Three Dimensional Geometry JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Three Dimensional Geometry

1. The image of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ in the plane $2x-y+z+3=0$ is the line **(+4, -1)**

   a. $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

   b. $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z-2}{5}$

   c. $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

   d. $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

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2. The equation of conic is $19x^2+15y^2=285$. A concentric circle with radius 4 units is given then angle of common tangent made by minor axis of ellipse is **(+4, -1)**

   a. $\frac{\Pi}{3}$

   b. $\frac{\Pi}{2}$

   c. $\frac{\Pi}{6}$

   d. $\frac{\Pi}{4}$

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3. The equation of the plane containing the $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is : **(+4, -1)**

   a. x + 2y - 2z = 0

   b. x - 2y + z = 0

   c. 5x + 2y - 4z = 0

   d. 3x + 2y - 3z = 0

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4. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **(+4, -1)**

   a. $\frac{3}{2}$

   b. $\frac{5}{2}$

c. $\frac{7}{2}$

d. $\frac{9}{2}$

5. The equation of the plane containing the lines $2x - 5y + z = 3, x + y + z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is **(+4, -1)**

a. $2x + 6y + 12z = 13$

b. $x + 3y + 6z = -7$

c. $x + 3y + 6z = 27$

d. $2x + 6y + 12x = -13$

6. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is : **(+4, -1)**

a. $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 1$

b. $x + y + z = 6$

c. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{6}$

d. $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

7. ABC is a triangle in a plane with vertices $A(2,3,5), B(-1,3,2)$ and $C(\lambda,5,\mu)$. If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is : **(+4, -1)**

a. 1130

b. 1348

c. 676

d. 1077

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8. Equation of the line of the shortest distance between the lines $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$ is : **(+4, -1)**

a. $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$

b. $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{-2}$

c. $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{-1}$

d. $\frac{x}{-2}=\frac{y}{1}=\frac{z}{2}$

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9. The point of intersection C of the plane $8x+y+2z=0$ and the line joining the points $A(-3,-6,1)$ and $B(2,4,-3)$ divides the line segment AB internally in the ratio $k:1$ If $a,b,c(|a|,|b|,|c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$, then $|a+b+c|$ is equal to ___ **(+4, -1)**

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10. Let $\alpha x+\beta y+yz=1$ be the equation of a plane passing through the point $(3,-2,5)$ and perpendicular to the line joining the points $(1,2,3)$ and $(-2,3,5)$ Then the value of $\alpha\beta y$ is equal to ____ **(+4, -1)**

Answers

1. **Answer: a**

Explanation:

Here, plane, line and its image are parallel to each other. So, find any point on the normal to the plane from which the image line will be passed and then find equation of image line.
Here, plane and line are parallel to each other.
Equation of normal to the plane through the point (1, 3, 4) is
$ \hspace40mm \frac{x-1}{2}=\frac{y-3}{-1} = \frac{z-4}{1} = k \, \, \, \, \, $ [say]
Any point in this normal is (2k + 1, - k + 3,4 + k).
Then, $\left(\frac{2k+1+1}{2}, \frac{3-k+3}{2}, \frac{4+k+4}{2}\right)$ lies on plane.
$\Rightarrow 2(k+1) - \left(\frac{6-k}{2}\right) + \left(\frac{8+k}{2}\right) + 3 = 0 \;\Rightarrow\; k = -2$
Hence, point through which this image pass is
$(2k+1, 3-k, 4+k)$
$i.e. \; [2(-2)+1, 3+2, 4-2] = (-3, 5, 2)$
Hence, equation of image line is $ \hspace40mm\frac{x+3}{3}=\frac{y-5}{1}= \frac{z-2}{-5}$

Concepts:

1. Introduction to Three Dimensional Geometry:

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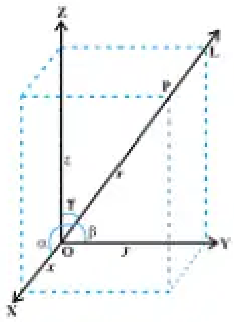
Direction Cosines and Direction Ratios of Line

Let's consider line 'L' is passing through the three-dimensional plane. Now, x,y, and z are the axes of the plane, and α,β, and γ are the three angles the line making with these axes. These are called the plane's direction angles. So, correspondingly, we can very well say that cosα, cosβ, and cosγ are the direction cosines of the given line L.



Read More: Introduction to Three-Dimensional Geometry

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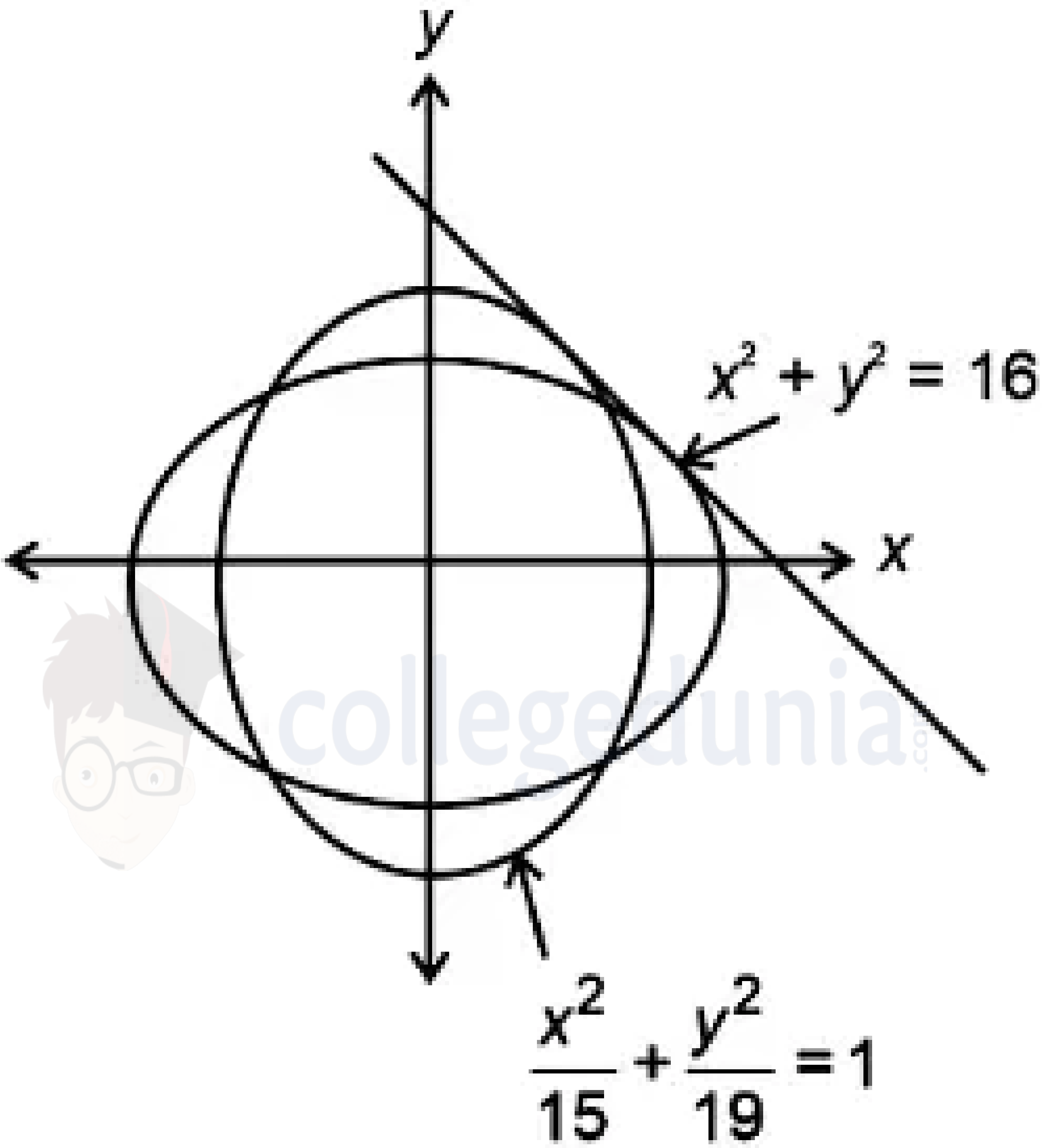
2. **Answer: a**

Explanation:

The correct option is (A): $\frac{\Pi}{3}$



y
x
$x^2 + y^2 = 16$
$\frac{x^2}{15} + \frac{y^2}{19} = 1$

$$19x^2 + 15y^2 = 285$$

$$\Rightarrow \quad \frac{x^2}{15} + \frac{y^2}{19} = 1$$

Let equation of tangent of ellipse be

$$y = mx \pm \sqrt{15m^2 + 19} \quad \ldots(i)$$

If it is tangent to $x^2 + y^2 = 16$ then

$$\left|\frac{\sqrt{15m^2 + 19}}{\sqrt{m^2 + 1}}\right| = 4$$

$$15m^2 + 19 = 16m^2 + 16$$

$$m^2 = 3$$

$$\boxed{m = \pm\sqrt{3}}$$

$\therefore$ angle made by the tangent by minor axis (x- axis) will be = $\pi/3$

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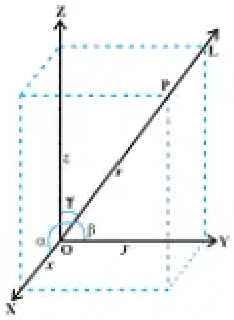

Read More: Introduction to Three-Dimensional Geometry

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3. **Answer: b**

Explanation:

Vector along the normal to the plane containing the lines
$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$
is $\left(8\hat{i} - \hat{j} - 10\hat{k}\right)$
vector perpendicular to the vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $8\hat{i} - \hat{j} - 10\hat{k}$ is $26\hat{i} - 52\hat{j} + 26\hat{k}$
so, required plane is $26x - 52y + 26z = 0$
$x - 2y + z = 0$

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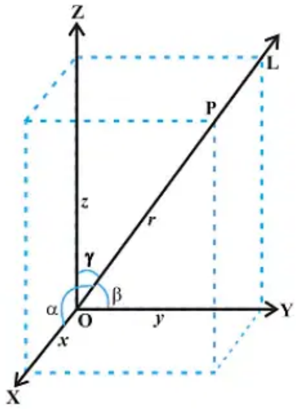

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4. **Answer: c**

Explanation:

Given planes are

$4x + 2y + 4z + 5 = 0$ ....(1)

$4x + 2y + 4z - 16 = 0$ ....(2)

$\left[d = \frac{|c_1 - c_2|}{\sqrt{16+4+116}}\right] \Rightarrow d = \left|\frac{5+16}{\sqrt{16+4+16}}\right|$

$= \frac{21}{6} = \frac{7}{2}$

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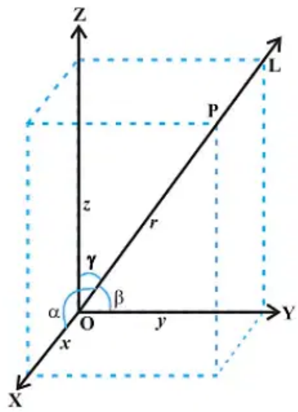

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5. **Answer: c**

Explanation:

The correct option is(C): $x+3y+6z=27$.

The given lines are:

1. 2x - 5y + z = 3
2. x + y + z = 5

$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$,

where (A, B, C) is the direction vector of the plane, and (x_0, y_0, z_0) is a point on the plane.

Plugging in the values, we get:

$1(x-3)+3(y+2)+6(z-4)=0$,

which simplifies to:

$x+3y+6z=27$.

So, the equation of the plane containing the lines 2x - 5y + z = 3 and x + y + z = 5, and parallel to the plane x + 3y + 6z = 1, is

$x+3y+6z=27$.

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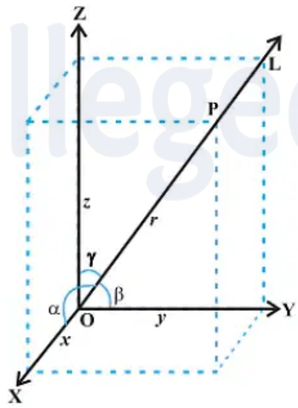

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6. **Answer: d**

Explanation:

Let plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
it passes through (3,2,1) $\therefore \frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$
Now A (a,0,0), B (0, b, 0), C (0,0,c)
$\therefore$ Locus of point of intersection of planes x = a
y = b, z = c is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

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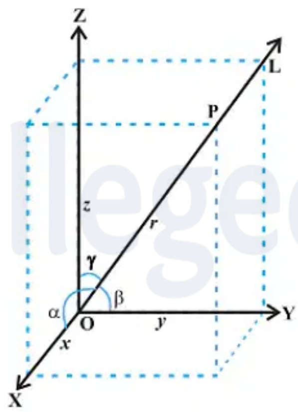

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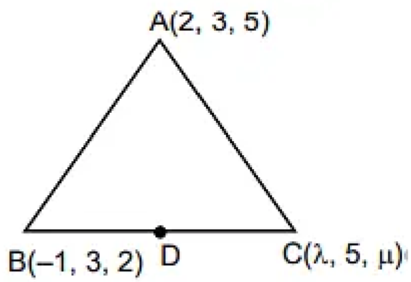
7. Answer: b

Explanation:



$D \equiv \left(\frac{-1+\lambda}{2}, 4, \frac{2+\mu}{2}\right)$

direction cosine of $AD = \left\{\frac{-1+\lambda}{2} - 2, 4-3, \frac{2+\mu}{2} - 5\right\}$

$\{\frac{-1+\lambda}{2}-2, 4-3, \frac{2+\mu}{2}-5\}$
$\overline{AD}=\frac{\lambda-5}{2}i+j+\frac{\mu-8}{2}\hat{k}$
$\Rightarrow \frac{\left(\frac{\lambda-5}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2+1^2+\left(\frac{\mu-8}{2}\right)^2}}=\frac{1}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}}=\frac{\left(\frac{\mu-8}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}}$
$\overline{AD}\cdot i=\overline{AD}\cdot\hat{j}=\overline{AD}\cdot\hat{k}$
$\lambda=7, \mu=10$
$\lambda^3+\mu^3+5=343+1000+5=1348$

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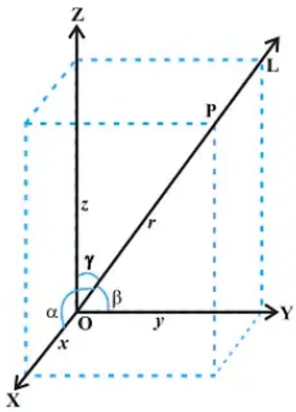

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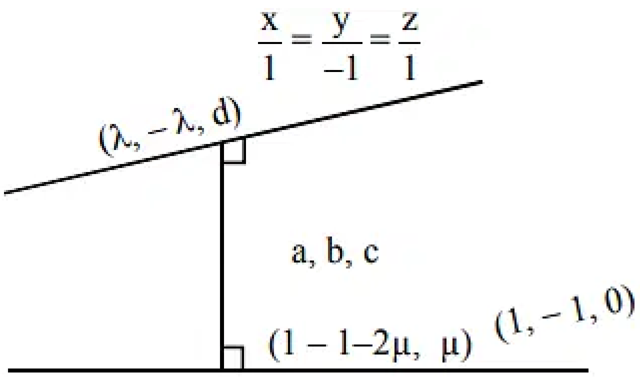
8. **Answer: b**

Explanation:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 0 & -2 & 1 \end{vmatrix}$$

$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$



$\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$

$\frac{1-\lambda}{1} = \frac{-1-2\mu+\lambda}{-1} = \frac{\mu-\lambda}{-2}$

$-2 = 3\lambda + \mu$

$-1 + \lambda = -1 - 2\mu + \lambda$

$\mu = 0$

$\lambda = -\frac{2}{3}$

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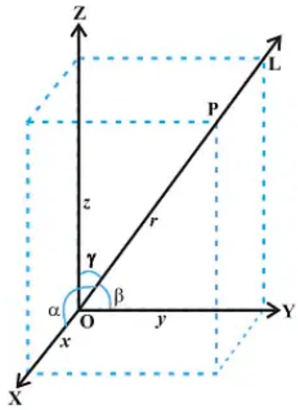


9. **Answer: 10 - 10**

Explanation:

The correct answer is 10.

Plane : $8x + y + 2z = 0$

Given line $AB : \frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

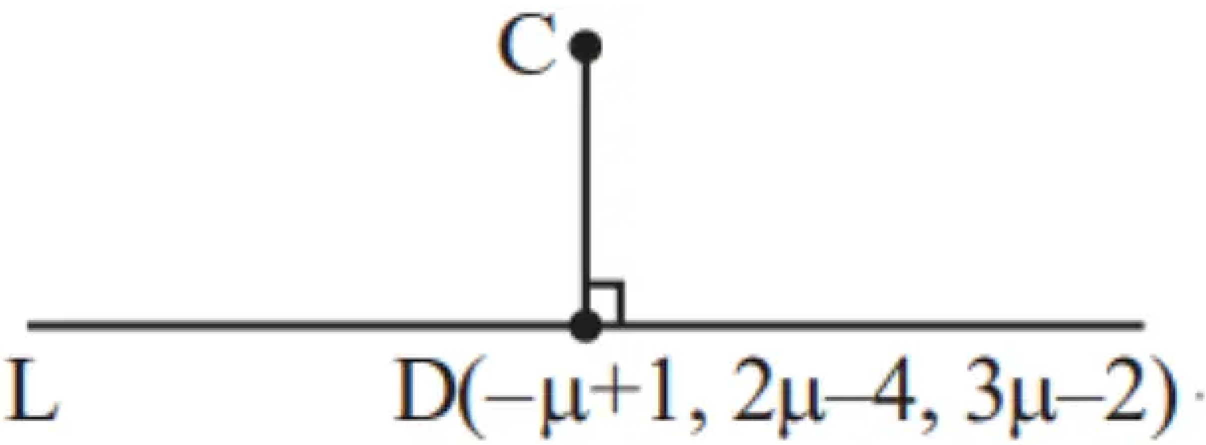
Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

Point of intersection of line and plane $8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$L : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overrightarrow{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

$\text{Direction ratios } \rightarrow (-1, -26, 17)$

$la + b + cl = 10$

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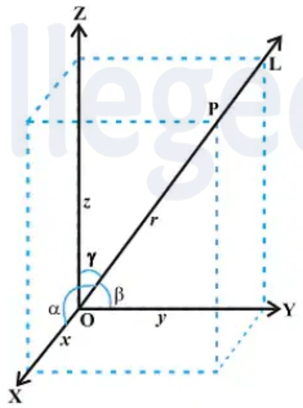

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10. Answer: 6 - 6

Explanation:

The correct answer is 6.
Given Equation is not equation of plane as yy is present. If we consider y is γ then answer would be 6 .
Normal vector of plane $= 3\hat{i} - \hat{j} - 2\hat{k}$
Plane : $3x - y - 2z + \lambda = 0$
Point $(3, -2, 5)$ satisfies the plane
$\lambda = -1$

$$3x - y - 2z = 1$$
$$\alpha\beta y = 6$$

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