Three Dimensional Geometry JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Three Dimensional Geometry

1. If the image of point P(1, 2, 3) about the plane 2x - y + 3z = 2 is Q, then the area of triangle PQR, where coordinates of R is (4, 10, 12) **(+4, -1)**

   a. $\sqrt{\frac{1531}{2}}$

   b. $\sqrt{\frac{1675}{2}}$

   c. $\sqrt{\frac{2443}{2}}$

   d. $\sqrt{\frac{1784}{2}}$

---

2. A rectangular parallelepiped with edges along x, y, z axis has length 3, 4, 5 respectively. Fiind the shortest distance of the body diagonal from one of the edges parallel to z-axis which is skew to the diagonal **(+4, -1)**

   a. $\frac{16}{5}$

   b. $\frac{15}{\sqrt{34}}$

   c. $\frac{12}{5}$

   d. $\frac{9}{5}$

---

3. If $A(1,1,1), B(0,\lambda,0), C(\lambda+1,0,1), D(2,-2,2)$ are co-planer then $\sum(\lambda_i+2)^2$ is equal to **(+4, -1)**

   a. $\frac{80}{3}$

   b. $\frac{320}{9}$

   c. $\frac{160}{9}$

   d. $\frac{160}{3}$

---

4. Plane P_3 is passing through (1,1,1) and line of intersection of P_1 and P_2 where $P_1 : 2x - y + z = 5$ and $P_2 : x + 3y + 2z + 2 = 0$. Then distance of (1,1,10) from P_3 is: **(+4, -1)**

a. $\frac{53}{85}$

b. $\sqrt{85}$

c. $\frac{52}{\sqrt{85}}$

d. 53

---

5. Shortest distance between lines $\frac{(x-5)}{4} = \frac{(y-3)}{6} = \frac{(z-2)}{4}$ and $\frac{(x-3)}{7} = \frac{(y-2)}{5} = \frac{(z-9)}{6}$ is ? **(+4, -1)**

a. $\frac{190}{37}$

b. $\frac{190}{\sqrt{756}}$

c. $\frac{37}{190}$

d. $\frac{756}{\sqrt{190}}$

---

6. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$ Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ______ **(+4, -1)**

---

7. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4), a \in N$ If the volume of the tetrahedron $OABC$ is 144 unit ^{3}, then which of the following points is NOT on P ? **(+4, -1)**

a. $(0, 4, 4)$

b. $(3, 0, 4)$

c. $(0, 6, 3)$

d. $(2, 2, 4)$

---

8. The area of the quadrilateral having vertices as (1,2), (5,6), (7,6), (-1,-6) is? **(+4, -1)**

---

9. Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$ If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to : **(+4, -1)**

a. $\frac{25}{83}$

b. $\frac{250}{83}$

c. $\frac{15}{53}$

d. $\frac{250}{82}$

---

10. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line. **(+4, -1)**

a. $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

b. $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

c. $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

d. $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

Answers

1. **Answer: a**

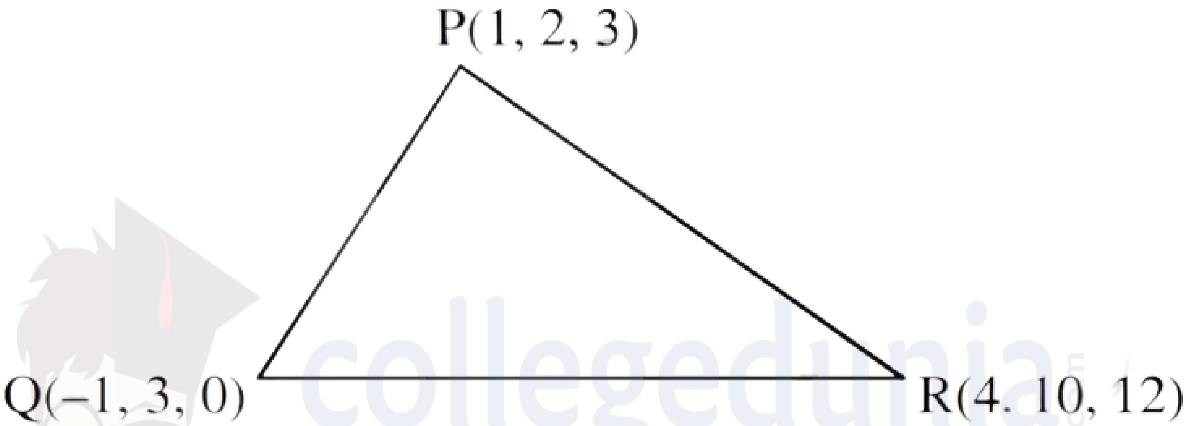
Explanation:

The correct option is (A): $\sqrt{\frac{1531}{2}}$

Image formula w.r.t P

$\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{3}=-2\frac{(2\times1-2+3\times3-2)}{1^2+2^2+3^2}$

$\Rightarrow Q(-1,3,0)$



$Area=\frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 7 & 12\\ 2 & -1 & 3\end{vmatrix}\right|$

$=\frac{1}{2}|33\hat{i}+9\hat{j}-19\hat{k}|$

$Area=\frac{1}{2}\sqrt{(33)^2+9^2+(19)^2}$

$=\frac{1}{2}\sqrt{1089+81+361}=\frac{1}{2}\sqrt{1531}$

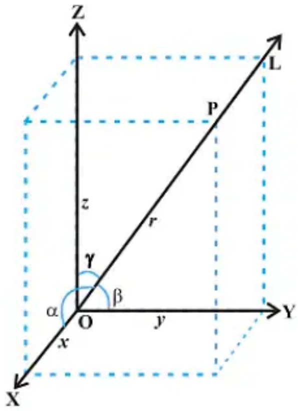
Concepts:

1. Three Dimensional Geometry:

Mathematically, Geometry is one of the most important topics. The concepts of Geometry are derived w.r.t. the planes. So, Geometry is divided into three major categories based on its dimensions which are one-dimensional geometry, two-dimensional geometry, and three-dimensional geometry.

Direction Cosines and Direction Ratios of Line:

Consider a line L that is passing through the three-dimensional plane. Now, x,y and z are the axes of the plane and α,β, and γ are the three angles the line makes with these axes. These are commonly known as the direction angles of the plane. So, appropriately, we can say that cosα, cosβ, and cosγ are the direction cosines of the given line L.

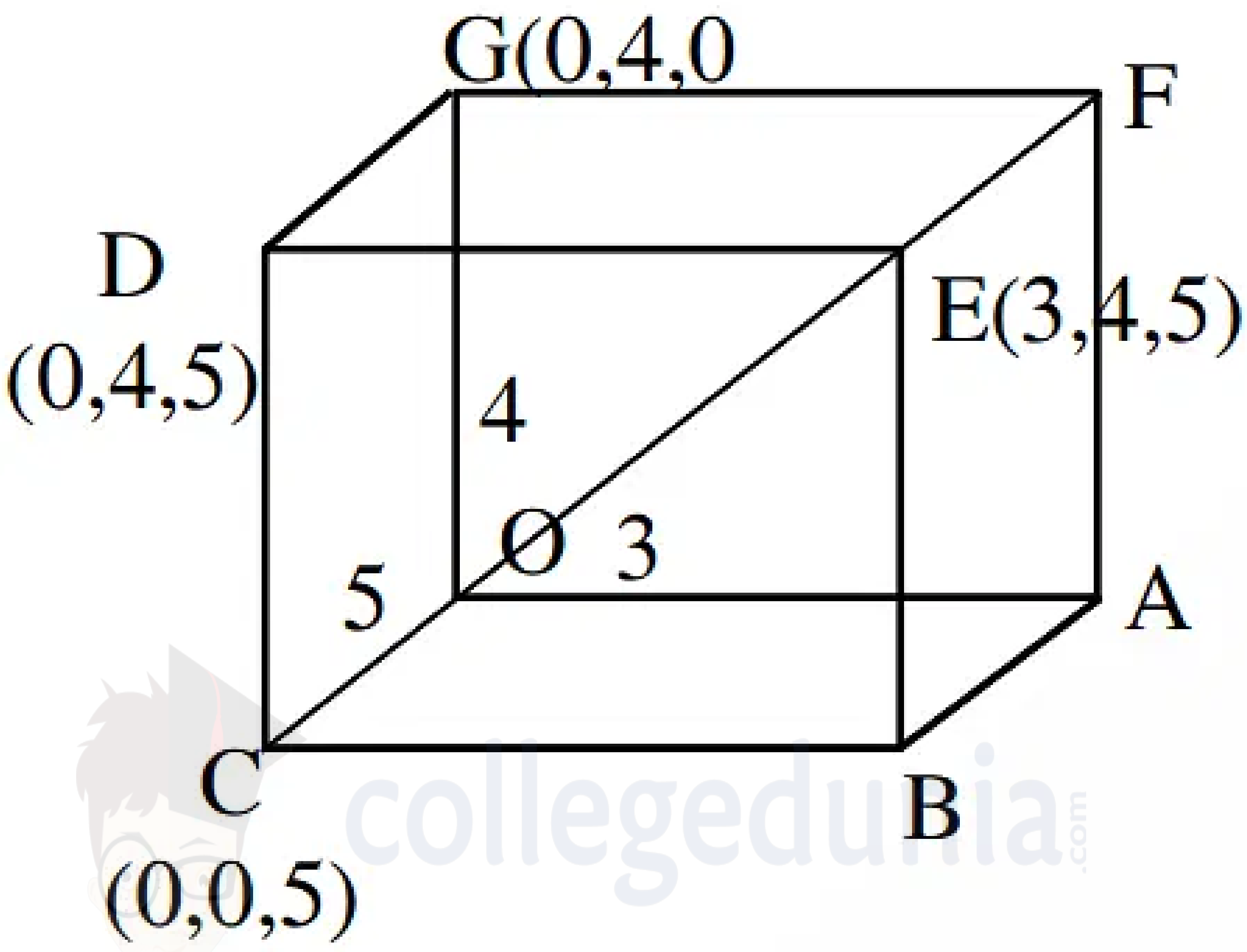


---

2. Answer: c

Explanation:

The equation of diagonal OE $\vec{r}$ = 0+λ($3\hat{i}+4\hat{j}+5\hat{k}$)



equation of edge GD

$\vec{r} = 4\hat{j} + \mu\hat{k}$

shortest distance = |projection of $4\hat{i}$ on $(3\hat{j} - 4\hat{i})$|

$= \frac{12}{\sqrt{9+16}} = \frac{12}{5}$

So, the correct answer is (C): $\frac{12}{5}$

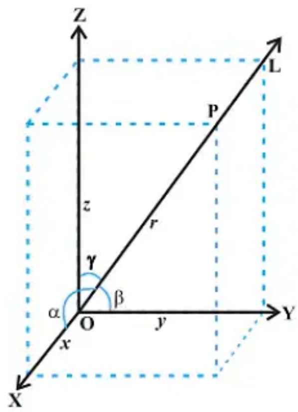
Concepts:

1. Three Dimensional Geometry:

Mathematically, Geometry is one of the most important topics. The concepts of Geometry are derived w.r.t. the planes. So, Geometry is divided into three major categories based on its dimensions which are one-dimensional geometry, two-dimensional geometry, and three-dimensional geometry.

Direction Cosines and Direction Ratios of Line:

Consider a line L that is passing through the three-dimensional plane. Now, x,y and z are the axes of the plane and α,β, and γ are the three angles the line makes with these axes. These are commonly known as the direction angles of the plane. So, appropriately, we can say that cosα, cosβ, and cosγ are the direction cosines of the given line L.



---

3. **Answer: c**

Explanation:

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$=0
$\Rightarrow \lambda = 2, \frac{-2}{3}$
$\sum(\lambda_i + 2)^2$ = 16+ $\frac{16}{9}$ = $\frac{160}{9}$

So, the correct answer is (C): $\frac{160}{9}$

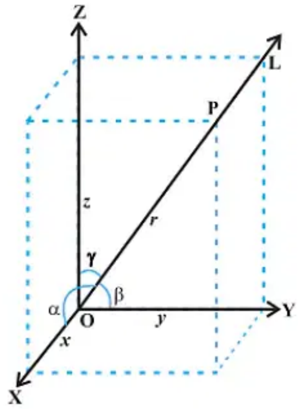
Concepts:

1. Three Dimensional Geometry:

Mathematically, Geometry is one of the most important topics. The concepts of Geometry are derived w.r.t. the planes. So, Geometry is divided into three major categories based on its dimensions which are one-dimensional geometry, two-dimensional geometry, and three-dimensional geometry.

Direction Cosines and Direction Ratios of Line:

Consider a line L that is passing through the three-dimensional plane. Now, x,y and z are the axes of the plane and α,β, and γ are the three angles the line makes with these axes. These are commonly known as the direction angles of the plane. So, appropriately, we can say that cosα, cosβ, and cosγ are the direction cosines of the given line L.



---

4. **Answer: c**

Explanation:

The correct option is (C): $\frac{52}{\sqrt{85}}$

Concepts:

1. Distance of a Point from a Plane:

The shortest perpendicular distance from the point to the given plane is **the distance between point and plane**. In simple terms, the shortest distance from a point to a plane is the length of the perpendicular parallel to the normal vector dropped from the particular point to the particular plane. Let's see the formula for the distance between point and plane.

The distance from (x_0, y_0, z_0) to the plane $Ax + By + Cz + D = 0$ is

$$\text{Distance} = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Read More: Distance Between Two Points

5. **Answer: b**

Explanation:

The correct option is (B): $\frac{190}{\sqrt{756}}$

Concepts:

1. Shortest Distance Between Two Parallel Lines:

Formula to find distance between two parallel line:

Consider two parallel lines are shown in the following form :

$y = mx + c_1$ ...(i)

$y = mx + c_2$ ....(ii)

Here, m = slope of line

Then, the formula for shortest distance can be written as given below:

$d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}$

If the equations of two parallel lines are demonstrated in the following way :

$ax + by + d_1 = 0$

$ax + by + d_2 = 0$

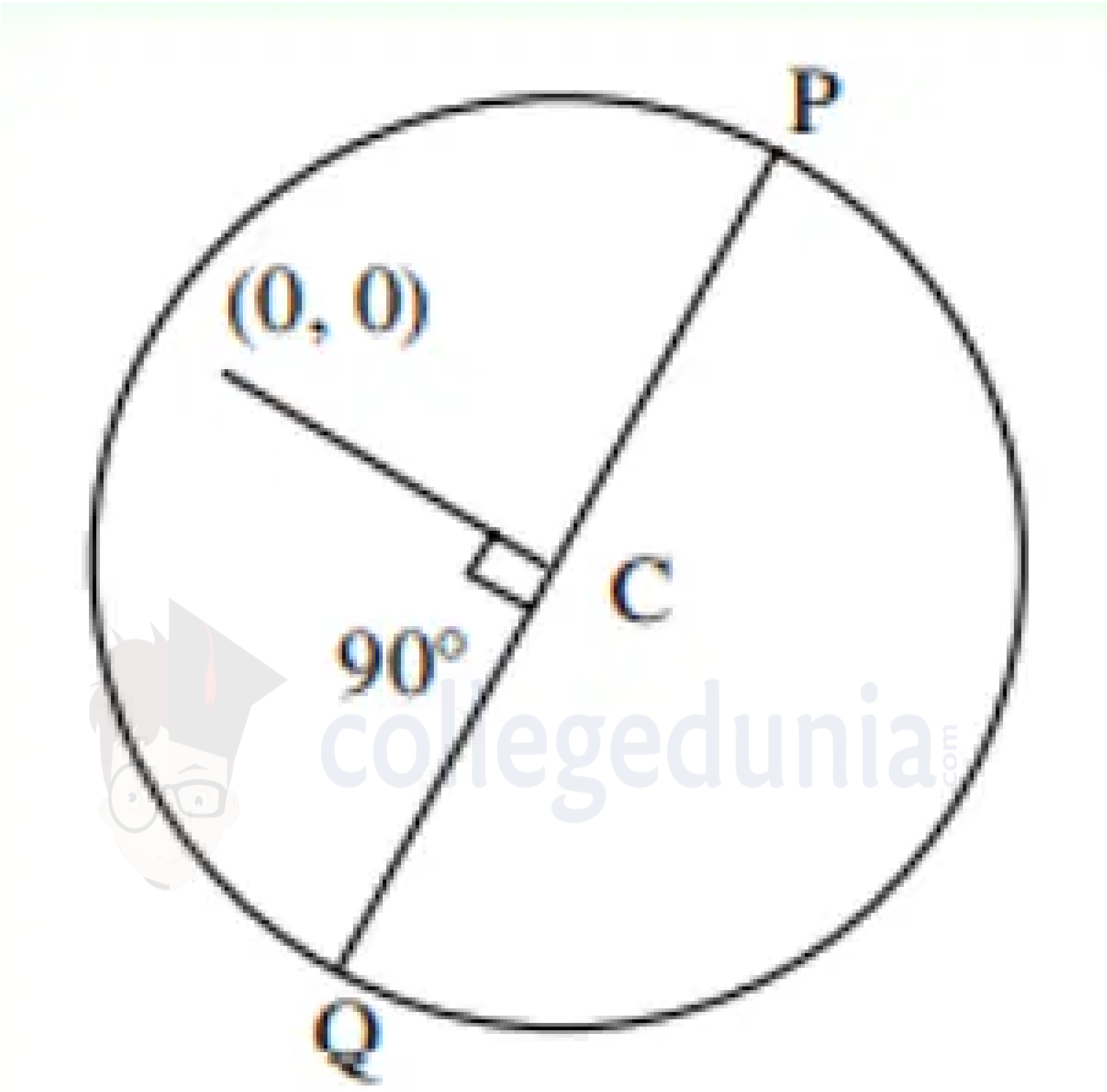
then there is a little change in the formula.

$d = \frac{|d_2 - d_1|}{\sqrt{a^2+b^2}}$

6. **Answer: 24 - 24**

Explanation:

$\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}; PC = \sqrt{7}$



$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$
=2(5+7)=24
So , the correct answer is 24.

Concepts:

1. Circle:

A circle can be geometrically defined as a combination of all the points which lie at an equal distance from a fixed point called the centre. The concepts of the circle are very important in building a strong foundation in units likes mensuration and coordinate geometry. We use **circle formulas** in order to calculate the area, diameter, and circumference of a circle. The length between any point on the circle and its centre is its radius.

Any line that passes through the centre of the circle and connects two points of the circle is the diameter of the circle. The radius is half the length of the diameter of the circle. The area of the circle describes the amount of space that is covered by the circle and the circumference is the length of the boundary of the circle.

Also Check:

Areas Related to Circles	Perimeter and Area of Circle	Circles Revision Notes

---

7. Answer: b

Explanation:

Equation of Plane:
$(2\hat{i} + aj + 4\hat{k}) \cdot [(x-2)\hat{i} + (y-a)\hat{j} + (z-4)\hat{k}] = 0$
$\Rightarrow 2x + ay + 4z = 20 + a^2$
$\Rightarrow A \equiv \left(\frac{20+a^2}{2}, 0, 0\right)$
$B \equiv \left(0, \frac{20+a^2}{a}, 0\right)$
$C \equiv \left(0, 0, \frac{20+a^2}{4}\right)$
$\Rightarrow$ Volume of tetrahedron
$= \frac{1}{6}[\vec{a}\vec{b}\vec{c}]$
$= \frac{1}{6}\vec{a} \cdot (\vec{b} \times \vec{c})$
$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right) \cdot \left(\frac{20+a^2}{a}\right) \cdot \left(\frac{20+a^2}{4}\right) = 144$
$\Rightarrow \left(20 + a^2\right)^3 = 144 \times 48 \times a$
$\Rightarrow a = 2$
$\Rightarrow$ Equation of plane is $2x + 2y + 4z = 24$
Or $x + y + 2z = 12$
$\Rightarrow (3, 0, 4)$ Not lies on the Plane $x + y + 2z = 12$
So , the correct option is (B) : $(3, 0, 4)$

Concepts:

1. Coordinates of a Point in Space:

Three-dimensional space is also named 3-space or tri-dimensional space.

It is a geometric setting that carries three values needed to set the position of an element. In Mathematics and Physics, a sequence of ‘n’ numbers can be acknowledged as a location in ‘n-dimensional space’. When n = 3 it is named a three-dimensional Euclidean space.

The Distance Formula Between the Two Points in Three Dimension is as follows;

The distance between two points P_1 and P_2 are (x_1, y_1) and (x_2, y_2) respectively in the XY-plane is expressed by the distance formula,

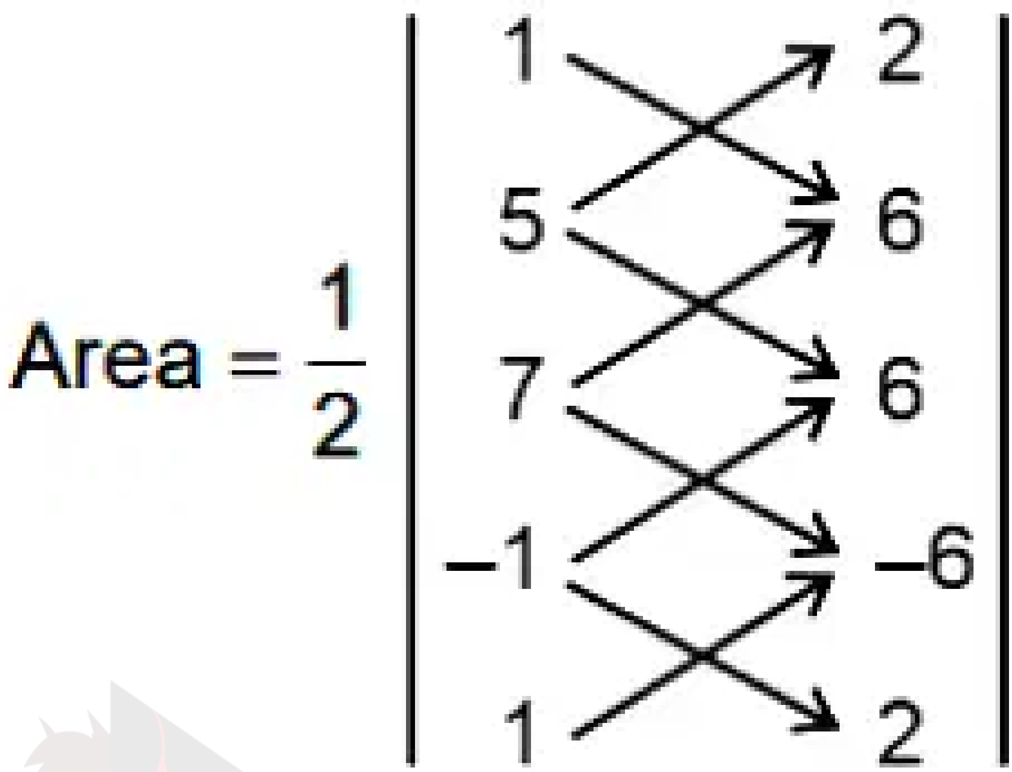
$$d\,(P1, P2) = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

Read More: Coordinates of a Point in Three Dimensions

---

8. **Answer: 24 - 24**

   Explanation:



$= \frac{1}{2}[6 + 30 - 42 - 2 - 10 - 42 + 6 + 6]$

$= \frac{1}{2}[48]$

$= 24$

The correct answer is 24.

Concepts:

1. Coordinates of a Point in Space:

Three-dimensional space is also named 3-space or tri-dimensional space.

It is a geometric setting that carries three values needed to set the position of an element. In Mathematics and Physics, a sequence of 'n' numbers can be acknowledged as a location in 'n-dimensional space'. When n = 3 it is named a three-dimensional Euclidean space.

The Distance Formula Between the Two Points in Three Dimension is as follows;

The distance between two points P_1 and P_2 are (x_1, y_1) and (x_2, y_2) respectively in the XY-plane is expressed by the distance formula,

$$d(P1, P2) = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

Read More: Coordinates of a Point in Three Dimensions

---

9. **Answer: b**

Explanation:

$P \equiv P_1 + \lambda P_2 = 0$

$(2+\lambda)x + (3+2\lambda)y + (3\lambda - 1)z - 2 - 6\lambda = 0$

Plane P is perpendicular to P_3

$\therefore \vec{n} \cdot \vec{n}_3 = 0$

$2(\lambda + 2) + (2\lambda + 3) - (3\lambda - 1) = 0$

$\lambda = -8$

$P \equiv -6x - 13y - 25z + 46 = 0$

$6x + 13y + 25z - 46 = 0$

Dist from $(-7, 1, 1)$

$d = \left|\frac{-42+13+25-46}{\sqrt{36+169+625}}\right| = \frac{50}{\sqrt{830}}$

$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$

Concepts:

1. Distance between Two Points:

The distance between any two points is the length or distance of the line segment joining the points. There is only one line that is passing through two points. So, the distance between two points can be obtained by detecting the length of this line segment joining these two points. The distance between two points using the given coordinates can be obtained by applying the distance formula.

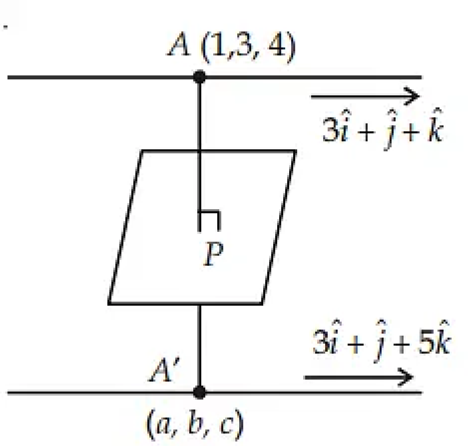
The formula for the distance, d, between two points whose coordinates are (x_1, y_1) and (x_2, y_2) is:

$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

This is called the **Distance Formula.**

10. **Answer: a**

Explanation:



$\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda \Rightarrow a=2\lambda+1$ $b=3-\lambda$ $c=4+\lambda$ $P\equiv\left(\lambda+1, 3-\frac{\lambda}{2}, 4+\frac{\lambda}{2}\right)$ $2(\lambda+1)-\left(3-\frac{\lambda}{2}\right)+\left(4+\frac{\lambda}{2}\right)+3=0$ $2\lambda+2-3+\frac{\lambda}{2}+4+\frac{\lambda}{2}+3=0$ $3\lambda+6=0 \Rightarrow \lambda=-2$ $a=-3, b=5, c=2$ So the equation of the required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

Concepts:

1. **Equation of a Line in Space:**

In a plane, the equation of a line is given by the popular equation **y = m x + C**. Let's look at how the equation of a line is written in vector form and Cartesian form.

Vector Equation

Consider a line that passes through a given point, say 'A', and the line is parallel to a given vector '$\vec{b}$'. Here, the line 'l' is given to pass through 'A', whose position vector is given by '$\vec{a}$'. Now, consider another arbitrary point 'P' on the given line, where the position vector of 'P' is given by '$\vec{r}$'.

$\vec{AP}=\lambda\vec{b}$

Also, we can write vector AP in the following manner:

$\vec{AP}=\vec{OP}-\vec{OA}$

$\lambda\vec{b}=\vec{r}-\vec{a}$

$\vec{a}=\vec{a}+\lambda\vec{b}$

$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$