Three Dimensional Geometry JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Three Dimensional Geometry

1. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is: **(+4, -1)**

a. $\frac{1}{4\sqrt{2}}$

b. $\frac{1}{3\sqrt{2}}$

c. $\frac{1}{2\sqrt{2}}$

d. $\frac{1}{\sqrt{2}}$

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2. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is **(+4, -1)**

a. $20\sqrt{2}$

b. 31

c. $13\sqrt{2}$

d. 26

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3. The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is : **(+4, -1)**

a. $5\sqrt{2}$

b. $4\sqrt{2}$

c. 4

d. 5

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4. Let the image of the point $P(2, -1, 3)$ in the plane $x + 2y - z = 0$ be Q Then the distance of the plane $3x + 2y + z + 29 = 0$ from the point Q is **(+4, -1)**

a. $2\sqrt{14}$

b. $\frac{22\sqrt{2}}{7}$

c. $3\sqrt{14}$

d. $\frac{24\sqrt{2}}{7}$

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5. Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and $P_2 : \hat{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$ Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_4 If α is the angle between L and P_2, then $(\tan^2 \theta)(\cot^2 \alpha)$ is equal to **(+4, -1)**

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6. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L If α is the area of triangle PQR, then α^2 is equal to **(+4, -1)**

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7. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _____ **(+4, -1)**

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8. Let the plane containing the line of intersection of the planes $P1 : x + (\lambda + 4)y + z = 1$ and $P2 : 2x + y + z = 2$ pass through the points $(0, 1, 0)$ and $(1, 0, 1)$ Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P2$ is **(+4, -1)**

a. $5\sqrt{6}$

b. $2\sqrt{6}$

c. $3\sqrt{6}$

d. $4\sqrt{6}$

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9. A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point $P(1, -1, 1)$ If the distance of the plane E from the point $Q(a, a, 2)$ is $3\sqrt{2}$, then $(PQ)^2$ is equal to **(+4, -1)**

a. 9

b. 12

c. 21

d. 33

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10. The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to : **(+4, -1)**

a. $2\sqrt{3}$

b. $\sqrt{14}$

c. 3

d. $\sqrt{6}$

Answers

1. Answer: b

Explanation:

L_1 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j}$

L_2 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$

Also, L_2 passes through $\left(\frac{5}{7}, \frac{8}{7}, 0\right)$

So, required plane is $\begin{vmatrix} x - \frac{5}{7} & y - \frac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$

$\Rightarrow 7x - 7y + 8z + 3 = 0$

Now, perpendicular distance $= \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

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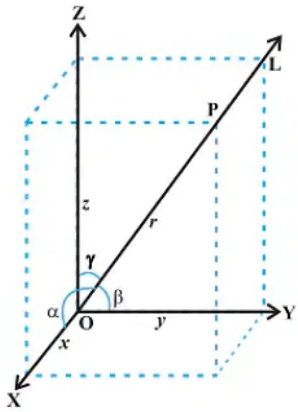
Direction Cosines and Direction Ratios of Line:

Consider a line L that is passing through the three-dimensional plane. Now, x,y and z are the axes of the plane and α,β, and γ are the three angles the line makes with these axes. These are commonly known as the direction angles of the plane. So, appropriately, we can say that cosα, cosβ, and cosγ are the direction cosines of the given line L.



2. **Answer: d**

Explanation:

Equation of line
$3x+1=-4y-9=12z+16$
G.P on line $(3\lambda-1,-4\lambda+9,12\lambda-16)$
point of intersection of line plane
$6\lambda-2-12\lambda+27-12\lambda+16=5$
$\lambda=2$
Point $(5,1,8)$
Distance $=36+64+576=26$

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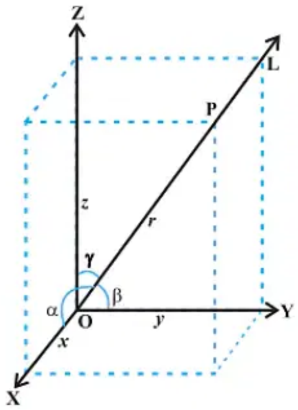

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3. **Answer: a**

Explanation:

$$= \begin{vmatrix} x-2 & -3 & 4 \\ y+3 & 4 & -5 \\ z-1 & -3 & 4 \end{vmatrix} = 0$$

$x - z - 1 = 0$

Distance of $P(7, -3, -4)$ from Plane is

$d = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$

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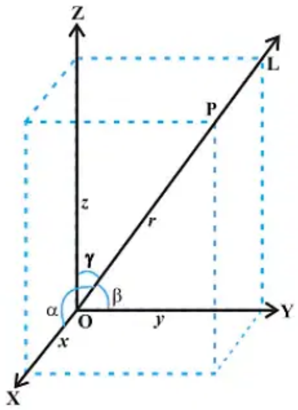
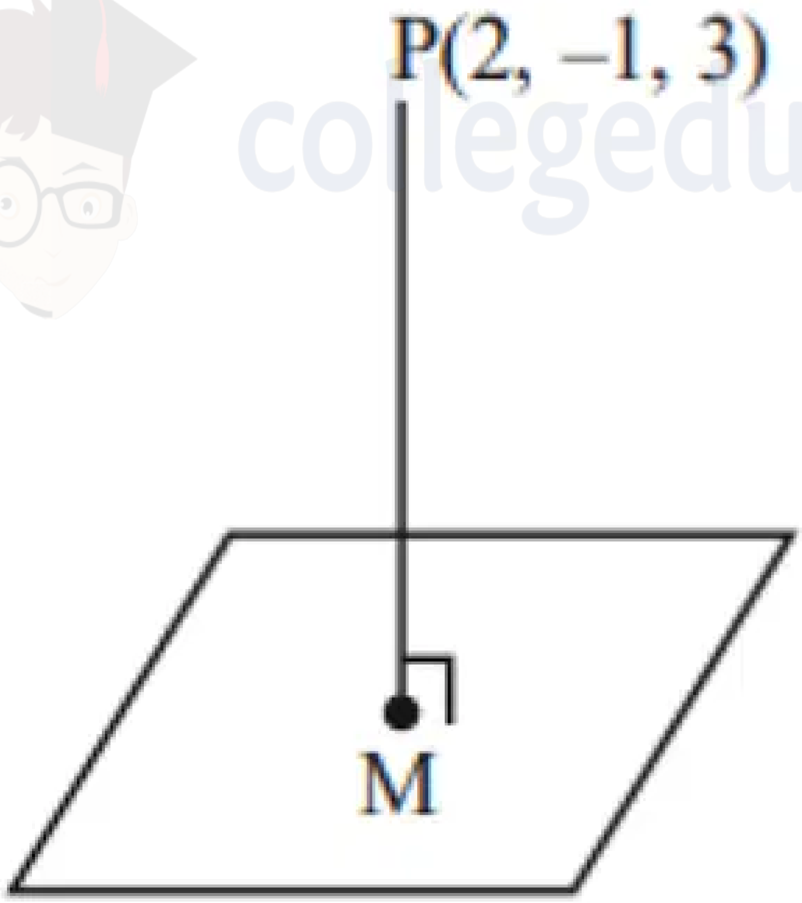


4. **Answer: c**

Explanation:



eq. of line PM 1x−2=2y+1=−1z−3=λ
any point on line=(λ+2,2λ−1,−λ+3)
for point ' m ′ (λ+2)+2(2λ−1)−(3−λ)=0
$\lambda=\frac{1}{2}$

Point m$\left(\frac{1}{2}+2, 2\times\frac{1}{2}, \frac{-1}{2}+3\right)$
=$\left(\frac{5}{2}, 0, \frac{5}{2}\right)$
For Image Q(α,β,γ)
$\frac{\alpha+2}{2}=\frac{5}{2}, \frac{\beta-1}{2}=0$
$\frac{\gamma+3}{2}=\frac{5}{2}$
Q:(3,1,2)
$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$

$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$

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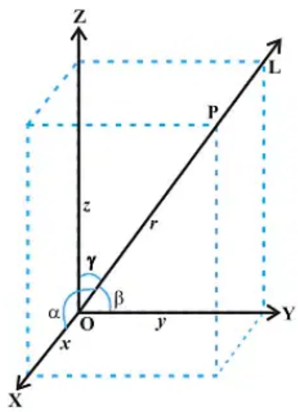

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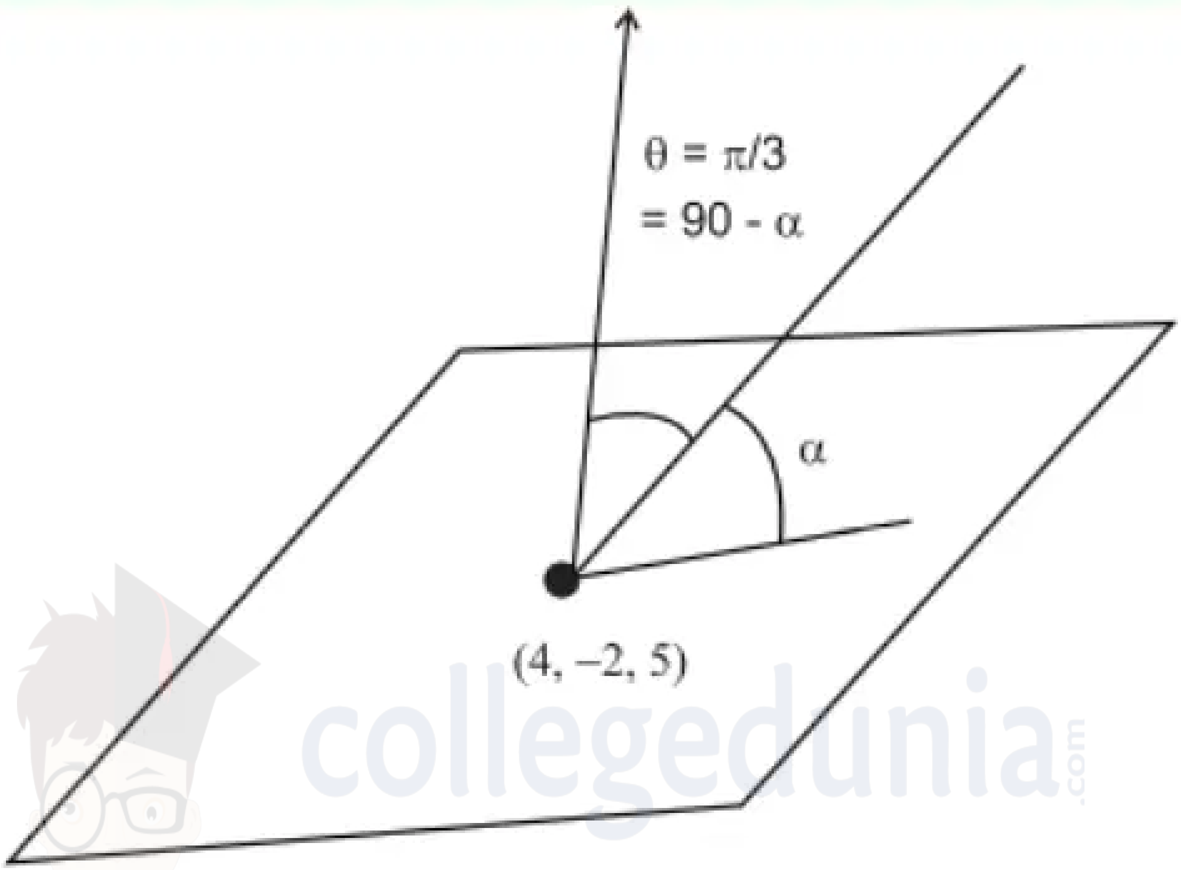
5. **Answer: 9 - 9**

Explanation:

The correct answer is 9.



$\cos\theta = \frac{(\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})}{6} = \frac{2-1+2}{6} = \frac{1}{2}$

$\theta = \pi/3$

$\alpha = \pi/6$

$\left(\tan^2\theta\right)\left(\cot^2\alpha\right)$

$(3)(3) = 9$

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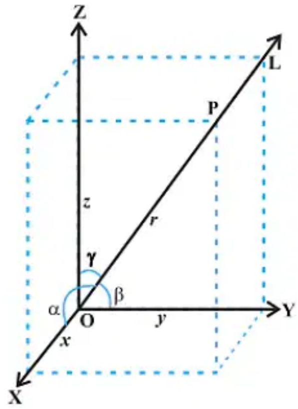

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6. **Answer: 180 - 180**

Explanation:

The correct answer is 180.

Any point on $L((2\lambda+1),(-\lambda-1),(\lambda+3))$

$2(2\lambda+1)+(-\lambda-1)+3(\lambda+3)=16$

$6\lambda+10=16 \Rightarrow \lambda=1$

$\therefore P=(3,-2,4)$

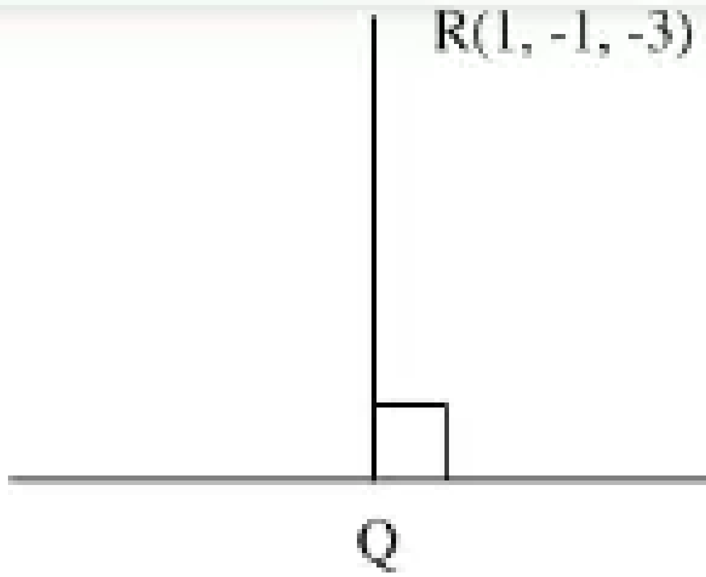
DR of $QR=\langle 2\lambda,-\lambda,\lambda+6\rangle$

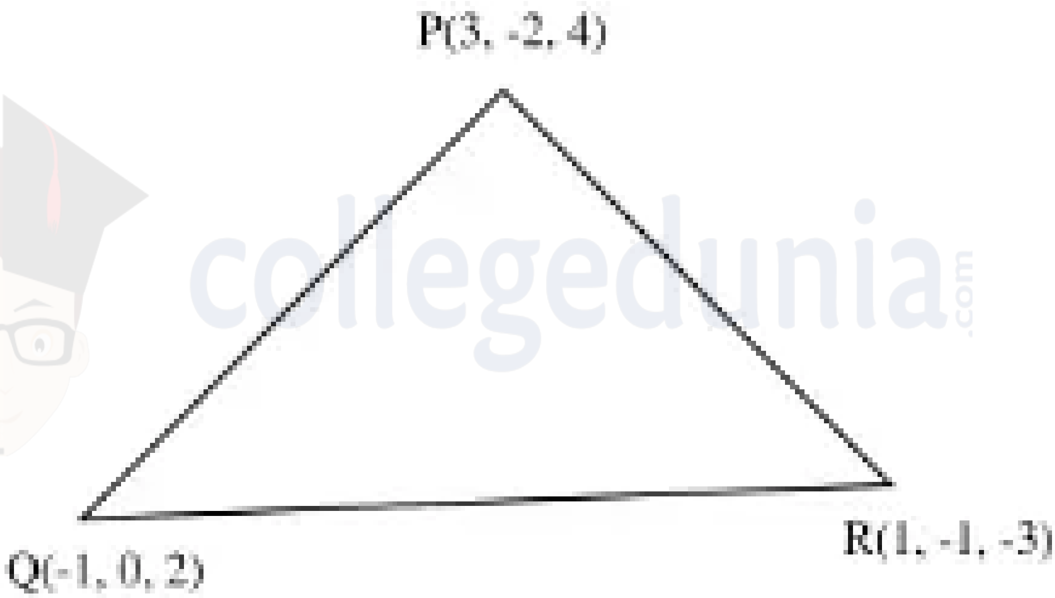
DR of $L=\langle 2,-1,1\rangle$

$4\lambda+\lambda+\lambda+6=0$

$6\lambda+6=0 \Rightarrow \lambda=-1$

$Q=(-1,0,2)$





$\overline{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$ $\overline{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$$

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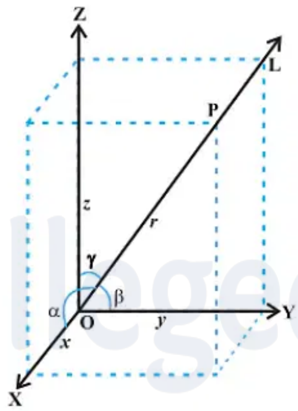

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7. **Answer: 315 - 315**

Explanation:

The correct answer is 315.

$P_1 = \vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 = \vec{r}.(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$

$\theta = sin^{-1}(\frac{2\sqrt{6}}{5})$

$\Rightarrow sin\theta = \frac{2\sqrt{6}}{5}$

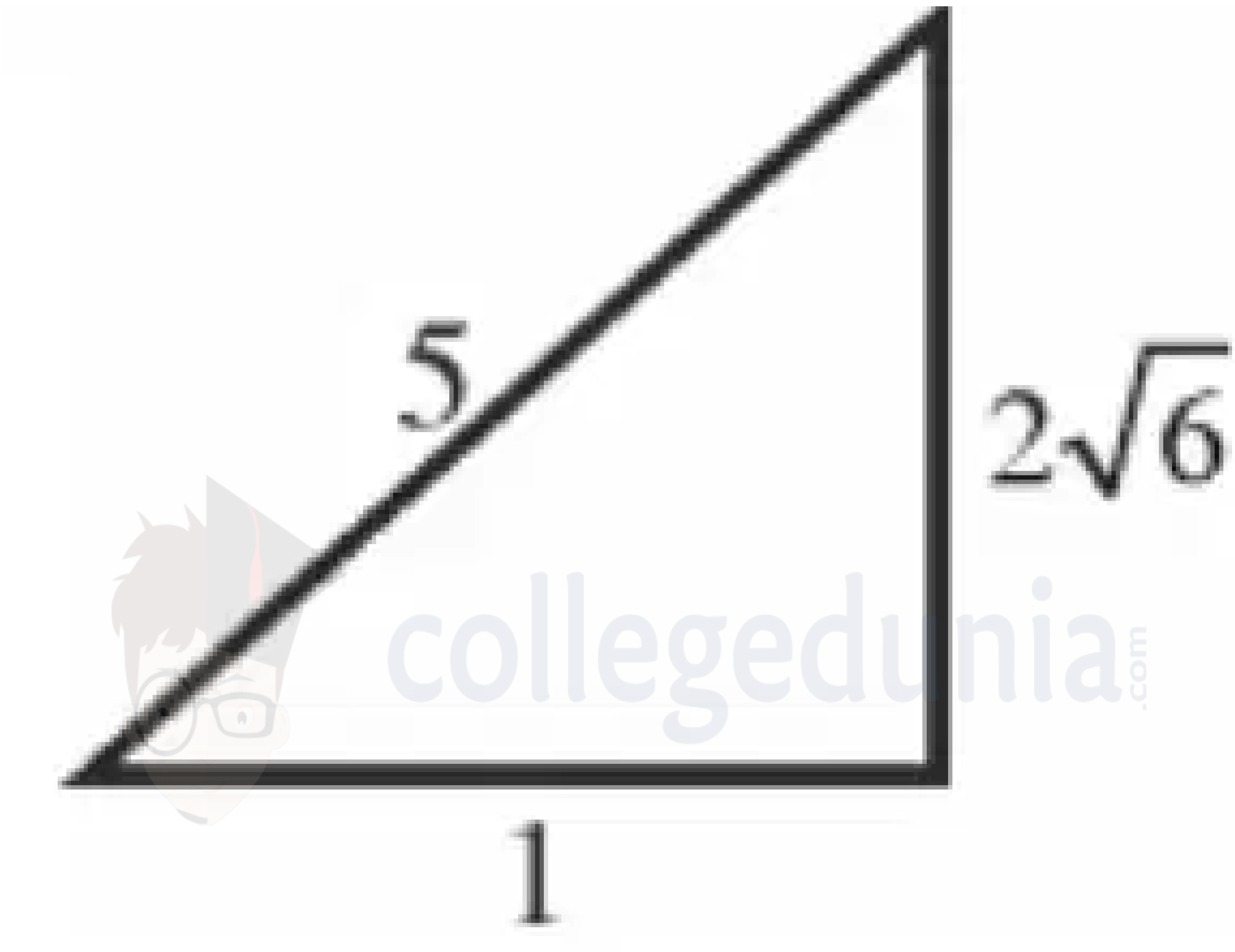
$\therefore cos\theta = \frac{1}{5}.$

$cos\theta = \frac{\vec{r}.\vec{r}}{|\vec{r}||\vec{r_2}|}$

$= \frac{(3i-5j+k)(\lambda i+j-3k)}{\sqrt{35}.\sqrt{\lambda^2+10}}$

$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125$=0

$\Rightarrow x = 5, \frac{25}{19}$



Perpendicular distance of point

$(38\lambda_1, 10\lambda_2, 2) = (50, 50, 2)$ from plane P_1

$= \frac{|30\times50-5\times50+2-7}{\sqrt{35}}$

$Square = \frac{105\times105}{35} = 315$

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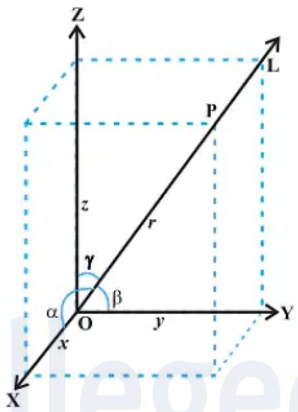

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8. Answer: c

Explanation:

The correct answer is (C) : $3\sqrt{6}$

Equation of plane passing through point of intersection of $P1$ and $P2$

$P = P1 + kP2$

$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$

Passing through $(0, 1, 0)$ and $(1, 0, 1)$

$(\lambda + 4 - 1) + k(1 - 2) = 0$

$(\lambda + 3) - k = 0.....$(1)

Also passing $(1, 0, 1)$

$(1 + 1 - 1) + k(2 + 1 - 2) = 0$

$1 + k = 0$

$k = -1$

put in (1)

$\lambda + 3 + 1 = 0$

$\lambda = -4$

Then point $(2\lambda, \lambda, -\lambda)$

$d = \left|\frac{-16-4,-4,4)}{\sqrt{6}}\right|$

$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$

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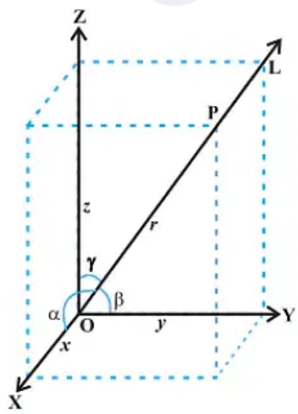

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9. Answer: c

Explanation:

Let equation of plane be $a(x-1)+b(y+1)+c(z-1)=0$.....(1)

It is perpendicular to the given two planes

$2a - 2b + c = 0$

$a-b+2c=0$
$\Rightarrow \frac{a}{3}=\frac{b}{3}=\frac{c}{0}$
Equation of plane be $x+y=0$
Now $\frac{|a+a|}{\sqrt{2}}=3\sqrt{2}\Rightarrow|2a|=6\Rightarrow a=\pm 3$
$P(3,3,2)$ or $P(-3,-3,2),Q(1,-1,1)$
$PQ^2=(3-1)^2+(3+1)^2+(2-1)^2=21$

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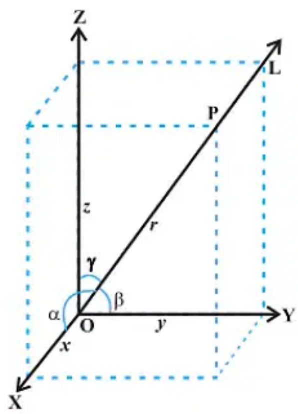

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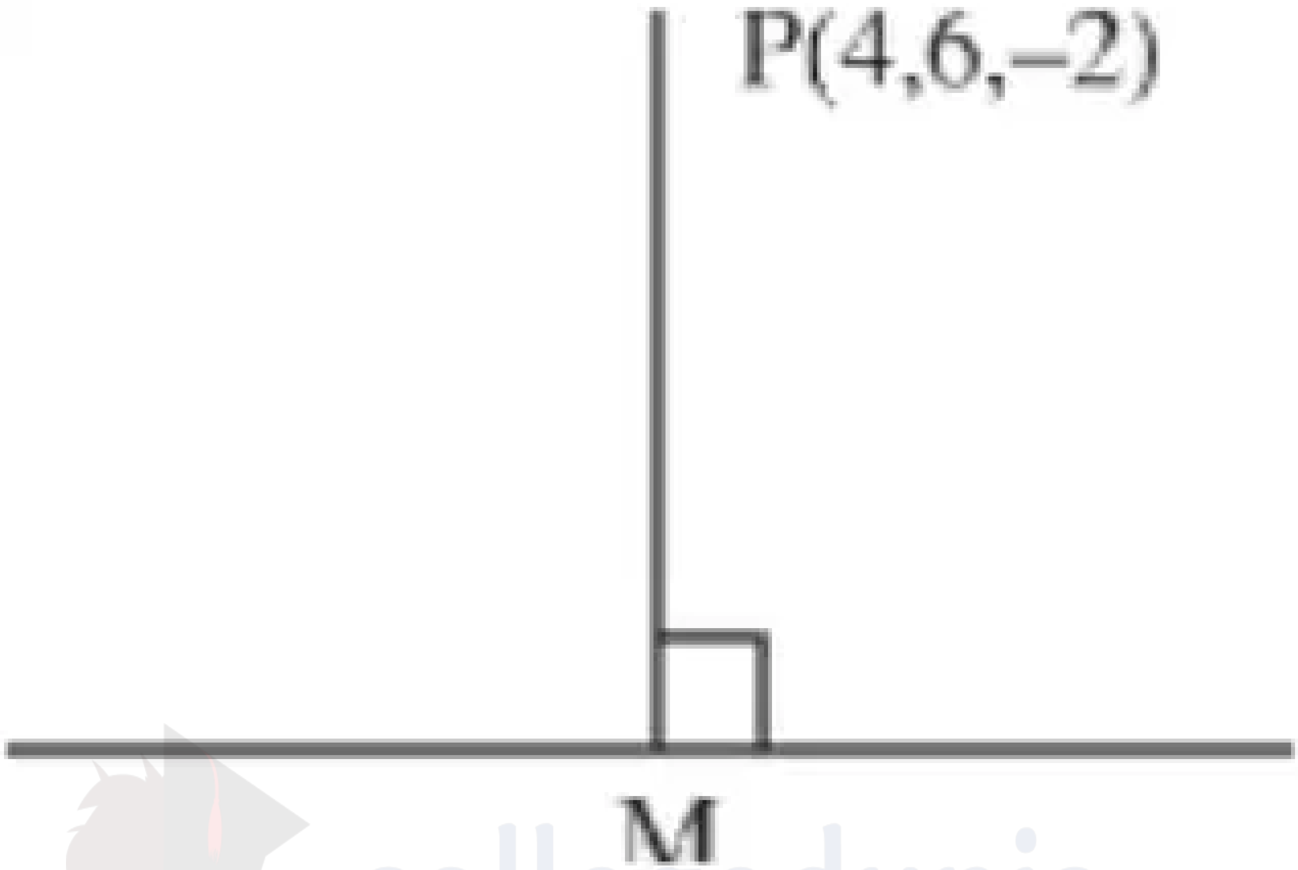
10. **Answer: b**

Explanation:



Equation of line is $\frac{x+3}{3}=\frac{y-2}{3}=\frac{z-3}{-1}=\lambda$

$M(3\lambda-3, 3\lambda+2, 3-\lambda)$

D.R of PM $(3\lambda-7, 3\lambda-4, 5-\lambda)$

Since PM is perpendicular to line

$\Rightarrow 3(3\lambda-7)+3(3\lambda-4)-1(5-\lambda)=0$

$\Rightarrow \lambda=2$

$\Rightarrow M(3,8,1) \Rightarrow PM=\sqrt{14}$

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