

Hyperbola JEE Main PYQ -1

Total Time: 20 Minute

Total Marks: 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Hyperbola

1. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations $ax + 2ay - 3az = 1$ $(2a + 1)x + (2a + 3)y + (a + 1)z = 2$ $(3a + 5)x + (a + 5)y + (a + 2)z = 3$ has unique solution and infinitely many solutions Then **(+4)**
- $S_1 = \Phi$ and $S_2 = R - \{0\}$
 - S_1 is an infinite set and $n(S_2) = 2$
 - $S_1 = R - \{0\}$ and $S_2 = \Phi$
 - $n(S_1) = 2$ and S_2 is an infinite set
-
2. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations $ax + 2ay - 3az = 1$ $(2a + 1)x + (2a + 3)y + (a + 1)z = 2$ $(3a + 5)x + (a + 5)y + (a + 2)z = 3$ has unique solution and infinitely many solutions. Then **(+4)**
- $S_1 = \Phi$ and $S_2 = R - \{0\}$
 - S_1 is an infinite set and $n(S_2) = 2$
 - $S_1 = R - \{0\}$ and $S_2 = \Phi$
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-
3. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6,$ be in AP and $a_1 + a_3 = 10$ If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to : **(+4)**
- 105
 - 210
 - 200
 - 220

-
4. Orthocentre of triangle having vertices as A (1,2), B(3,-4), C(0,6) is (+4)
- a. (-129, -37)
 - b. (9, -1)
 - c. (7, -3)
 - d. (28, -16)
-
5. A rectangle is drawn by lines $x=0$, $x=2$, $y=0$ and $y=5$. Points A and B lie on coordinate axes. If line AB divides the area of rectangle in 4:1, then the locus of mid-point of AB is? (+4)
- a. Circle
 - b. Hyperbola
 - c. Ellipse
 - d. Straight line
-
6. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus? (+4)
- a. (-3, -9)
 - b. (-3, -8)
 - c. $(\frac{1}{3}, -\frac{8}{3})$
 - d. $(-\frac{10}{3}, -\frac{7}{3})$
-
7. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes, then (+4)
- a. $2bc - 3ad = 0$

b. $2bc + 3ad = 0$

c. $3bc - 2ad = 0$

d. $3bc + 2ad = 0$

8. Let the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, pass through the point $(2, 3)$ and have eccentricity equal to $\frac{1}{2}$. Then equation of the normal to this ellipse at $(2, 3)$ is : (+4)

a. $2y - x = 4$

b. $2x - y = 1$

c. $3x - 2y = 0$

d. $3x - y = 3$

9. Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ to the tangent drawn at a point P on the ellipse. If F₁ and F₂ are two foci of the ellipse, then $(PF_1 - PF_2)^2$ is equal to; (+4)

a. (A) $4 - \frac{2}{2}$

b. (B) $4 - \frac{2}{2}$ Your Answer

c. (C) $4 - \frac{2}{2}$

d. (D) None of these

10. If the maximum distance of normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1, b < 2$, from the origin is 1, then the eccentricity of the ellipse is (+4)

a. $\frac{\sqrt{3}}{4}$

b. $\frac{\sqrt{3}}{2}$

c. $\frac{1}{\sqrt{2}}$

d. $\frac{1}{2}$

Answers

1. Answer: c

Explanation:

$$\begin{aligned}
 ? &= \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix} \\
 &= a(15a^2 + 31a + 36) = 0 \quad ? \quad a = 0 \\
 ? &\neq 0 \text{ for all } a \in R - \{0\} \\
 \text{Hence } S_1 &= R - \{0\} \quad S_2 = F
 \end{aligned}$$

Concepts:

1. Straight lines:

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Standard Form : $ax + by = c$

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Hence, the equation becomes:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y -axis at a distance c from the origin, and that the distance c is referred to as the line L's y -intercept. As a result, the coordinates of the spot on the y -axis where the line intersects are $(0, c)$. As a result, the slope of the line L is m , and it passes through a fixed point $(0, c)$. The equation of the line L thus obtained from the slope – point form is given by

$$y - c = m(x - 0)$$

As a result, the point (x, y) on the line with slope m and y -intercept c lies on the line, if and only if

$$y = mx + c$$

2. Answer: c

Explanation:

The correct answer is (C) : $S_1 = R - \{0\}$ and $S_2 = \Phi$

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$
$$= a(15a^2 + 31a + 36) = 0 \Rightarrow a = 0$$

$\Delta \neq 0$ for all $a \in R - \{0\}$

Hence $S_1 = R - \{0\}$, $S_2 = \Phi$

Concepts:

1. Solution of System of Linear Inequalities in Two Variables:

A System of Linear Inequalities is a set of 2 or more linear inequalities which have the same variables.

Example

$$x + y \geq 5$$

$$x - y \leq 3$$

Here are two inequalities having two same variables that are, x and y.

Solution of System of Linear Inequalities in Two Variables

The solution of a system of a linear inequality is the ordered pair which is the solution of all inequalities in the studied system and the graph of the system of a linear inequality is the graph of the common solution of the system.

Therefore, the Solution of the System of Linear Inequalities could be:

Graphical Method:

For the Solution of the System of Linear Inequalities, the Graphical Method is the easiest method. In this method, the process of making a graph is entirely similar to the graph of linear inequalities in two variables.

Non-Graphical Method:

In the Non-Graphical Method, there is no need to make a graph but we can find the solution to the system of inequalities by finding the interval at which the system persuades all the inequalities.

In this method, we have to find the point of intersection of the two inequalities by resolving them. It could be feasible that there is no intersection point between them.

3. Answer: b

Explanation:

The correct answer is (C) : 210

$$a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$$

$$\Rightarrow \frac{6}{2} [a_1 + a_6] = 57$$

$$\Rightarrow a_1 + a_6 = 19$$

$$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5$$

$$\Rightarrow a_1 = 2, d = 3$$

Numbers : 2, 5, 8, 11, 14, 17

Variance = $\sigma^2 = \text{mean of squares} - \text{square of mean}$

$$= \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$$8\sigma^2 = 210$$

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The slope of $P_2P =$ The slope of P_1P_2 , i.e.

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Hence, the equation becomes:

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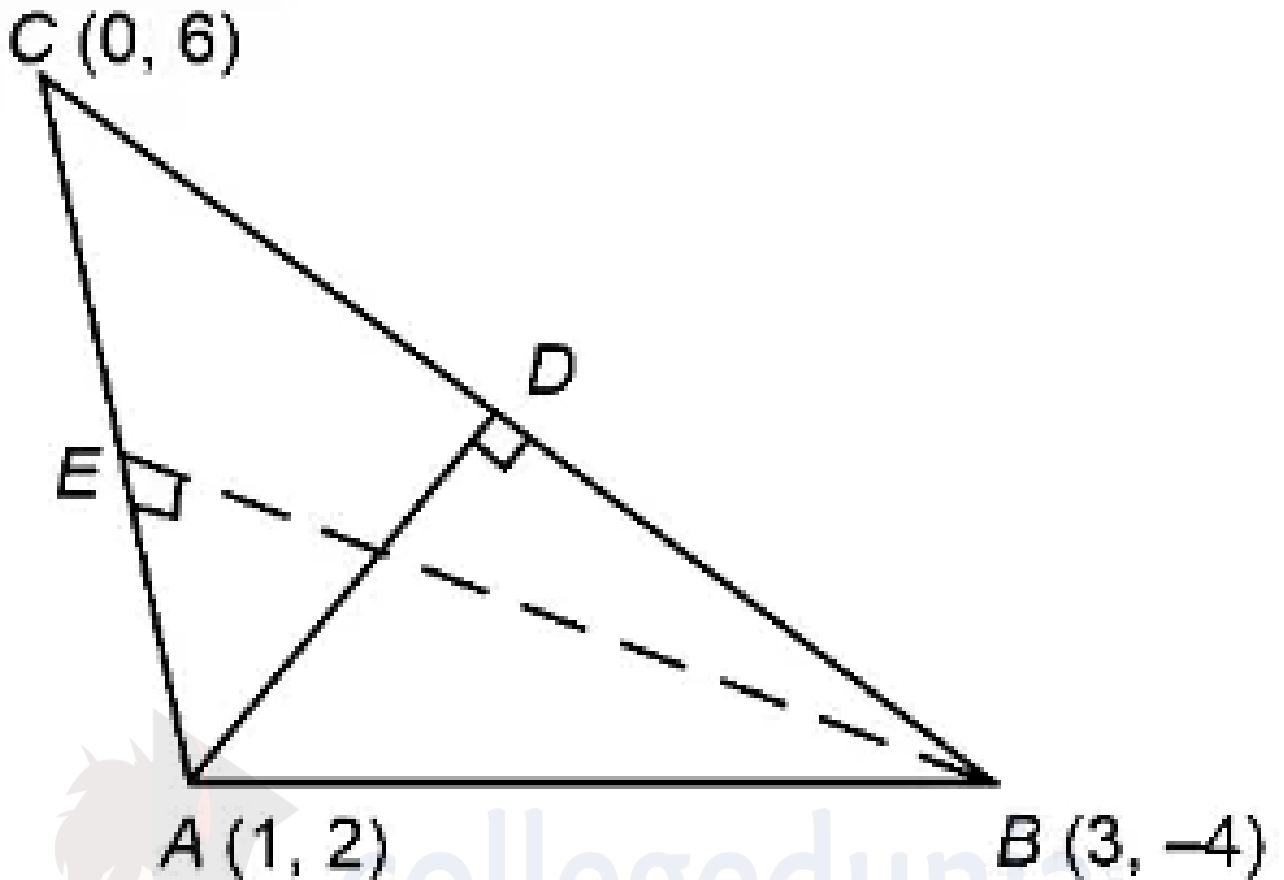
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4. **Answer: a**

Explanation:

Correct answer is (A). $(-129, -37)$



$$AD = (y-2) = \frac{3}{10}(x-1)$$

$$3x-10y+17=0 \dots (i)$$

$$BE = (y+4) = \frac{1}{4}(x-3)$$

$$x-4y=19 \dots (ii)$$

solving (i) and (ii),

$(-129, -37)$ is Orthocentre

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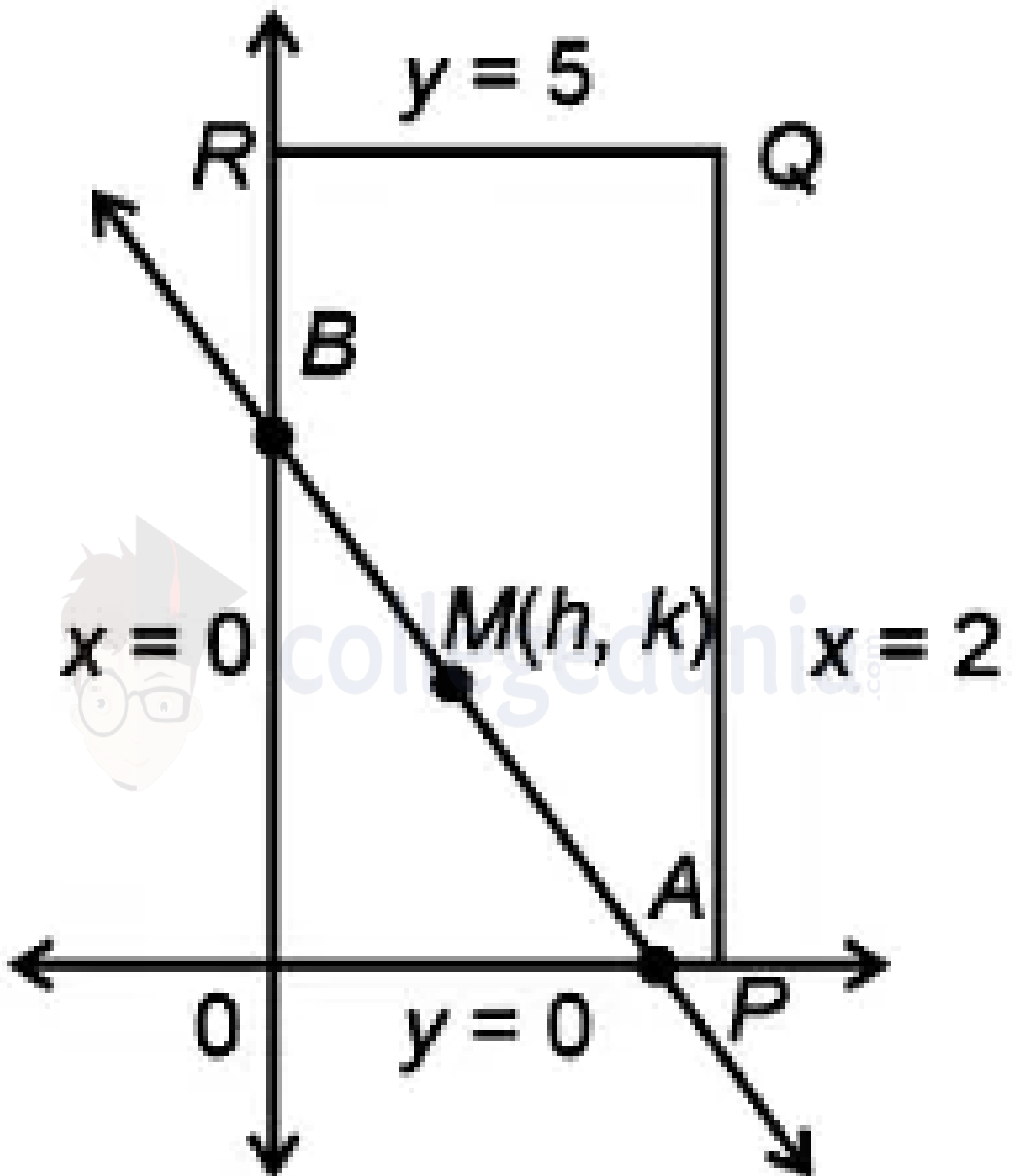
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5. **Answer: b**

Explanation:

The correct answer is option (B): Hyperbola



$A(2h,0), B(0,2k)$

Area of $\triangle OAB = 8$

$\frac{1}{2} \times 2h \times 2k = 8$

$$hk = 4$$

Locus of $XY = 4$

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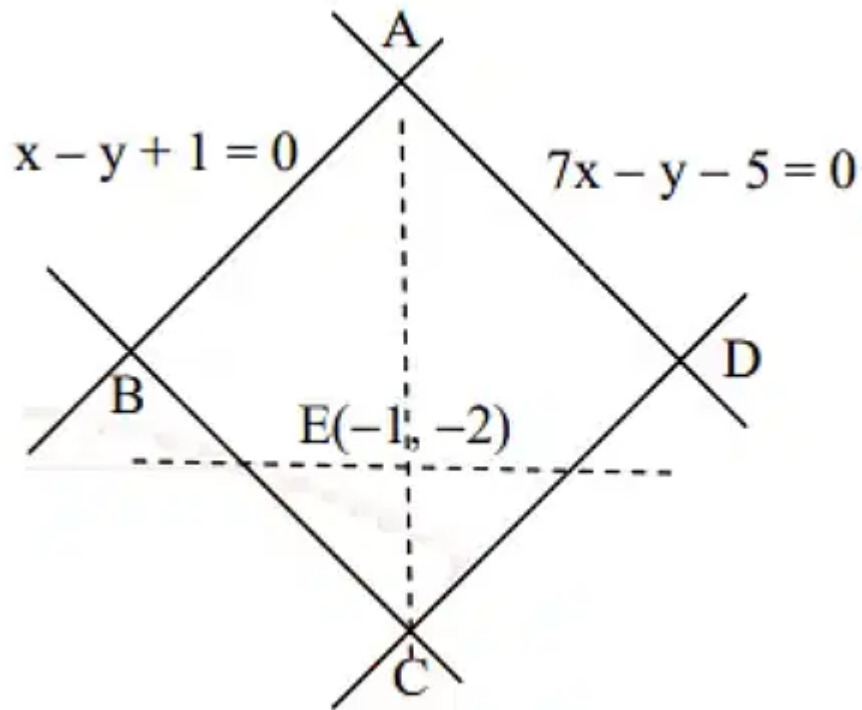
$$y = mx + c$$



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6. Answer: c

Explanation:



Coordinates of $A \equiv (1, 2)$

\therefore Slope of $AE = 2$

\Rightarrow Slope of $BD = -\frac{1}{2} \Rightarrow$

E of BD is $\frac{y+2}{x+1} = -\frac{1}{2}$

$\Rightarrow x + 2y + 5 = 0$

\therefore Co-ordinates of $D = \left(\frac{1}{3}, -\frac{8}{3}\right)$

So, the correct option is (C): $\left(\frac{1}{3}, -\frac{8}{3}\right)$

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7. Answer: c

Explanation:

Let $(\alpha, -\alpha)$ be the point of intersection

$$\therefore 4a\alpha - 2a\alpha + c = 0$$

$$\Rightarrow \alpha = -\frac{c}{2a}$$

and $5b\alpha - 2b\alpha + d = 0$

$$\Rightarrow \alpha = -\frac{d}{3b}$$

$$\Rightarrow 3bc = 2ad$$

$$\Rightarrow 3bc - 2ad = 0$$

:

The point of intersection will be

$$\frac{x}{2ad-2bc} = \frac{-y}{4ad-5bc} = \frac{1}{8ab-10ab}$$

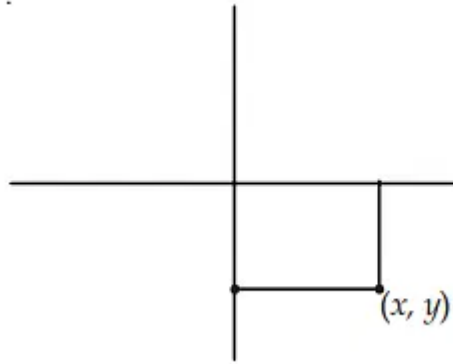
$$\Rightarrow x = \frac{2(ad-bc)}{-2ab}$$

$$\Rightarrow y = \frac{5bc-4ad}{-2ab}$$

\therefore Point of intersection is in fourth quadrant so x is positive and y is negative

Also distance from axes is same

So $x = -y$ (\therefore distance from x -axis is $-y$ as y is negative)



$$\frac{2(ad-bc)}{-2ab} = \frac{-(5bc-4ad)}{-2ab}$$

$$2ad - 2bc = -5bc + 4ad$$

$$\Rightarrow 3bc - 2ad = 0 \quad \dots (i)$$

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8. Answer: b

Explanation:

Answer (b) $2x - y = 1$

Concepts:

1. Ellipse:

Ellipse Shape

An **ellipse** is a locus of a point that moves in such a way that its distance from a fixed point (focus) to its perpendicular distance from a fixed straight line (directrix) is constant. i.e. eccentricity(e) which is less than unity

Properties

- Ellipse has two focal points, also called foci.
- The fixed distance is called a directrix.
- The eccentricity of the ellipse lies between 0 to 1. $0 \leq e < 1$
- The total sum of each distance from the locus of an ellipse to the two focal points is constant
- Ellipse has one major axis and one minor axis and a center

Read More: [Conic Section](#)

Eccentricity of the Ellipse

The ratio of distances from the center of the ellipse from either focus to the semi-major axis of the ellipse is defined as the eccentricity of the ellipse.

The **eccentricity** of ellipse, $e = c/a$

Where c is the focal length and a is length of the semi-major axis.

Since $c \leq a$ the eccentricity is always greater than 1 in the case of an ellipse.

Also,

$$c^2 = a^2 - b^2$$

Therefore, eccentricity becomes:

$$e = \sqrt{(a^2 - b^2)}/a$$

$$e = \sqrt{[(a^2 - b^2)/a^2]} \quad e = \sqrt{[1 - (b^2/a^2)]}$$

Area of an ellipse

The **area of an ellipse** = πab , where a is the semi major axis and b is the semi minor axis.

Position of point related to Ellipse

Let the point $p(x_1, y_1)$ and ellipse

$$(x^2 / a^2) + (y^2 / b^2) = 1$$

$$\text{If } [(x_1^2 / a^2) + (y_1^2 / b^2) - 1]$$

$$= 0 \text{ \{on the curve\}}$$

$$< 0 \text{ \{inside the curve\}}$$

$$> 0 \text{ \{outside the curve\}}$$

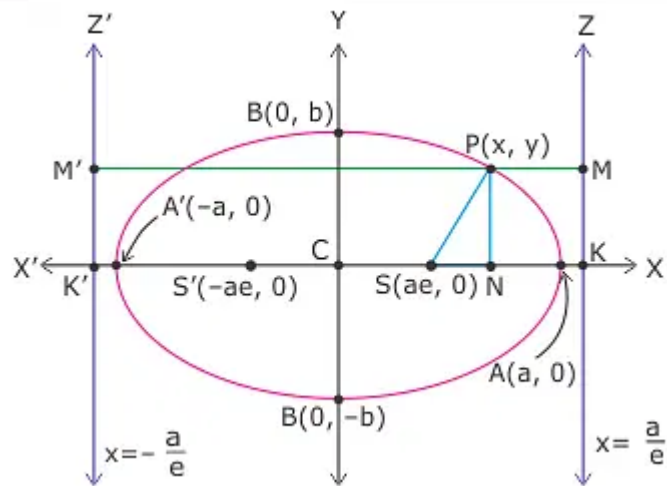
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9. Answer: c

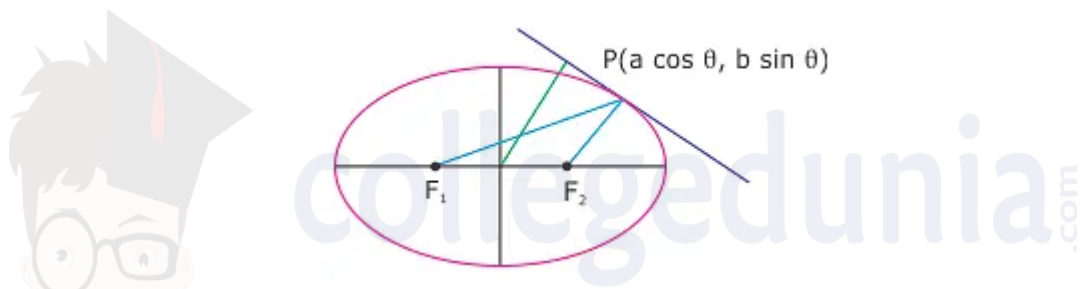
Explanation:

Explanation:

Given: 'd' is the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse.-- F1 and F are two foci of the ellipse. We have to find the value of $(PF_1 - PF)^2$. Consider,



Let $P(x, y)$ be any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (As shown in Fig). Then, by definition of ellipse, we have $SP = e PM$ and $S'P = e PM' \Rightarrow S = e(NK)$ and $S'P = e(NK') \Rightarrow SP = e(CK - CN)$ and $S'P = e(CK' + CN) \Rightarrow SP = a - ex$ and $S'P = a + ex$



Consider, co-ordinate of point P in parametric form as $(a \cos \theta, b \sin \theta)$. Since $PF_1 = a + ex$ and $PF_2 = a - ex$, therefore $PF_1 - PF_2 = 2ex = 2ea \cos \theta \therefore (PF_1 - PF_2)^2 = 4a^2 e^2 \cos^2 \theta$

(i) Equation of tangent to ellipse at $P(\theta)$ is, $-\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ $d = \frac{1}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}}$ [Using perpendicular distance of a line from a point and trigonometric identities]

$$\frac{1}{d^2} = \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \quad \frac{x^2}{a^2} = \frac{x^2}{a^2} \cos^2 \theta + \sin^2 \theta$$

$$\text{or } 1 - \frac{x^2}{a^2} = 1 - \frac{x^2}{a^2} \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - \frac{x^2}{a^2} \cos \theta$$

$$= \cos^2 \theta \left(1 - \frac{x^2}{a^2}\right)$$

$$= \cos^2 \theta \left(1 - \frac{x^2}{a^2}\right) \quad [\text{Using eccentricity of ellipse}] \quad 4a^2 \left(1 - \frac{x^2}{a^2}\right) = 4a^2 \cos^2 \theta \quad \text{So,}$$

$$\left(1 - \frac{x^2}{a^2}\right)^2 = 4a^2 \cos^2 \theta = 4a^2 \left(1 - \frac{x^2}{a^2}\right) \quad [\text{Using (i)}] \text{ Hence, the correct option is (B).}$$

10. Answer: b

Explanation:

Equation of normal is

$$2x \sec \theta - by \operatorname{cosec} \theta = 4 - b^2$$

$$\text{Distance from } (0, 0) = \frac{4-b^2}{\sqrt{4 \sec^2 \theta + b^2 \operatorname{cosec}^2 \theta}}$$

Distance is maximum if

$$4 \sec^2 \theta + b^2 \operatorname{cosec}^2 \theta \text{ is minimum}$$

$$\Rightarrow \tan^2 \theta = \frac{b}{2}$$

$$\Rightarrow \frac{4-b^2}{\sqrt{4 \cdot \frac{b+2}{2} + b^2 \cdot \frac{b+2}{b}}} = 1$$

$$\Rightarrow 4 - b^2 = b + 2 \Rightarrow b = 1 \Rightarrow e = \frac{\sqrt{3}}{2}$$

Concepts:

1. Ellipse:

Ellipse Shape

An **ellipse** is a locus of a point that moves in such a way that its distance from a fixed point (focus) to its perpendicular distance from a fixed straight line (directrix) is constant. i.e. eccentricity(e) which is less than unity

Properties

- Ellipse has two focal points, also called foci.
- The fixed distance is called a directrix.
- The eccentricity of the ellipse lies between 0 to 1. $0 \leq e < 1$
- The total sum of each distance from the locus of an ellipse to the two focal points is constant
- Ellipse has one major axis and one minor axis and a center

Read More: [Conic Section](#)

Eccentricity of the Ellipse

The ratio of distances from the center of the ellipse from either focus to the semi-major axis of the ellipse is defined as the eccentricity of the ellipse.

The **eccentricity** of ellipse, $e = c/a$

Where c is the focal length and a is length of the semi-major axis.

Since $c \leq a$ the eccentricity is always greater than 1 in the case of an ellipse.

Also,

$$c^2 = a^2 - b^2$$

Therefore, eccentricity becomes:

$$e = \sqrt{(a^2 - b^2)}/a$$

$$e = \sqrt{[(a^2 - b^2)/a^2]} \quad e = \sqrt{[1 - (b^2/a^2)]}$$

Area of an ellipse

The **area of an ellipse** = πab , where a is the semi major axis and b is the semi minor axis.

Position of point related to Ellipse

Let the point $p(x_1, y_1)$ and ellipse

$$(x^2 / a^2) + (y^2 / b^2) = 1$$

$$\text{If } [(x_1^2 / a^2) + (y_1^2 / b^2) - 1]$$

$$= 0 \text{ \{on the curve\}}$$

$$< 0 \text{ \{inside the curve\}}$$

$$> 0 \text{ \{outside the curve\}}$$