Differential Equations JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Differential Equations

1. The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1) = 0$ is **(+4, -1)**
[27-Jan-2024 Shift 1]

a. $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$

b. $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$

c. $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

d. $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

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2. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has : **(+4, -1)**
[29-Jan-2024•Shift•1]

a. no solution

b. exactly two solutions in $(-\infty, \infty)$

c. a unique solution in $(-\infty, 1)$

d. a unique solution in $(-\infty, \infty)$

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3. Let $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$ Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha \in s} \alpha\right)^2 x + \sum_{\alpha \in s}(\alpha+1)^2\beta = 0$ has real roots, is _____ **(+4, -1)**

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4. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is **(+4, -1)**
[27-Jan-2024 Shift 1]

a. 4

b. 3

c. 2

d. 0

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5. Let $y = y(x)$ be the solution of the differential equation $\left(x^2 - 3y^2\right)dx + 3xydy = 0, y(1) = 1$.Then $6y^2(e)$ is equal to [1-Feb-2024•Shift•1] (+4, -1)

a. e^2

b. $\frac{3}{2}e^2$

c. $3e^2$

d. $2e^2$

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6. Let a smooth curve $y = f(x)$ be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$ If the curve passes through the point $(1, 2)$ and $(8, 1)$, then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to [1-Feb-2024•Shift•1] (+4, -1)

a. $2\log_e 2$

b. 4

c. 1

d. $4\log_e 2$

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7. Let αx=exp$(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2ydy-(1-xy^2)\,dx = 0$, $x>0$, $y(2)=\sqrt{log_e^2}$. Then $\alpha+\beta-\gamma$ equals : [29-Jan-2024•Shift•1] (+4, -1)

a. 0

b. -1

c. 1

d. 3

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8. Let $y = y(x)$ be the solution of the differential equation $x^3dy + (xy - 1)dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - e$ Then $y(1)$ is equal to [1-Feb-2023 Shift 2] (+4, -1)

a. $2 - e$

b. e

c. 1

d. 3

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9. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2\left(1 + \log_e x\right)\right), x > 0, y(1) = 3$ Then $\frac{y^2(x)}{9}$ is equal to : **(+4, -1)**

[1-Feb-2023 Shift 2]

a. $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

b. $\frac{x^2}{3x^3(1+\log_e x^2)-2}$

c. $\frac{x^2}{7-3x^3(2+\log_e x^2)}$

d. $\frac{x^2}{2x^3(2+\log_e x^3)-3}$

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10. If $\frac{dy}{dx}$ = y+7 and y(0) = 0, then the value of y(1) is? **(+4, -1)**

[6-Apr-2023•shift•2]

a. 7(e-1)

b. 2(e-1)

c. 7e

d. None of these

Answers

1. Answer: d

Explanation:

Put $y = vx$

$v + x\frac{dv}{dx} = -\left(\frac{1+3v^2}{3+v^2}\right)$

$x\frac{dv}{dx} = -\frac{(v+1)^3}{3+v^2}$

$\frac{(3+v^2)dv}{(v+1)^3} + \frac{dx}{x} = 0$

$\int \frac{4dv}{(v+1)^3} + \int \frac{dv}{v+1} - \int \frac{2dv}{(v+1)^2} + \int \frac{dx}{x} = 0$

$\frac{-2}{(v+1)^2} + \ln(v+1) + \frac{2}{v+1} + \ln x = c$

$\frac{-2x^2}{(x+y)^2} + \ln\left(\frac{x+y}{x}\right) + \frac{2x}{x+y} + \ln x = c$

$\frac{2xy}{(x+y)^2} + \ln(x+y) = c$

$\therefore c = 0, \text{ as } x = 1, y = 0$

$\therefore \frac{2xy}{(x+y)^2} + \ln(x+y) = 0$

So, the correct answer is (D) : $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

Concepts:

1. Differential Equations:

A **differential equation** is an equation that contains one or more functions with its derivatives. The derivatives of the function define the rate of change of a function at a point. It is mainly used in fields such as physics, engineering, biology and so on.

Orders of a Differential Equation

First Order Differential Equation

The first-order differential equation has a degree equal to 1. All the linear equations in the form of derivatives are in the first order. It has only the first derivative such as dy/dx, where x and y are the two variables and is represented as: dy/dx = f(x, y) = y'

Second-Order Differential Equation

The equation which includes second-order derivative is the second-order differential equation. It is represented as; d/dx(dy/dx) = d^2y/dx^2 = f''(x) = y''.

Types of Differential Equations

Differential equations can be divided into several types namely

- Ordinary Differential Equations
- Partial Differential Equations
- Linear Differential Equations
- Nonlinear differential equations
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- Nonhomogeneous Differential Equations

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2. Answer: d

Explanation:

$x^2 - 4x + [x] + 3 = x[x]$
$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$
$\Rightarrow (x-1)(x-3) = [x].(x-1)$
$\Rightarrow x = 1 \text{ or } x - 3 = [x]$
$\Rightarrow x - [x] = 3$
$\Rightarrow \{x\} = 3 \text{ (Not Possible)}$
Only one solution $x = 1$ in $(-\infty, \infty)$

Concepts:

1. Complex Numbers and Quadratic Equations:

Complex Number: Any number that is formed as a+ib is called a complex number. For example: 9+3i,7+8i are complex numbers. Here i = -1. With this we can say that i^2 = 1. So, for every equation which does not have a real solution we can use i = -1.

Quadratic equation: A polynomial that has two roots or is of the degree 2 is called a quadratic equation. The general form of a quadratic equation is $y=ax^2+bx+c$. Here a≠0, b and c are the real numbers.

Equations	Detailed Explanations
$3x^2 + 4x + 6 = 0$	In this expression, the known values a = 3, b = 4 and c = 6; while x remains the unknown factor.
$2x^2 - 6x = 0$	Here, the known factors a = 2 and b = 6. However, can you ascertain the value of c? Well, the value of c = 0 as it is not present.
$7x - 4 = 0$	Here the value of a is equal to zero since the equation is not quadratic.

3. **Answer: 25 - 25**

Explanation:

The correct answer is 25.

$\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right)=2$

$\Rightarrow \frac{9^{2\alpha-4}+13}{\frac{5}{2}3^{2\alpha-4}+1}=4$

$\Rightarrow \alpha = 2$ or

$\sum_{\alpha\in S}\alpha = 5$ and $\sum_{\alpha\in S}(\alpha+1)^2=25$

$\Rightarrow x^2-50x+25\beta=0$ has real roots

$\Rightarrow \beta \le 25$

$\Rightarrow \beta_{\max}=25$

Concepts:

1. **Types of Differential Equations:**

There are various types of Differential Equation, such as:

Ordinary Differential Equations:

Ordinary Differential Equations is an equation that indicates the relation of having one independent variable x, and one dependent variable y, along with some of its other derivatives.

$$F(\tfrac{dy}{dt},y,t)=0$$

Partial Differential Equations:

A partial differential equation is a type, in which the equation carries many unknown variables with their partial derivatives.

1. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
2. $u_{xx} + u_{yy} = 0$
3. $ux\frac{\partial^2 u}{\partial x^2} + u^2 xy\frac{\partial^2 u}{\partial x \partial y} + uy\frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + u^3 = 0$
4. $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$

Linear Differential Equations:

It is the linear polynomial equation in which derivatives of different variables exist. Linear Partial Differential Equation derivatives are partial and function is dependent on the variable.

Linear Differential Equation in y

$$\frac{dy}{dx} + Py = Q$$

Linear Differential Equation in x

$$\frac{dx}{dy} + P_1 x = Q_1$$

Homogeneous Differential Equations:

When the degree of f(x,y) and g(x,y) is the same, it is known to be a homogeneous differential equation.

$$\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$$

Read More: Differential Equations

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4. Answer: d

Explanation:

The correct answer is (D) : 0

$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Let $x + \frac{1}{x} = t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$
$3t(t-1) + 1(t-1) = 0$
$(t-1)(3t+1) = 0$
$t = 1, -\frac{1}{3}$
$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow$ No solution.

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5. Answer: d

Explanation:

The correct answer is (D) : $2e^2$
$(x^2 - 3y^2)\, dx + 3xydy = 0$
$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y} \ldots\ldots(1)$
Put $y = vx$
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
(1) $\Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$
$\Rightarrow vdv = \frac{-1}{3x}$

Integrating both side

$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$

$y(1) = 1$

$\Rightarrow \frac{1}{2} = c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$

$\Rightarrow y^2 = -\frac{2}{3}x^2\ln x + x^2$

$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow 6y^2(e) = 2e^2$

Concepts:

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Orders of a Differential Equation

First Order Differential Equation

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Second-Order Differential Equation

The equation which includes second-order derivative is the second-order differential equation. It is represented as; d/dx(dy/dx) = d^2y/dx^2 = f″(x) = y″.

Types of Differential Equations

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6. **Answer: b**

Explanation:

$\frac{dy}{dx} = -\frac{\alpha y}{x}$

$\frac{dy}{y} = -\frac{\alpha}{x}dx$

$\Rightarrow \frac{dy}{y} + \frac{\alpha}{x}dx = 0$

$\Rightarrow \ell ny + \alpha \ell nx = \ell nc$

$\Rightarrow yx^{\alpha} = c$

For $(1, 2) \Rightarrow 2.1^{\alpha} = c \Rightarrow c = 2$

For $(8, 1) \Rightarrow 1.8^{\alpha} = 2 \Rightarrow \alpha = \frac{1}{3}$

$\therefore$ curve is $y = 2x^{-1/3}$

At $x = 1/8, y(1/8) = 2\left(\frac{1}{8}\right)^{-\frac{1}{3}} \Rightarrow y = 4$

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7. **Answer: c**

Explanation:

$\alpha x = e^{x^{\beta} \cdot y^{7}}$

$2x^2 y \frac{dy}{dx} = 1 - x \cdot y^2 \quad y^2 = t$

$x^2 \frac{dt}{dx} = 1 - xt$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \quad \text{I.F.} = e^{(nx} = x$

$t(x) = \int \frac{1}{x^2} \cdot x dx$

$y^2 \cdot x = \ell nx + C$

$\therefore 2 \cdot \ln 2 = \ell n2 + C$

$\therefore C = \ln 2$

Hence, $xy^2 = \ell n2x$

$\therefore 2x = e^{x \cdot y^2}$

Hence $\alpha = 2, \beta = 1, \gamma = 2$

Concepts:

1. General Solutions to Differential Equations:

A relation between involved variables, which satisfy the given differential equation is called its solution. The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution and the solution free from arbitrary constants is called particular solution.

For example,

Read More: Formation of a Differential Equation

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8. **Answer: c**

Explanation:

$\frac{dy}{dx}=\frac{1-xy}{x^3}=\frac{1}{x^3}-\frac{y}{x^2}$
$\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$
$\text{If }=e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$
$y\cdot e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}\cdot\frac{1}{x^3}dx\left(\text{put }-\frac{1}{x}=t\right)$
$y\cdot e^{-\frac{1}{x}}=-\int e^t\cdot tdt$
$y=\frac{1}{x}+1+Ce^{\frac{1}{x}}$
Where C is constant
Put $x=\frac{1}{2}$
$3-e=2+1+Ce^2$
$C=-\frac{1}{e}$
$y(1)=1$

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9. Answer: a

Explanation:

$$\frac{dy}{dx} - \frac{y}{x} = y^3\left(1 + \log_e x\right)$$
$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{xy^2} = 1 + \log_e x$$
$$\text{Let } -\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$
$$\therefore \frac{dt}{dx} + \frac{2t}{x} = 2\left(1 + \log_e x\right)$$
$$\text{I.F. } = e^{\int \frac{2}{x}dx} = x^2$$
$$\frac{-x^2}{y^2} = \frac{2}{3}\left(\left(1 + \log_e x\right)x^3 - \frac{x^3}{3}\right) + C$$
$$y(1) = 3$$
$$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3\left(2 + \log_e x^3\right)}$$

OR

$$xdy = ydx + x^3\left(1 + \log_e x\right)dx$$
$$\frac{xdy - ydx}{y^3} = x\left(1 + \log_e x\right)dx$$
$$-\frac{x}{y}d\left(\frac{x}{y}\right) = x^2\left(1 + \log_e x\right)dx$$
$$-\left(\frac{x}{y}\right)^2 = 2\int x^2\left(1 + \log_e x\right)dx$$

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10. **Answer: a**

Explanation:

The correct option is (A): 7(e-1)

$$\frac{dy}{y+7} = dx$$

$$\Rightarrow \quad \log|y+7| = x + c$$

$$y(0) = 0$$

$$\Rightarrow \quad c = \log 7$$

$$\log|y+7| = x + \log 7$$

Now put $x = 1$

$$\log|y+7| = 1 + \log 7$$

$$|y+7| = 7e$$

$$\therefore \quad y = 7(e-1)$$

Concepts:

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