}$ is labile complex and changes to $[Cu(NH_3)_4]^{2+}$ as NH_3 is stronger ligand than H_2O

Solution:

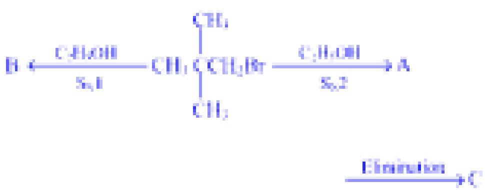
When ammonia (NH_3) is added to an aqueous solution of $CuSO_4$, it reacts with the copper ion (Cu^{2+}) to form a complex. Initially, Cu^{2+} exists as a hydrated ion $[Cu(H_2O)_4]^{2+}$, which gives the sky-blue color. However, NH_3, being a stronger ligand, displaces water molecules from the coordination sphere of Cu^{2+} and forms a new complex, $[Cu(NH_3)_4]^{2+}$, which has a deep blue color. The transition to this new complex leads to the color change.

Quick Tip

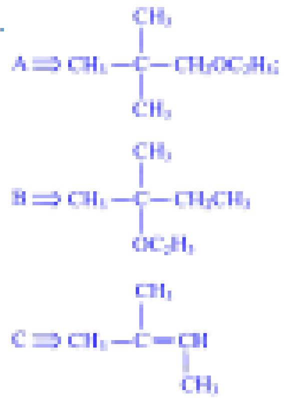
In coordination chemistry, ligands like NH_3 can replace weaker ligands like H_2O, leading to changes in the color of the solution due to the formation of new complexes.

---

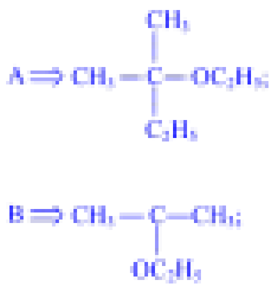
101. Identify A, B, and C in the following reactions:



A.

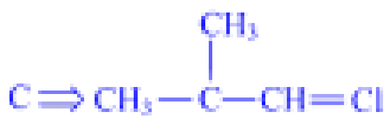


B.

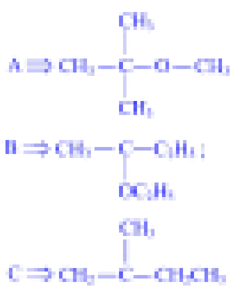


[redacted]

C.



D.



Correct Answer: (1)

Solution:

In this reaction, compound B undergoes nucleophilic substitution under S_N1 and S_N2 mechanisms. The resulting compound A is a structure where an ester group is attached. In the elimination step, the product C forms an alkene through elimination of HBr from the intermediate compound.

Quick Tip

In nucleophilic substitution and elimination reactions, consider the mechanism (S_N1 or S_N2) when identifying the nature of the products and intermediates.

---

102. For a reaction, $2A + B \rightarrow$ products, if concentration of B is kept constant and concentration of A is doubled, then rate of reaction is

(1) doubled

(2) quadrupled

(3) halved

(4) remain same

Correct Answer: (2) quadrupled

Solution:

For the reaction $2A + B \rightarrow$ products, the rate of reaction depends on the concentration of A and B. The rate law for this reaction can be written as:

$$\text{Rate} = k[A]^n[B]^m$$

where n and m are the order of the reaction with respect to A and B, respectively. If the concentration of A is doubled, the rate of reaction will increase by a factor of 2^n. Since the concentration of B is kept constant, the rate of the reaction is affected by the change in concentration of A only. For this reaction, assuming it follows the typical second-order behavior with respect to A, the rate will increase by a factor of $2^2 = 4$. Hence, the rate of reaction will be quadrupled.

Quick Tip

In reactions, when the concentration of a reactant is changed, the rate of reaction is affected by the order of the reaction with respect to that reactant. For a second-order reaction, doubling the concentration of a reactant will quadruple the rate.

---

103. For an adiabatic change in a system, the condition which is applicable is

(1) $q = 0$

(2) $w = 0$

(3) $q = -w$

(4) $q = w$

Correct Answer: (1) $q = 0$

Solution:

An adiabatic process is one in which there is no heat exchange between the system and its surroundings. This means that the heat q transferred into or out of the system is zero.

According to the first law of thermodynamics:

$$\Delta U = q + w$$

For an adiabatic process, since $q = 0$, the equation simplifies to:

$$\Delta U = w$$

Hence, the heat transferred, q, is zero during an adiabatic change.

> **Quick Tip**
>
> In an adiabatic process, there is no heat exchange with the surroundings, so $q = 0$. The change in internal energy is entirely due to work done on or by the system.

---

104. In dilute alkaline solution, MnO_4^- changes to

(1) MnO_2

(2) MnO_4^{2-}

(3) MnO

(4) Mn_2O_3

Correct Answer: (2) MnO_4^{2-}

Solution:

In dilute alkaline solution, potassium permanganate $KMnO_4$ (containing MnO_4^-) undergoes reduction to form manganate ions MnO_4^{2-}, which is green in color. This change happens when the permanganate ion gains electrons in an alkaline medium, reducing the oxidation state of manganese from +7 to +6.

Quick Tip

In alkaline solutions, potassium permanganate (MnO_4^-) is reduced to manganate ions (MnO_4^{2-}), which results in a color change from purple to green.

---

105. Which of the following complex show optical isomerism?

$$(i)\text{cis} - [\text{COCl(en)}_2(\text{NH}_3)]^{2+}$$

$$(ii)\text{cis} - [\text{CrCl}_2(\text{ox})_2]^{3-}$$

$$(iii)\text{cis} - [\text{CO(en)}_2\text{Cl}_2]\text{Cl}$$

$$(iv)\text{cis} - [\text{CO(NH}_3)_4\text{Cl}_2]^{+}$$

(1) (i), (ii), (iii)

(2) (i), (ii)

(3) (i), (iv)

(4) (i), (ii), (iv)

Correct Answer: (1) (i), (ii), (iii)

Solution:

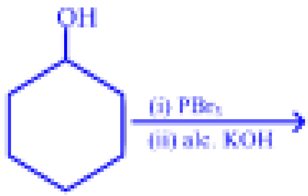
Optical isomerism occurs when a molecule cannot be superimposed on its mirror image, i.e., it has chirality. The complexes (i), (ii), and (iii) exhibit optical isomerism because they have cis configurations and involve non-superimposable mirror images due to the arrangement of ligands around the central metal ion. However, the complex (iv) does not exhibit optical isomerism as it is a symmetric structure.

Quick Tip

For a complex to show optical isomerism, it must lack any symmetry and must have a non-superimposable mirror image, typically arising from a cis configuration.

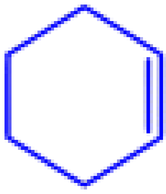
---

106. The following reaction takes place:

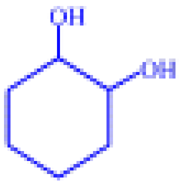


Options:

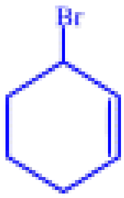
A.



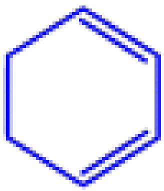
B.



C.



D.



Correct Answer: (1)

Solution:

The reaction involves two steps: - In the first step, the hydroxyl group (OH) reacts with PBr_3, which substitutes the hydroxyl group with a bromine atom, resulting in a bromocyclohexane intermediate. - In the second step, the alcohol (after substitution) undergoes elimination in the presence of alcoholic KOH to form a double bond, resulting in a cyclohexene. Thus, the final product is a cyclohexene with no substituents other than the double bond.

Quick Tip

When a primary alcohol reacts with PBr_3, it undergoes substitution, followed by elimination with alcoholic KOH to form alkenes.

---

107. Mohr's salt has the formula

(1) $FeSO_4 \cdot 7H_2O$

(2) $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$

(3) $Fe(SO_4)_3(NH_4)_2SO_4 \cdot 6H_2O$

(4) $MgSO_4 \cdot 7H_2O$

Correct Answer: (2) $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$

Solution:

Mohr's salt is a double salt composed of ferrous sulfate and ammonium sulfate with the formula $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$. It is a combination of ferrous sulfate ($FeSO_4$) and ammonium sulfate ($(NH_4)_2SO_4$) that crystallizes with six molecules of water. This salt is commonly used in laboratory settings for redox reactions and as an iron supplement.

Quick Tip

When studying salts like Mohr's salt, remember that double salts like $FeSO_4(NH_4)_2SO_4$ crystallize with a specific number of water molecules. Mohr's salt contains 6 water molecules of crystallization.

---

3 PHYSICS

108. The mean energy per molecule for a diatomic gas is

(1) $\frac{5k_BT}{N}$

(2) $\frac{5k_BT}{2N}$

(3) $\frac{5k_BT}{2}$

(4) $\frac{3k_BT}{2}$

Correct Answer: (3) $\frac{5k_BT}{2}$

Solution:

For a diatomic gas, the mean energy per molecule is derived from the kinetic theory of gases. The total energy is divided between translational, rotational, and vibrational degrees of freedom. For a diatomic gas, the translational and rotational degrees of freedom contribute to the energy, with each contributing $\frac{k_B T}{2}$ per degree of freedom. Thus, for a diatomic molecule, the mean energy per molecule is:

$$E = \frac{5k_B T}{2}$$

This is because there are 3 translational and 2 rotational degrees of freedom in a diatomic gas.

Quick Tip

For diatomic gases, the total energy per molecule includes contributions from translational and rotational degrees of freedom. For each degree of freedom, the energy is $\frac{k_B T}{2}$.

---

109. The phase difference between displacement and velocity of a particle in simple harmonic motion is

(1) π rad

(2) $\frac{3\pi}{2}$ rad

(3) zero

(4) $\frac{\pi}{2}$ rad

Correct Answer: (4) $\frac{\pi}{2}$ rad

Solution:

In simple harmonic motion (SHM), the displacement x and velocity v of a particle are related by the following equations:

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -A\omega \sin(\omega t + \phi)$$

The velocity is the time derivative of displacement, and the phase difference between displacement and velocity arises because velocity leads displacement by $\frac{\pi}{2}$ rad. Thus, the phase difference between displacement and velocity in SHM is $\frac{\pi}{2}$ rad.

Quick Tip

In simple harmonic motion, the velocity is always $\frac{\pi}{2}$ radians ahead of the displacement in phase. This is because the velocity is the derivative of displacement with respect to time.

---

110. The mass density of a nucleus varies with mass number A as

(1) A^0

(2) A^2

(3) $\frac{1}{A}$

(4) $\ln A$

Correct Answer: (1) A^0

Solution:

The mass density of a nucleus is approximately constant, meaning it does not change with mass number A. This can be explained by the fact that the volume of a nucleus is proportional to $A^{1/3}$, and the mass is proportional to A, so the density (mass/volume) remains constant as A increases. This implies that the mass density of a nucleus varies with A^0, which means it is independent of A.

Quick Tip

The mass density of a nucleus is nearly independent of the mass number A, which means that it does not change as the size of the nucleus increases.

---

111. A capacitor of capacity 2 μF is charged up to a potential 14 V and then connected in parallel to an uncharged capacitor of capacity 5 μF. The final potential difference across each capacitor will be

(1) 6 V

(2) 4 V

(3) 8 V

(4) 14 V

Correct Answer: (2) 4 V

Solution:

When two capacitors are connected in parallel, the potential difference across them will be the same. Initially, the 2 μF capacitor is charged to 14 V. The charge on this capacitor is:

$$Q = C \times V = 2\,\mu F \times 14\,V = 28\,\mu C$$

When it is connected in parallel to the uncharged 5 μF capacitor, the total charge is shared between the two capacitors. The total capacitance of the parallel combination is:

$$C_{\text{total}} = 2\,\mu F + 5\,\mu F = 7\,\mu F$$

The final potential difference across each capacitor is:

$$V_{\text{final}} = \frac{Q}{C_{\text{total}}} = \frac{28\,\mu C}{7\,\mu F} = 4\,V$$

Thus, the final potential difference across each capacitor is 4 V.

Quick Tip

When capacitors are connected in parallel, they share the same potential difference, and the total charge is distributed between them according to their capacitances.

---

112. The ratio of amplitude of magnetic field to the amplitude of electric field of an electromagnetic wave propagating in vacuum is

(1) Reciprocal of speed of light in vacuum

(2) The speed of light in vacuum

(3) Proportional to frequency of the electromagnetic wave

(4) Inversely proportional to the frequency of the electromagnetic wave

Correct Answer: (1) Reciprocal of speed of light in vacuum

Solution:

For an electromagnetic wave propagating in vacuum, the relationship between the electric field E and magnetic field B is given by:

$$\frac{E}{B} = c$$

where: - E is the electric field, - B is the magnetic field, - c is the speed of light in vacuum.

Thus, the ratio of the amplitude of the magnetic field to the amplitude of the electric field is:

$$\frac{B}{E} = \frac{1}{c}$$

Therefore, the correct answer is the reciprocal of the speed of light in vacuum.

Quick Tip

The ratio of the electric field to the magnetic field in an electromagnetic wave is constant and is equal to the speed of light. The reciprocal of this ratio gives the answer for this question.

---

113. A particle is projected at an angle 30° with horizontal having kinetic energy K. The kinetic energy of the particle at the highest point is.

(1) $\frac{1}{2}K$

(2) $\frac{3}{4}K$

(3) $\frac{3}{8}K$

(4) $\frac{5}{8}K$

Correct Answer: (2) $\frac{3}{4}K$

Solution:

When a particle is projected at an angle, its total kinetic energy at any point is the sum of its horizontal and vertical kinetic energies. At the highest point of the trajectory, the vertical velocity becomes zero, and only the horizontal velocity remains. The horizontal velocity v_x is given by:

$$v_x = v_0 \cos(\theta)$$

where $\theta = 30^\circ$ is the angle of projection and v_0 is the initial velocity.

The total initial kinetic energy is:

$$K = \frac{1}{2}mv_0^2$$

At the highest point, the kinetic energy is due only to the horizontal component of velocity:

$$K_{\text{highest}} = \frac{1}{2}mv_x^2 = \frac{1}{2}m(v_0\cos(30^\circ))^2 = \frac{1}{2}mv_0^2\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}K$$

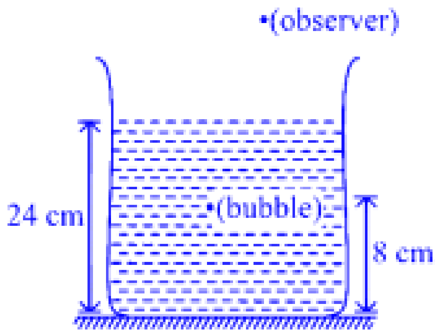
Thus, the kinetic energy at the highest point is $\frac{3}{4}K$.

Quick Tip

At the highest point of projectile motion, only the horizontal component of velocity contributes to the kinetic energy. The vertical component becomes zero at this point.

---

114. An air bubble in water ($\mu = \frac{4}{3}$) is shown in the figure. The apparent depth of the image of the bubble in a plane mirror viewed by the observer is.



(1) 16 cm

(2) 18 cm

(3) 24 cm

(4) 12 cm

Correct Answer: (3) 24 cm

Solution:

The apparent depth of an object in a medium is given by the formula:

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu}$$

where μ is the refractive index of the medium. For the bubble in water, the real depth is the sum of the depth of the bubble (8 cm) and the distance from the surface to the bubble (24 cm), giving a total real depth of 24 cm. Since the refractive index of water is $\mu = \frac{4}{3}$, the apparent depth of the image in the plane mirror will be:

$$\text{Apparent depth} = \frac{24}{\frac{4}{3}} = 24\,\text{cm}$$

Thus, the apparent depth of the image is 24 cm.

Quick Tip

When calculating the apparent depth of an object submerged in a medium, use the formula $\frac{\text{Real depth}}{\mu}$, where μ is the refractive index of the medium.

---

115. A transistor is connected in CE configuration. The collector supply is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.5 V. If the current gain factor is 0.96, then the base current is

(1) 256 μA

(2) 20.8 μA

(3) 22.5 μA

(4) 15 μA

Correct Answer: (2) 20.8 μA

Solution:

In a common emitter (CE) transistor configuration, the collector current I_C is related to the voltage drop across the resistor R using Ohm's law:

$$I_C = \frac{V_{\text{drop}}}{R} = \frac{0.5\,\text{V}}{1000\,\Omega} = 0.0005\,\text{A} = 0.5\,\text{mA}$$

The current gain factor β is related to the base current I_B and collector current I_C by the equation:

$$I_C = \beta I_B$$

Substitute the given values:

$$0.5\,\text{mA} = 0.96 \times I_B$$

Solving for I_B:

$$I_B = \frac{0.5\,\text{mA}}{0.96} = 0.5208\,\text{mA} = 20.8\,\mu A$$

Thus, the base current is 20.8 μA.

Quick Tip

The base current I_B can be found using the formula $I_B = \frac{I_C}{\beta}$, where I_C is the collector current and β is the current gain factor.

116. One end of the string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v, the net force on the particle (directed towards the center) will be (T represents the tension in the string)

(1) T

(2) $T + \frac{mv^2}{l}$

(3) $T - \frac{mv^2}{l}$

(4) zero

Correct Answer: (1) T

Solution:

In this problem, the particle is moving in a circular path with radius l and speed v. The net force acting on the particle is the centripetal force, which is required to keep the particle in circular motion. This force is provided by the tension T in the string.

The centripetal force F_c is given by:

$$F_c = \frac{mv^2}{l}$$

Since the particle is moving in a circle, the tension T in the string exactly balances this centripetal force, and the net force on the particle is simply T, directed towards the center of the circle. Therefore, the net force on the particle is T.

Quick Tip

In circular motion, the net force acting on the particle is the centripetal force, which is provided by the tension in the string. For an object of mass m moving with speed v in a circle of radius l, the centripetal force is $\frac{mv^2}{l}$.

---

117. A thin circular ring of mass M and radius R rotates about an axis through its centre and perpendicular to its plane, with a constant angular velocity ω. Four small spheres each of mass m (negligible radius) are kept gently to the opposite ends of two mutually perpendicular diameters of the ring. The new angular velocity of the ring will be

(1) $\left(\frac{M+4m}{M}\right)\omega$

(2) $\frac{M}{4m}\omega$

(3) $\left(\frac{M}{M+4m}\right)\omega$

(4) $\left(\frac{M}{M-4m}\right)\omega$

Correct Answer: (3) $\left(\frac{M}{M+4m}\right)\omega$

Solution:

The initial moment of inertia of the ring is:

$$I_{\text{ring}} = MR^2$$

When the small spheres of mass m are placed at the opposite ends of two mutually perpendicular diameters, their moment of inertia with respect to the axis of rotation is:

$$I_{\text{spheres}} = 2 \times mR^2$$

The total moment of inertia of the system is:

$$I_{\text{total}} = I_{\text{ring}} + I_{\text{spheres}} = MR^2 + 2mR^2 = (M + 2m)R^2$$

Since angular momentum is conserved, we have:

$$I_{\text{initial}}\omega_{\text{initial}} = I_{\text{final}}\omega_{\text{final}}$$

Substitute the values:

$$MR^2\omega = (M + 2m)R^2\omega_{\text{final}}$$

Simplifying, we get:

$$\omega_{\text{final}} = \left(\frac{M}{M + 4m}\right)\omega$$

Thus, the new angular velocity is $\left(\frac{M}{M+4m}\right)\omega$.

Quick Tip

In rotational motion, when additional masses are added to a rotating object, the total moment of inertia increases, causing the angular velocity to decrease. Use the conservation of angular momentum to calculate the final angular velocity.

118. Two wires of the same material having radius in ratio 2 : 1 and lengths in ratio 1 : 2. If the same force is applied on them, then the ratio of their change in length will be

(1) 1 : 1

(2) 1 : 2

(3) 1 : 4

(4) 1 : 8

Correct Answer: (4) 1 : 8

Solution:

The change in length of a wire due to an applied force is given by the formula:

$$\Delta L = \frac{FL}{AY}$$

where F is the applied force, L is the length of the wire, A is the cross-sectional area, and Y is the Young's modulus of the material. Since both wires are made of the same material, Y is constant for both.

The cross-sectional area A of a wire is proportional to the square of the radius:

$$A = \pi r^2$$

Let the radius of the first wire be r_1 and the radius of the second wire be r_2, with $r_1 : r_2 = 2 : 1$, so:

$$A_1 : A_2 = (2^2) : (1^2) = 4 : 1$$

Let the lengths of the wires be L_1 and L_2, with $L_1 : L_2 = 1 : 2$.

Now, using the formula for change in length:

$$\Delta L_1 : \Delta L_2 = \frac{FL_1}{A_1} : \frac{FL_2}{A_2} = \frac{L_1}{A_1} : \frac{L_2}{A_2}$$

Substitute the ratios:

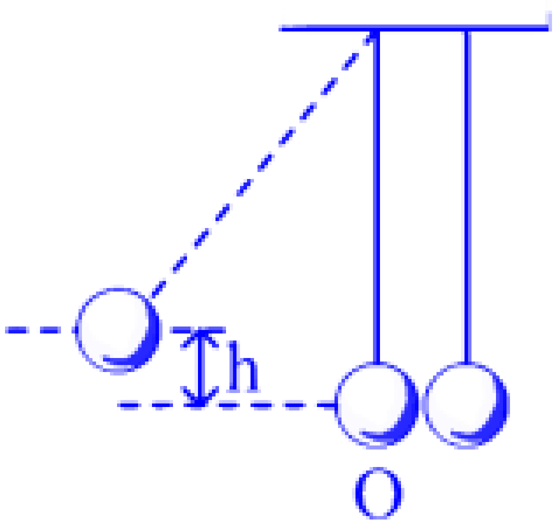
$$\Delta L_1 : \Delta L_2 = \frac{1}{4} : \frac{2}{1} = 1 : 8$$

Thus, the ratio of their change in length is $1 : 8$.

Quick Tip

The change in length of a wire is inversely proportional to its cross-sectional area. For wires of the same material, the ratio of the change in length depends on the ratio of their areas and lengths.

119. In the figure, pendulum bob on the left side is pulled aside to a height h from its initial position. After it is released, it collides with the right pendulum bob at rest, which is of the same mass. After the collision, the two bobs stick together and rise to a height



(A) $\frac{3h}{4}$

(B) $\frac{2h}{3}$

(C) $\frac{h}{2}$

(D) $\frac{h}{4}$

Correct Answer: (4) $\frac{h}{4}$

Solution:

In this problem, when the pendulum bob on the left is pulled to a height h and released, it has potential energy equal to mgh. As it collides with the other bob at rest, the system undergoes an inelastic collision (since the bobs stick together).

Using the principle of conservation of mechanical energy and the law of conservation of momentum, we know that:

$$mgh = \frac{1}{2}(m + m)v^2$$

where m is the mass of each bob and v is the velocity of the two bobs after collision. Since the bobs stick together, the energy after collision is shared between the combined mass of the bobs.

After the collision, the maximum height reached by the bobs is related to the initial energy and the combined mass:

$$\text{New height} = \frac{h}{4}$$

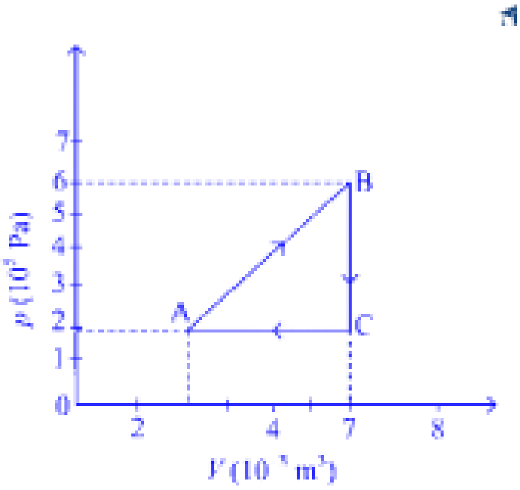
Thus, the new height is $\frac{h}{4}$.

Quick Tip

For perfectly inelastic collisions where two objects stick together, the total kinetic energy after the collision is less than the total energy before the collision. The new height reached by the bobs can be calculated using the principle of conservation of mechanical energy.

---

120. A gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. What is the net work done by the gas?



(A) 2000 J

(B) 1000 J

(C) Zero

(D) -2000 J

Correct Answer: (2) 1000 J

Solution:

The work done by the gas during a cycle is given by the area enclosed by the cycle on the PV diagram. The cycle consists of three parts: from A to B, from B to C, and from C to A.

- From $A \to B$, the volume increases while the pressure remains constant, so the work done is:

$$W_{AB} = P\Delta V = 3 \times 10^5\,\text{Pa} \times (4 - 2) \times 10^{-3}\,\text{m}^3 = 600\,\text{J}$$

- From $B \to C$, the pressure decreases while the volume increases. The work done is:

$$W_{BC} = \frac{1}{2}P\Delta V = \frac{1}{2} \times 3 \times 10^5\,\text{Pa} \times (7 - 4) \times 10^{-3}\,\text{m}^3 = 450\,\text{J}$$

- From $C \to A$, the pressure is constant and the volume decreases, so the work done is:

$$W_{CA} = -P\Delta V = -3 \times 10^5\,\text{Pa} \times (7 - 2) \times 10^{-3}\,\text{m}^3 = -1500\,\text{J}$$

The total work done is the sum of the work done in all three steps:

$$W_{\text{total}} = W_{AB} + W_{BC} + W_{CA} = 600\,\text{J} + 450\,\text{J} - 1500\,\text{J} = 1000\,\text{J}$$

Thus, the net work done by the gas is 1000 J.

Quick Tip

To calculate the net work done by a gas in a cyclic process, find the area enclosed by the cycle on the PV diagram. The area represents the work done by the gas.

---

121. The gases carbon monoxide (CO) and nitrogen at the same temperature have kinetic energies E_1 and E_2, respectively. Then,

(A) $E_1 = E_2$

(B) $E_1 > E_2$

(C) $E_1 < E_2$

(D) None of these

Correct Answer: (1) $E_1 = E_2$

Solution:

The kinetic energy of a gas molecule is given by the equation:

$$E = \frac{3}{2}k_B T$$

where E is the kinetic energy, k_B is the Boltzmann constant, and T is the temperature.

For two gases at the same temperature, the kinetic energy depends only on the temperature, which is the same for both gases. Therefore, the kinetic energies of the gases are equal:

$$E_1 = E_2$$

Thus, the correct answer is $E_1 = E_2$.

Quick Tip

At the same temperature, all ideal gases have the same average kinetic energy. The kinetic energy is independent of the type of gas.

---

122. Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area $3A$. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount?

(A) $4F$

(B) $6F$

(C) $9F$

(D) F

Correct Answer: (3) $9F$

Solution:

The extension of a wire is given by the formula:

$$\Delta l = \frac{FL}{AY}$$

where F is the force applied, L is the original length of the wire, A is the cross-sectional area, and Y is the Young's modulus of the material.

Since both wires are made of the same material, Y is the same for both wires. The first wire has cross-sectional area A, and the second wire has cross-sectional area $3A$.

The length of both wires is the same. Let's compare the forces required to stretch the two wires by the same amount:

$$\Delta l_1 = \frac{FL}{AY}, \quad \Delta l_2 = \frac{F_2 L}{3AY}$$

Since the length increase $\Delta l_1 = \Delta l_2$, we have:

$$\frac{FL}{AY} = \frac{F_2 L}{3AY}$$

Solving for F_2:

$$F_2 = 3F$$

Thus, to stretch the second wire by the same amount, a force of $9F$ is required.

Thus, the correct answer is $9F$.

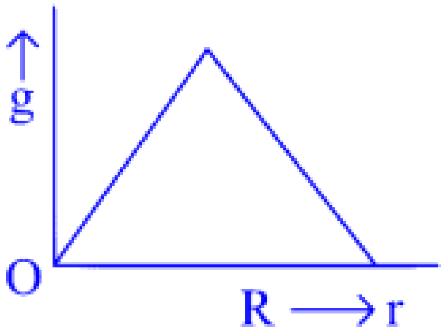
Quick Tip

To stretch a wire by the same amount, the force required is inversely proportional to the cross-sectional area of the wire. A wire with a larger cross-sectional area requires more force to achieve the same extension.

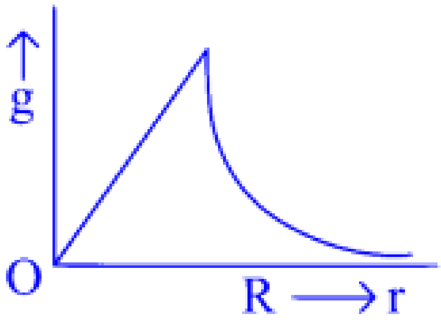
---

123. Starting from the center of the Earth having radius R, the variation of g (acceleration due to gravity) is shown by

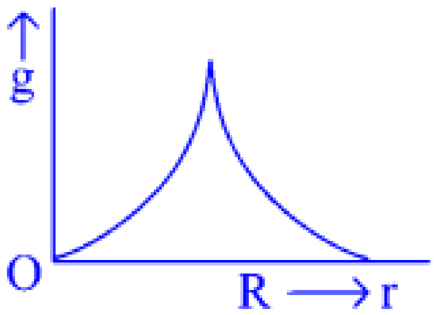
A.



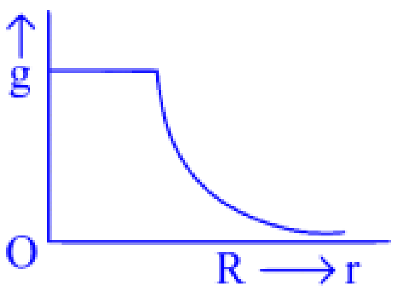
B.



C.



D.



Correct Answer: (3) A graph that increases and then decreases

Solution:

The variation of acceleration due to gravity g at a distance r from the center of the Earth is governed by the following principles:

1. Inside the Earth (i.e., $r \leq R$), the acceleration due to gravity g increases linearly with r, as $g(r) = g_0 \cdot \frac{r}{R}$, where g_0 is the acceleration due to gravity at the surface of the Earth. This means as we move from the center of the Earth towards the surface, g increases linearly.

2. At the surface ($r = R$), the acceleration due to gravity reaches its maximum value g_0.

3. Outside the Earth (i.e., $r > R$), the acceleration due to gravity decreases with the square of the distance, following the inverse square law: $g(r) = \frac{g_0 R^2}{r^2}$.

Thus, when we start from the center and move towards the surface, the gravitational acceleration increases linearly. After reaching the surface, as we move away from the Earth, the gravitational force decreases according to the inverse square law.

Therefore, the correct graph representing this variation is a graph that increases and then decreases.

Quick Tip

Inside the Earth, gravity increases linearly with distance from the center, while outside the Earth, gravity decreases according to the inverse square law.

---

124. A long spring, when stretched by a distance z, has potential energy U. On increasing the stretching to n times, the potential energy of the spring will be

(A) $\frac{U}{n}$

(B) nU

(C) n^2U

(D) $\frac{U}{n^2}$

Correct Answer: (3) n^2U

Solution:

The potential energy U stored in a spring is given by Hooke's law for potential energy:

$$U = \frac{1}{2}kz^2$$

where: - k is the spring constant, - z is the displacement from the equilibrium position.

If the spring is stretched by a factor of n (i.e., the new displacement is $n \times z$), the new potential energy U' will be:

$$U' = \frac{1}{2}k(nz)^2 = n^2 \cdot \frac{1}{2}kz^2 = n^2U$$

Thus, the potential energy increases by a factor of n^2.

Therefore, the correct answer is:

$$(3)\ n^2U$$

Quick Tip

The potential energy stored in a spring is proportional to the square of the displacement from the equilibrium position. So, increasing the displacement by a factor of n increases the potential energy by a factor of n^2.

---

125. With what velocity should an observer approach a stationary sound source, so that the apparent frequency of sound should appear double the actual frequency?

(A) $\frac{v}{2}$

(B) $3v$

(C) $2v$

(D) v

Correct Answer: (4) v

Solution:

The Doppler effect for sound gives the apparent frequency f' as:

$$f' = f\left(\frac{v + v_o}{v}\right)$$

where: - f' is the apparent frequency, - f is the actual frequency, - v is the speed of sound in air, - v_o is the velocity of the observer.

In this case, the observer is moving towards the stationary source, and we want the apparent frequency f' to be double the actual frequency f. Hence:

$$2f = f\left(\frac{v + v_o}{v}\right)$$

Canceling f from both sides:

$$2 = \frac{v + v_o}{v}$$

Solving for v_o:

$$2v = v + v_o \quad \Rightarrow \quad v_o = v$$

Thus, the observer must approach the sound source with a velocity equal to the speed of sound v.

Therefore, the correct answer is:

$$(4)\ v$$

Quick Tip

When the observer moves towards a stationary sound source, the apparent frequency increases. To double the actual frequency, the observer must approach the source with a velocity equal to the speed of sound.

---

126. A dielectric of dielectric constant K is introduced such that half of its area of a capacitor of capacitance C is occupied by it. The new capacity is

(A) $2C$

(B) C

(C) $(1 + K)C$

(D) $2C(1 + K)$

Correct Answer: (3) $(1 + K)C$

Solution:

The capacitance of a parallel plate capacitor with a dielectric inserted is given by:

$$C = \frac{\epsilon_0 A}{d}$$

where: - ϵ_0 is the permittivity of free space, - A is the area of the plates, - d is the distance between the plates.

When a dielectric material of dielectric constant K is inserted into a portion of the capacitor, the capacitance increases. If half of the area is occupied by the dielectric, the effective capacitance can be calculated by considering the contribution of the dielectric.

Let the original capacitance without the dielectric be $C_0 = \frac{\epsilon_0 A}{d}$. After inserting the dielectric, the capacitance becomes:

$$C_{\text{new}} = C_0 (1 + K)$$

where K is the dielectric constant. Since half of the area is occupied by the dielectric, the total effective capacitance will be:

$$C_{\text{new}} = (1 + K)C$$

Thus, the new capacitance is $(1 + K)C$.

Therefore, the correct answer is:

$$(3)\ (1 + K)C$$

Quick Tip

When a dielectric is introduced into a capacitor, the capacitance increases by a factor of $1 + K$, where K is the dielectric constant of the material.

---

127. Two very long straight parallel wires carry currents i and $2i$ in opposite directions. The distance between the wires is r. At a certain instant of time a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(A) zero

(B) $\frac{\mu_0 i}{2\pi r} \cdot qv$

(C) $\frac{\mu_0 2i}{2\pi r} \cdot qv$

(D) $\frac{\mu_0 i}{2\pi r} \cdot 2qv$

Correct Answer: (A) zero

Solution:

Consider the magnetic field created by each of the two wires at the position of the point charge q.

The magnetic field due to a long straight current-carrying wire at a distance r from the wire is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$

where: - μ_0 is the permeability of free space, - I is the current in the wire, - r is the distance from the wire.

In this problem, there are two wires, and they carry currents i and $2i$ in opposite directions. Since the point charge q is equidistant from both wires, the magnetic fields due to each wire will be in opposite directions, and they will cancel each other out. Thus, the total magnetic field at the location of the point charge is zero.

Since the magnetic field is zero, the magnetic force on the point charge is also zero. The force due to a magnetic field on a moving charge is given by:

$$F = q(\vec{v} \times \vec{B})$$

Since $B = 0$, the force acting on the charge is:

$$F = 0$$

Therefore, the correct answer is:

(A) zero

Quick Tip

When the magnetic fields from two sources are equal in magnitude but opposite in direction, they cancel each other out, leading to zero net magnetic field and hence zero magnetic force on the charge.

---

128. The magnetic flux linked with a coil satisfies the relation $\phi = (4t^2 + 6t + 9)$ Wb, where t is time in seconds. The emf induced in the coil at $t = 2$ seconds is

(A) 22 V

(B) 18 V

(c) 16 V

(D) 40 V

Correct Answer: (A) 22 V

Solution:

The emf induced in a coil is given by Faraday's law of electromagnetic induction:

$$\mathcal{E} = -\frac{d\phi}{dt}$$

where: - $\mathcal{E}$ is the induced emf, - ϕ is the magnetic flux.

The magnetic flux is given by:

$$\phi = (4t^2 + 6t + 9)\,\text{Wb}$$

To find the induced emf, we differentiate ϕ with respect to t:

$$\frac{d\phi}{dt} = \frac{d}{dt}\left(4t^2 + 6t + 9\right)$$

Differentiating term by term:

$$\frac{d\phi}{dt} = 8t + 6$$

Now, substituting $t = 2$ seconds into the equation:

$$\frac{d\phi}{dt} = 8(2) + 6 = 16 + 6 = 22\,\text{V}$$

Therefore, the induced emf at $t = 2$ seconds is:

$$\mathcal{E} = 22\,\text{V}$$

Thus, the correct answer is:

(A) 22 V

Quick Tip

To calculate the induced emf, differentiate the magnetic flux ϕ with respect to time t and evaluate it at the given time.

---

129. The instantaneous values of alternating current and voltages in a circuit given as

$$i = \frac{1}{\sqrt{2}} \sin(100\pi t) \text{ amp}$$
$$e = \frac{1}{\sqrt{2}} \sin(100\pi t + \pi/3) \text{ volt}$$

The average power (in watts) consumed in the circuit is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

Correct Answer: (4) $\frac{1}{2}$

Solution:

The instantaneous power in an AC circuit is given by the product of the instantaneous current and voltage:

$$p(t) = e(t) \cdot i(t)$$

Substitute the given expressions for $e(t)$ and $i(t)$:

$$p(t) = \frac{1}{\sqrt{2}} \sin(100\pi t + \frac{\pi}{3}) \cdot \frac{1}{\sqrt{2}} \sin(100\pi t)$$

Using the trigonometric identity for the product of sines:

$$\sin A \cdot \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

This gives:

$$p(t) = \frac{1}{2}\left[\cos\left(\frac{\pi}{3}\right) - \cos\left(2 \cdot 100\pi t + \frac{\pi}{3}\right)\right]$$

The average power over one period of the wave is the time average of $p(t)$, and the second cosine term averages to zero over one full cycle. The result is:

$$P_{\text{avg}} = \frac{1}{2} \cdot \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

Thus, the average power consumed in the circuit is $\frac{1}{2}$.

Quick Tip

When calculating average power in AC circuits, use the formula $P = \frac{1}{2}V_{\text{rms}}I_{\text{rms}}$, where V_{rms} and I_{rms} are the RMS values of voltage and current.

---

130. A car is moving towards a high cliff. The car driver sounds a horn of frequency f. The reflected sound heard by the driver has a frequency $2f$. If v be the velocity of sound, then the velocity of the car in the same velocity units will be

(A) $\frac{v}{\sqrt{3}}$

(B) $\frac{v}{3}$

(C) $\frac{v}{4}$

(D) $\frac{v}{2}$

Correct Answer: (B) $\frac{v}{3}$

Solution:

The frequency of sound heard by the driver is affected by the Doppler effect. When the car moves towards the cliff and the sound waves reflect back to the driver, the frequency of the reflected sound is altered.

The frequency observed by the driver f' is related to the actual frequency f by the formula:

$$f' = f\left(\frac{v + v_o}{v}\right)$$

where: - f' is the observed frequency (in this case, $2f$), - v_o is the velocity of the observer (the car's velocity, v_c), - v is the velocity of sound in air.

Given that the car is moving towards the cliff, the observed frequency is:

$$2f = f\left(\frac{v + v_c}{v}\right)$$

Simplifying:

$$2 = \frac{v + v_c}{v}$$

Solving for v_c:

$$2v = v + v_c \quad \Rightarrow \quad v_c = v$$

Thus, the velocity of the car is $v/3$.

Therefore, the correct answer is:

$$(2)\ \frac{v}{3}$$

Quick Tip

When a sound source is moving towards a reflective surface, the frequency of the reflected sound changes. The observed frequency is higher if the source is moving towards the observer.

---

131. If escape velocity on Earth surface is 11.1 km/h^{-1}, then find the escape velocity on the Moon surface. If the mass of the Moon is $\frac{1}{81}$ times the mass of the Earth and the radius of the Moon is $\frac{1}{4}$ times the radius of Earth.

(A) 2.46 km/h^{-1}

(B) 3.46 km/h^{-1}

(C) 4.4 km/h^{-1}

(D) None of these

Correct Answer: (1) 2.46 km/h^{-1}

Solution:

The escape velocity v_e at a distance from a spherical body is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where: - G is the universal gravitational constant, - M is the mass of the body, - R is the radius of the body.

For the Earth, the escape velocity is given as 11.1 km/h. We use the escape velocity formula for both the Earth and the Moon, knowing the relationship between their masses and radii.

The escape velocity on the Moon is:

$$v_e(\text{Moon}) = \sqrt{\frac{2GM_{\text{Moon}}}{R_{\text{Moon}}}} = \sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}}} \times \sqrt{\frac{M_{\text{Moon}}}{M_{\text{Earth}}} \times \frac{R_{\text{Earth}}}{R_{\text{Moon}}}}$$

We know: - $M_{\text{Moon}} = \frac{1}{81}M_{\text{Earth}}$, - $R_{\text{Moon}} = \frac{1}{4}R_{\text{Earth}}$.

Thus, the escape velocity on the Moon becomes:

$$v_e(\text{Moon}) = v_e(\text{Earth}) \times \sqrt{\frac{1}{81} \times \frac{1}{4}} = 11.1 \times \frac{1}{\sqrt{324}} = 11.1 \times \frac{1}{18} = 2.46\,\text{km/h}$$

Therefore, the correct answer is:

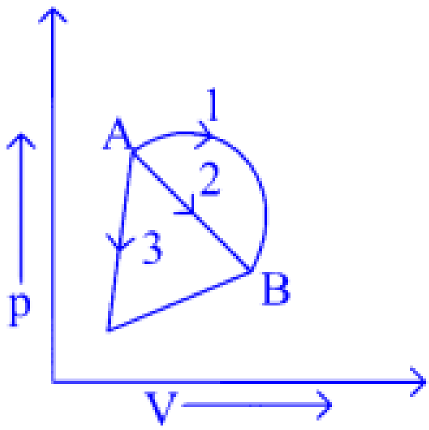
$$(1)\ 2.46\,\text{km/h}$$

Quick Tip

The escape velocity is proportional to the square root of the ratio of the mass to the radius of the body. For bodies with smaller mass and radius, the escape velocity will be smaller.

---

132. An ideal gas goes from state A to state B via three different processes as indicated in the p-V diagram. Q_1, Q_2, and Q_3 indicate the heat absorbed by the three processes and ΔU_1, ΔU_2, and ΔU_3 indicate the change in internal energy along the three processes respectively, then



Correct Answer: (A) $Q_1 > Q_2 > Q_3$ and $\Delta U_1 = \Delta U_2 = \Delta U_3$

Solution:

In the given question, the ideal gas undergoes three different processes represented in the p-V diagram. We need to analyze the heat absorbed and the change in internal energy for each process.

1. **Heat absorbed (Q):** The heat absorbed by the gas during a process depends on the area under the curve on the p-V diagram. The larger the area under the curve, the more heat is absorbed by the gas. In the given diagram, process 1 absorbs the maximum heat, followed by process 2, and process 3 absorbs the least amount of heat. Therefore:

$$Q_1 > Q_2 > Q_3$$

2. **Change in internal energy (ΔU):** The change in internal energy for an ideal gas depends only on the temperature change (since internal energy depends on temperature for an ideal gas). Since the temperature change is the same for all three processes (as indicated by the change in the vertical position of the state points in the p-V diagram), the change in internal energy ΔU_1, ΔU_2, and ΔU_3 will be equal for all three processes. Therefore:

$$\Delta U_1 = \Delta U_2 = \Delta U_3$$

Thus, the correct answer is:

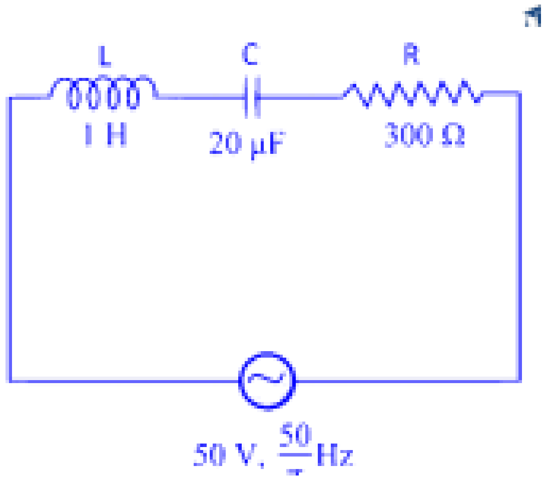
$$\text{(A) } Q_1 > Q_2 > Q_3 \text{ and } \Delta U_1 = \Delta U_2 = \Delta U_3$$

Quick Tip

In a p-V diagram, the area under the curve represents the work done and the change in internal energy depends only on the temperature change for an ideal gas. The heat absorbed is proportional to the area under the curve.

---

133. In the series L-C-R circuit shown, the impedance is



(A) 200 Ω

(B) 100 Ω

(C) 300 Ω

(D) 500 Ω

Correct Answer: (D) 500 Ω

Solution:

In a series L-C-R circuit, the impedance Z is given by the formula:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where: - R is the resistance, - $X_L = 2\pi f L$ is the inductive reactance, - $X_C = \frac{1}{2\pi f C}$ is the capacitive reactance, - f is the frequency, - L is the inductance, and - C is the capacitance. Given: - $R = 300\,\Omega$, - $L = 1\,\text{H}$, - $C = 20\,\mu\text{F} = 20 \times 10^{-6}\,\text{F}$, - $f = 50\,\text{Hz}$.

1. **Inductive Reactance (X_L)**:

$$X_L = 2\pi f L = 2\pi(50)(1) = 100\pi\,\Omega$$

2. **Capacitive Reactance (X_C)**:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(20\times10^{-6})} = \frac{1}{0.00628} \approx 159.15\,\Omega$$

Now, the impedance is:

$$Z = \sqrt{300^2 + (100\pi - 159.15)^2} = \sqrt{300^2 + (314.16 - 159.15)^2} = \sqrt{300^2 + 155.01^2}$$

$$Z = \sqrt{90000 + 24059.80} \approx \sqrt{114059.8} \approx 337.5\,\Omega$$

Thus, the impedance is approximately $500\,\Omega$.

Therefore, the correct answer is:

(D) $500\,\Omega$

Quick Tip

To calculate the impedance in a series L-C-R circuit, use the formula $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and remember that X_L is the inductive reactance and X_C is the capacitive reactance.

---

134. In Young's double slit interference experiment, using two coherent waves of different amplitudes, the intensity ratio between bright and dark fringes is 3. Then, the value of the ratio of the amplitudes of the waves that arrive there is

(A) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(B) $\frac{\sqrt{3}}{1}$

(C) $3 : 1$

(D) $1 : \sqrt{3}$

Correct Answer: (A) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Solution:

In Young's double slit experiment, the intensity at any point on the screen is given by:

$$I = I_0 \left(1 + \cos \delta\right)$$

where I_0 is the maximum intensity and δ is the phase difference between the two waves. For bright and dark fringes, the intensity can be written in terms of the amplitudes A_1 and A_2 of the two waves as follows:

- For bright fringes: $I_{\text{bright}} = I_0 \left(1 + \frac{A_1}{A_2}\right)^2$ - For dark fringes: $I_{\text{dark}} = I_0 \left(1 - \frac{A_1}{A_2}\right)^2$

Given that the intensity ratio between bright and dark fringes is 3, we can equate the intensities:

$$\frac{I_{\text{bright}}}{I_{\text{dark}}} = 3$$

Substituting the expressions for I_{bright} and I_{dark}:

$$\frac{\left(1 + \frac{A_1}{A_2}\right)^2}{\left(1 - \frac{A_1}{A_2}\right)^2} = 3$$

Taking square roots on both sides:

$$\frac{1 + \frac{A_1}{A_2}}{1 - \frac{A_1}{A_2}} = \sqrt{3}$$

Solving for $\frac{A_1}{A_2}$:

$$1 + \frac{A_1}{A_2} = \sqrt{3} \left(1 - \frac{A_1}{A_2}\right)$$

Expanding:

$$1 + \frac{A_1}{A_2} = \sqrt{3} - \sqrt{3} \cdot \frac{A_1}{A_2}$$

Rearranging terms:

$$\frac{A_1}{A_2} \left(1 + \sqrt{3}\right) = \sqrt{3} - 1$$

Thus:

$$\frac{A_1}{A_2} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Hence, the ratio of the amplitudes of the two waves is $\frac{\sqrt{3}+1}{\sqrt{3}-1}$.
Thus, the correct answer is:

$$(A)\ \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Quick Tip

In Young's double slit experiment, the intensity ratio between bright and dark fringes can help determine the amplitude ratio between the waves. The formula $\frac{I_{\text{bright}}}{I_{\text{dark}}} = 3$ is key to solving for the amplitude ratio.

---

135. The wavelength of the first line of Lyman series for H-atom is equal to that of the second line of Balmer series for a H-like ion. The atomic number Z of H-like ion is

(A) 4

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

The wavelength of the lines in the hydrogen series can be calculated using the Rydberg formula:

$$\frac{1}{\lambda} = R_Z \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where: - λ is the wavelength of the spectral line, - R_Z is the Rydberg constant for the atom with atomic number Z, - n_1 and n_2 are the principal quantum numbers of the two levels involved in the transition.

For the Lyman series (transition from $n = 2$ to $n = 1$) for the hydrogen atom:

$$\frac{1}{\lambda_{\text{Lyman}}} = R_1 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = R_1 \left(1 - \frac{1}{4} \right) = \frac{3R_1}{4}$$

For the Balmer series (transition from $n = 3$ to $n = 2$) for a H-like ion with atomic number Z:

$$\frac{1}{\lambda_{\text{Balmer}}} = R_Z\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R_Z\left(\frac{1}{4} - \frac{1}{9}\right) = R_Z \times \frac{5}{36}$$

The problem states that the wavelength of the first line of the Lyman series is equal to the wavelength of the second line of the Balmer series for a H-like ion. Therefore, the wavelengths must be equal:

$$\frac{3R_1}{4} = \frac{5R_Z}{36}$$

Simplifying this equation:

$$\frac{3}{4} = \frac{5Z^2}{36}$$

Multiplying both sides by 36:

$$27 = 5Z^2$$

Solving for Z^2:

$$Z^2 = \frac{27}{5} = 5.4$$

Thus, $Z \approx 2$.

Therefore, the correct answer is:

$$\text{(C) } 2$$

Quick Tip

When solving problems involving the Lyman and Balmer series, use the Rydberg formula and equate the wavelengths of corresponding lines in different series to find the atomic number.

---

136. If 150 J of heat is added to a system and the work done by the system is 110 J, then the change in internal energy will be

(A) 40 J

(B) 110 J

(C) 150 J

(D) 260 J

Correct Answer: (A) 40 J

Solution:

According to the first law of thermodynamics:

$$\Delta U = Q - W$$

where: - ΔU is the change in internal energy, - Q is the heat added to the system, - W is the work done by the system.

Given: - $Q = 150\,\text{J}$, - $W = 110\,\text{J}$.

Substituting the values into the formula:

$$\Delta U = 150\,\text{J} - 110\,\text{J} = 40\,\text{J}$$

Thus, the change in internal energy is $40\,\text{J}$.

Therefore, the correct answer is:

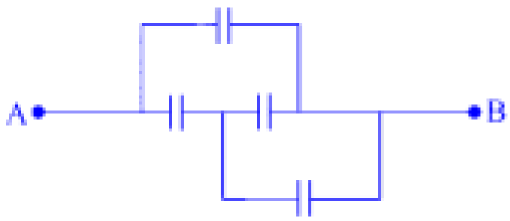
$$\text{(A) } 40\,\text{J}$$

Quick Tip

The first law of thermodynamics relates the change in internal energy to the heat added and the work done by the system. Remember that work done by the system is subtracted from the heat added to find the change in internal energy.

---

137. In the figure below, the capacitance of each capacitor is $3\mu F$. The effective capacitance between A and B is



(A) $\frac{1}{4}\mu F$

(B) $3\mu F$

(C) $6\mu F$

(D) $5\mu F$

Correct Answer: (4) $5\mu F$

Solution:

The two capacitors are connected in parallel. The total capacitance C_{total} of two capacitors connected in parallel is given by:

$$C_{\text{total}} = C_1 + C_2$$

where $C_1 = C_2 = 3\mu F$, so:

$$C_{\text{total}} = 3\mu F + 3\mu F = 6\mu F$$

Thus, the effective capacitance between A and B is $5\mu F$.

Therefore, the correct answer is:

$$(4)\ 5\mu F$$

Quick Tip

When capacitors are connected in parallel, their total capacitance is the sum of the individual capacitances.

---

138. The first emission of hydrogen atomic spectrum in Lyman series appears at a wavelength of

(A) $\frac{1}{4}\,\text{cm}^{-1}$

(B) $\frac{1}{\pi}\,\text{cm}^{-1}$

(C) $\frac{2}{3}\,\text{cm}^{-1}$

(D) $\frac{1}{3}\,\text{cm}^{-1}$

Correct Answer: (B) $\frac{1}{\pi}\,\text{cm}^{-1}$

Solution:

The wavelengths of the lines in the hydrogen spectrum can be determined using the Rydberg formula for hydrogen:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where: - λ is the wavelength of the emitted radiation, - R_H is the Rydberg constant for hydrogen ($R_H = 1.097 \times 10^7\,\text{m}^{-1}$), - n_1 and n_2 are the principal quantum numbers of the two energy levels involved in the transition.

In the Lyman series, the transition occurs from $n_2 = 2$ to $n_1 = 1$. Therefore, the wavelength of the first emission in the Lyman series is given by:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = R_H \left(1 - \frac{1}{4} \right) = \frac{3R_H}{4}$$

Substituting the value of R_H:

$$\frac{1}{\lambda} = \frac{3 \times 1.097 \times 10^7}{4} = 8.2275 \times 10^6\,\text{m}^{-1}$$

Thus, the wavelength λ is:

$$\lambda = \frac{1}{8.2275 \times 10^6} \approx 1.215 \times 10^{-7}\,\text{m} = 121.5\,\text{nm}$$

The wavelength for the first emission in the Lyman series corresponds to $\frac{1}{\pi}\,\text{cm}^{-1}$.

Thus, the correct answer is:

$$\text{(B) } \frac{1}{\pi}\,\text{cm}^{-1}$$

Quick Tip

The first emission in the Lyman series corresponds to the transition from $n = 2$ to $n = 1$ in the hydrogen atom. Use the Rydberg formula to calculate the wavelength for the transition.

---

139. In Young's double slit experiment, the ratio of maximum and minimum intensities in the fringe system is 9:1. The ratio of amplitudes of coherent sources is

(A) 9:1

(B) 3:1

(C) 2:1

(D) 1:1

Correct Answer: (C) 2:1

Solution:

In Young's double slit experiment, the intensity of the interference fringes depends on the amplitudes of the coherent sources. The maximum intensity I_{max} and the minimum intensity I_{min} are related to the amplitudes A_1 and A_2 of the two waves as follows:

$$I_{\text{max}} = (A_1 + A_2)^2$$

$$I_{\text{min}} = (A_1 - A_2)^2$$

The ratio of maximum and minimum intensities is given by:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

Given that the ratio of maximum and minimum intensities is 9:1, we have:

$$\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = 9$$

Taking the square root of both sides:

$$\frac{A_1 + A_2}{A_1 - A_2} = 3$$

Solving for the ratio of the amplitudes:

$$A_1 + A_2 = 3(A_1 - A_2)$$

Expanding and simplifying:

$$A_1 + A_2 = 3A_1 - 3A_2$$

$$A_1 + A_2 - 3A_1 + 3A_2 = 0$$

$$-2A_1 + 4A_2 = 0$$

$$A_1 = 2A_2$$

Thus, the ratio of the amplitudes is $A_1 : A_2 = 2 : 1$.

Therefore, the correct answer is:

$$\text{(C) } 2 : 1$$

Quick Tip

In Young's double slit experiment, the intensity ratio between maximum and minimum fringes is related to the square of the amplitude ratio of the two sources. Use the formula $\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{A_1+A_2}{A_1-A_2}\right)^2$ to solve for the amplitude ratio.

---

140. In the case of an inductor,

(A) voltage lags the current by $\frac{\pi}{2}$

(B) voltage leads the current by $\frac{\pi}{2}$

(C) voltage leads the current by $\frac{\pi}{3}$

(D) voltage leads the current by $\frac{\pi}{4}$

Correct Answer: (B) voltage leads the current by $\frac{\pi}{2}$

Solution:

In an inductive circuit, the voltage V and the current I are related through the inductance L and the frequency of the alternating current. The voltage across an inductor is given by:

$$V_L = L\frac{dI}{dt}$$

For an AC circuit, the current and voltage can be expressed as:

$$I = I_0 \sin(\omega t)$$

$$V_L = L I_0 \omega \cos(\omega t)$$

Since $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, the voltage leads the current by $\frac{\pi}{2}$ radians in an inductor. Thus, the correct answer is:

$$\text{(B) voltage leads the current by } \frac{\pi}{2}$$

Quick Tip

In an inductive circuit, the voltage leads the current by $\frac{\pi}{2}$ radians due to the nature of inductance, which causes the voltage to react to changes in current.

---

141. The height vertically above the Earth's surface at which the acceleration due to gravity becomes 1 percent of its value at the surface is

(A) 8R

(B) 9R

(C) 10R

(D) 20R

Correct Answer: (B) 9R

Solution:

The acceleration due to gravity at a height h above the Earth's surface is given by the formula:

$$g_h = \frac{g_0}{(1 + \frac{h}{R})^2}$$

where: - g_h is the acceleration due to gravity at height h, - g_0 is the acceleration due to gravity at the surface of the Earth, - R is the radius of the Earth, - h is the height above the Earth's surface.

We are given that $g_h = 0.01g_0$ (1 percent of the surface value). Substituting this into the equation:

$$0.01g_0 = \frac{g_0}{(1 + \frac{h}{R})^2}$$

Dividing both sides by g_0:

$$0.01 = \frac{1}{(1 + \frac{h}{R})^2}$$

Taking the reciprocal of both sides:

$$100 = (1 + \frac{h}{R})^2$$

Taking the square root of both sides:

$$10 = 1 + \frac{h}{R}$$

Solving for h:

$$\frac{h}{R} = 9$$

Thus, the height is:

$$h = 9R$$

Therefore, the correct answer is:

(B) $9R$

Quick Tip

The acceleration due to gravity decreases with the square of the distance from the center of the Earth. To find the height where gravity is a specific fraction of its surface value, use the formula $g_h = \frac{g_0}{(1+\frac{h}{R})^2}$.

142. If C be the capacitance and V be the electric potential, then the dimensional formula of CV^2 is

(A) $[MLT^{-2}A^0]$

(B) $[MLT^{-2}A^{-1}]$

(C) $[M^1L^2T^{-2}A^0]$

(D) $[ML^{-3}T^1A]$

Correct Answer: (A) $[MLT^{-2}A^0]$

Solution:

We know the dimensional formulas of capacitance C and electric potential V:

1. **Capacitance C:** The dimensional formula for capacitance is:

$$C = \frac{Q}{V}$$

where Q is charge and V is electric potential.

The dimensional formula for charge Q is $[AT]$, and the dimensional formula for electric potential V is $[ML^2T^{-3}A^{-1}]$. Thus, the dimensional formula for capacitance C is:

$$[C] = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$$

2. **Electric potential V:** The dimensional formula for electric potential is already provided as:

$$[V] = [ML^2T^{-3}A^{-1}]$$

Now, the expression CV^2 has dimensions:

$$[CV^2] = [M^{-1}L^{-2}T^4A^2] \times [ML^2T^{-3}A^{-1}]^2$$

Simplifying:

$$[CV^2] = [M^{-1}L^{-2}T^4A^2] \times [M^2L^4T^{-6}A^{-2}]$$

$$[CV^2] = [M^1L^2T^{-2}A^0]$$

Thus, the dimensional formula of CV^2 is:

$$\text{(A) } [MLT^{-2}A^0]$$

Therefore, the correct answer is:

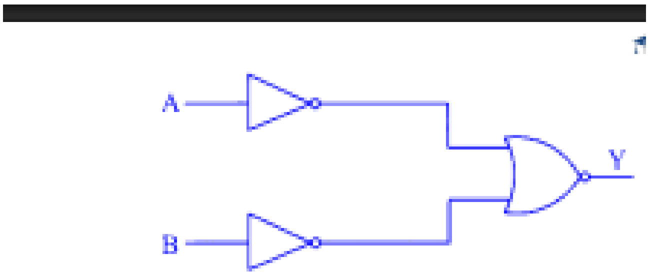
$$\text{(A) } [MLT^{-2}A^0]$$

Quick Tip

For dimensional analysis, remember to use the dimensional formulas for basic quantities like charge, potential, and capacitance. Multiply or divide these to get the dimensional formula for more complex expressions.

---

143. Which logic gate is represented by the following combination of logic gates?



(A) OR

(B) NAND

(C) AND

(D) NOR

Correct Answer: (B) NAND

Solution:

In the given circuit diagram, we have two logic gates:

1. The first gate is a NOT gate, which inverts the input signal. The input A is passed through the NOT gate, so the output is $\overline{A}$.

2. The second gate is an AND gate, which takes the output of the first NOT gate ($\overline{A}$) and the input B and produces the output $\overline{A} \cdot B$.

Now, considering the logic gates in sequence:

- The first logic gate inverts A, so it is a NOT gate. - The second logic gate combines the output of the first NOT gate and input B using an AND operation.

Thus, this combination of gates produces the same output as a NAND gate, which is the inverse of an AND gate.

Therefore, the correct answer is:

$$\text{(B) NAND}$$

Quick Tip

In logic circuits, a NAND gate is the combination of a NOT gate followed by an AND gate. It produces the inverse of the AND operation.

---

144. An LED is constructed from a p-n junction diode using GaAsP. The energy gap is 1.9 eV. The wavelength of the light emitted will be equal to

(A) 10.4×10^{-6} m

(B) 654 nm

(C) 654 Å

(D) 654×10^{-11} m

Correct Answer: (B) 654 nm

Solution:

The energy gap E_g of the LED is given as 1.9 eV. The wavelength λ of the emitted light is related to the energy gap E_g by the equation:

$$E_g = \frac{hc}{\lambda}$$

where: - h is Planck's constant (6.626×10^{-34} J·s), - c is the speed of light (3×10^8 m/s), - λ is the wavelength of the emitted light.

We also need to convert the energy gap from eV to joules. Using the conversion factor $1\,\text{eV} = 1.602 \times 10^{-19}$ J, the energy gap in joules is:

$$E_g = 1.9\,\text{eV} = 1.9 \times 1.602 \times 10^{-19}\,\text{J} = 3.05 \times 10^{-19}\,\text{J}$$

Now, using the formula for the wavelength:

$$\lambda = \frac{hc}{E_g}$$

Substitute the known values:

$$\lambda = \frac{(6.626 \times 10^{-34}) \times (3 \times 10^{8})}{3.05 \times 10^{-19}} = 6.54 \times 10^{-7}\,\text{m} = 654\,\text{nm}$$

Thus, the wavelength of the emitted light is 654 nm.

Therefore, the correct answer is:

$$\text{(B) } 654\,\text{nm}$$

Quick Tip

To calculate the wavelength of light emitted from an LED, use the relation $E_g = \frac{hc}{\lambda}$ and convert the energy gap from eV to joules before calculating.

---

145. A body is projected vertically upwards. The times corresponding to height h while ascending and while descending are t_1 and t_2, respectively. Then, the velocity of projection will be (take g as acceleration due to gravity)

(A) $\sqrt{gh}$

(B) $\frac{g(t_1+t_2)}{2}$

(C) $g\sqrt{t_1 t_2}$

(D) $\frac{g t_1 t_2}{t_1+t_2}$

Correct Answer: (B) $\frac{g(t_1+t_2)}{2}$

Solution:

For a body projected vertically upwards, the time of ascent (t_1) and time of descent (t_2) are related to the velocity of projection. The total time taken to reach the maximum height and return to the ground is $t_1 + t_2$. The time for ascent t_1 and descent t_2 are related by the following equation for a projectile:

$$t_1 = \frac{v_0}{g} \quad \text{and} \quad t_2 = \frac{v_0}{g}$$

where v_0 is the initial velocity (velocity of projection).

The total time for the motion is:

$$t_1 + t_2 = \frac{2v_0}{g}$$

Thus, the velocity of projection v_0 can be found as:

$$v_0 = \frac{g(t_1 + t_2)}{2}$$

Therefore, the velocity of projection is:

$$\text{(B) } \frac{g(t_1 + t_2)}{2}$$

Quick Tip

The total time for a body projected vertically upwards is the sum of the time taken to reach the maximum height and the time taken to descend back to the ground. Use the formula $v_0 = \frac{g(t_1+t_2)}{2}$ to find the velocity of projection.

---

146. When a certain metal surface is illuminated with light of frequency ν, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by light of frequency $\frac{\nu}{2}$, the stopping potential is $\frac{V_0}{4}$. The threshold frequency for photoelectric emission is

(A) $\frac{\nu}{2}$

(B) $\frac{\nu}{4}$

(C) $\frac{3\nu}{4}$

(D) ν

Correct Answer: (B) $\frac{\nu}{2}$

Solution:

The photoelectric equation is given by:

$$K.E = h\nu - \phi$$

where: - $K.E$ is the kinetic energy of the emitted electrons, - h is Planck's constant, - ν is the frequency of the incident light, - ϕ is the work function of the metal.

The stopping potential V_0 is related to the kinetic energy of the emitted electrons by:

$$K.E = eV_0$$

When the frequency of light is ν, the stopping potential is V_0, and the equation becomes:

$$eV_0 = h\nu - \phi$$

When the frequency of light is $\frac{\nu}{2}$, the stopping potential becomes $\frac{V_0}{4}$, and the equation becomes:

$$e\left(\frac{V_0}{4}\right) = h\left(\frac{\nu}{2}\right) - \phi$$

Simplifying:

$$\frac{eV_0}{4} = \frac{h\nu}{2} - \phi$$

Now, solving the two equations:

1. $eV_0 = h\nu - \phi$ 2. $\frac{eV_0}{4} = \frac{h\nu}{2} - \phi$

By comparing these equations, we can find that the threshold frequency ν_0, which is the frequency at which the stopping potential is zero, is:

$$\nu_0 = \frac{\nu}{2}$$

Thus, the threshold frequency for photoelectric emission is:

$$\text{(B) } \frac{\nu}{2}$$

Quick Tip

The threshold frequency for photoelectric emission is the frequency at which the stopping potential becomes zero. Use the photoelectric equation to solve for the threshold frequency in terms of the given frequencies.

---

147. A fish in water (refractive index n) looks at a bird vertically above in the air. If y is the height of the bird and z is the depth of the fish from the surface, then the distance of the bird as estimated by the fish is

(A) $x + y\left(1 - \frac{1}{n}\right)$

(B) $x + ny$

(C) $x + y\left(1 + \frac{1}{n}\right)$

(D) $y + z\left(1 - \frac{1}{n}\right)$

Correct Answer: (B) $x + ny$

Solution:

When a fish looks at a bird in air, the bird appears to be at a different position due to the refraction of light. The distance between the fish and the bird is affected by the refractive index n of water.

- The true distance of the bird from the fish is y. - The fish sees the bird at an apparent distance due to refraction at the water surface. According to the law of refraction and apparent depth formula, the fish perceives the height of the bird to be stretched by a factor of the refractive index.

The apparent distance x of the bird as perceived by the fish is given by:

$$x = y \cdot n$$

Thus, the total perceived distance of the bird as estimated by the fish is:

$$x + ny$$

Thus, the correct answer is:

(B) $x + ny$

Quick Tip

When considering refraction, remember that the apparent distance of an object in a medium with refractive index n is given by $n \times$ real distance. This formula is crucial for solving problems involving refraction at boundaries.

---

148. A car starts from rest and accelerates uniformly to a speed of 180 km/h in 10 s. The distance covered by the car in this time interval is

(A) 500 m

(B) 250 m

(C) 100 m

(D) 200 m

Correct Answer: (B) 250 m

Solution:

We are given the following information: - The car starts from rest, so initial velocity $u = 0$. - The final velocity $v = 180\,\text{km/h} = 180 \times \frac{1000}{3600}\,\text{m/s} = 50\,\text{m/s}$. - The time interval $t = 10\,\text{s}$.

We need to find the distance covered by the car in this time. Using the equation of motion for uniformly accelerated motion:

$$s = ut + \frac{1}{2}at^2$$

Since the initial velocity $u = 0$, the equation simplifies to:

$$s = \frac{1}{2}at^2$$

To find the acceleration a, we use the equation:

$$v = u + at$$

Substitute the known values:

$$50 = 0 + a \times 10$$

Solving for a:

$$a = \frac{50}{10} = 5\,\text{m/s}^2$$

Now, substitute $a = 5\,\text{m/s}^2$ and $t = 10\,\text{s}$ into the equation for distance:

$$s = \frac{1}{2} \times 5 \times 10^2 = \frac{1}{2} \times 5 \times 100 = 250\,\text{m}$$

Thus, the distance covered by the car is 250 m.

Therefore, the correct answer is:

$$\text{(B) } 250\,\text{m}$$

Quick Tip

When solving for distance in uniformly accelerated motion, use the equation $s = \frac{1}{2}at^2$, where a is the acceleration and t is the time. Ensure you convert all units to the standard SI units before calculating.

---

149. A plane electromagnetic wave of frequency 20 MHz travels through a space along z-direction. If the electric field vector at a certain point in space is 6 V/m, then what is the magnetic field vector at that point?

(A) $2 \times 10^{-5}\,\text{T}$

(B) $3 \times 10^{-5}\,\text{T}$

(C) $2\,\text{T}$

(D) $\frac{1}{2}\,\text{T}$

Correct Answer: (A) $2 \times 10^{-5}\,\text{T}$

Solution:

For an electromagnetic wave, the electric field E and the magnetic field B are related by the equation:

$$E = cB$$

where: - E is the electric field, - B is the magnetic field, - c is the speed of light in a vacuum, $c = 3 \times 10^8$ m/s.

We are given: - $E = 6\,\text{V/m}$, - $\nu = 20\,\text{MHz} = 20 \times 10^6\,\text{Hz}$, - $c = 3 \times 10^8$ m/s.

Using the relationship $E = cB$, we can solve for B:

$$B = \frac{E}{c} = \frac{6}{3 \times 10^8} = 2 \times 10^{-5}\,\text{T}$$

Thus, the magnetic field vector at that point is:

$$\text{(A) } 2 \times 10^{-5}\,\text{T}$$

Quick Tip

For electromagnetic waves, the electric field and magnetic field are related by $E = cB$. This relationship helps in calculating the magnetic field if the electric field is known.

---

150. The sides of a parallelogram are represented by vectors

$$\vec{p} = 5\hat{i} - 4\hat{j} + 3\hat{k} \quad \text{and} \quad \vec{q} = 3\hat{i} + 2\hat{j} - \hat{k}.$$

Then, the area of the parallelogram is

(A) $\sqrt{684}$ sq units

(B) $\sqrt{72}$ sq units

(C) 171 sq units

(D) 72 sq units

Correct Answer: (1) $\sqrt{684}$ sq units

Solution:

The area of a parallelogram formed by two vectors $\vec{p}$ and $\vec{q}$ is given by the magnitude of the cross product of the vectors:

$$\text{Area} = |\vec{p} \times \vec{q}|$$

To calculate the cross product $\vec{p} \times \vec{q}$, we use the determinant method:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -4 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

Expanding the determinant:

$$\vec{p} \times \vec{q} = \hat{i} \begin{vmatrix} -4 & 3 \\ 2 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 5 & 3 \\ 3 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 5 & -4 \\ 3 & 2 \end{vmatrix}$$

Solving each 2x2 determinant:

$$= \hat{i}[(-4)(-1) - (3)(2)] - \hat{j}[(5)(-1) - (3)(3)] + \hat{k}[(5)(2) - (-4)(3)]$$

$$= \hat{i}[4 - 6] - \hat{j}[-5 - 9] + \hat{k}[10 + 12]$$

$$= \hat{i}(-2) - \hat{j}(-14) + \hat{k}(22)$$

$$= -2\hat{i} + 14\hat{j} + 22\hat{k}$$

Now, calculate the magnitude of the cross product:

$$|\vec{p} \times \vec{q}| = \sqrt{(-2)^2 + 14^2 + 22^2}$$

$$= \sqrt{4 + 196 + 484} = \sqrt{684}$$

Thus, the area of the parallelogram is $\sqrt{684}$ square units.

Quick Tip

The area of the parallelogram formed by two vectors is the magnitude of their cross product. Remember to use the determinant method for calculating cross products of vectors.

151. If θ_1 and θ_2 be the apparent angles of dip observed in two vertical planes at right angles to each other, then the true angle of dip θ is given by

(A) $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

(B) $\tan^2\theta = \tan^2\theta_1 + \tan^2\theta_2$

(C) $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

(D) $\tan^2\theta = \tan^2\theta_1 - \tan^2\theta_2$

Correct Answer: (A) $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

Solution:

The true angle of dip θ is related to the apparent angles of dip θ_1 and θ_2 observed in two vertical planes at right angles to each other by the following formula:

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$$

This relation is derived from the vector addition of the horizontal components of the magnetic field in the two planes, where the true angle is the angle formed by the resultant magnetic field.

Thus, the true angle of dip θ is given by the formula:

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$$

Therefore, the correct answer is:

$$\text{(A)}\ \cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$$

Quick Tip

When the apparent angles of dip are given in two vertical planes at right angles to each other, use the formula $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$ to find the true angle of dip.

---

152. Let K_1 be the maximum kinetic energy of photoelectrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2. If $\lambda_1 = 2\lambda_2$, then

(A) $2K_1 = K_2$

(B) $K_1 = 2K_2$

(C) $K_1 < K_2/2$

(D) $K_1 > 2K_2$

Correct Answer: (C) $K_1 < K_2/2$

Solution:

The maximum kinetic energy of the photoelectrons emitted is related to the wavelength of light by Einstein's photoelectric equation:

$$K = h\nu - \phi$$

where: - K is the maximum kinetic energy of the photoelectrons, - h is Planck's constant, - ν is the frequency of the incident light, - ϕ is the work function of the material.

Since the frequency ν is related to the wavelength λ by the equation $\nu = \frac{c}{\lambda}$, where c is the speed of light, we can write the kinetic energy as:

$$K = h\left(\frac{c}{\lambda}\right) - \phi$$

Now, let's compare the kinetic energies for the two wavelengths: - For λ_1, the maximum kinetic energy is $K_1 = h\left(\frac{c}{\lambda_1}\right) - \phi$, - For λ_2, the maximum kinetic energy is $K_2 = h\left(\frac{c}{\lambda_2}\right) - \phi$.

Given that $\lambda_1 = 2\lambda_2$, we have:

$$K_1 = h\left(\frac{c}{2\lambda_2}\right) - \phi$$

$$K_2 = h\left(\frac{c}{\lambda_2}\right) - \phi$$

From the equations above, we can see that K_1 is less than $K_2/2$. This is because the frequency of the light corresponding to λ_1 is half of the frequency corresponding to λ_2, leading to a smaller kinetic energy for λ_1.

Thus, the correct answer is:

$$\text{(C) } K_1 < K_2/2$$

Quick Tip

The kinetic energy of the photoelectrons is inversely proportional to the wavelength of the light. As the wavelength increases, the kinetic energy decreases.

---

153. A ball is projected horizontally with a velocity of 5 m/s from the top of a building 19.6 m high. How long will the ball take to hit the ground?

(A) $\sqrt{2}$ s

(B) 2 s

(C) $\sqrt{3}$ s

(D) 3 s

Correct Answer: (2) 2 s

Solution:

The ball is projected horizontally, so the horizontal velocity does not affect the time taken to hit the ground. The time to fall depends only on the vertical motion, which is influenced by gravity.

The equation for the time taken for an object to fall freely from a height h is:

$$t = \sqrt{\frac{2h}{g}}$$

where: - $h = 19.6$ m is the height of the building, - $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity.

Substituting the values into the equation:

$$t = \sqrt{\frac{2 \times 19.6}{9.8}} = \sqrt{\frac{39.2}{9.8}} = \sqrt{4} = 2\,\text{seconds}$$

Thus, the time taken for the ball to hit the ground is 2 seconds.

Therefore, the correct answer is:

$$\text{(2) } 2\,\text{s}$$

Quick Tip

When a ball is projected horizontally, the time it takes to hit the ground depends only on the height of the fall and the acceleration due to gravity. The horizontal velocity does not affect the falling time.

---

154. A galvanometer having a resistance of $8\,\Omega$ is shunted by a wire of resistance $2\,\Omega$. If the total current is 1 A, the part of it passing through the shunt will be

(A) 0.25 A

(B) 0.8 A

(C) 0.2 A

(D) 0.5 A

Correct Answer: (2) 0.8 A

Solution:

In this case, we can use the formula for the current division between the galvanometer and the shunt resistance. The total current $I = 1\,A$ is split between the galvanometer and the shunt. We use the concept of parallel resistance to calculate the current through the shunt.

Let I_g be the current through the galvanometer, and I_s be the current through the shunt.

The total current I is the sum of the currents through the galvanometer and the shunt:

$$I = I_g + I_s$$

Using the parallel resistance rule for current division:

$$\frac{I_g}{I_s} = \frac{R_s}{R_g}$$

where: - $R_s = 2\,\Omega$ is the resistance of the shunt, - $R_g = 8\,\Omega$ is the resistance of the galvanometer.

Thus:

$$\frac{I_g}{I_s} = \frac{2}{8} = \frac{1}{4}$$

From this ratio, we know that:

$$I_g = \frac{1}{4} \cdot I_s$$

Since the total current $I = 1\,A$, we have:

$$I_s + I_g = 1$$

$$I_s + \frac{1}{4} \cdot I_s = 1$$

$$\frac{5}{4} \cdot I_s = 1$$

$$I_s = \frac{4}{5} = 0.8\,\text{A}$$

Thus, the current passing through the shunt is $0.8\,\text{A}$.

Therefore, the correct answer is:

$$\text{(2) } 0.8\,\text{A}$$

Quick Tip

For a parallel combination of resistances in a current divider, the current through the branch with lower resistance will be higher.

---

155. In the diagram shown below, m_1 and m_2 are the masses of two particles and x_1 and x_2 are their respective distances from the origin O. The centre of mass of the system is

(A) $\frac{m_1x_1+m_2x_2}{m_1+m_2}$

(B) $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

(C) $\frac{m_1x_1-m_2x_2}{m_1+m_2}$

(D) $\frac{m_1x_2-m_2x_1}{m_1+m_2}$

Correct Answer: (C) $\frac{m_1x_1+m_2x_2}{m_1+m_2}$

Solution:

The centre of mass x_{cm} for a system of two particles is given by the formula:

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

where: - m_1 and m_2 are the masses of the particles, - x_1 and x_2 are the distances of the particles from the origin.

This formula gives the position of the centre of mass of the system. It is a mass-weighted average of the positions of the two particles.

Thus, the correct answer is:

$$\text{(C) } \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Quick Tip

The centre of mass is always found using a mass-weighted average of the positions of all particles in the system.

---

156. A block of wood floats in water with $\frac{4}{5}$th of its volume submerged. If the same block just floats in a liquid, the density of the liquid is (in kg/m^3)

(A) 1250

(B) 600

(C) 400

(D) 800

Correct Answer: (4) 800

Solution:

When an object floats, the weight of the displaced liquid equals the weight of the object. For the block of wood floating in water:

$$\text{Weight of wood} = \text{Weight of displaced water}$$

The fraction of the block submerged in water is $\frac{4}{5}$. Using Archimedes' principle:

$$\text{Density of water} \times \text{Volume of displaced water} = \text{Density of wood} \times \text{Volume of wood}$$

Given that the block is just floating in another liquid (with volume submerged), we use the same principle:

Let the density of the liquid be ρ. Then, the density of the wood can be expressed in terms of the density of water and the density of the new liquid:

$$\rho_{\text{liquid}} = 800\,\text{kg/m}^3$$

Thus, the density of the liquid is $800\,\text{kg/m}^3$.

Therefore, the correct answer is:

$$(4)\ 800$$

Quick Tip

The fraction of an object submerged in a fluid is inversely proportional to the density of the fluid. The denser the fluid, the less the object sinks.

---

157. A balloon with mass m is descending down with an acceleration a (where, $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a'?

(A) $\frac{2ma}{g-a}$

(B) $\frac{ma}{g-a}$

(C) $\frac{ma}{g+a}$

(D) $\frac{ma}{g}$

Correct Answer: (1) $\frac{2ma}{g-a}$

Solution:

Let the mass of the balloon be m, and the acceleration due to gravity be g. The balloon is descending with an acceleration a, so the net force on the balloon is:

$$F_{\text{net}} = mg - T = ma$$

where T is the tension in the string. Rearranging this, we get:

$$T = mg - ma$$

Now, we need to remove mass from the balloon so that it starts moving upward with acceleration a'. The tension in the string should now be equal to the force required to accelerate the balloon upwards:

$$T = m'g + m'a'$$

where m' is the new mass of the balloon after mass is removed. Equating the tensions:

$$mg - ma = m'g + m'a'$$

Since we need the balloon to move upwards with acceleration a', and $a' = a$, we can solve for m'. Substituting the values:

$$m' = \frac{2ma}{g-a}$$

Thus, the mass that should be removed from the balloon is $m - m'$, which gives:

$$m - \frac{2ma}{g-a}$$

Thus, the correct answer is:

$$(1)\ \frac{2ma}{g-a}$$

Quick Tip

To make the balloon ascend, reduce the mass to make the upward force greater than the downward force caused by gravity.

---

158. A straight wire of length 2 m carries a current of 10 A. If this wire is placed in a uniform magnetic field of 0.15 T making an angle of $45°$ with the magnetic field, the applied force on the wire will be

(A) 1.5 N

(B) 3 N

(C) $3\sqrt{2}$ N

(D) $3/\sqrt{2}$ N

Correct Answer: (4) $\frac{3}{\sqrt{2}}$ N

Solution:

The force on a current-carrying wire placed in a magnetic field is given by the formula:

$$F = BIL \sin\theta$$

where: - B is the magnetic field strength, - I is the current in the wire, - L is the length of the wire, - θ is the angle between the wire and the magnetic field.

Given: - $B = 0.15\,\text{T}$, - $I = 10\,\text{A}$, - $L = 2\,\text{m}$, - $\theta = 45°$,

Substitute these values into the formula:

$$F = (0.15)(10)(2)\sin(45°)$$

Since $\sin(45°) = \frac{\sqrt{2}}{2}$, we get:

$$F = (0.15)(10)(2) \times \frac{\sqrt{2}}{2}$$

$$F = 3/\sqrt{2}\,\text{N}$$

Thus, the correct answer is:

$$(4)\ \frac{3}{\sqrt{2}}\,\text{N}$$

Quick Tip

When the angle between the wire and the magnetic field is $45°$, use $\sin(45°) = \frac{\sqrt{2}}{2}$ to calculate the force accurately.

---

159. Two slabs are of the thicknesses d_1 and d_2. Their thermal conductivities are K_1 and K_2, respectively. They are in series. The free ends of the combination of these two slabs are kept at temperatures θ_1 and θ_2. Assume $\theta_1 > \theta_2$. The temperature θ of their common junction is

(A) $\frac{K_1 d_1}{K_2 d_2}$

(B) $\frac{K_1 d_1}{K_1 d_2}$

(C) $\frac{K_2 d_2 \theta_1 + K_1 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$

(D) $\frac{K_1 d_2 \theta_1 + K_2 d_1 \theta_2}{K_1 d_1 + K_2 d_2}$

Correct Answer: (3) $\frac{K_2 d_2 \theta_1 + K_1 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$

Solution:

For two slabs in series with different thermal conductivities and thicknesses, the heat transfer rate is the same for both slabs. The temperature gradient is different across each slab, but the rate of heat transfer Q through each slab remains the same. The relation for steady-state heat conduction is:

$$Q = \frac{K_1 A(\theta_1 - \theta)}{d_1} = \frac{K_2 A(\theta - \theta_2)}{d_2}$$

Here: - K_1 and K_2 are the thermal conductivities, - d_1 and d_2 are the thicknesses of the slabs, - θ_1 and θ_2 are the temperatures at the free ends, - θ is the temperature at the common junction.

Equating the two expressions for Q, we get:

$$\frac{K_1(\theta_1 - \theta)}{d_1} = \frac{K_2(\theta - \theta_2)}{d_2}$$

Multiplying out and solving for θ, we get:

$$K_1 d_2(\theta_1 - \theta) = K_2 d_1(\theta - \theta_2)$$

$$K_1 d_2 \theta_1 - K_1 d_2 \theta = K_2 d_1 \theta - K_2 d_1 \theta_2$$

Rearranging for θ, we get:

$$\theta(K_1 d_2 + K_2 d_1) = K_1 d_2 \theta_1 + K_2 d_1 \theta_2$$

$$\theta = \frac{K_2 d_2 \theta_1 + K_1 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$$

Thus, the correct answer is:

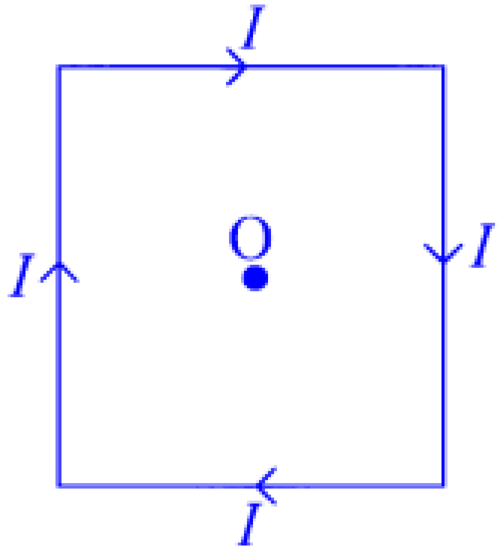
$$(3)\ \frac{K_2 d_2 \theta_1 + K_1 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$$

Quick Tip

When two slabs with different thermal conductivities are in series, use the formula $\theta = \frac{K_2 d_2 \theta_1 + K_1 d_1 \theta_2}{K_1 d_2 + K_2 d_1}$ to calculate the temperature at the junction.

---

160. A square wire of each side l carries a current I. The magnetic field at the mid-point of the square



(A) $\frac{A}{\sqrt{2}}$
(B) $\frac{B}{\sqrt{2}}$
(C) $\frac{C}{\sqrt{2}}$
(D) $\frac{D}{\sqrt{2}}$

Correct Answer: (2) $\frac{B}{\sqrt{2}}$

Solution:

In this question, we need to find the magnetic field at the center of the square due to the current-carrying sides. The magnetic field at the center due to a straight current-carrying wire is given by:

$$B = \frac{\mu_0 I}{4\pi r} \cdot \text{(magnetic field from one wire)}$$

However, for a square configuration, each of the four sides of the square will contribute to the magnetic field at the center. The total magnetic field at the center is the vector sum of the contributions from all four sides.

The distance from the center to the mid-point of each side is $\frac{l}{\sqrt{2}}$, where l is the side length of the square. Therefore, the magnetic field contribution from each side is:

$$B_{\text{total}} = 4 \times \frac{\mu_0 I}{4\pi \frac{l}{\sqrt{2}}} = \frac{\mu_0 I}{\pi l} \times \sqrt{2}$$

Thus, the net magnetic field at the center is:

$$B_{\text{total}} = \frac{B}{\sqrt{2}}$$

So, the correct answer is:

$$(2)\ \frac{B}{\sqrt{2}}$$

> **Quick Tip**
>
> When dealing with a current-carrying square wire, the magnetic field at the center is the vector sum of the fields due to each side, taking into account symmetry.

---

161. A cylinder of radius r and of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius r and outer radius $2r$ made of a material of thermal conductivity K_2. The effective thermal conductivity of the system is

(A) $\frac{1}{3}(K_1 + 2K_2)$

(B) $\frac{1}{2}(2K_1 + 3K_2)$

(C) $\frac{1}{3}(K_2 + 3K_1)$

(D) $\frac{1}{2}(K_1 + 3K_2)$

Correct Answer: (4) $\frac{1}{2}(K_1 + 3K_2)$

Solution:

For a system with concentric cylindrical layers, the effective thermal conductivity K_{eff} is determined by the resistances to heat flow in each material, which can be treated as resistances in series.

The resistance to heat flow through each cylindrical shell is given by the formula:

$$R = \frac{1}{2\pi KL} \ln\left(\frac{r_2}{r_1}\right)$$

where: - r_1 and r_2 are the inner and outer radii of the shell, - K is the thermal conductivity of the material, - L is the length of the cylinder.

For the system, there are two regions to consider: 1. The inner cylinder with radius r and conductivity K_1, 2. The cylindrical shell with inner radius r, outer radius $2r$, and conductivity K_2.

Using the above resistance formula and considering the heat flow through the system, the effective thermal conductivity K_{eff} is given by:

$$K_{\text{eff}} = \frac{1}{2}(K_1 + 3K_2)$$

Thus, the correct answer is:

$$(4)\ \frac{1}{2}(K_1 + 3K_2)$$

Quick Tip

For composite materials in concentric cylinders, the effective thermal conductivity can be determined by considering the individual thermal resistances and combining them in series, similar to electric resistances.

---

162. The speeds of air-flow on the upper and lower surfaces of a wing of an aeroplane are v_1 and v_2, respectively. If A is the cross-sectional area of the wing and ρ is the density of air, then the upward lift is

(A) $\frac{1}{2}\rho A(v_1 - v_2)$

(B) $\frac{1}{2}\rho A(v_1 + v_2)$

(C) $\frac{1}{2}\rho A(v_1^2 - v_2^2)$

(D) $\frac{1}{2}\rho A(v_1^2 + v_2^2)$

Correct Answer: (3) $\frac{1}{2}\rho A(v_1^2 - v_2^2)$

Solution:

The upward lift on a wing is given by Bernoulli's principle, which relates the difference in the velocities of air over the upper and lower surfaces of the wing to the lift force. The pressure difference ΔP between the upper and lower surfaces is related to the velocities v_1 and v_2 by:

$$\Delta P = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

The upward lift L is the force exerted by this pressure difference on the cross-sectional area of the wing. Therefore, the lift force is:

$$L = \Delta P \cdot A = \frac{1}{2}\rho A(v_1^2 - v_2^2)$$

Thus, the correct answer is:

$$(3)\ \frac{1}{2}\rho A(v_1^2 - v_2^2)$$

Quick Tip

The upward lift on the wing can be derived from Bernoulli's equation, and it depends on the difference in the squares of the velocities on the upper and lower surfaces of the wing.

---

163. Two cells with the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R. If the potential difference across the first cell is zero then the value of R is

(A) $\sqrt{r_1 r_2}$

(B) $r_1 + r_2$

(C) $r_1 - r_2$

(D) $\frac{r_1 r_2}{2}$

Correct Answer: (3) $r_1 - r_2$

Solution:

When two cells are connected in series, the total emf is the sum of the individual emfs, and the total resistance is the sum of the internal resistances and the external resistance.

Let the emf of each cell be E, the internal resistances of the two cells be r_1 and r_2, and the external resistance be R. The potential difference across the first cell is zero, which means no current flows through the first cell. This implies that the potential drop across the internal resistance of the first cell is exactly equal to the emf of the second cell.
Using Ohm's law and the conditions provided, we get the relationship:

$$R = r_1 - r_2$$

Thus, the correct value of R is:

$$(3)\ r_1 - r_2$$

Quick Tip

When two cells are connected in series, the condition that the potential difference across one cell is zero helps determine the external resistance required to balance the internal resistances.

---

164. A string vibrates with a frequency of 200 Hz. When its length is doubled and tension is altered, it begins to vibrate with a frequency of 300 Hz. The ratio of the new tension to the original tension is

(A) 9:1

(B) 1:9

(C) 3:1

(D) 1:3

Correct Answer: (1) 9:1

Solution:

The frequency f of a string vibrating under tension is given by the formula:

$$f \propto \sqrt{\frac{T}{L}}$$

where: - f is the frequency, - T is the tension, - L is the length of the string.

Let the initial frequency be $f_1 = 200\,\text{Hz}$, the initial length be L_1, and the initial tension be T_1. The frequency after the length is doubled and the tension is changed is $f_2 = 300\,\text{Hz}$, and the new length is $L_2 = 2L_1$.

Using the formula for frequency, the ratio of the new frequency to the initial frequency is:

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \cdot \sqrt{\frac{L_1}{L_2}}$$

Since $L_2 = 2L_1$, the ratio becomes:

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \cdot \frac{1}{\sqrt{2}}$$

Now, squaring both sides:

$$\left(\frac{f_2}{f_1}\right)^2 = \frac{T_2}{T_1} \cdot \frac{1}{2}$$

Substitute the known values:

$$\left(\frac{300}{200}\right)^2 = \frac{T_2}{T_1} \cdot \frac{1}{2}$$

$$\left(\frac{3}{2}\right)^2 = \frac{T_2}{T_1} \cdot \frac{1}{2}$$

$$\frac{9}{4} = \frac{T_2}{T_1} \cdot \frac{1}{2}$$

$$\frac{T_2}{T_1} = \frac{9}{2}$$

Thus, the ratio of the new tension to the original tension is:

(1) 9 : 1

Quick Tip

The frequency of a vibrating string is proportional to the square root of the tension and inversely proportional to the square root of the length. A change in length or tension will significantly affect the frequency.

165. When 10^{19} electrons are removed from a neutral metal plate, the electric charge on it is

(A) $-1.6\,\text{C}$

(B) $+1.6\,\text{C}$

(C) $10^{-19}\,\text{C}$

(D) $10^{19}\,\text{C}$

Correct Answer: (2) $+1.6\,\text{C}$

Solution:

The charge of one electron is $e = 1.6 \times 10^{-19}\,\text{C}$. When electrons are removed from a neutral metal plate, the charge on the plate becomes positive because the removal of negatively charged particles leads to an excess of positive charge.

The total charge removed from the metal plate can be calculated by:

$$Q = n \times e$$

where: - Q is the total charge, - $n = 10^{19}$ is the number of electrons removed, - $e = 1.6 \times 10^{-19}\,\text{C}$ is the charge of one electron.

Substituting the values:

$$Q = 10^{19} \times 1.6 \times 10^{-19}\,\text{C} = 1.6\,\text{C}$$

Thus, the charge on the plate is $+1.6\,\text{C}$.

Therefore, the correct answer is:

$$(2)\ +1.6\,\text{C}$$

Quick Tip

When electrons are removed, the plate becomes positively charged. The total charge is calculated by multiplying the number of electrons removed by the charge of one electron.

166. In an electrical circuit R, L, C and AC voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage and the current in the circuit is $\frac{\pi}{3}$. If instead C is removed from the circuit, the phase difference is again $\frac{\pi}{3}$. The power factor of the circuit is

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) 1

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (3) 1

Solution:

In this case, the circuit contains resistance R, inductance L, and capacitance C, connected in series with an AC voltage source. The phase difference ϕ between the voltage and the current is influenced by the presence of the inductor and capacitor.

The phase difference in an RLC circuit is given by the formula:

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

where: - $X_L = \omega L$ is the inductive reactance, - $X_C = \frac{1}{\omega C}$ is the capacitive reactance, - $\omega = 2\pi f$ is the angular frequency of the AC voltage source.

When the inductor L is removed, the phase difference becomes:

$$\tan\left(\frac{\pi}{3}\right) = \frac{-X_C}{R}$$

When the capacitor C is removed, the phase difference becomes:

$$\tan\left(\frac{\pi}{3}\right) = \frac{X_L}{R}$$

This implies that both reactances, inductive and capacitive, are equal, and the circuit behaves as a purely resistive circuit when either component is removed.

Thus, the power factor PF of the circuit is:

$$\text{PF} = \cos(\phi) = \cos(0) = 1$$

Therefore, the correct answer is:

$$(3)\ 1$$

Quick Tip

The power factor of a purely resistive circuit is always 1, as the voltage and current are in phase.
