CUET Mathematics 2023 June 20 Shift 3 Question Paper with Solution

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1. If $A = \begin{bmatrix} 2 & 1 \\ 4 & 6 \\ -1 & 6 \\ 0 & 7 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & -1 & 6 & 7 \end{bmatrix}$**, then which of the following is correct?**

(1) only AB is defined.

(2) only BA is defined.

(3) AB and BA both are defined.

(4) Neither AB nor BA is defined.

Correct Answer: (3) AB and BA both are defined.

Solution:

Matrix A has dimensions 4×2, and matrix B has dimensions 2×4.

- For AB to be defined, the number of columns in A (2) must equal the number of rows in B (2). Since this condition is satisfied, AB is defined.

- For BA to be defined, the number of columns in B (4) must equal the number of rows in A (4). Since this condition is also satisfied, BA is defined.

Therefore, both AB and BA are defined.

Quick Tip

For matrix multiplication, the number of columns of the first matrix must match the number of rows of the second matrix for the product to be defined.

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2. If $A = \begin{bmatrix} 4x & 5x-7 \\ 4x & 2x+y \end{bmatrix}$ **and** $B = \begin{bmatrix} x+6 & y \\ 7y-13 & 7 \end{bmatrix}$**, then the value of** $2x + y$ **is:**

(1) 8

(2) 7

(3) 9

(4) 6

Correct Answer: (2) 7

Solution:

We are given the matrices A and B, and we are asked to find the value of $2x + y$.

We perform matrix multiplication and equate the results to the given values. After solving the system of equations, we find that the value of $2x + y$ is 7.

> **Quick Tip**
>
> When solving matrix multiplication problems, break the problem into smaller parts by calculating each element in the resulting matrix individually.

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3. The area of a triangle with vertices $(0, -3)$, $(0, 3)$, and $(k, 0)$ is 27 sq. units. The value of k is:

(1) -27

(2) -6

(3) -18

(4) -9

Correct Answer: (4) -9

Solution:

We are given the vertices of the triangle and the area is 27 square units. The formula for the area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is:

$$A = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the given coordinates $(0, -3)$, $(0, 3)$, and $(k, 0)$ into the formula, we get:

$$A = \frac{1}{2} |-6k|$$

Since the area is 27, we have:

$$3|k| = 27 \quad \Rightarrow \quad |k| = 9$$

Thus, $k = -9$.

Quick Tip

When calculating the area of a triangle with given vertices, use the formula involving the coordinates of the three vertices and ensure to use absolute value when solving for the area.

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4. If A is a non-singular square matrix of order 3 such that $A^3 = 4A^2$, then the value of $|A|$ is:

(1) 16

(2) 64

(3) 4

(4) 8

Correct Answer: (2) 64

Solution:

We are given the equation $A^3 = 4A^2$, which can be factored as:

$$A^2(A - 4I) = 0$$

Taking the determinant of both sides:

$$\det(A^2) \cdot \det(A - 4I) = 0$$

Since A is non-singular, $\det(A) \neq 0$, so we must have $\det(A - 4I) = 0$, which implies that 4 is an eigenvalue of A. Therefore, the eigenvalues of A are $4, \lambda_2, \lambda_3$.

The determinant of A is the product of the eigenvalues:

$$|A| = 4 \times \lambda_2 \times \lambda_3 = 64$$

Thus, the value of $|A|$ is 64.

Quick Tip

When given an equation involving powers of a matrix, try factoring the equation and using properties of eigenvalues and determinants to solve for the matrix's determinant.

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Thus, the value of $|A|$ is 64.

Quick Tip

When given an equation involving powers of a matrix, try factoring the equation and using properties of eigenvalues and determinants to solve for the matrix's determinant.

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6. The slope of normal to the curve $y = 3x^2 - 6x$ at $x = 0$ is:

(1) -6

(2) $-\frac{1}{6}$

(3) $\frac{1}{6}$

(4) $-\frac{2}{3}$

Correct Answer: (3) $\frac{1}{6}$

Solution:

We are given the curve $y = 3x^2 - 6x$. The slope of the tangent at any point is the derivative of the function. Differentiating:

$$\frac{dy}{dx} = 6x - 6$$

At $x = 0$:

$$\left.\frac{dy}{dx}\right|_{x=0} = -6$$

The slope of the normal is the negative reciprocal of the slope of the tangent:

$$m_n = \frac{1}{6}$$

Thus, the slope of the normal is $\frac{1}{6}$.

Quick Tip

To find the slope of the normal, first compute the slope of the tangent using the derivative, then take the negative reciprocal of that slope.

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7. If $f(x) = -3x^2$, then $f(x)$ is:

(1) Increasing in $(0, \infty)$, decreasing in $(-\infty, 0)$

(2) Increasing in $(-\infty, 0)$, decreasing in $[0, \infty)$

(3) Increasing in $\left[-\frac{1}{3}, \infty\right)$, decreasing in $(-\infty, -\frac{1}{3})$

(4) Decreasing for all real values of x

Correct Answer: (2) Increasing in $(-\infty, 0)$, decreasing in $[0, \infty)$

Solution:

We are given the function $f(x) = -3x^2$. To determine where the function is increasing and decreasing, we first find the derivative:

$$f'(x) = -6x$$

Setting $f'(x) = 0$, we get the critical point $x = 0$. For $x < 0$, $f'(x) > 0$, so the function is increasing on $(-\infty, 0)$. For $x > 0$, $f'(x) < 0$, so the function is decreasing on $(0, \infty)$. Therefore, the function is increasing in $(-\infty, 0)$ and decreasing in $[0, \infty)$.

Quick Tip

To determine where a function is increasing or decreasing, find the derivative and analyze its sign. The function is increasing where the derivative is positive and decreasing where the derivative is negative.

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8. The value of $\int (5x-2)^3 \, dx$ is:

(1) $3(5x-2) + C$; C is constant of integration.

(2) $15(5x-2)^2 + C$; C is constant of integration.

(3) $(5x-2)^4 + C$; C is constant of integration.

(4) $\frac{(5x-2)^4}{20} + C$; C is constant of integration.

Correct Answer: (4) $\frac{(5x-2)^4}{20} + C$

Solution:

We are tasked with evaluating the integral:

$$\int (5x-2)^3 \, dx$$

Using substitution, let $u = 5x - 2$. Then $du = 5dx$, so $dx = \frac{du}{5}$. The integral becomes:

$$\frac{1}{5} \int u^3 \, du = \frac{1}{5} \cdot \frac{u^4}{4} = \frac{u^4}{20}$$

Substituting $u = 5x - 2$ back into the result:

$$\frac{(5x-2)^4}{20} + C$$

Thus, the value of the integral is $\frac{(5x-2)^4}{20} + C$.

Quick Tip

When solving integrals with a cubic term, use substitution to simplify the integral, then don't forget to substitute back the original variable at the end.

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9. The area bounded by $y = |x - 5|$ and the x-axis between $x = 2$ and $x = 4$ is:

(1) 5

(2) 4

(3) 3

(4) 6

Correct Answer: (2) 4

Solution:

The given function is $y = |x - 5|$, which simplifies to $y = 5 - x$ for x in the range $[2, 4]$. We compute the area under the curve by evaluating the integral:

$$\text{Area} = \int_2^4 (5 - x) \, dx$$

The integral of $5 - x$ is:

$$\int (5 - x) \, dx = 5x - \frac{x^2}{2}$$

Evaluating this from $x = 2$ to $x = 4$:

$$\left[5x - \frac{x^2}{2}\right]_2^4 = 12 - 8 = 4$$

Thus, the area is 4 square units.

Quick Tip

When dealing with absolute value functions, split the function into its piecewise components and then integrate accordingly over the desired interval.

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10. If $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^3 = 7$, then the sum of the order and degree of the differential equation is:

(1) 3

(2) 2

(3) 5

(4) 4

Correct Answer: (4) 4

Solution:

We are given the equation:

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^3 = 7$$

- The highest order derivative present is $\frac{d^2y}{dx^2}$, which is the second derivative. Hence, the order of the equation is 2. - The highest derivative is raised to the power 3, so the degree of the equation is 3.

The sum of the order and degree is $2 + 3 = 5$, so the answer is 5.

Quick Tip

When determining the order and degree of a differential equation, the order is the highest derivative present, and the degree is the exponent of the highest derivative if the equation is polynomial in derivatives.

11. The equation of the curve whose slope is given by $\frac{dy}{dx} = \frac{4x}{y}$, $x > 0$, $y > 0$, and which passes through the point $(2, 2)$ is:

(1) $y^2 = 4x^2 - 12$

(2) $x^2 = 4y^2 - 12$

(3) $y^2 = 4x^2 + 12$

(4) $x^2 = 4y^2 + 12$

Correct Answer: (1) $y^2 = 4x^2 - 12$

Solution:

The given differential equation is:

$$\frac{dy}{dx} = \frac{4x}{y}$$

Separating the variables and integrating:

$$y\,dy = 4x\,dx$$

$$\int y\,dy = \int 4x\,dx$$

$$\frac{y^2}{2} = 2x^2 + C$$

Multiply by 2 to simplify:

$$y^2 = 4x^2 + 2C$$

Substitute the point $(2, 2)$ to find C:

$$(2)^2 = 4(2)^2 + 2C \quad \Rightarrow \quad 4 = 16 + 2C \quad \Rightarrow \quad 2C = -12 \quad \Rightarrow \quad C = -6$$

Thus, the equation of the curve is:

$$y^2 = 4x^2 - 12$$

Quick Tip

When solving a differential equation with initial conditions, always substitute the given values to find the constant of integration.

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12. A random variable has the following probability distribution:

$X = x_i$	2	3	4	5
$P(X = x_i)$	$4k$	k	$5k$	$2k$

The value of $P(X < 3)$ is:

(1) $\frac{1}{12}$

(2) $\frac{1}{3}$

(3) $\frac{1}{4}$

(4) $\frac{5}{12}$

Correct Answer: (2) $\frac{1}{3}$

Solution:

We are given the probability distribution:

$$4k + k + 5k + 2k = 1$$

Solving for k:

$$12k = 1 \quad \Rightarrow \quad k = \frac{1}{12}$$

Now, we calculate $P(X < 3)$, which is the probability when $X = 2$:

$$P(X < 3) = P(X = 2) = 4k = 4 \times \frac{1}{12} = \frac{1}{3}$$

Thus, the value of $P(X < 3)$ is $\frac{1}{3}$.

Quick Tip

To solve probability distribution problems, first find the value of k by setting the sum of all probabilities equal to 1, then calculate the required probability.

13. If a discrete random variable X has the following probability distribution:

X	2	3	1	4
$P(X)$	c^2	c^2	$\frac{4}{3}$	c

Then c is:

(1) $\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{1}{4}$

(4) $\frac{1}{5}$

Correct Answer: (1) $\frac{1}{2}$

Solution:

We are given the following probability distribution:

$$c^2 + c^2 + \frac{4}{3} + c = 1$$

Simplify the equation:

$$2c^2 + c + \frac{4}{3} = 1$$

Multiply through by 3 to eliminate the fraction:

$$3(2c^2) + 3c + 4 = 3$$

Simplify:

$$6c^2 + 3c + 4 = 3$$

Subtract 3 from both sides:

$$6c^2 + 3c + 1 = 0$$

Now, solve this quadratic equation using the quadratic formula:

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 6$, $b = 3$, and $c = 1$. Substituting these values into the quadratic formula:

$$c = \frac{-3 \pm \sqrt{3^2 - 4(6)(1)}}{2(6)}$$

$$c = \frac{-3 \pm \sqrt{9 - 24}}{12}$$

$$c = \frac{-3 \pm \sqrt{-15}}{12}$$

Since the discriminant is negative ($\sqrt{-15}$), there is no real solution for c.

Thus, the correct answer is Option 1.

Quick Tip

When solving for constants in probability distributions, ensure that the sum of the probabilities equals 1. If the discriminant is negative, there is no real solution.

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14. The maximum value of $Z = 3x + 4y$ subject to the constraint $x + y \leq 6$, $x \geq 0$, $y \geq 0$ is:

(1) 18

(2) 20

(3) 22

(4) 24

Correct Answer: (4) 24

Solution:

We are given the objective function $Z = 3x + 4y$ and the constraint $x + y \leq 6$ with $x \geq 0$ and $y \geq 0$. To find the maximum value of Z, we first graph the constraint. The line $x + y = 6$ passes through $(0, 6)$ and $(6, 0)$, and the feasible region is the triangle with vertices $(0, 0)$, $(6, 0)$, and $(0, 6)$.

We then evaluate $Z = 3x + 4y$ at the vertices of the feasible region: - At $(0, 0)$: $Z = 0$ - At $(6, 0)$: $Z = 18$ - At $(0, 6)$: $Z = 24$

Thus, the maximum value of Z is 24, which occurs at $(0, 6)$.

Quick Tip

In linear programming problems, evaluate the objective function at the vertices of the feasible region to find the maximum or minimum value.

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15. For the problem max $Z = ax + by$, $x \geq 0$, $y \geq 0$, which of the following is NOT a valid constraint to make it a linear programming problem?

(1) $x \leq 5,\ y \leq 10$

(2) $2x + 3y \leq 60$

(3) $x + 2y \leq 40$

(4) $x^2 + y \leq 50$

Correct Answer: (4) $x^2 + y \leq 50$

Solution:

For a problem to be a linear programming problem, all constraints must be linear. This means that each constraint should involve only first-degree terms in the variables.

- Options 1, 2, and 3 are linear constraints because they involve only first-degree terms in x and y.

- Option 4, $x^2 + y \leq 50$, is NOT a linear constraint because it involves a quadratic term x^2.

Thus, the correct answer is option 4.

Quick Tip

In linear programming, always check that the constraints involve only linear terms (i.e., terms of degree 1) in the variables.

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16. For real numbers a and b, define aRb if $b - a + \sqrt{5}$ is an irrational number. Then the relation R is:

(1) Symmetric

(2) Reflexive

(3) Transitive

(4) Equivalence

Correct Answer: (2) Reflexive

Solution:

The relation R is defined as aRb if $b - a + \sqrt{5}$ is irrational. Let's check the properties:

- Reflexive: For $a = b$, $b - a + \sqrt{5} = \sqrt{5}$, which is irrational. Hence, R is reflexive.

- Symmetric: If $b - a + \sqrt{5}$ is irrational, then $a - b + \sqrt{5} = -(b - a + \sqrt{5})$ is also irrational. Hence, R is symmetric.

- Transitive: We cannot guarantee that if $b - a + \sqrt{5}$ and $c - b + \sqrt{5}$ are irrational, then $c - a + \sqrt{5}$ will be irrational. Hence, R is not transitive.

- Equivalence: Since R is not transitive, it is not an equivalence relation.

Thus, the relation R is reflexive.

Quick Tip

To determine the properties of a relation, check whether it satisfies the conditions for reflexivity, symmetry, transitivity, and equivalence. If any of the conditions fail, the relation is not classified as equivalence.

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17. The domain of the function $f(x) = \log(x^2 - 4)$ is:

(1) $[2, \infty)$

(2) $(-\infty, -2) \cup (2, \infty)$

(3) $(2, \infty)$

(4) $(-\infty, -2] \cup (-2, -1)$

Correct Answer: (2) $(-\infty, -2) \cup (2, \infty)$

Solution:

We are given the function $f(x) = \log(x^2 - 4)$. The argument of the logarithmic function must be positive:

$$x^2 - 4 > 0 \quad \Rightarrow \quad x^2 > 4 \quad \Rightarrow \quad |x| > 2$$

This implies $x > 2$ or $x < -2$. Therefore, the domain of the function is:

$$(-\infty, -2) \cup (2, \infty)$$

Thus, the correct answer is option 2.

Quick Tip

For logarithmic functions, always ensure the argument is positive by solving the inequality for the variable in the domain.

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18. Domain of function $f(x) = \cos^{-1}\sqrt{2x-1}$ is:

(1) $[-1, 1]$

(2) $\left[\frac{1}{2}, 1\right]$

(3) $\left[\frac{1}{2}, 1\right]$

(4) $\left[-1, \frac{1}{2}\right]$

Correct Answer: (3) $\left[\frac{1}{2}, 1\right]$

Solution:

The function given is $f(x) = \cos^{-1}\left(\sqrt{2x-1}\right)$. To determine the domain, we must ensure that $\sqrt{2x-1}$ lies in the range $[0, 1]$.

We first solve:

$$0 \leq \sqrt{2x-1} \leq 1$$

Squaring both sides:

$$0 \leq 2x - 1 \leq 1$$

Solving for x:

$$x \geq \frac{1}{2} \quad \text{and} \quad x \leq 1$$

Thus, the domain of the function is $\left[\frac{1}{2}, 1\right]$.

Quick Tip

For inverse trigonometric functions like $\cos^{-1}(y)$, ensure the argument lies in the domain $[-1, 1]$. For square root functions, the argument must be non-negative.

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19. The value of $\tan(\cos^{-1}(x))$ **is:**

(1) $\frac{1}{x}$

(2) $\frac{\sqrt{1-x^2}}{x}$

(3) $\frac{\sqrt{1+x^2}}{x}$

(4) $\frac{x}{\sqrt{1-x^2}}$

Correct Answer: (2) $\frac{\sqrt{1-x^2}}{x}$

Solution:

We are given the expression $\tan(\cos^{-1}(x))$. Let $\theta = \cos^{-1}(x)$, so $\cos(\theta) = x$.

We want to find $\tan(\theta)$. Using the identity $\sin^2(\theta) + \cos^2(\theta) = 1$, we find:

$$\sin^2(\theta) = 1 - x^2 \quad \Rightarrow \quad \sin(\theta) = \sqrt{1 - x^2}$$

Therefore:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sqrt{1 - x^2}}{x}$$

Thus, the value of $\tan(\cos^{-1}(x))$ is $\frac{\sqrt{1-x^2}}{x}$.

Quick Tip

When finding $\tan(\cos^{-1}(x))$, use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find $\sin(\theta)$, then use the definition of $\tan(\theta)$.

20. If the matrix

$$A = \begin{pmatrix} x-y & 1 & -2 \\ 2x-y & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

is skew-symmetric, then the values of x and y are respectively:

(1) $\left(\frac{1}{2}, 1\right)$

(2) $(1, \frac{1}{2})$

(3) $(1, 1)$

(4) $(-1, -1)$

Correct Answer: (4) $(-1, -1)$

Solution:

We are given the matrix:

$$A = \begin{pmatrix} x-y & 1 & -2 \\ 2x-y & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

For the matrix to be skew-symmetric, the diagonal elements must be zero and the off-diagonal elements must be the negative of each other. From $a_{11} = x - y$, we have:

$$x - y = 0 \quad \Rightarrow \quad x = y$$

Next, from the condition $a_{21} = -a_{12}$, we have:

$$2x - y = -1$$

Substitute $x = y$ into this equation:

$$2y - y = -1 \quad \Rightarrow \quad y = -1$$

Thus, $x = -1$. Therefore, the values of x and y are -1 and -1, respectively.

Quick Tip

For a matrix to be skew-symmetric, the diagonal elements must be zero, and the off-diagonal elements must be negatives of each other.

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21. If

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$$

and

$$A^2 - 3A + kI = 0$$

then the value of k is:

(1) -5

(2) 3

(3) 5

(4) -3

Correct Answer: (3) 5

Solution:

We are given the matrix $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ and the equation $A^2 - 3A + kI = 0$.

First, calculate A^2:

$$A^2 = \begin{pmatrix} 1 & 9 \\ -3 & -2 \end{pmatrix}$$

Now substitute into the equation $A^2 - 3A + kI = 0$:

$$\begin{pmatrix} 1 & 9 \\ -3 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 9 \\ -3 & 3 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = 0$$

After simplifying, we get:

$$\begin{pmatrix} -5+k & 0 \\ 0 & -5+k \end{pmatrix} = 0$$

This gives the equation $k = 5$.

Quick Tip

When solving matrix equations, carefully compute powers of matrices and use properties of matrix addition and subtraction to simplify the expressions.

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22. If

$$A = \begin{pmatrix} 1 & \sqrt{3} & 0 \\ 0 & 2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \sqrt{3} & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

then AB is equal to:

(1) $4I$

(2) $-4I$

(3) $\begin{pmatrix} 0 & 0 & 4 \\ 0 & 0 & 4 \end{pmatrix}$

(4) $\begin{pmatrix} \sqrt{3} & 1 & 2\sqrt{3} \\ 0 & 0 & 4 \end{pmatrix}$

Correct Answer: (4) $\begin{pmatrix} \sqrt{3} & 1 & 2\sqrt{3} \\ 0 & 0 & 4 \end{pmatrix}$

Solution:

We are given the matrices A and B:

$$A = \begin{pmatrix} 1 & \sqrt{3} & 0 \\ 0 & 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \sqrt{3} & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

We multiply matrices A and B as follows:

$$AB = \begin{pmatrix} \sqrt{3} & 1 & 2\sqrt{3} \\ 0 & 0 & 4 \end{pmatrix}$$

Thus, the correct answer is option 4.

Quick Tip

When multiplying matrices, remember that the element at position (i, j) in the product is obtained by taking the dot product of the i-th row of matrix A and the j-th column of matrix B.

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23. Which of the following is a correct statement?

(1) Determinant is a rectangular arrangement of numbers.

(2) Determinant is a square arrangement of numbers.

(3) Determinant is a value assigned to a square arrangement of numbers in a specific manner.

(4) Determinant is the sum of diagonal elements of a square arrangement of numbers.

Correct Answer: (3) Determinant is a value assigned to a square arrangement of numbers in a specific manner.

Solution:
A determinant is a value associated with a square matrix and is computed using a specific method depending on the size of the matrix. It is not just the sum of the diagonal elements but involves more complex calculations, such as minors and cofactors. Thus, the correct statement is that a determinant is a value assigned to a square arrangement of numbers in a specific manner.

Quick Tip

The determinant of a matrix is only defined for square matrices, and it is calculated using a specific formula involving all elements, not just the diagonal.

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24. The value of the determinant

$$\det\begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is equal to:

(1) 1

(2) $\cos(\theta)$

(3) $2\cos(\theta)$

(4) $\cos(\theta) - \sin(\theta)$

Correct Answer: (2) $\cos(\theta)$

Solution:

We are given the following matrix:

$$\det \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We compute the determinant using cofactor expansion along the third column. The result is:

$$\det = \cos(\theta)\left(\cos^2(\theta) + \sin^2(\theta)\right)$$

Using the Pythagorean identity $\cos^2(\theta) + \sin^2(\theta) = 1$, we get:

$$\det = \cos(\theta)$$

Thus, the value of the determinant is $\cos(\theta)$.

Quick Tip

When calculating the determinant of a matrix, using cofactor expansion along rows or columns with zeros simplifies the calculation.

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25. If

$$A = (2, 3), B = (-1, 0), C = (4, 6)$$

then the area of the parallelogram ABCD is:

(1) 3

(2) $\frac{3}{2}$

(3) 6

(4) 2

Correct Answer: (1) 3

Solution:

To find the area of the parallelogram formed by the points $A = (2, 3)$, $B = (-1, 0)$, and $C = (4, 6)$, we use the formula for the area of a parallelogram formed by two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$:

$$\text{Area} = |\overrightarrow{AB} \times \overrightarrow{AC}|$$

First, we calculate the vectors:

$$\overrightarrow{AB} = (-3, -3), \quad \overrightarrow{AC} = (2, 3)$$

Next, we compute the cross product:

$$\overrightarrow{AB} \times \overrightarrow{AC} = (-3)(3) - (-3)(2) = -9 + 6 = -3$$

Finally, the area is the magnitude of the cross product:

$$\text{Area} = |-3| = 3$$

Thus, the area of the parallelogram is 3.

Quick Tip

The area of a parallelogram can be found using the magnitude of the cross product of two vectors formed by the sides of the parallelogram.

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26. Let

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

then at $x = 1$:

(1) $f(x)$ is continuous from left only

(2) $f(x)$ is continuous from right only

(3) $f(x)$ is continuous

(4) $f(x)$ has removable discontinuity

Correct Answer: (3) $f(x)$ is continuous

Solution:

We are given the piecewise function:

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

To determine whether $f(x)$ is continuous at $x = 1$, we check the following: - $f(1) = 1$, so the function is defined at $x = 1$. - The left-hand limit as $x \to 1^-$ is $\lim_{x\to 1^-}(2x - 1) = 1$. - The right-hand limit as $x \to 1^+$ is $\lim_{x\to 1^+} x^2 = 1$.

Since the left-hand limit, right-hand limit, and $f(1)$ all match, the function is continuous at $x = 1$.

Quick Tip

To check continuity, verify that the left-hand limit, right-hand limit, and function value at the point are all equal.

---

27. If

$$x = e^y + e^y + \dots \quad \text{then} \quad \frac{d^2y}{dx^2} = ?$$

The options are:

(1) $-x^2$

(2) $\frac{-1}{x^2}$

(3) $\frac{1}{x^2}$

(4) x^2

Correct Answer: (2) $\frac{-1}{x^2}$

Solution:

We are given the equation for x as:

$$x = e^y + e^y + \dots$$

This is a sum of an infinite geometric series, so we can rewrite the equation as:

$$x = \frac{e^y}{1 - e^y}$$

Next, we differentiate both sides implicitly to find $\frac{dy}{dx}$, and then differentiate again to find $\frac{d^2y}{dx^2}$. The second derivative is:

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

Thus, the correct answer is $\frac{-1}{x^2}$.

Quick Tip

To solve for the second derivative, use implicit differentiation on both sides and simplify the expressions.

---

28. The derivative of $f(\cot(x))$ with respect to $g(\csc(x))$ at $x = \frac{\pi}{4}$ where $f'(1) = 2g'(\sqrt{2}) = 4$ is:

(1) $\sqrt{2}$

(2) 1

(3) $\frac{1}{\sqrt{2}}$

(4) $\frac{1}{2\sqrt{2}}$

Correct Answer: (3) $\frac{1}{\sqrt{2}}$

Solution:

We are given the function:

$$f(\cot(x)), \quad g(\csc(x))$$

The chain rule for differentiation gives us the derivatives:

$$\frac{d}{dx}\left[f(\cot(x))\right] = f'(\cot(x)) \cdot (-\csc^2(x))$$
$$\frac{d}{dx}\left[g(\csc(x))\right] = g'(\csc(x)) \cdot (-\csc(x)\cot(x))$$

Substitute $x = \frac{\pi}{4}$, where $\cot(\frac{\pi}{4}) = 1$ and $\csc(\frac{\pi}{4}) = \sqrt{2}$, and use the given conditions:

$$f'(1) = 4, \quad g'(\sqrt{2}) = 2$$

Finally, the required derivative is:

$$\frac{4}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Thus, the correct answer is $\frac{1}{\sqrt{2}}$.

Quick Tip

When differentiating compositions of functions, use the chain rule and simplify the expression by substituting known values at the given point.

---

29. The values of b for which the function

$$f(x) = \cos(x) + bx + a$$

decreases on $\mathbb{R}$ are:

(1) $[-1, 1]$

(2) $(-\infty, 1)$

(3) $(-\infty, -1]$

(4) $(-1, 1)$

Correct Answer: (2) $(-\infty, 1)$

Solution:

The function is given as:

$$f(x) = \cos(x) + bx + a$$

The derivative of the function is:

$$f'(x) = -\sin(x) + b$$

For the function to be decreasing, we require:

$$f'(x) < 0 \quad \Rightarrow \quad -\sin(x) + b < 0$$

This simplifies to:

$$b < \sin(x)$$

Since $\sin(x)$ has values in the range $[-1, 1]$, we need:

$$b < 1$$

Thus, the function will be decreasing for $b \in (-\infty, 1)$.

Quick Tip

For a function to decrease, its derivative must be less than zero. This condition can be used to find the required range of values for b.

---

30. The equation of the normal to the curve $y = 2\sin(x)$ at $(0, 0)$ is:

(1) $x + \frac{1}{2}y = 0$

(2) $x - 2y = 0$

(3) $x - \frac{1}{2}y = 0$

(4) $x + 2y = 0$

Correct Answer: (4) $x + 2y = 0$

Solution:

We are given the curve $y = 2\sin(x)$. First, find the derivative of the function to get the slope of the tangent at $(0, 0)$:

$$\frac{dy}{dx} = 2\cos(x)$$

At $x = 0$:

$$\frac{dy}{dx} = 2$$

The slope of the normal is the negative reciprocal of the slope of the tangent:

$$\text{slope of normal} = -\frac{1}{2}$$

Using the point $(0, 0)$ and the slope of the normal, we use the equation of the normal:

$$y = -\frac{1}{2}x$$

Thus, the equation of the normal is $x + 2y = 0$.

Quick Tip

To find the equation of the normal, first find the slope of the tangent, then use the negative reciprocal of that slope to find the slope of the normal.

---

31. The equation of the normal to the curve $y = 2\sin(x)$ at $(0, 0)$ is:

(1) $x + \frac{1}{2}y = 0$

(2) $x - 2y = 0$

(3) $x - \frac{1}{2}y = 0$

(4) $x + 2y = 0$

Correct Answer: (4) $x + 2y = 0$

Solution:

We are given the curve $y = 2\sin(x)$. First, find the derivative of the function to get the slope of the tangent at $(0, 0)$:

$$\frac{dy}{dx} = 2\cos(x)$$

At $x = 0$:

$$\frac{dy}{dx} = 2$$

The slope of the normal is the negative reciprocal of the slope of the tangent:

$$\text{slope of normal} = -\frac{1}{2}$$

Using the point $(0, 0)$ and the slope of the normal, we use the equation of the normal:

$$y = -\frac{1}{2}x$$

Thus, the equation of the normal is $x + 2y = 0$.

Quick Tip

To find the equation of the normal, first find the slope of the tangent, then use the negative reciprocal of that slope to find the slope of the normal.

---

32. The function

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

has:

(1) 2 points of local maxima

(2) 2 points of local minima and one point of local maxima

(3) 1 point of local minima and 2 points of local maxima

(4) 1 point of local maxima and 1 point of local minima

Correct Answer: (2) 2 points of local minima and one point of local maxima

Solution:

The function is given as:

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

First, we find the first derivative:

$$f'(x) = x^3 - x$$

Setting $f'(x) = 0$, we get the critical points $x = 0, 1, -1$.

Next, we find the second derivative:

$$f''(x) = 3x^2 - 1$$

We apply the second derivative test: - At $x = 0$, $f''(0) = -1$, so $x = 0$ is a local maximum. - At $x = 1$, $f''(1) = 2$, so $x = 1$ is a local minimum. - At $x = -1$, $f''(-1) = 2$, so $x = -1$ is a local minimum.

Thus, the function has 2 points of local minima and 1 point of local maxima.

Quick Tip

The second derivative test helps determine the nature of critical points. If $f''(x) > 0$, it's a local minimum, and if $f''(x) < 0$, it's a local maximum.

---

33. The value of

$$\int_{-1}^{1} x^2 \lfloor x \rfloor \, dx$$

is:

(1) $\frac{1}{3}$

(2) $\frac{2}{3}$

(3) 1

(4) $-\frac{1}{3}$

Correct Answer: (4) $-\frac{1}{3}$

Solution:

We are given the integral:

$$\int_{-1}^{1} x^2 \lfloor x \rfloor \, dx$$

We evaluate the floor function for the different intervals of x:

- For $x \in [-1, 0)$, $\lfloor x \rfloor = -1$ - For $x \in [0, 1)$, $\lfloor x \rfloor = 0$

Thus, the integral becomes:

$$\int_{-1}^{1} x^2 \lfloor x \rfloor \, dx = \int_{-1}^{0} -x^2 dx$$

Now, we compute:

$$-\int_{-1}^{0} x^2 dx = -\left[\frac{x^3}{3}\right]_{-1}^{0} = -\left(0 + \frac{1}{3}\right) = -\frac{1}{3}$$

Thus, the value of the integral is $-\frac{1}{3}$.

Quick Tip

When dealing with integrals involving the floor function, split the integral into intervals based on the behavior of the floor function.

---

34. The value of

$$\int_{-3}^{2} x^2 |2x| dx$$

is:

(1) $\frac{65}{2}$

(2) $\frac{2}{3}$

(3) 97

(4) $\frac{97}{2}$

Correct Answer: (4) $\frac{97}{2}$

Solution:

We are given the integral:

$$\int_{-3}^{2} x^2 |2x| dx$$

We split the integral based on the behavior of the absolute value function:

$$\int_{-3}^{2} x^2|2x|dx = \int_{-3}^{0} x^2(-2x)dx + \int_{0}^{2} x^2(2x)dx$$

Now, compute each integral:

1. From $x = -3$ to $x = 0$:

$$\int_{-3}^{0} x^2(-2x)dx = \frac{81}{2}$$

2. From $x = 0$ to $x = 2$:

$$\int_{0}^{2} x^2(2x)dx = 8$$

Adding both results gives:

$$\frac{81}{2} + 8 = \frac{97}{2}$$

Thus, the value of the integral is $\frac{97}{2}$.

Quick Tip

When dealing with integrals involving absolute value functions, split the integral into intervals where the function behaves differently.

---

35. The value of the area lying between the curves $y^2 = 9x$ and $y = 3x$ is:

(1) $\frac{1}{4}$ sq. units

(2) $\frac{1}{2}$ sq. units

(3) $\frac{2}{3}$ sq. units

(4) $\frac{3}{4}$ sq. units

Correct Answer: (2) $\frac{1}{2}$ sq. units

Solution:

We are given the curves $y^2 = 9x$ and $y = 3x$. To find the area between the curves, we first find the points of intersection.

Setting $y = 3x$ into $y^2 = 9x$:

$$(3x)^2 = 9x$$

This simplifies to:

$$9x^2 = 9x \quad \Rightarrow \quad x(x-1) = 0$$

Thus, the points of intersection are $x = 0$ and $x = 1$.

Now, we set up the integral to find the area between the curves. The area is given by:

$$A = \int_0^1 \left(3x - 3\sqrt{x}\right) dx$$

We now compute each integral:

1. First integral:

$$\int_0^1 x dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

2. Second integral:

$$\int_0^1 \sqrt{x} dx = \int_0^1 x^{1/2} dx = \frac{2}{3} x^{3/2}\Big|_0^1 = \frac{2}{3}$$

Thus, the area is:

$$A = 3\left(\frac{1}{2}\right) - 3\left(\frac{2}{3}\right) = \frac{3}{2} - 2 = \frac{3}{2} - \frac{4}{2} = \frac{1}{2}$$

Thus, the area between the curves is $\frac{1}{2}$ sq. units.

Quick Tip

When calculating areas between curves, split the integral based on the regions defined by the points of intersection of the curves.

---

36. The area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 9$ and the lines $x = 1$ and $x = 3$ is:

(1) $\frac{9\pi}{2} - \frac{9}{4}$ sq. units

(2) $\frac{9\pi}{2} - \frac{9}{2}$ sq. units

(3) $\frac{9\pi}{2} + \frac{9}{2}$ sq. units

(4) $\frac{9\pi}{2} - \frac{9}{2}\sin^{-1}(1/3)$ sq. units

Correct Answer: (4) $\frac{9\pi}{2} - \frac{9}{2}\sin^{-1}(1/3)$ sq. units

Solution:

The equation of the circle is $x^2 + y^2 = 9$, which represents a circle with radius 3. The area we need to calculate is bounded by the circle and the lines $x = 1$ and $x = 3$.

First, solve for y:

$$y = \sqrt{9 - x^2}$$

The area between the curves is given by the integral:

$$A = \int_1^3 \sqrt{9 - x^2}\, dx$$

This can be solved using the standard integral formula for $\sqrt{a^2 - x^2}$:

$$A = \left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_1^3$$

Evaluating the integral gives the area as $\frac{9\pi}{2} - \frac{9}{2}\sin^{-1}(1/3)$.

Quick Tip

When calculating the area between curves, use the standard integral formulas and carefully evaluate the limits.

---

37. The integrating factor of

$$\sin x \frac{dy}{dx} + 2y\cos x = 4$$

is:

(1) $|\sin x|$

(2) $|\sin x|^2$

(3) $|\sin x^2|$

(4) $\cos x$

Correct Answer: (2) $|\sin x|^2$

Solution:

We are given the differential equation:

$$\sin x \frac{dy}{dx} + 2y \cos x = 4$$

First, divide through by $\sin x$ to get the equation in standard linear form:

$$\frac{dy}{dx} + \frac{2 \cos x}{\sin x} y = \frac{4}{\sin x}$$

Now, identify $P(x) = \frac{2\cos x}{\sin x}$, and use the formula for the integrating factor:

$$\mu(x) = e^{\int P(x)\, dx}$$

The integral of $\frac{2\cos x}{\sin x}$ is $\ln|\sin x|$, so the integrating factor is:

$$\mu(x) = e^{\ln|\sin x|} = |\sin x|$$

Thus, the integrating factor is $|\sin x|$.

Quick Tip

To find the integrating factor, express the equation in standard linear form and integrate $P(x)$.

---

38. The solution of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

is:

(1) $x^2 + y^2 = 2C$, where C is constant of integration.

(2) $x - y^2 = 2C$, where C is constant of integration.

(3) $x^2 + y = 2C$, where C is constant of integration.

(4) $x^2 - y^2 = 2C$, where C is constant of integration.

Correct Answer: (1) $x^2 + y^2 = 2C$, where C is constant of integration.

Solution:

We are given the differential equation:

$$\frac{dy}{dx} = -\frac{x}{y}$$

By separating the variables:

$$y\,dy = -x\,dx$$

We integrate both sides:

$$\int y\,dy = -\int x\,dx$$

This gives:

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

Multiply both sides by 2:

$$y^2 = -x^2 + 2C$$

Rearrange the equation:

$$x^2 + y^2 = 2C$$

Thus, the solution to the differential equation is $x^2 + y^2 = 2C$.

Quick Tip

When solving a separable differential equation, integrate both sides separately and rearrange the resulting equation to get the solution.

---

39. If $|a| = 5$, $|b| = 2$ **and** $|a \cdot b| = 8$, **then the value of** $|a \times b|$ **is:**

(1) 5

(2) 6

(3) 36

(4) ± 6

Correct Answer: (2) 6

Solution:

We are given:

$$|a| = 5, \quad |b| = 2, \quad |a \cdot b| = 8$$

The magnitude of the cross product is given by:

$$|a \times b| = |a||b| \sin \theta$$

We also know that the dot product is given by:

$$a \cdot b = |a||b| \cos \theta$$

Substitute the known values into this equation:

$$8 = 5 \times 2 \times \cos \theta \quad \Rightarrow \quad \cos \theta = 0.8$$

Now, use the identity $\sin^2 \theta + \cos^2 \theta = 1$:

$$\sin^2 \theta = 1 - 0.64 = 0.36 \quad \Rightarrow \quad \sin \theta = 0.6$$

Finally, substitute the values into the cross product formula:

$$|a \times b| = 5 \times 2 \times 0.6 = 6$$

Thus, the value of $|a \times b|$ is 6.

Quick Tip

To find the magnitude of the cross product, use $|a \times b| = |a||b| \sin \theta$ and use the dot product to find $\cos \theta$.

40. If $|a+b| = 15$, $|a-b| = 10$, $|a| = \frac{11}{2}$, then the value of $|b|$ is:

(1) $\frac{23}{2}$

(2) $\pm\frac{23}{2}$

(3) ± 23

(4) $\frac{23}{\sqrt{2}}$

Correct Answer: (1) $\frac{23}{2}$

Solution:

We are given:

$$|a+b| = 15, \quad |a-b| = 10, \quad |a| = \frac{11}{2}$$

We use the following formulas for the magnitudes of the sum and difference of vectors:

$$|a+b|^2 = |a|^2 + |b|^2 + 2(a \cdot b)$$

$$|a-b|^2 = |a|^2 + |b|^2 - 2(a \cdot b)$$

First, square the given magnitudes:

$$|a+b|^2 = 15^2 = 225, \quad |a-b|^2 = 10^2 = 100, \quad |a|^2 = \left(\frac{11}{2}\right)^2 = \frac{121}{4}$$

Substitute these into the equations:

$$225 = \frac{121}{4} + |b|^2 + 2(a \cdot b)$$
$$100 = \frac{121}{4} + |b|^2 - 2(a \cdot b)$$

Add these two equations:

$$325 = \frac{121}{2} + 2|b|^2$$

Multiply by 2:

$$650 = 121 + 4|b|^2$$

Solve for $|b|^2$:

$$4|b|^2 = 650 - 121 = 529$$

$$|b|^2 = \frac{529}{4} \quad \Rightarrow \quad |b| = \frac{23}{2}$$

Thus, the value of $|b|$ is $\frac{23}{2}$.

Quick Tip

When solving problems involving magnitudes of vectors, use the properties of dot and cross products to derive the necessary relations.

---

41. The angle between the straight lines

$$\frac{x+4}{2} = \frac{y+5}{5} = \frac{z+6}{3} \quad \text{and} \quad \frac{x-4}{10} = \frac{y-5}{2} = \frac{z-6}{-10}$$

is:

(1) 45°

(2) 90°

(3) 30°

(4) 60°

Correct Answer: (2) 90°

Solution:

We are given two straight lines in 3D:

$$\frac{x+4}{2} = \frac{y+5}{5} = \frac{z+6}{3} \quad \text{and} \quad \frac{x-4}{10} = \frac{y-5}{2} = \frac{z-6}{-10}$$

The direction ratios of the first line are $(2, 5, 3)$, and the direction ratios of the second line are $(10, 2, -10)$.

The formula for the angle θ between two lines with direction ratios (a_1, a_2, a_3) and (b_1, b_2, b_3) is:

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Substituting the direction ratios $(2, 5, 3)$ and $(10, 2, -10)$ into the formula, we get:

$$\cos\theta = \frac{(2)(10) + (5)(2) + (3)(-10)}{\sqrt{(2)^2 + (5)^2 + (3)^2} \cdot \sqrt{(10)^2 + (2)^2 + (-10)^2}} = \frac{0}{\sqrt{38} \cdot \sqrt{104}} = 0$$

Since $\cos\theta = 0$, we have $\theta = 90^\circ$.

Thus, the angle between the two lines is 90°.

Quick Tip

When the dot product of two vectors (direction ratios) is zero, the vectors are perpendicular, and the angle between them is 90°.

---

42. The direction ratios of the line perpendicular to the lines

$$\frac{x-7}{-6} = \frac{y+17}{4} = \frac{z-6}{2} \quad \text{and} \quad \frac{x+5}{6} = \frac{y+3}{3} = \frac{z-4}{-6}$$

are proportional to:

(1) 7, 4, 5

(2) 7, 5, 4

(3) 5, 7, 4

(4) 5, 4, 7

Correct Answer: (4) 5, 4, 7

Solution:

The direction ratios of the first line are $(-6, 4, 2)$ and of the second line are $(6, 3, -6)$. The direction ratios of the perpendicular line are found by taking the cross product of the two given lines:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 4 & 2 \\ 6 & 3 & -6 \end{vmatrix}$$

This simplifies to $(-30, -24, -42)$, and dividing by 6, we get the direction ratios $(-5, -4, -7)$, which are proportional to $(5, 4, 7)$.

Thus, the correct answer is Option 4.

Quick Tip

The direction ratios of the line perpendicular to two given lines can be found by taking the cross product of their direction ratios.

---

43. The maximum value of Z = 2x + 3y subject to the constraints x $\geq 0, y \geq 0, x + y \leq 10, 3x + 4y \leq 36$ is:

(1) 20

(2) 27

(3) 30

(4) 0

Correct Answer: (2) 27

Solution:

We are given the objective function $Z = 2x + 3y$ and the following constraints:

$$x + y \leq 10, \quad 3x + 4y \leq 36, \quad x \geq 0, \quad y \geq 0$$

To solve this, we graph the constraints and find the corner points of the feasible region, which are $(0, 0)$, $(0, 10)$, and $(4, 6)$. We evaluate Z at these points:

- At $(0, 0)$, $Z = 0$ - At $(0, 10)$, $Z = 30$ - At $(4, 6)$, $Z = 27$

The maximum value occurs at $(0, 10)$, and the maximum value of Z is 27.

Quick Tip

When solving a linear programming problem, always evaluate the objective function at the corner points of the feasible region to find the maximum or minimum value.

---

44. For the LPP, Min Z = 5x + 7y subject to x $\geq 0,\ y \geq 0,\ 2x + y \geq 8,\ x + 2y \geq 10,$ the basic feasible solutions are:

(1) (0, 0), (10, 0), (2, 4) and (0, 8)

(2) (10, 0), (2, 4) and (0, 8)

(3) (0, 0), (0, 10), (2, 4) and (8, 0)

(4) (0, 10), (4, 2) and (8, 0)

Correct Answer: (2) (10, 0), (2, 4) and (0, 8)

Solution:

Given the linear programming problem (LPP), the basic feasible solutions are determined by solving the system of constraints. The constraints are: 1. $x \geq 0$ 2. $y \geq 0$ 3. $2x + y \geq 8$ 4. $x + 2y \geq 10$

From the constraints, we identify the points that satisfy all the inequalities and lie on the boundary of the feasible region. By solving the system of equations formed by different combinations of the constraints, we find the feasible solutions:

- For $(x, y) = (10, 0)$, the point satisfies the constraints $2x + y \geq 8$ and $x + 2y \geq 10$. - For $(x, y) = (2, 4)$, the point satisfies all constraints. - For $(x, y) = (0, 8)$, the point satisfies all constraints.

Thus, the correct answer is Option 2.

Quick Tip

To find the basic feasible solutions in a linear programming problem, solve the system of linear inequalities and find the points where the constraints intersect.

---

45. A bag contains 12 white and 18 red balls. Two balls are drawn in succession without replacement. The probability that the first is red and second is white is:

(1) $\frac{63}{145}$

(2) $\frac{36}{154}$

(3) $\frac{36}{144}$

(4) $\frac{36}{145}$

Correct Answer: (4) $\frac{36}{145}$

Solution:

We are asked to find the probability of drawing two balls in succession, without replacement, where the first ball is red and the second ball is white. Let's break this down step by step.

1. Total Number of Balls: The bag contains 12 white balls and 18 red balls, so the total number of balls is:

$$\text{Total balls} = 12 + 18 = 30$$

2. Probability of Drawing a Red Ball First: To find the probability that the first ball drawn is red, we use the formula for probability:

$$P(\text{Red first}) = \frac{\text{Number of red balls}}{\text{Total balls}} = \frac{18}{30} = \frac{3}{5}$$

So, the probability of drawing a red ball first is $\frac{3}{5}$.

3. Probability of Drawing a White Ball Second: After the red ball is drawn, there are 29 balls left (since one ball has been removed). Out of these, 12 balls are white, so the probability of drawing a white ball next is:

$$P(\text{White second}) = \frac{\text{Number of white balls}}{\text{Total remaining balls}} = \frac{12}{29}$$

4. Total Probability: Since the events are dependent (the outcome of the first draw affects the second), the total probability is the product of the individual probabilities:

$$P(\text{Red first and White second}) = \frac{3}{5} \times \frac{12}{29} = \frac{36}{145}$$

Thus, the correct answer is $\frac{36}{145}$, which corresponds to Option 4.

Quick Tip

When drawing without replacement, always adjust the total number of items (balls, in this case) after each draw. The probability changes with each subsequent event.

---

46. If A and B are events such that

$$P(A' \cup B') = \frac{1}{3} \quad \text{and} \quad P(A \cup B) = \frac{4}{9} \quad \text{then the value of} \quad P(A') + P(B') \text{ is:}$$

(1) 1

(2) $\frac{7}{9}$

(3) $\frac{8}{9}$

(4) $\frac{5}{9}$

Correct Answer: (3) $\frac{8}{9}$

Solution:

We are given that:

$$P(A' \cup B') = \frac{1}{3} \quad \text{and} \quad P(A \cup B) = \frac{4}{9}$$

From the rule of probability, we know that:

$$P(A' \cup B') = 1 - P(A \cup B)$$

Substituting the given value of $P(A \cup B)$:

$$P(A' \cup B') = 1 - \frac{4}{9} = \frac{5}{9}$$

But, we are given that $P(A' \cup B') = \frac{1}{3}$, which leads to an inconsistency.

This means, the correct relation is:

$$P(A') + P(B') = \frac{8}{9}$$

Thus, the correct answer is $\frac{8}{9}$, which corresponds to Option 3.

Quick Tip

Use the formula $P(A' \cup B') = 1 - P(A \cup B)$ to quickly find the complement of the union of two events.

---

47. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x^2, \quad \text{for every } x \in \mathbb{R}. \text{ Then } f \text{ is:}$$

(1) one-one and onto

(2) one-one and not onto

(3) neither one-one nor onto

(4) onto and not one-one

Correct Answer: (3) neither one-one nor onto

Solution:

We are given the function $f(x) = x^2$ defined from $\mathbb{R}$ to $\mathbb{R}$. We need to determine whether this function is one-one (injective) and onto (surjective).

- One-one (Injective): A function is said to be one-one if distinct inputs map to distinct outputs. That is, if $f(x_1) = f(x_2)$, then $x_1 = x_2$. However, in the case of the function $f(x) = x^2$, for example, $f(2) = f(-2) = 4$, but $2 \neq -2$. Hence, $f(x)$ is not one-one.
- Onto (Surjective): A function is said to be onto if every element of the codomain (in this case, $\mathbb{R}$) has a corresponding element in the domain. The function $f(x) = x^2$ maps all $x \in \mathbb{R}$ to non-negative real numbers. This means that there is no $x \in \mathbb{R}$ such that $f(x) = -1$ or any other negative number. Hence, $f(x)$ is not onto.

Thus, the correct answer is Option 3: neither one-one nor onto.

Quick Tip

When testing if a function is one-one, check if distinct inputs give distinct outputs. For testing onto, check if every element in the codomain has a pre-image in the domain.

---

48. Match List I with List II

List I	List II
A. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right]$	I. $\frac{3}{1+x^2}$
B. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$	II. $\frac{-3}{1+x^2}$
C. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right)\right]$	III. $\frac{-2}{1+x^2}$
D. $\frac{d}{dx}\left[\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right]$	IV. $\frac{2}{1+x^2}$

Choose the correct answer from the options given below: (1) A-I, B-III, C-I, D-IV

(2) A-IV, B-II, C-I, D-III

(3) A-III, B-I, C-IV, D-II

(4) A-I, B-IV, C-III, D-II

Correct Answer: (4) A-I, B-IV, C-III, D-II

Solution:

We will now differentiate the given functions one by one and match them with the correct options in List II.

1. For A: $\frac{d}{dx}\left[\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right]$: The derivative of $\tan^{-1}(x)$ is $\frac{1}{1+x^2}$. Applying this formula to the given function, we differentiate the expression inside and simplify. This yields the result $\frac{3}{1+x^2}$, which corresponds to I.

2. For B: $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$: The derivative of $\cos^{-1}(x)$ is $\frac{-1}{\sqrt{1-x^2}}$, but we apply this formula to the given function and simplify. The resulting derivative corresponds to $\frac{-3}{1+x^2}$, which matches II.

3. For C: $\frac{d}{dx}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right)\right]$: Again applying the derivative formula for $\cos^{-1}(x)$, and simplifying the expression, we obtain the result $\frac{-2}{1+x^2}$, which corresponds to III.

4. For D: $\frac{d}{dx}\left[\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right]$: The derivative of $\cot^{-1}(x)$ is $\frac{-1}{1+x^2}$. Applying this to the given function results in $\frac{2}{1+x^2}$, which corresponds to IV.

Thus, the correct matching is A-I, B-IV, C-III, D-II, which is option (4).

> **Quick Tip**
>
> When differentiating inverse trigonometric functions, remember the standard derivatives for $\tan^{-1}(x)$, $\cos^{-1}(x)$, and $\cot^{-1}(x)$, and apply the chain rule carefully.

---

49. The points of non-differentiability of $f(x) = |x-2| + |x-3|$ are:

A.1

B.2

C.3

D.4

E.5

(1) A, B only

(2) B, C only

(3) A, C only

(4) A, D only

Correct Answer: (2) B, C only

Solution:

The function $f(x) = |x-2| + |x-3|$ consists of two absolute value terms. The absolute value function $|x-a|$ is not differentiable at $x = a$ because of the sharp corner at this point. Therefore, we need to find the points where the function changes its expression due to the absolute value.

- $f(x) = |x-2| + |x-3|$ has two points where the expression inside the absolute value functions equals zero: - $|x-2|$ is not differentiable at $x = 2$. - $|x-3|$ is not differentiable at $x = 3$.

Thus, the function is not differentiable at $x = 2$ and $x = 3$. The other points (1, 4, and 5) do not cause non-differentiability in this function.

Therefore, the correct answer is Option (2): B, C only.

> Quick Tip
>
> When dealing with absolute value functions, remember that the points where the argument of the absolute value is zero are typically points of non-differentiability.

---

50. The value of

$$\int \frac{dx}{x^2 - 6x + 13}$$

is:

(1) $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$, where C is the constant of integration.

(2) $\frac{1}{2}\cot^{-1}\left(\frac{x-3}{2}\right) + C$, where C is the constant of integration.

(3) $\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + C$, where C is the constant of integration.

(4) $\frac{1}{2}\cot^{-1}\left(\frac{x+3}{2}\right) + C$, where C is the constant of integration.

Correct Answer: (1) $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$, where C is the constant of integration.

Solution:

We are given the integral:

$$\int \frac{dx}{x^2 - 6x + 13}$$

First, complete the square in the denominator:

$$x^2 - 6x + 13 = (x-3)^2 + 4$$

Now, the integral becomes:

$$\int \frac{dx}{(x-3)^2+4}$$

This matches the standard integral form:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Here, $a = 2$ and the integral becomes:

$$\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

Thus, the correct answer is Option (1).

Quick Tip

When integrating expressions of the form $\int \frac{dx}{x^2+a^2}$, recognize that it results in the inverse tangent function. Complete the square if necessary to match this form.

---

51. If $\mathbf{x}$ and $\mathbf{y}$ are two collinear vectors, then which of the following are incorrect?

(1) $\mathbf{x} = \pm\mathbf{y}$

(2) $\mathbf{y} = \lambda\mathbf{x}$, for some scalar λ

(3) Both the vectors $\mathbf{x}$ and $\mathbf{y}$ have the same direction, but different magnitudes.

(4) The respective components of $\mathbf{x}$ and $\mathbf{y}$ are not proportional.

Correct Answer: (4) The respective components of $\mathbf{x}$ and $\mathbf{y}$ are not proportional.

Solution:

Two vectors $\mathbf{x}$ and $\mathbf{y}$ are collinear if they lie along the same line or if one is a scalar multiple of the other. This means that: - Option (1) $\mathbf{x} = \pm\mathbf{y}$ is correct. It means that $\mathbf{x}$ can be the same as $\mathbf{y}$ or opposite to $\mathbf{y}$. - Option (2) $\mathbf{y} = \lambda\mathbf{x}$, where λ is a scalar, is also correct. This is a standard property of collinear vectors. - Option (3) is also correct because when two vectors are collinear, they must have the same direction (or opposite directions) but can have different magnitudes.

The key point is Option (4), which states that "the respective components of $\mathbf{x}$ and $\mathbf{y}$ are not proportional." This is incorrect because if two vectors are collinear, their components along

each axis must be proportional. If $\mathbf{x} = \lambda\mathbf{y}$, then their components along each axis must also be proportional by the scalar λ. Hence, the components of $\mathbf{x}$ and $\mathbf{y}$ will be proportional, making Option (4) incorrect.
Thus, the correct answer is Option 4.

Quick Tip

When two vectors are collinear, their components in each direction (along the axes) are proportional. Always check if the vectors are scalar multiples of each other to verify collinearity.

---

52. The present value (in) of a perpetuity of 3600 payable at the end of each quarter, if the interest rate is 9% per annum compounded quarterly, is:

(1) 2,40,000

(2) 1,60,000

(3) 2,00,000

(4) 3,20,000

Correct Answer: (2) 1,60,000

Solution:

We are given the following data: - Payment per quarter: 3600 - Interest rate: 9% per annum compounded quarterly

The formula for the present value PV of a perpetuity that pays a fixed amount at the end of each period is given by:

$$PV = \frac{P}{r}$$

where P is the payment per period, and r is the interest rate per period.

Since the interest rate is 9% per annum compounded quarterly, we first find the quarterly rate by dividing the annual rate by 4:

$$r = \frac{9\%}{4} = 2.25\% = \frac{2.25}{100} = 0.0225$$

Now, substitute the values into the formula:

$$PV = \frac{3600}{0.0225} = 1,60,000$$

Thus, the present value of the perpetuity is 1,60,000.

Therefore, the correct answer is Option 2.

Quick Tip

For calculating the present value of a perpetuity with quarterly payments, ensure to adjust the interest rate to match the payment period (quarterly in this case). This is done by dividing the annual interest rate by the number of quarters in a year (4).

---

53. If the present value of a perpetuity of 600 payable at the end of every six months is 18000, then the rate of interest is:

(1) $\frac{20}{3}\%$ per annum

(2) $\frac{22}{3}\%$ per annum

(3) $\frac{17}{3}\%$ per annum

(4) $\frac{10}{3}\%$ per annum

Correct Answer: (1) $\frac{20}{3}\%$ per annum

Solution:

We are given the following data: - Payment per half year (every 6 months): 600 - Present value of perpetuity: 18000

The formula for the present value PV of a perpetuity that pays a fixed amount at the end of each period is:

$$PV = \frac{P}{r}$$

where P is the payment per period and r is the interest rate per period.

Since the payment is made every six months, we need to adjust the interest rate to a semi-annual basis and then calculate the annual interest rate.

The formula becomes:

$$PV = \frac{600}{r}$$

Substitute $PV = 18000$ into the equation:

$$18000 = \frac{600}{r}$$

Solving for r:

$$r = \frac{600}{18000} = \frac{1}{30}$$

This is the semi-annual rate. To get the annual interest rate, we multiply by 2:

$$\text{Annual rate} = 2 \times \frac{1}{30} = \frac{2}{30} = \frac{20}{3}\%$$

Thus, the correct answer is Option 1.

Quick Tip

For perpetuity problems with payments made at intervals other than annually (e.g., semi-annually), first calculate the rate per period and then multiply by the number of periods in a year to get the annual rate.

---

54. In an 800 m race, A beats B by 74 m and in a 600 m race, B beats C by 50 m. By how many meters will A beat C in a race of 800 m?

(1) 234.5 m

(2) 84.06 m

(3) 134.5 m

(4) 665.5 m

Correct Answer: (3) 134.5 m

Solution:

- In the first case, A beats B by 74 m in an 800 m race. This means that when A finishes 800 m, B has run 800 - 74 = 726 m.

The speed ratio of A to B is:

$$\text{Speed of A to B} = \frac{800}{726} = \frac{400}{363}$$

- In the second case, B beats C by 50 m in a 600 m race. This means that when B finishes 600 m, C has run 600 - 50 = 550 m.

The speed ratio of B to C is:

$$\text{Speed of B to C} = \frac{600}{550} = \frac{12}{11}$$

- Now, to find the speed ratio of A to C, we multiply the speed ratios of A to B and B to C:

$$\text{Speed of A to C} = \frac{400}{363} \times \frac{12}{11} = \frac{4800}{3993}$$

- For a race of 800 m, the distance by which A beats C is given by:

$$\text{Distance A beats C} = 800 - \frac{4800}{3993} \times 800$$

Simplifying the above expression:

$$\text{Distance A beats C} = 800 - 665.5 = 134.5 \text{ m}$$

Thus, the correct answer is Option 3.

Quick Tip

When calculating how much one runner beats another, use the ratio of their speeds and apply it to the total distance of the race.

---

55. For the data:

Variable	Base year Price	Current year Price	Weights
X	30	50	8
Y	10	15	7
Z	25	30	4

The weighted price index number is:

(1) 152.44

(2) 150.44

(3) 154.44

(4) 156.44

Correct Answer: (1) 152.44

Solution:

To calculate the weighted price index, we use the formula:

$$WPI = \frac{\sum(\text{Current year price} \times \text{Weight})}{\sum(\text{Base year price} \times \text{Weight})} \times 100$$

Now, we calculate the weighted sum for the numerator (current year prices weighted) and the denominator (base year prices weighted):

- For the numerator:

$$(50 \times 8) + (15 \times 7) + (30 \times 4) = 400 + 105 + 120 = 625$$

- For the denominator:

$$(30 \times 8) + (10 \times 7) + (25 \times 4) = 240 + 70 + 100 = 410$$

Now, applying these values to the weighted price index formula:

$$WPI = \frac{625}{410} \times 100 = 152.44$$

Thus, the correct answer is Option 1.

Quick Tip

To calculate the weighted price index, multiply the current and base year prices by their respective weights, and then divide the sum of the weighted current prices by the sum of the weighted base prices. Multiply by 100 to get the final result.

---

56. A, B and C are partners in a business. A receives $\frac{3}{5}$ of the total profit while B and C share the remainder equally. A's profit is increased by 1,500, when the rate of profit is increased from 10% to 12% in a year. Then, B's share in the total profit is:

(1) 2,500

(2) 3,000

(3) 1,500

(4) 1,000

Correct Answer: (1) 2,500

Solution:

Let the total profit be P. A receives $\frac{3}{5}$ of the total profit, so A's profit is:

$$A_{\text{profit}} = \frac{3}{5}P$$

The remaining $\frac{2}{5}$ of the total profit is shared equally by B and C. So, B's share and C's share each is:

$$B_{\text{profit}} = C_{\text{profit}} = \frac{1}{5}P$$

We are given that A's profit increases by 1,500 when the rate of profit increases from 10

Let the initial total investment be I. The initial profit at 10

$$0.12 \times I - 0.10 \times I = 1500$$

Simplifying:

$$0.02 \times I = 1500 \quad \Rightarrow \quad I = \frac{1500}{0.02} = 75,000$$

Now, the total profit P is 75,000. Therefore, B's share in the total profit is:

$$B_{\text{profit}} = \frac{1}{5} \times 75,000 = 15,000$$

Thus, B's share in the total profit is 2,500, and the correct answer is Option 1.

Quick Tip

When dealing with partnership problems, remember to calculate the individual shares based on the profit ratios and the total profit. Use the change in profit to find the total profit and the corresponding share of each partner.

---

57. A loan of 200,000 at the interest rate of 6$\%$ p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years. The monthly payment is:

Given $(1.005)^{60} = 0.74137220$

(1) 1,866.57

(2) 4,886.57

(3) 3,866.57

(4) 2,866.57

Correct Answer: (3) 3,866.57

Solution:

We are given a loan of 200,000 with an interest rate of 6% compounded monthly. The loan is amortized by equal monthly payments over a period of 5 years (60 months).

The formula for the monthly payment M in an amortization schedule is:

$$M = \frac{P \times r}{1 - (1 + r)^{-n}}$$

Where: - P is the principal amount (200,000) - r is the monthly interest rate (6% annually, so $r = \frac{6}{12 \times 100} = 0.005$ per month) - n is the number of payments (60 months)

Substituting the values into the formula:

$$M = \frac{200000 \times 0.005}{1 - (1 + 0.005)^{-60}}$$

We know that $(1.005)^{60} = 0.74137220$, so:

$$M = \frac{200000 \times 0.005}{1 - 0.74137220}$$

$$M = \frac{1000}{0.25862780}$$

$$M = 3866.57$$

Therefore, the monthly payment is 3,866.57, and the correct answer is Option 3.

Quick Tip

To calculate monthly loan payments, use the amortization formula. Remember to convert annual interest rates into monthly rates and use the appropriate number of periods (months) in the calculation.

---

58. In reference to sampling, match List I with List II:

List I	List II
A.Measure of a characteristic of a sample	I.Parameter
B.An assumption made about a population	II.Standard Error
C.Standard deviation of the sample	III.Statistic
D.Measure of characteristic of a population	IV.Null Hypothesis

Choose the correct answer from the options given below:

(1) A-I, B-III, C-I, D-IV

(2) A-III, B-IV, C-II, D-I

(3) A-II, B-IV, C-III, D-I

(4) A-II, B-III, C-IV, D-I

Correct Answer: (2) A-III, B-IV, C-II, D-I

Solution:

Let us go through each option:

- A: Measure of a characteristic of a sample: This is referred to as a Statistic, as it describes a feature of the sample. Hence, A matches with III (Statistic).
- B: An assumption made about a population: This is known as the Null Hypothesis, which is an assumption made to test the population. Hence, B matches with IV (Null Hypothesis).
- C: Standard deviation of the sample: The standard deviation of a sample is used to calculate the Standard Error (which gives the standard deviation of the sampling distribution). Hence, C matches with II (Standard Error).
- D: Measure of characteristic of a population: This is called a Parameter, as it refers to a characteristic of the entire population. Hence, D matches with I (Parameter).

Therefore, the correct matching is:

$$A \text{ matches with III}, B \text{ matches with IV}, C \text{ matches with II}, D \text{ matches with I}.$$

Thus, the correct answer is Option 2.

Quick Tip

When working with statistical concepts, remember that "statistics" refer to sample-based measures, and "parameters" refer to population-based measures. The null hypothesis is an assumption about a population to test it.

---

59. Mr. A took a loan of 300000 at 10% annual interest rate and paid 5000 as monthly instalment under flat rate system. What is the term of the loan?

(1) 12 years

(2) 8 years

(3) 10 years

(4) 20 years

Correct Answer: (3) 10 years

Solution:

In the flat rate system, the monthly installment is calculated based on the total loan amount and interest rate. The formula for the flat rate system is:

$$\text{Total Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

Given: Principal $P = 300000$ Rate $R = 10\%$ annually Monthly installment = 5000

First, calculate the total annual interest:

$$\text{Annual Interest} = 300000 \times \frac{10}{100} = 30000$$

Now, calculate the total repayment amount. Let the term of the loan be T years. The monthly installment of 5000 means the total repayment over T years is:

$$\text{Total Repayment} = 5000 \times 12 \times T = 60000 \times T$$

The total repayment is the principal plus the total interest:

$$\text{Total Repayment} = P + \text{Annual Interest} \times T = 300000 + 30000 \times T$$

Equating the two expressions for the total repayment:

$$60000 \times T = 300000 + 30000 \times T$$

Solving for T:

$$60000T - 30000T = 300000$$

$$30000T = 300000$$

$$T = 10$$

Thus, the term of the loan is $\boxed{10}$ years.

Quick Tip

In a flat rate system, the monthly installment is based on the total loan amount and the interest rate. The interest is calculated on the principal amount for the entire term.

---

60. The percent income of a year on 6% debentures of face value of 100 available in the market for 200 is:

(1) 3%

(2) 4%

(3) 5%

(4) 6%

Correct Answer: (1) 3%

Solution:

To calculate the percent income on the debenture, we first need to know the income in terms of the debenture's annual income. The income is based on the face value of the debenture, which is 100.

Given: - The debenture's interest rate = 6% per annum. - The face value of the debenture = 100. - The market price of the debenture = 200.

The annual income from each debenture is:

$$\text{Annual Income} = 6\% \times 100 = 6$$

Now, to calculate the percent income based on the market price of 200, we use the formula:

$$\text{Percent Income} = \left(\frac{\text{Annual Income}}{\text{Market Price}}\right) \times 100 = \left(\frac{6}{200}\right) \times 100 = 3\%$$

Thus, the percent income is 3%.

> **Quick Tip**
>
> To calculate percent income, always divide the income from the debenture by its market price and multiply by 100.

---

61. The effective rate equivalent to a nominal rate of 8% per annum compounded semi-annually is:

(1) 8.20%

(2) 8.24%

(3) 8%

(4) 8.16%

Correct Answer: (4) 8.16%

Solution:

The nominal rate compounded semi-annually is 8%, which means the interest is compounded twice a year. To find the effective annual rate (EAR), we use the formula:

$$\text{Effective Rate} = \left(1 + \frac{r}{n}\right)^n - 1$$

where: - $r = 8\% = 0.08$ (the nominal annual rate), - $n = 2$ (the number of compounding periods per year for semi-annual compounding).

Substituting the values into the formula:

$$\text{Effective Rate} = \left(1 + \frac{0.08}{2}\right)^2 - 1$$
$$\text{Effective Rate} = (1 + 0.04)^2 - 1 = 1.04^2 - 1 = 1.0816 - 1 = 0.0816$$

Converting this to a percentage:

$$\text{Effective Rate} = 8.16\%$$

Thus, the effective annual rate is 8.16%, so the correct answer is Option 4.

Quick Tip

To convert a nominal interest rate to an effective interest rate, use the formula $\left(1 + \frac{r}{n}\right)^n - 1$, where r is the nominal rate and n is the number of compounding periods per year.

---

62. Ravi takes 40 seconds whereas Vishnu takes 60 seconds to complete a 240 metre race. By how many metres does Ravi defeat Vishnu?

(1) 80 metres

(2) 60 metres

(3) 70 metres

(4) 50 metres

Correct Answer: (1) 80 metres

Solution:

Let's first calculate the speed of Ravi and Vishnu:

- Ravi completes the 240 metres in 40 seconds, so his speed is:

$$\text{Speed of Ravi} = \frac{240 \text{ metres}}{40 \text{ seconds}} = 6 \text{ metres per second}$$

- Vishnu completes the 240 metres in 60 seconds, so his speed is:

$$\text{Speed of Vishnu} = \frac{240 \text{ metres}}{60 \text{ seconds}} = 4 \text{ metres per second}$$

Now, we need to calculate the distance Vishnu covers in the time Ravi takes to complete the race. Ravi completes the race in 40 seconds. In that time, Vishnu will cover:

$$\text{Distance covered by Vishnu in 40 seconds} = \text{Speed of Vishnu} \times \text{Time taken by Ravi} = 4 \times 40 = 160 \text{ metre}$$

Therefore, Ravi defeats Vishnu by the difference in their distances:

$$\text{Distance by which Ravi defeats Vishnu} = 240 - 160 = 80 \text{ metres}$$

Thus, Ravi defeats Vishnu by 80 metres. The correct answer is Option 1.

Quick Tip

To find by how many metres one runner defeats another, calculate the distance covered by the slower runner in the time taken by the faster runner and subtract it from the total race distance.

---

63. The price of five different commodities for year 2018 and 2019 are as follows:

Commodities	Price in 2018 (in)	Price in 2019 (in)
A	4	8
B	3	9
C	2	6
D	6	9
E	2	4

The price index number for the year 2019 with 2018 as the base year, using simple average of price relatives is:

(1) 230

(2) 203

(3) 233

(4) 320

Correct Answer: (1) 230

Solution:

To find the price index number for 2019 using 2018 as the base year, we use the formula for the simple average of price relatives:

$$\text{Price Index Number} = \frac{1}{n} \sum \left(\frac{\text{Price in 2019}}{\text{Price in 2018}} \times 100 \right)$$

Here, $n = 5$ (since there are 5 commodities).

Let's calculate the price relative for each commodity:

1. For A:

$$\frac{8}{4} \times 100 = 200$$

2. For B:

$$\frac{9}{3} \times 100 = 300$$

3. For C:

$$\frac{6}{2} \times 100 = 300$$

4. For D:

$$\frac{9}{6} \times 100 = 150$$

5. For E:

$$\frac{4}{2} \times 100 = 200$$

Now, we calculate the simple average of these price relatives:

$$\text{Price Index Number} = \frac{1}{5}(200 + 300 + 300 + 150 + 200) = \frac{1150}{5} = 230$$

Thus, the price index number for 2019 with 2018 as the base year is 230. The correct answer is Option 1.

Quick Tip

When calculating the price index number using the simple average method, compute the price relative for each commodity and then take the average.

---

64. A boat can row upstream at 10 km/h and downstream at 18 km/h. If m is the speed of the boat in still water and n is the speed of the stream, then $m + n$ is:

(1) 16 km/h

(2) 18 km/h

(3) 20 km/h

(4) 14 km/h

Correct Answer: (2) 18 km/h

Solution:

Let:

$$m = \text{speed of boat in still water (in km/h)}$$

n = speed of stream (in km/h)

We know:

$$\text{Upstream speed} = m - n = 10 \quad (1)$$

$$\text{Downstream speed} = m + n = 18 \quad (2)$$

To find $m + n$, simply use equation (2):

$$m + n = \boxed{18 \text{ km/h}}$$

However, let's also verify by solving both equations:

Add (1) and (2):

$$(m + n) + (m - n) = 18 + 10 \Rightarrow 2m = 28 \Rightarrow m = 14$$

Substitute $m = 14$ into equation (2):

$$14 + n = 18 \Rightarrow n = 4$$

Thus,

$$m = 14 \text{ km/h}, \quad n = 4 \text{ km/h}, \quad m + n = 18 \text{ km/h}$$

Quick Tip

Use the downstream and upstream formulas:

$$\text{Downstream} = m + n, \quad \text{Upstream} = m - n$$

Add the two equations to find m, then substitute to get n, if needed.

---

65. Pipes A and B can fill a tank in 40 hours and 50 hours respectively and pipe C can empty the full tank in 60 hours. If all the pipes are opened together, how much time (in hours) will be needed to fill the tank?

(1) $33\frac{5}{17}$

(2) $35\frac{5}{17}$

(3) $31\frac{5}{17}$

(4) $30\frac{5}{17}$

Correct Answer: (2) $35\frac{5}{17}$

Solution:

Let's calculate the net work done by all three pipes in 1 hour:

- Pipe A can fill the tank in 40 hours

$$\Rightarrow \text{Work by A in 1 hour} = \frac{1}{40}$$

- Pipe B can fill the tank in 50 hours

$$\Rightarrow \text{Work by B in 1 hour} = \frac{1}{50}$$

- Pipe C can empty the tank in 60 hours

$$\Rightarrow \text{Work by C in 1 hour} = -\frac{1}{60}$$

Now, total work done by all three pipes in 1 hour:

$$\frac{1}{40} + \frac{1}{50} - \frac{1}{60}$$

Take LCM of 40, 50, and 60:

$$\text{LCM} = 600$$

Convert all terms with denominator 600:

$$\frac{1}{40} = \frac{15}{600}, \quad \frac{1}{50} = \frac{12}{600}, \quad \frac{1}{60} = \frac{10}{600}$$

So, total work in 1 hour:

$$\frac{15 + 12 - 10}{600} = \frac{17}{600}$$

$$\Rightarrow \text{Time to fill the tank} = \frac{1}{\left(\frac{17}{600}\right)} = \frac{600}{17} \text{ hours}$$

Convert $\frac{600}{17}$ into mixed fraction:

$$600 \div 17 = 35 \text{ (quotient)}, \quad \text{remainder} = 5 \Rightarrow \frac{600}{17} = 35\frac{5}{17}$$

$$\boxed{35\frac{5}{17} \text{ hours}}$$

Quick Tip

In pipe and cistern problems, treat filling as positive work and emptying as negative. Use LCM for ease of calculation and convert the final answer to a mixed fraction for clarity.

---

66. For the formula $t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, consider the following statements:

A. μ_1 and μ_2 are sample mean and population mean respectively.

B. n_1 and n_2 are sample sizes of two samples from same population.

C. S_1 and S_2 are sample means of two samples from same population.

D. S_1 and S_2 are standard error of two samples from same population.

E. n_1 is the sample size and n_2 is the population size.

Choose the correct answer from the options given below:

(1) B and D only

(2) A, B and D only

(3) B, C and D only

(4) A and E only

Correct Answer: (1) B and D only

Solution:

The given formula:

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

is the formula used for the two-sample t-test assuming unequal variances.

Let's analyze the components:

- μ_1 and μ_2 represent the sample means of the two independent samples, not a sample and a population mean.

$$\Rightarrow \textbf{A is incorrect}$$

- n_1 and n_2 are the sample sizes of the two samples.

$$\Rightarrow \textbf{B is correct}$$

- S_1 and S_2 represent the sample standard deviations, not sample means.

$$\Rightarrow \textbf{C is incorrect}$$

- The values $\frac{S_1^2}{n_1}$ and $\frac{S_2^2}{n_2}$ are used to compute the standard error of the difference in means. So, S_1 and S_2 are the standard deviations used to compute standard error.

$$\Rightarrow \textbf{D is correct}$$

- n_1 and n_2 are both sample sizes; n_2 is not the population size.

$$\Rightarrow \textbf{E is incorrect}$$

Hence, only statements B and D are correct.

$$\boxed{\text{Correct answer is Option 1: B and D only}}$$

Quick Tip

In statistical hypothesis testing, always distinguish between sample statistics (like sample mean, sample SD) and population parameters. Also, understand the role of standard error in the t-test.

67. 10 litres of concentrated acid containing 75$\%$ of acid and rest water is mixed with X litres of diluted acid containing 40$\%$ of acid and rest water. If the final mixture contains 60$\%$ of acid then the value of X is:

(1) 7.5 litres

(2) 9 litres

(3) 6 litres

(4) 8 litres

Correct Answer: (1) 7.5 litres

Solution:

Let us calculate the amount of acid in each mixture and then apply the concept of weighted average.

- Acid in 10 litres of 75% solution:

$$= 75\% \text{ of } 10 = \frac{75}{100} \times 10 = 7.5 \text{ litres}$$

- Acid in X litres of 40% solution:

$$= 40\% \text{ of } X = \frac{40}{100} \times X = 0.4X \text{ litres}$$

- Total volume after mixing:

$$= 10 + X \text{ litres}$$

- Final concentration of acid = 60% of total:

$$\Rightarrow \text{Total acid} = 60\% \text{ of } (10 + X) = \frac{60}{100}(10 + X) = 0.6(10 + X)$$

Now equating the total acid:

$$7.5 + 0.4X = 0.6(10 + X)$$

Simplify the right-hand side:

$$7.5 + 0.4X = 6 + 0.6X$$

Bring terms together:

$$7.5 - 6 = 0.6X - 0.4X \Rightarrow 1.5 = 0.2X \Rightarrow X = \frac{1.5}{0.2} = 7.5$$

$$\boxed{X = 7.5 \text{ litres}}$$

Quick Tip

In mixture problems, use the weighted average formula:

$$\text{Total Acid} = \text{Acid from solution 1} + \text{Acid from solution 2}$$

and then relate it to the final percentage.

---

68. Fisher's price index number is:

(1) A.M. of Laspeyres's and Paasche's index

(2) G.M. of Laspeyres's and Paasche's index

(3) H.M. of Laspeyres's and Paasche's index

(4) Average of Laspeyres's and Paasche's index

Correct Answer: (2) G.M. of Laspeyres's and Paasche's index

Solution:

Fisher's price index is a well-known formula used in index number theory that provides an ideal measure by combining two commonly used indices:

- Laspeyres's Price Index (LPI): Based on base year quantities - Paasche's Price Index (PPI): Based on current year quantities

Fisher's index is designed to eliminate the bias present in each of these when used alone.

The formula for Fisher's Ideal Price Index is:

$$Fisher's\ Index = \sqrt{LPI \times PPI}$$

This represents the Geometric Mean (G.M.) of Laspeyres's and Paasche's indices.

$$\boxed{Fisher's\ Index = \sqrt{\text{Laspeyres's Index} \times \text{Paasche's Index}}}$$

Thus, Fisher's index is not the arithmetic mean (A.M.), harmonic mean (H.M.), or just any average—it is specifically the geometric mean of the two.

$$\boxed{\text{Correct answer is Option 2: G.M. of Laspeyres's and Paasche's index}}$$

Quick Tip

Fisher's index is often referred to as the "Ideal Index" because it satisfies both time reversal and factor reversal tests. Always remember it is the geometric mean of Laspeyres's and Paasche's indices.

---

69. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Then value of $E(X)$ is:

(1) 1.66

(2) 2.66

(3) 3.66

(4) 4.66

Correct Answer: (3) 3.66

Solution:

We are given the probability distribution. To find the expected value $E(X)$, use:

$$E(X) = \sum X \cdot P(X)$$

First, sum all probabilities and equate to 1:

$$\Rightarrow P(X=0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7)=1$$

$$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+(7k^2+k)=1$$

$$\Rightarrow (k+2k+2k+3k+k)+(k^2+2k^2+7k^2)=1$$

$$\Rightarrow 9k+10k^2=1$$

Solve the equation:

$$10k^2+9k-1=0$$

Use the quadratic formula:

$$k=\frac{-9\pm\sqrt{9^2-4\cdot 10\cdot(-1)}}{2\cdot 10}=\frac{-9\pm\sqrt{81+40}}{20}=\frac{-9\pm\sqrt{121}}{20}\Rightarrow\frac{-9\pm 11}{20}$$

So,

$$k=\frac{2}{20}=\frac{1}{10}=0.1 \quad \text{(reject } -1 \text{ as probability can't be negative)}$$

Now compute $E(X)=\sum X\cdot P(X)$

$$\begin{aligned} E(X)&=0\cdot 0+1\cdot k+2\cdot 2k+3\cdot 2k+4\cdot 3k+5\cdot k^2+6\cdot 2k^2+7\cdot(7k^2+k)\\ &=k+4k+6k+12k+5k^2+12k^2+7(7k^2+k)\\ &=(k+4k+6k+12k+7k)+(5k^2+12k^2+49k^2)\\ &=30k+66k^2 \end{aligned}$$

Substitute $k=0.1$:

$$E(X)=30\cdot 0.1+66\cdot(0.1)^2=3+66\cdot 0.01=3+0.66=3.66$$

$$\boxed{E(X)=3.66}$$

Quick Tip

To find $E(X)$ for a discrete random variable, always check the total probability equals 1 to find unknown constants before applying $E(X)=\sum X\cdot P(X)$.

70. With reference to time series, match List I with List II:

List I	List II
A.Secular movements	I.Price increase before Deepavali
B.Seasonal variations	II.Increase in price of gold during a major war
C.Cyclic variations	III.Long term trends
D.Irregular variations	IV.Recurring rise and decline in production

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II

(2) A-II, B-I, C-IV, D-III

(3) A-III, B-II, C-IV, D-I

(4) A-II, B-III, C-IV, D-I

Correct Answer: (1) A-III, B-I, C-IV, D-II

Solution:

We analyze each item:

- A. Secular Movements $\rightarrow$ III. Long term trends These represent persistent, long-term growth or decline patterns in data (e.g., population growth, technological progress).
- B. Seasonal Variations $\rightarrow$ I. Price increase before Deepavali These are periodic and occur at regular intervals due to seasonal factors or festivals.
- C. Cyclic Variations $\rightarrow$ IV. Recurring rise and decline in production These are fluctuations occurring over long periods, often tied to economic cycles (booms and recessions).
- D. Irregular Variations $\rightarrow$ II. Increase in price of gold during a major war These are unpredictable, one-time variations due to unforeseen events like wars, pandemics, etc.

$$\boxed{\text{A} \rightarrow \text{III}, \quad \text{B} \rightarrow \text{I}, \quad \text{C} \rightarrow \text{IV}, \quad \text{D} \rightarrow \text{II}}$$

$$\boxed{\text{Correct answer is Option 1: A-III, B-I, C-IV, D-II}}$$

Quick Tip

Time series components include: - Secular Trends (long-term direction), - Seasonal Variations (fixed periodic), - Cyclic Variations (business/economic cycles), - Irregular Variations (unexpected events).

---

71. The region represented by the system of inequalities $x \geq 0,\ y \geq 0,\ y \leq 8,\ x + y \leq 4$ **is:**

(1) unbounded in first quadrant

(2) unbounded in first and second quadrant

(3) bounded in first quadrant

(4) bounded in first and second quadrants

Correct Answer: (3) bounded in first quadrant

Solution:

We are given the following system of inequalities:

$$x \geq 0 \quad (1)$$

y ≥ 0 (2)

y ≤ 8 (3)

x + y ≤ 4 (4)

Let us interpret these:

- Inequalities (1) and (2) define the first quadrant, where $x \geq 0$ and $y \geq 0$ - Inequality (3): $y \leq 8$ restricts the height of the region, but note that $x + y \leq 4$ (inequality 4) is more restrictive in the relevant range - Inequality (4): $x + y \leq 4$ represents the region below and including the line $x + y = 4$

Let's identify the boundary points by solving:

- When $x = 0 \Rightarrow y = 4$ - When $y = 0 \Rightarrow x = 4$

Thus, the region is enclosed between the x-axis, y-axis, and the line $x + y = 4$ in the first quadrant.

Also, this region is bounded because all inequalities collectively restrict the feasible area to a triangle with vertices:

$$(0,0),\ (4,0),\ (0,4)$$

$$\boxed{\text{Region is bounded in the first quadrant}}$$

Quick Tip

For inequality region questions, graph all boundaries and check where all conditions overlap. If a closed polygon forms within one quadrant, the region is bounded.

---

72. The solution of $7x \equiv 3 \pmod 5$ is:

(1) $x = 5 \pmod 5$

(2) $x \equiv 4 \pmod 5$

(3) $x \equiv 1 \pmod 5$

(4) $x \equiv 2 \pmod 5$

Correct Answer: (2) $x \equiv 4 \pmod 5$

Solution:

We are given the congruence:

$$7x \equiv 3 \pmod 5$$

Step 1: Reduce coefficient modulo 5:

$$7 \equiv 2 \pmod 5 \Rightarrow 7x \equiv 2x \pmod 5 \Rightarrow 2x \equiv 3 \pmod 5$$

Step 2: Multiply both sides by the multiplicative inverse of 2 modulo 5.

We need to find a such that:

$$2a \equiv 1 \pmod 5$$

Try values of a:

- $2 \times 1 = 2 \not\equiv 1$ - $2 \times 2 = 4 \not\equiv 1$ - $2 \times 3 = 6 \equiv 1 \pmod 5 \Rightarrow$ Inverse of 2 is 3

Step 3: Multiply both sides of $2x \equiv 3 \pmod 5$ by 3:

$$3 \cdot 2x \equiv 3 \cdot 3 \pmod 5 \Rightarrow 6x \equiv 9 \pmod 5 \Rightarrow x \equiv 4 \pmod 5$$

$$\boxed{x \equiv 4 \pmod 5}$$

Quick Tip

When solving linear congruences, always reduce coefficients modulo m, and find the modular inverse to isolate the variable.

---

73. For the LPP Max $Z = 3x + 4y$, subject to the constraints:

$$x + y \leq 40, \quad x + 2y \leq 60, \quad x \geq 0, \quad y \geq 0$$

The solution is:

(1) $x = 20, y = 20,\ \text{Max } Z = 140$

(2) $x = 40, y = 0,\ \text{Max } Z = 120$

(3) $x = 0, y = 60,\ \text{Max } Z = 240$

(4) $x = 10, y = 30,\ \text{Max } Z = 130$

Correct Answer: (1) $x = 20, y = 20,\ \text{Max } Z = 140$

Solution:

We are given a Linear Programming Problem:

$$\text{Maximize } Z = 3x + 4y$$

Subject to constraints:

$$\begin{cases} x + y \leq 40 \\ x + 2y \leq 60 \\ x \geq 0,\ y \geq 0 \end{cases}$$

Step 1: Find corner points (vertices) of the feasible region.

From the constraints:

1. $x + y = 40 \rightarrow$ Line 1 2. $x + 2y = 60 \rightarrow$ Line 2

Now, solve for the intersection point of Line 1 and Line 2:

$$x + y = 40 \quad \text{(i)}$$

x + 2y = 60   (ii)

Subtract (i) from (ii):

$$(x + 2y) - (x + y) = 60 - 40 \Rightarrow y = 20$$

$\Rightarrow x = 40 - y = 20$

So, point of intersection is $(20, 20)$

Other intercepts:

- $x + y = 40$, if $y = 0 \Rightarrow x = 40 \Rightarrow (40, 0)$ - $x + 2y = 60$, if $y = 0 \Rightarrow x = 60 \Rightarrow (60, 0) \leftarrow$ Not valid as it violates $x + y \leq 40$ - $x + 2y = 60$, if $x = 0 \Rightarrow 2y = 60 \Rightarrow y = 30 \Rightarrow (0, 30)$

Feasible region vertices: - $(0, 0)$ - $(40, 0)$ - $(20, 20)$ - $(0, 30)$

Step 2: Evaluate $Z = 3x + 4y$ at these points:

- At $(0, 0)$: $Z = 0$ - At $(40, 0)$: $Z = 3(40) + 4(0) = 120$ - At $(20, 20)$: $Z = 3(20) + 4(20) = 60 + 80 = 140$ - At $(0, 30)$: $Z = 3(0) + 4(30) = 120$

Maximum Z = 140 at $\boxed{(20, 20)}$

$$\boxed{x = 20, \quad y = 20, \quad \text{Max } Z = 140}$$

Quick Tip

Always graph constraints and find intersection (corner) points. Evaluate the objective function Z at each vertex to find max/min value.

---

74. If the mean of the probability distribution is 5, then the value of k is:

X	2	k	5
$P(X)$	0.2	0.4	0.6

(1) 1

(2) 2

(3) 4

(4) 3

Correct Answer: (3) 4

Solution:

We are given a probability distribution with three values of the random variable X:

$$X = 2,\ k,\ 5$$

P(X = 2) = 0.2,   P(X = k) = 0.4,   P(X = 5) = 0.6

To find the mean (expected value) $E(X)$, use the formula:

$$E(X) = \sum X \cdot P(X)$$

Given:

$$E(X) = 5$$

Apply the values:

$$E(X) = 2 \cdot 0.2 + k \cdot 0.4 + 5 \cdot 0.6 = 5$$

Now simplify:

$$0.4 + 0.4k + 3 = 5 \Rightarrow 0.4k + 3.4 = 5 \Rightarrow 0.4k = 1.6 \Rightarrow k = \frac{1.6}{0.4} = 4$$

$$\boxed{k = 4}$$

Quick Tip

In expected value problems, always multiply each value of X with its corresponding probability and sum them up. Solve for unknowns using the given mean.

75. If $A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & -1 & 7 \end{bmatrix}$, then the sum of the elements of the matrix $AA^\top$ is:

(1) 156

(2) 164

(3) 146

(4) 136

Correct Answer: (3) 146

Solution:

Given matrix:

$$A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & -1 & 7 \end{bmatrix}$$

First, compute $A^\top$ (transpose of A):

$$A^\top = \begin{bmatrix} 4 & 3 \\ 5 & -1 \\ 2 & 7 \end{bmatrix}$$

Now, compute $AA^\top$:

$$AA^\top = \begin{bmatrix} 4 & 5 & 2 \\ 3 & -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 5 & -1 \\ 2 & 7 \end{bmatrix}$$

Matrix multiplication:

$$AA^\top = \begin{bmatrix} (4 \cdot 4 + 5 \cdot 5 + 2 \cdot 2) & (4 \cdot 3 + 5 \cdot (-1) + 2 \cdot 7) \\ (3 \cdot 4 + (-1) \cdot 5 + 7 \cdot 2) & (3 \cdot 3 + (-1) \cdot (-1) + 7 \cdot 7) \end{bmatrix}$$

Step-by-step:

- Row 1, Col 1: $16 + 25 + 4 = 45$ - Row 1, Col 2: $12 - 5 + 14 = 21$ - Row 2, Col 1: $12 - 5 + 14 = 21$ - Row 2, Col 2: $9 + 1 + 49 = 59$

So,

$$AA^\top = \begin{bmatrix} 45 & 21 \\ 21 & 59 \end{bmatrix}$$

Sum of elements:

$$45 + 21 + 21 + 59 = \boxed{146}$$

$$\boxed{\text{Sum of elements in } AA^\top = 146}$$

Quick Tip

To compute $AA^\top$, always multiply the original matrix with its transpose. Double-check each product and sum all the elements if required.

---

76. If $x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}}$ (where a is a constant), then $\frac{dy}{dx}$ is equal to:

(1) $\left(\frac{y}{x}\right)^{\frac{1}{4}}$

(2) $\left(\frac{x}{y}\right)^{\frac{1}{4}}$

(3) $-\left(\frac{y}{x}\right)^{\frac{1}{4}}$

(4) $-\left(\frac{x}{y}\right)^{\frac{1}{4}}$

Correct Answer: (3) $-\left(\frac{y}{x}\right)^{\frac{1}{4}}$

Solution:

Given:

$$x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}} \quad \text{(a is a constant)}$$

Differentiate both sides with respect to x:

$$\frac{d}{dx}\left(x^{\frac{3}{4}}\right) + \frac{d}{dx}\left(y^{\frac{3}{4}}\right) = \frac{d}{dx}\left(a^{\frac{3}{4}}\right)$$

$$\Rightarrow \frac{3}{4}x^{-\frac{1}{4}} + \frac{3}{4}y^{-\frac{1}{4}} \cdot \frac{dy}{dx} = 0$$

Rearranging:

$$\frac{3}{4}y^{-\frac{1}{4}} \cdot \frac{dy}{dx} = -\frac{3}{4}x^{-\frac{1}{4}}$$

Divide both sides by $\frac{3}{4}y^{-\frac{1}{4}}$:

$$\frac{dy}{dx} = -\frac{x^{-\frac{1}{4}}}{y^{-\frac{1}{4}}} = -\left(\frac{y}{x}\right)^{\frac{1}{4}}$$

$$\boxed{\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{4}}}$$

Quick Tip

For equations involving both x and y, use implicit differentiation, and apply chain rule when differentiating terms like y^n with respect to x.

---

77. Let A and B be symmetric matrices of same order, then which of the following statements is true?

(1) $(A + B)$ is a symmetric matrix

(2) (AB) is a symmetric matrix

(3) $(A + B)$ is a skew-symmetric matrix

(4) (AB) is a skew-symmetric matrix

Correct Answer: (1) $(A + B)$ is a symmetric matrix

Solution:

We are given: - A and B are symmetric matrices. This means:

$$A^\top = A \quad \text{and} \quad B^\top = B$$

Check Option 1: $(A + B)$ is symmetric

Take transpose:

$$(A + B)^\top = A^\top + B^\top = A + B \Rightarrow \text{Hence, } A + B \text{ is symmetric.}$$

True.

Check Option 2: AB is symmetric

Take transpose:

$$(AB)^\top = B^\top A^\top = BA$$

But in general, $AB \neq BA$. So,

$$(AB)^\top \neq AB \Rightarrow \text{Not symmetric unless } A \text{ and } B \text{ commute}$$

Not necessarily true.

Check Option 3: $A + B$ is skew-symmetric

If a matrix is skew-symmetric, $(A+B)^\top = -(A+B)$. But we just showed $(A+B)^\top = A+B$

False.

Check Option 4: AB is skew-symmetric

This would require:

$$(AB)^\top = -AB \Rightarrow BA = -AB \Rightarrow AB = -BA$$

Again, this doesn't hold unless under very special conditions.

False.

$$\boxed{(A+B) \text{ is a symmetric matrix}}$$

Quick Tip

The sum of symmetric matrices is always symmetric. The product of symmetric matrices is symmetric only if they commute: $AB = BA$.

---

78. For matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, the values of x and y such that $A^2 + xI = yA$ are:

(1) $x = 6, y = 8$

(2) $x = 8, y = 6$

(3) $x = 8, y = 8$

(4) $x = 6, y = 6$

Correct Answer: (3) $x = 8, y = 8$

Solution:

We are given:

$$A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}, \quad \text{and} \quad A^2 + xI = yA$$

Step 1: Compute A^2

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \cdot 3 + 1 \cdot 7) & (3 \cdot 1 + 1 \cdot 5) \\ (7 \cdot 3 + 5 \cdot 7) & (7 \cdot 1 + 5 \cdot 5) \end{bmatrix} = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

Step 2: Let I be the identity matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad xI = x \cdot I = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

So,

$$A^2 + xI = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$$

Step 3: Compute yA:

$$yA = y \cdot \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

Now, equate matrices:

$$\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

Compare corresponding elements:

- $16 + x = 3y \rightarrow$ (i) - $8 = y \rightarrow$ (ii) - $56 = 7y \rightarrow$ (check) - $32 + x = 5y \rightarrow$ (iii)

From (ii): $y = 8$ Substitute into (i):

$$16 + x = 3 \cdot 8 = 24 \Rightarrow x = 8$$

Also verify (iii):

$$32 + x = 32 + 8 = 40 = 5 \cdot 8 = 40$$

$$\boxed{x = 8, \quad y = 8}$$

Quick Tip

In matrix identity problems, compute both sides explicitly and match individual entries. Always verify all elements for confirmation.

---

79. If $8 + 3x < |8 + 3x|$, $x \in \mathbb{R}$, then x lies in:

(1) $(-\infty, \infty)$

(2) $\left(-\infty, -\frac{3}{8}\right)$

(3) $\left(-\infty, -\frac{8}{3}\right)$

(4) $\left(-\infty, \frac{8}{3}\right)$

Correct Answer: (3) $\left(-\infty, -\frac{8}{3}\right)$

Solution:

We are given:

$$8 + 3x < |8 + 3x|$$

Let us recall the property of modulus:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Let $a = 8 + 3x$. Now analyze the inequality:

Case 1: If $8 + 3x \geq 0 \Rightarrow |8 + 3x| = 8 + 3x$

Then the inequality becomes:

$$8 + 3x < 8 + 3x \Rightarrow \text{False}$$

So no values satisfy the inequality when $8 + 3x \geq 0$.

Case 2: If $8 + 3x < 0 \Rightarrow |8 + 3x| = -(8 + 3x)$

Then inequality becomes:

$$8 + 3x < -(8 + 3x) \Rightarrow 8 + 3x < -8 - 3x \Rightarrow 6x < -16 \Rightarrow x < -\frac{8}{3}$$

Hence, the solution is:

$$x \in \left(-\infty, -\frac{8}{3}\right)$$

$$\boxed{x \in \left(-\infty, -\frac{8}{3}\right)}$$

Quick Tip

When solving inequalities with modulus, always split the equation into two cases based on the definition of modulus and solve each logically.

---

80. For $x + y = 8$, the maximum value of xy is:

(1) 4

(2) 8

(3) 32

(4) 16

Correct Answer: (4) 16

Solution:

We are given:

$$x + y = 8 \quad (1)$$

We want to maximize the expression:

$$xy = ?$$

Step 1: Express one variable in terms of the other using equation (1):

$$y = 8 - x \Rightarrow xy = x(8 - x) = 8x - x^2$$

Step 2: This is a quadratic expression in x:

$$f(x) = -x^2 + 8x$$

This parabola opens downward, and the maximum value occurs at the vertex:

$$x = \frac{-b}{2a} = \frac{-8}{2 \cdot (-1)} = \frac{8}{2} = 4 \Rightarrow y = 8 - 4 = 4$$

So:

$$xy = 4 \cdot 4 = 16$$

$$\boxed{\text{Maximum value of } xy = 16}$$

Quick Tip

Use the identity $xy = x(8 - x)$ and treat it as a quadratic. The maximum product of two numbers with a fixed sum occurs when both are equal.

---

81. The maximum profit that a company can make if the profit function is given by:

$$P(x) = 32 + 24x - 18x^2$$

is:

(1) 48

(2) 40

(3) 36

(4) 42

Correct Answer: (2) 40

Solution:

The given function is:

$$P(x) = -18x^2 + 24x + 32$$

This is a quadratic equation of the form:

$$P(x) = ax^2 + bx + c \quad \text{where} \quad a = -18,\ b = 24,\ c = 32$$

Since $a < 0$, the parabola opens downward, so the vertex gives the maximum value.

The formula for the x-coordinate of the vertex is:

$$x = \frac{-b}{2a} = \frac{-24}{2 \cdot (-18)} = \frac{-24}{-36} = \frac{2}{3}$$

Now substitute $x = \frac{2}{3}$ into the function:

$$P\left(\frac{2}{3}\right) = 32 + 24 \cdot \frac{2}{3} - 18 \cdot \left(\frac{2}{3}\right)^2$$

$$= 32 + 16 - 18 \cdot \frac{4}{9} = 48 - 8 = 40$$

$$\boxed{\text{Maximum profit } = 40}$$

Quick Tip

In a quadratic profit or revenue function, the maximum (or minimum) occurs at $x = \frac{-b}{2a}$. Always check the sign of a to know if it's a max or min.

---

82. A motor boat goes 48 km downstream and comes back to the starting point in 16 hours. If the speed of the stream is 4 km/hr, then the speed of the motor boat in still water is:

(1) 6 km/hr

(2) 8 km/hr

(3) 10 km/hr

(4) 5 km/hr

Correct Answer: (2) 8 km/hr

Solution:

Let the speed of the boat in still water be x km/hr. Speed of stream = 4 km/hr

Then, - Downstream speed = $x + 4$ km/hr - Upstream speed = $x - 4$ km/hr

Distance (one way) = 48 km

Time downstream:

$$t_1 = \frac{48}{x+4}$$

Time upstream:

$$t_2 = \frac{48}{x-4}$$

Total time:

$$t_1 + t_2 = 16 \Rightarrow \frac{48}{x+4} + \frac{48}{x-4} = 16$$

Divide both sides by 16:

$$\frac{3}{x+4} + \frac{3}{x-4} = 1$$

Take LCM and simplify:

$$\frac{3(x-4)+3(x+4)}{(x+4)(x-4)} = 1 \Rightarrow \frac{3x-12+3x+12}{x^2-16} = 1 \Rightarrow \frac{6x}{x^2-16} = 1$$

Multiply both sides:

$$6x = x^2 - 16 \Rightarrow x^2 - 6x - 16 = 0$$

Solve the quadratic:

$$x = \frac{6 \pm \sqrt{36+64}}{2} = \frac{6 \pm \sqrt{100}}{2} = \frac{6 \pm 10}{2}$$

$$x = 8 \quad \text{(only positive root considered)}$$

$$\boxed{\text{Speed of the boat in still water } = 8 \text{ km/hr}}$$

Quick Tip

In upstream-downstream problems, total time = time downstream + time upstream. Use basic algebra and quadratic equations to solve.

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83. If $x = \frac{1}{t^2}$ and $y = \frac{1}{t^3}$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is:

(1) $\frac{3}{2}$

(2) $\frac{1}{2}$

(3) $\frac{2}{3}$

(4) $\frac{3}{4}$

Correct Answer: (4) $\frac{3}{4}$

Solution:

Given:

$$x = \frac{1}{t^2} \Rightarrow x = t^{-2}, \quad y = \frac{1}{t^3} \Rightarrow y = t^{-3}$$

We are to find:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Since both x and y are in terms of t, we use the parametric differentiation formula:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$$

Step 1: First derivatives

$$\frac{dy}{dt} = \frac{d}{dt}(t^{-3}) = -3t^{-4}$$
$$\frac{dx}{dt} = \frac{d}{dt}(t^{-2}) = -2t^{-3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3t^{-4}}{-2t^{-3}} = \frac{3}{2t}$$

Step 2: Differentiate $\frac{dy}{dx}$ with respect to t

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{3}{2t}\right) = -\frac{3}{2t^2}$$

Step 3: Apply formula for second derivative

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2t^2}}{-2t^{-3}} = \frac{3}{2t^2} \cdot \frac{1}{2t^{-3}} = \frac{3}{2t^2} \cdot \frac{t^3}{2} = \frac{3t}{4}$$

Now substitute $t = 1$:

$$\frac{d^2y}{dx^2} = \frac{3 \cdot 1}{4} = \boxed{\frac{3}{4}}$$

$$\boxed{\frac{d^2y}{dx^2} \text{ at } t = 1 \text{ is } \frac{3}{4}}$$

Quick Tip

When both x and y are expressed in terms of a third variable t, use the parametric formulas:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

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84. A coin is tossed 5 times. The probability of getting at least one head is:

(1) $\frac{31}{32}$

(2) $\frac{15}{16}$

(3) $\frac{1}{32}$

(4) $\frac{63}{64}$

Correct Answer: (1) $\frac{31}{32}$

Solution:

Let us find the probability of getting at least one head in 5 tosses of a fair coin.

It is easier to find the complement:

$$\text{P(at least one head)} = 1 - \text{P(no head)}$$

If no head appears in 5 tosses, that means we get all tails.

$$\text{P(all tails)} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

So,

$$\text{P(at least one head)} = 1 - \frac{1}{32} = \frac{31}{32}$$

$$\boxed{\text{Probability of getting at least one head} = \frac{31}{32}}$$

Quick Tip

When finding probability of "at least one", always try using the complement rule:

$$P(\text{at least one}) = 1 - P(\text{none})$$

It simplifies the calculation.

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85. The following data are from a simple random sample: 1, 4, 7

The point estimate of the population standard deviation is:

(1) 2

(2) 5

(3) 1.5

(4) 3

Correct Answer: (4) 3

Solution:

We are given a simple random sample:

$$x_1 = 1, \quad x_2 = 4, \quad x_3 = 7$$

Step 1: Compute the sample mean:

$$\bar{x} = \frac{1+4+7}{3} = \frac{12}{3} = 4$$

Step 2: Compute squared deviations from the mean:

$$(x_1 - \bar{x})^2 = (1-4)^2 = 9$$

$(x_2 - \bar{x})^2 = (4-4)^2 = 0$

$(x_3 - \bar{x})^2 = (7-4)^2 = 9$

Sum of squared deviations:

$$\sum (x_i - \bar{x})^2 = 9 + 0 + 9 = 18$$

Step 3: Compute the point estimate of the population standard deviation (use denominator n):

$$\sigma = \sqrt{\frac{18}{3}} = \sqrt{6} \approx 2.45 \quad \text{(sample SD)}$$

BUT we are asked for population standard deviation, and since this is a point estimate, we take the sample as entire population, and standard deviation is:

$$\sqrt{\frac{(1-4)^2 + (4-4)^2 + (7-4)^2}{3}} = \sqrt{\frac{18}{3}} = \sqrt{6} \approx 2.45$$

Now reconsider: if this is sample SD as estimator for population SD, we must use sample SD formula (n - 1):

$$s = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

So, point estimate of population standard deviation is:

$$\boxed{3}$$

$$\boxed{\text{Point estimate of population standard deviation} = 3}$$

Quick Tip

To estimate population standard deviation from a sample, use:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

This is called the unbiased estimator of the population standard deviation.

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86. A car costing 8,50,000 has scrap value of 1,25,000. If annual depreciation charge is 1,45,000, then useful life of the car is:

(1) 12 years

(2) 5 years

(3) 7 years

(4) 10 years

Correct Answer: (2) 5 years

Solution:

We are given:

- Cost of the car (initial value) = 8,50,000 - Scrap value (residual value) = 1,25,000 - Annual depreciation = 1,45,000

The formula for useful life using straight-line depreciation method is:

$$\text{Useful Life} = \frac{\text{Cost} - \text{Scrap Value}}{\text{Annual Depreciation}}$$

Substitute values:

$$\text{Useful Life} = \frac{8,50,000 - 1,25,000}{1,45,000} = \frac{7,25,000}{1,45,000} = 5 \text{ years}$$

$$\boxed{\text{Useful life of the car} = 5 \text{ years}}$$

Quick Tip

In straight-line depreciation, the asset depreciates uniformly. Use:

$$\text{Useful Life} = \frac{\text{Purchase Price} - \text{Scrap Value}}{\text{Annual Depreciation}}$$
